Anthropic states in the multi-sphere Einstein-Maxwell Landscape

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Benasque — September 9, 2013

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arXiv:1207.4908 [hep-th] Phys.Rev.Lett. 110 (2013) 041602,

arXiv:1207.4662 [hep-th] Phys.Rev. D87 (2013) 023503

Overview

In order to understand the actual Universe a toy landscape model is studied, with the following program:

- The Cosmological Constant problem: A recipe
- The Bousso-Polchinski (BP) model: A brief presentation
- The Multi-Sphere Einstein-Maxwell (MS-EM) model
- Identifying anthropic states in the MS-EM Landscape

A Huge Discrepancy

Standard Model of Particle Physics: $\rho_{\mathsf{vac}} = \Lambda \approx 1$

$+$

Standard Model of Cosmology: $\Lambda \approx 10^{-120}$

$=$

Cosmological Constant Problem!

We use units in which $8\pi G = \hbar = c = 1$.

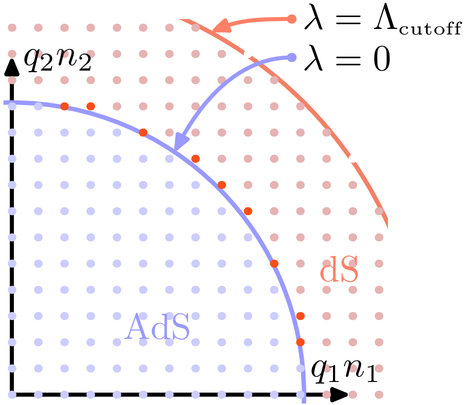
A recipe

To approach a solution of the CC problem we need three ingredients:

- A huge landscape of vacua with a dense spectrum (*the discretuum*) able to accommodate a vacuum energy like the observed one
- A cosmological scenario that explores all possible universes (a multiverse), for example Eternal Inflation
- An environmental criterion (also named *Anthropic Principle*) to justify the observed universe

Each ingredient is accompanied by a probability measure. We study mainly the first one

The Bousso-Polchinski (BP) model



The states of the BP Landscape are represented by the nodes $(n_1, \cdots, n_J)$ of an J-dimensional integer lattice, whose effective cosmological constant λ is given by

$$\lambda = \Lambda + \frac{1}{2} \sum_{i=1}^{J} q_i^2 n_i^2$$

The parameters of the model are $\Lambda < 0$, a bare cosmological constant, and the charges $\{q_j\}$.

Properties of the BP Landscape

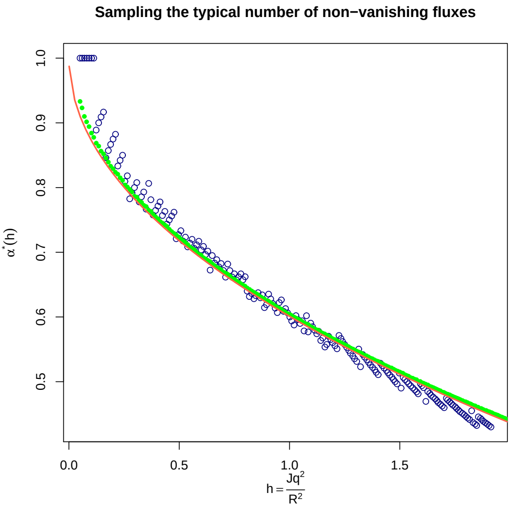
Virtues

1. If J is large the BP Landscape can accommodate states with a very small λ
2. Inspired by String/M-Theory

Pitfalls

1. The stability of the moduli is not known
2. The percentage of anthropic states is very small
3. The number of occupied fluxes $\alpha^* J$ has $\alpha^* < 1$: The "α^*-problem"
4. KKLT mechanism requires a large flux

A tension



At the end we have encountered a tension:

Large J to provide a small CC

and

Small J to have stable moduli

Looking for a resolution

Properties of a model to address the α^*-problem

1. Exactly (with controlled approximations) solvable
2. With a mechanism of stabilization as in the $5+1$ Einstein-Maxwell compactifications:
   - $\mathrm{(A)dS}_2 \times \mathrm{S}^4$
   - $\mathrm{(A)dS}_4 \times \mathrm{S}^2$

   with a single modulus (the sphere radius)
3. Many moduli $\Rightarrow$ many spheres

$$\mathrm{(A)dS}_2 \times \prod_{i=1}^{J} \mathrm{S}^2$$

One Flux Compactification

The metric splits

$$\mathrm{d}s^2 = e^{2\phi}(-\,\mathrm{d}t^2 + \,\mathrm{d}x^2) + e^{2\psi}(\,\mathrm{d}u^2 + \,\mathrm{d}v^2)$$

- The **cosmological part** $\phi(t,x)$ represents a two-dimensional spacetime of constant curvature λ, which is de Sitter (dS_2) if $\lambda > 0$, Minkowski (M_2) if $\lambda = 0$ and anti-de Sitter (AdS_2) if $\lambda < 0$. λ can be interpreted as the cosmological constant of the dimensionally reduced cosmological model.
- The **compact part** $\psi(u,v)$ represents a compact surface of constant curvature K. We can choose K positive, and then the surface will be a S^2 sphere of radius $1/\sqrt{K}$.

Einstein-Liouville equations

The model has a cosmological constant Λ and a magnetic field

$$\mathbf{F} = \frac{Q}{V}\, e^{2\psi(u,v)}\, \mathrm{d}u \wedge \mathrm{d}v$$

with V the area of the sphere and Q the magnetic flux:

$$V = \frac{4\pi}{K}\,, \quad \int_{\mathsf{S}^2} \mathbf{F} = Q$$

ϕ and ψ are uncoupled and satisfy Liouville equations of $-+$ and $++$ signatures: The Gaussian curvature of each surface is constant (λ and K resp.)

$$(\phi_{tt} - \phi_{xx})e^{-2\phi} = \Lambda - \left(\frac{Q}{V}\right)^2 = \lambda$$

$$-(\psi_{uu} + \psi_{vv})e^{-2\psi} = \Lambda + \left(\frac{Q}{V}\right)^2 = K$$

The two branches

The relation $V = \frac{4\pi}{K}$ leads to an algebraic equation for K, with two solutions:

$$K_{\pm} = 2\Lambda\left(\frac{Q_{\mathsf{max}}}{Q}\right)^2\left[1 \pm \sqrt{1-\left(\frac{Q}{Q_{\mathsf{max}}}\right)^2}\right], \qquad Q_{\mathsf{max}} = \frac{2\pi}{\sqrt{\Lambda}}.$$

The two-dimensional cosmological constant has also two branches

$$\lambda_{\pm} = 2\Lambda - K_{\mp}.$$

Dirac quantization condition implies $Qe = 2\pi n$ with $n \in \mathbb{Z}$ that translates in a maximum value of the integer n,

$$n_{\mathsf{max}} = \left\lfloor \frac{e}{\sqrt{\Lambda}} \right\rfloor$$

Modulus Stabilization

Allow a dependence of the radius of S^2 on t, x

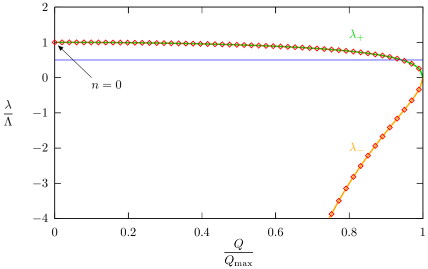
$$ds^2 = e^{2\phi(t,x)-2\xi(t,x)}\left(-\,dt^2 + dx^2\right) + e^{2\psi(u,v)+2\xi(t,x)}\left(du^2 + dv^2\right).$$

- The previous ϕ, ψ and $\mathbf{F}$ are backgrounds for ξ
- Integrating u, v we obtain a $1+1$ eff. theory with a self-interacting radion
- The stability condition is $2K - 3\Lambda > 0$.
  - $K_+ > \frac{3}{2}\Lambda$ always and therefore **all AdS_2 states are stable**
  - For the K_- branch $K_- > \frac{3}{2}\Lambda$ if $Q > Q_{\text{min}} = \frac{2\sqrt{2}}{3} Q_{\text{max}}$. Upon quantization all states in the dS_2 branch characterized by a quantum number $n < n_{\text{min}} = \left\lceil \frac{Q_{\text{min}} e}{2\pi} \right\rceil$ are unstable: **Stability of dS_2 requires large flux**

The one flux EM Landscape

The number of stable states ($n_{\text{min}} \leq n \leq n_{\text{max}}$) in this one-flux landscape is $\mathcal{N}_1 = \underbrace{n_{\text{max}}}_{\text{AdS}_2} + \underbrace{n_{\text{max}} - n_{\text{min}} + 1}_{\text{dS}_2}$ that is

$$\mathcal{N}_1 = 2\left\lfloor \frac{e}{\sqrt{\Lambda}} \right\rfloor - \left\lceil \frac{2\sqrt{2}}{3} \frac{e}{\sqrt{\Lambda}} \right\rceil + 1 \approx \frac{e}{\sqrt{\Lambda}} \left(2 - \frac{2\sqrt{2}}{3} \right) .$$



With $\Lambda/e^2 = 3.915 \times 10^{-4}$, $\mathcal{N}_1 = 53$ (50 AdS_2 and 3 dS_2 states)

Multi-sphere compactification

The ansatz representing a manifold of the form (A)dS$_2$ $\times$ $\left[\mathrm{S}^2\right]^J$ is

$$\mathrm{d}s^2 = e^{2\phi(t,x)}\left(-\,\mathrm{d}t^2 + \,\mathrm{d}x^2\right) + \sum_{i=1}^{J} e^{2\psi_i(u_i,v_i)}\left(\,\mathrm{d}u_i^2 + \,\mathrm{d}v_i^2\right).$$

The moduli are the J radii of the spheres
The uncoupled Liouville equations are

$$\lambda = \left(\phi_{tt} - \phi_{xx}\right)e^{-2\phi}\,, \qquad K_i = -\left(\partial_{u_i}^2\psi_i + \partial_{v_i}^2\psi_i\right)e^{-2\psi_i}\,,$$

where

- λ is the curvature of the AdS$_2$ ($\lambda < 0$) or dS$_2$ part ($\lambda > 0$)
- K_i is the Gaussian curvature of the i^{th} sphere S^2.

The MS-EM Landscape

- As before the only parameter of the model is $\Lambda > 0$
- The Einstein equations distributes the curvatures

$$\Lambda = \frac{1}{2}\left(J\lambda + \sum_{i=1}^{J} K_i\right).$$

- We also have two branches for each curvature

$$K_j^{(\pm)} = \frac{4\pi^2}{Q_j^2}\left[1 \pm \sqrt{1 - 2\lambda\,\frac{Q_j^2}{4\pi^2}}\,\right].$$

- The Maxwell field on each S^2 has a magnetic flux Q_j which satisfies the Dirac quantization condition $Q_j e = 2\pi n_j$ with $n_j \in \mathbb{Z}$

The constitutive equation

Inserting the curvatures $K_j^{(\pm)}$ into the Einstein equations gives (redefining $\frac{\Lambda}{e^2} \to \Lambda$)

$$\Lambda = L_n(\lambda) \equiv \frac{1}{2}\Big[J\lambda + \sum_{j=1}^{J} \frac{1}{n_j^2}\Big(1 + s_j\sqrt{1 - 2\lambda\, n_j^2}\,\Big)\Big]\,.$$

- $s_j = \pm$ gives different equations (branches) for each given node $n = (n_1, \cdots, n_J)$
- For AdS states ($\lambda < 0$), positivity of all curvatures K_j implies $s_j = +$ for all J: the **principal branch**
- Reality of λ implies the existence of a branching point $\lambda_{\rm b} = \frac{1}{2\,\text{máx}_{1\le j\le J}\{n_j^2\}}$ with $\lambda < \lambda_{\rm b}$

The stability criterion

Therefore, states might exist if adequate solutions are found to the constitutive equation, but they will be true physical states only if they are stable. We proceed:

1. Perturbing the ansatz with a multiradion (J moduli $\xi_i(t,x)$)

$$ds^2 = e^{2\phi - 2\sum_{i=1}^{J}\xi_i}\left(-\mathrm{d}t^2 + \mathrm{d}x^2\right) + \sum_{i=1}^{J} e^{2\psi_i + 2\xi_i}\left(\mathrm{d}u_i^2 + \mathrm{d}v_i^2\right).$$

2. Determine the eom's of the perturbed metric
3. Insert the curvatures λ, K_i of the unperturbed solution, thus neglecting the backreaction of the perturbations on the cosmological part
4. Linearize the equations of motion about the unperturbed solution $\xi_i = 0$

The stable states

The multiradion satisfies a Klein-Gordon equation

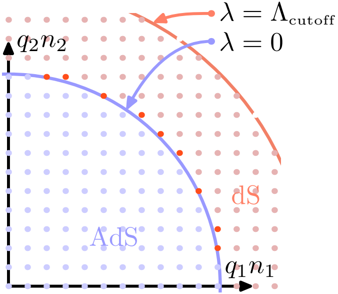
$$e^{-2\phi}\left[\partial_{tt}\boldsymbol{\xi} - \partial_{xx}\boldsymbol{\xi}\right] = -H\boldsymbol{\xi}\,,$$

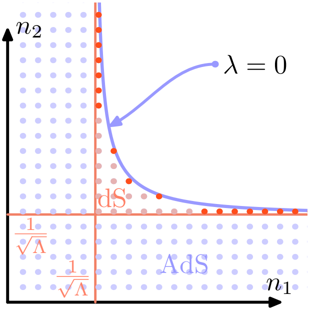
where $\boldsymbol{\xi}$ is the column vector of the radions and $H(\lambda, \{n_i\})$ is a constant matrix formed out of a solution of $\Lambda = L_n(\lambda)$. Stability implies $H > 0$.
We find:

- All AdS states are stable.
- All dS states having at least a vanishing flux number $n_i = 0$ are unstable.
- All dS states coming from a non-principal branch are unstable. This leaves the principal branch as the only source of AdS and stable dS states.

No α^* problem and state chains

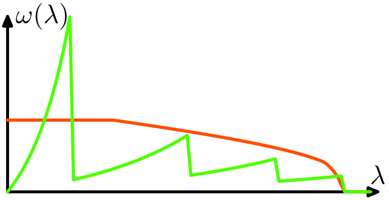
- Minkowski states with $\lambda = 0$ can exist if and only if $\sum_{i=1}^{J} \frac{1}{n_i^2} = \Lambda$ (in the BP case $2|\Lambda| = \sum_{i=1}^{J} n_i^2 q_i^2$)
- The $\lambda = 0$ hypersurface has asymptotic hyperplanes given by $|n_i| = \frac{1}{\sqrt{\Lambda}}$, and no dS state can exist below this value. The α^*-problem **is absent** in the MS-EM landscape.
- The null-λ hypersurface allows the existence of **state chains**





Distribution of the Cosmological Constant ($J = 2$)

- Effective cosmological constant distribution of a $J = 2$ BP landscape (flat curve), and a $J = 2$ MS-EM landscape (jagged curve)
- The BP λ distribution is approximately constant near $\lambda = 0$
- The MS-EM λ distribution has a dominant peak coming from the longest state chains, and subdominant peaks separated by a gap from the dominant one



Anthropic states in the MS-EM landscape

Step-by-step construction of anthropic states. Calling $\Lambda \equiv \Lambda_1$ for the start of a recurrence

$$\sum_{j=2}^{J} \frac{1}{n_j^2} = \Lambda_1 - \frac{1}{n_1^2} \equiv \Lambda_2 > 0 \quad \Rightarrow \quad n_1 = \left\lceil \frac{1}{\sqrt{\Lambda_1}} \right\rceil .$$

The recurrence relation $\Lambda_{j+1} = \Lambda_j - \frac{1}{n_j^2}\,, \qquad n_j = \left\lceil \frac{1}{\sqrt{\Lambda_j}} \right\rceil$ gives

the best integer choice at each step for getting the smallest possible difference between the two sides of the formula

$$\sum_{i=j}^{J} \frac{1}{n_i^2} = \Lambda_j \,.$$

For the last step the floor function is taken instead of the ceiling

$$n_J = \left\lfloor \frac{1}{\sqrt{\Lambda_J}} \right\rfloor ,$$

A fixed point iteration

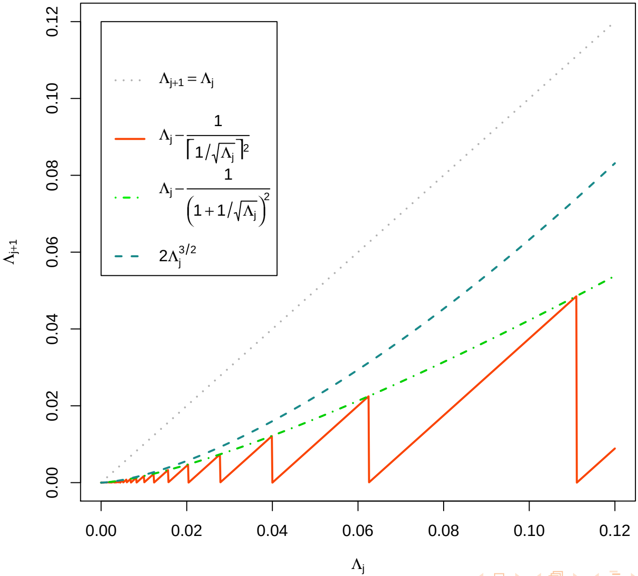
- Before the final step closes the algorithm, we can rewrite the recurrence relation as a fixed-point iteration:

$$\Lambda_{j+1} = f(\Lambda_j)\,, \quad \text{with} \quad f(x) = x - \frac{1}{\left\lceil \frac{1}{\sqrt{x}} \right\rceil^2}$$

- Superlinear behaviour of $f(x)$ near $x \to 0$ implies a fast convergence
- First-order approximation of $f(x)$ ($\Lambda_{j+1} = f(\Lambda_j) < 2\Lambda_j^{\frac{3}{2}}$) exactly solvable giving a double-exponential rate

$$\Lambda_j = 2^{\sum_{k=0}^{j-2} \left(\frac{3}{2}\right)^k} \Lambda_1^{\left(\frac{3}{2}\right)^{j-1}} \qquad (j \geq 2)$$

Iteration function



The anthropic chains

- The $\{n_j\}$ obtained in this way represents always an stable state
- $\{n_j\}$ is the end of a very long state chain given a narrow peak in the λ-distribution
- The number of states in the interval $[0, \Lambda_J/2]$ is $\sim n_J$
- We can find the whole peak inside the anthropic range $0 \leq \lambda \leq \lambda_{\mathrm{A}}$ for a given Λ by equating $\lambda_{\mathrm{A}} = \Lambda_J/2$ and solving for J, resulting in $J = 1 + \log_{\frac{3}{2}}\left(\frac{\log(\lambda_{\mathrm{A}}/2)}{\log(4\Lambda)}\right)$.

Examples

- Example 1: With $\lambda_{\rm A} = 10^{-120}$, we can obtain an anthropic peak using $\Lambda = 0.1$ and $J = 15$, yielding $\sim 10^{57}$ states
- Example 2: Using $\Lambda = 0.0002$ and $J = 10$ we obtain 10^{59} anthropic states with the same $\lambda_{\rm A}$
- The MS-EM landscape contains a huge amount of anthropic states with moderate values of J for any Λ. No fine-tuning is needed
- The anthropic states are non-generic despite being very numerous
- Nevertheless, the peak in the prior distribution can be made very narrow when compared with the full anthropic range, and thus the anthropic factor influencing the cosmological constant prediction can be considered as almost constant

Conclusions

- We have studied the MS-EM landscape where stable (A)dS can be formed
- The α^*-problem is absent in our model
- Extrapolating the results obtained in the MS-EM model to the BP one, would dramatically change their conclusions
- The MS-EM landscape has long chains with low λ states enlarging the proportion of anthropic states
- All these features extend to the $d+1$ case, with a different density of states favouring even smaller λ values
- Future work is projected to explore the probability distribution of a multiverse setup

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