QUANTUM CORRELATIONS ACROSS THE DE SITTER HORIZON

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CSIC
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Encuentros Relativistas Españoles 2013

INTRODUCTION

- Our aim is to study the possible quantum influence across cosmological horizons

  - We construct a model of a spacetime with two regions, separeted by a cosmological horizon

  - We will find an exact quantization of the this spacetime

- The chosen model derives from de Sitter spacetime

- The result is closely related with black hole models and quantum wormholes

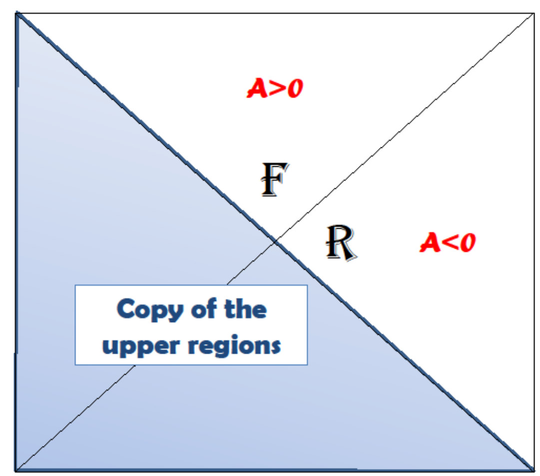
THE MODEL

➢ **We consider a model provided with a cosmological constant and described by this metric**

$$ds^2 = -\frac{N^2(T)}{A(T)}dT^2 + A(T)dR^2 + b^2(T)d\Omega_2^2$$

Classically

- **A>0 ➔ b²>Λ/3 Static de Sitter spacetime**
- **A<0 ➔ b²<Λ/3 Kantowski-Sachs spacetime**



Penrose Diagram

CONFIGURATION SPACE

- The Lagrangian in each region i=R,F is given by

$$L_i(N_i, A_i, b_i, \dot{A}_i, \dot{b}_i) = \frac{1}{2}\left(\frac{b_i \dot{b}_i \dot{A}_i}{N_i} + \frac{A_i \dot{b}_i^{\,2}}{N_i} - N_i + \Lambda b^2 N_i\right)$$

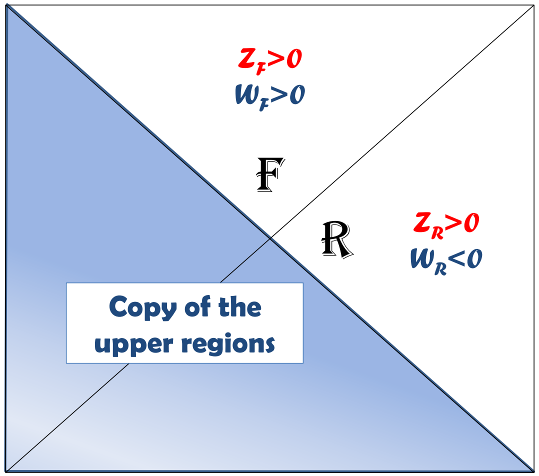
- Identifying an appropriately rescaled lapse function N'_i in each region, we can write the action in the Hamiltonian formalism as

$$S = \sum_i \int dt \left[\Pi_{A_i} \dot{A}_i + \Pi_{b_i} \dot{b}_i - N' H_i\right]$$

➤ **New configuration variables**

$$w_i = A_i b_i^2 \left| \frac{\Lambda b_i^2}{3} - 1 \right| , \quad z_i = \frac{A_i}{\left(\frac{\Lambda b_i^2}{3} - 1 \right)}$$

- **The classical solutions are** $z_i = 1$



Penrose Diagram

HAMILTONIAN CONSTRAINT

$$H = [w_R\partial_{w_R}w_R\partial_{w_R} + \frac{|w_R|}{4} + w_F\partial_{w_F}w_F\partial_{w_F} + \frac{|w_F|}{4} - \\ -z_R\partial_{z_R}z_R\partial_{z_R} - z_F\partial_{z_F}z_F\partial_{z_F}]$$

➢ **We choose the kinematical Hilbert space to be the tensor product spanned by**

$$\phi_{k_R}(z_R)\phi_{k_F}(z_F)\varphi_{K_R}(w_R)\varphi_{K_F}(w_F)$$

with $k_R, k_F, K_R, K_F > 0$

where

$$\begin{aligned}\phi_{k_i}(z_i) &= e^{\pm i k_i \ln z_i} \\ \varphi_{K_i}(w_i) &= C_{K_i} J_{2iK_i}(\sqrt{|w_i|}) + C'_{K_i} J_{-2iK_i}(\sqrt{|w_i|})\end{aligned}$$

PHYSICAL HILBERT SPACE

INDEPENDENT QUANTIZATION OF THE REGIONS

- We consider the Hilbert space as $\boxed{\mathcal{H} = \mathcal{H}^{\mathcal{R}} \oplus \mathcal{H}^{\mathcal{F}}}$

- Defining $\boxed{\alpha_i = \ln|w_i| \ , \quad \beta_i = \ln z_i}$

The Wheeler-De Witt equation for each region reads

$$\left[-\partial^2_{\alpha_i} + \partial^2_{\beta_i} - \frac{e^{\alpha_i}}{4}\right]\phi_{k_i}(z_i)\varphi_{K_i}(w_i) = 0$$

where $\boxed{k_i = K_i}$

- The physical states are given by

$$\Psi = \prod_{i=R,F} \int dk_i C_{k_i}\phi_{k_i}(z_i)\varphi_{k_i}(w_i)$$

QUANTIZATION WITH CORRELATIONS BETWEEN THE REGIONS

- **We consider the Hilbert space as** $\mathcal{H} = \mathcal{H}^{\mathcal{R}} \otimes \mathcal{H}^{\mathcal{F}}$

- **The Wheeler-De Witt equation takes the form**

$$\left[-\partial^2_{\alpha_R} + \partial^2_{\beta_R} - \frac{e^{\alpha_R}}{4} - \partial^2_{\alpha_F} + \partial^2_{\beta_F} - \frac{e^{\alpha_F}}{4} \right] \Psi = 0$$

- **ln z_i behaves like an internal time in each region**

- **We define a common internal time (ln z) with the identification**

$$\ln z_R = \ln z_F$$

So now the states are spanned by $\phi_k(z) \varphi_{K_R}(w_R) \varphi_{K_F}(w_F)$

- **The physical Hilbert space is characterized by the dispersion relation**

$$\boxed{2k^2 = K_R^2 + K_F^2}$$

- **The physical states are given by**

$$\boxed{\Psi = \int dK_R dK_F C_{K_R K_F} \phi_k(z) \varphi_{K_R}(w_R) \varphi_{K_F}(w_F)}$$

- **The solutions k=0 that imply K_i=0 have the asymptotic form**

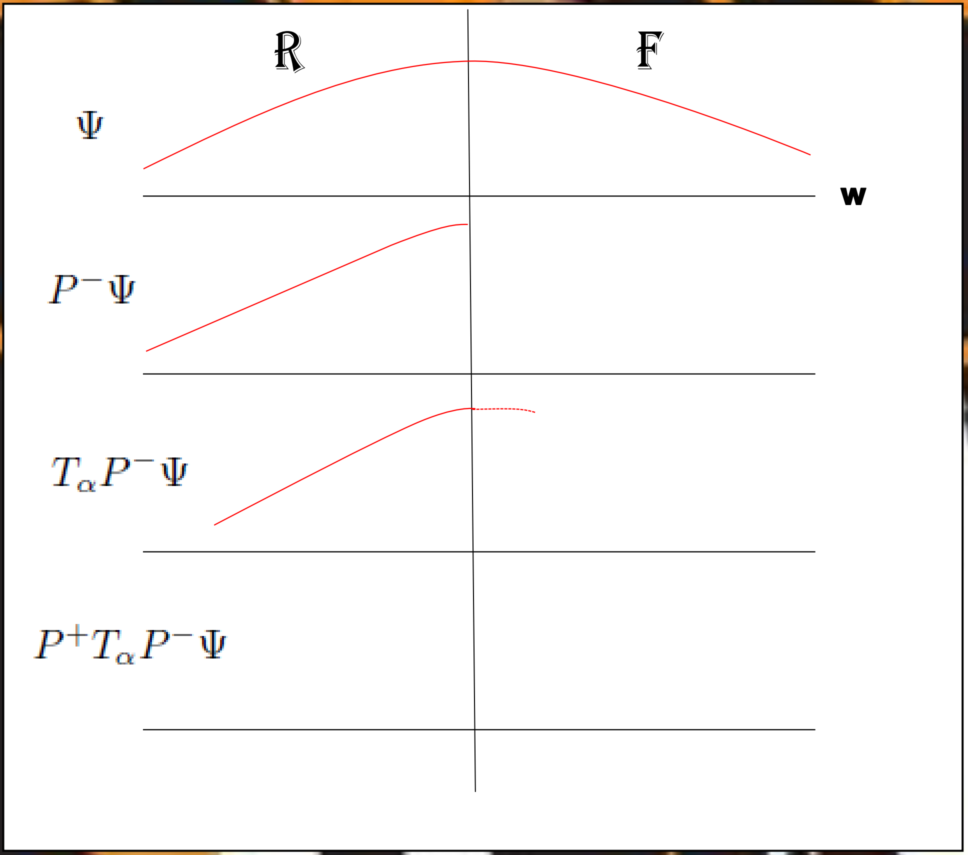
$$\boxed{\Psi \sim (|w_R||w_F|)^{-1/4} \cos\sqrt{|w_R|} \cos\sqrt{|w_F|}}$$

which is sharply peaked around the semiclassical solutions

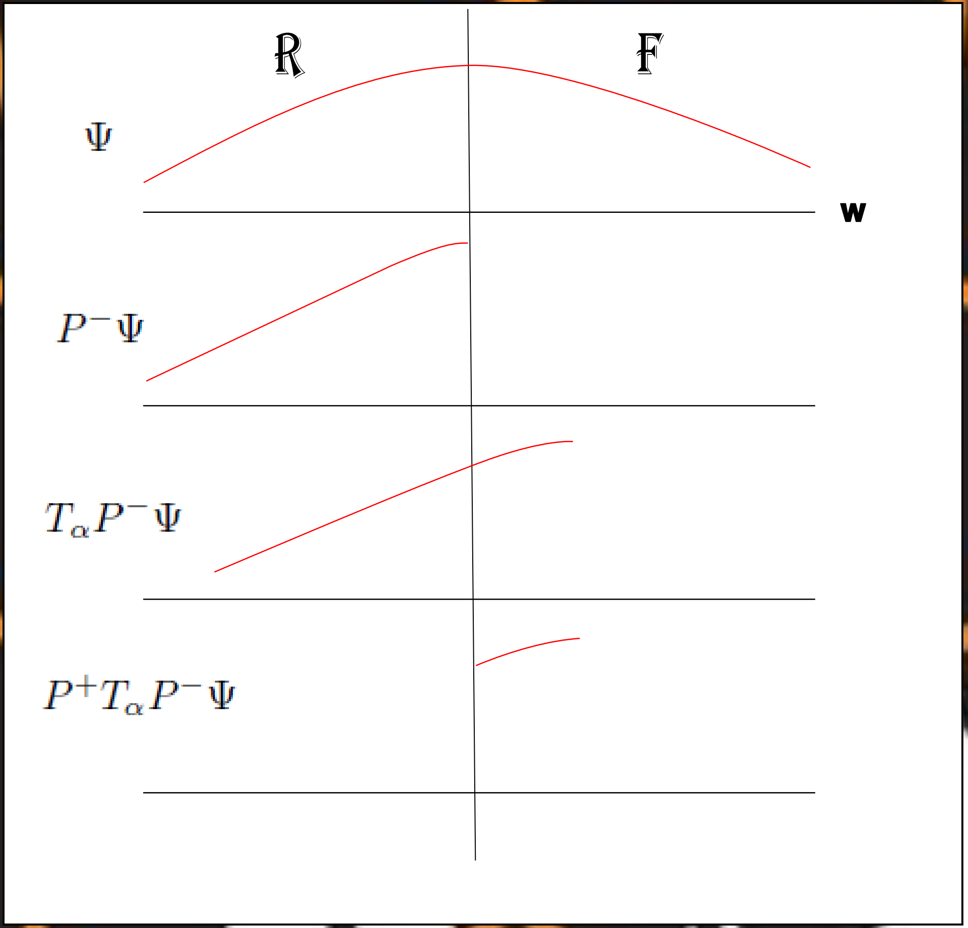
QUANTUM CORRELATIONS

- **We define a translation operator (T_α) in the variable w**

INDEPENDENT QUANTIZATION OF THE REGIONS



QUANTIZATION WITH CORRELATIONS BETWEEN THE REGIONS



- We have a new contribution coming from the region R in the region F

CONCLUSIONS

- **Exact quantization of the (maximal) extension of the de Sitter spacetime**

- **Recover semiclassical behavior**

- **We construct a quantization that establishes quantum correlations between both regions, mixing them**

- **We can find effects in one region by tracing out the other one, which is not accessible**

- **The analysis of the correlation in the case of the translation operator can be extended to other (more physical) operators mixing both regions**

- **Work in progress…**

In memory of Pedro F. González Díaz