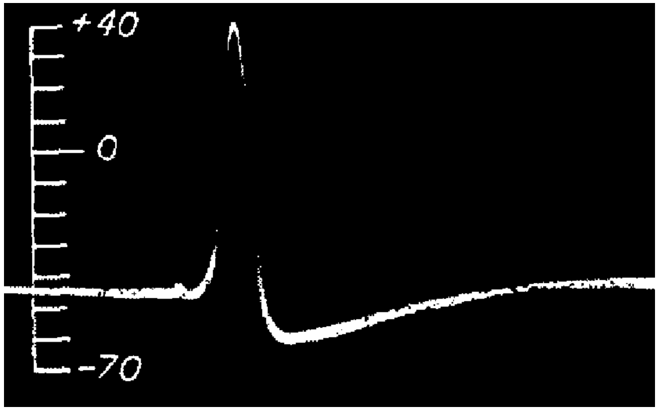
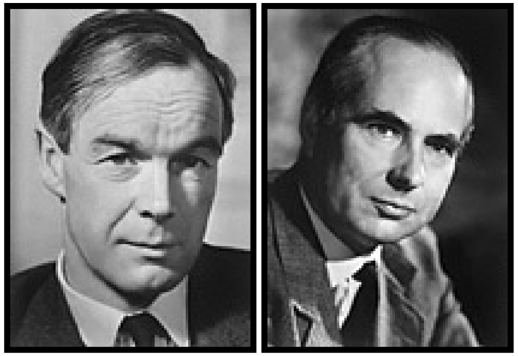
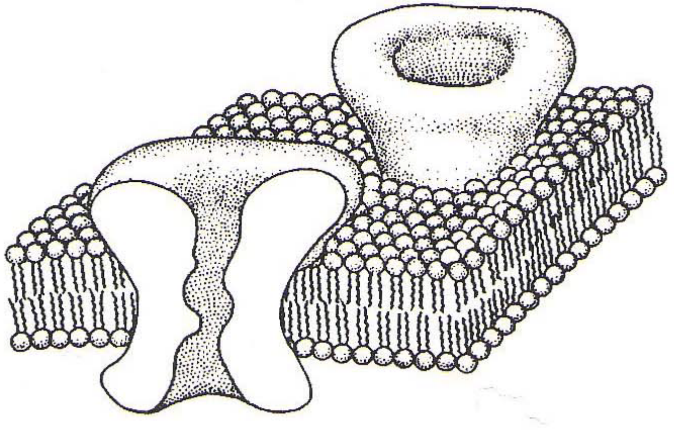
Hodgkin-Huxley model

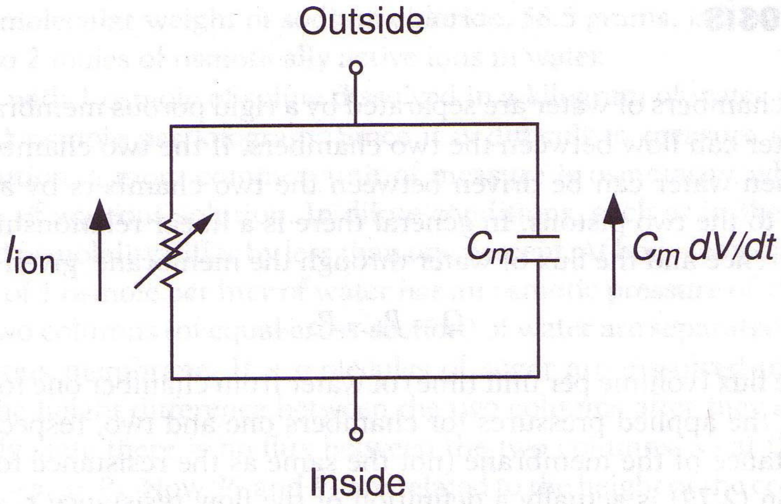




$$
\begin{aligned}
C_m \frac{dv}{dt} &= -\bar{g}_{\mathrm{K}} n^4 (v - v_{\mathrm{K}}) - \bar{g}_{\mathrm{Na}} m^3 h (v - v_{\mathrm{Na}}) - \bar{g}_{\mathrm{L}} (v - v_{\mathrm{L}}) + I_{\mathrm{app}}, \\
\frac{dm}{dt} &= \alpha_m (1 - m) - \beta_m m, \\
\frac{dn}{dt} &= \alpha_n (1 - n) - \beta_n n, \\
\frac{dh}{dt} &= \alpha_h (1 - h) - \beta_h h.
\end{aligned}
$$

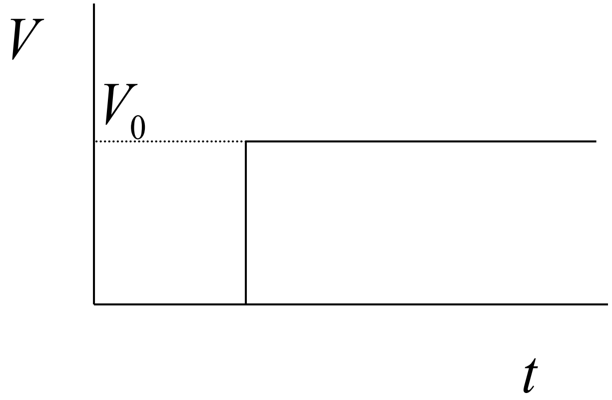
HH thought of neuronal membrane as a circuit

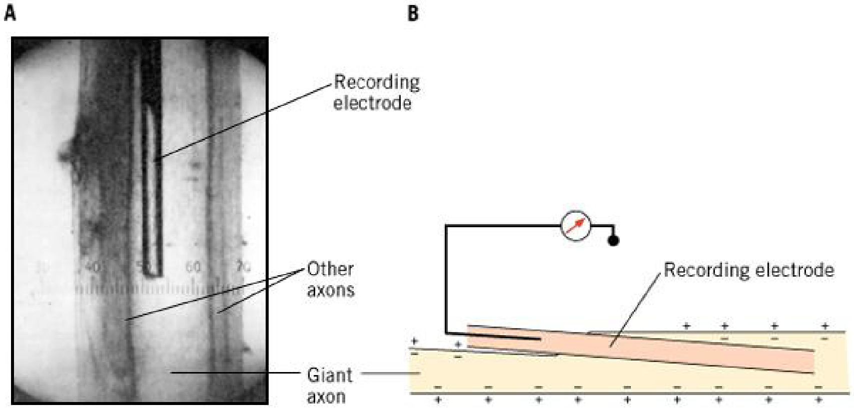




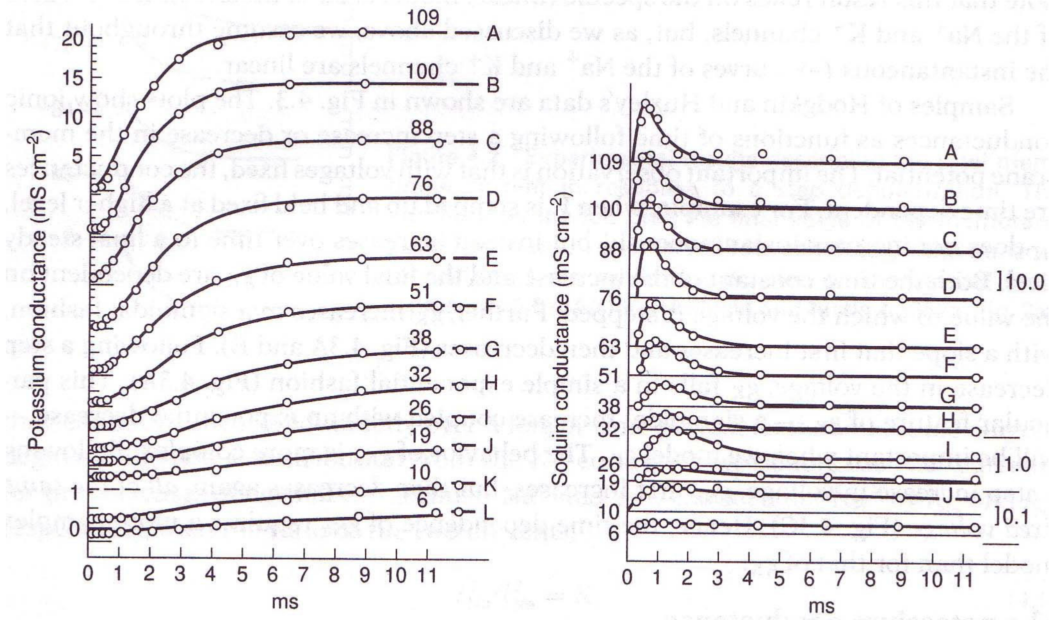
$$C_m \frac{dV}{dt} = -g_{Na^+}(V - V_{Na^+}) - g_{K^+}(V - V_{K^+})$$

Conductances depend on V and t (voltage-clamp experiments)





$$g_{K^+}(V_0,t)=\frac{V_0-V_{K^+}}{I_{medido}(t)}$$



$g_{K^+}(V,t)$

HH could have assumed a differential equation like

$$\frac{dg_{K^+}}{dt} = f(V,t)$$

But they chose to assume the existence of channels with ‘gates’

$$g_{K^+} = \overline{g}_{K^+}\, n^4 \qquad ; \qquad \frac{dn}{dt} = f(V,t)$$

What does this mean?

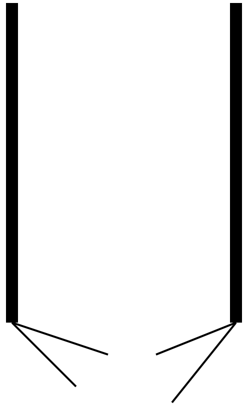
$$g_{K^+} = \overline{g}_{K^+}\; n^4 \qquad ; \qquad \frac{dn}{dt} = f(V,t)$$

Maximum conductance

Dynamics of probability that
a single gate is open
(depending on de voltage and time)

Probability that the four gates of the channel are open

n: Probabilidad de que las 4 puertas del canal
estén abiertas al paso del ion



We assume a first-order kinetics for the gate

n : probability that a gate is open

$1-n$: probabilioty that a gate is closed

$$n \underset{\alpha_n(V)}{\overset{\beta_n(V)}{\rightleftarrows}} 1-n$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)\,n$$

Rewriting the dynamics of gate opening

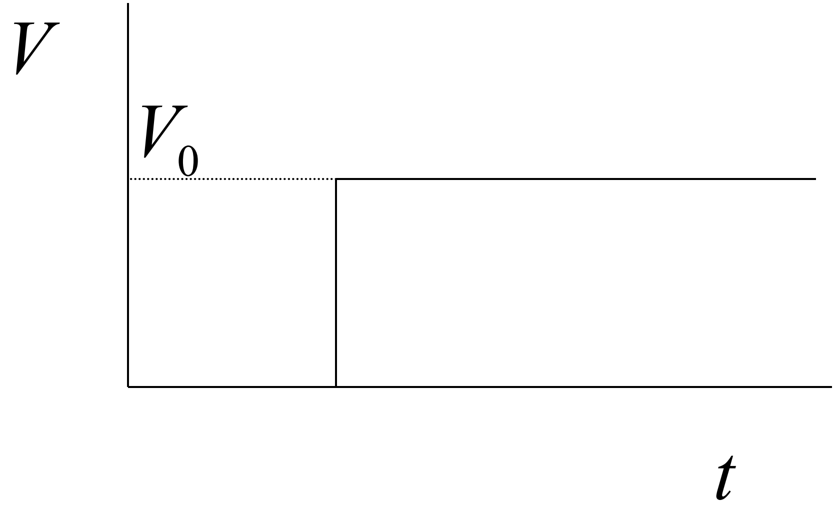
$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)\,n$$

$$n_\infty(V) \equiv \frac{\alpha_n(V)}{\alpha_n(V)+\beta_n(V)} \quad ; \quad \tau_n(V) \equiv \frac{1}{\alpha_n(V)+\beta_n(V)}$$

$$\frac{dn}{dt} = \frac{n_\infty(V)-n}{\tau_n(V)}$$

Dynamics of gate opening in voltage-clamp experiment

$$\frac{dn}{dt} = \frac{n_\infty(V) - n}{\tau_n(V)}$$

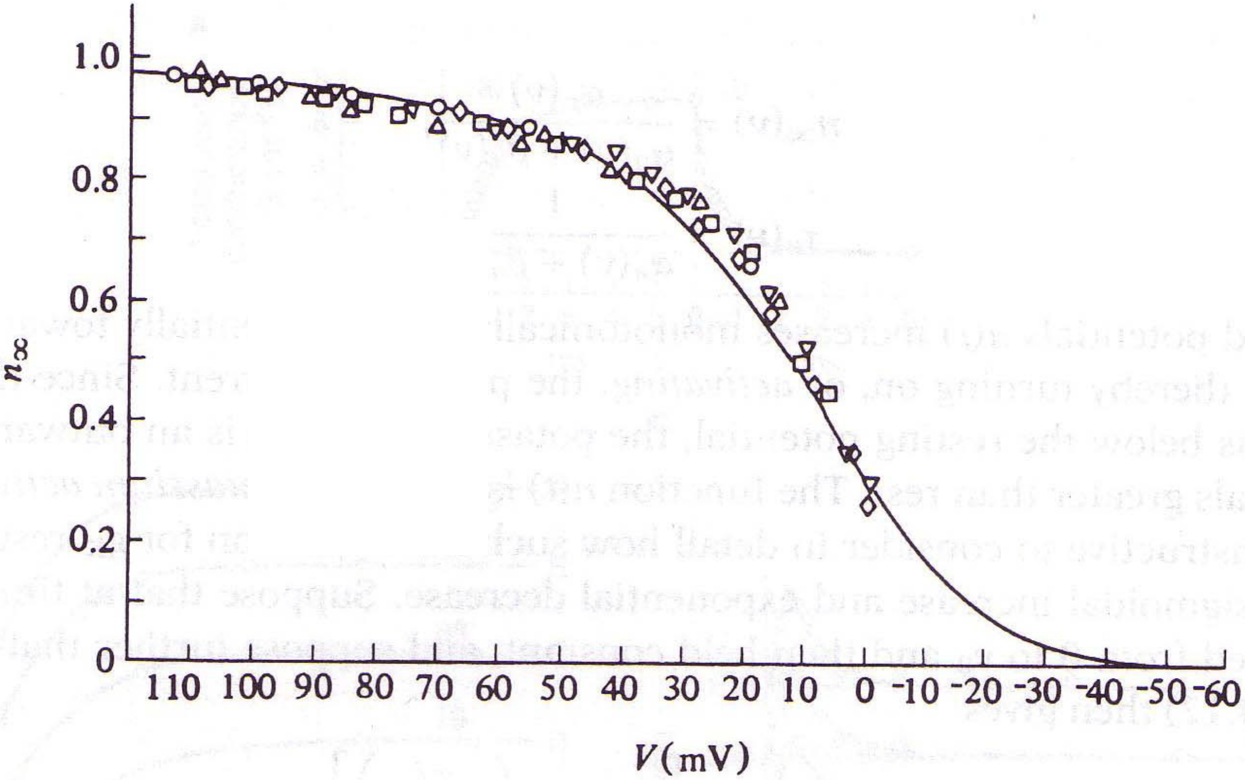


$$n(V_0, t) = n_\infty(V_0)\left[1 - \exp\left(-\frac{t}{\tau_n(V_0)}\right)\right]$$

Fit to experiments

$$n(V_0,t) = n_\infty(V_0)\left[1 - \exp\left(-\frac{t}{\tau_n(V_0)}\right)\right]$$

Fitting $n_\infty(V_0)$



The same for $\tau_n(V_0)$

We then obtain

$$\alpha_n = 0.01\left(\frac{10-V}{\exp\left(\frac{10-V}{10}\right)-1}\right) \qquad \beta_n = 0.125\left(\frac{-V}{80}\right)$$

$$\overline{g}_{K^+} = 36$$

$$V_{K^+} = -12$$

Really nice fit to voltage-clamp experiments

$$g_{K^+} = \overline{g}_{K^+} \; n^4$$

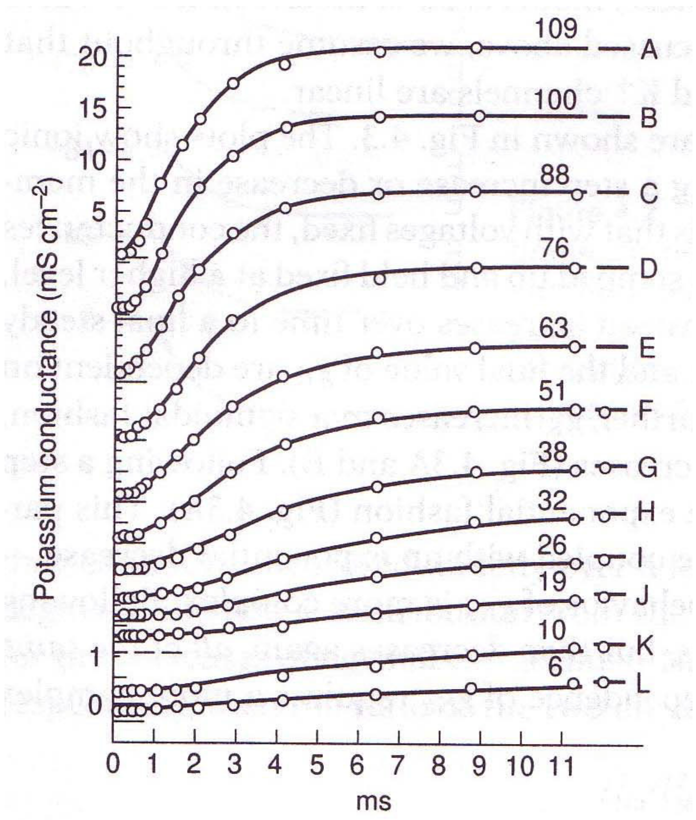
$$n(V_0,t) = \frac{\alpha_n(V)}{\alpha_n(V)+\beta_n(V)}\left[1-\exp\left(-(\alpha_n(V)+\beta_n(V))t\right)\right]$$

$$\alpha_n = 0.01\left(\frac{10-V}{\exp\left(\frac{10-V}{10}\right)-1}\right)$$

$$\beta_n = 0.125\left(\frac{-V}{80}\right)$$

$$\overline{g}_{K^+} = 36$$

$$V_{K^+} = -12$$



What do we have so far?

$$C_m \frac{dV}{dt} = -g_{Na^+}(V - V_{Na^+}) - g_{K^+}(V - V_{K^+})$$

$$C_m \frac{dV}{dt} = g_{Na^+}(V - V_{Na^+}) + \overline{g}_{K^+} n^4 (V - V_{K^+})$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)\, n$$

$$\alpha_n = 0.01\left(\frac{10-V}{\exp\left(\frac{10-V}{10}\right)-1}\right) \qquad \beta_n = 0.125\left(\frac{-V}{80}\right)$$

$$\overline{g}_{K^+} = 36$$

$$V_{K^+} = -12$$

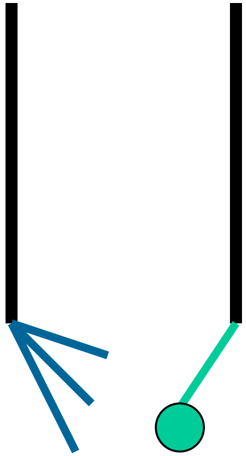
$g_{Na^+}(V,t)$

Fit to experiments assuming two types of gates

$$g_{Na^+} = \bar{g}_{Na^+}\ m^3 h$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)\,m$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)\,h$$



The complete model

$$C_m \frac{dV}{dt} = -\overline{g}_{Na^+} m^3 h (V - V_{Na^+}) - \overline{g}_{K^+} n^4 (V - V_{K^+}) - \overline{g}_L (V - V_L) + I_{ext}$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)\,n$$

$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)\,m$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)\,h$$

With functions (in (ms)$^{-1}$)

$$\alpha_m = 0.1\left(\frac{25-V}{\exp\left(\frac{25-V}{10}\right)-1}\right) \quad \alpha_h = 0.07\exp\left(\frac{-V}{20}\right) \quad \alpha_n = 0.01\left(\frac{10-V}{\exp\left(\frac{10-V}{10}\right)-1}\right)$$

$$\beta_m = 4\exp\left(\frac{-V}{18}\right) \quad \beta_h = \frac{1}{\exp\left(\frac{30-V}{10}\right)+1} \quad \beta_n = 0.125\exp\left(\frac{-V}{80}\right)$$

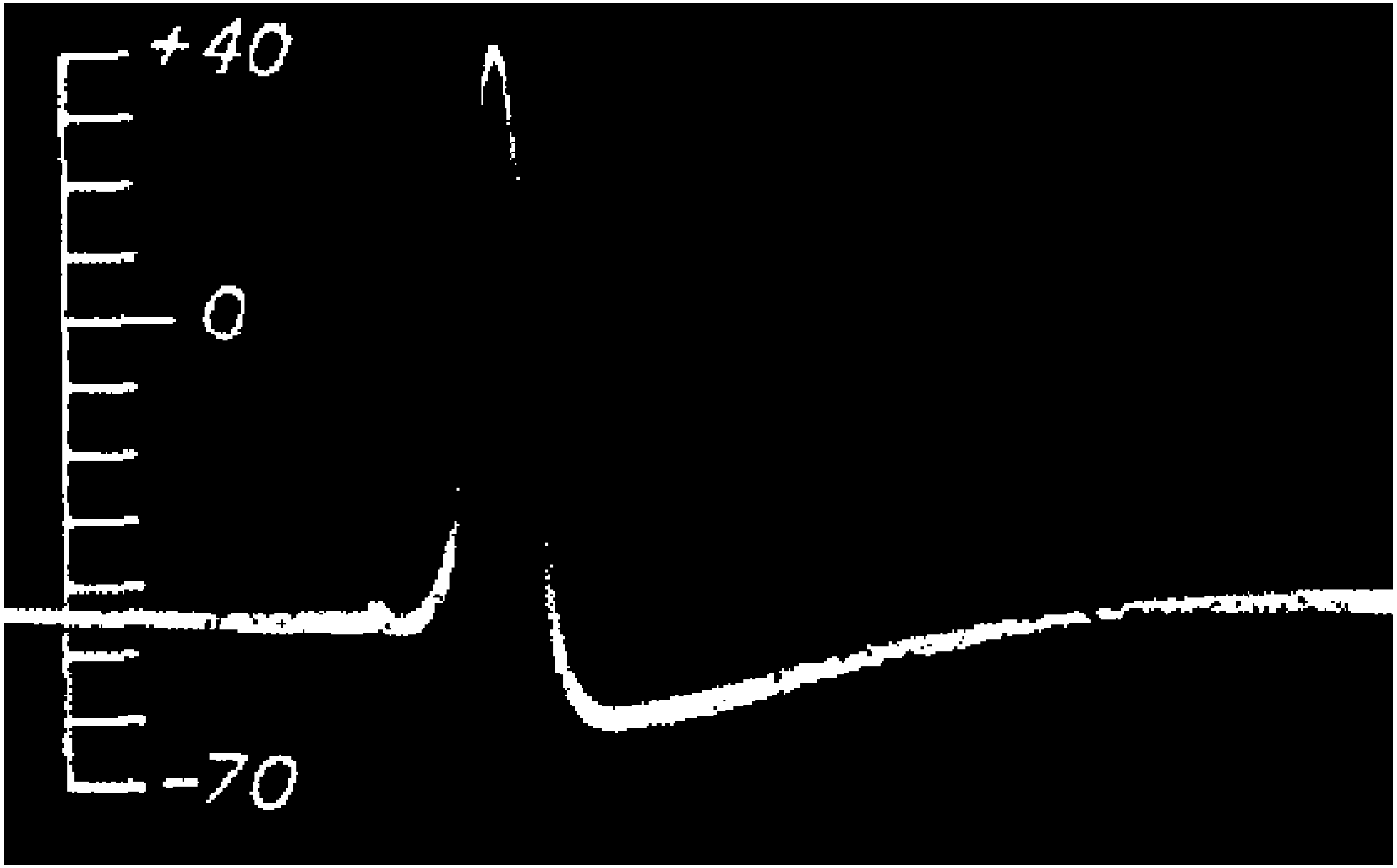
And constants

$$\overline{g}_{Na^+} = 120 \quad mS/cm^2 \qquad \overline{g}_{K^+} = 36 \quad mS/cm^2 \qquad \overline{g}_L = 0.3 \quad mS/cm^2$$

$$V_{Na^+} = 115 \quad mV \qquad V_{K^+} = -12 \quad mV \qquad V_L = 10.6 \quad mV$$

$$C_m = 1 \quad \mu F/cm^2$$

Does the HH model reproduce the action potential?

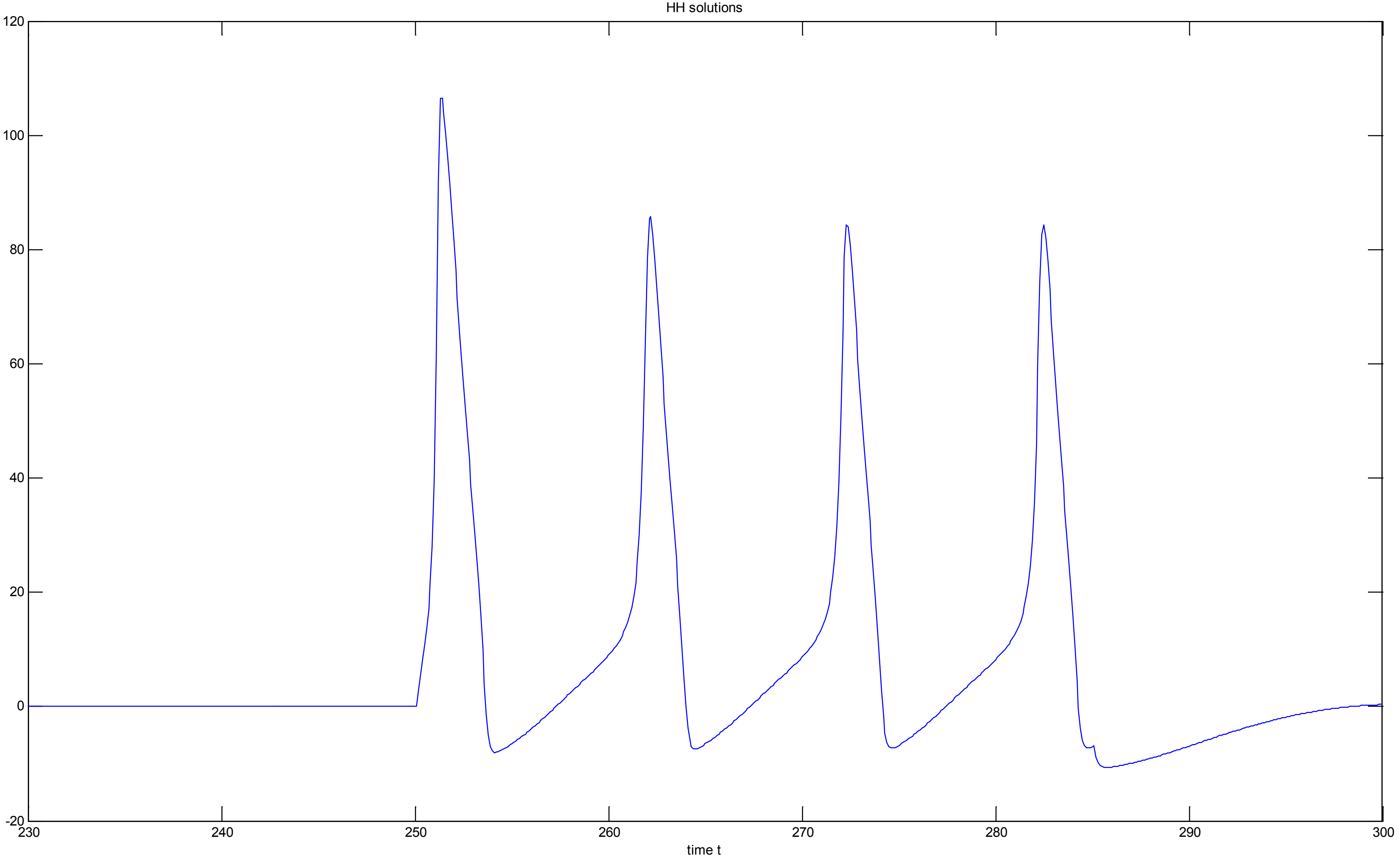


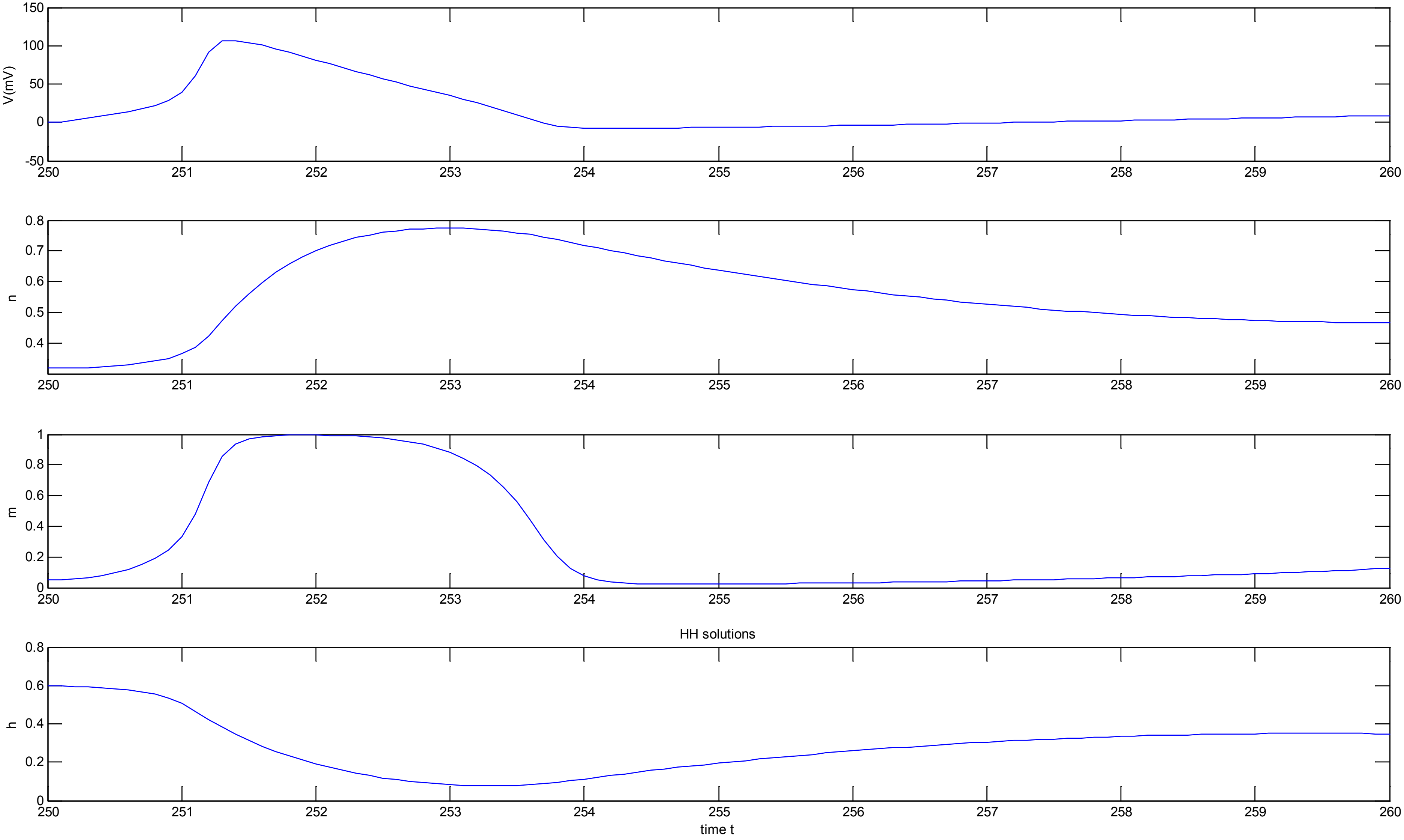
```
function  hh
%
% Hodgkin-Huxley equations solved using ODE23s
%
clear all
time_length=500;                            % duration of complete experiment in ms
time_resol=0.1;                             % number of points in a ms
tspan = [0:time_resol:time_length];         % time points for solutions
%
% Initial values for voltage and conductances V0, n0, m0 and h0
%
yinit=[0;0.1;0.1;0.1];                      % initial values for integration
%
% Call to integration routine
%
[t,y] = ode23(@hodgkinhuxley,tspan,yinit);
%
% Plotting
%
```

```
figure;
plot([230:0.1:300],y(2300:3000,1));
title(['HH solutions']);
xlabel('time t');

function dydt = hodgkinhuxley(t,y)

%
% External current step I(t)
%
if t<250; I=0; elseif t>=250&t<285; I=30; else; I=0; end
%
% Parameters for the HH equations
%
cap=1; gK=36;gNa=120;gl=0.3;VK=-12;VNa=115;Vl=10.6;
%
% Functions for HH equations
%
alpha_n=(0.1-0.01*y(1))/(exp(1-0.1*y(1))-1);
beta_n=0.125*exp(-y(1)/80);
alpha_m=(2.5-0.1*y(1))/(exp(2.5-0.1*y(1))-1);
beta_m=4*exp(-y(1)/18);
alpha_h=0.07*exp(-y(1)/20);
beta_h=1/( exp(3-0.1*y(1))+1 );
```

```
%
% The HH equations
%
dydt = [            (I-gK*y(2)^4*(y(1)-VK)-gNa*y(3)^3*y(4)*(y(1)-VNa)-gl*(y(1)-Vl))/cap
                alpha_n*(1-y(2))-beta_n*y(2)
                alpha_m*(1-y(3))-beta_m*y(3)
                alpha_h*(1-y(4))-beta_h*y(4)];
```

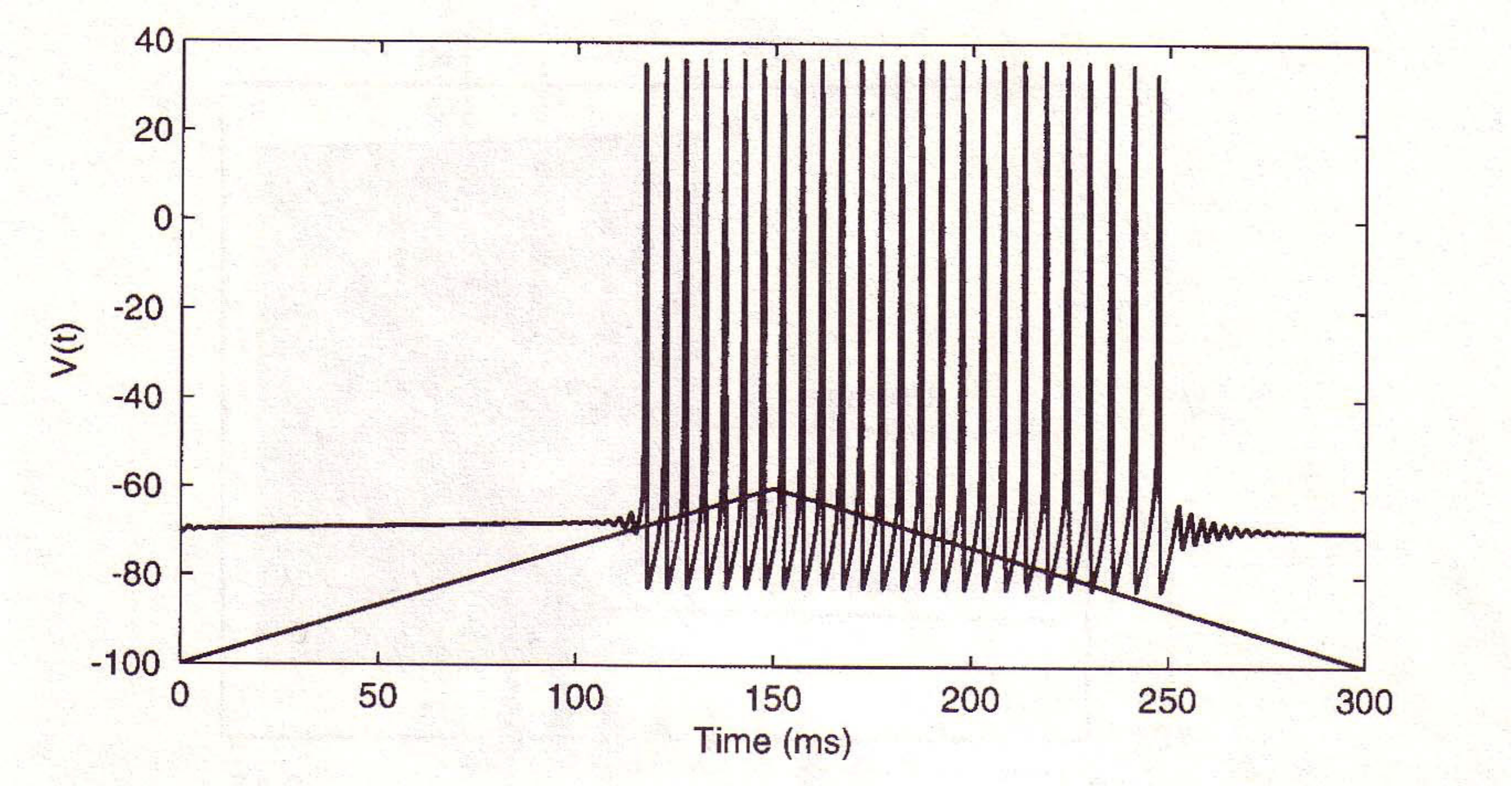
Response of model to injection of current step




150
100
50
0
-50
V(mV)
250
251
252
253
254
255
256
257
258
259
260
0.8
0.7
0.6
0.5
0.4
n
1
0.8
0.6
0.4
0.2
0
m
HH solutions
0.8
0.6
0.4
0.2
0
h
time t

1. HH model
2. Simplyfications of HH model
    2.1. Rinzel model
    2.2. Wilson model
    2.3. FitzHugh-Nagumo model
3. Adaptation and bursts
4. Networks

How would you explain this?



Rinzel model

HH model (4 variables):

$$C_m \frac{dV}{dt} = -\overline{g}_{Na^+} m^3 h (V - V_{Na^+}) - \overline{g}_{K^+} n^4 (V - V_{K^+}) - \overline{g}_L (V - V_L) + I_{ext}$$

$$\frac{dn}{dt} = \alpha_n(V)(1-n) - \beta_n(V)\, n$$

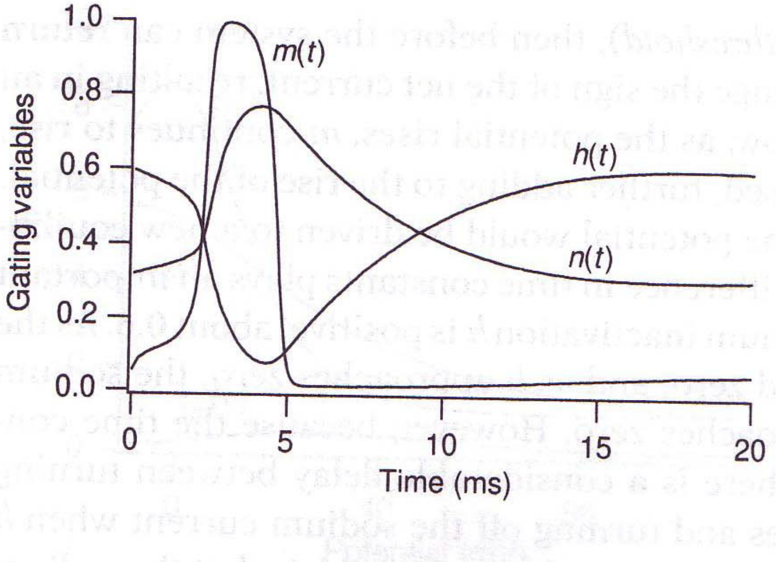
$$\frac{dm}{dt} = \alpha_m(V)(1-m) - \beta_m(V)\, m$$

$$\frac{dh}{dt} = \alpha_h(V)(1-h) - \beta_h(V)\, h$$

1. τ_m is small

$$m(V,t) = m_\infty(V)\left[1 - \exp\left(-\frac{t}{\tau_m(V)}\right)\right] \approx m_\infty(V)$$

2. Symmetry between h and n

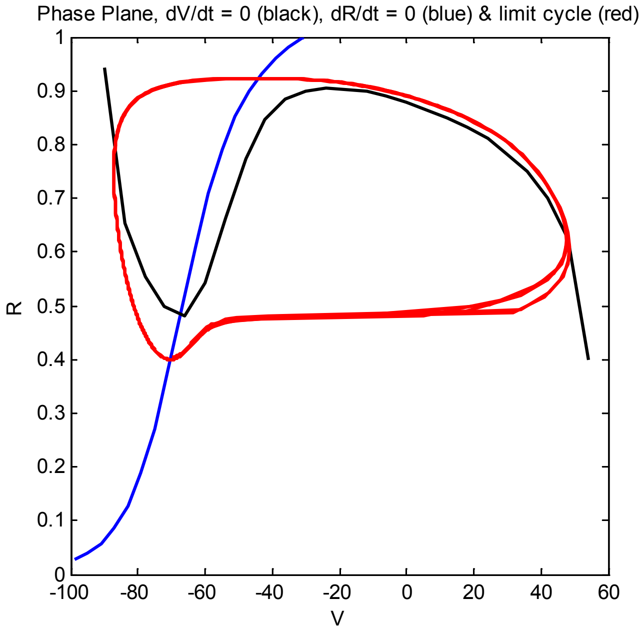


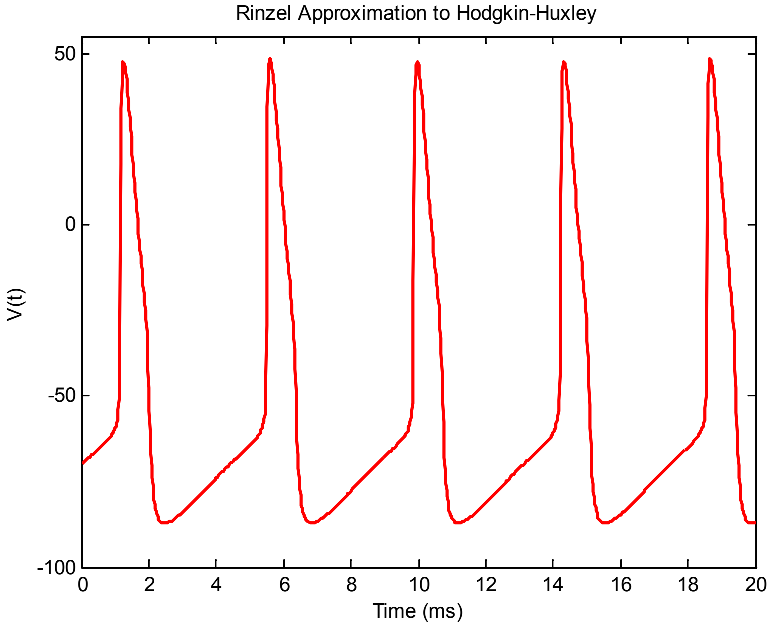
$$h(V,t) + n(V,t) \approx 1.0$$

Rinzel model: approxs 1 & 2 on HH

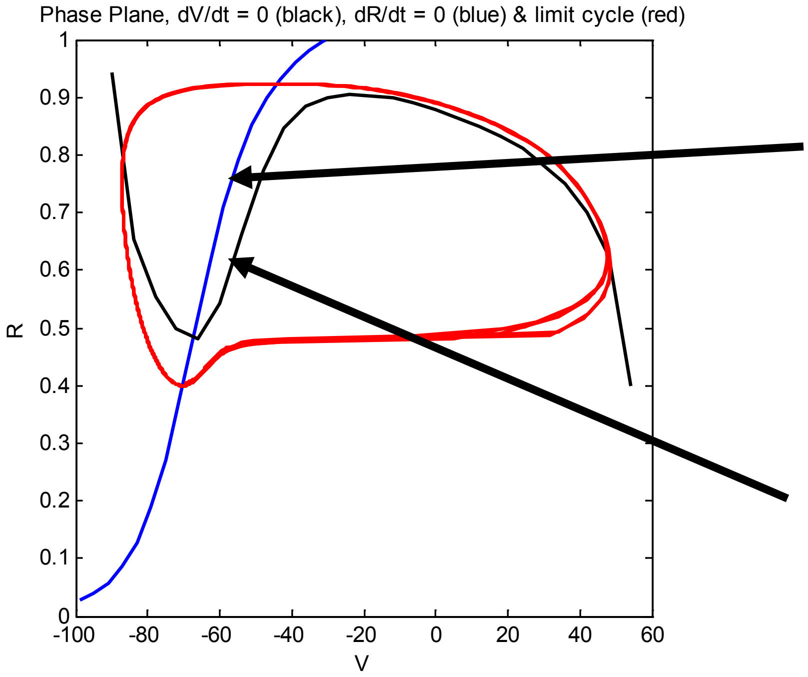
$$C_m \frac{dV}{dt} = -\overline{g}_{Na^+} m_\infty^{\ 3} (1-R)(V - V_{Na^+}) - \overline{g}_{K^+} R^4 (V - V_{K^+}) - \overline{g}_L (V - V_L) + I_{ext}$$

$$\frac{dR}{dt} = \alpha_n(V)(1-R) - \beta_n(V) R$$





Wilson model simplifies Rinzel a bit more



1. $\frac{dR}{dt} = 0$

$$\frac{dR}{dt} = \frac{1}{\tau_R}(-R + G(V))\,;\, G(V) = 1.35V + 1.03$$

$$\frac{dR}{dt} = 0 \Rightarrow R = G(V) = 1.35V + 1.03$$

2. $\frac{dV}{dt} = 0$

$$C\frac{dV}{dt} = -g_{Na^+}(V - V_{Na^+}) - R(V - V_{K^+}) + I_{ext}$$

$$\frac{dV}{dt} = 0 \Rightarrow R = \frac{-g_{Na^+}(V - V_{Na^+}) + I_{ext}}{(V - V_{K^+})} \quad ; \quad g_{Na^+} = 17.81 + 47.71V + 32.63V^2$$

Wilson model simplifies Rinzel a bit more

$$0.8\frac{dV}{dt} = -(17.81 + 47.71V + 32.63V^2)(V - 0.55) - 26.0R(V + 0.92) + I_{ext}$$

$$\frac{dR}{dt} = \frac{1}{1.9}(-R + 1.35V + 1.03)$$

V en decivoltios, I en $\mu A/100$ y t en ms

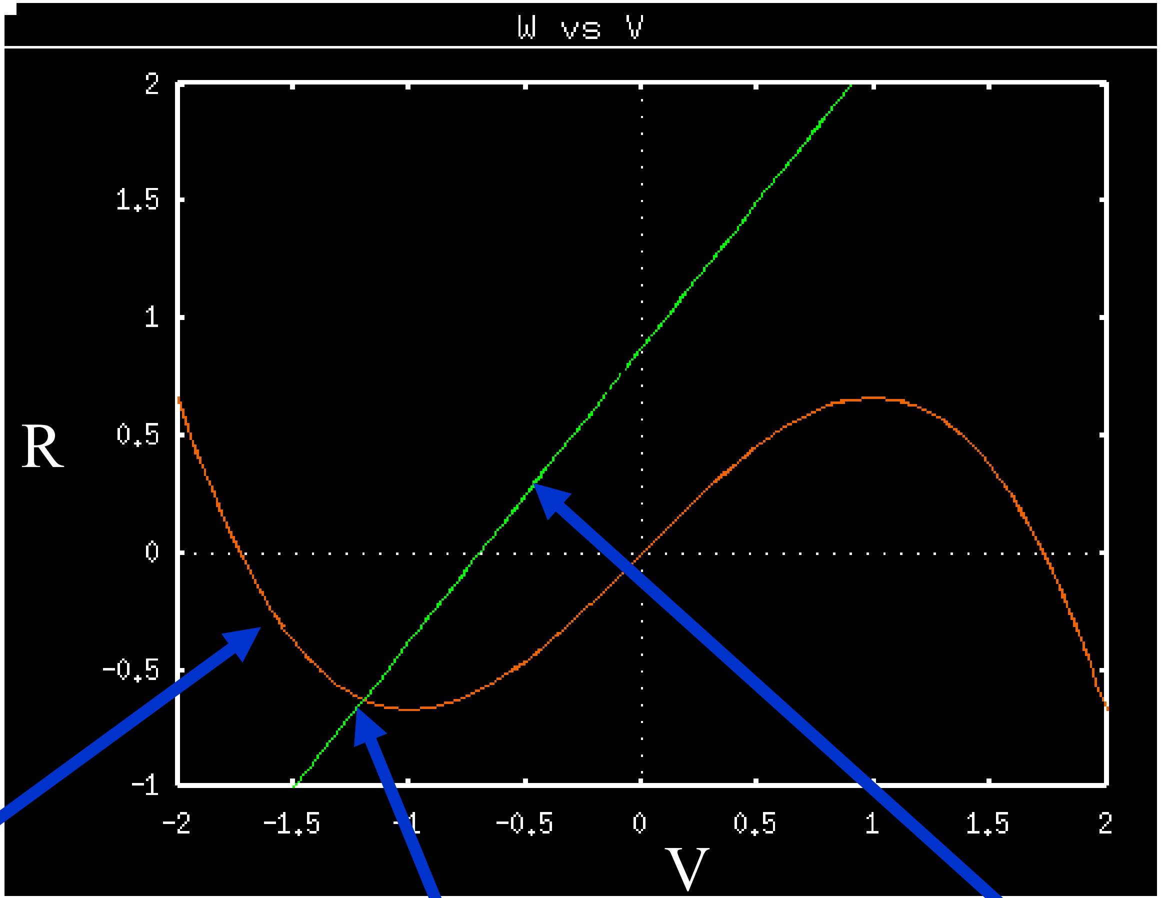
FitzHugh-Nagumo model

Skeleton version of Wilson

$$\frac{dV}{dt} = V - V^3/3 - R + I_{ext}$$

$$\frac{dR}{dt} = \phi(V + a - bR)$$

We will use $\phi = 0.08 \quad ; \quad a = 0.7 \quad ; \quad b = 0.8$



V nullcline

$$\frac{dV}{dt} = 0$$

Equilibrium point

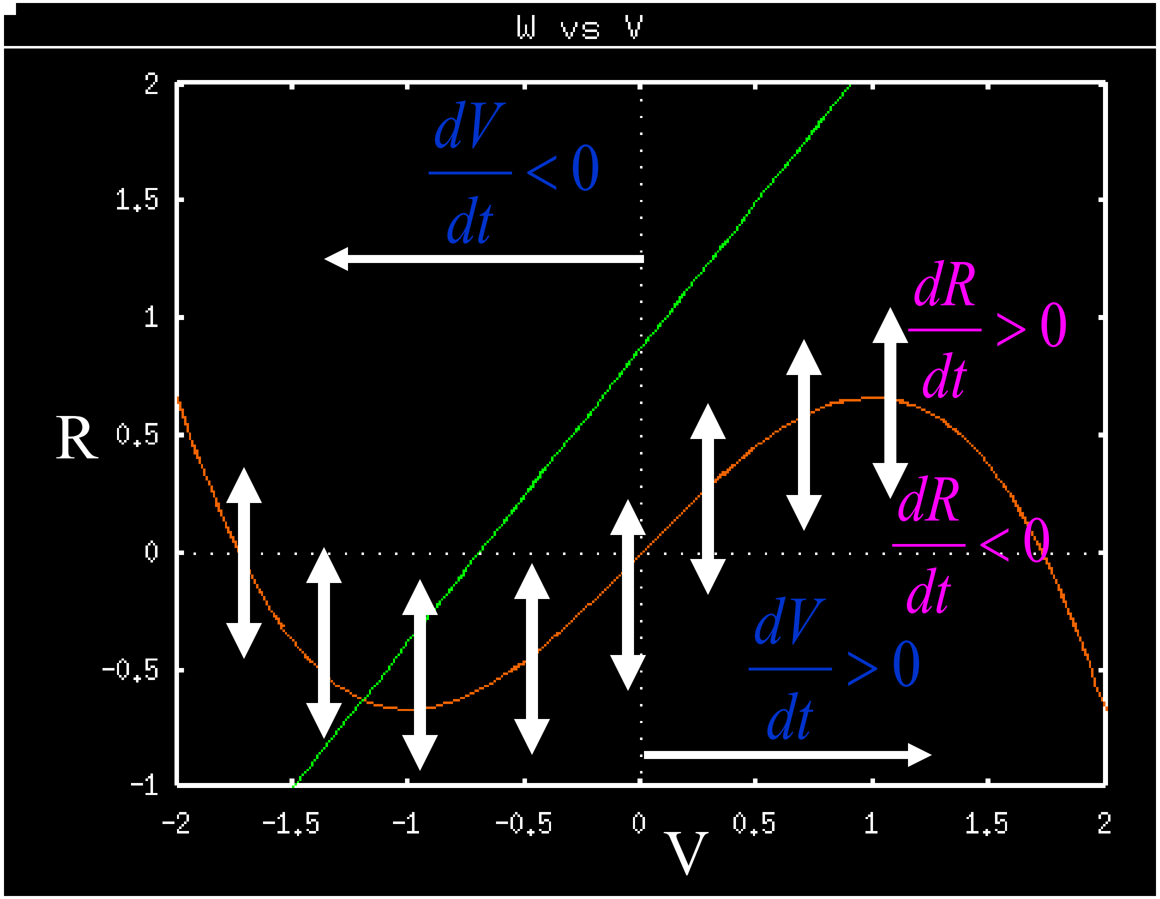
$$\frac{dV}{dt} = \frac{dR}{dt} = 0$$

R nullcline

$$\frac{dR}{dt} = 0$$

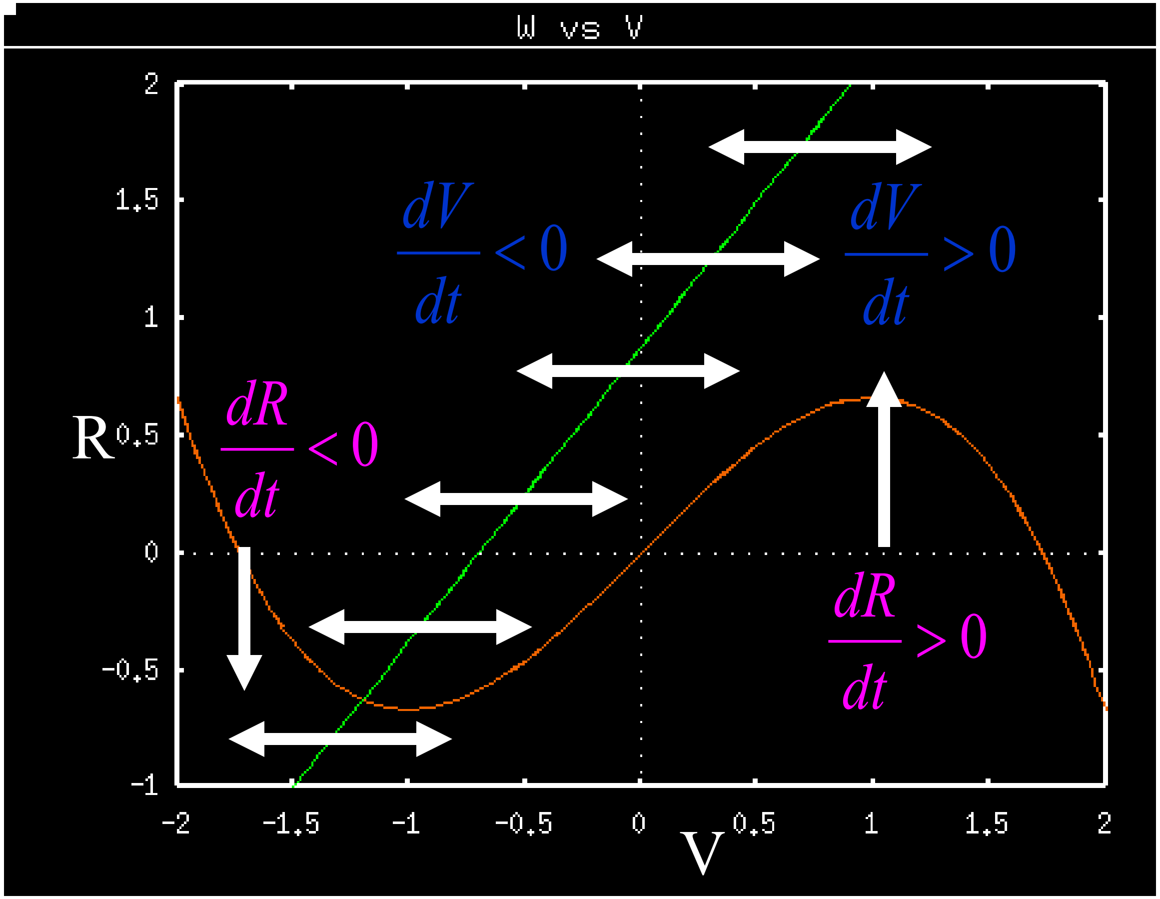
Flow implied by V nullcline (for I_{ext}=0)

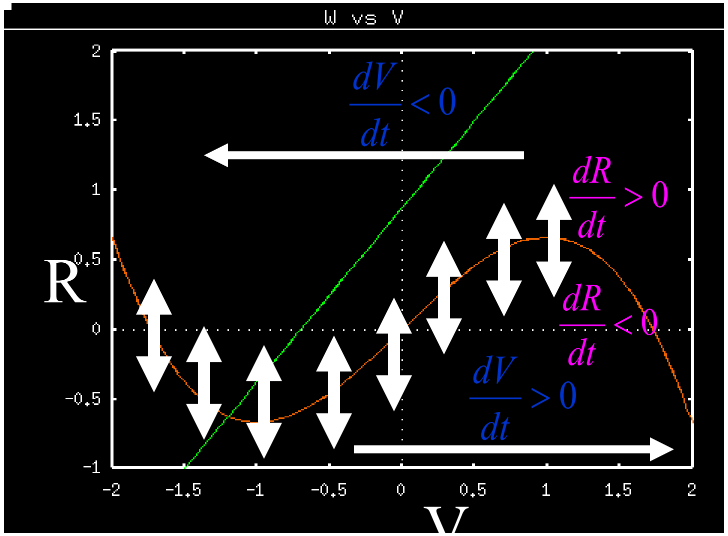
$$\frac{dV}{dt} = V - V^3/3 - R + I_{ext}$$

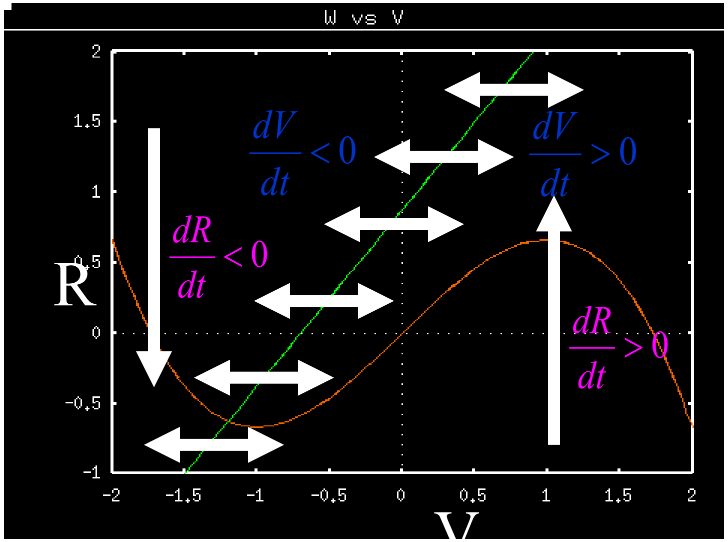


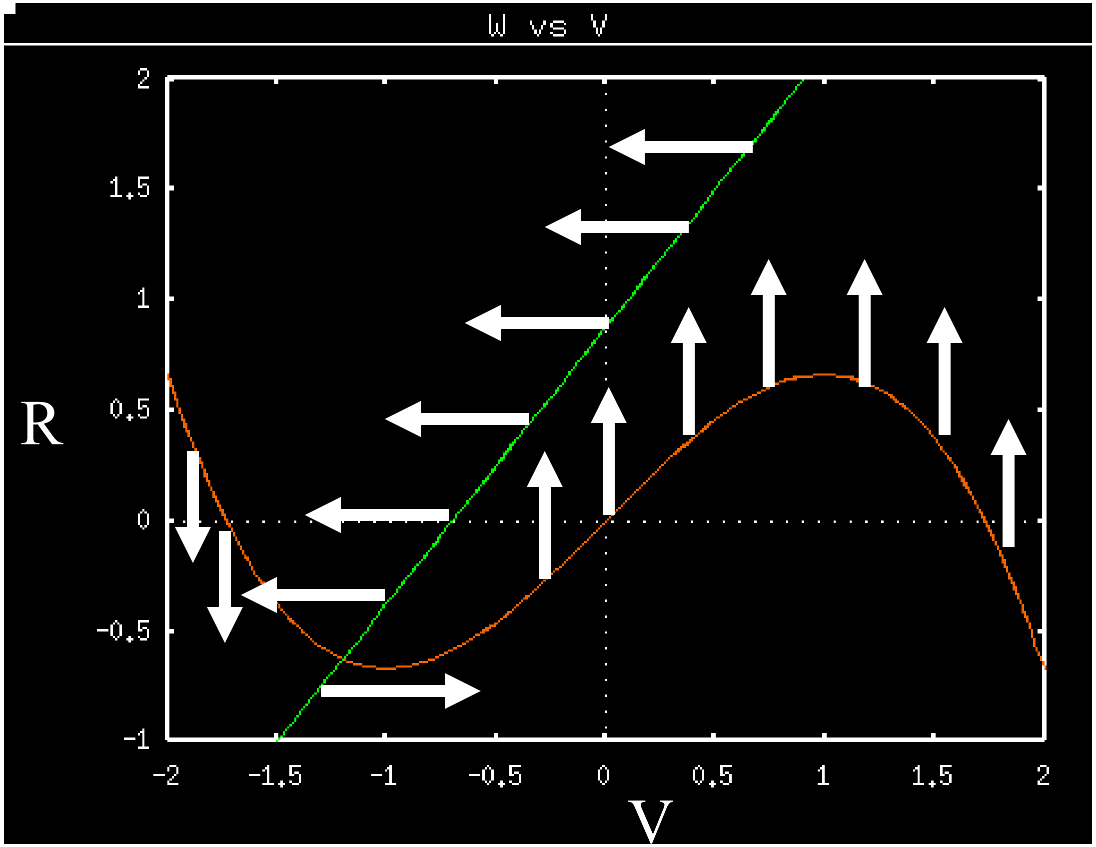
Flow implied by R nullcline

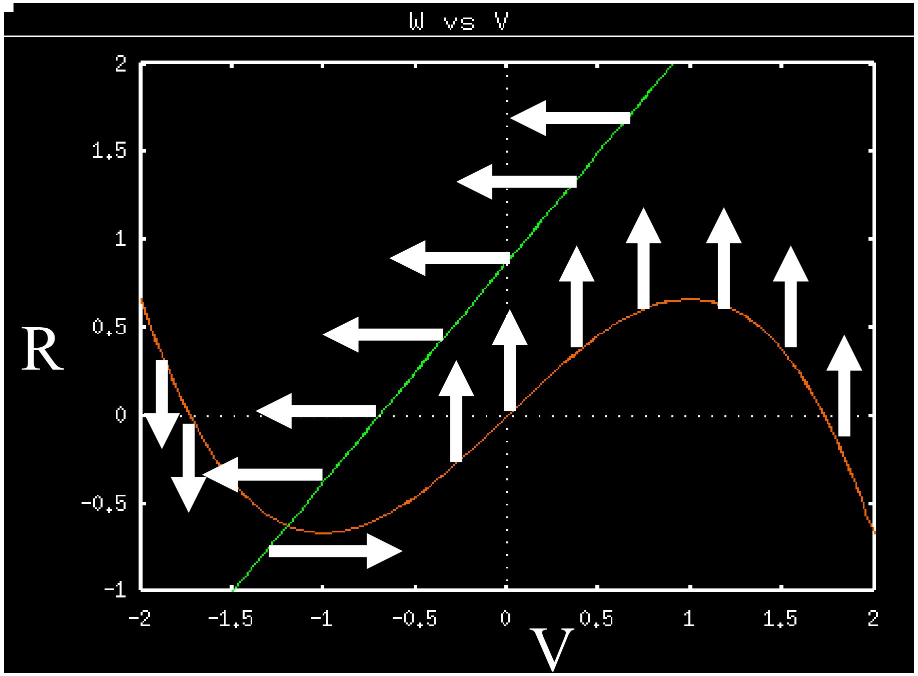
$$\frac{dR}{dt} = \phi(V + a - bR)$$

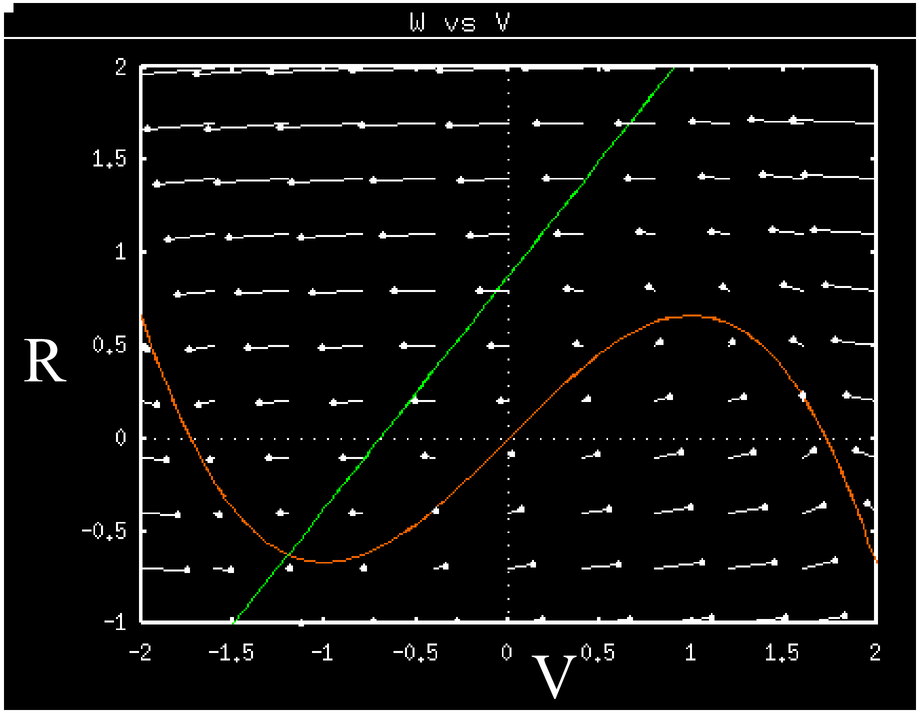



W vs V
$\frac{dV}{dt} < 0$
$\frac{dR}{dt} > 0$
$\frac{dR}{dt} < 0$
$\frac{dV}{dt} > 0$
R
V
2
1.5
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2


W vs V
$\frac{dV}{dt} < 0$
$\frac{dV}{dt} > 0$
$\frac{dR}{dt} < 0$
$\frac{dR}{dt} > 0$
R
V
2
1.5
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

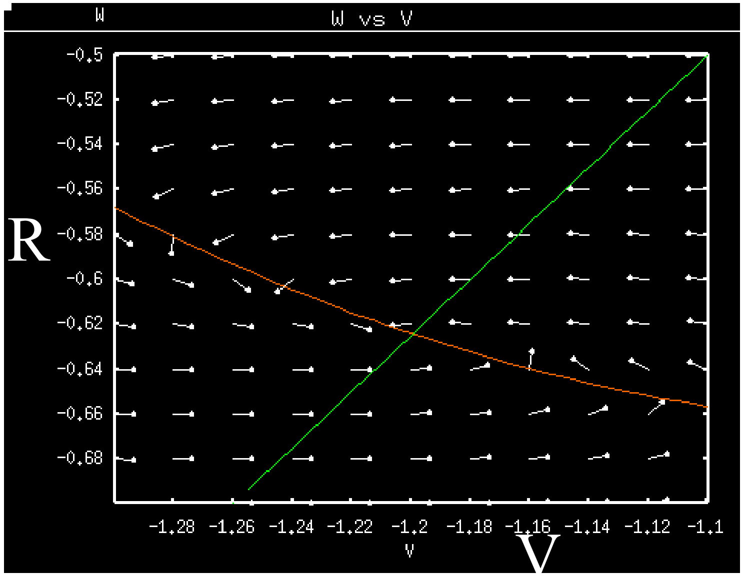

W vs V
R
V
2
1.5
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

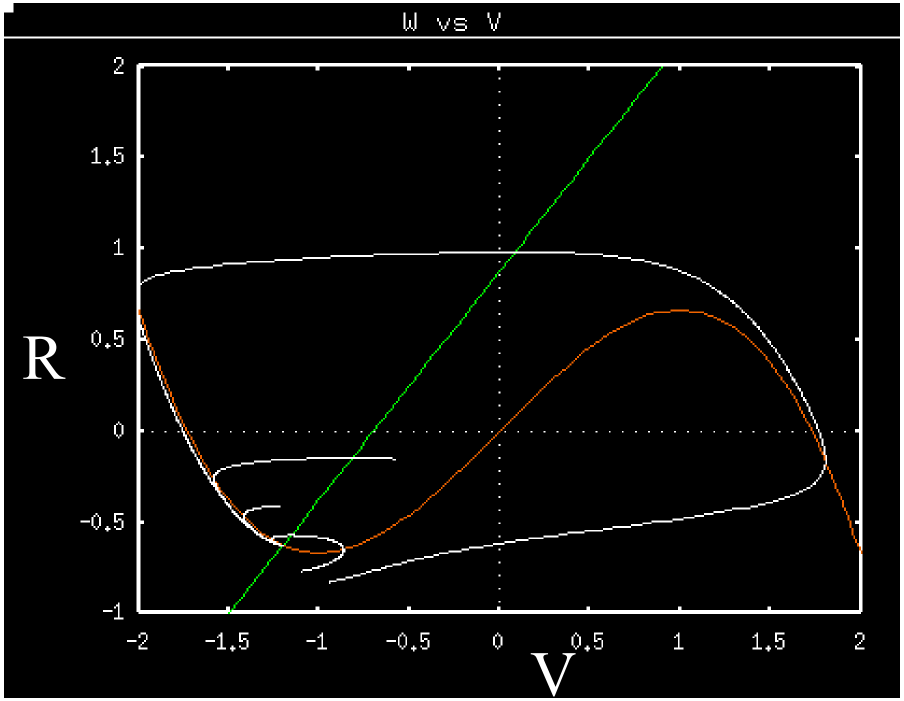

W vs V
2
1.5
1
0.5
0
-0.5
-1
R
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
V


W vs V
2
1.5
1
0.5
0
-0.5
-1
R
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
V

Stability of equilibrium point

For I=0, numerically we see:





But which types of equilibrium can we have?

Let us study the neighbourhood of equilibrium point $(\overline{V}, \overline{R})$

$$\dot{\overline{V}} + \delta\dot{V} = (\overline{V} + \delta V) - (\overline{V} + \delta V)^3/3 - (\overline{R} + \delta R) + I_{ext}$$
$$\dot{\overline{R}} + \delta\dot{R} = \phi((\overline{V} + \delta V) + a - b(\overline{R} + \delta R))$$

$(\overline{V}, \overline{R})$ obeys:

1. $\dot{\overline{V}} = 0 \Rightarrow \overline{V} - \overline{V}^3/3 - \overline{R} + I_{ext} = 0$
2. $\dot{\overline{R}} = 0 \Rightarrow \overline{V} + a - b\overline{R} = 0$
3. $\dot{\overline{V}} = \dot{\overline{R}} = 0$

Substituting conditions 1,2 y 3 and taking linear terms only:

$$\delta\dot{V} = (1 - \overline{V}^2)\delta V - \delta R$$
$$\delta\dot{R} = \phi(\delta V - b\delta R)$$

In matrix form:

$$\delta \dot{V} = (1-\overline{V}^2)\delta V - \delta R$$
$$\delta \dot{R} = \phi(\delta V - b\delta R)$$

$$\begin{pmatrix} \delta \dot{V} \\ \delta \dot{R} \end{pmatrix} = \begin{pmatrix} 1-\overline{V}^2 & -1 \\ \phi & -b\phi \end{pmatrix} \begin{pmatrix} \delta V \\ \delta R \end{pmatrix}$$

With solution: $\delta \overline{r}(t) = c_1 \overline{r}_1 \exp(\lambda_1 t) + c_2 \overline{r}_2 \exp(\lambda_2 t)$

with $\overline{r}_1, \overline{r}_2$ The eigenvectors, c_1, c_2 constants depending on i.c.

and λ_1, λ_2 the eigenvalues of the characteristic equation

$$(1-\overline{V}^2-\lambda)(-b\phi-\lambda)+\phi=0$$

$$\lambda_{1,2} = \frac{1-\overline{V}^2-b\phi \pm \left((-1+\overline{V}^2+b\phi)^2-4b\phi(1-\overline{V}^2)\right)^{1/2}}{2}$$

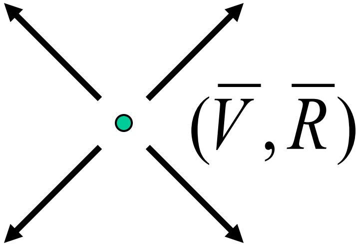
The type of solution depends on the eigenvalues

$$\delta\,\bar{r}(t) = c_1\bar{r}_1 \exp(\lambda_1 t) + c_2\bar{r}_2 \exp(\lambda_2 t)$$

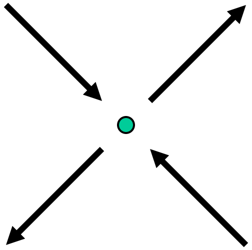
$$\lambda_{1,2} = \frac{1-\bar{V}^2 - b\phi \pm \left((-1+\bar{V}^2+b\phi)^2 - 4b\phi(1-\bar{V}^2)\right)^{1/2}}{2}$$

Types of solutions:

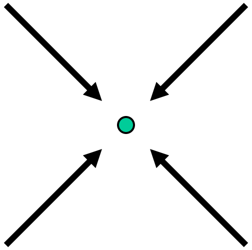
1. “Origin”: $\lambda_1 > 0 \quad \lambda_2 > 0$



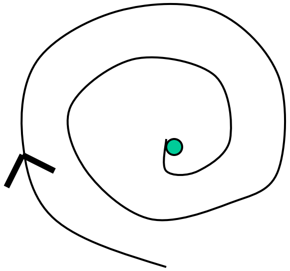
2. “Saddle”: $\lambda_1 > 0 \quad \lambda_2 < 0$



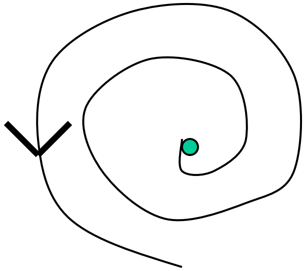
3. “Node”: $\lambda_1 < 0 \quad \lambda_2 < 0$



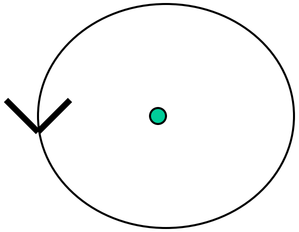
4. “Spiral” or “Attractor”: $\lambda_{1,2} = \alpha \pm i\omega \quad ; \quad \alpha < 0$



5. “Unstable spiral” or “focus”: $\lambda_{1,2} = \alpha \pm i\omega \quad ; \quad \alpha > 0$



6. “Center”: $\lambda_{1,2} = \alpha \pm i\omega \quad ; \quad \alpha = 0$

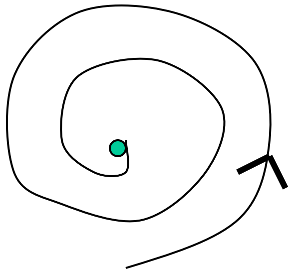


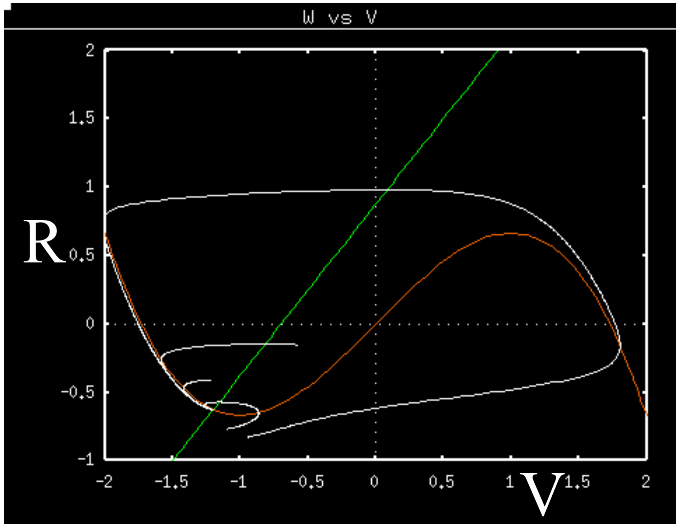
So what do we get in FitzHugh-Nagumo?

(a) $I_{ext}=0$

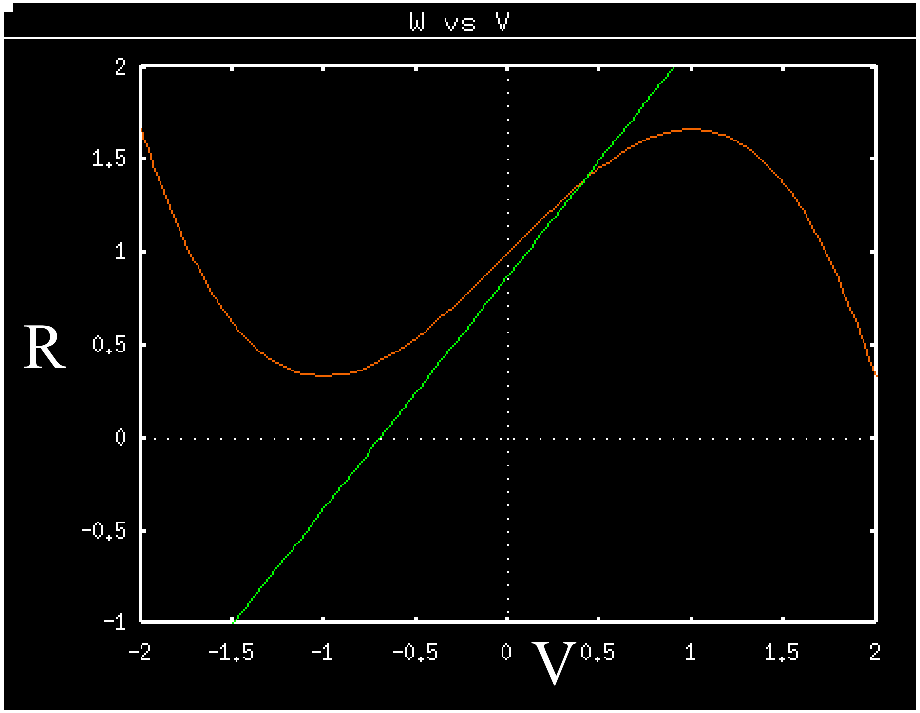
$$(\bar{V},\bar{R})=(-1.2,-0.625)$$

$$\lambda_{1,2}=\frac{1-\bar{V}^2-b\phi\pm\left((-1+\bar{V}^2+b\phi)^2-4b\phi(1-\bar{V}^2)\right)^{1/2}}{2}=-0.5\pm0.42i$$

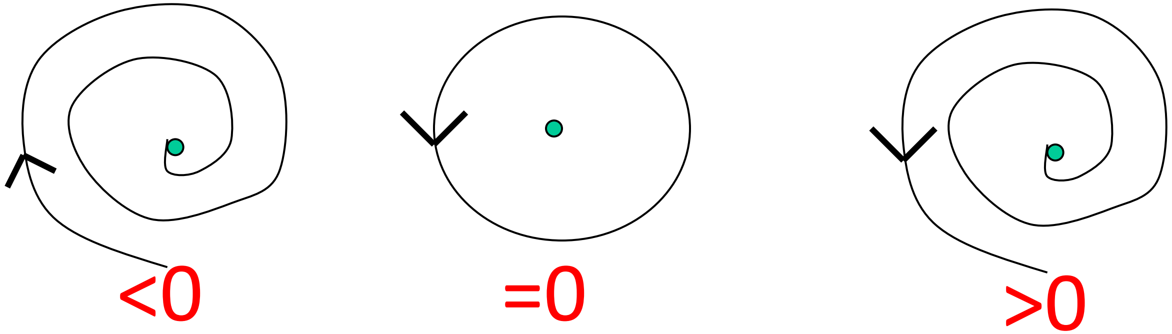




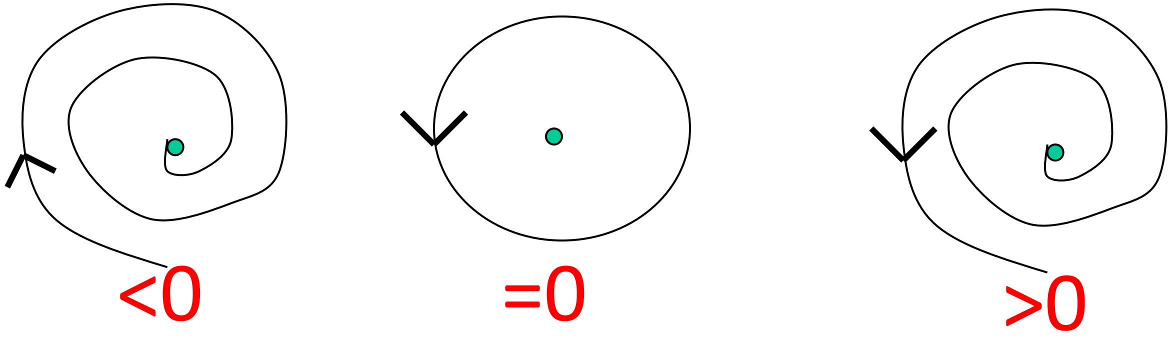
(b) $I_{ext}>0$



$$\lambda_{1,2}=\frac{\boxed{1-\overline{V}^2-b\phi}\pm\left((-1+\overline{V}^2+b\phi)^2-4b\phi(1-\overline{V}^2)\right)^{1/2}}{2}$$



$$\lambda_{1,2} = \frac{\boxed{1-\bar{V}^2 - b\phi} \pm \left((-1+\bar{V}^2+b\phi)^2 - 4b\phi(1-\bar{V}^2)\right)^{1/2}}{2}$$



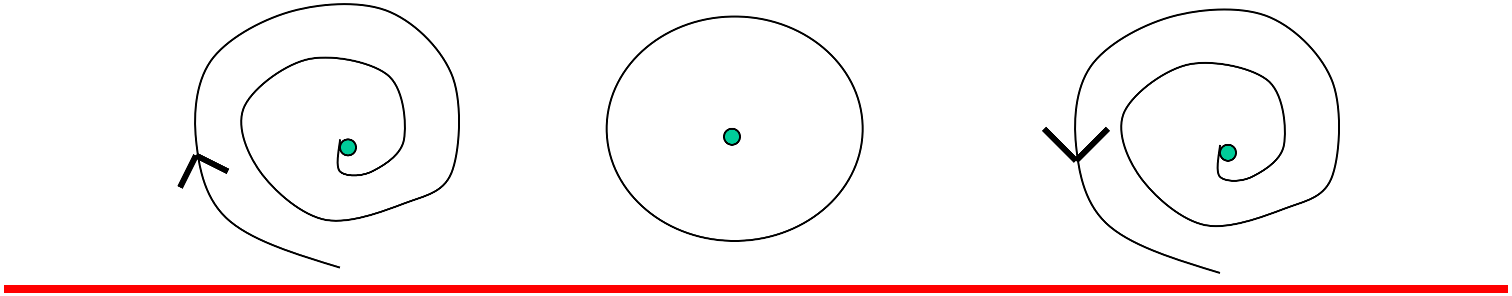
$$\bar{V}_{\pm} = \pm(1-b\phi)^{1/2} = \pm 0.967$$

Sustituyendo en

$$\frac{dV}{dt} = 0 \Rightarrow R = V - V^3/3 + I_{ext}$$

$$\frac{dR}{dt} = 0 \Rightarrow R = \frac{V+a}{b}$$

Obtenemos $I_{-} = 0.3313 \quad I_{+} = 1.42$

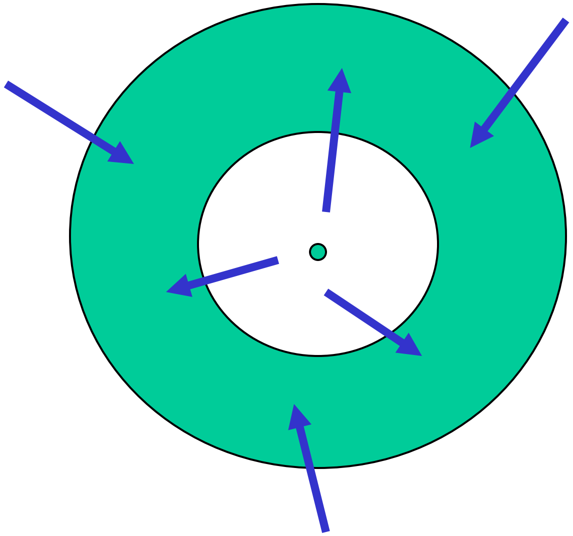


$I=0.3313$

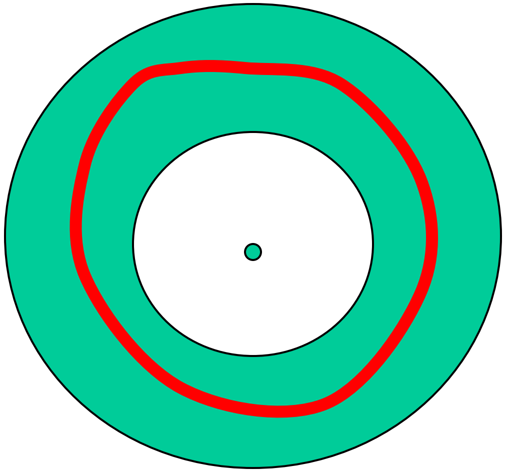
What type of solution for $I>0.3313$?

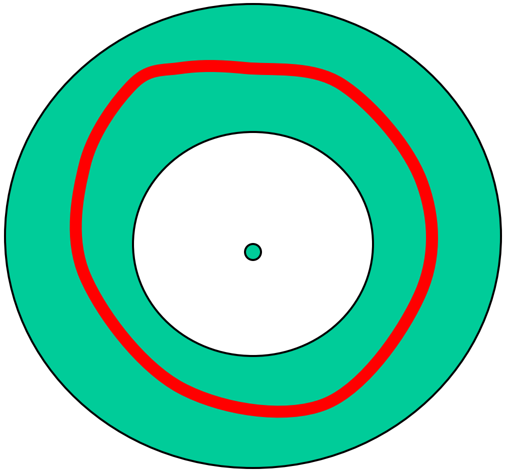
Poincaré-Bendixon theorem:

If

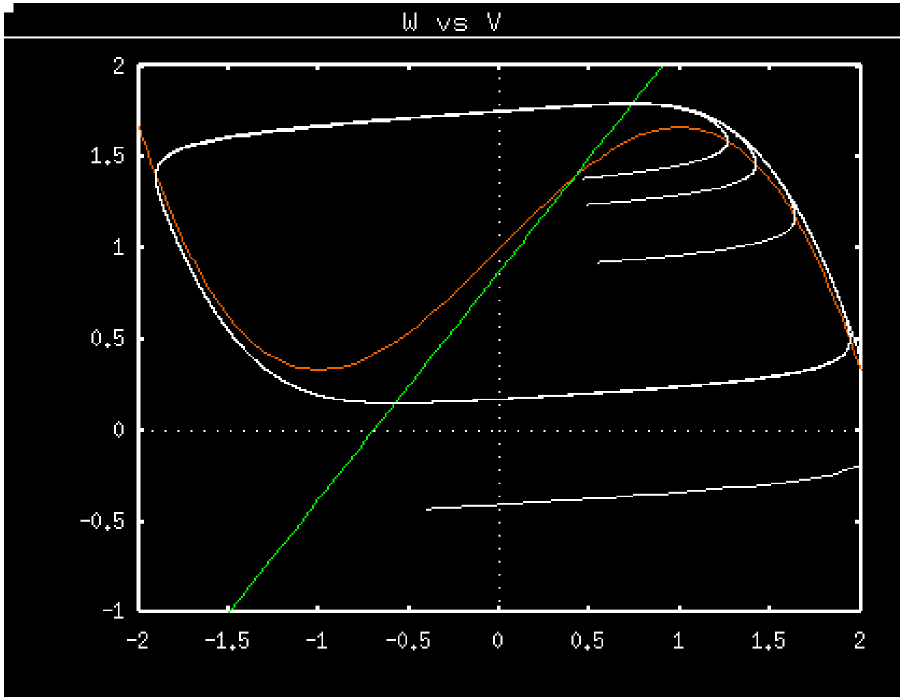


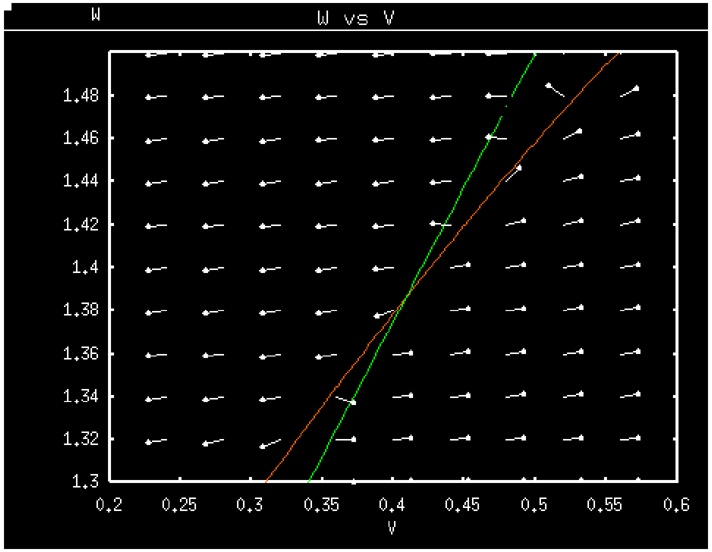
then

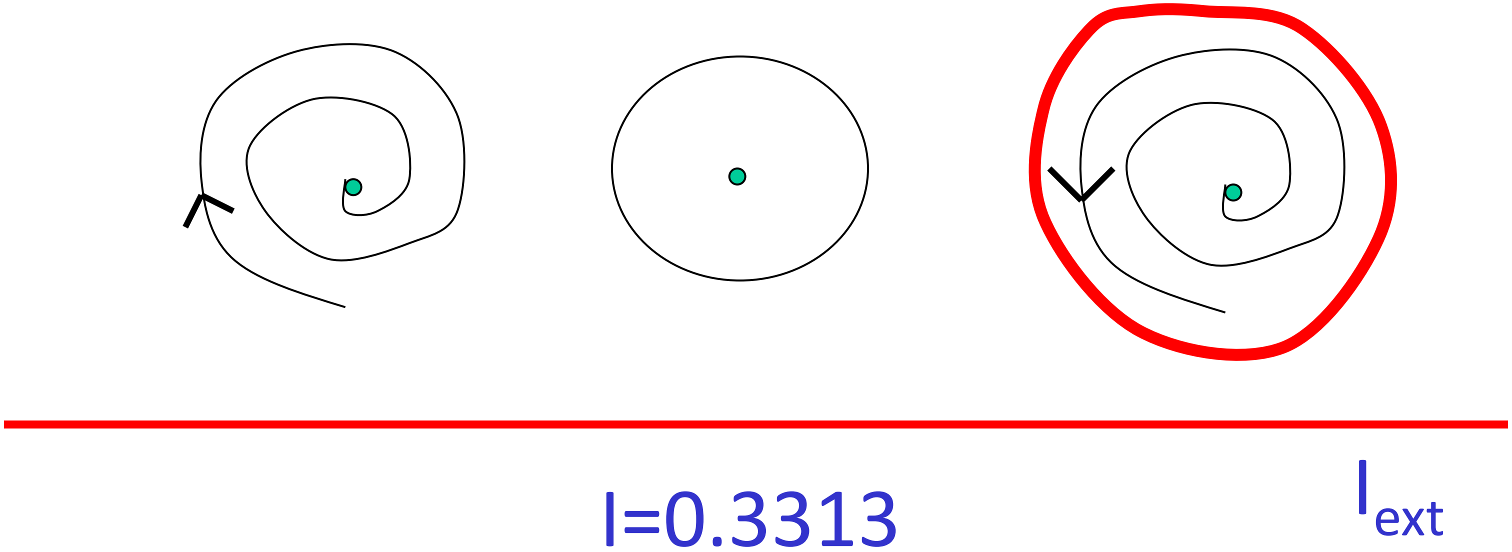




I=1


W vs V
2
1.5
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2

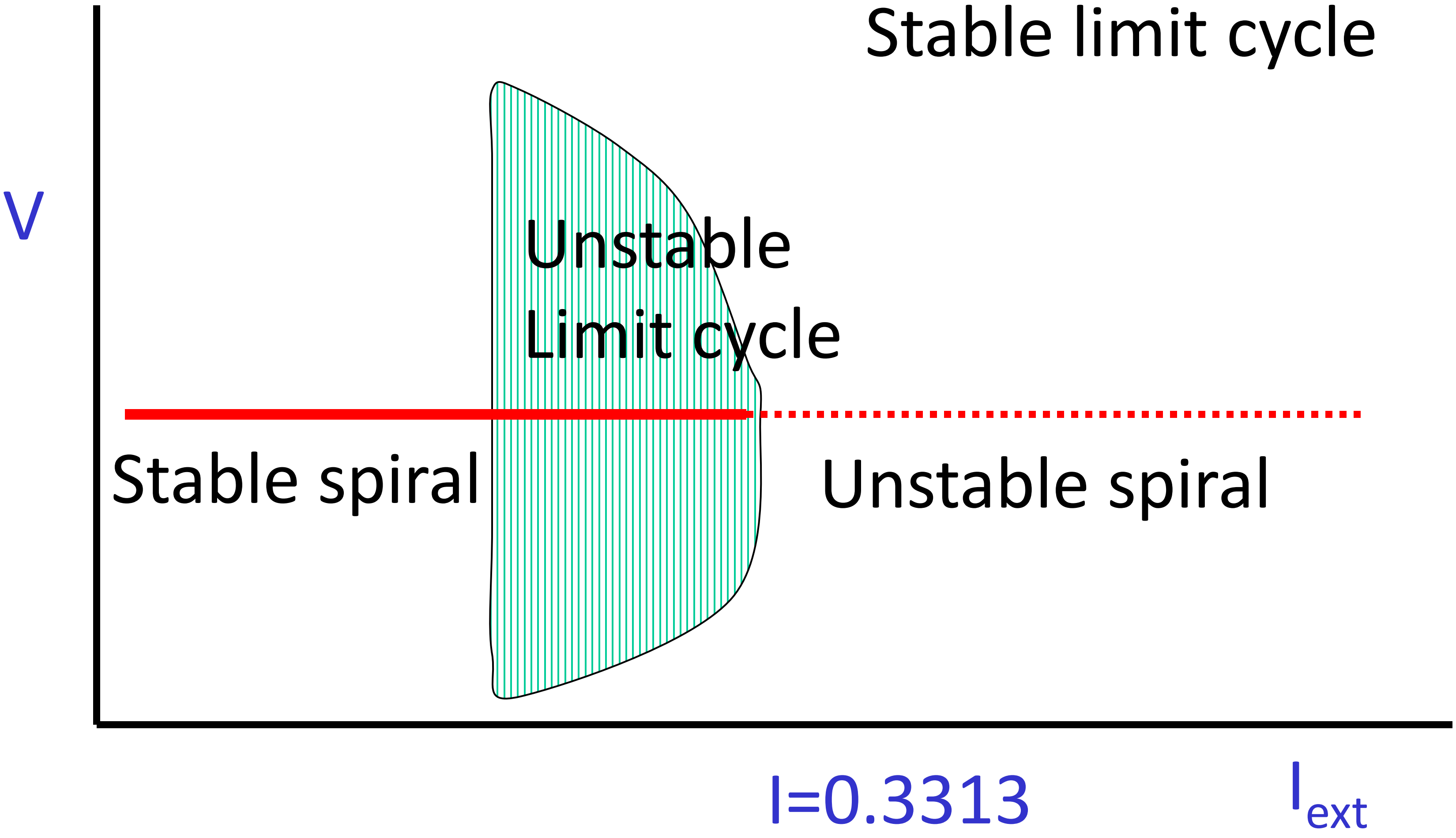

W
W vs V
1.48
1.46
1.44
1.42
1.4
1.38
1.36
1.34
1.32
1.3
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
V



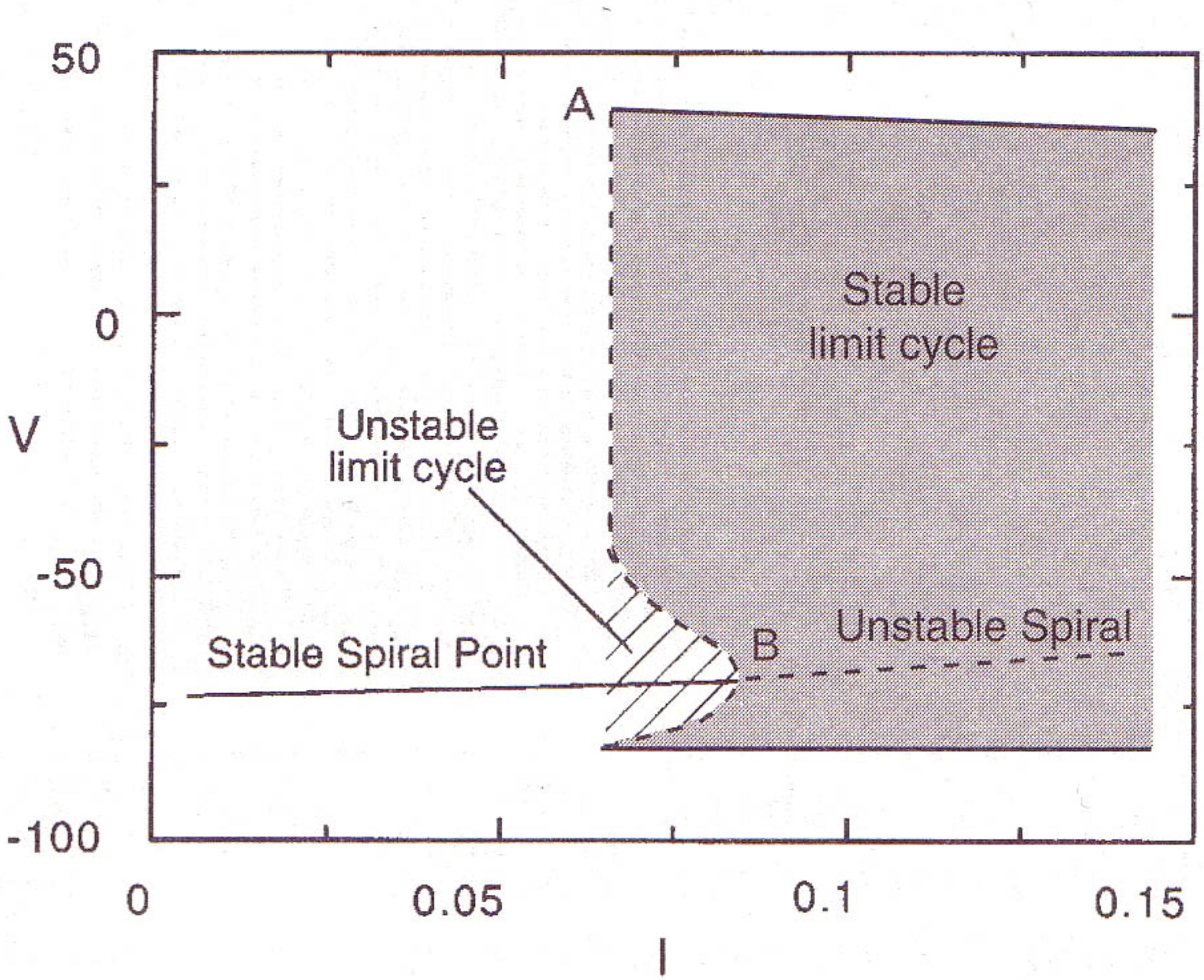
Large oscillations (action potentials) suddenly apprear at I=0.3313

→ Subcritical Hopf bifurcation

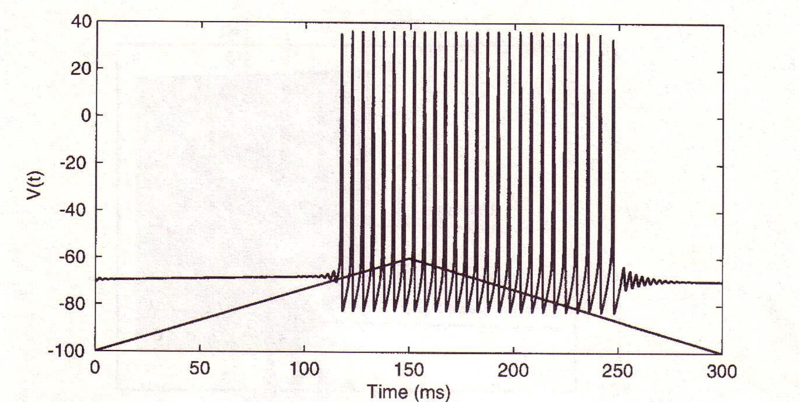
Subcritical Hopf bifurcation

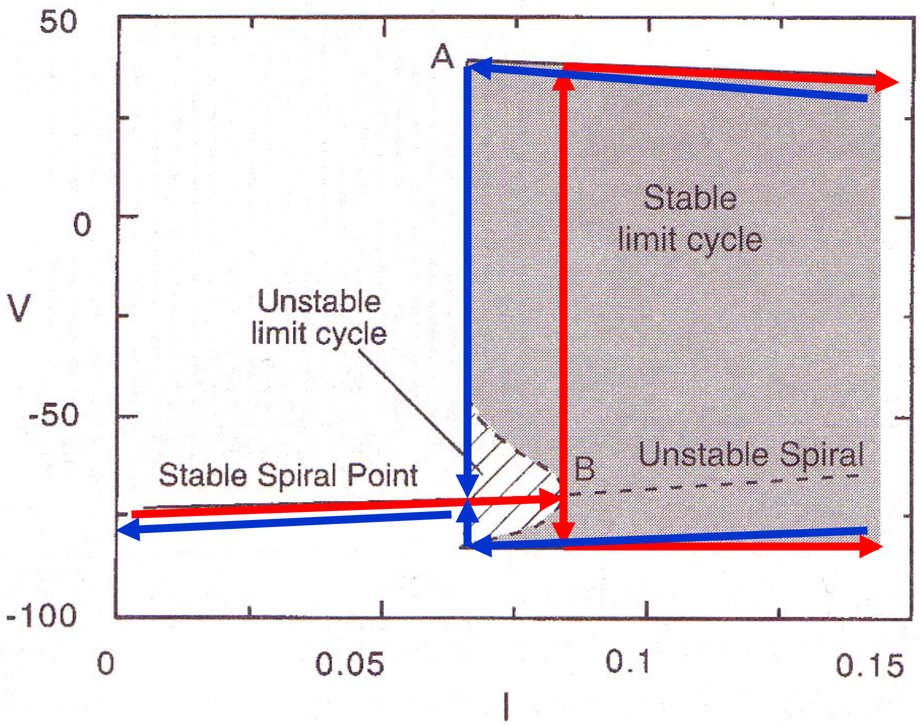


Bifurcation diagram of Wilson model



Hysteresis in an axon





1. HH model
2. Simplyfications of HH model
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3. Adaptation and bursts
4. Networks

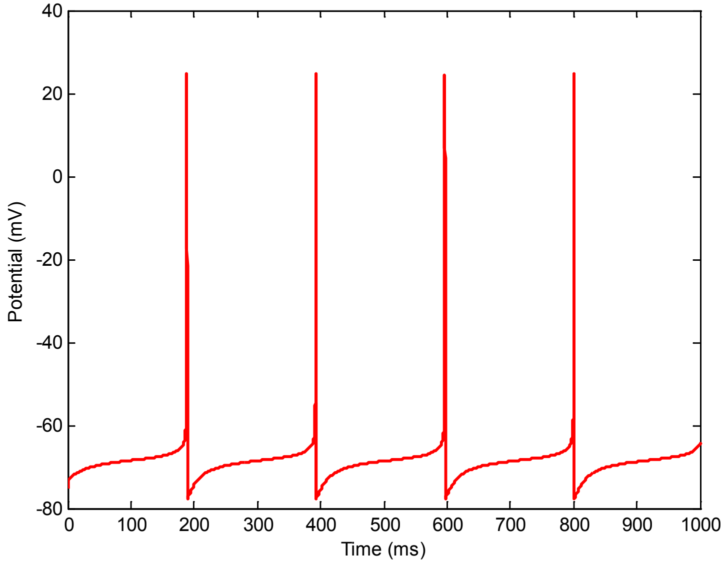
Low frequency firing (class I neuron)

Squid axon fires at 150-300 AP/sec.

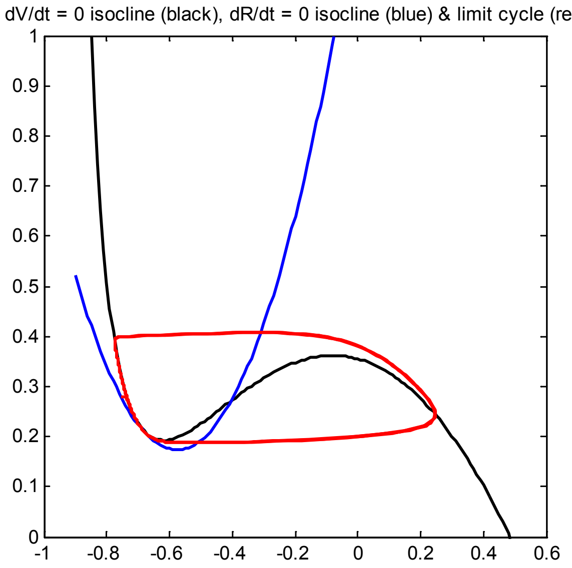
Potassium current I_A allows for much lower firing rates

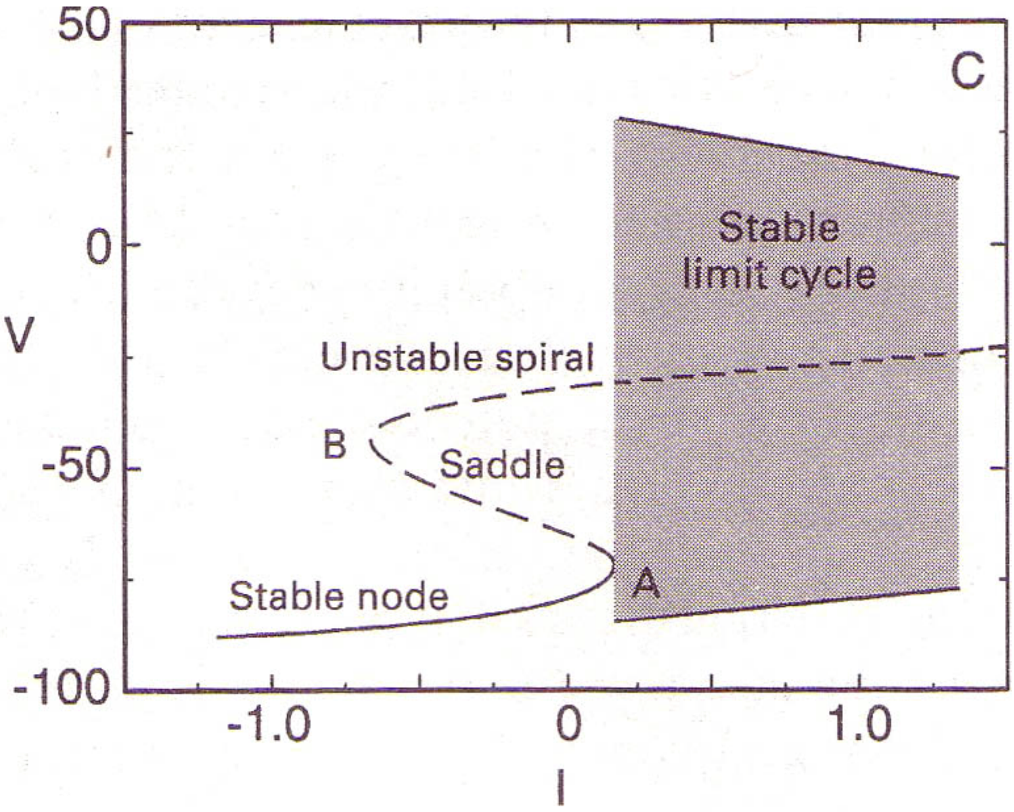
$$\frac{dV}{dt} = -(17.81 + 47.58\,V + 33.8\,V^2)(V - 0.48) - 26R(V + 0.95) + I_{ext}$$

$$\frac{dR}{dt} = \frac{1}{5.6}(-R + 1.29\,V + 0.79 + 3.3(V + 0.38)^2)$$



$$I = 0.25\,nA$$


dV/dt = 0 isocline (black), dR/dt = 0 isocline (blue) & limit cycle (red
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6


C
50
0
-50
-100
V
Stable
limit cycle
Unstable spiral
B
Saddle
A
Stable node
-1.0
0
1.0
I

Adaptation of firing rates

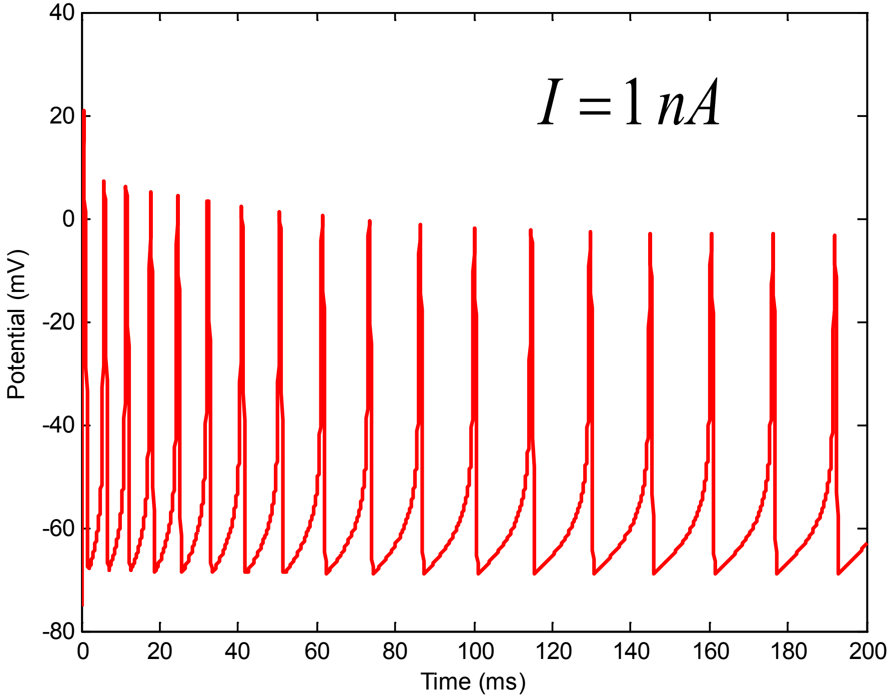
Squid axon fires without adaptation.
Potassium current I_{AHP} (afterhyperpolarization):

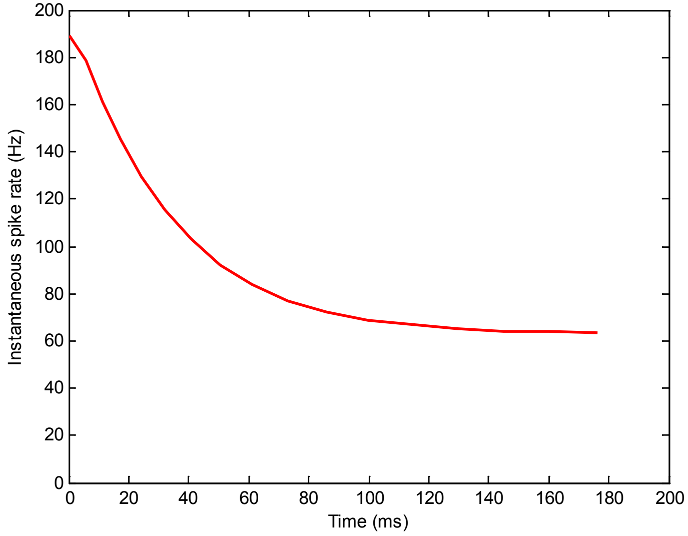
1. Slow: 99 ms (does not affect AP shape)
2. Acts only when the neuron is firing (zero at –0.754)
3. Its job is to eliminate the effect of I_{ext}

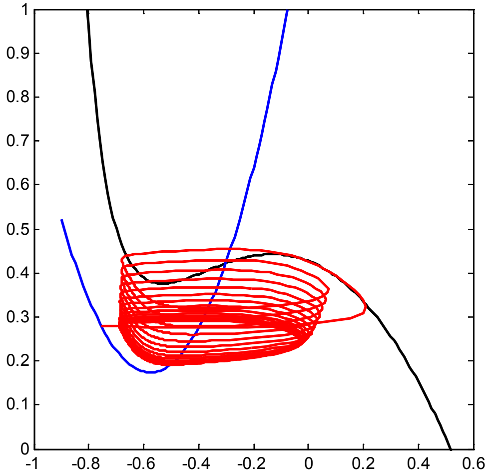
$$\frac{dV}{dt} = -(17.81+47.58V+33.8V^2)(V-0.48)-26R(V+0.95)$$
$$\boxed{-13H(V+0.95)+}I_{ext}$$

$$\frac{dR}{dt} = \frac{1}{5.6}(-R+1.29V+0.79+3.3(V+0.38)^2)$$

$$\frac{dH}{dt} = \frac{1}{99}(-H+11(V+0.754)(V+0.69))$$

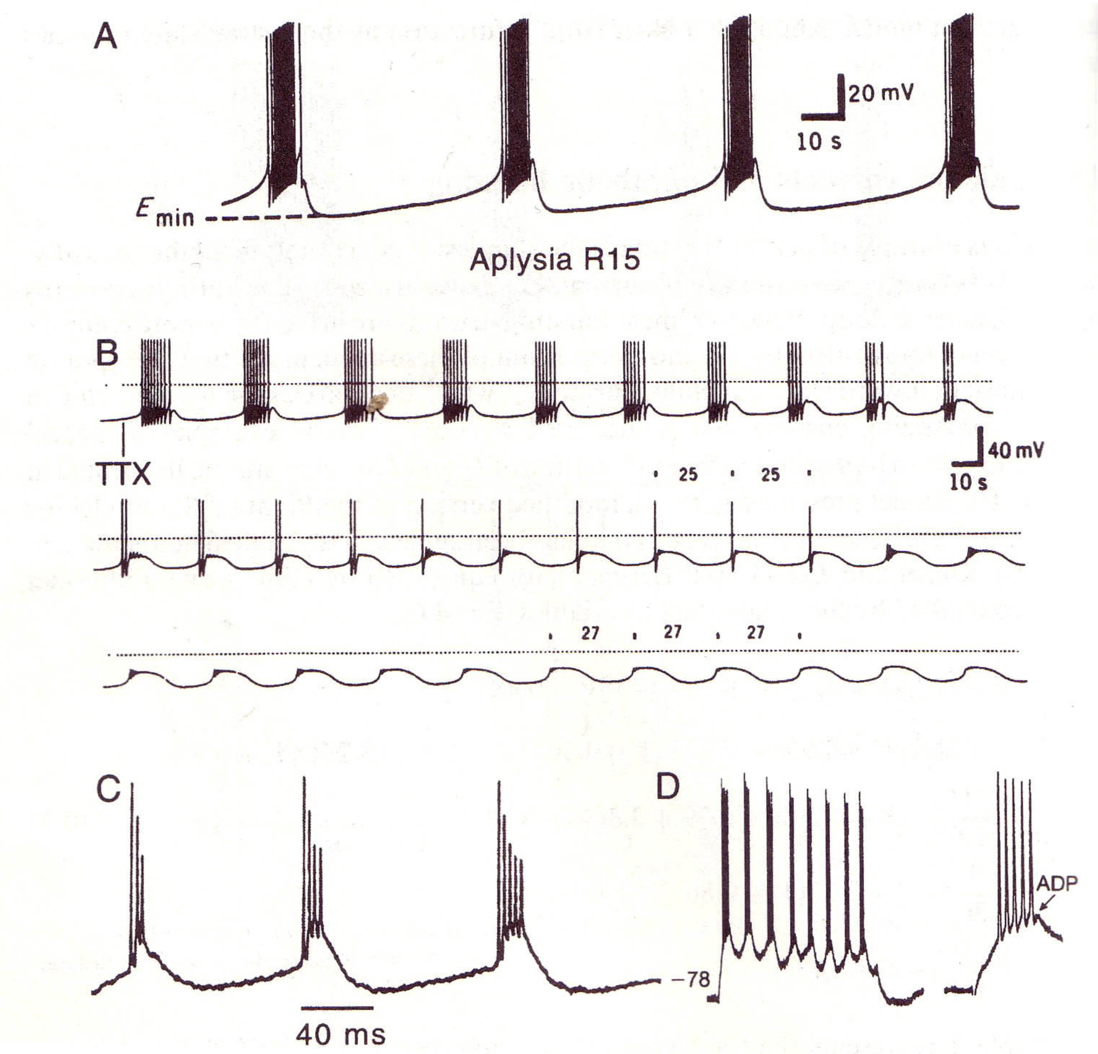

$I = 1\,nA$
Potential (mV)
40
20
0
-20
-40
-60
-80
0
20
40
60
80
100
120
140
160
180
200
Time (ms)


Instantaneous spike rate (Hz)
200
180
160
140
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160
180
200
Time (ms)


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

Firing in burts

Examples



Neuron R15
in Aplysia

Applying
TTX to R15

Cortical neurons

Model for bursty neuron

2 currents:

Corriente de potasio I_{AHP} (afterhyperpolarization, dependiente de calcio)

Corriente de calcio I_{T} (depolarización)

$$\frac{dV}{dt} = -(17.81+47.58V+33.8V^2)(V-0.48)-26R(V+0.95)$$

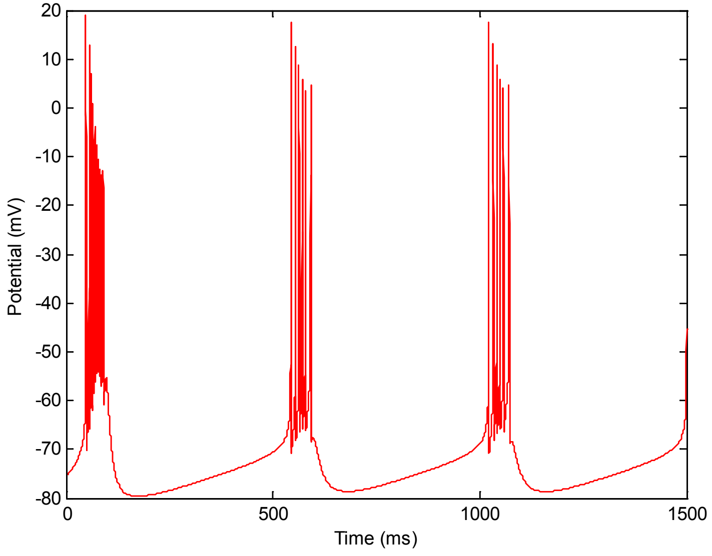
$$-\underset{I_{\text{T}}}{1.93X(1-0.5C)(V-1.4)}-\underset{I_{\text{AHP}}}{3.25C(V+0.95)}$$

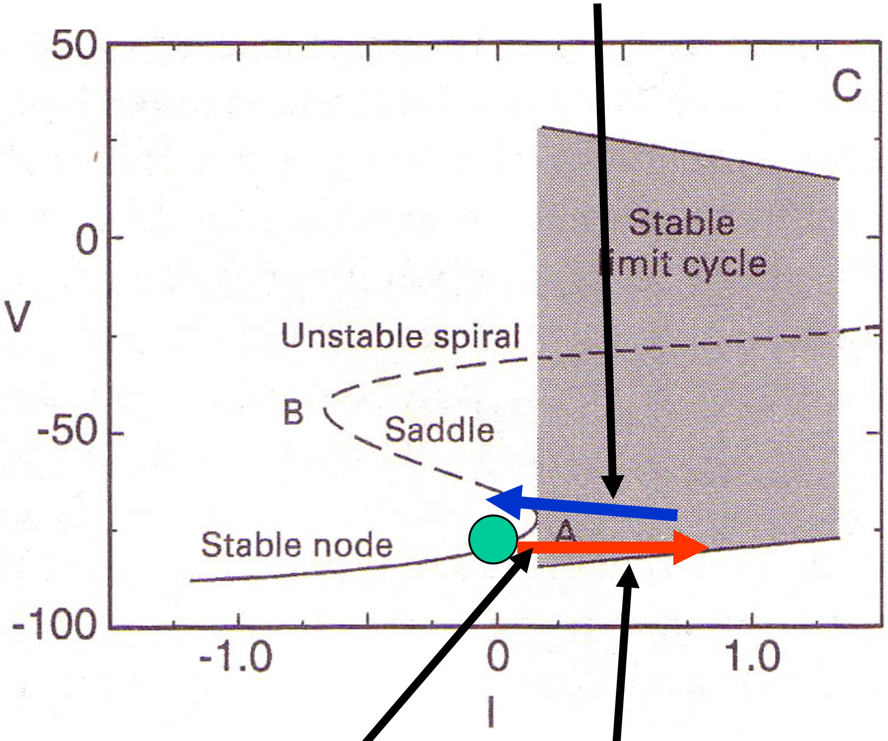
$$\frac{dR}{dt} = \frac{1}{5.6}(-R+1.29V+0.79+3.3(V+0.38)^2)$$

$$\frac{dX}{dt} = \frac{1}{30}(-X+7.33(V+0.86)(V+0.84))$$

$$\frac{dC}{dt} = \frac{1}{100}(-C+3X)$$

Concentración de calcio



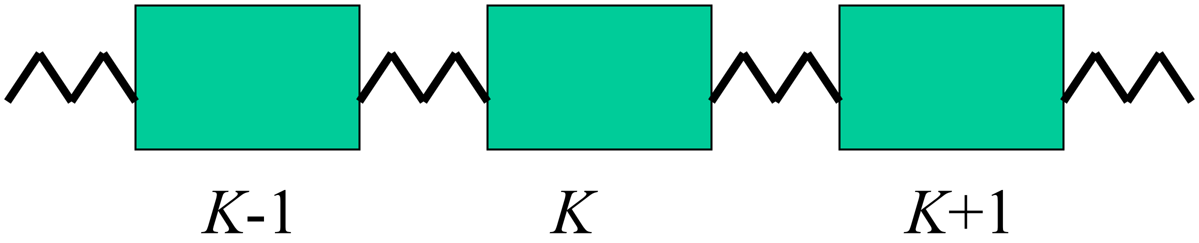


C increases &
V decreases &
X inactivates

X increases V

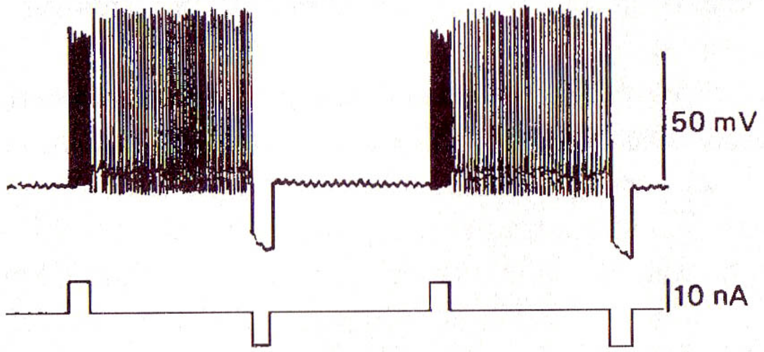
Firing

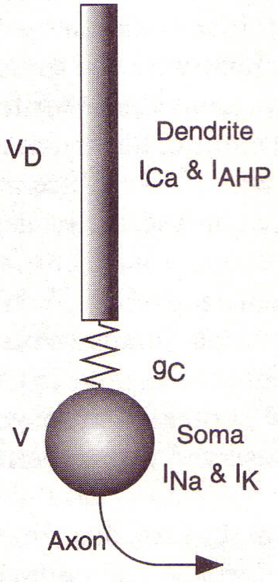
Compartments



$$C\frac{dV_K}{dt}=\sum_{j=1}^{N} I_j + g_{K-1,K}(V_{K-1}-V_K)+g_{K,K+1}(V_{K+1}-V_K)$$

Example: dendrite+soma

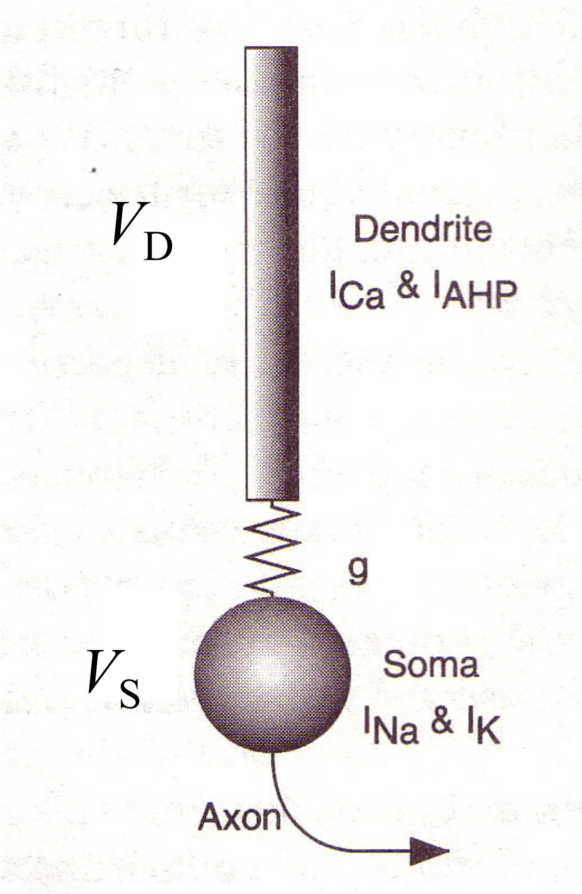




$$C_S \frac{dV_S}{dt} = -I_{Na^+} - I_{K^+} + I_{ext} + \frac{g}{p}(V_D - V_S)$$

$$C_D \frac{dV_D}{dt} = -I_{Ca^{2+}} - I_{AHP} + \frac{g}{1-p}(V_S - V_D)$$

p: proporción de membrana en el soma



$$\frac{dV_S}{dt} = -(17.81 + 47.58\, V_S + 33.8\, {V_S}^2)(V_S - 0.48) - 26R(V + 0.95)$$

$$+ I_{ext} + \frac{g}{p}(V_D - V_S)$$

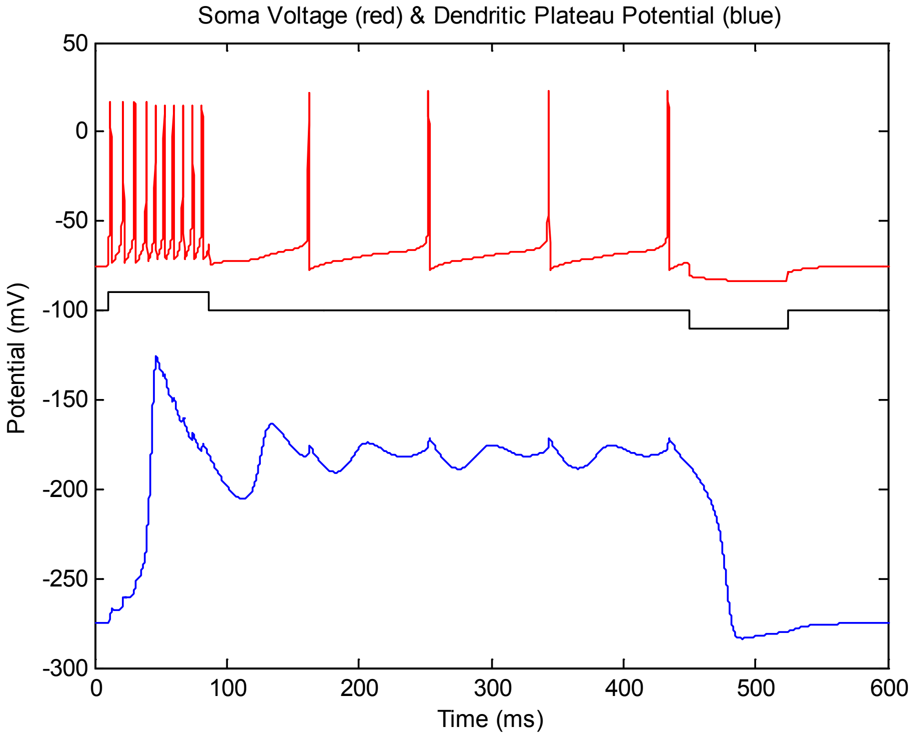
$$\frac{dR}{dt} = \frac{1}{5.6}(-R + 1.29\, V_S + 0.79 + 3.3(V_S + 0.38)^2)$$

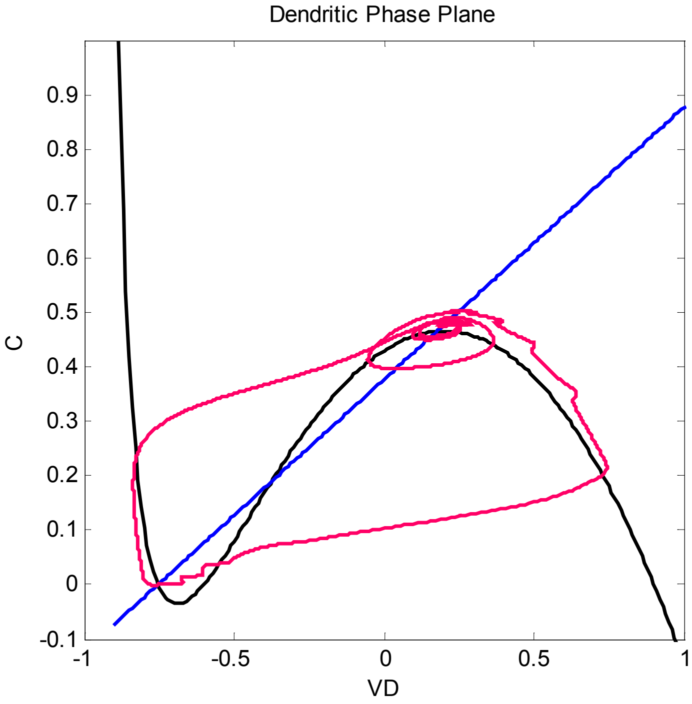
$$\frac{dV_D}{dt} = -(V_D + 0.754)(V_D + 0.7)(V_D - 1.0) - g_{AHP}\, C\,(V_D + 0.95)$$

$$+ \frac{g}{1-p}(V_S - V_D)$$

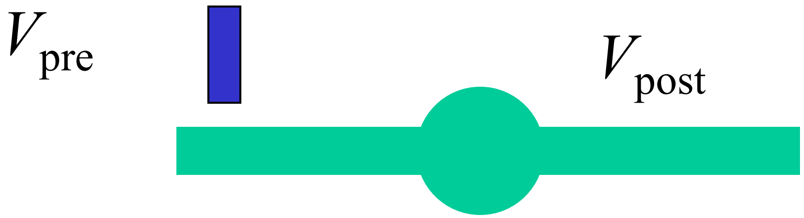
$$\frac{dC}{dt} = \frac{1}{20}(-C + 0.5(V_D + 0.754))$$

For g_{AHP}=1.0, g=0.1, p=0.37


Soma Voltage (red) & Dendritic Plateau Potential (blue)


Dendritic Phase Plane

A very simple model of a synapsis



$$C\frac{dV_{post}}{dt} = \sum_{j=1}^{N} I_j + k\, g_{syn}\,(V_{post} - E_{syn}) \quad ; \quad E_{syn} = \begin{cases} 0 & (exc) \\ -0.92 & (inh) \end{cases}$$

Post-synaptic factor

Synaptic conductance

$$\frac{dg_{syn}}{dt} = \frac{1}{\tau_{syn}}(-g_{syn} + f)$$

$$\frac{df}{dt} = \frac{1}{\tau_{syn}}(-f + H(V_{pre} - \Theta)) \quad ; \quad H(X) = \begin{cases} 0 & X \le 0 \\ 1 & X > 0 \end{cases}$$

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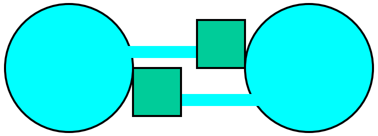
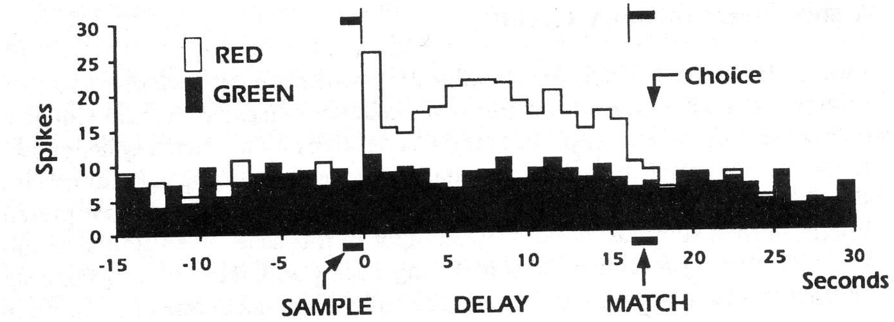
Simple memory circuit





$$\dot{E}_1 = \frac{1}{\tau}\left(-E_1 + S(3E_2;100,2,120)\right)$$

$$\dot{E}_2 = \frac{1}{\tau}\left(-E_2 + S(3E_1;100,2,120)\right)$$

τ: 20ms

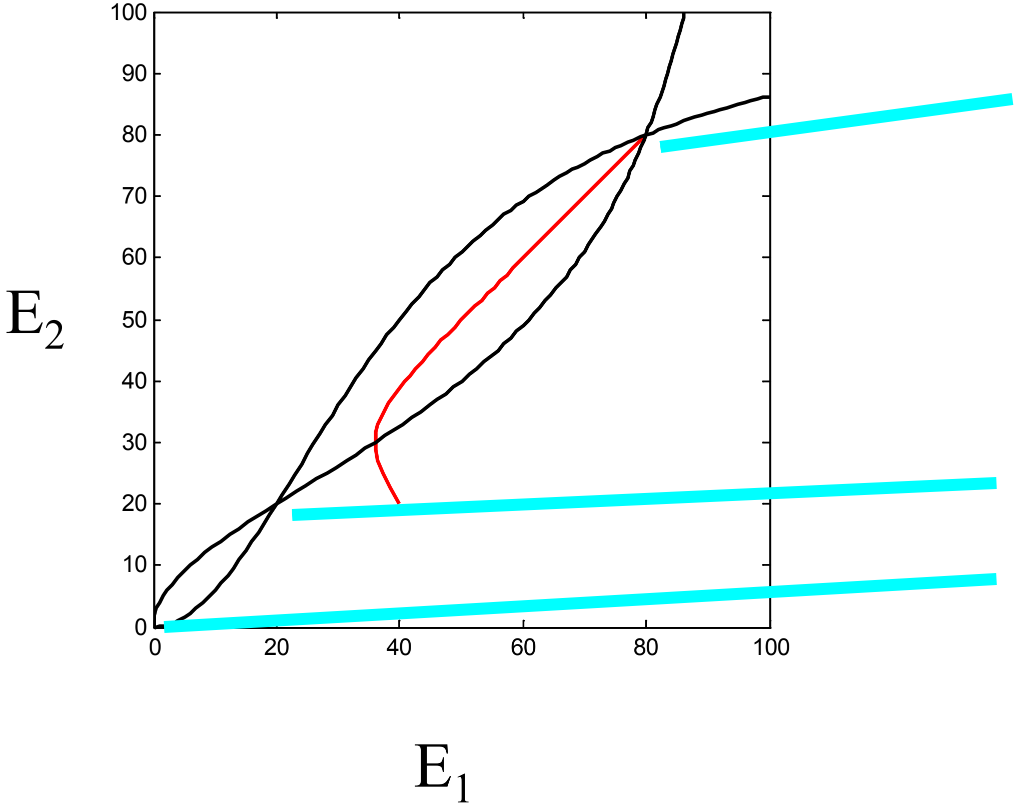
Synaptic weight: 3

$$S(P;M,N,\sigma) = \frac{MP^N}{\sigma^N + P^N}$$

M: maximum firing rate

N:determines maximum slope

σ: point with S(P)/2



Stable node

Unstable node

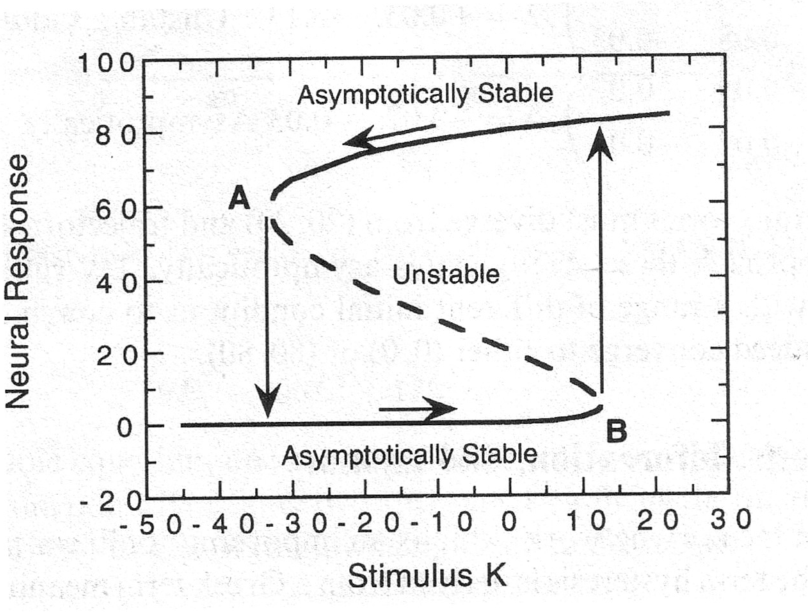
Stable node

Memory and hysteresis

$$\dot{E}_1 = \frac{1}{\tau}\left(-E_1 + S(3E_2 + K;100,2,120)\right)$$

$$\dot{E}_2 = \frac{1}{\tau}\left(-E_2 + S(3E_1 + K;100,2,120)\right)$$

K: stimulus



Memory decay by adaptation

Adaptation thorough an increase of σ

$$\dot{E}_1 = \frac{1}{\tau}\left(-E_1 + S(3E_2;100,2,120 + A_1)\right)$$

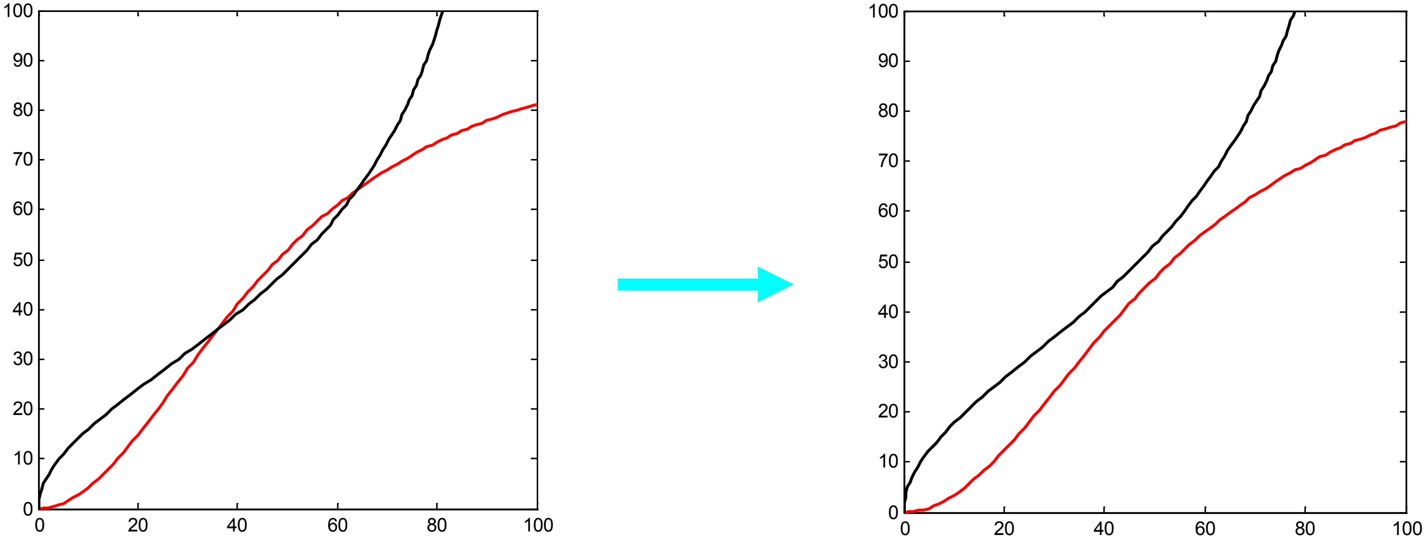
$$\dot{E}_2 = \frac{1}{\tau}\left(-E_2 + S(3E_1;100,2,120 + A_2)\right)$$

$$\dot{A}_1 = \frac{1}{\tau_a}\left(-A_1 + 0.7E_1\right)$$

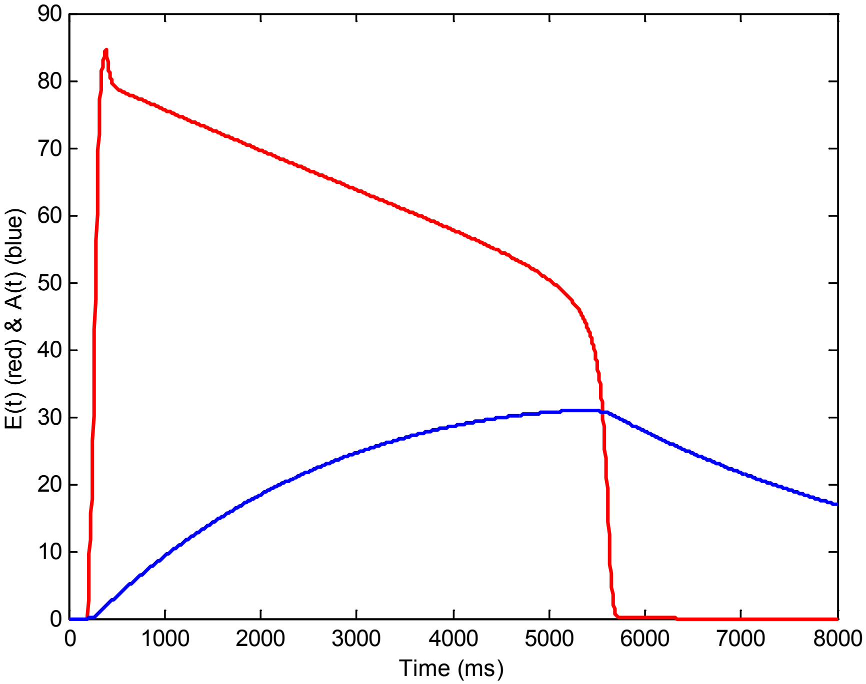
$$\dot{A}_2 = \frac{1}{\tau_a}\left(-A_2 + 0.7E_2\right)$$

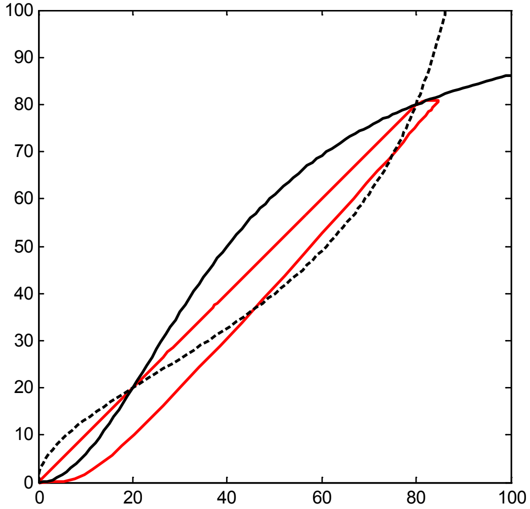
τ: 20ms
τ_a: 4000ms

Adaptation affects nullclines

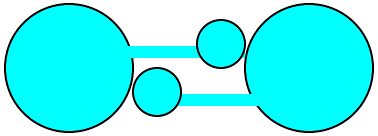


K=50 for 200ms and then K=0





Competition and decisions by neurons

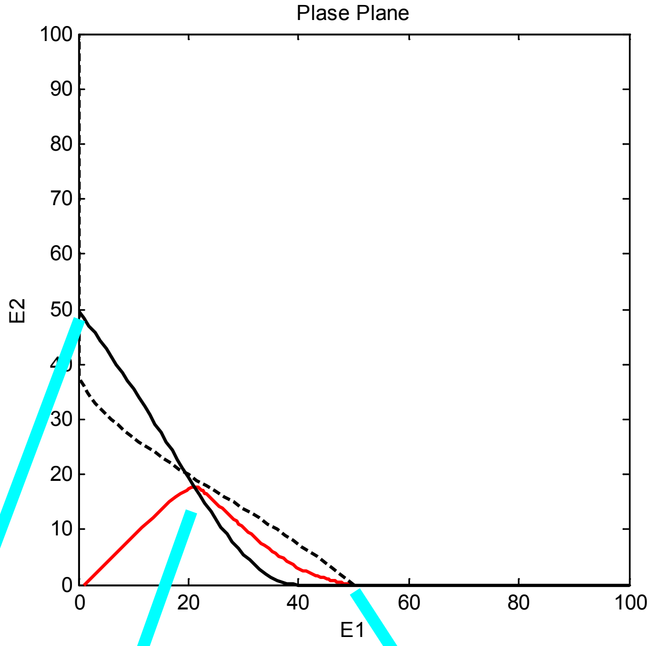


$$\dot{E}_1 = \frac{1}{\tau}\left(-E_1 + S(K_1 - 3E_2;100,2,120)\right)$$

$$\dot{E}_2 = \frac{1}{\tau}\left(-E_2 + S(K_2 - 3E_1;100,2,120)\right)$$

$$S(P;M,N,\sigma) = \frac{MP^N}{\sigma^N + P^N}$$

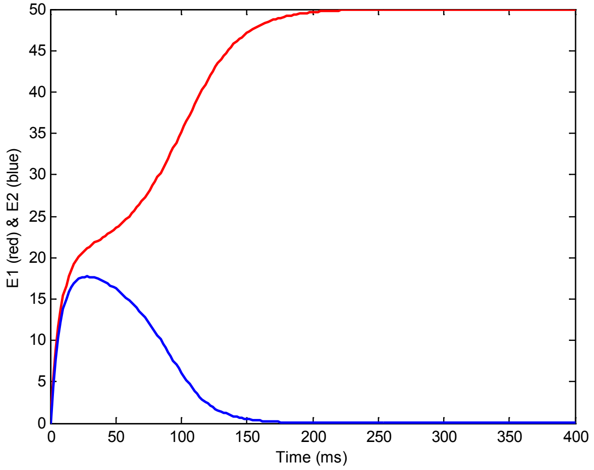
Wins the neuron receiving more stimulation (WTA)



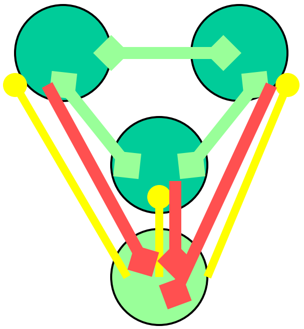
Saddle (unstable)

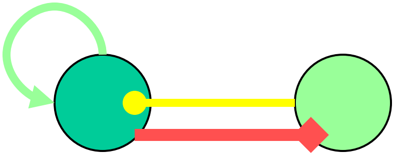
Stable node

Stable node



Oscillatory network (Wilson-Cowan)



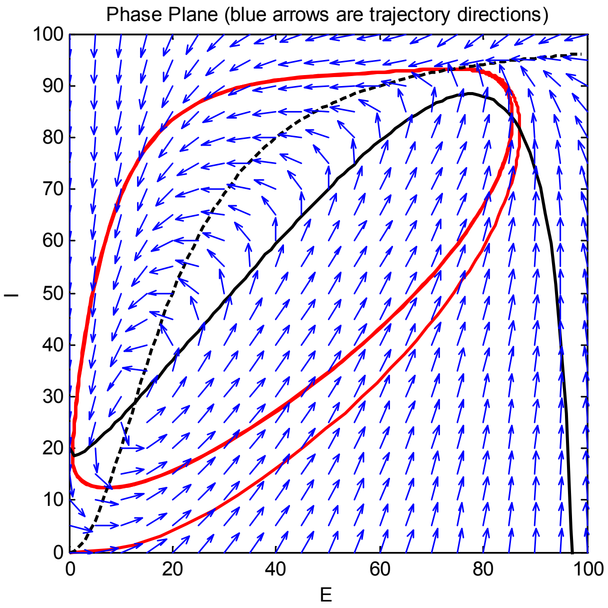


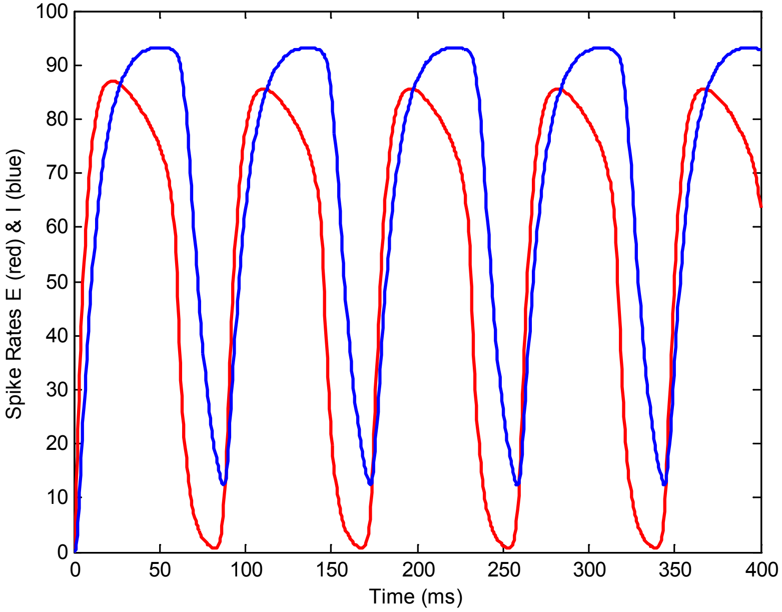
Reduced system doing the same when Excitatory neurons receive and send with same weights

$$\dot{E} = \frac{1}{5}\left(-E + S\left(1.6E - I + K;100,2,30\right)\right)$$

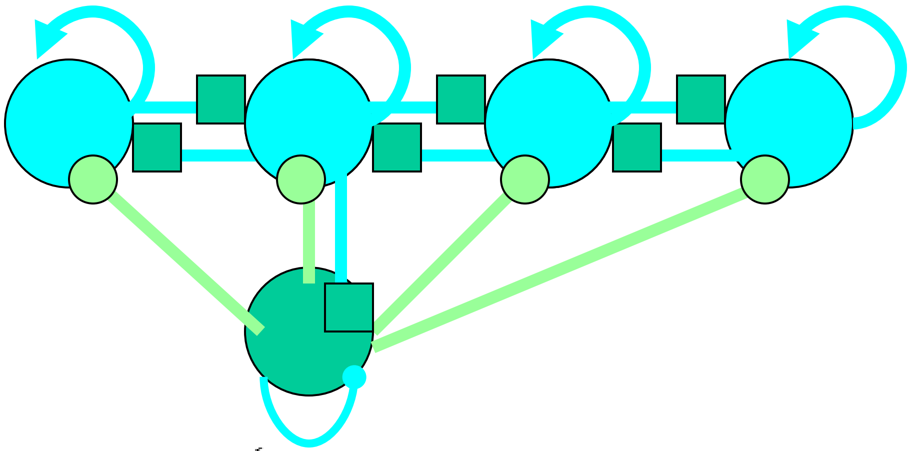
$$\dot{I} = \frac{1}{10}\left(-I + S(1.5E;100,2,30)\right)$$

K=20


Phase Plane (blue arrows are trajectory directions)
I
E
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100


Spike Rates E (red) & I (blue)
Time (ms)
0
10
20
30
40
50
60
70
80
90
100
0
50
100
150
200
250
300
350
400

Cortical dynamics of Wilson-Cowan type



$$\dot{E}(x)=\frac{1}{\tau}\left(-E(x)+S\left(\sum_{x} w_{EE}E(x)-\sum_{x} w_{IE}I(x)+P(x);100,2,\sigma_E\right)\right)$$

$$\dot{I}(x)=\frac{1}{\tau}\left(-I(x)+S\left(\sum_{x} w_{EI}E(x)-\sum_{x} w_{II}I(x)+Q(x);100,2,\sigma_I\right)\right)$$

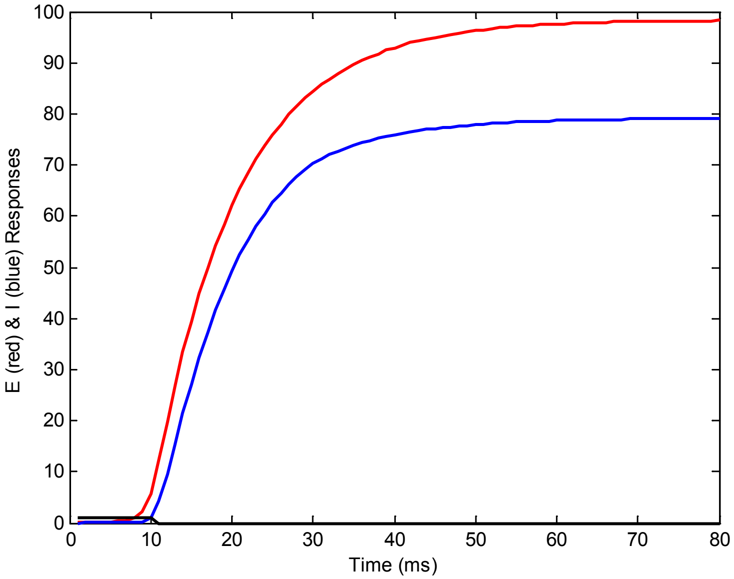
$$w_{ij}=b_{ij}\exp\left(-\left|x-x'\right|/\sigma_{ij}\right)$$

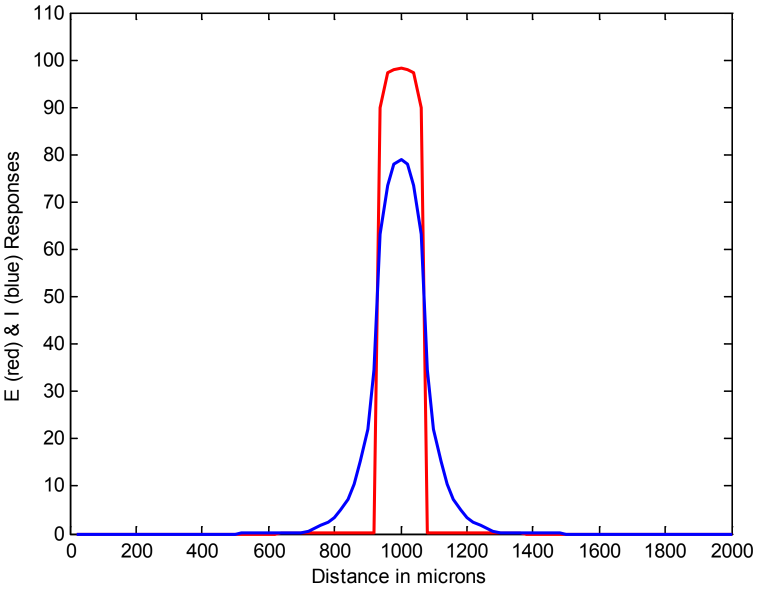
σ_{EE}=40 μm; σ_{IE}= σ_{IE} =60 μm; σ_{II} =30 μm;
σ_E =20; σ_I =40;
$b_{EI} = b_{IE}$

Memory mode of Wilson-Cowan model

$b_{EE} = 1.95$, $b_{EI} = 1.4$, $b_{II} = 2.2$

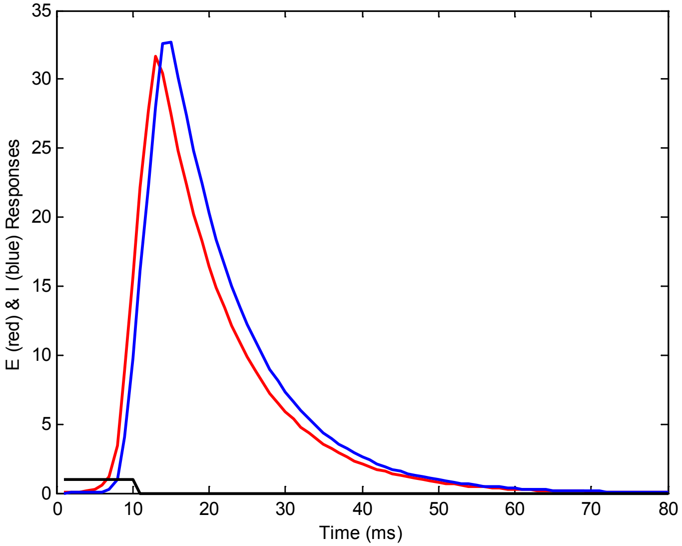
Stimulus of value *P*=1 for10ms and affecting 100μm

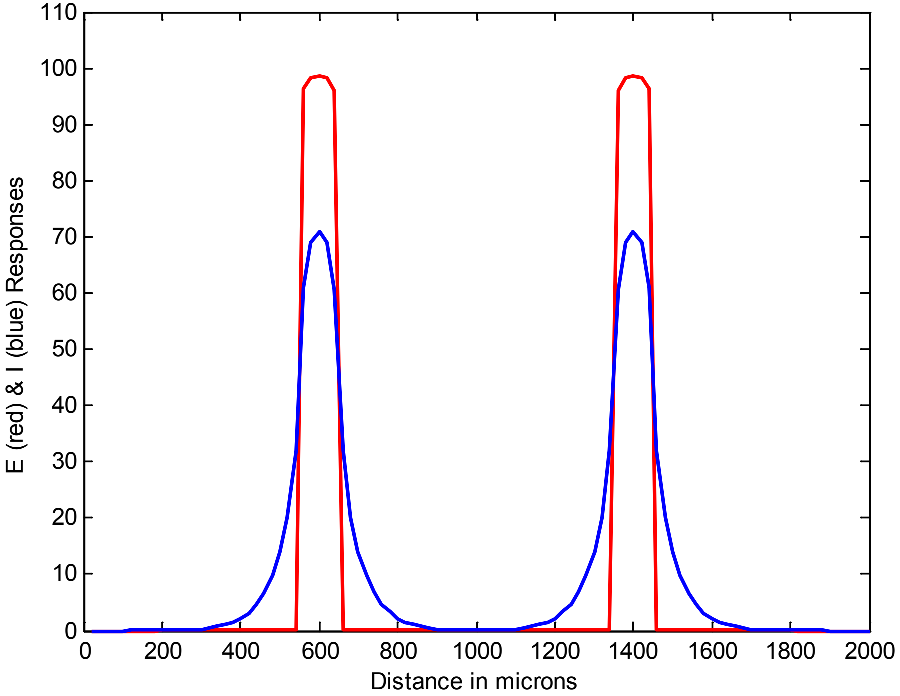




$b_{EE} = 1.95$, $b_{EI} = 1.4$, $b_{II} = 2.2$

Stimulus of value P=1 for 10ms and affecting 1000μm

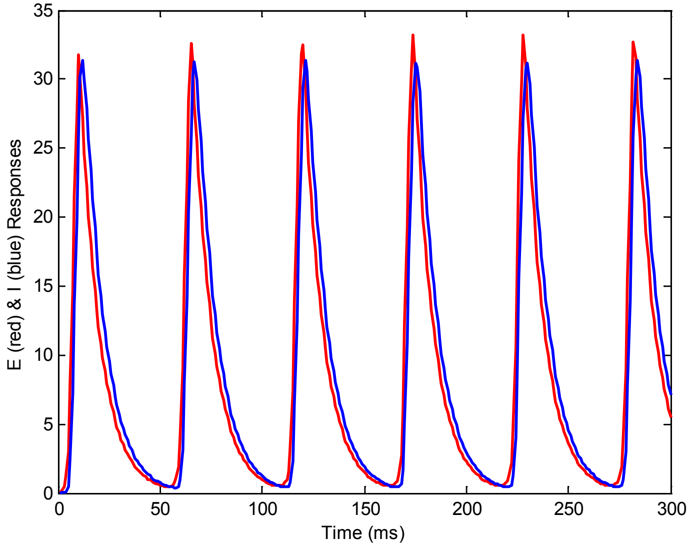


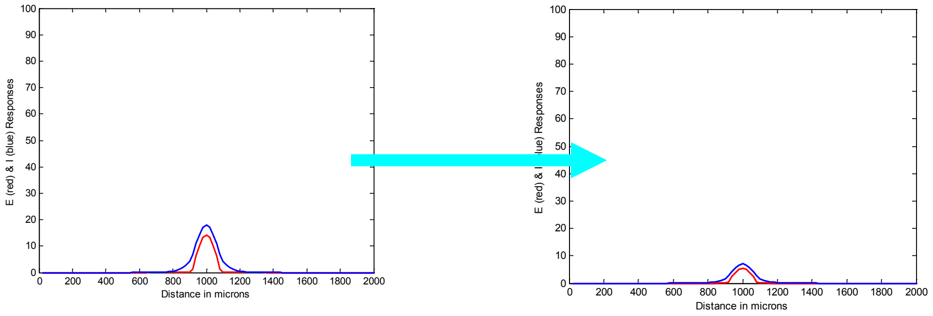


Oscillations in a localized region

$b_{EE} = 1.9, b_{EI} = 1.5, b_{II} = 1.5$

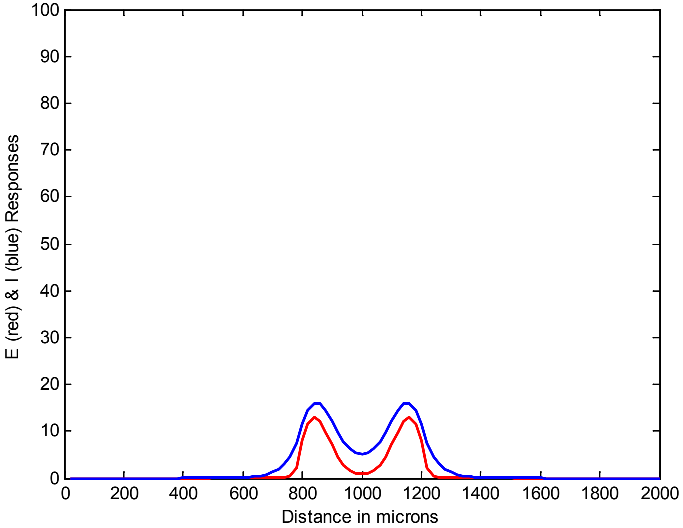
Intensity of stimulus of *P*=1 for 5ms and a width of 100μm





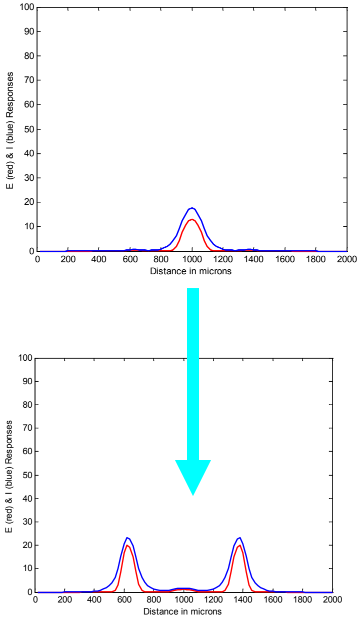
$b_{EE} = 1.9$, $b_{EI} = 1.5$, $b_{II} = 1.5$

Stimulus of intensity *P*=1
for 5ms and a width of 400μm



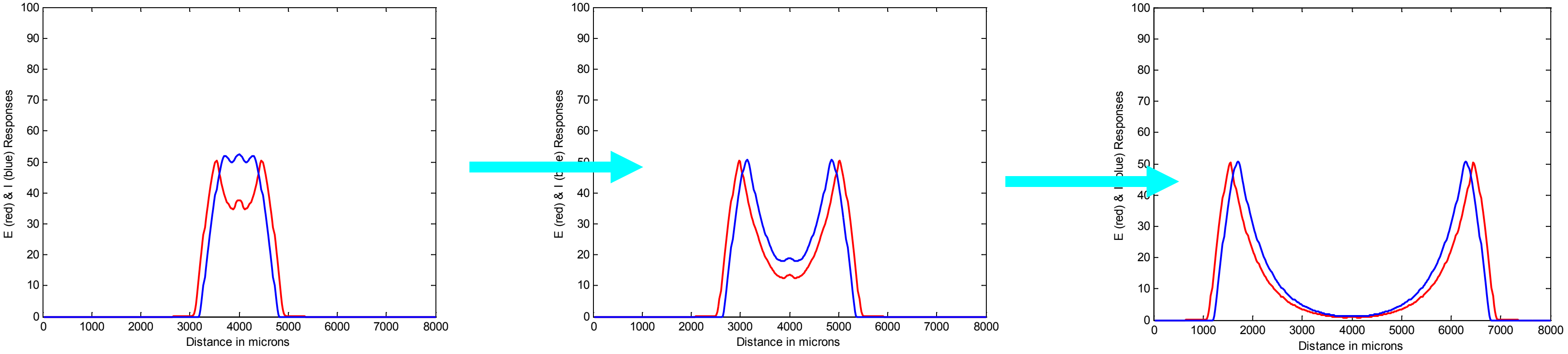
$b_{EE} = 1.9$, $b_{EI} = 1.5$, $b_{II} = 1.5$

Stimulus of intensity *P*=1
for 5ms and a width of 800μm



Travelling waves mode (epilepsia)

As in oscilaltory mode but with Q=-90 (inhibition gets reduced)



Visual hallucinations

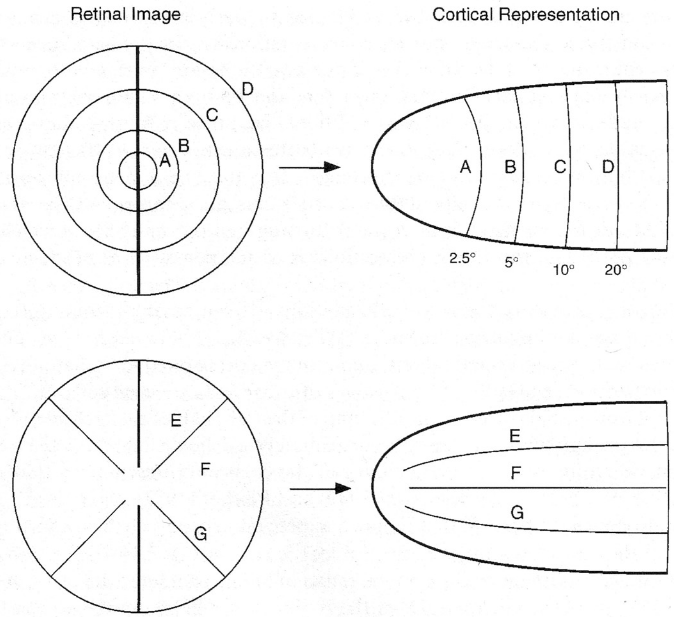
Extension of WC to 2D with more weight among excitatory neurons

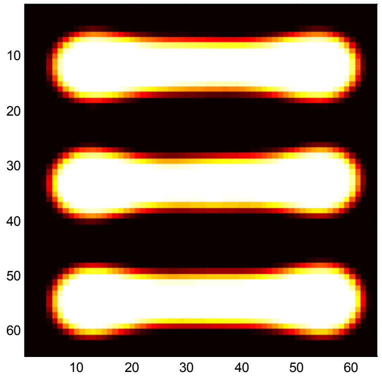
Remember that polar corrdinates in retina (R,θ) are trasnformed to cortical coordinates (1+R) and θ

This means that concentric circles in retina transform to vertical lines in cortex

and

Radial excitation in retina into horizontal lines in cortex




10
20
30
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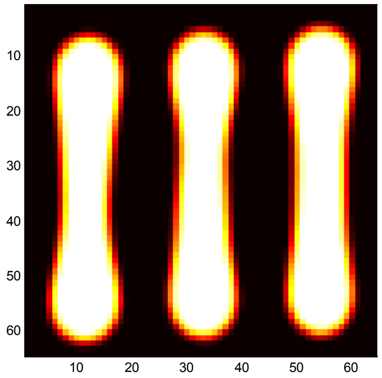
10
20
30
40
50
60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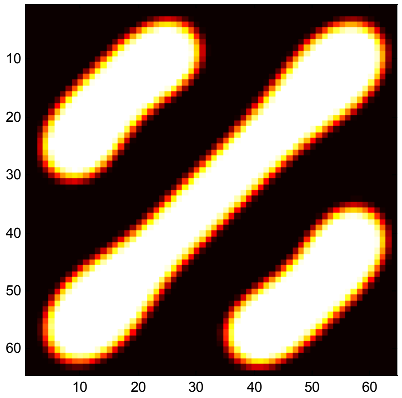
50
60


10
20
30
40
50
60
10
20
30
40
50
60

Learning in networks

Hebb´s rule

If neurons *i* and *j* fire simultaneously at frequencies E_i y E_j, respectively, the synaptic weight between them is

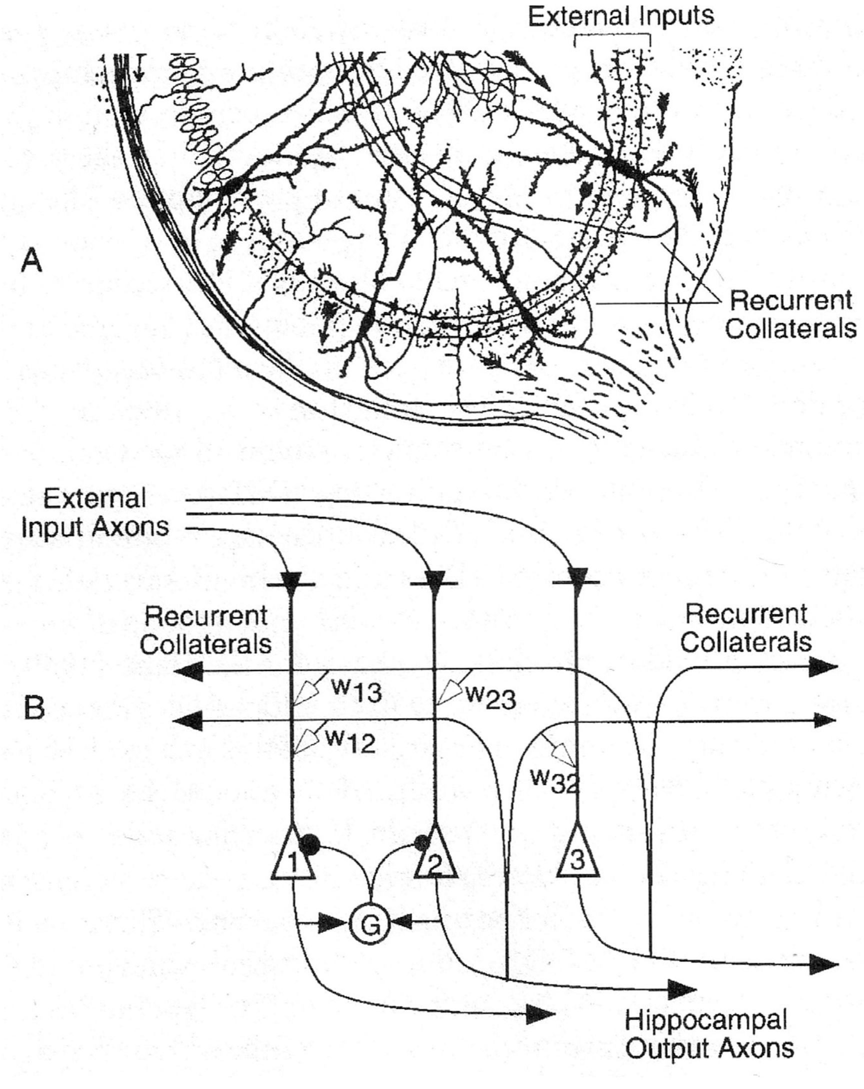
$$w_{ij} = kE_i E_j$$

A more practical rule is

$$w_{ij} = kH\left(E_i - 0.5M\right)H\left(E_j - 0.5M\right); \quad H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \leq 0 \end{cases}$$

A synapse goes from value 0 to value *k* only when neurons *i* and *j* fire at more than half their maximum firing frequency *M*

Modelización de la red CA3 del hipocampo



256 excitatory neurons (16x16 matrix):

Each of them has a Webb synapse with the other 255

1 inbibitory neuron gets excited by excitatory neurons and inhibits them

Model

$$\dot{E}_i = \frac{1}{10}\left(-E_i + S\left(\sum_{j=1}^{255} w_{ij} E_j - 0.1I; 100, 2, 10\right)\right)$$

$$\dot{I} = \frac{1}{10}\left(-I + 0.076\sum_{i=1}^{256} E_i\right)$$

$$w_{ij} = kH\left(E_i - 0.5M\right)H\left(E_j - 0.5M\right); \quad H(x) = \begin{cases} 1 & x > 0 \\ 0 & x \le 0 \end{cases}$$

1. **Training phase**

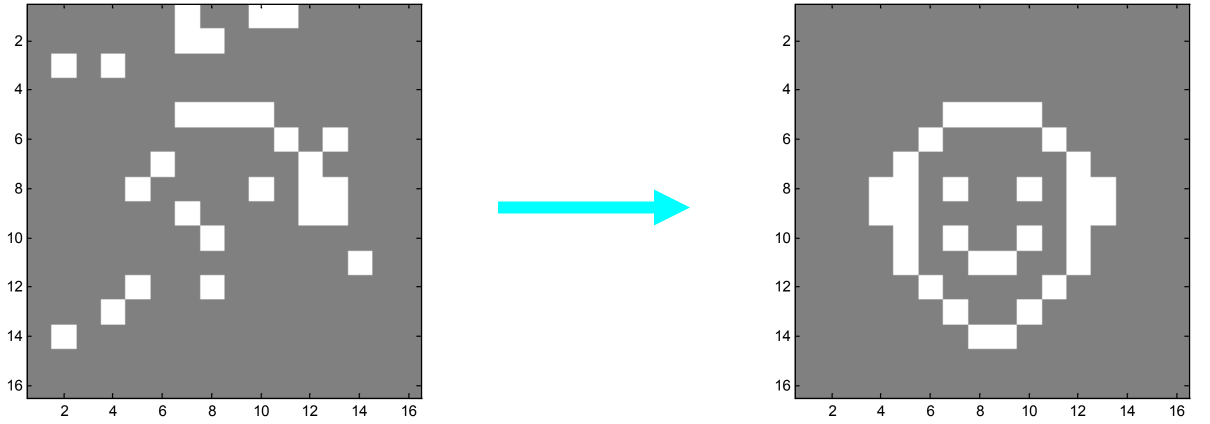
   4 patterns get chosen as stimuli (32 pixel images en in a 16x16 matrix)

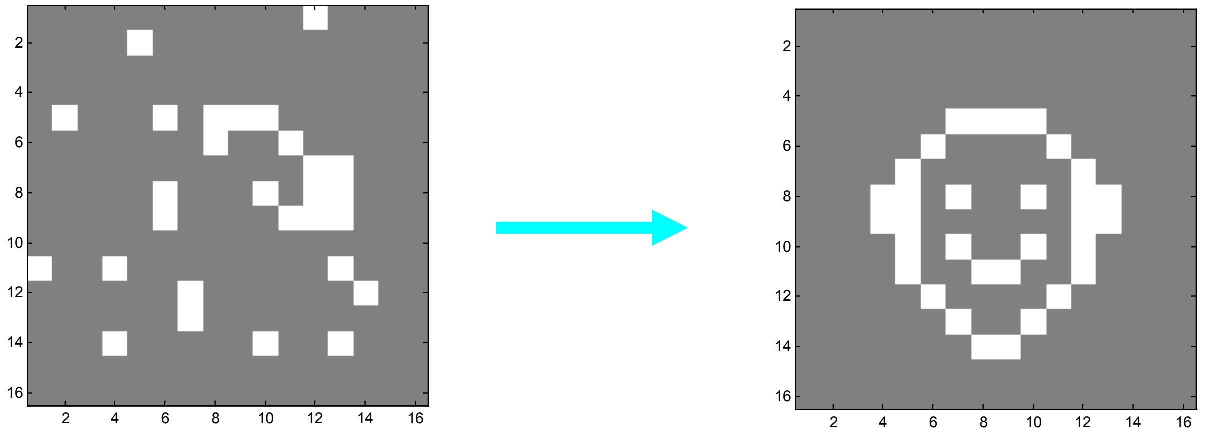
   Each pattern excites 32 excitatory neurons . The rest of neurons are not excited initially.

   We run the model and synapses change accordingly

2. **Recognition phase**

   Show to the network an image made up of 1/3 of the pixels of one of the trained images and 2/3 of random pixels


2
4
6
8
10
12
14
16
2 4 6 8 10 12 14 16
2
4
6
8
10
12
14
16
2 4 6 8 10 12 14 16


2
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2 4 6 8 10 12 14 16
2
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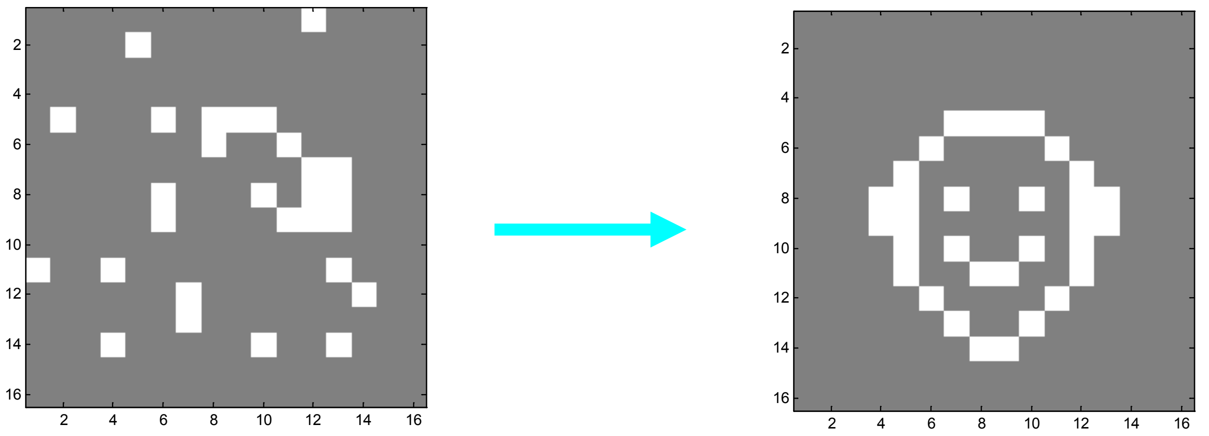
2 4 6 8 10 12 14 16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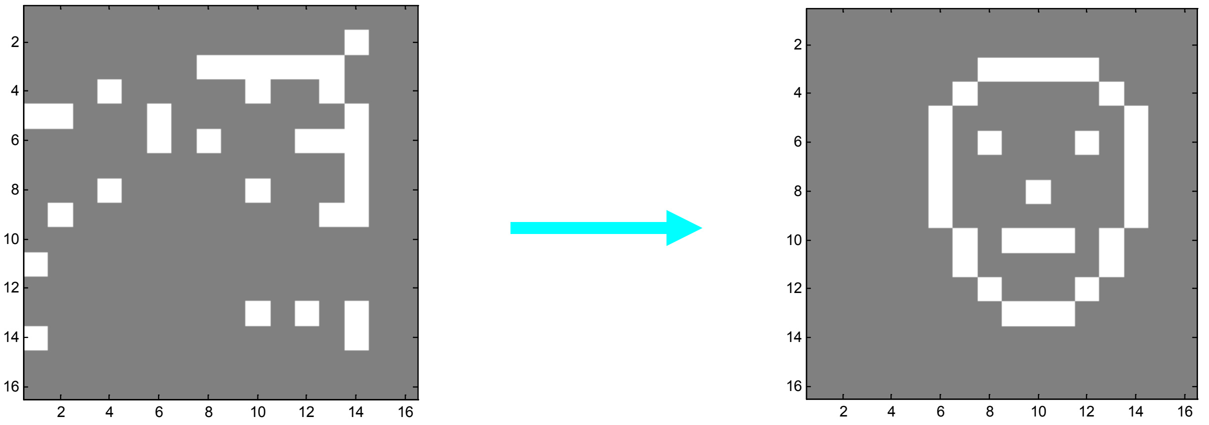
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