Relativistic Bondi-type accretion in cosmological spacetimes

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Motivation

- A recent proposition by R. Penrose advocates a cyclic cosmological model in which each cycle in the history of the universe (aeon) starts and ends in a state where only radiative matter is present (Conformal Cyclic Cosmology). The late stage of the evolution of each aeon is dominated by the cosmological constant Λ.
- This allows for an introduction of a conformal factor that realizes a transition from a late part of a given aeon to the early part of the next aeon. A conformal structure of spacetime survives in such a transition, with a regular (finite) Weyl tensor.
- A key point in this scenario is to get rid of non-radiative matter in the late epoch of each aeon. There are some possibilities to do this, e.g., particle decay. An appealing idea: all matter falls into black holes, that can subsequently evaporate, producing mostly radiation.
- Here we investigate the possibility of an efficient accretion of matter onto black holes in the Λ-dominated universe.

- We will work in the framework of Einstein–Straus mode.
- A spherical region with Schwarzschild–(anti-)de Sitter metric is glued to an exterior FLRW solution. This gluing was worked out by Balbinot, Bergamini and Comastri Phys. Rev. D38, 2415 (1988).
- Then, it is enough to consider accretion occurring in the Schwarzschild–(anti)de Sitter spacetime.
- Here we consider the simpliest, spherically symmetric, steady Bondi-type accretion flows.
- One can also investigate a similar situation taking into account the self-gravity of the accreting fluid – a numerical work of Karkowski and Malec, Phys. Rev. D87, 04407 (2013).

Schwarzschild–(anti-)de Sitter metric in polar coordinates:

$$ds^2 = -\left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \frac{dr^2}{\left(1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2\right)} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right).$$

The horizons are located at zeros of $1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2$. For $0 < \Lambda < \frac{1}{9m^2}$ they are given by

$$\begin{aligned} r_a &= \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{\pi}{3} + \frac{1}{3}\arccos\left(3m\sqrt{\Lambda}\right)\right], \\ r_b &= \frac{2}{\sqrt{\Lambda}}\cos\left[\frac{\pi}{3} - \frac{1}{3}\arccos\left(3m\sqrt{\Lambda}\right)\right]. \end{aligned}$$

It is easy to show that $0 < 2m < r_a < 3m < \frac{1}{\sqrt{\Lambda}} < r_b < \frac{3}{\sqrt{\Lambda}}$. For $\Lambda < 0$

$$r_h = \frac{2}{\sqrt{|\Lambda|}}\sinh\left[\frac{1}{3}\operatorname{ar}\sinh\left(3m\sqrt{|\Lambda|}\right)\right].$$

It is convenient to introduce new Eddington–Finkelstein type coordinates. We define Eddington–Finkelstein time t_{EF} by

$$dt = dt_{\mathrm{EF}} - \frac{\frac{2m}{r} + \frac{\Lambda}{3} r^2}{1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2} dr.$$

In the new coordinates the metric can be written as

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right) dt_{\mathrm{EF}}^2 + 2\left(\frac{2m}{r} + \frac{\Lambda}{3} r^2\right) dt_{\mathrm{EF}}\, dr \\ &\quad + \left(1 + \frac{2m}{r} + \frac{\Lambda}{3} r^2\right) dr^2 + r^2 \left(d\theta^2 + \sin^2\theta\, d\phi^2\right). \end{aligned}$$

- The motion of the fluid is described by standard conservation laws

$$\nabla_\mu(\rho u^\mu) = \frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\rho u^\mu\right) = 0, \tag{1}$$

$$\nabla_\mu\left((e+p)u^\mu u^\nu + pg^{\mu\nu}\right) = 0. \tag{2}$$

Here u^μ denotes the four-velocity of the fluid, ρ is the baryonic density, p is the pressure, and e denotes the energy density. We will also use the specific enthalpy $h = (e+p)/\rho$ and the local speed of sound a.

- For a spherically symmetric, steady flow:

$$r^2\rho u^r = \mathrm{const}, \tag{3}$$

$$h\sqrt{1 - \frac{2m}{r} - \frac{\Lambda}{3}r^2 + (u^r)^2} = \mathrm{const}. \tag{4}$$

Remarkably, the above equations have the same form in the standard polar gauge, and in the Eddington–Finkelstein-type coordinates introduced in the previous slide.

- Analytic solutions can be found for equations of state $p = ke$, where $k = 1/3, 1/2, 1$.
- For polytropic fluids ($p = K\rho^\Gamma$) one needs simple numerics.
- We assume that the fluid region extends up to $r = r_\infty$, where the baryonic density is ρ_∞, and the speed of sound a_∞ (for polytropes). For polytropes we also fix Γ. For equations of state of the form $p = ke$ it makes sense to fix e_∞. We also fix m and Λ.
- We search for transonic solutions for which $a = |u^r/u_t|$ at some radius $r \equiv r_*$ (sonic point). They maximize accretion rate.

An example – solution for $p = e/3$. Define

$$
\begin{aligned}
X_2 &= -1 + \frac{2m}{r} + \frac{\Lambda}{3} r^2 + \frac{(1 - 9\Lambda m^2) r^2}{(3m)^2} \\
&\quad \times \cos\left\{ \frac{\pi}{3} - \frac{1}{3} \arccos\left[\frac{3^3 m^2 \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right)}{(1 - 9\Lambda m^2) r^2} \right] \right\}, \\
X_3 &= -1 + \frac{2m}{r} + \frac{\Lambda}{3} r^2 + \frac{(1 - 9\Lambda m^2) r^2}{(3m)^2} \\
&\quad \times \cos\left\{ \frac{\pi}{3} + \frac{1}{3} \arccos\left[\frac{3^3 m^2 \left(1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2\right)}{(1 - 9\Lambda m^2) r^2} \right] \right\}.
\end{aligned}
$$

The solutions are given by

$$
(u^r)^2 = \begin{cases} X_3, & r \geq 3m, \\ X_2, & r < 3m, \end{cases} \quad (u^r)^2 = \begin{cases} X_2, & r \geq 3m, \\ X_3, & r < 3m. \end{cases} \tag{5}
$$

They are subsonic (supersonic) outside the sonic sphere, respectively. The sonic sphere is located at $r_* = 3m$, irrespectively of Λ.

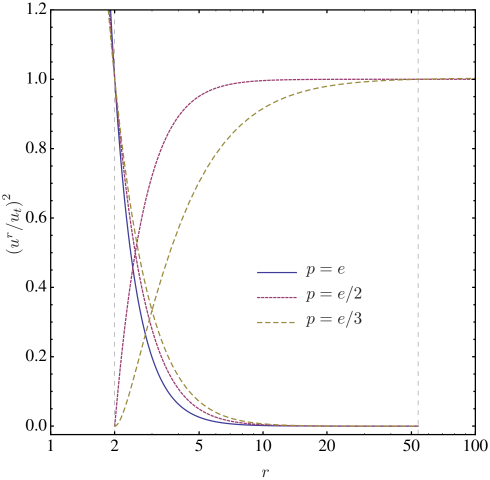

Figure : Solutions obtained for equations of state $p = ke$, with $k = 1/3, 1/2, 1$, $\Lambda = 1/1000$ and $m = 1$. Dotted vertical lines denote the locations of the horizons.

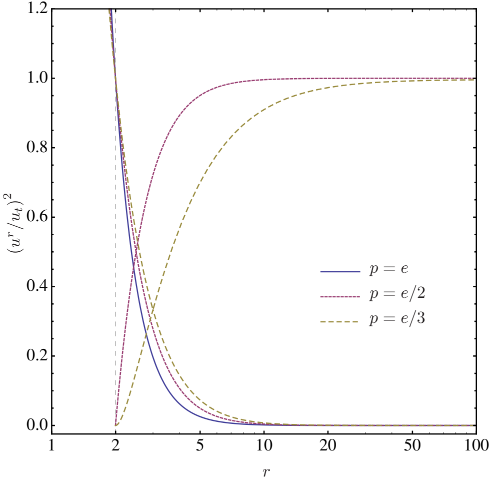

Figure : Solutions obtained for equations of state $p = ke$, with $k = 1/3, 1/2, 1$, $\Lambda = -1/1000$ and $m = 1$. The dotted vertical line denotes the location of the horizon.

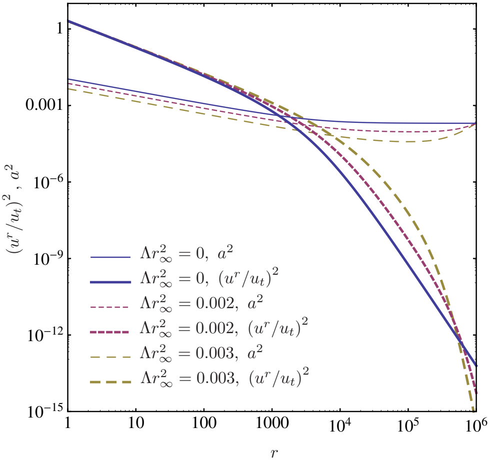

Figure : Transonic solutions obtained for the polytropic equation of state with $\Gamma = 4/3$, $m = 1$, $r_\infty = 10^6$ and $a_\infty^2 = 2 \times 10^{-4}$. The plot shows graphs of both a^2 and $(u^r/u_t)^2$ for three different values of the cosmological constant $\Lambda r_\infty^2 = 0, 2 \times 10^{-3}$ and 3×10^{-3}.

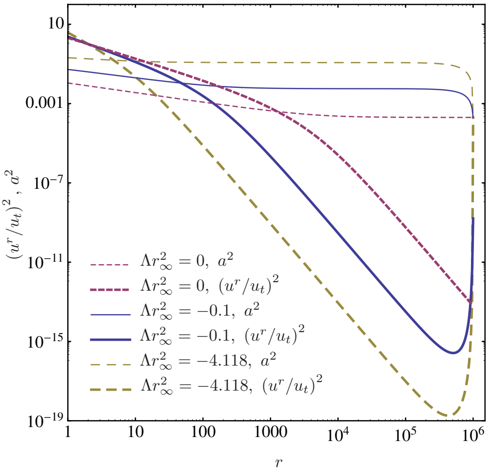

Figure : Same as in the previous figure, but for $\Lambda r_\infty^2 = 0, -0.1$ and -4.118.

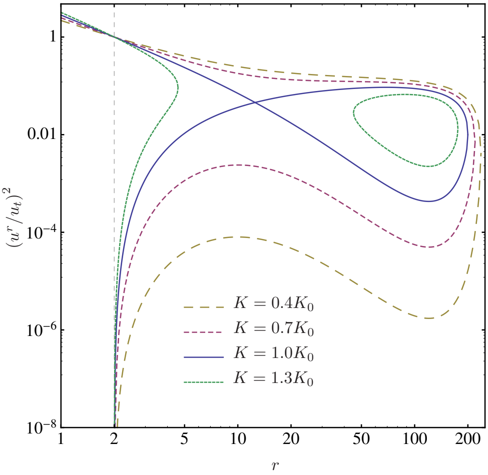

Figure : Solutions obtained for the polytropic equation of state with $\Gamma = 4/3$. Different curves correspond to solutions with different polytropic (entropy) constants K. The "homoclinic" solution (polytropic constant K_0) is characterized by the square of the speed of sound equal $a_\infty^2 = 1/100$ at $r_\infty = 200$. In this example $m = 1$, $\Lambda \approx -0.3328$.

Here we are mainly concerned with the efficiency of the accretion process. There are at least two sensible measures of the mass accretion rate:

$$\dot{m} = -4\pi r^2 \sqrt{1 - \frac{2m}{r} - \frac{\Lambda}{3} r^2 + (u^r)^2}\, u^r (e + p),$$

corresponding to the quasilocal mass

$$m(r) = 4\pi \int^r dr' \, r'^2 e,$$

and the baryonic mass accretion rate

$$\dot{B} = -4\pi r^2 u^r \rho,$$

corresponding to the baryonic mass

$$m_B = 4\pi \int^r dr' \, r'^2 u^t \rho.$$

For isothermal equations of state $p = ke$ with $k = 1/3, 1/2$ and 1, both accretion rates can be computed analytically. For $p = e/3$ we have

$$\dot{m} = \frac{16\pi}{3\sqrt{3}\sqrt[3]{2m^2}} \left(1 - 9\Lambda m^2\right)^{\frac{1}{3}} \left(r_\infty^2 |u_\infty^r|\right)^{\frac{4}{3}} e_\infty,$$

where

$$\begin{aligned} r_\infty^2 |u_\infty^r| \quad &= \quad r_\infty^2 \left\{ -1 + \frac{2m}{r_\infty} + \frac{\Lambda}{3} r_\infty^2 + \frac{(1 - 9\Lambda m^2) r_\infty^2}{(3m)^2} \right. \\ &\quad \left. \times \cos\left[\frac{\pi}{3} + \frac{1}{3}\arccos\left(\frac{3^3 m^2 \left(1 - \frac{2m}{r_\infty} - \frac{\Lambda}{3} r_\infty^2\right)}{(1 - 9\Lambda m^2) r_\infty^2}\right)\right] \right\}^{\frac{1}{2}}. \end{aligned}$$

- The gluing conditions of the Einstein–Straus vacuole impose constraints on the total mass contained in the vacuole. It makes sense to parametrize the solutions so that the total mass of the accretion system is kept constant.
- We define the mass of the fluid as

$$m_f = 4\pi \int_{r_h}^{r_\infty} dr r^2 e.$$

- One can show that for equations of state of the form $p = ke$ the mass of the fluid can be expressed as

$$m_f = 4\pi e_\infty (r_\infty^2 |u_\infty^r|)^{1+k} \int_{r_h}^{r_\infty} \frac{dr'}{r'^{2k}|u^r|^{1+k}}.$$

This gives the following formula for the mass accretion rate

$$\dot{m} = \frac{1+k}{\sqrt{k} r_*^{2k}} \left(\frac{m}{2r_*} - \frac{\Lambda}{6} r_*^2 \right)^{\frac{1-k}{2}} \left(\int_{r_h}^{r_\infty} \frac{dr'}{r'^{2k}|u^r|^{1+k}} \right)^{-1} m_f.$$

- An analogous result can be also obtained in the test-fluid polytropic case. Expressions for $\dot{B}$ can be also found.

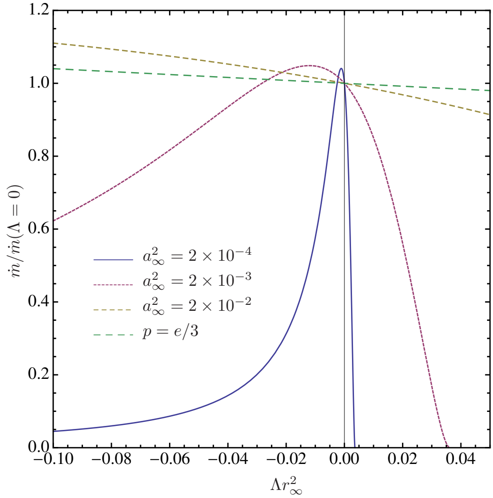

Figure : Dependence of the accretion rate $\dot{m}$ on Λ for systems with fixed mass m_f. We plot data corresponding to polytropic fluids with $\Gamma = 4/3$, $m = 1$, $r_\infty = 10^6$ and $a_\infty^2 = 2 \times 10^{-4}, 2 \times 10^{-3}$, and 2×10^{-2}. The last graph depicts data obtained for the equation of state $p = e/3$.

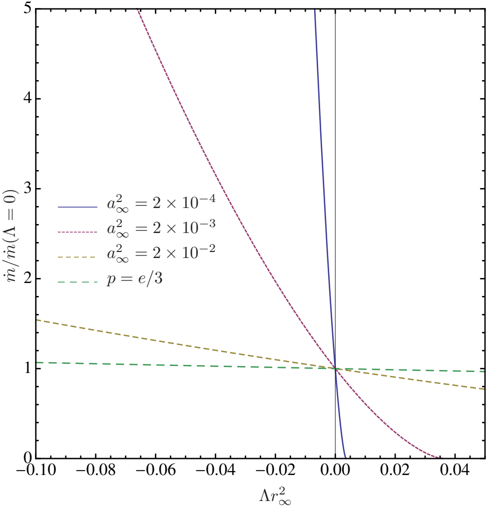

Figure : Dependence of the accretion rate $\dot{m}$ on Λ for system with fixed boundary energy density e_∞. Here all parameters are exactly the same as in the previous figure.

- For positive Λ the accretion rate decreases with Λ.
- For negative Λ the situation is complex. The relation between the accretion rate and Λ depends on the parametrization of solutions.
  - Polytropic configurations with a fixed total mass exhibit a maximum of $\dot{m}$ for some negative Λ. The accretion rate $\dot{m}$ decreases with Λ when the boundary value of the density is kept fixed.
  - For isothermal fluids $\dot{m}$ is a decreasing function of Λ in both parametrizations. The exception is given by the ultra-hard equation of state $p = e$, where the *baryonic* accretion rate has a maximum for some negative Λ.
- The behavior of test-fluid polytropes agrees with numerical results obtained for self-gravitating polytropes by Karkowski and Malec (2013).

Conclusions

- Global expansion can affect local structures through the cosmological constant, even in the above extremely simple scenario
- In the late Λ-dominated stages of the evolution of the Universe, stationary accretion on black holes is probably very inefficient.
- Most interesting phenomena occur in the $\Lambda < 0$ sector. There exist “homoclinic-type” solutions. Results concerning the relation between accretion rates and Λ depend on the parametrization of solutions.
- Not discussed here — stability. Linear stability against perturbations satisfying the potential flow condition was proved in PM & E. Malec, arXiv:1309.1546 (2013).