

# Incorporating RNA-Protein Interactions into RNA Secondary Structure Prediction

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July 29, 2012

# Overview

- 1 Introduction
- 2 Physical extension
- 3 Interaction with proteins
- 4 Putting it all together
- 5 Summary

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# RNA secondary structure prediction

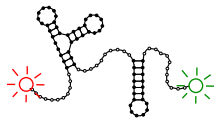
## Secondary structure prediction

Prediction of RNA secondary structures in solution is **mature field**:  
Nussinov, Waterman, McCaskill, MFOLD, Dynafold, Vienna package, Sfold, ...

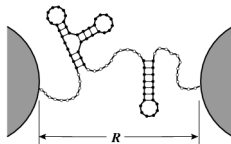
But ...

**Solution structure** may not always be what we want  $\Rightarrow$

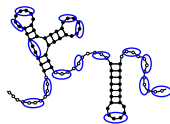
FRET



Force-extension



Protein-RNA interactions



# Partition function approach

## Partition function approach (McCaskill, 1990)

- Each secondary structure  $S$  is a **state**
- Each state  $S$  has a (free) **energy**

$$E[S] = \sum_{(i,j) \in S} \varepsilon_{i,j}$$

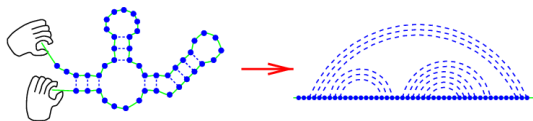
(in practice use much more detailed energy model)

- All thermodynamic quantities can be derived from **partition function**



$$Z = \sum_S \exp(-E[S]/k_B T)$$

# Arc diagrams

Secondary structures correspond one to one to arc diagrams



Partition functions as recursion equations

- Partition function  $Q_{i,j}$   for substrand from base  $i$  to base  $j$
- Partition function  $W_{i,j}$   for substrand from base  $i$  to base  $j$  where base  $i$  is **paired to the last paired base**

⇒ Recursion equations

$$\text{Diagram of double-strand } Q_{i,j} = \text{Diagram of single-strand } 1 \text{ from } i \text{ to } j + \text{Diagram of } W_{i,j} \text{ with arc from } i \text{ to last paired base} + \sum_{k=i}^{j-h-2} \text{Diagram of } Q_{i,k} \text{ from } i \text{ to } k \text{ followed by } W_{k+1,j} \text{ from } k+1 \text{ to } j$$

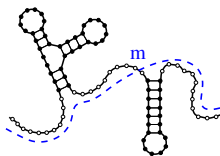
$$\text{Diagram of } W_{i,j} \text{ with arc from } i \text{ to last paired base} = \sum_{k=i+h+1}^j \text{Diagram of } Q_{i+1,k-1} \text{ from } i+1 \text{ to } k-1 \text{ followed by } 1 \text{ from } k \text{ to } j$$

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- 1 Introduction
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  - Restricted partition function
  - Polymer Physics
  - FRET
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## External bases

Physical extension of a given secondary structure depends only on the external bases  $m$ .



## Restricted partition functions

Define  $Q(m)$  to be the partition function over all structures with fixed number of external bases  $m$ .

- Can be calculated by similar recursion equation for  $Q_{1,j}(m)$
- $Z = \sum_{m=\Delta}^N Q(m)$
- $P(m) = Q(m)/Z$  is the probability to observe  $m$  external bases

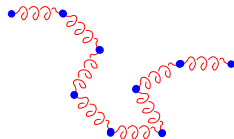
U. Gerland, RB, and T. Hwa, Biophys J., 2001



# Polymer Physics

## Backbone model

- For given  $m$  backbone is a **single-stranded RNA** molecule
- Well described by **extensible freely jointed chain**
- Distribution  $W(R|m)$  of **end-to-end-distance  $R$**  well known



## Including secondary structure

End-to-end distance distribution

$$P(R) = \sum_{m=\Delta}^N P(m)W(R|m)$$

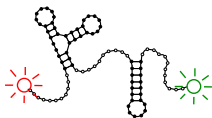
Force-distance relationship

$$F(R) = -k_B T \frac{\partial}{\partial R} \log \sum_{m=\Delta}^N Q(m)W(R|m)$$

# FRET

## Problem

- Energy transfer proportional to  $(R_0/R)^6$
  - Backbone **moves faster** than FRET time scale
- ⇒ Energy transfer is **not** given by  $(R_0/\langle R \rangle)^6$



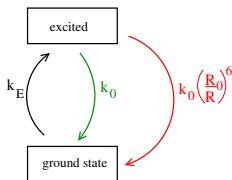
## Solution

- Monte-Carlo simulation of polymer dynamics

Murphy et al., Biophys. J. 2004

- Provides  $P(E_{FRET}|m)$
- Total expected FRET-distribution

$$P(E_{FRET}) = \sum_{m=\Delta}^N P(E_{FRET}|m)P(m)$$



# Overview


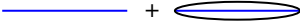

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  - Protein on single-stranded RNA
  - Including secondary structure
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# Protein on single-stranded RNA

## Main quantity

Define  $O_{i,j}$  to be the **partition function** of the single-stranded RNA from base  $i$  to base  $j$  in the presence of a single-stranded binding protein at **concentration**  $C$  with **footprint**  $F$  and dissociation constant  $K_D$ .

## Calculation

- $O_{i,j} = 1$  if  $j - i + 1 < F$  
- $O_{i,i+F-1} = 1 + C/K_D$  
- $O_{i,j} = O_{i,j-1} + C/K_D O_{i,j-F}$  
- Can include **sequence dependence** and **cooperativity**

# RNA secondary structure with protein interactions

## Secondary structure recursion

Protein-RNA interaction can be incorporated into secondary structure recursions by replacing 1 by  $O_{i,j}$

$$\begin{aligned}
 \overline{\overline{i \quad j}}_{Q_{i,j}} &= \overline{i \quad j}_{O_{i,j}} + \overline{i \quad j}_{W_{i,j}} + \sum_{k=i}^{j-h-2} \overline{\overline{i \quad k}}_{Q_{i,k}} \overline{k \quad k+1 \quad j}_{W_{k+1,j}} \\
 \overline{i \quad j}_{W_{i,j}} &= \sum_{k=i+h+1}^j \overline{\overline{i \quad k}}_{Q_{i+1,k-1}} \overline{k \quad k+1 \quad j}_{O_{k+1,j}}
 \end{aligned}$$

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# FRET prediction

## Implementation

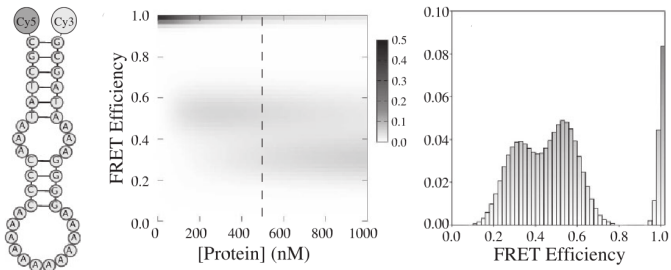
- Incorporate everything into **Vienna package**

Hofacker et al., Monatsh. Chemie, 1994

- Uses full **nearest neighbor model** and **Turner parameters**

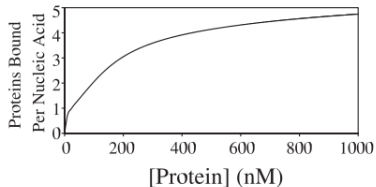
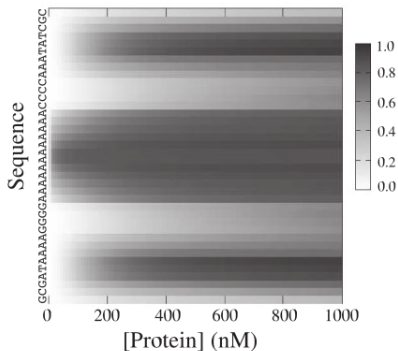
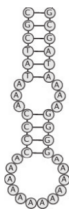
## Example

Protein with binding energy of 10kcal/mol at 1M and 6nt footprint



## Other observables

- Can extend ability to calculate **pair probabilities** to case with proteins
- Can calculate **protein occupancy** of every base
- Can calculate **number of bound proteins** by taking numerical derivative





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## Conclusions

- **Physical end-to-end distance** can be incorporated into RNA secondary structure prediction
- **Protein-RNA interactions** can be included in RNA secondary structure

## Future work

- Apply approach to **extract mechanistic parameters** from experiments
- **Double-stranded** RNA-binding proteins
- RNA **kinetics** in the presence of proteins

## Acknowledgements



Bob Forties

NSF DMR-0706002