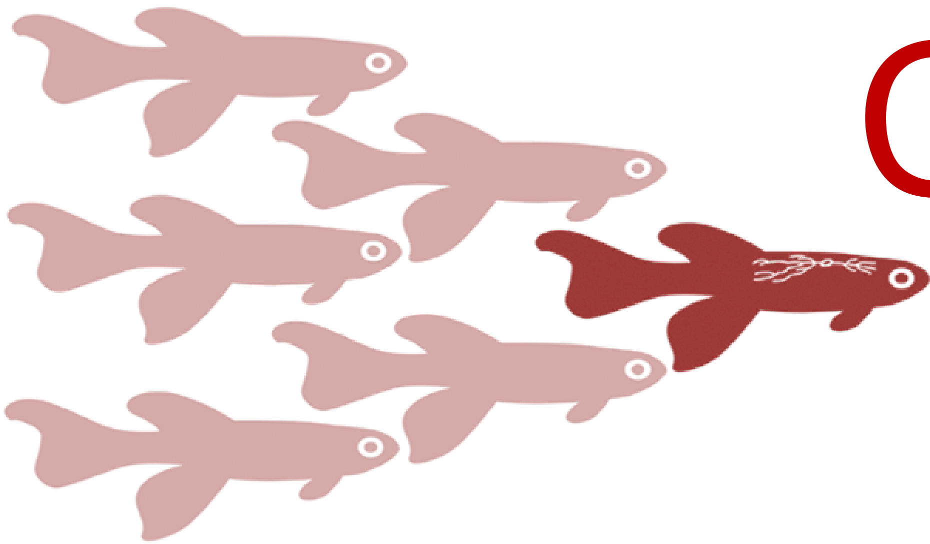
Optimal decision-making in

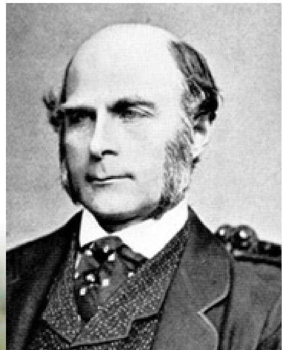


Collective Animal behavior

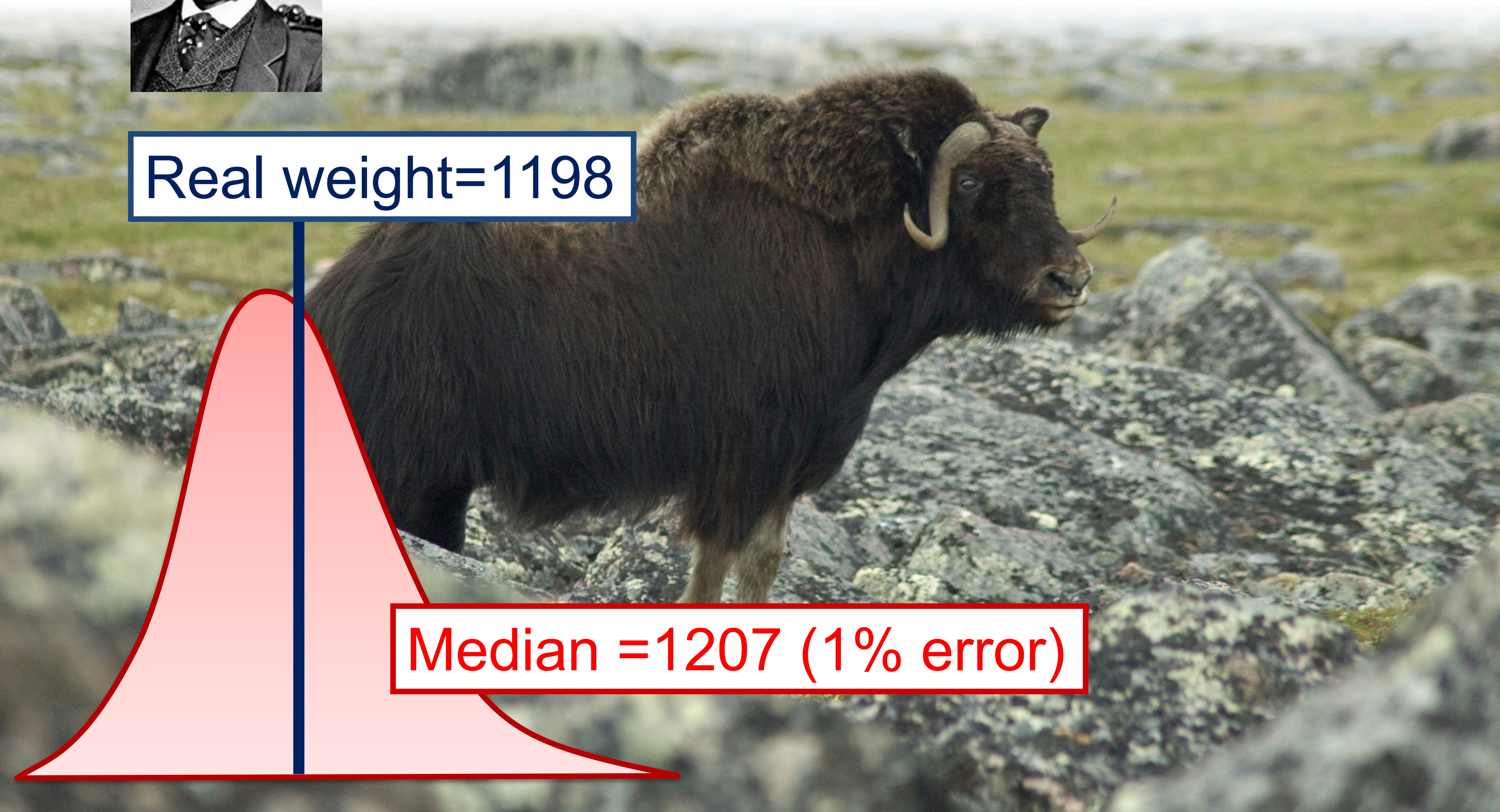
Gonzalo G. de Polavieja
Instituto Cajal, CSIC, Spain

1. Galton´s ‘wisdom of the crowd’
2. Condorcet decision by majority
3. Information cascades
4. A unified approach for animal decisions in groups

What is the weight of this ox?



What is the weight of this ox?



Condorcet majority case

1785.Essay on the applicability of probabilistic analysis to majority decisions

Task

n individuals choose between 2 options.
Each of them has a probability *p* of choosing the correct option.

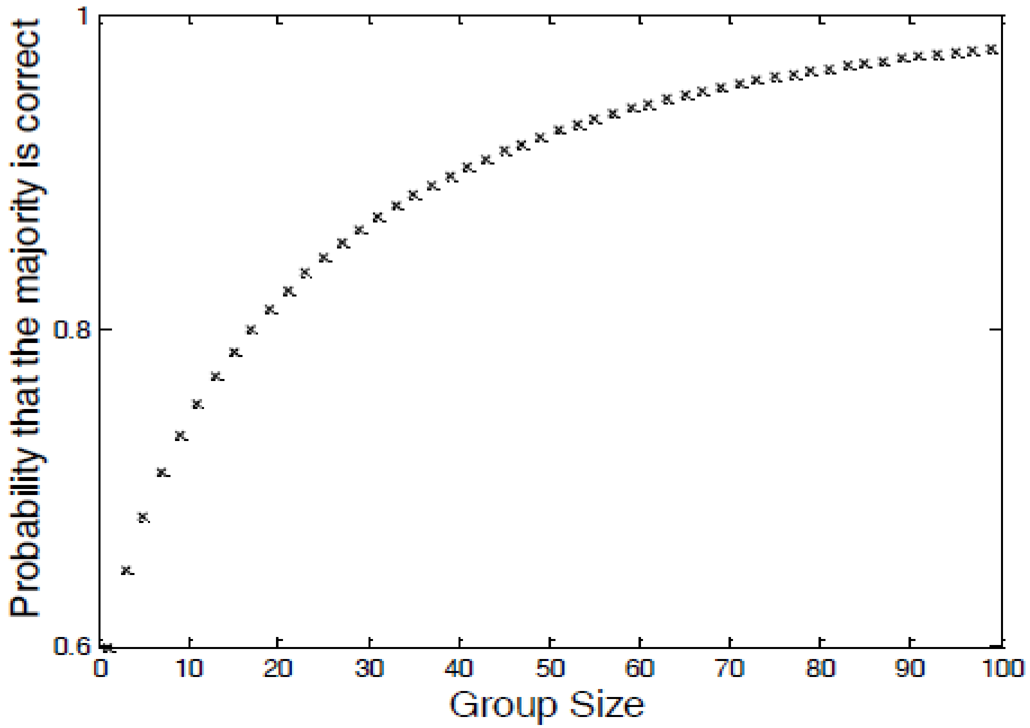
Probability of *k* correct choices in *n* $\frac{n!}{k!(n-k)}p^k p^{n-k}$

The probability that the majority makes the correct choice

$$\sum_{k=\frac{n}{2}+1}^{n} \frac{n!}{k!(n-k)} p^k p^{n-k}$$

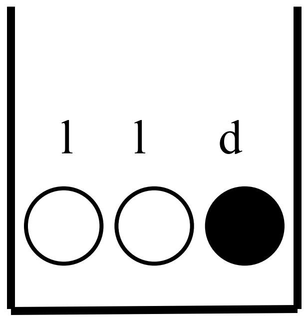
Plot the case with p=0.6

$$\sum_{k=\frac{n}{2}+1}^{n} \frac{n!}{k!(n-k)} p^{k} p^{n-k}$$



Information cascades

$$P(L) = 1/2$$

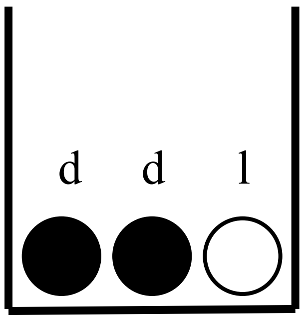


Urn A

$$P(l \mid L) = 2/3$$

$$P(d \mid L) = 1/3$$

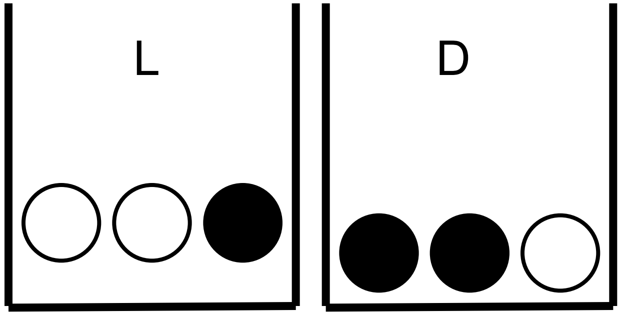
$$P(D) = 1/2$$



Urn B

$$P(l \mid D) = 1/3$$

$$P(d \mid D) = 2/3$$



Ideal estimation of urn (L or D) by an individual when taking a ball from an unknown urn

$$P(L \mid l) = \frac{P(l \mid L)P(L)}{P(l)} = \frac{\frac{2}{3}\frac{1}{2}}{1/2} = 2/3 \qquad P(D \mid l) = 1/3$$

$$P(D \mid d) = 2/3 \qquad P(D \mid l) = 1/3$$

The individual should estimate urn A (or B) when it takes a light (or dark) ball

Ideal estimation of urn (L or D) by the next individual

The second has two results: the ball it takes and the urn individual 1 said it was.

Knowing what individual 1 said, this individual knows the color of the ball he/she took.

So individual 2 recieves effectively has two independent draws from the unknown urn

$$P(L \mid s) = \frac{P(s \mid L)P(L)}{P(s)} = \frac{P(s \mid L)P(L)}{P(s \mid A)P(A) + P(s \mid B)P(B)}$$

$$s \quad \text{is} \quad n_1 \text{ of } l \,\&\, n_2 \text{ of } d$$

$$P(L \mid s) = \frac{\frac{2}{3}^{n_1} \frac{1}{3}^{n_2}}{\frac{2}{3}^{n_1} \frac{1}{3}^{n_2} + \frac{1}{3}^{n_1} \frac{2}{3}^{n_2}}$$

$$n_1 = 2; n_2 = 0$$ (the two got light balls)

$$P(L \mid s) = 4/5$$ Individual 2 should say it is urn L

$$n_1 = 0; n_2 = 2$$ (the two got dark balls)

$$P(L \mid s) = 1/5$$ Individual 2 should say it is urn D

$$n_1 = 1; n_2 = 1$$ (the two got different colors)

$$P(L \mid s) = 1/2$$ Individual 2 should choose at random

In summary, if individual coindices with individual 1 should choose that color
And if they not both optins have same probability (here we assume he/she follows private signal)

Ideal estimation of urn (L or D) by the third individual

Individual 3 knows that individuals 1 and 2 chose using their private signals

If 1 and 2 coincide, 3 should choose the same even if private signal contradicts
If 1 and 2 do not coincide, its private signal decides

If 1 and 2 coincide, person 4 receives no information from 3 so it needs to decide as if he/she was person 3, thus choosing the same color than 1 &2. A cascade starts…

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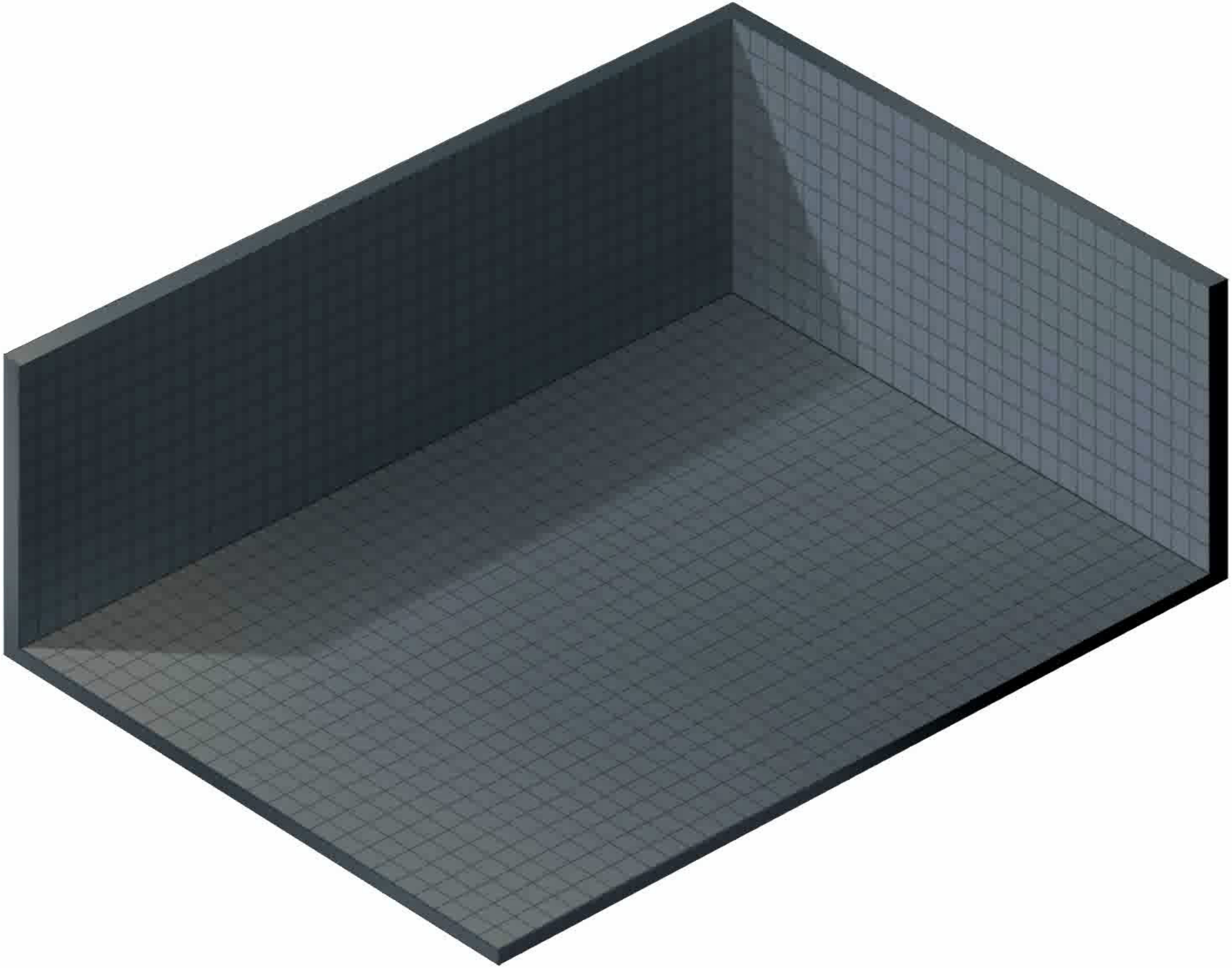

Models are empirical. Can we propose more general models?

Empirical model

$$P_L(t) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{(L-R)t}{(L+R)\sqrt{4D_2 t}}\right)\right]$$

Empirical model

$$p = 1 \,/\, (1 + \exp(-0.24 - b_1 X_1 + b_2 X_2))$$

Empirical model

$$a + (m - a) \times \frac{(L(t) - L(t-T))^k}{U(t)^k + (L(t) - L(t-T))^k + (R(t) - R(t-T))^k}$$

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A single model,
derived from a basic property

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A single model,
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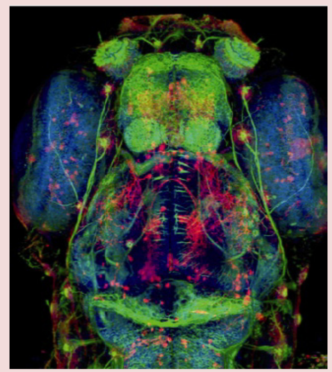
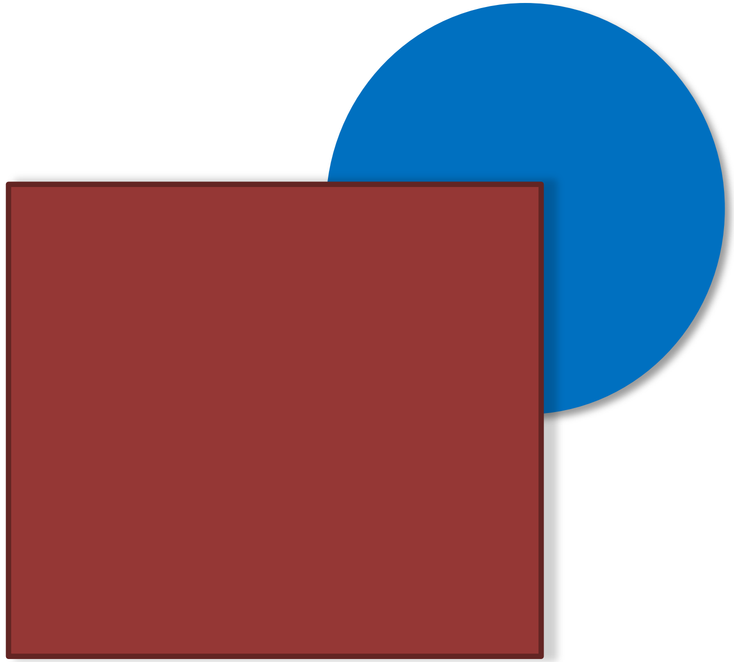


The basic property: brains are for estimation

What´s behind the square?



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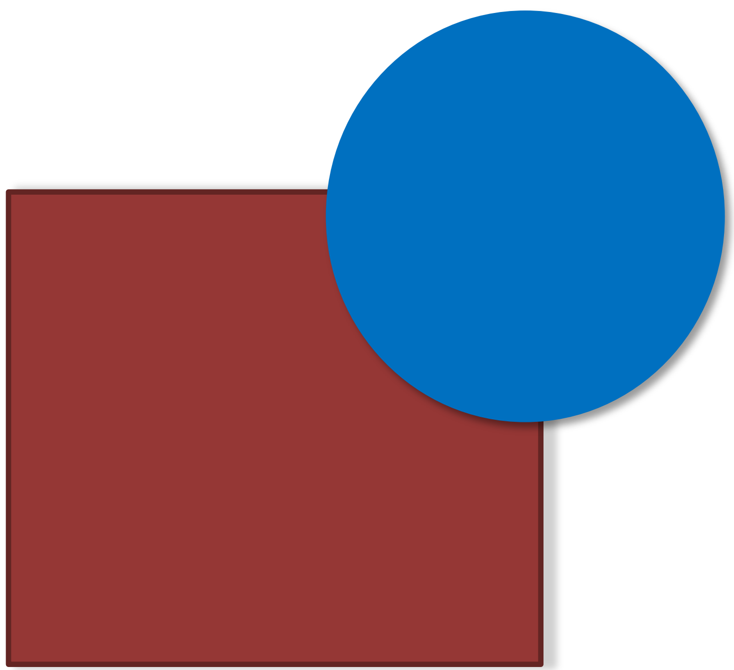

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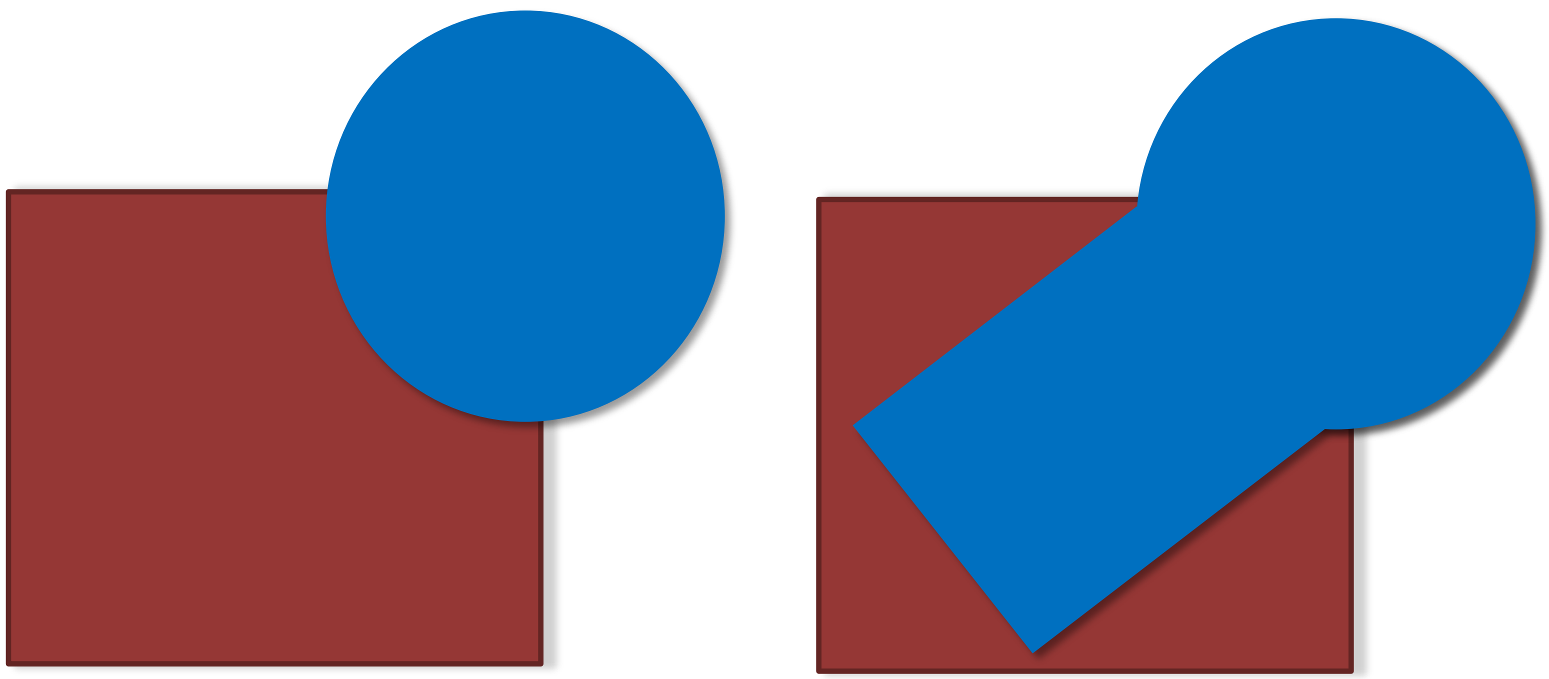


The basic property: brains are for estimation

What´s behind the vegetation?

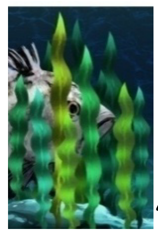
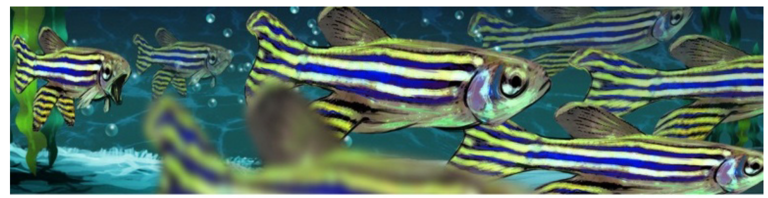
The basic property: brains are for estimation

What´s behind the vegetation?

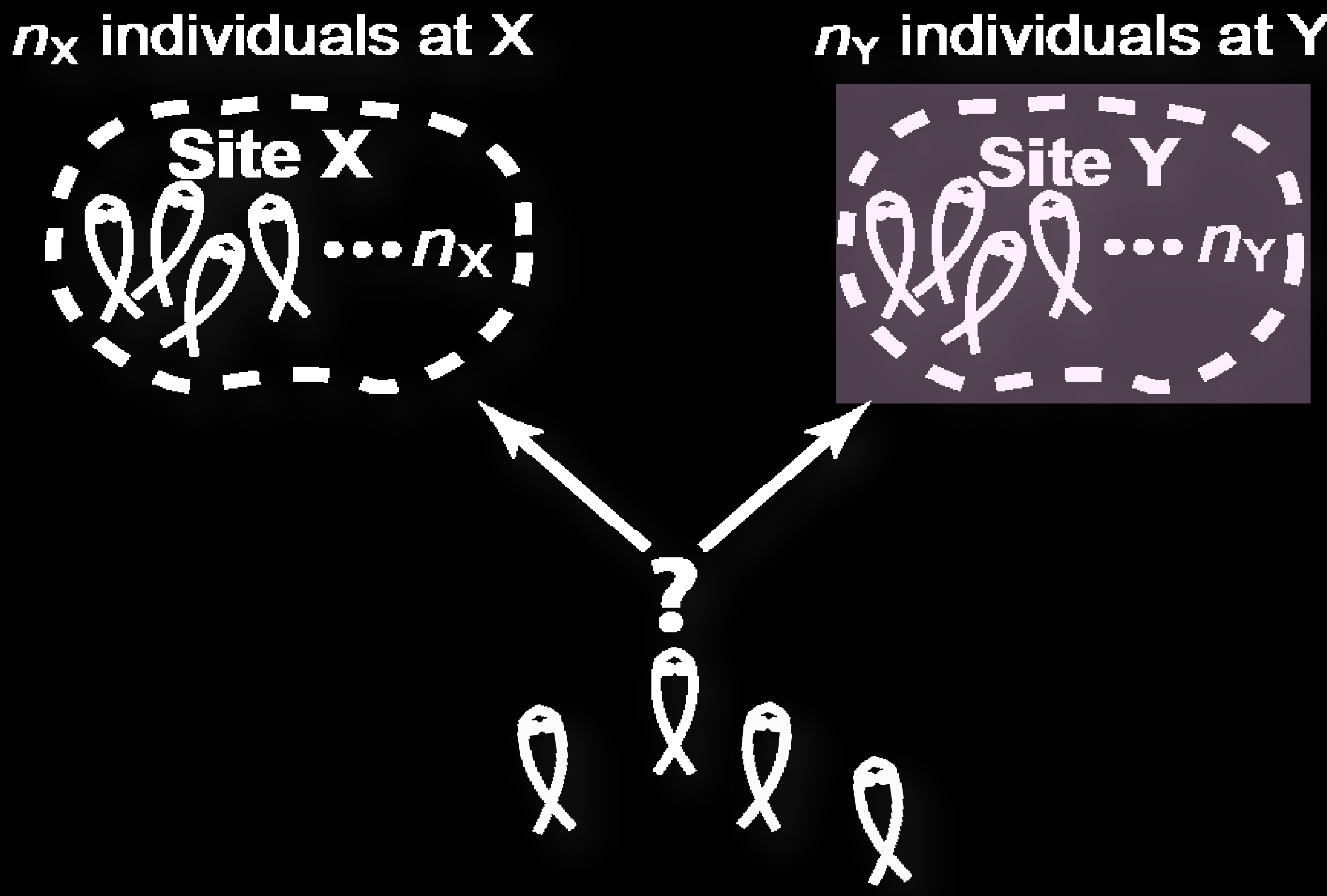
Concrete hypothesis

Rules of group behavior can be obtained from individuals making estimations about the world using

(a) non-social and **(b)** social data

$P($  $|$ ,  $)$

I will consider two-choice set-ups

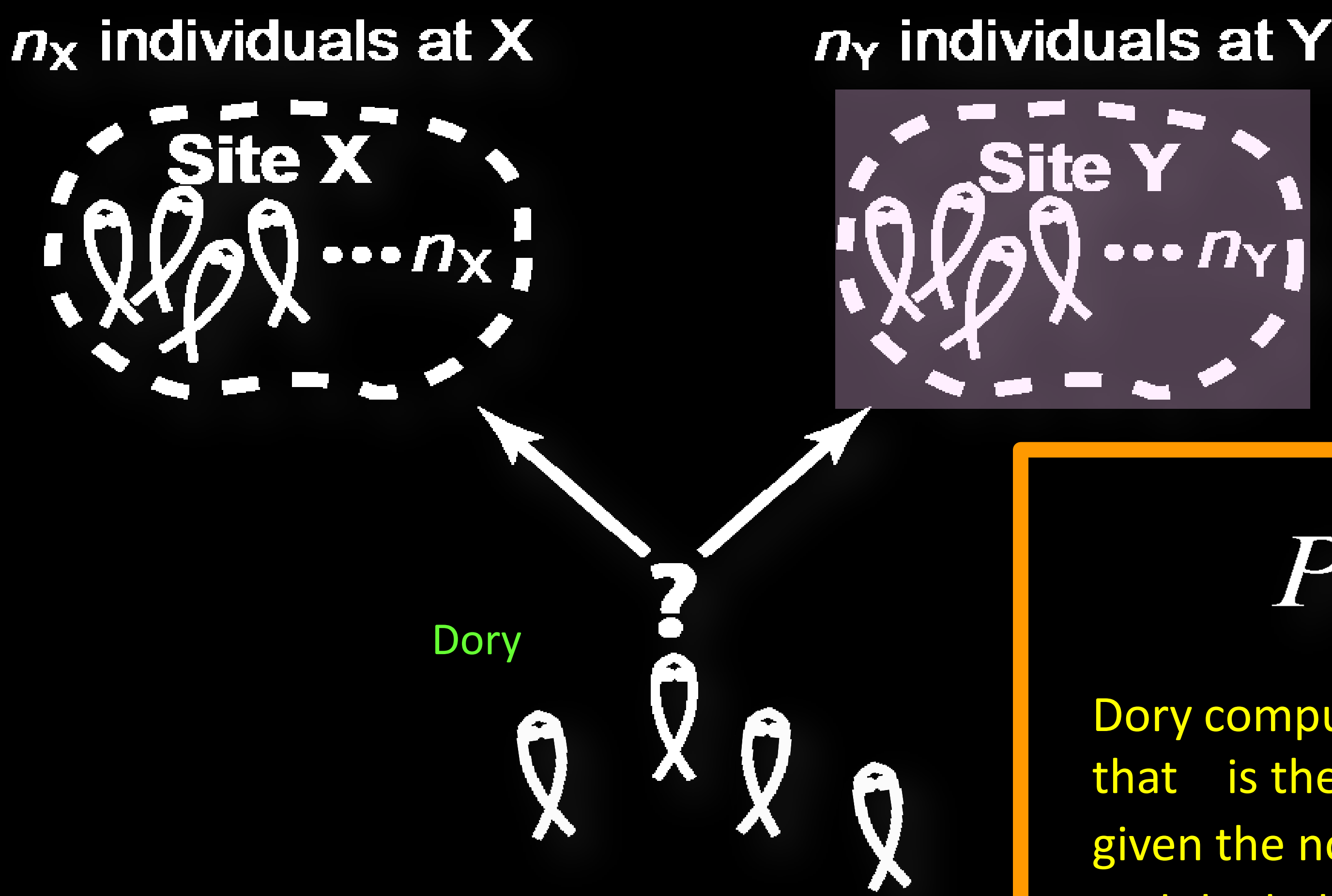


The cognitive model is derived in two steps:

Step 1. Each animal estimates which choice is best to take

Step 2. Each animal decides according to this estimation

Step 1. The brain estimates which option is best



$$P\left(Y \middle| s, B\right)$$

Dory computes the probability that is the best option given the nons-social stimulus and the behavior of other fish

Step 1. The brain estimates which option is best

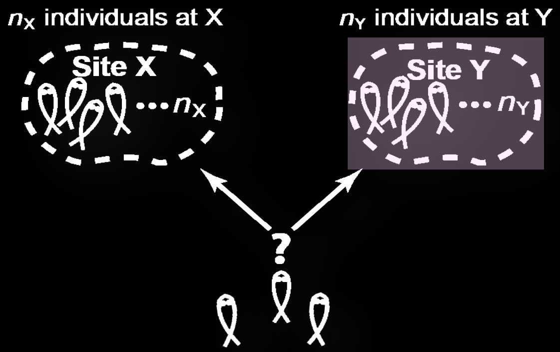
Bayes rule

$$P(Y|s,B) = \frac{P(B|Y,s)P(Y|s)}{P(B|X,s)P(X|s) + P(B|Y,s)P(Y|s)}$$

Step 1.

Bayes rule

$$P(Y|s,B)=\frac{P(B|Y,s)P(Y|s)}{P(B|X,s)P(X|s)+P(B|Y,s)P(Y|s)}$$

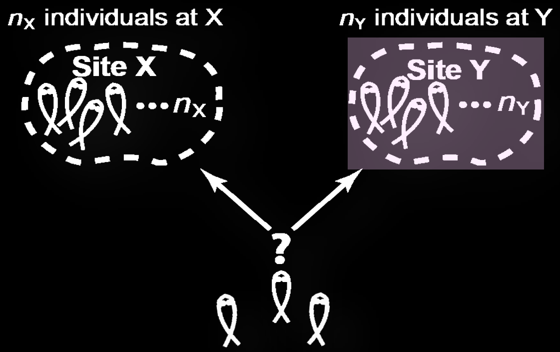


Dory does not use correlations among behaviors

$$P(B|Y,s)\approx \Omega P(B=X|Y,s)^{n_X} P(B=Y|Y,s)^{n_Y} P(B=U|Y,s)^{N-(n_X+n_Y)}$$

Step 1.

Bayes rule



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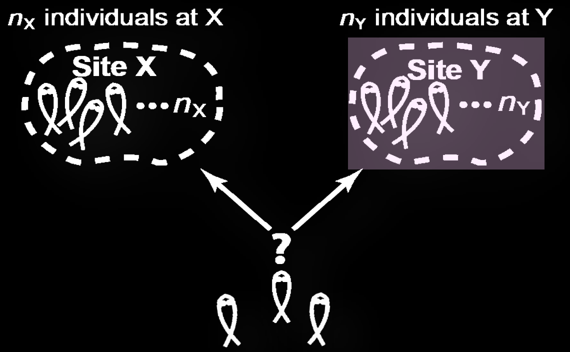
$$P(Y|s,B) = \left(1 + a_0 a_1^{n_X} a_2^{-n_Y} a_3^{N-(n_X+n_Y)}\right)^{-1}$$

Step 1.

Bayes rule



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$$P(Y|s,B) = \left(1 + a_0 a_1^{n_X} a_2^{-n_Y} a_3^{N-(n_X+n_Y)}\right)^{-1}$$

$a_0 = P(X \mid s) / P(Y \mid s)$ Reliability of non-social sensory data

$a_1 = P(B=X \mid X,s) / P(B=X \mid Y,s)$ Reliability of individuals going to *X*

$a_2 = P(B=Y \mid Y,s) / P(B=Y \mid X,s)$ Reliability of individuals going to *Y*

$a_3 = P(B=U \mid X,s) / P(B=U \mid Y,s)$ Reliability of undecided individuals

Step 2. Each individual decides according to its estimation

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Deterministic decision rule

Go to *Y* if

$$P(Y|s,B) > P(X|s,B)$$

Otherwise go to *X*

Step 2. Each individual decides according to its estimation

Deterministic decision rule

Go to *Y* if $$P(Y|s,B) > P(X|s,B)$$

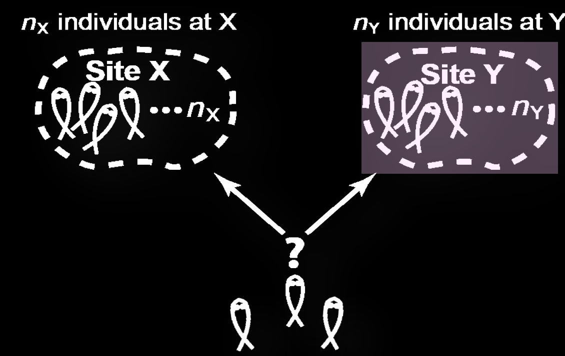
Otherwise go to *X*

Probabilistic matching

$$P_Y = P(Y|s,B)$$

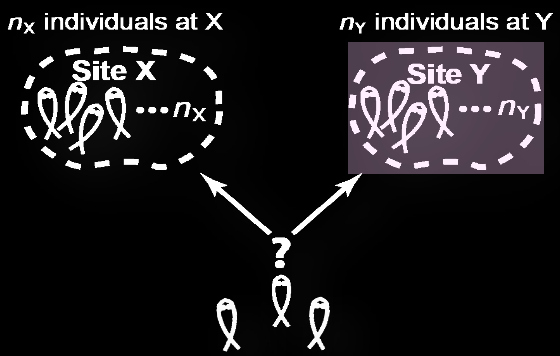
$$P_X = 1 - P_Y$$

Using probability matching



$$P_Y = P(Y|s,B) = \left(1 + a_0 a_1^{n_X} a_2^{-n_Y} a_3^{N-(n_X+n_Y)}\right)^{-1}$$

OK, so we got…



$$P_Y = P(Y|s,B) = \left(1 + a_0 a_1^{n_X} a_2^{-n_Y} a_3^{N-(n_X+n_Y)}\right)^{-1}$$

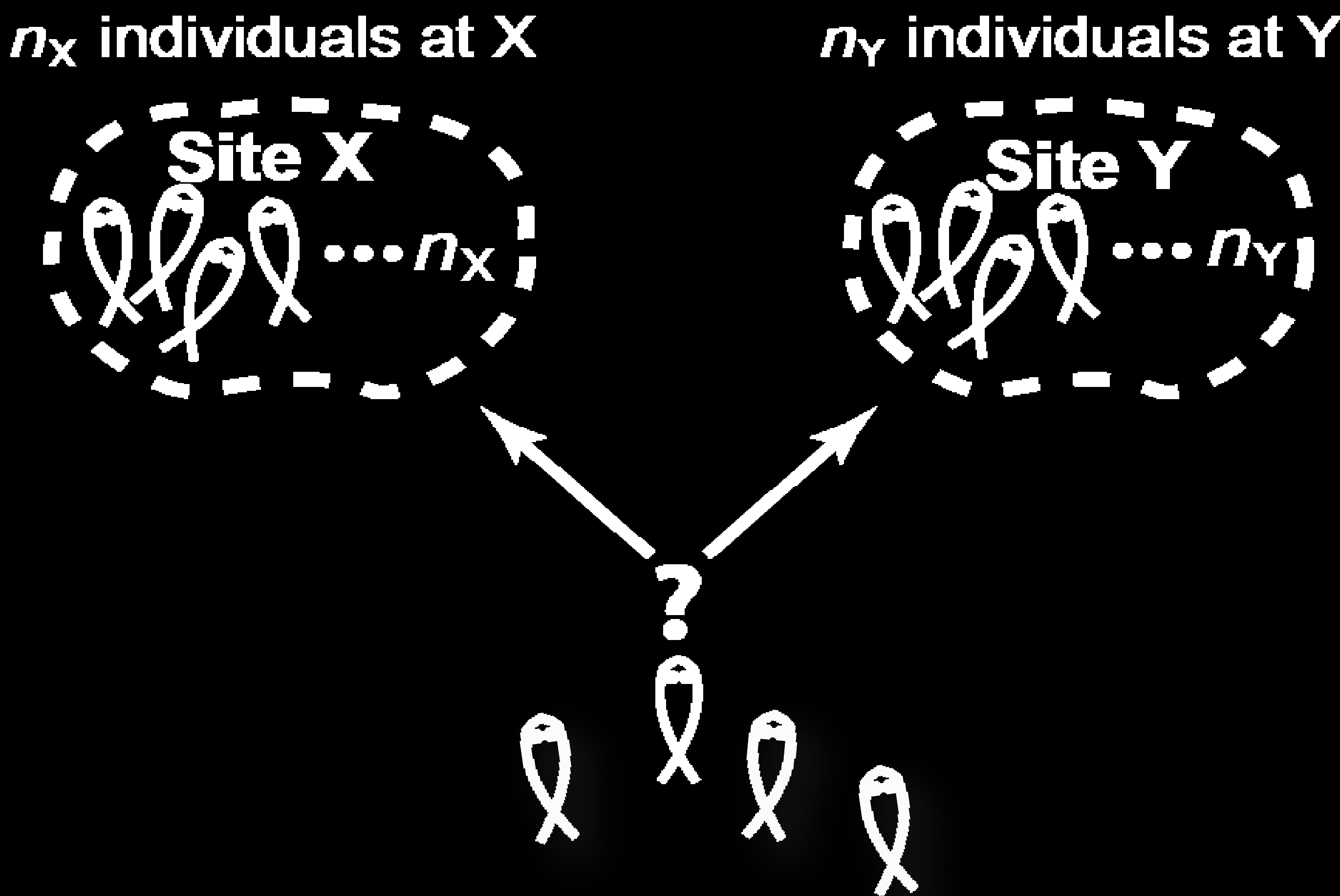
$$a_0 = P(X \mid s) / P(Y \mid s)$$

$$a_1 = P(B = X \mid X, s) / P(B = X \mid Y, s)$$

$$a_2 = P(B = Y \mid Y, s) / P(B = Y \mid X, s)$$

$$a_3 = P(B = U \mid X, s) / P(B = U \mid Y, s)$$

Now let´s focus on the simpler case of choice between two identical locations



In this case:

$$a_0 = a_3 = 1 \quad \& \quad a_2 = a_1$$

$$a_0 = P(X \mid s) / P(Y \mid s)$$
$$a_1 = P(B = X \mid X, s) / P(B = X \mid Y, s)$$
$$a_2 = P(B = Y \mid Y, s) / P(B = Y \mid X, s)$$
$$a_3 = P(B = U \mid X, s) / P(B = U \mid Y, s)$$

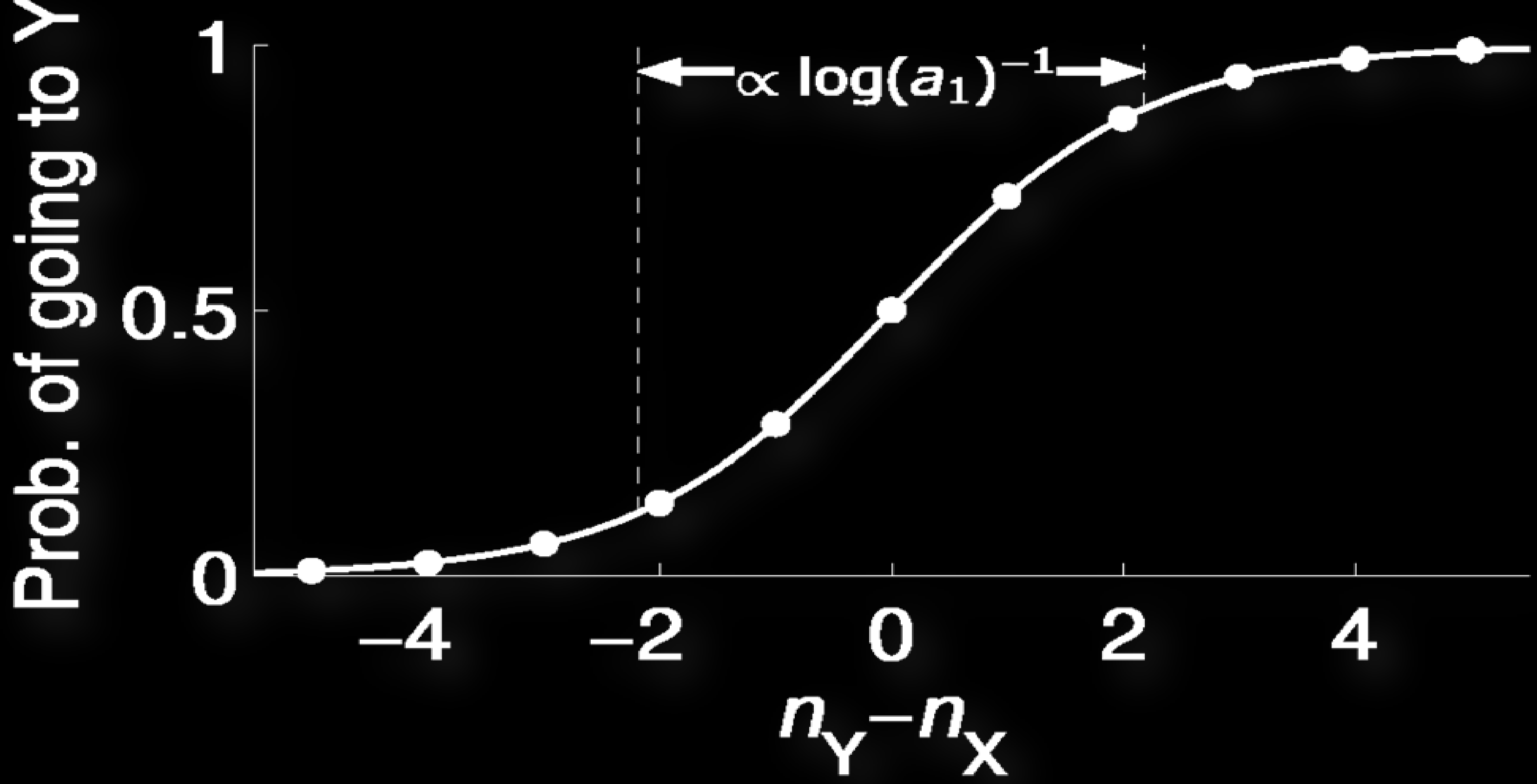
So the general expression

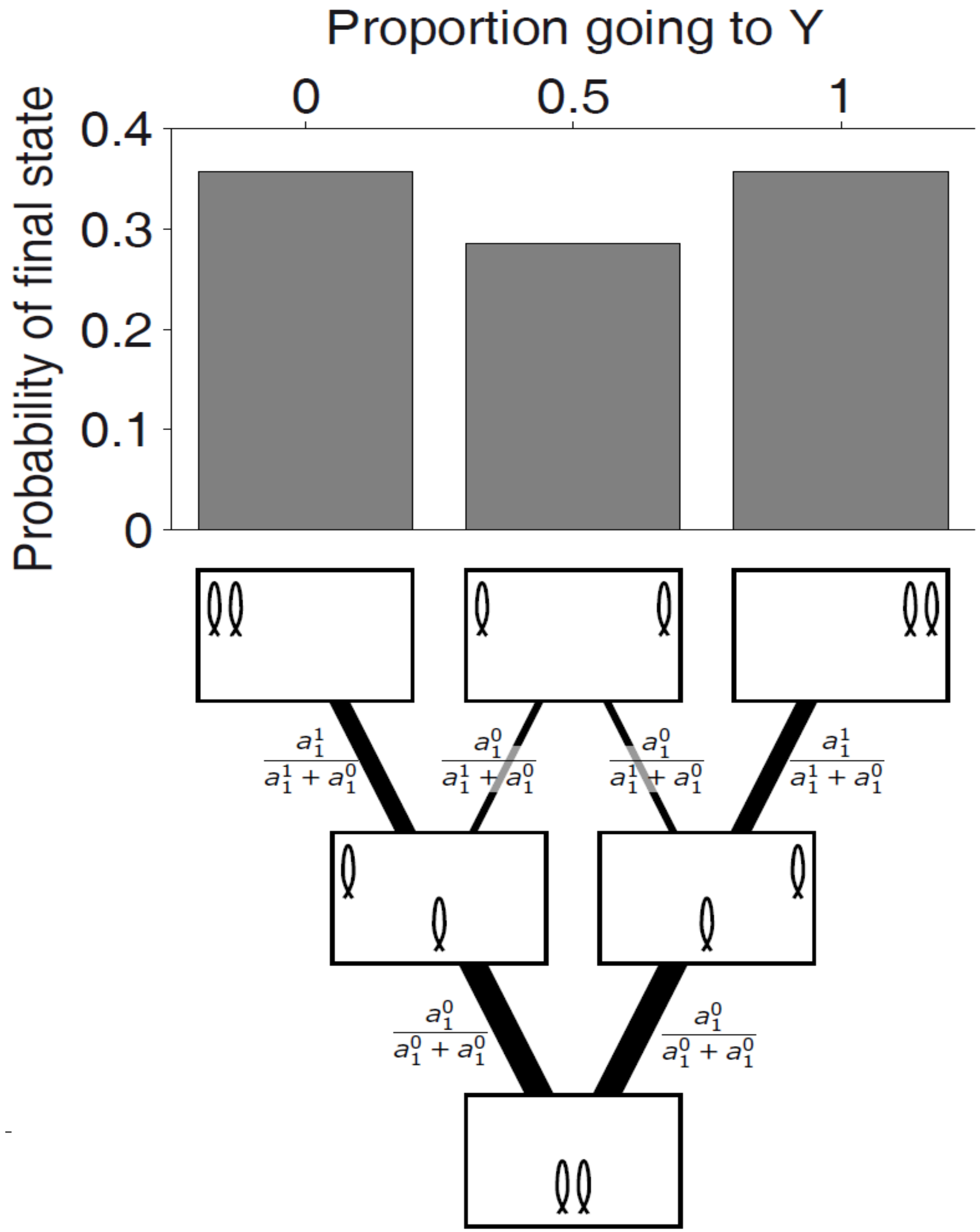
$$P_Y = P(Y \mid s, B) = \left(1 + a_0 a_1^{n_X} a_2^{-n_Y} a_3^{N-(n_X+n_Y)}\right)^{-1}$$

reduces to

$$P_Y = \left(1 + a_1^{\Delta n}\right)^{-1} = \frac{a_1^{n_Y}}{a_1^{n_x} + a_1^{n_Y}}$$

Let´s plot $P_Y = \left(1 + a_1^{\Delta n}\right)^{-1}$ to see what we got





Proportion going to Y
0
0.5
1
Probability of final state
0.4
0.3
0.2
0.1
0
$\frac{a_1^1}{a_1^1 + a_1^0}$
$\frac{a_1^0}{a_1^1 + a_1^0}$
$\frac{a_1^0}{a_1^1 + a_1^0}$
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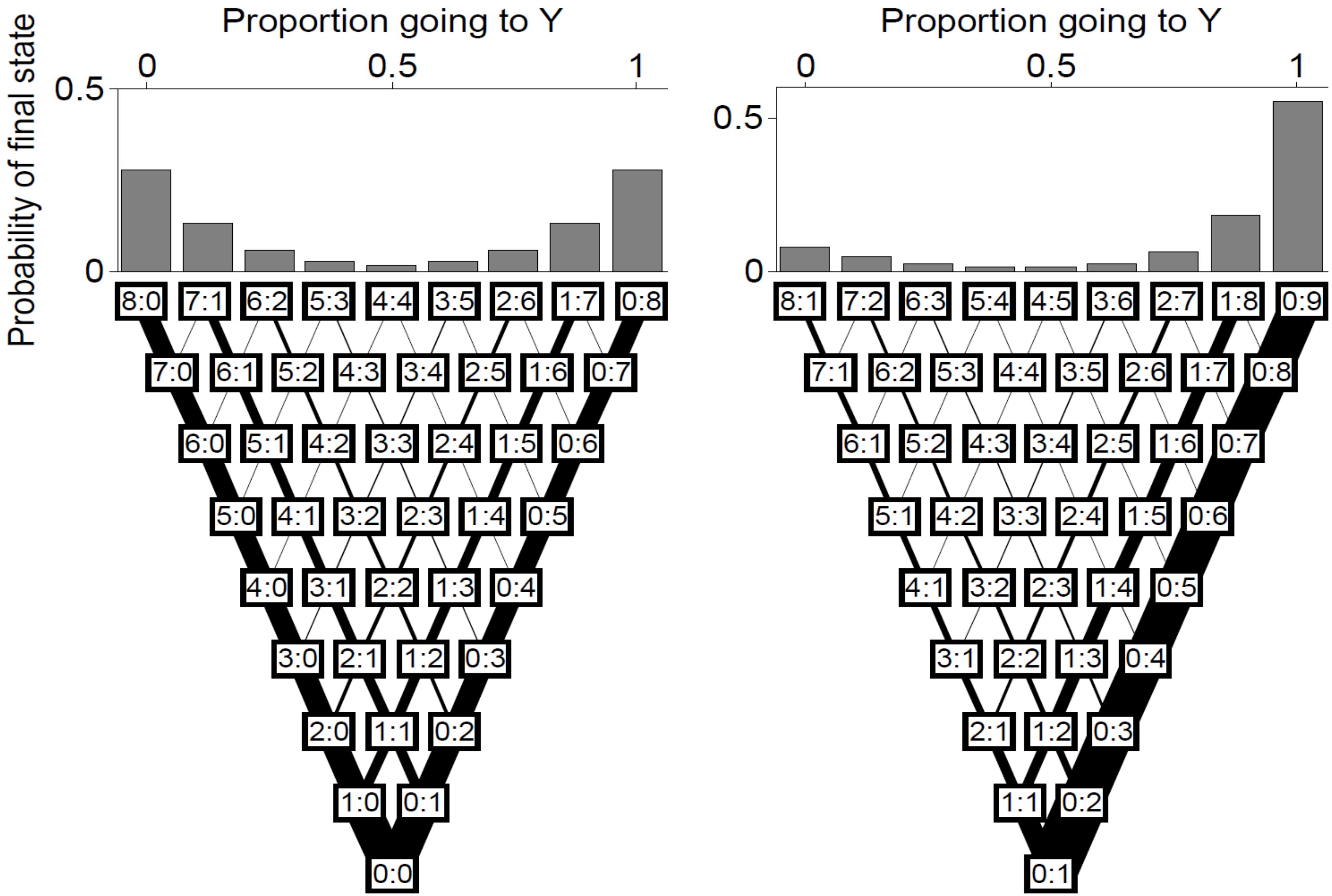
Proportion going to Y
0
0.5
1
Probability of final state
0.5
0
8:0 7:1 6:2 5:3 4:4 3:5 2:6 1:7 0:8
7:0 6:1 5:2 4:3 3:4 2:5 1:6 0:7
6:0 5:1 4:2 3:3 2:4 1:5 0:6
5:0 4:1 3:2 2:3 1:4 0:5
4:0 3:1 2:2 1:3 0:4
3:0 2:1 1:2 0:3
2:0 1:1 0:2
1:0 0:1
0:0
Proportion going to Y
0
0.5
1
0.5
0
8:1 7:2 6:3 5:4 4:5 3:6 2:7 1:8 0:9
7:1 6:2 5:3 4:4 3:5 2:6 1:7 0:8
6:1 5:2 4:3 3:4 2:5 1:6 0:7
5:1 4:2 3:3 2:4 1:5 0:6
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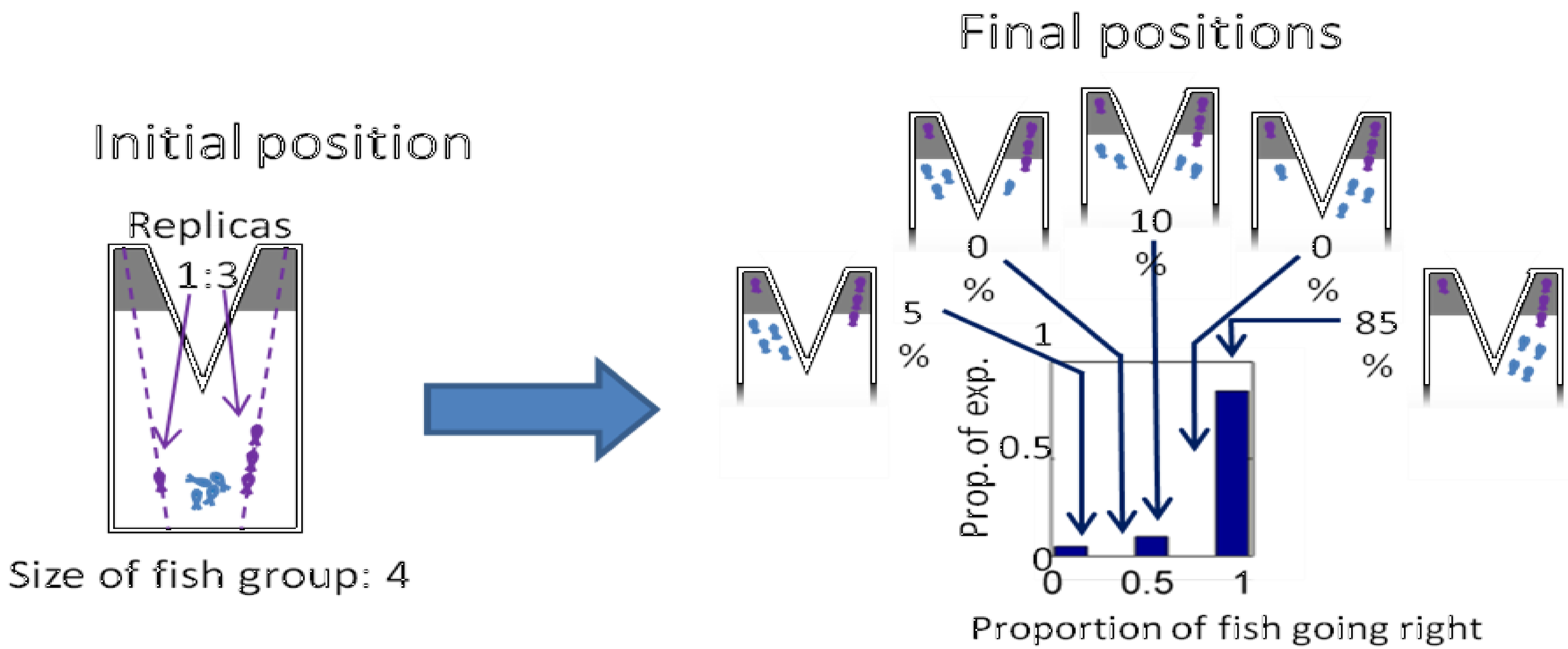
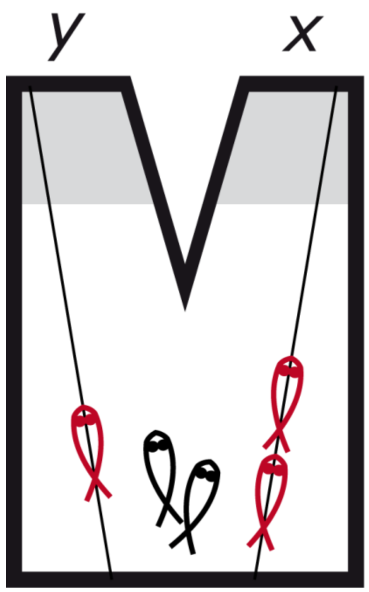
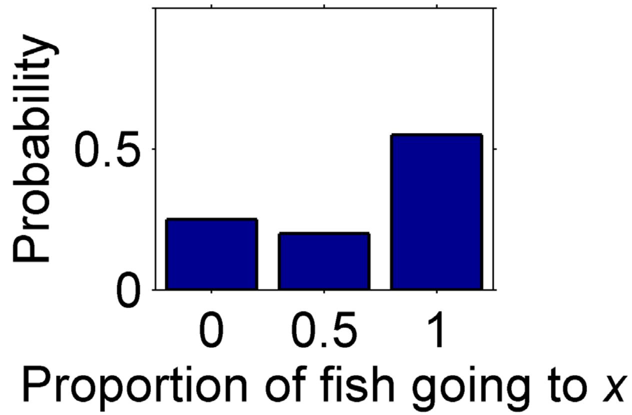
3:1 2:2 1:3 0:4
2:1 1:2 0:3
1:1 0:2
0:1



Initial position
Replicas
1:3
Size of fish group: 4
Final positions
0
%
10
%
0
%
5
%
85
%
1
0.5
0
Prop. of exp.
0
0.5
1
Proportion of fish going right

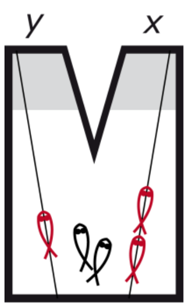
Stickleback experiments

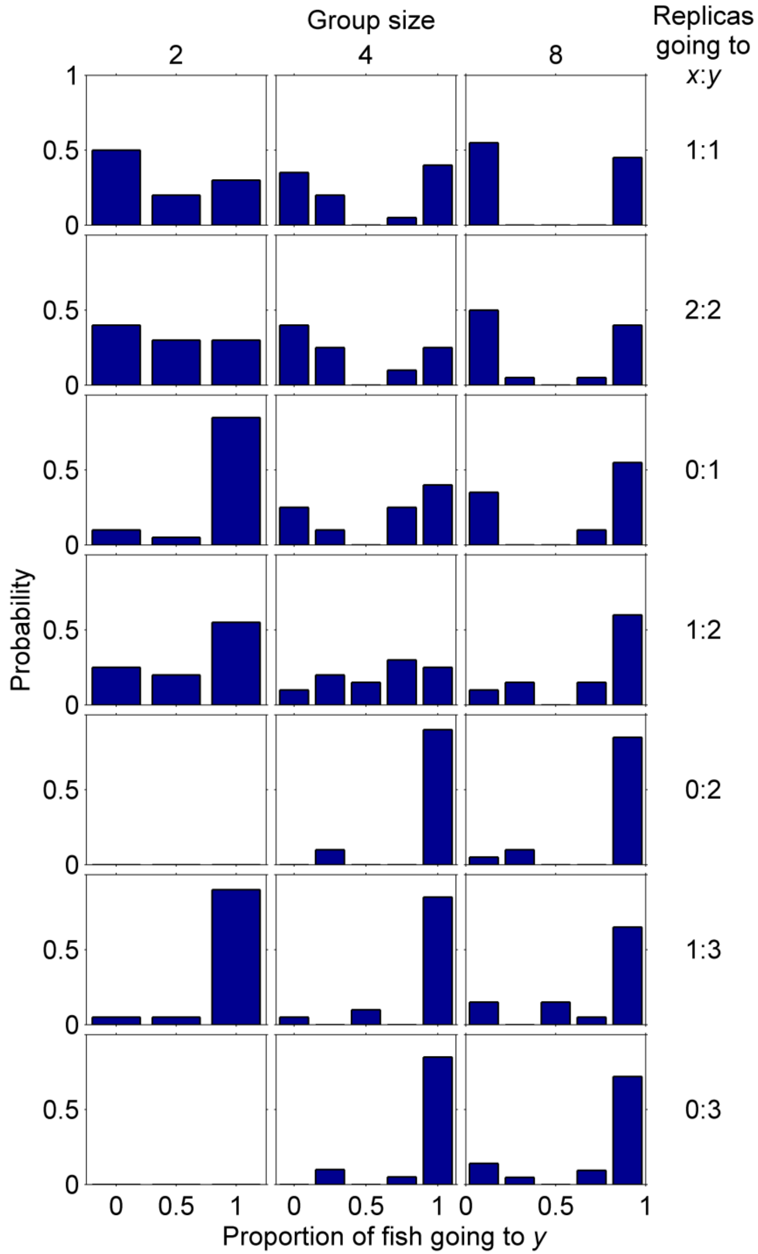




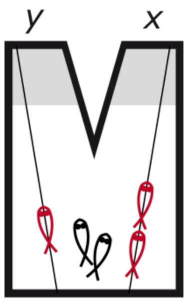
Data from Ward *et al.* (2005)
Sumpter et al. (2008)

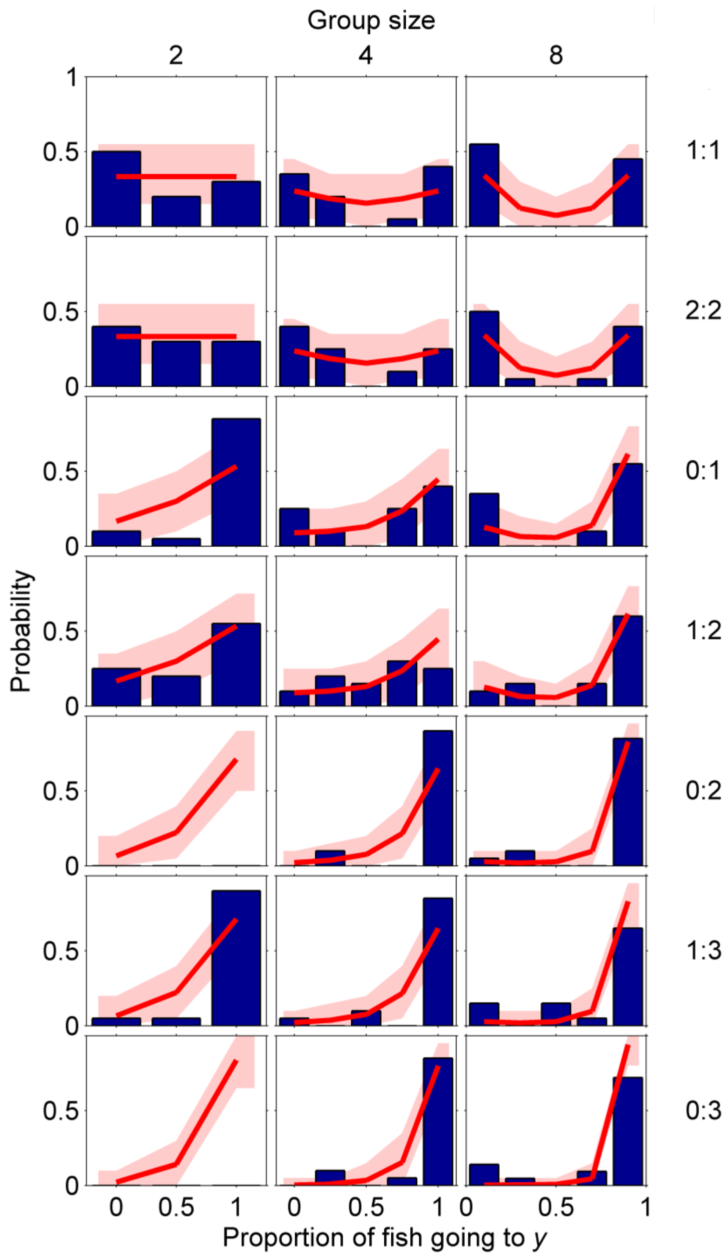
Stickleback experiments



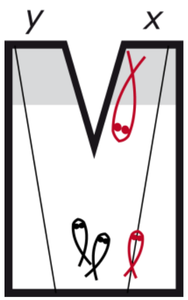


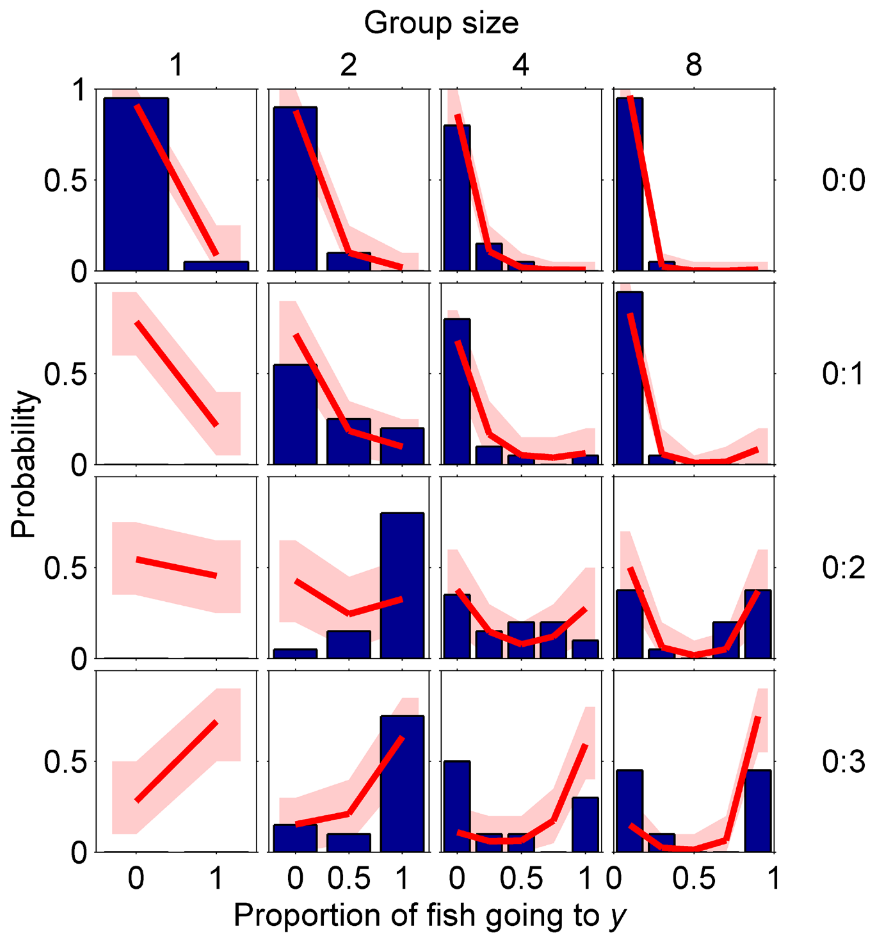
Stickleback experiments



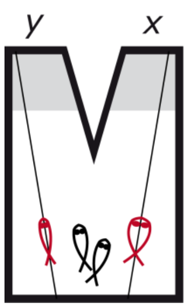


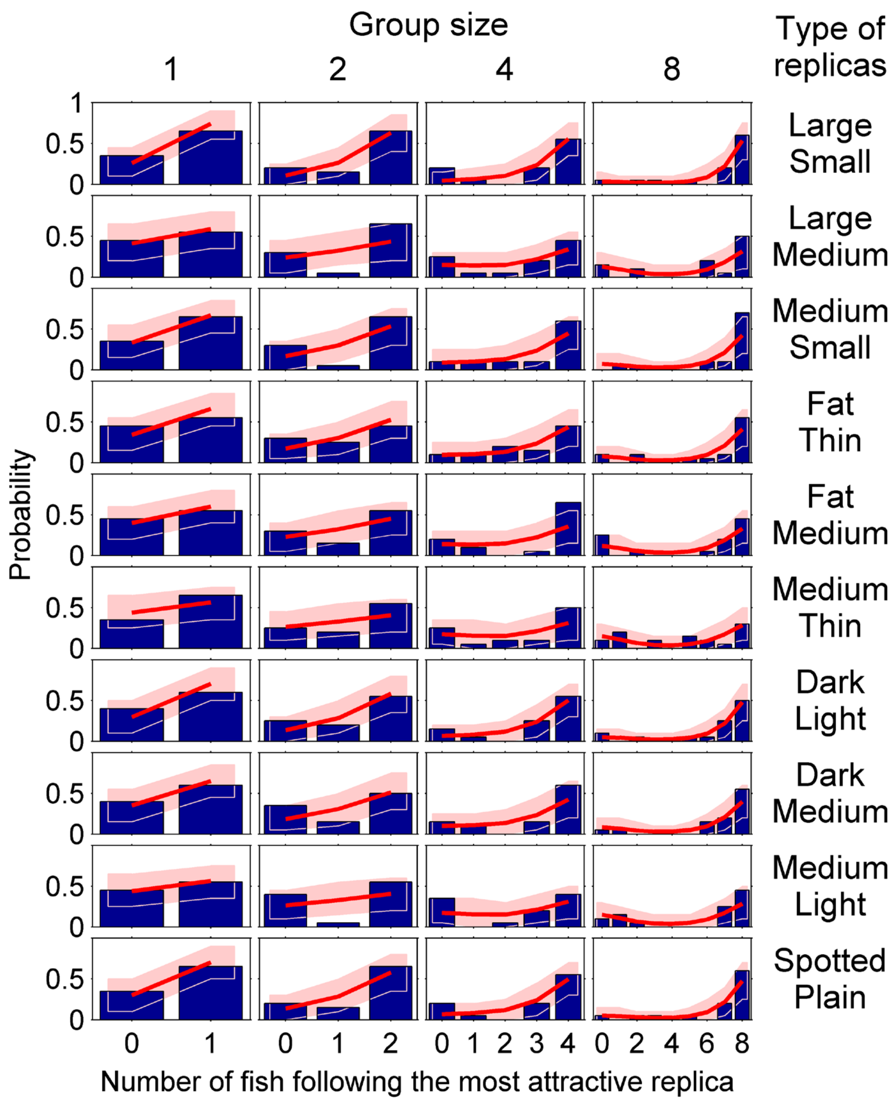
Stickleback experiments





Stickleback experiments



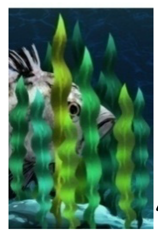
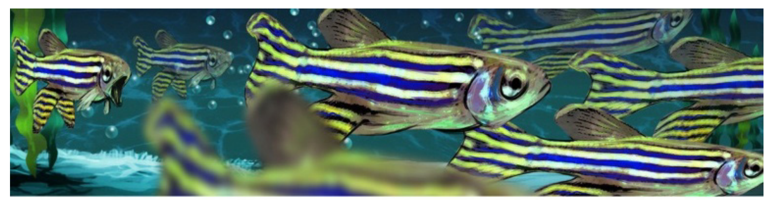


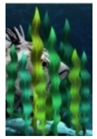
Concrete hypothesis

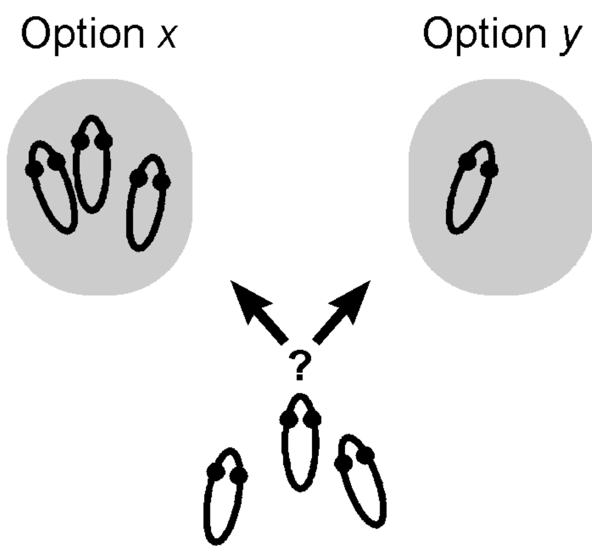
Group behavior from individuals estimating the world using

(a) non-social and

(b) social data

P( | , )

P( | , )



Animals estimate

$$P(X|C, B)$$

$$P(Y|C, B)$$

We get:

$$P(X|C,B) = \frac{1}{1 + a\, s^{-(n_x - k n_y)}}$$

$$P(Y|C,B) = \frac{1}{1 + a\, s^{-(n_y - k n_x)}}$$

Theoretical framework: Probability matching

From the estimation

$$P(X|C,B) = \frac{1}{1 + a\, s^{-(n_x - k n_y)}}$$

animals would apply probability matching

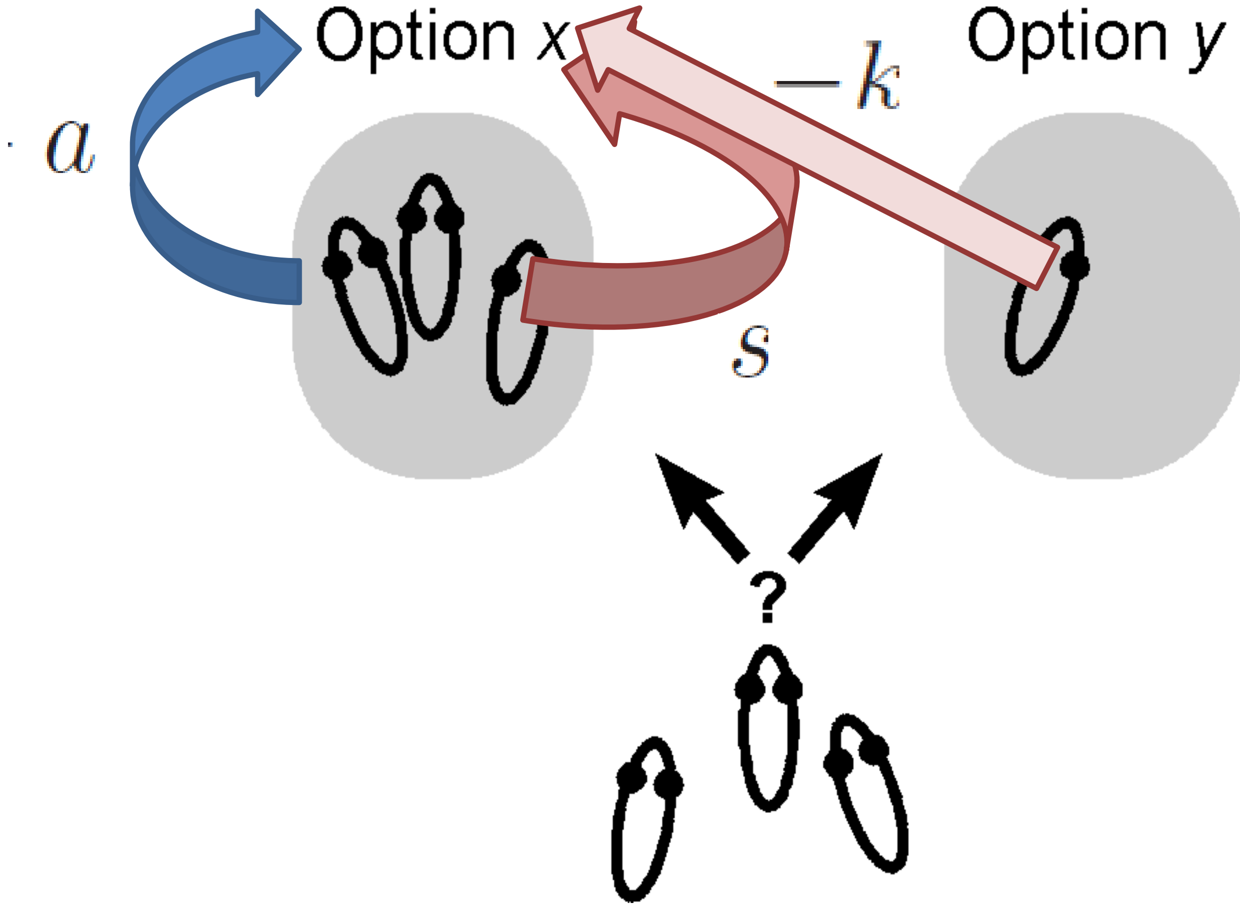
$$P_x = \frac{P(X|C,B)}{P(X|C,B) + P(Y|C,B)}$$

giving

$$P_x = \left(1 + \frac{1 + a s^{-(n_x - k n_y)}}{1 + a s^{-(n_y - k n_x)}}\right)^{-1}$$

Theoretical framework: parameters

$$P_x = \left(1 + \frac{1 + as^{-(n_x - kn_y)}}{1 + as^{-(n_y - kn_x)}}\right)^{-1}$$



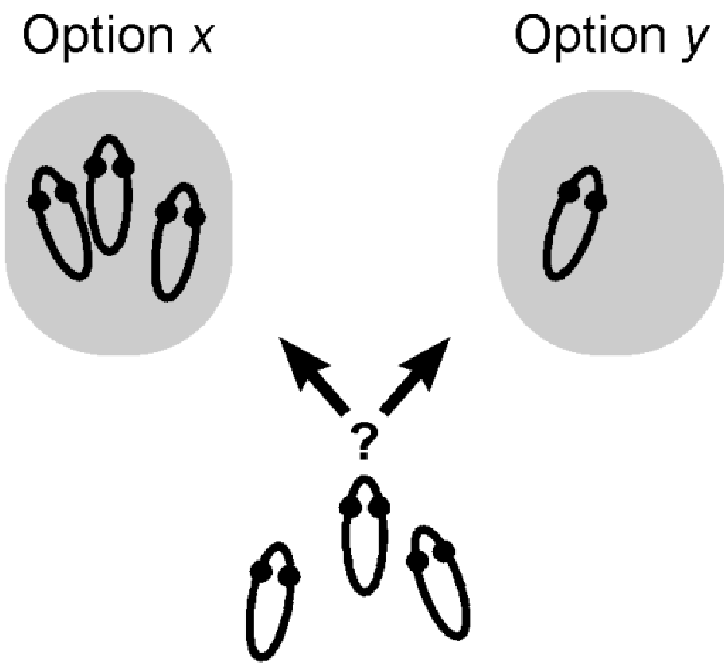
$$a = \frac{P(\bar{X}|C)}{P(X|C)} = \frac{P(\bar{Y}|C)}{P(Y|C)}$$

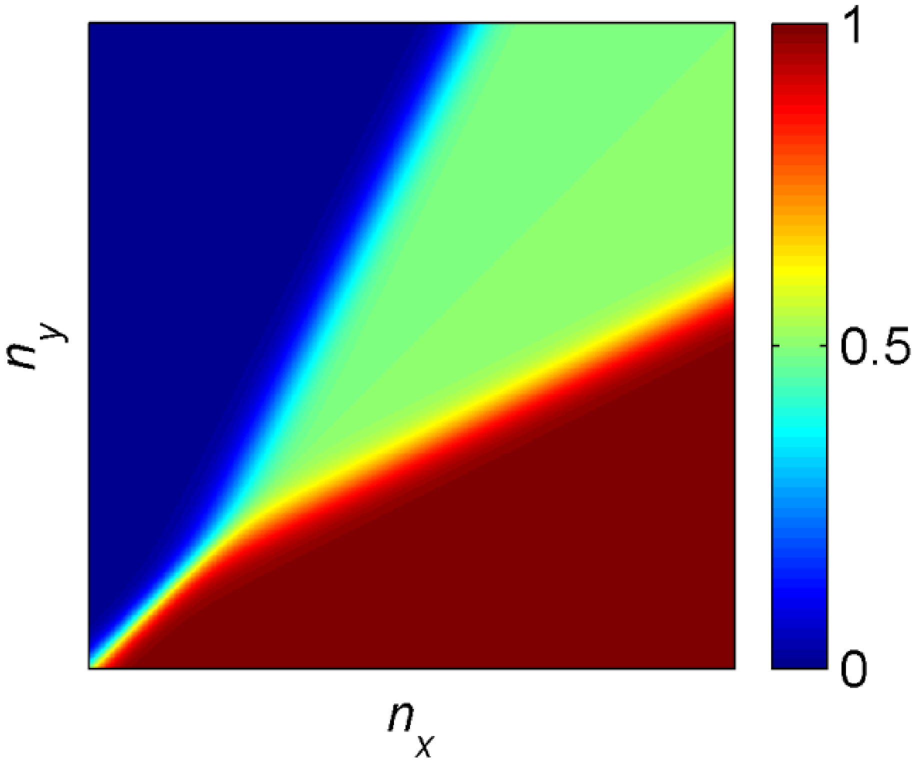
$$s = \frac{P(x|X,C)}{P(x|\bar{X},C)} = \frac{P(y|Y,C)}{P(y|\bar{Y},C)}$$

$$k = -\frac{\log \frac{P(x|Y,C)}{P(x|\bar{Y},C)}}{\log \frac{P(x|X,C)}{P(x|\bar{X},C)}}$$

Theoretical framework: plotting the result

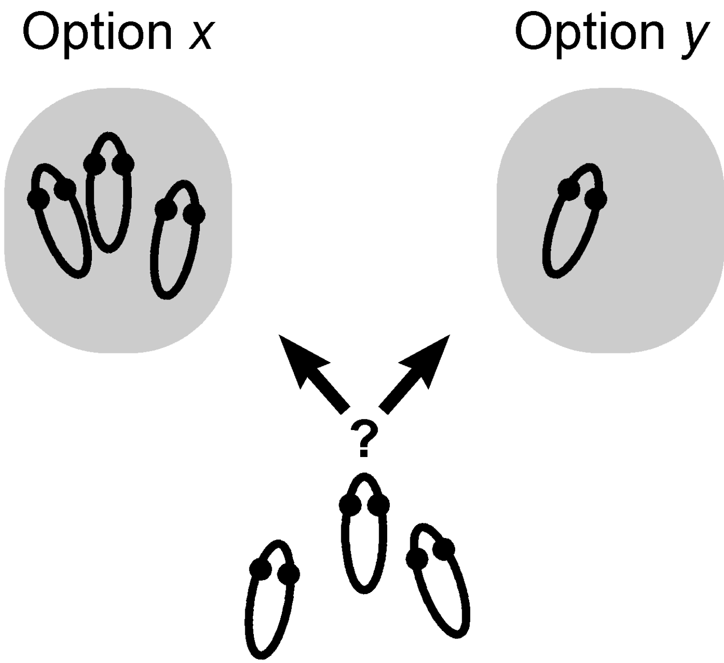
$$P_x = \left(1 + \frac{1 + as^{-(n_x - kn_y)}}{1 + as^{-(n_y - kn_x)}}\right)^{-1}$$

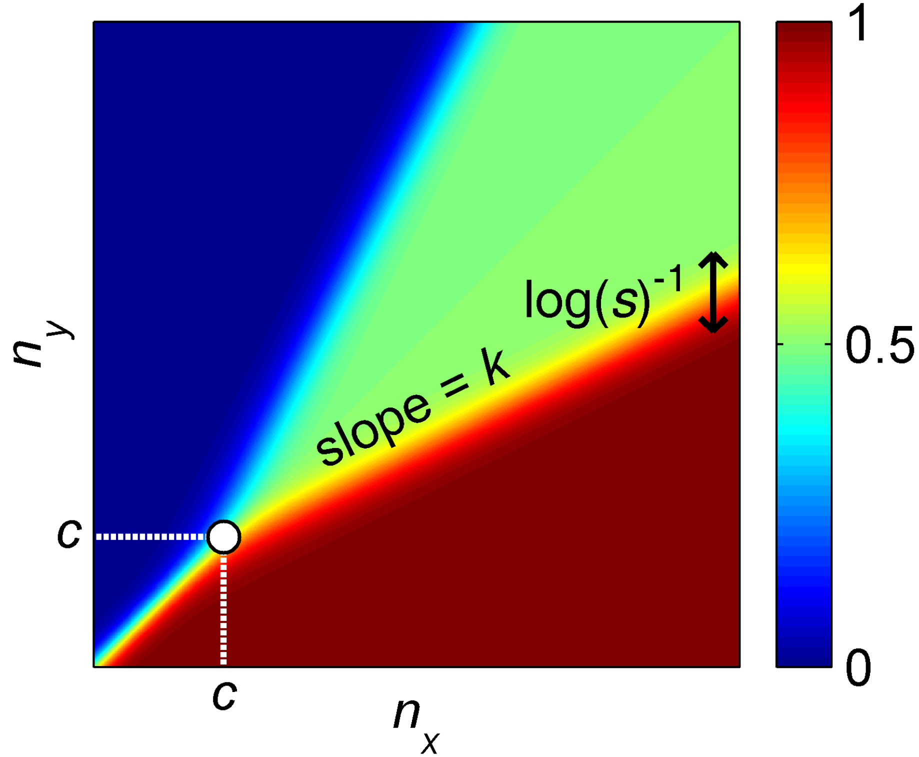




Theoretical framework: plotting the result

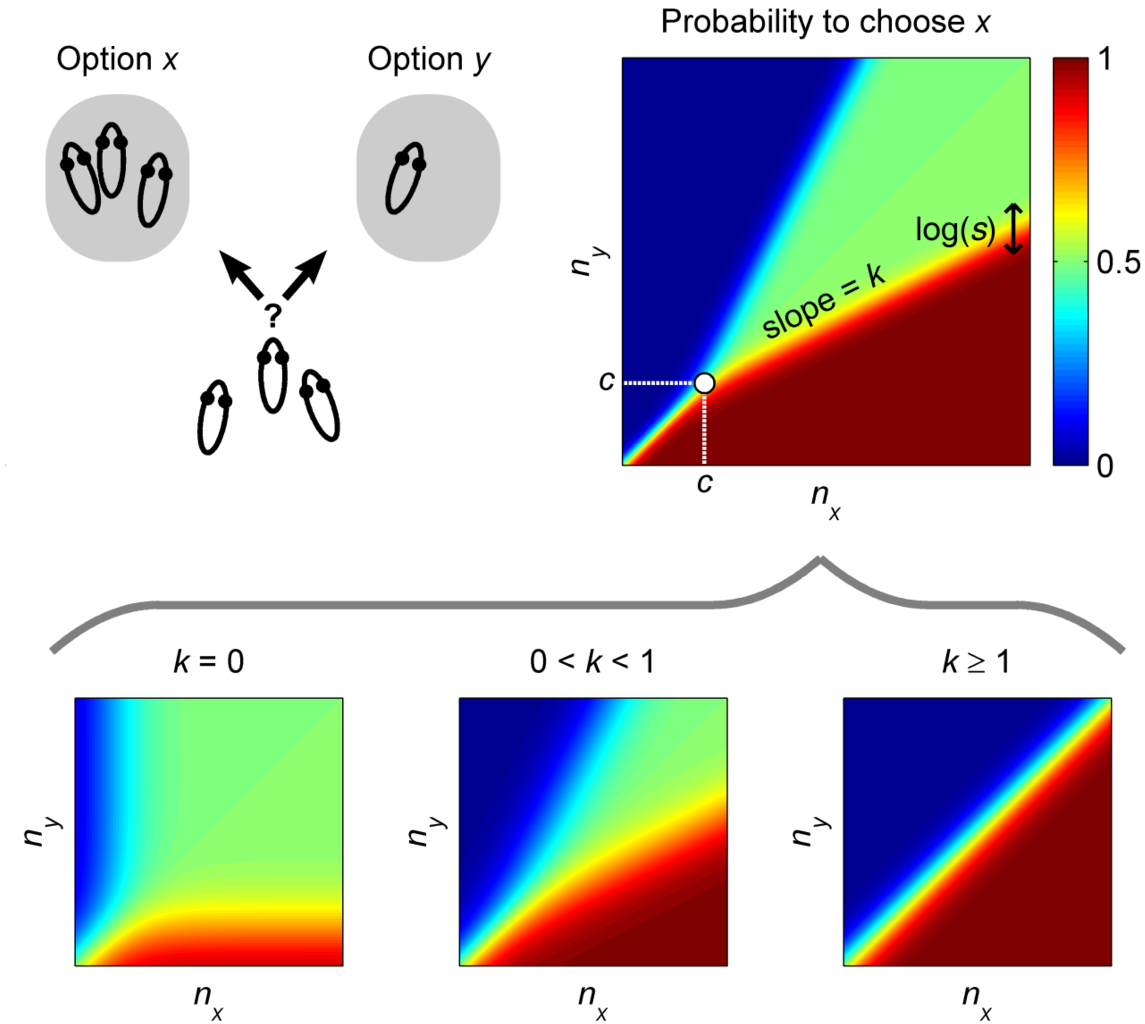
$$P_x = \left(1 + \frac{1 + as^{-(n_x - kn_y)}}{1 + as^{-(n_y - kn_x)}}\right)^{-1}$$





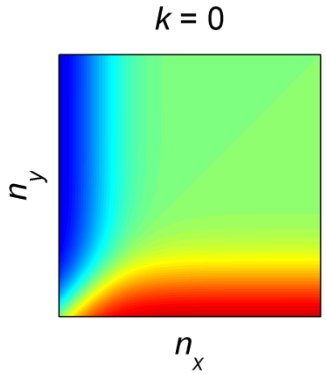
$$c = \frac{\log a}{(1-k)\log s}$$

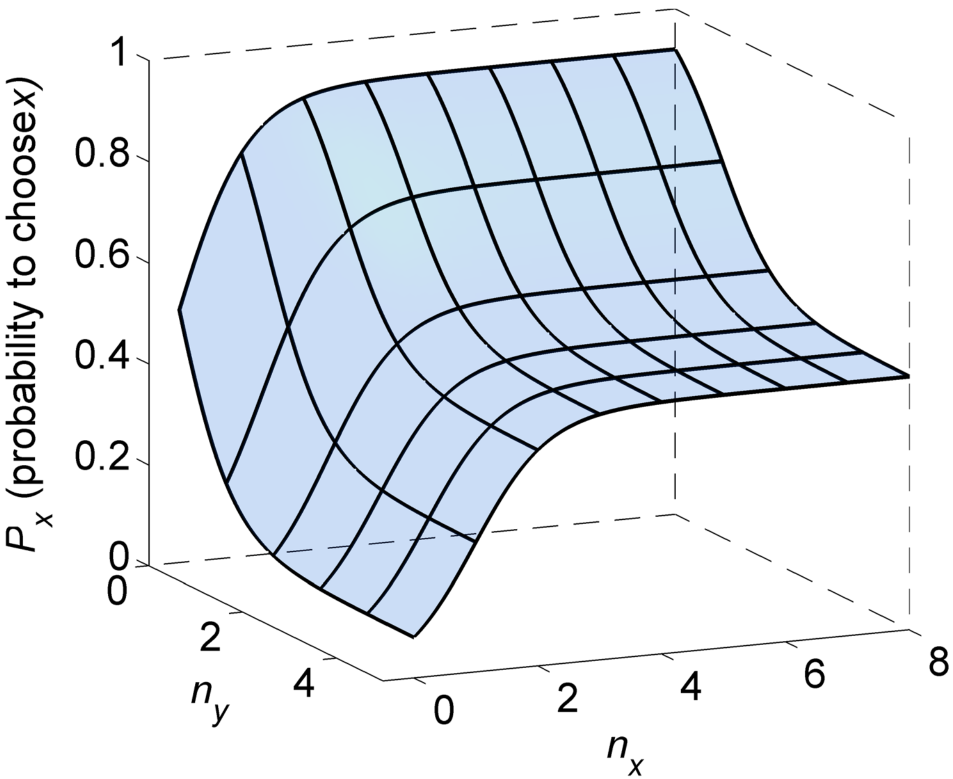
Theoretical framework: plotting the result



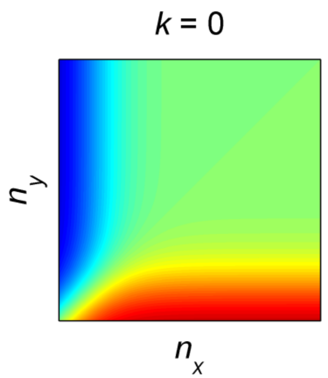
This approach should be valid for many animal species

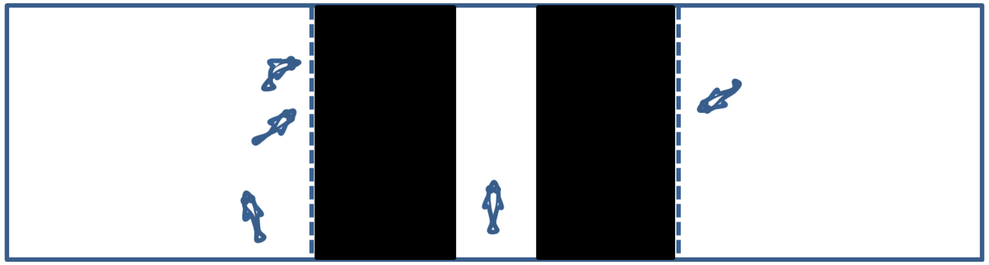
Zebrafish experiments correspond to case k=0

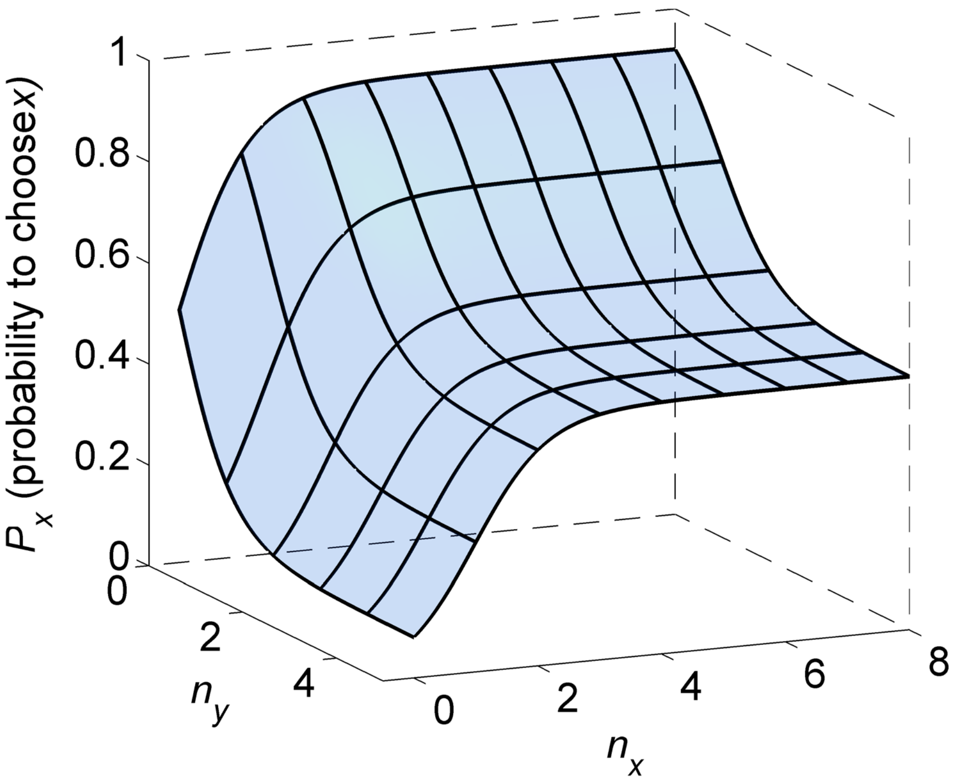




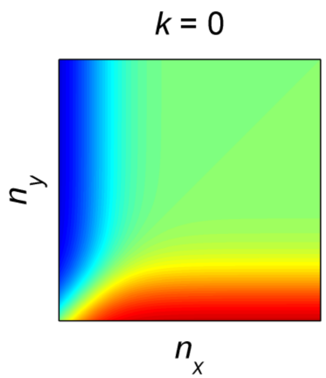
Zebrafish experiments correspond to case k=0

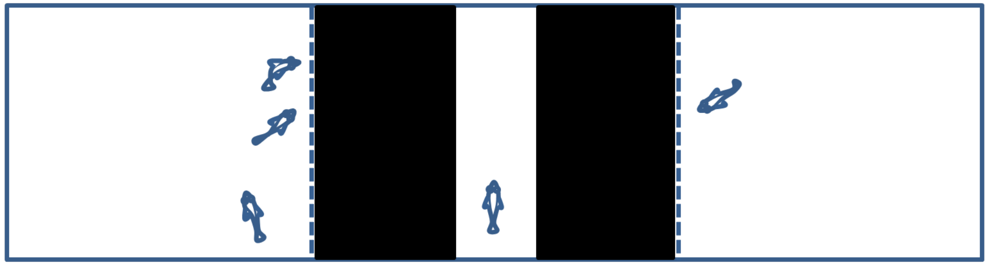


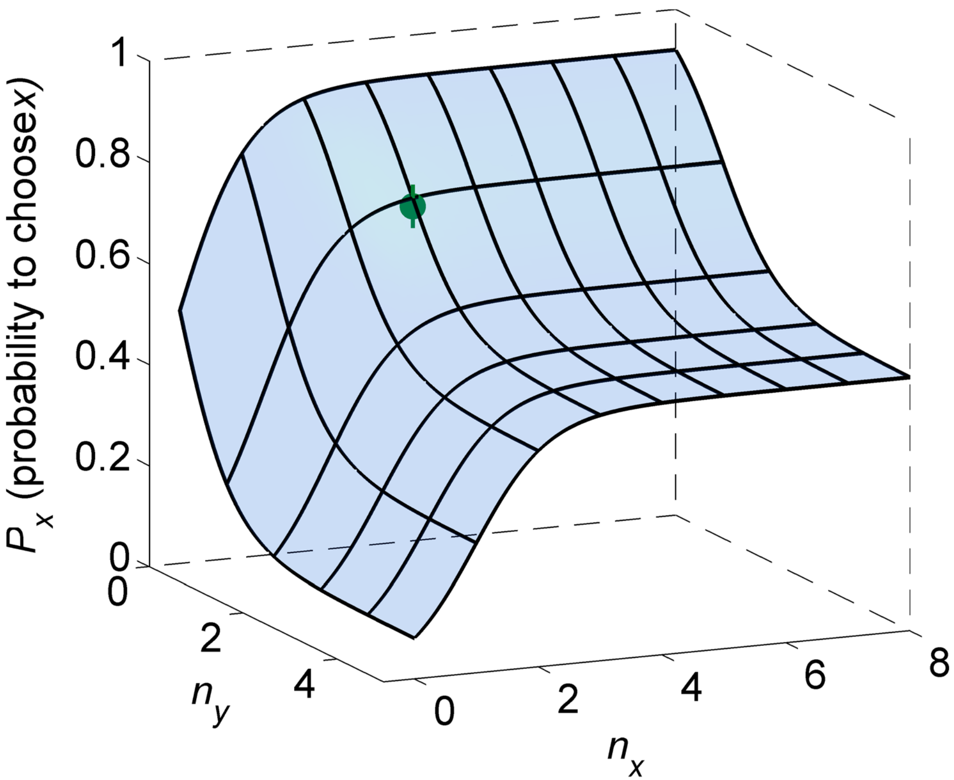




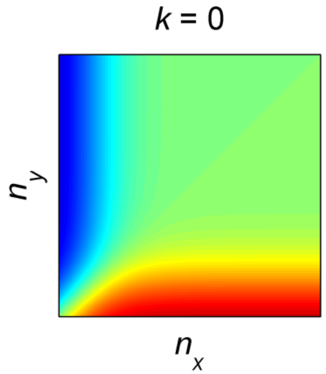
Zebrafish experiments correspond to case *k*=0

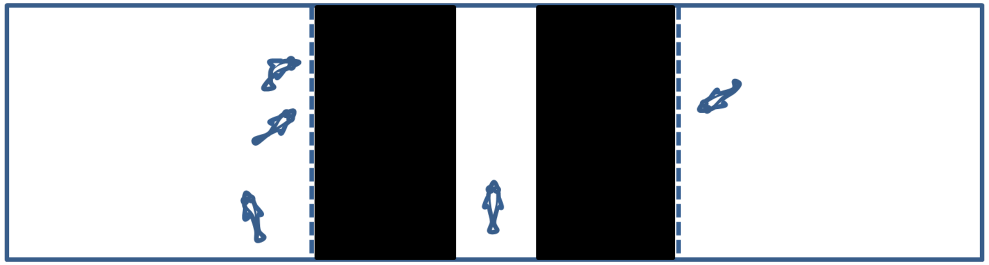


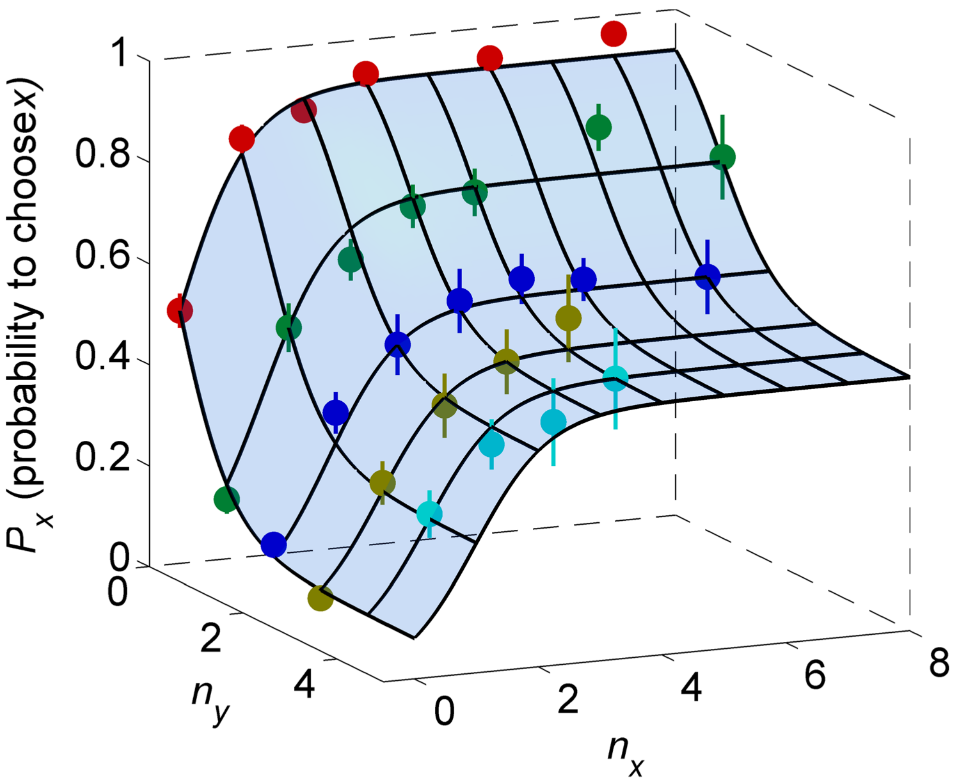




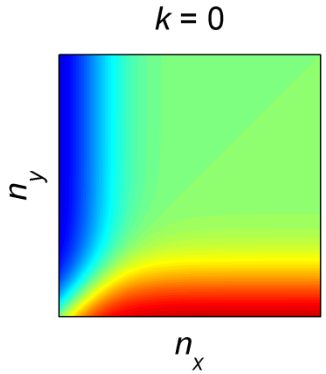
Zebrafish experiments correspond to case k=0

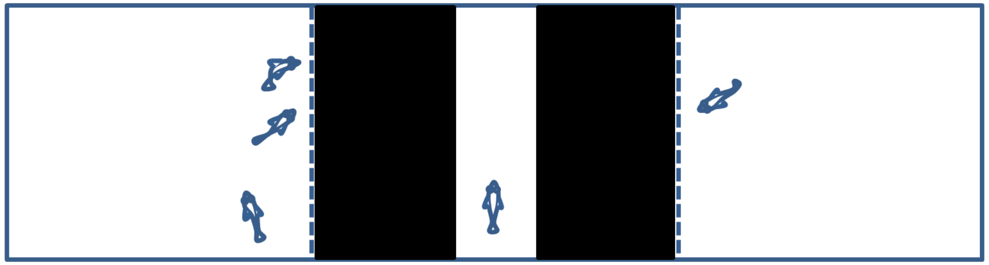


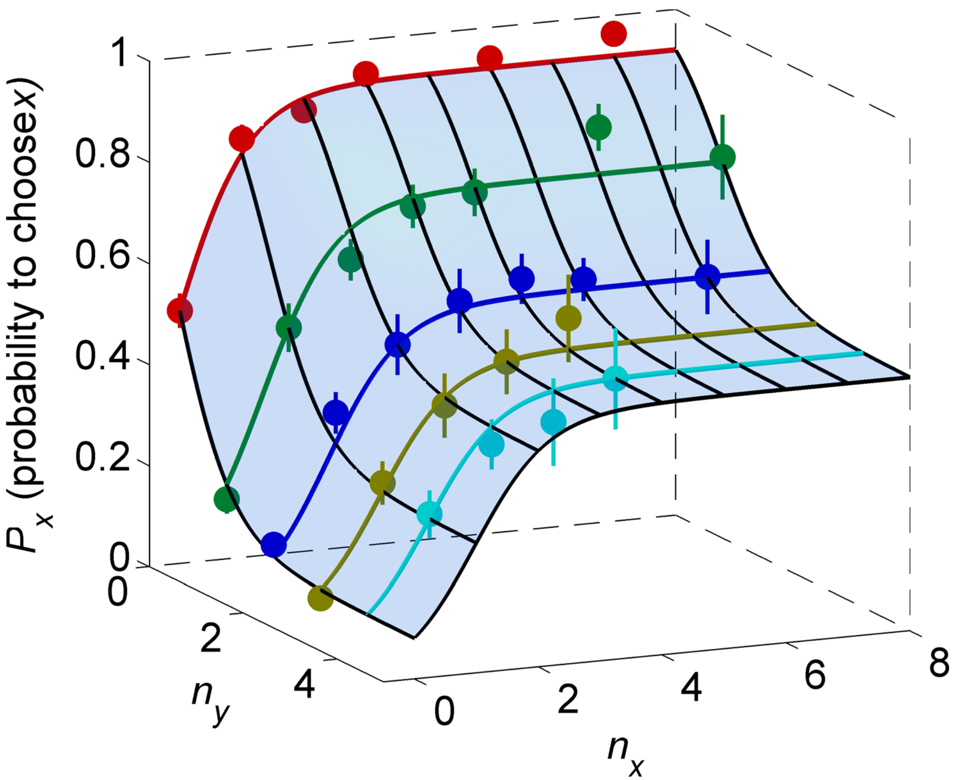




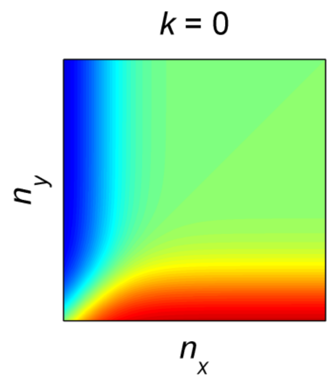
Zebrafish experiments correspond to case k=0

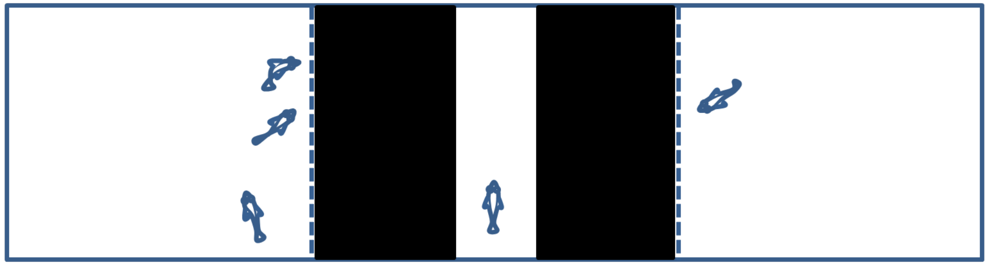


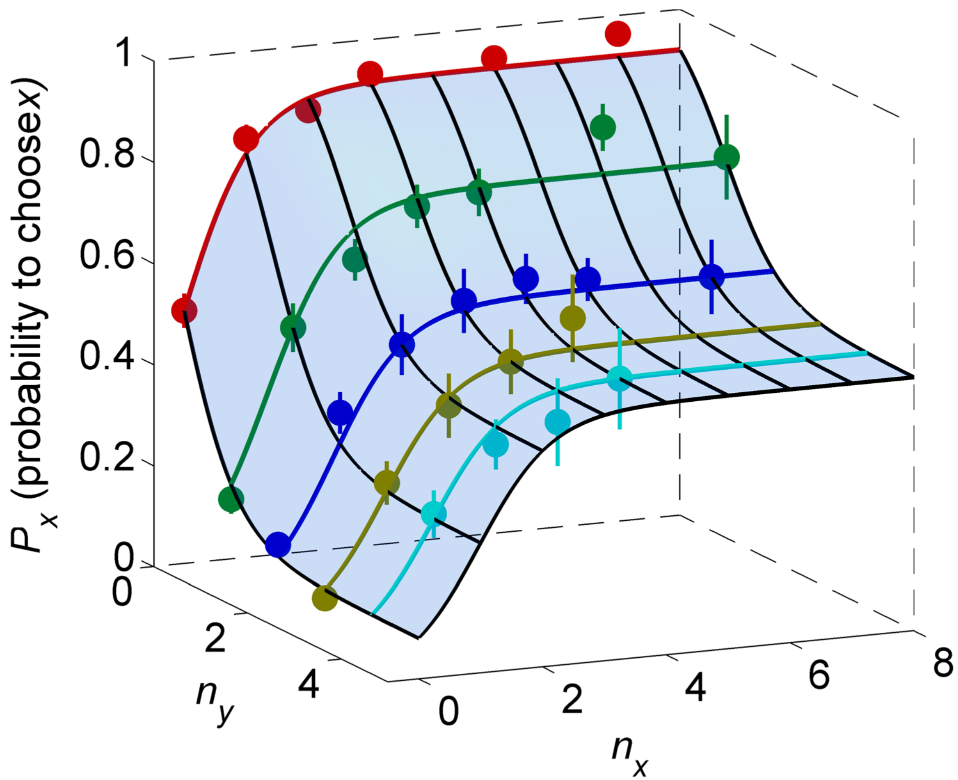


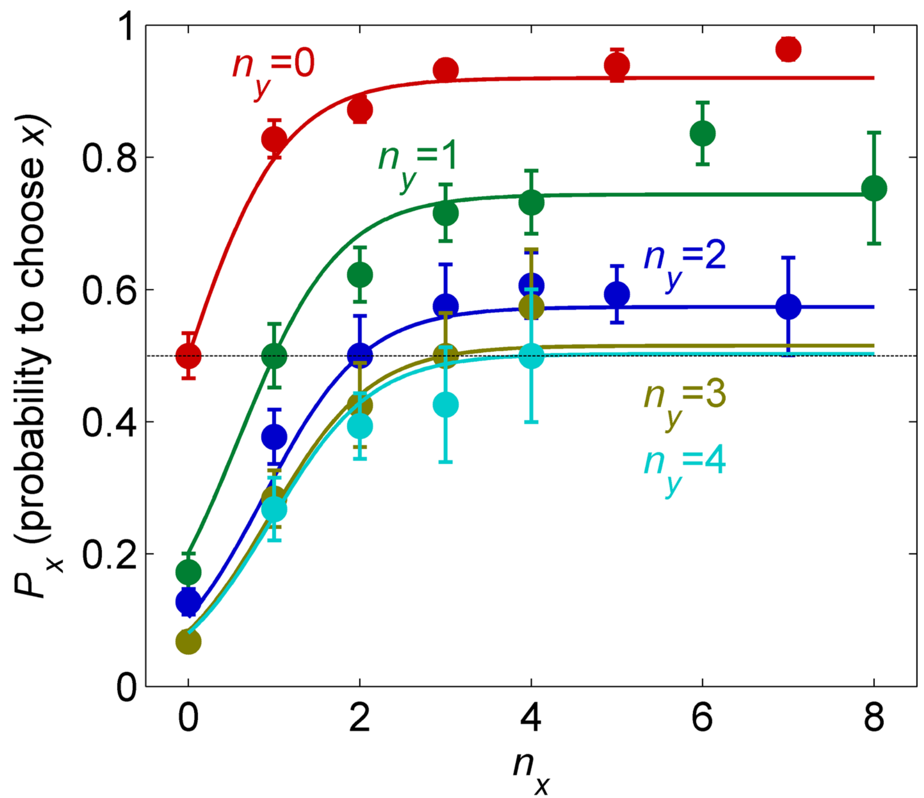


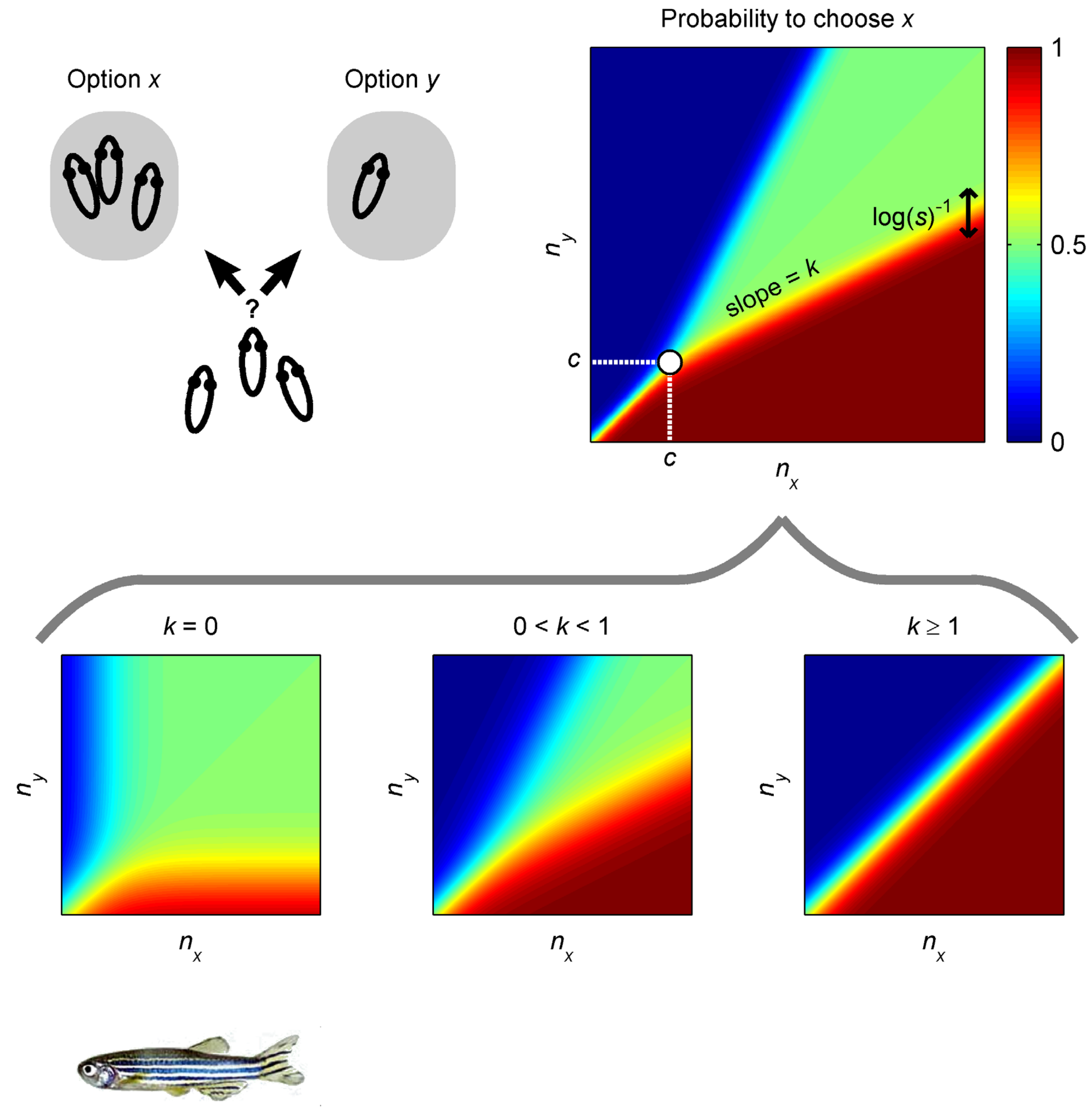
Zebrafish experiments correspond to case k=0





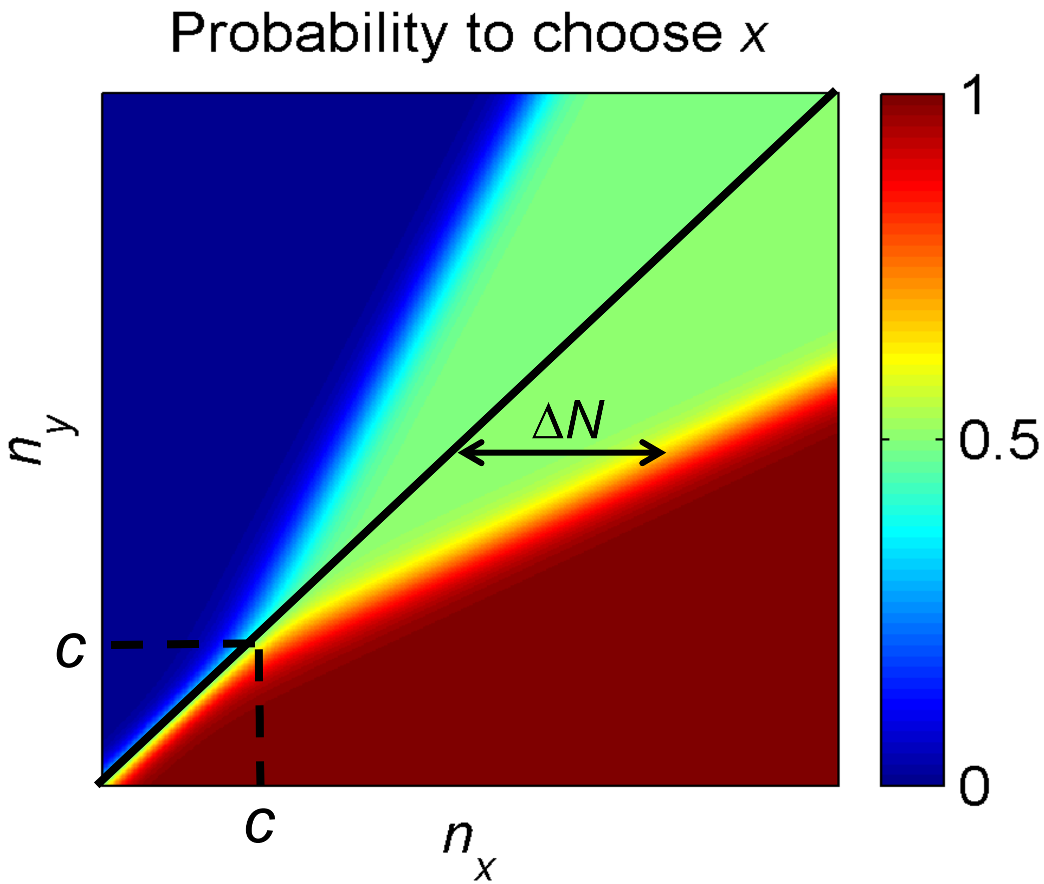


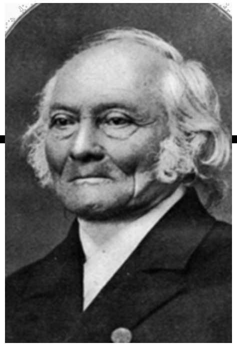




Option x
Option y
?
Probability to choose x
1
0.5
0
log(s)^-1
slope = k
c
c
n_y
n_x
k = 0
0 < k < 1
k ≥ 1
n_y
n_x
n_y
n_x
n_y
n_x

Ant experiments correspond to case 0<*k*<1





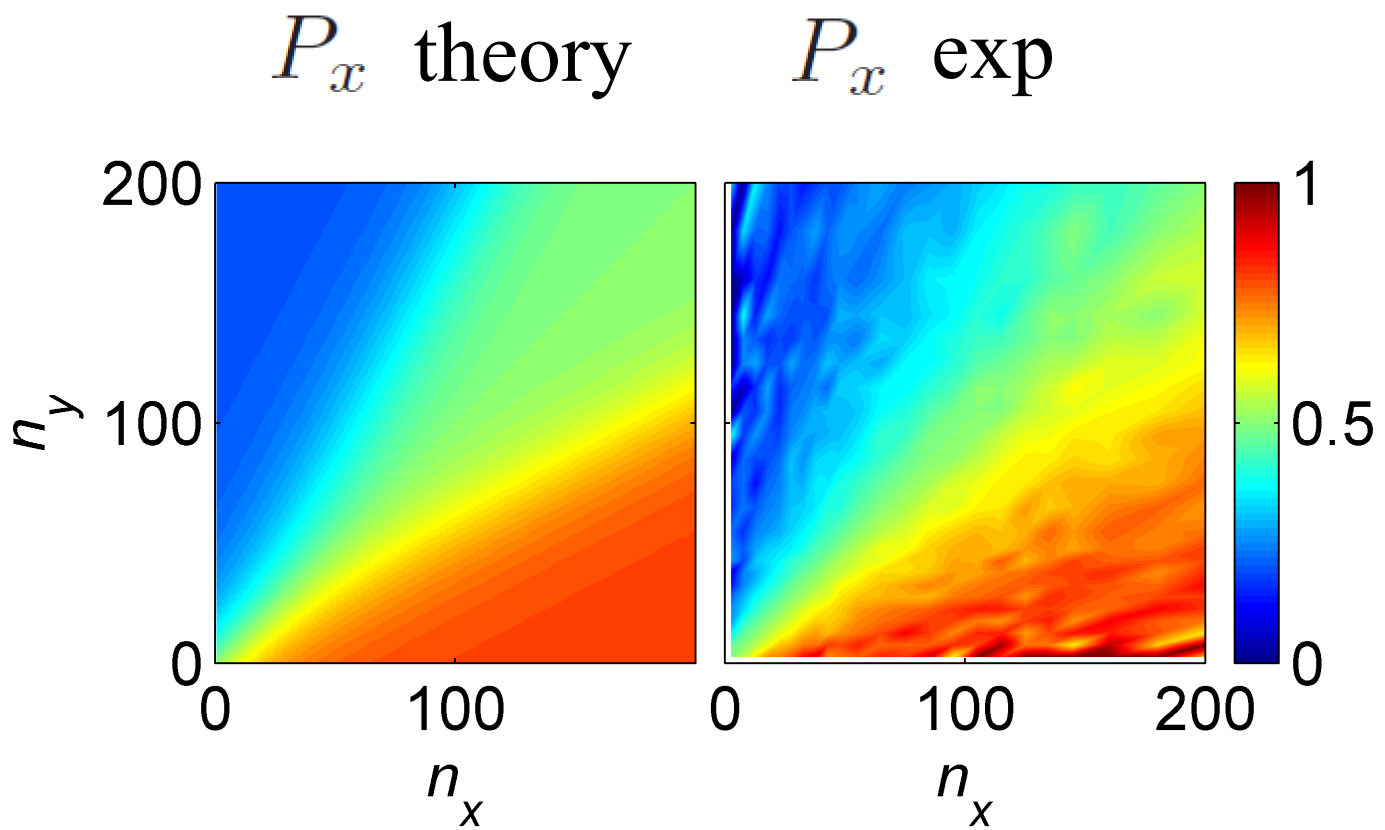
Weber's Law:

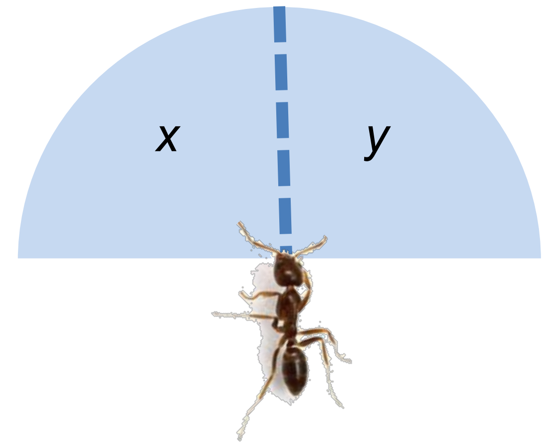
Two quantities: n_x, n_y

$$\Delta N / N \equiv (n_x - n_y)/(n_x + n_y) = \alpha$$

Our model follows Weber at high *n*, with $\alpha = (1-k)/(1+k)$

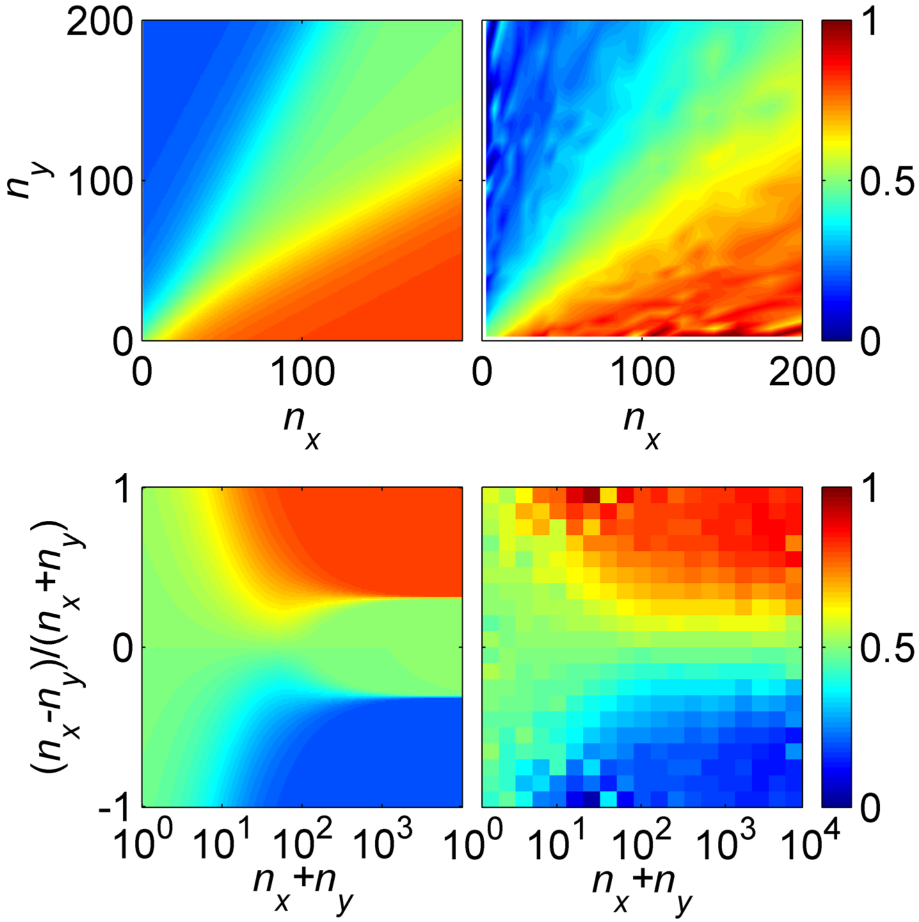
Ant experiments correspond to case $0<k<1$: high n follows Weber



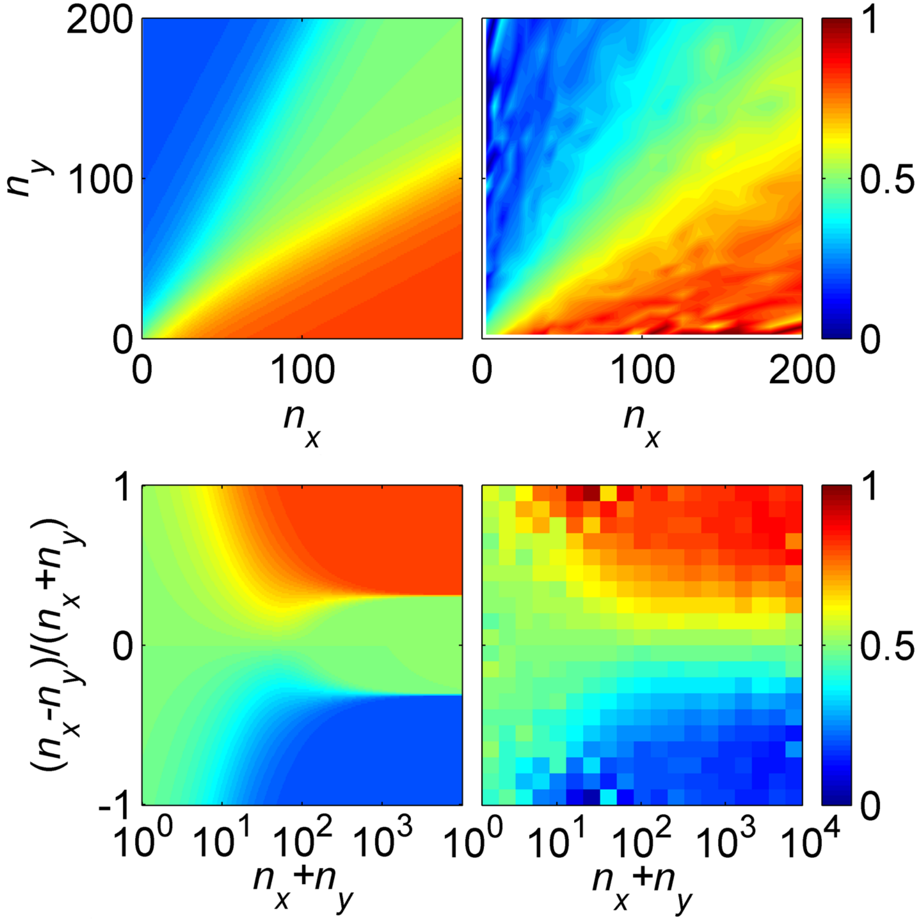


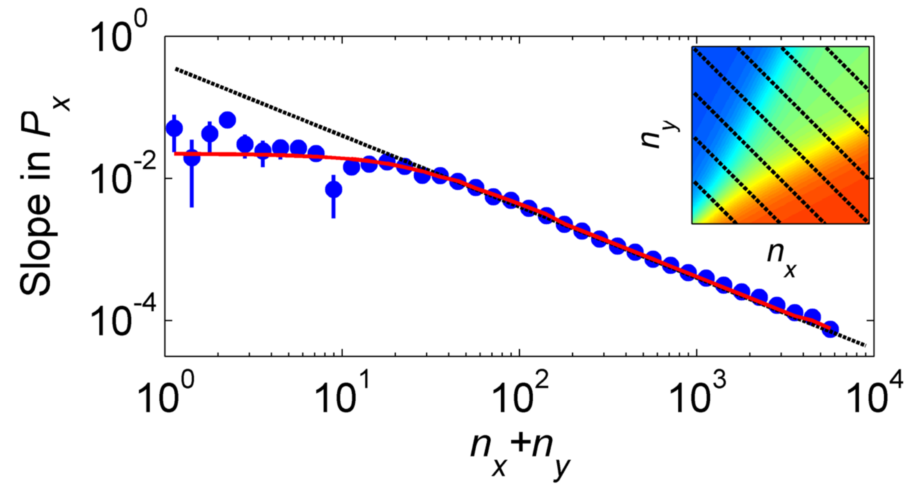
Data from Perna *et al.* (submitted)

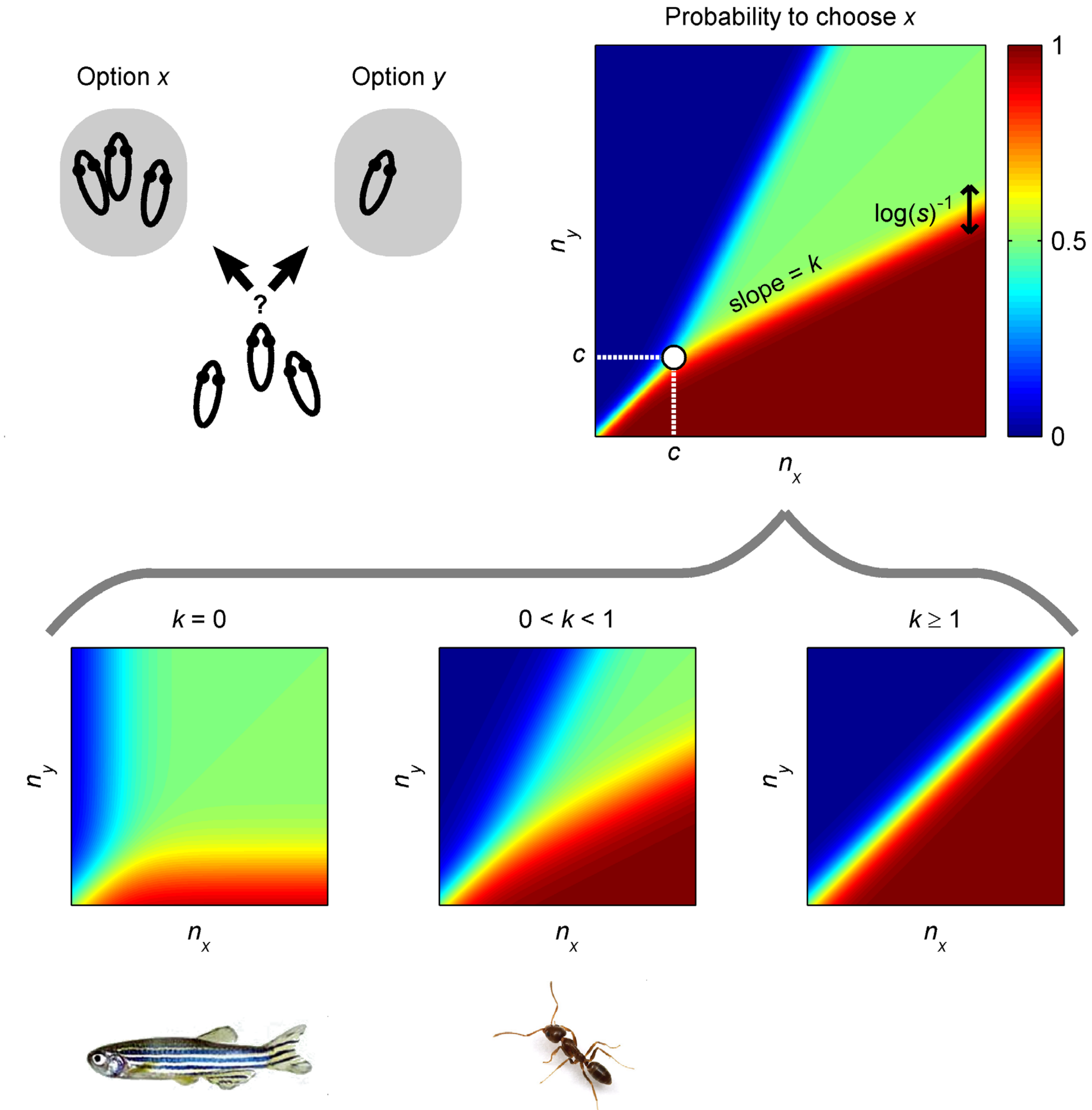
Ant experiments correspond to case 0<*k*<1: low *n* is also predicted

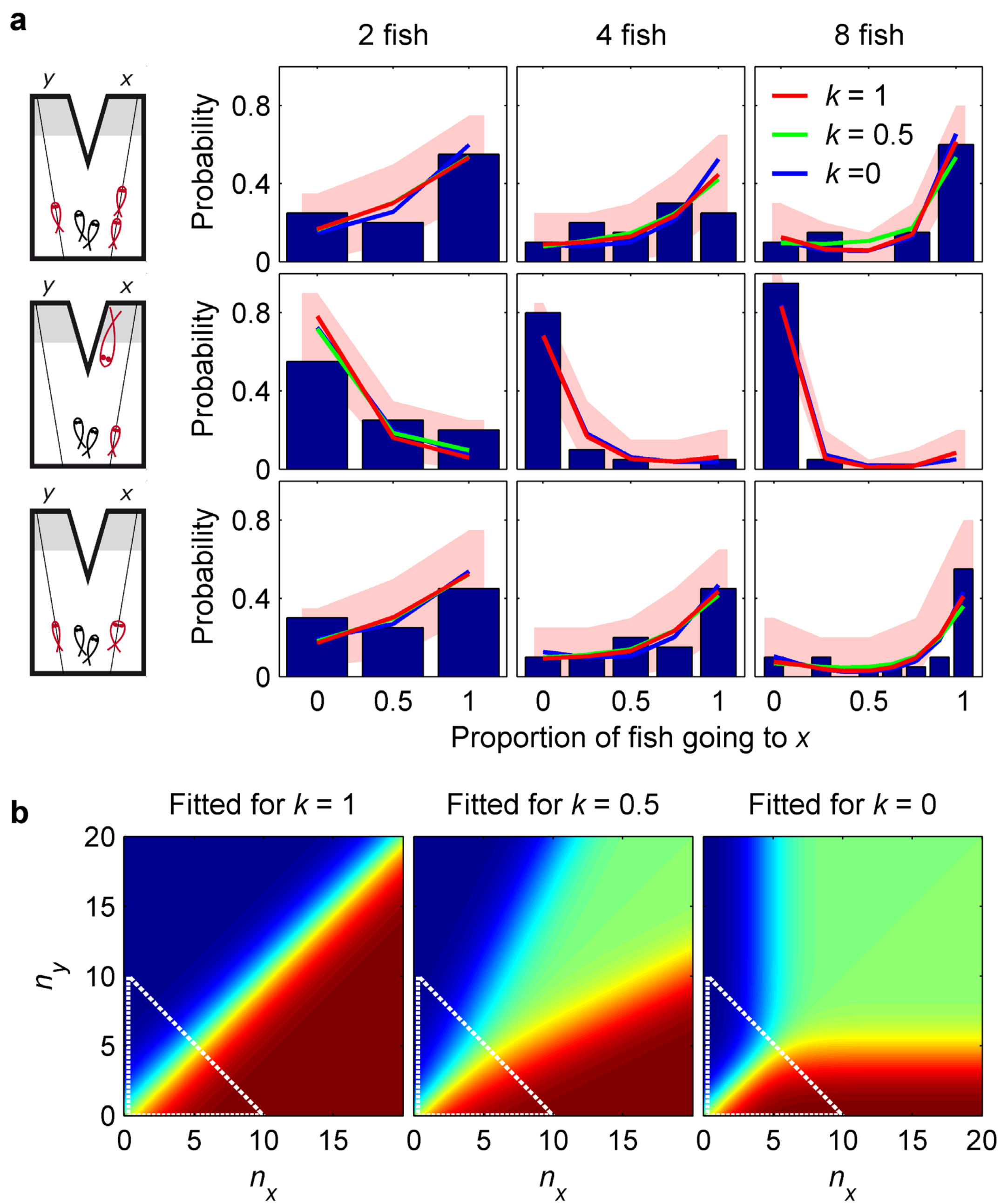


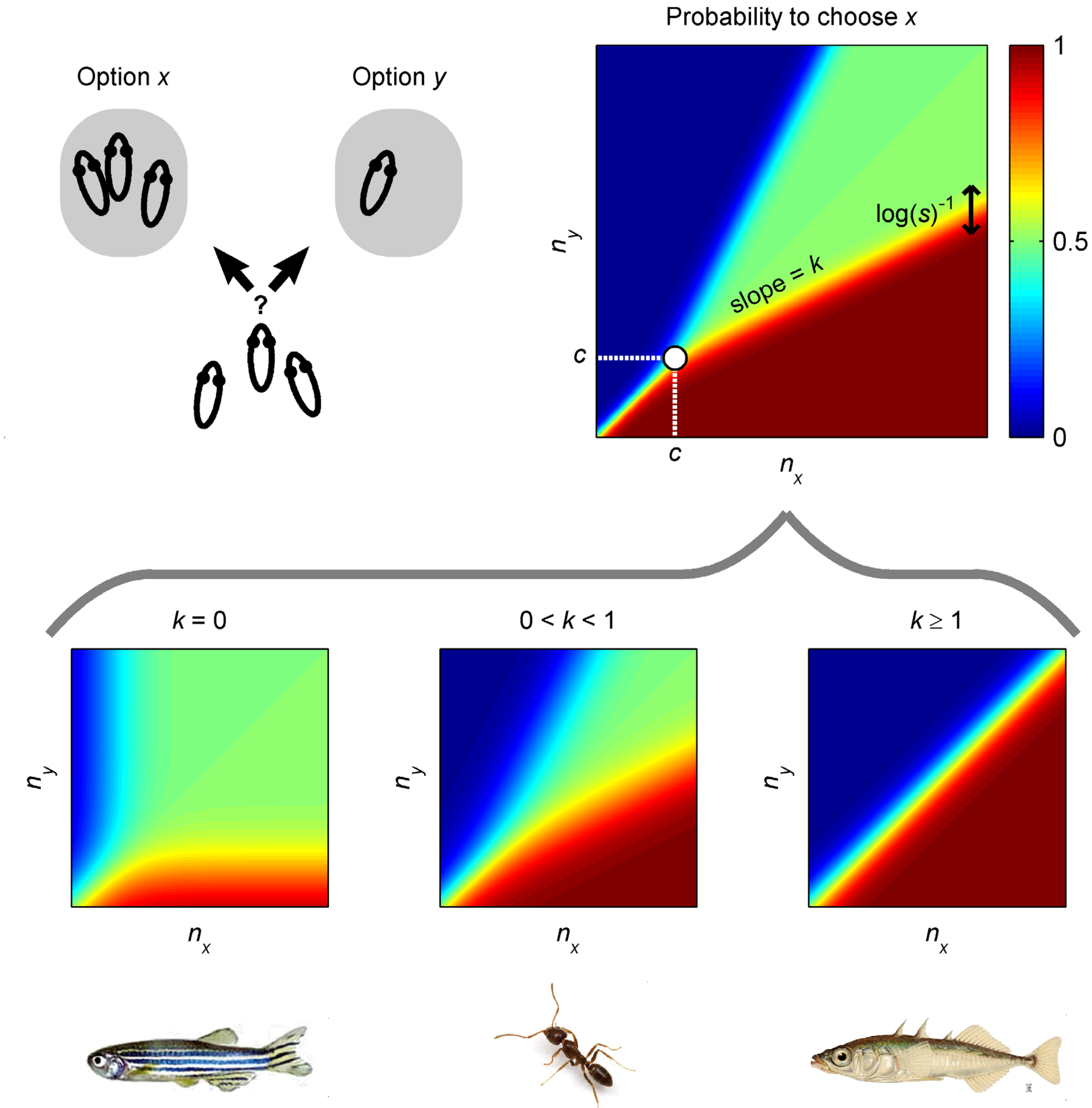
Ant experiments correspond to case 0<*k*<1: low *n* is also predicted






Option x
Option y
?
Probability to choose x
1
0.5
0
log(s)^-1
slope = k
c
c
n_y
n_x
k = 0
0 < k < 1
k ≥ 1
n_y
n_x
n_y
n_x
n_y
n_x


a
2 fish
4 fish
8 fish
y
x
Probability
0.8
0.4
0
k = 1
k = 0.5
k =0
0
0.5
1
Proportion of fish going to x
b
Fitted for k = 1
Fitted for k = 0.5
Fitted for k = 0
n_y
20
15
10
5
0
n_x



Option x
Option y
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Probability to choose x
1
0.5
0
n_y
n_x
c
c
slope = k
$\log(s)^{-1}$
$k = 0$
$0 < k < 1$
$k \geq 1$
n_y
n_x
n_y
n_x
n_y
n_x