Imprint of Exotic Matter on Quasi-normal modes of Neutron Stars

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1. Introduction

2. Numerical Method

3. Results:
   a) Axial and polar fundamental wI mode
   b) Axial wII mode
   c) Polar f mode
   d) Polar p mode
   e) Empirical relations

4. Conclusions

1. Introduction

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- *We consider an static spherical space-time*

$$ds^2 = - e^{2\psi(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2 \left[d\theta^2 + \sin^2\theta \, d\varphi^2 \right]$$

- *Two regions*

  Interior of the star perfect fluid with eos $p = p(\rho)$
  Empty outside

- *Following Chandrasekhar and Ferrari we make perturbations over this metric, expanding in tensor harmonics, and taking into account both axial and polar perturbations:*

  Under parity transformation $\theta \to \pi - \theta$, $\varphi \to \varphi + \pi$
  Axial transform like $(-1)^{l+1}$.
  Polar transform like $(-1)^{l+2}$

$$g_{\mu\nu} = g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu}$$

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu}$$

- We solve Einstein Equations up to first order in perturbation theory.

For static functions:

$$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \frac{dp}{dr} = \frac{(4\pi r^3 p + M)(\rho + p)}{(r - 2M)r} \qquad \frac{d\upsilon}{dr} = \frac{-1}{(\rho + p)}\frac{dp}{dr}$$

First order perturbation: AXIAL

Regge-Wheeler equation

$$\frac{d^2 Z^{lm}}{dr_*^2} + \left[\omega - V^l(r)\right] Z^{lm} = 0$$

$$V^l(r) = \frac{e^{2\upsilon}}{r^3}\left[l(l+1)r + 4\pi r^3(\rho + p) - 6M\right]$$

Axial modes do not couple to matter perturbations

First order perturbation: POLAR

Inside the star:

Lindblom equations $\left\{ V^{lm}, H_{(1)}{}^{lm}, K^{lm}, W^{lm} \right\}$

Outside the star:

Zerilli equation

$$\frac{d^2 Z^{lm}}{dr_*^2} + \left[\omega - V^l(r) \right] Z^{lm} = 0$$

$$V^l(r) = \frac{2(r-2M)}{r^4(nr+3M)^2}\left[n^2(n+1)r^3 + 3Mn^2r^2 + 9M^2nr + 9M^3 \right]$$

$$n = \frac{(l-1)(l-2)}{2}$$

Polar modes couple to pressure
and density perturbations

- *A general solution will be a superposition of an outgoing wave and an ingoing wave:*

$$\lim_{r\to\infty} Z_{out} \sim e^{-i\omega r_*}$$

$$\lim_{r\to\infty} Z_{in} \sim e^{i\omega r_*}$$

- *The outgoing contribution diverges at radial infinity*

- *The ingoing contribution tends exponentially to zero at radial infinity*

⇓

A small contamination of the outgoing wave behaviour faraway of the star gives rise to an important ingoing wave contribution nearby the star.

1. Introduction

Families of modes for static (realistic) stars.

Space-time modes: Axial and Polar

wI modes: curvature modes. Infinite in number. ~10 khz ~0.1 ms
wII modes: interface modes. ~1 khz ~0.1 ms

Fluid modes: Polar

f mode: fundamental mode. Only one. ~1 khz ~0.1 s
p modes: pressure modes. Infinite in number. ~1 khz ~1 s

Other modes we do not consider (gravity modes, rotational modes…)

2. Numerical Method

2. Numerical Method

Exterior part of the solution:

$$\lim_{r\to\infty} Z_{out} \sim e^{-i\omega r_*} = e^{-i\omega_R r_*} e^{r_*/\tau}$$

- *Two main problems for outgoing quasi-normal modes:*

  Infinite oscillations

  Divergence towards infinity

- *We can combine two methods in order to deal with these issues: (Andersson et al 1995)*

  Integration of the phase of the Regge-Wheeler/Zerilli function

  Exterior Complex Scaling

Studying the phase function we deal with the oscillations

$$g = \frac{Z'}{Z}$$

$$\partial_r g + g^2 \frac{\omega^2 r^2}{(2-2M)^2} + \frac{2M}{r(r-2M)} g - \frac{l(l+1)r - 6M}{r^4 (r-2M)} = 0 \qquad \textit{(axial)}$$

Without oscillations we can compactify the radial coordinate.

Far from the star:

$$g \sim i\omega \frac{Ae^{i\omega r} - Be^{-i\omega r}}{Ae^{i\omega r} + Be^{-i\omega r}}$$

Incoming wave: $g \sim i\omega$ *Mixture:* $g \sim -i\omega$

We can not distinguish between mixture and outgoing wave behaviour.

2. Numerical Method

Exterior complex scaling:
(J. Aguilar amd J. Combes, Commun. Math. Phys. 22 269 (1971))
(G. Alvarez et al, PRA 44, 3060 (1991))

$$r = a + y e^{-i\alpha}$$

Parametrize the radius along a line in the complex plane.

The boundary condition is dependent of α.

We use α as a free parameter, allowing us to make:

$$\omega_R \sin\alpha + \omega_I \cos\alpha > 0$$

Outgoing wave: $g \sim -i\omega$ *Mixture:* $g \sim i\omega$

Using α as a free parameter for the integration path, the boundary condition for the phase fixes completely the outgoing wave behaviour

2. Numerical Method

Realistic Equations of State:

Important constraint to realistic equations of state after the discovery of the 2 Msun pulsar PSR J164-2230

1. *Piese-wise polytrope interpolation (Read et al 2008)*
2. *Hermite interpolation of tabulated equations of state*

- *Hyperon matter.*

GNH3 - N. Glendenning, APJ 293, 470 (1985)
H4 - B. D. Lackey, M. Nayyar, and B. J. Owen, PRD 73 (2006)
WCS1-2 - S. Weissenborn, D. Chatterjee, and J. Schaner-Bielich, PRC 85 (2012)
BHZBM - Bednarek, I., Haensel, P., Zdunik, J. L., Bejger, M., and Manka, R., A&A 543 (2012)

2. Numerical Method

- *Quark matter*

Hybrid stars:

ALF2-4 - M. Alford, M. Braby, M. Paris, and S. Reddy, APJ 629 (2005)
WSPHS3 - S. Weissenborn, I. Sagert, G. Pagliara, M. Hempel. and J. Schaner-Bielich, APJL 740 (2011)

Hybrid stars with hyperons:

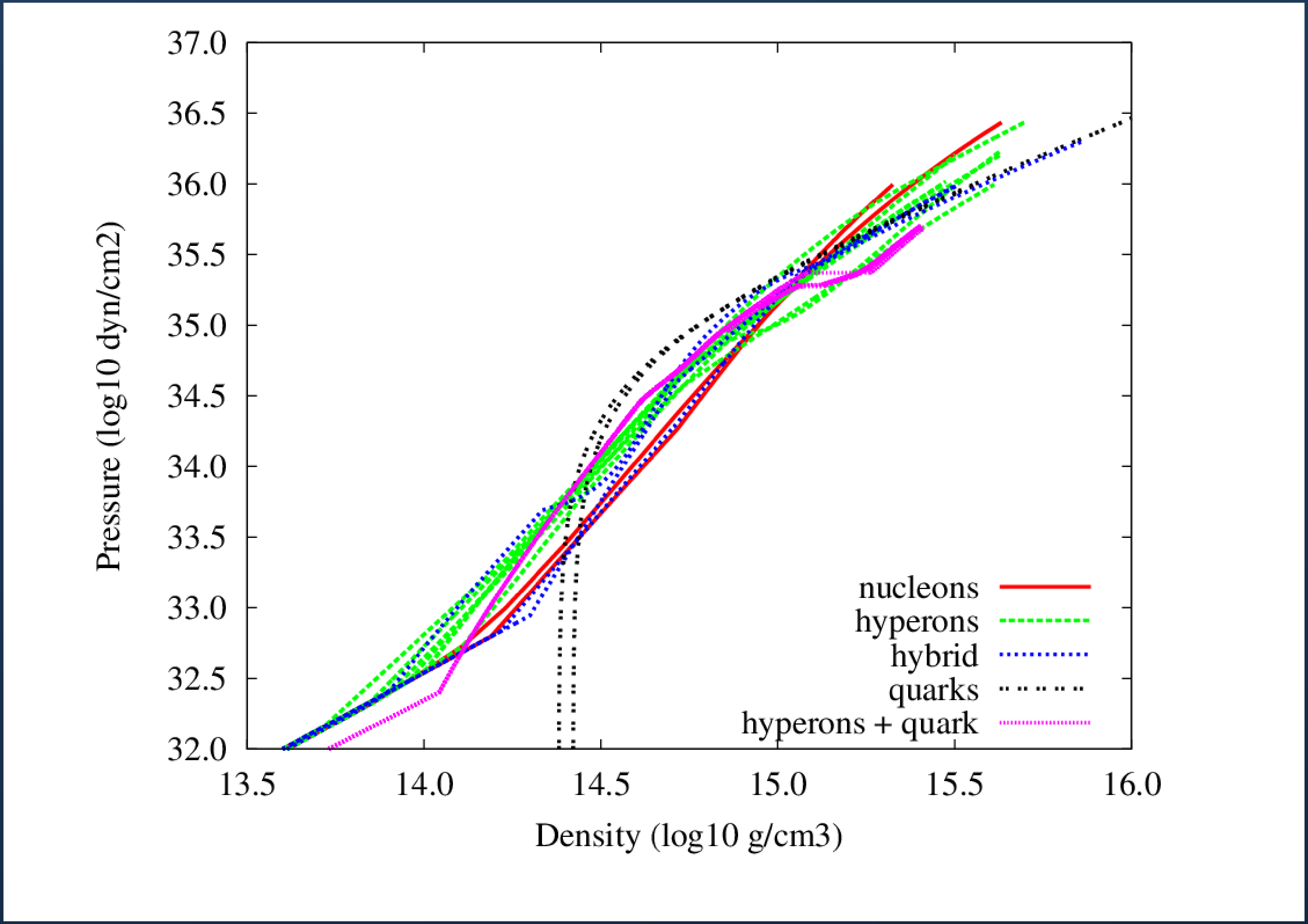
BS1-4 - Bonanno, Luca and Sedrakian, Armen, A&A 539 (2012)

Quark stars:

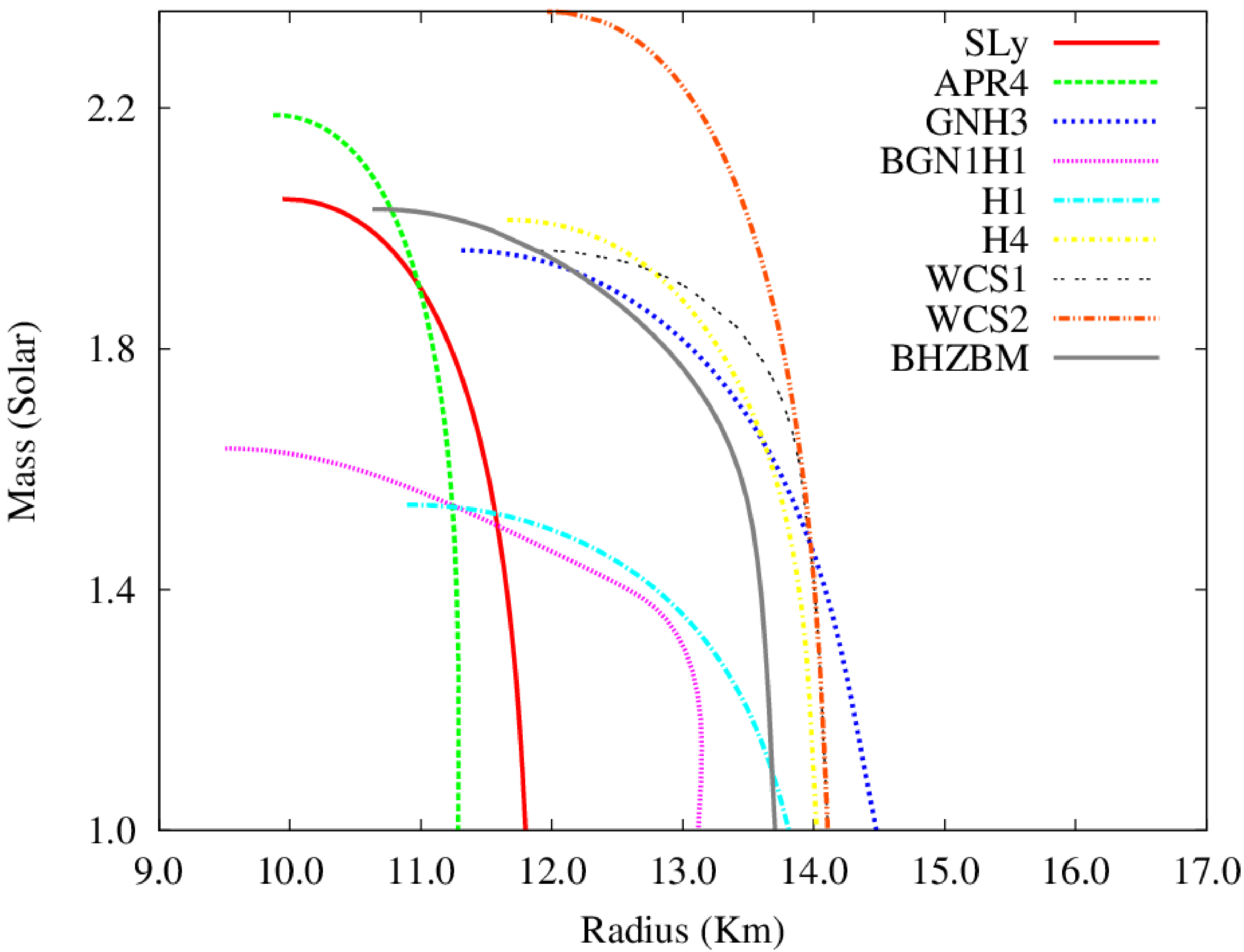
WSPHS1-2 - S. Weissenborn, I. Sagert, G. Pagliara, M. Hempel. and J. Schaner-Bielich, APJL 740 (2011)

- *Nuclear matter: SLy and APR4*

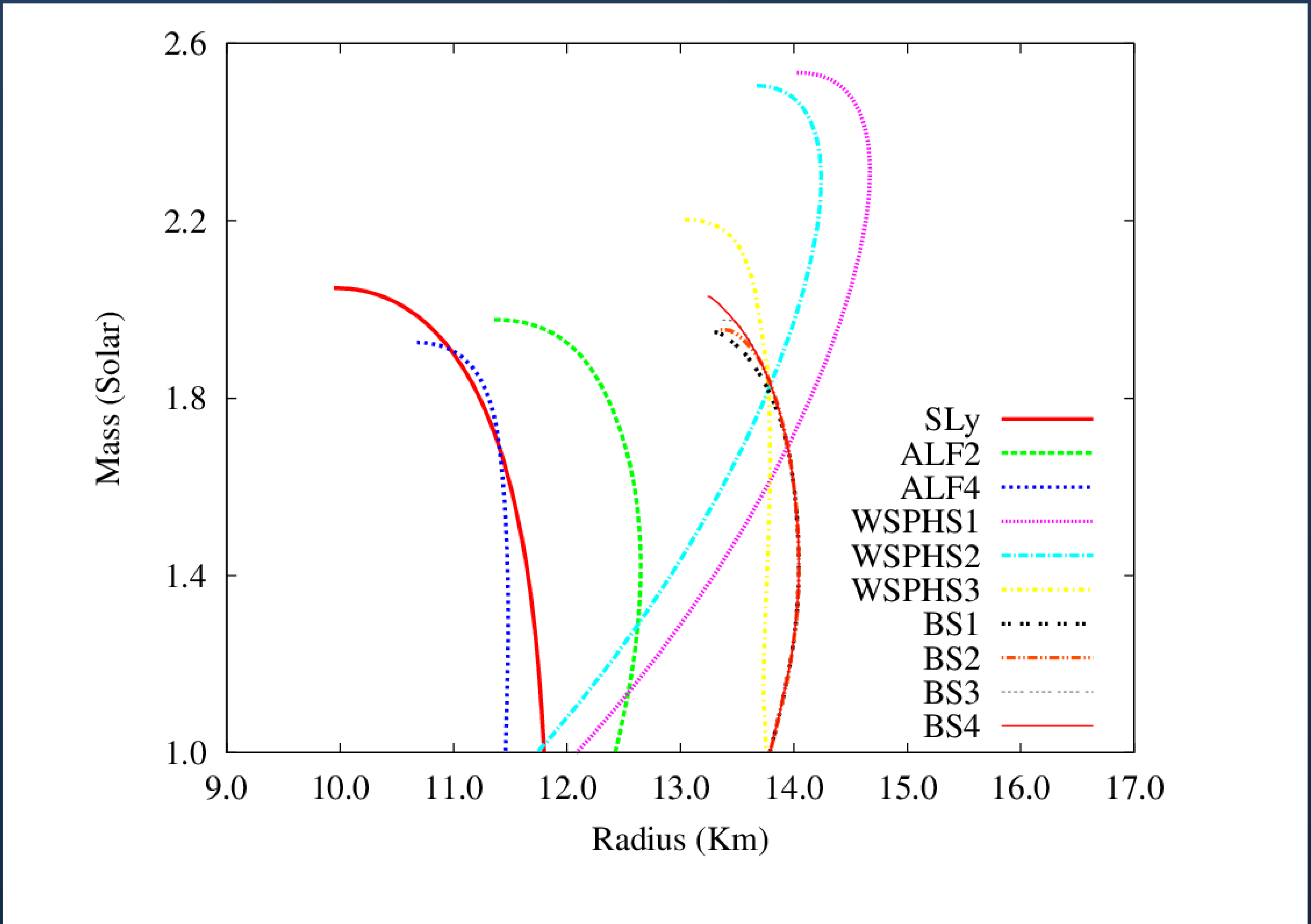
2. Numerical Method



2. Numerical Method



2. Numerical Method

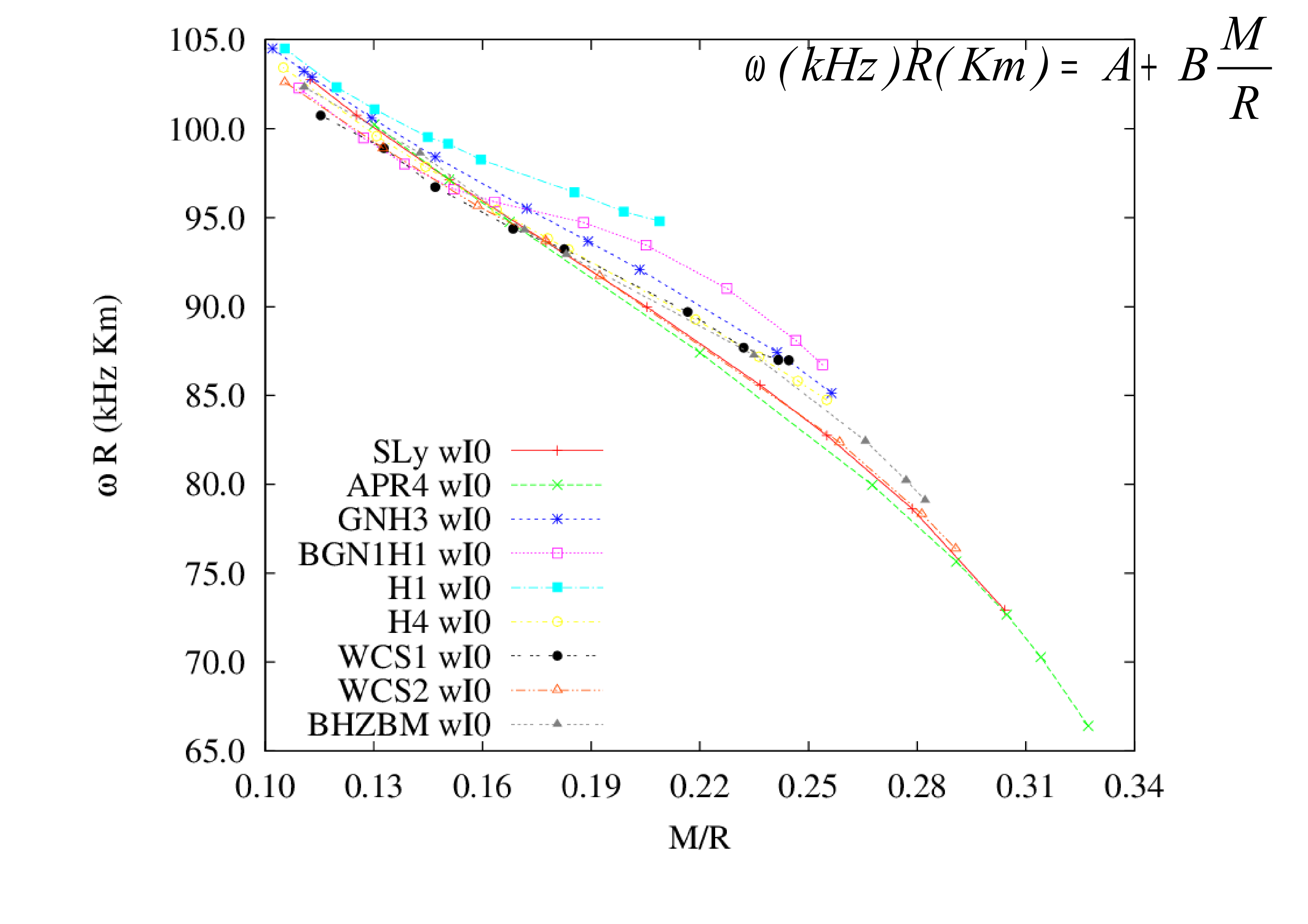


3. Results

a) Axial and polar fundamental wI mode

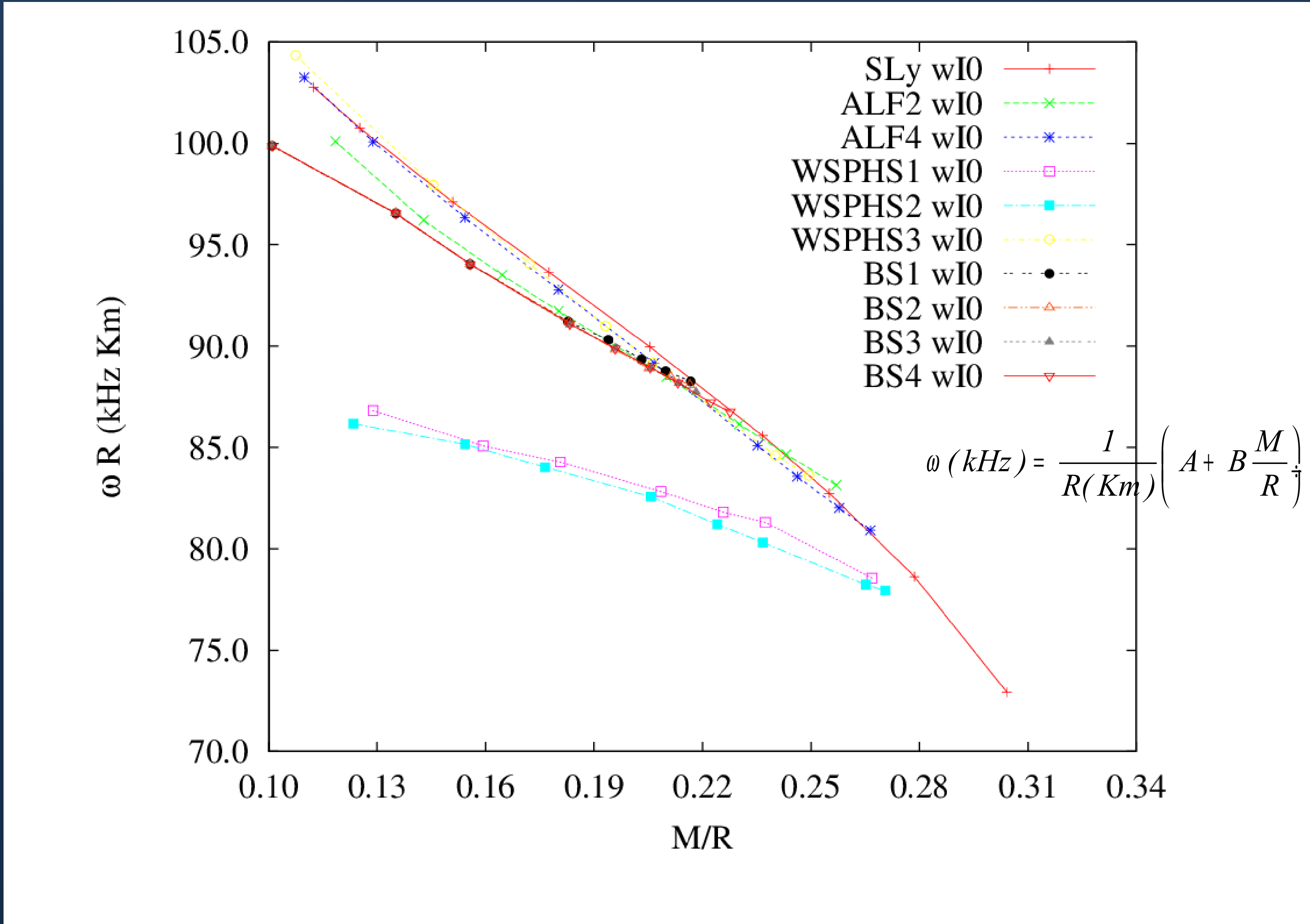
3. Results: a) wI0 modes

Axial fundamental wI mode: Hyperon matter



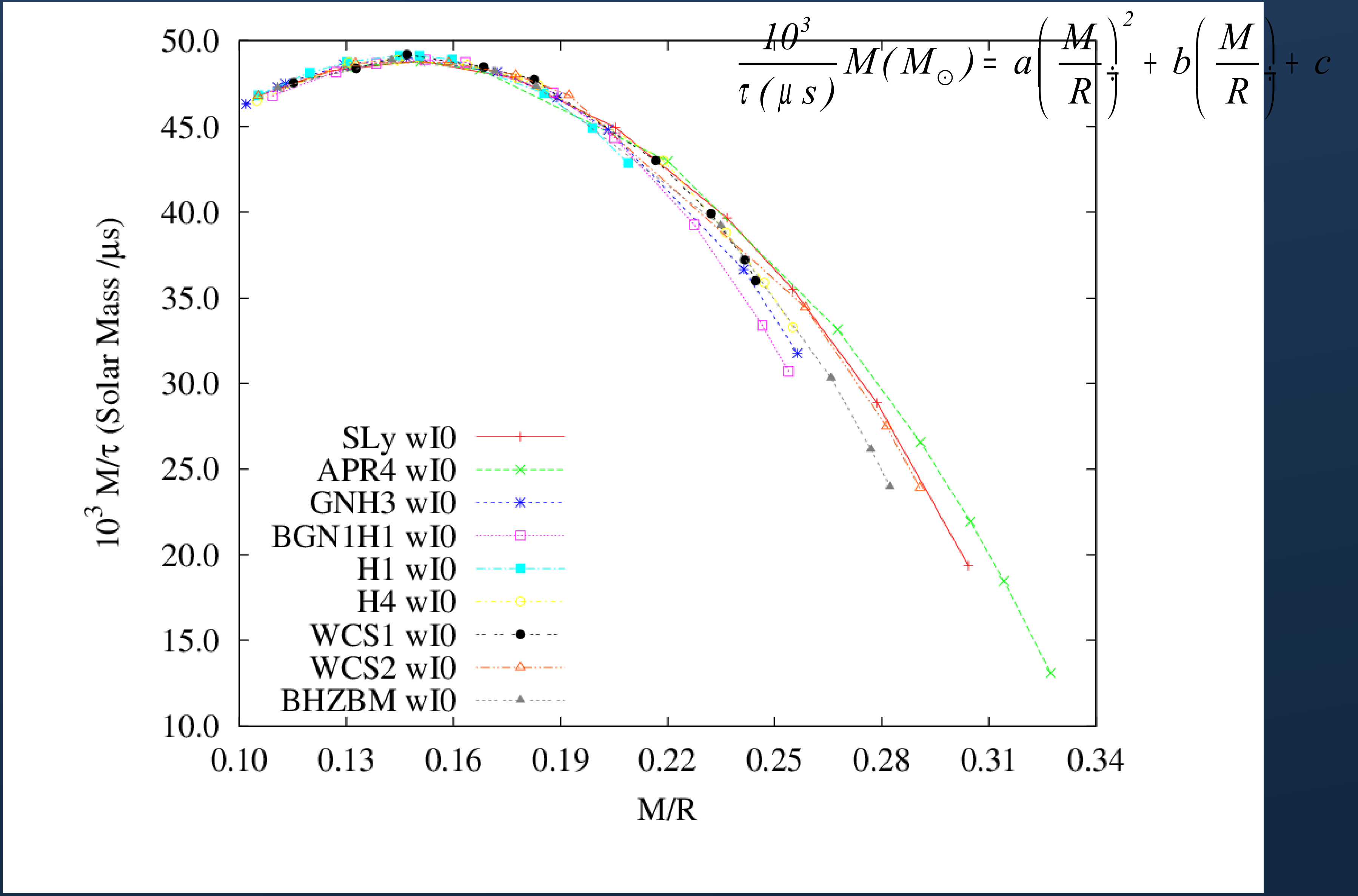
3. Results: a) wl0 modes

Axial fundamental wl mode: Hybrid – quark matter



3. Results: a) wI0 modes

Axial fundamental wI mode: Hyperon matter



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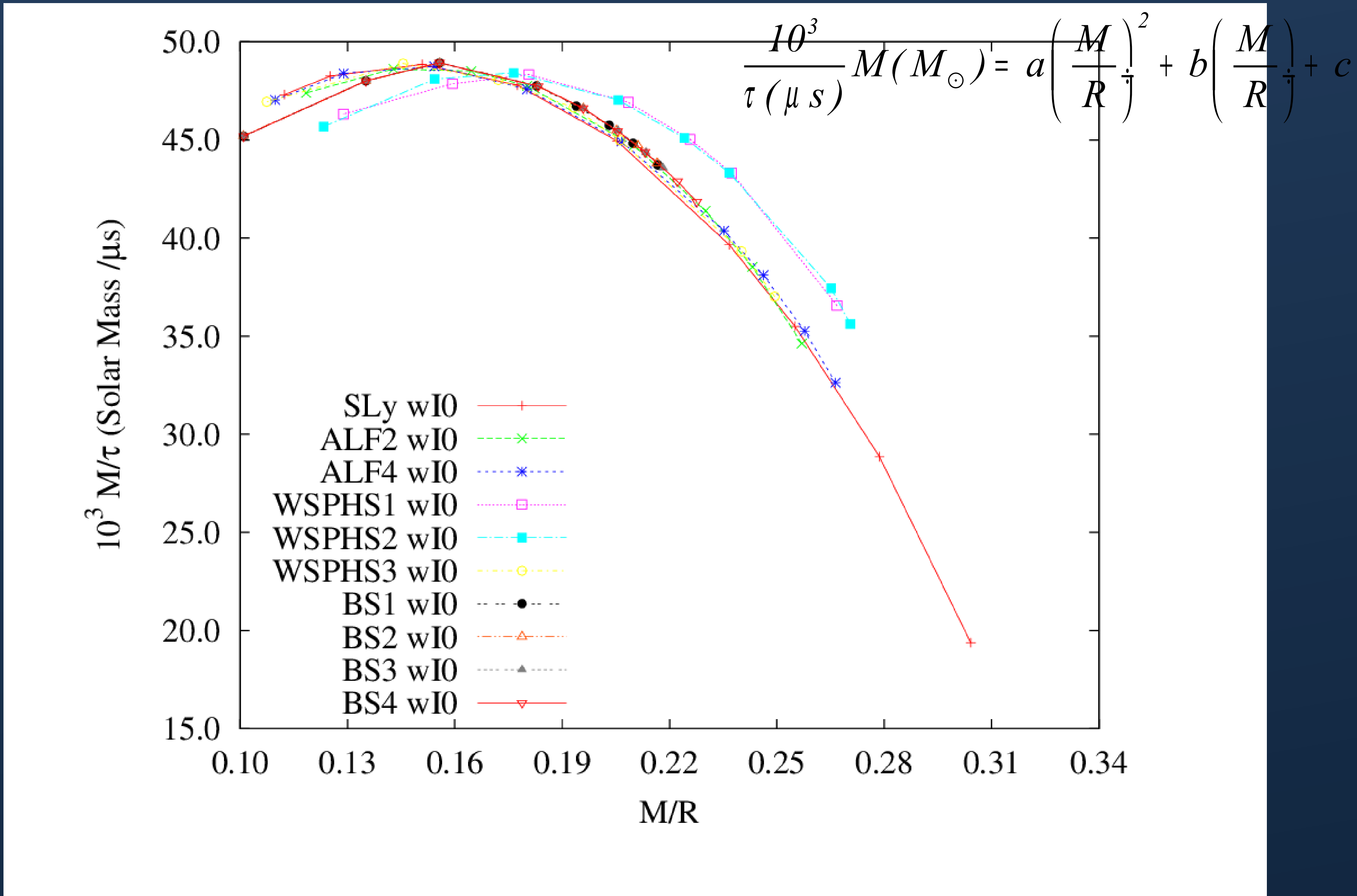
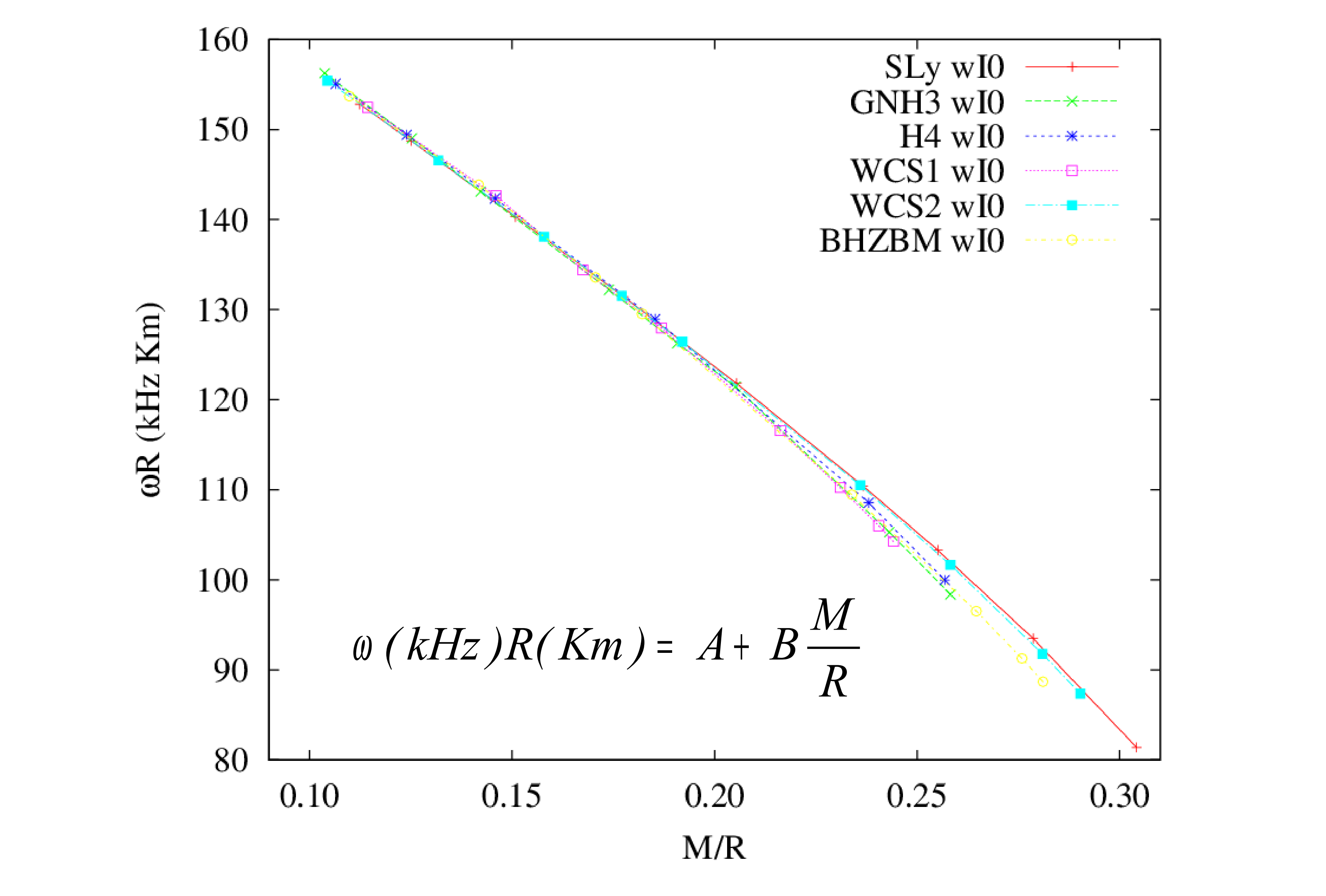
Axial fundamental wI mode: Hybrid – quark matter



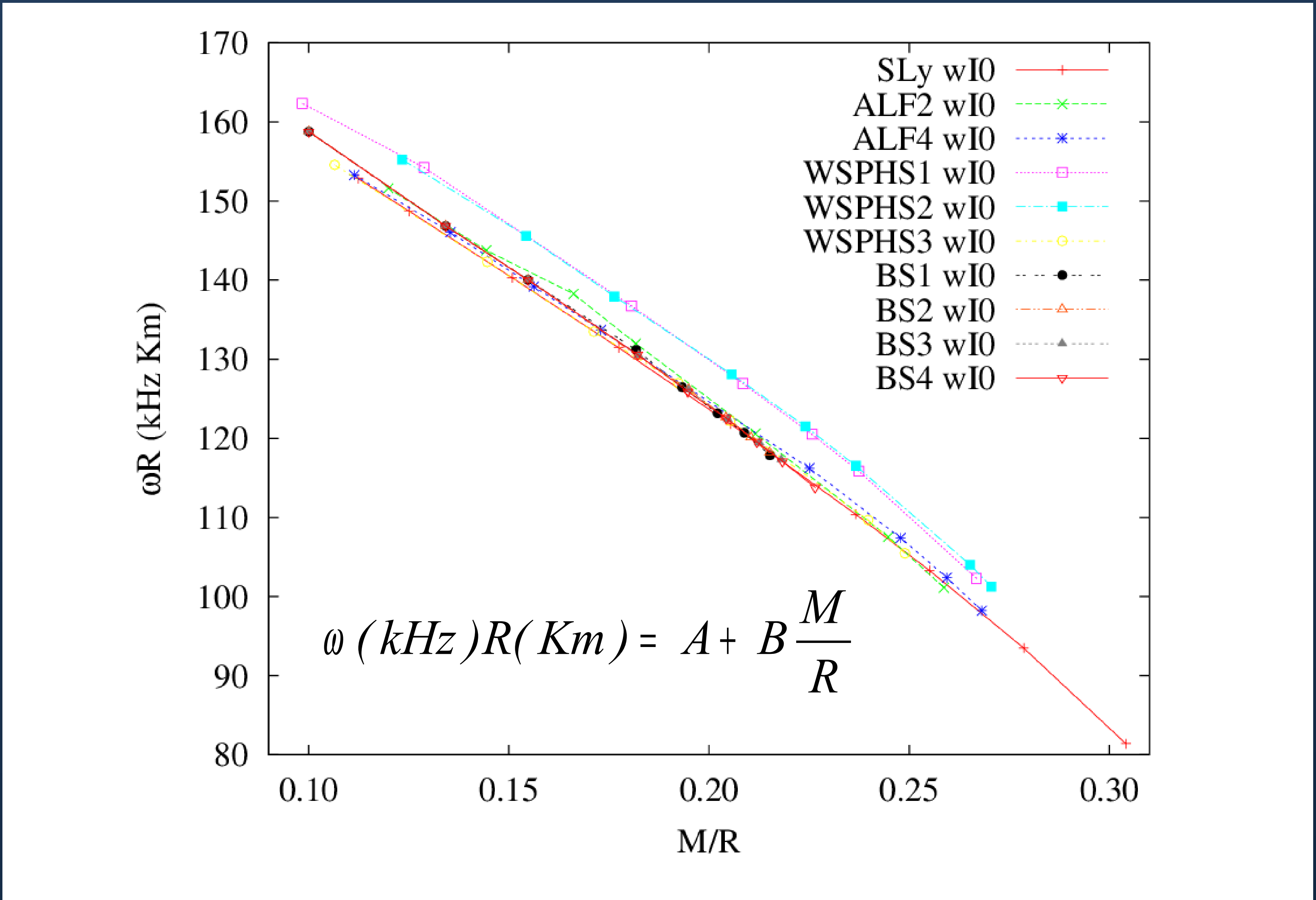
3. Results: a) wl0 modes

Polar fundamental wI mode: Hyperon matter

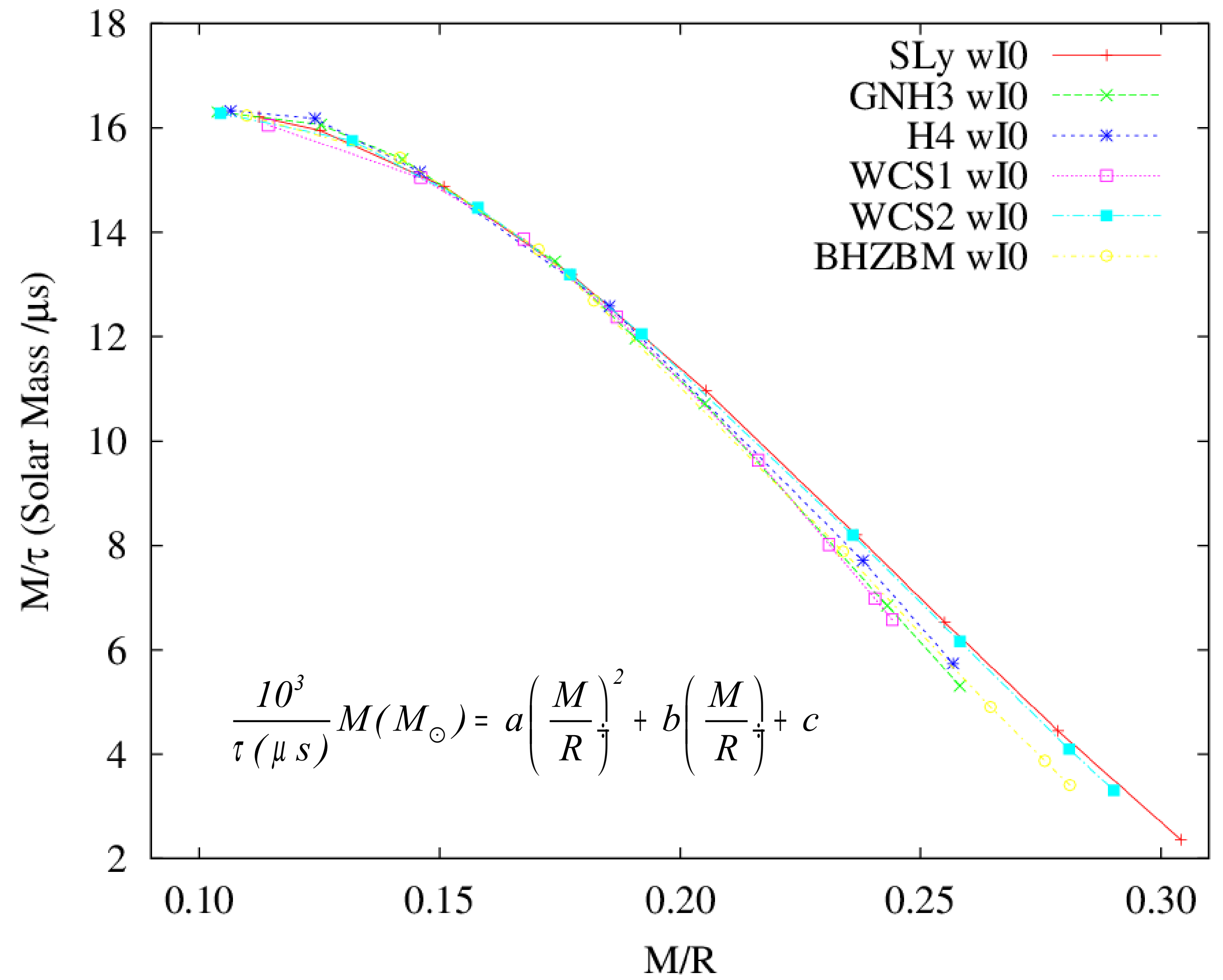


3. Results: a) wl0 modes

Polar fundamental wl mode: Hybrid – quark matter

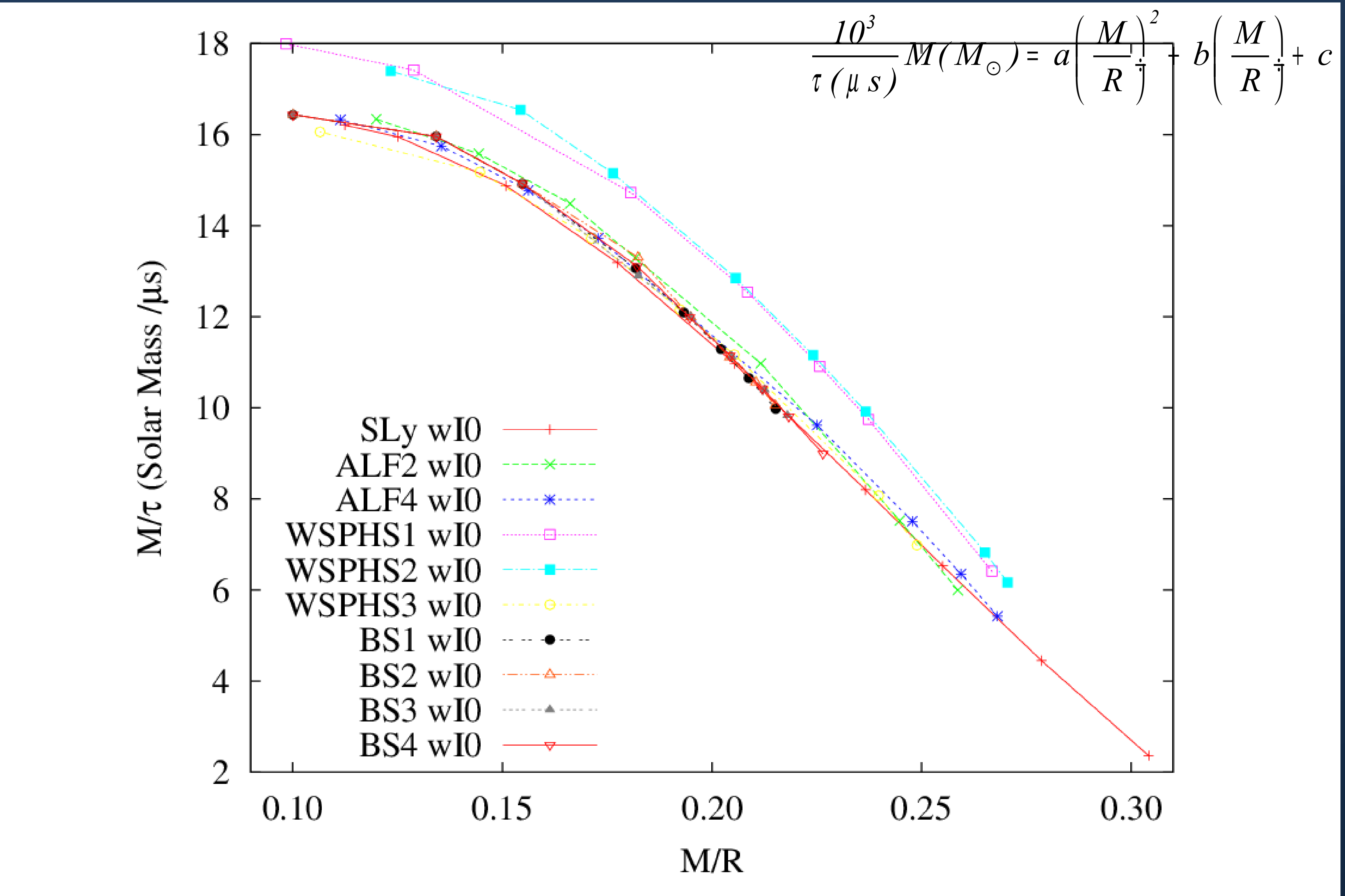


Polar fundamental wl mode: Hyperon matter



3. Results: a) wl0 modes

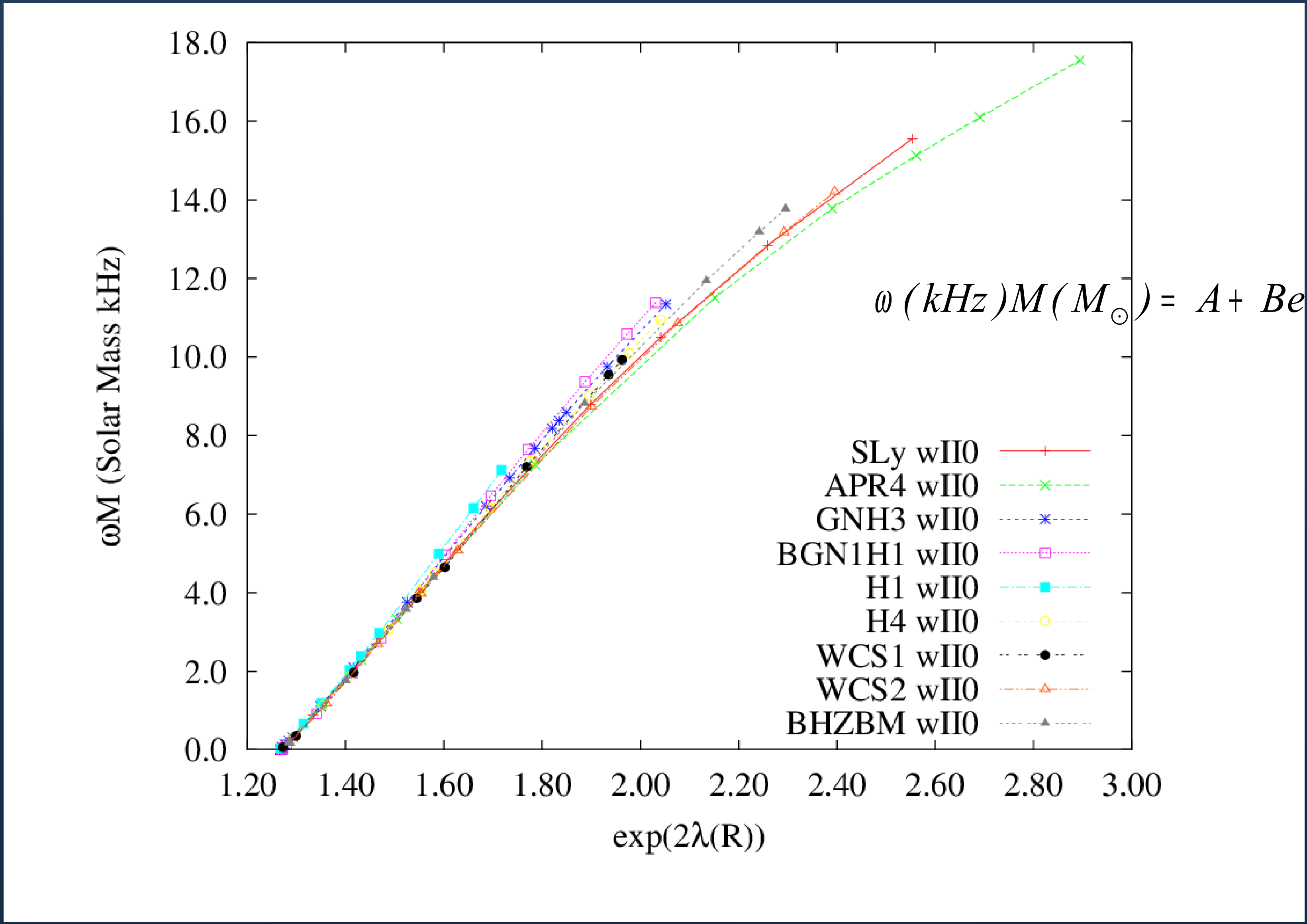
Polar fundamental wl mode: Hybrid – quark matter



b) Axial fundamental wII mode

3. Results: b) wII0 modes

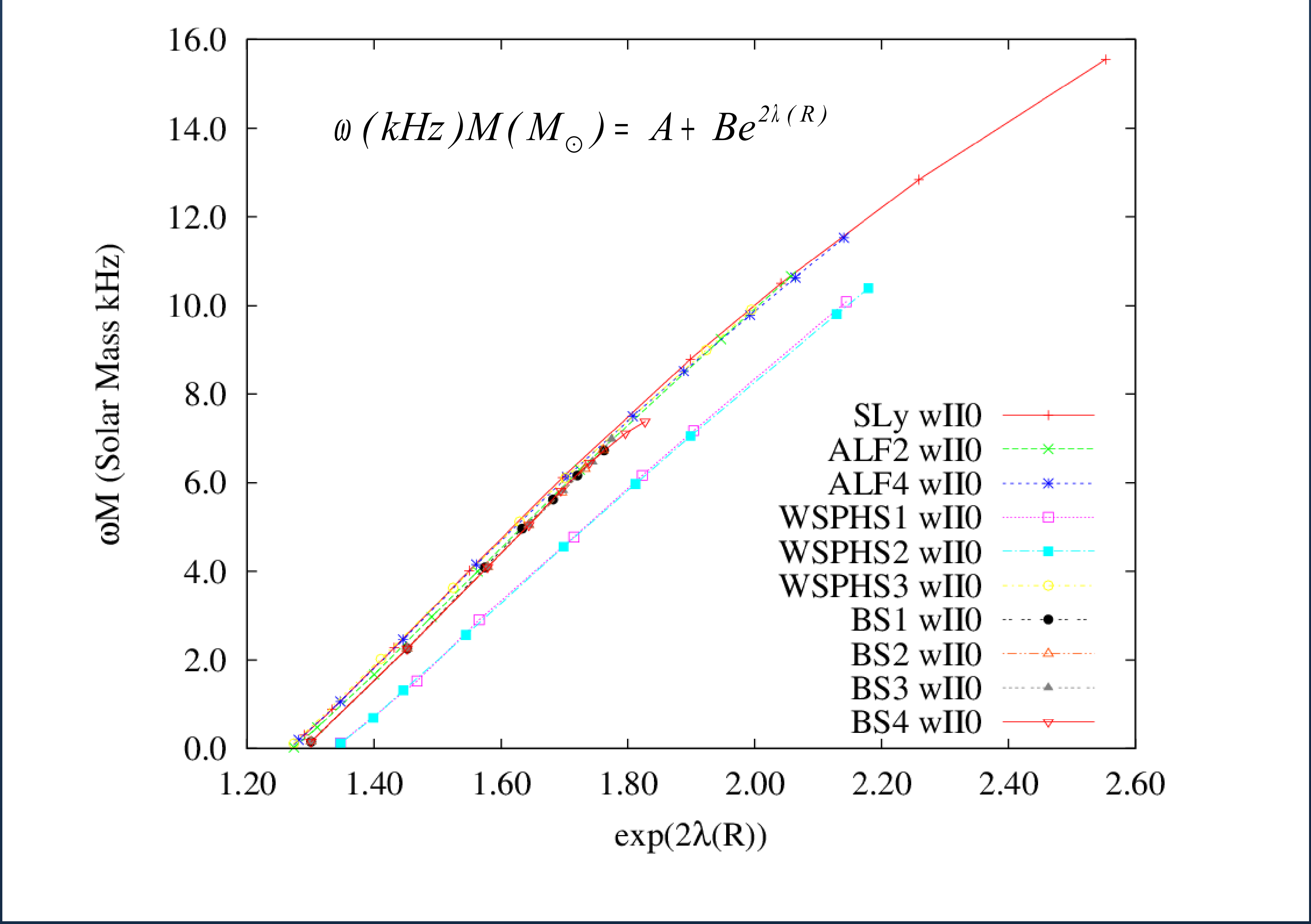
Axial fundamental wII mode: Hyperon matter



$$\omega\,(kHz)\,M\,(M_\odot) = A + B e^{2\lambda(R)}$$

3. Results: b) wII0 modes

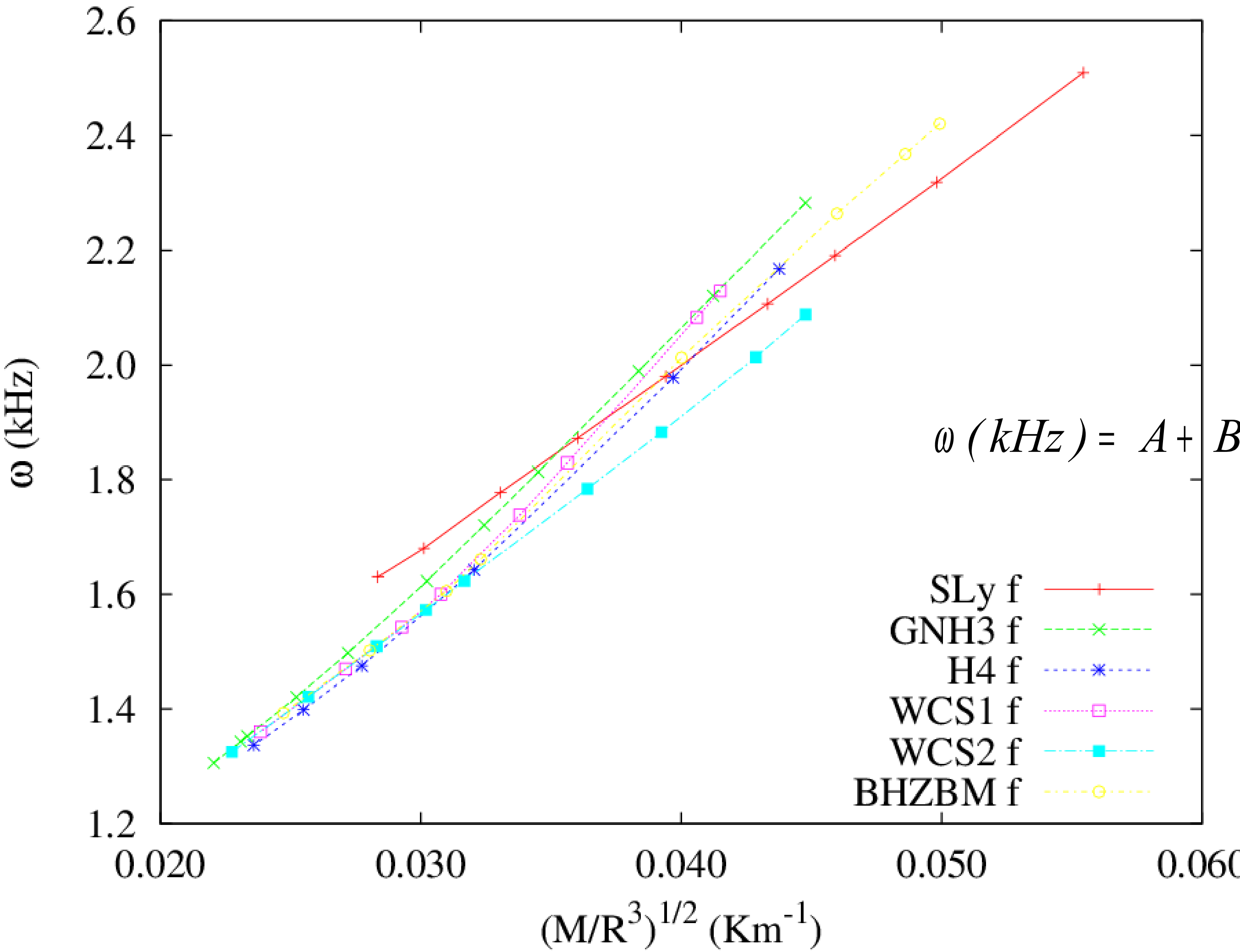
Axial fundamental wII mode: Hybrid – quark matter



c) Polar f mode

3. Results: c) f modes

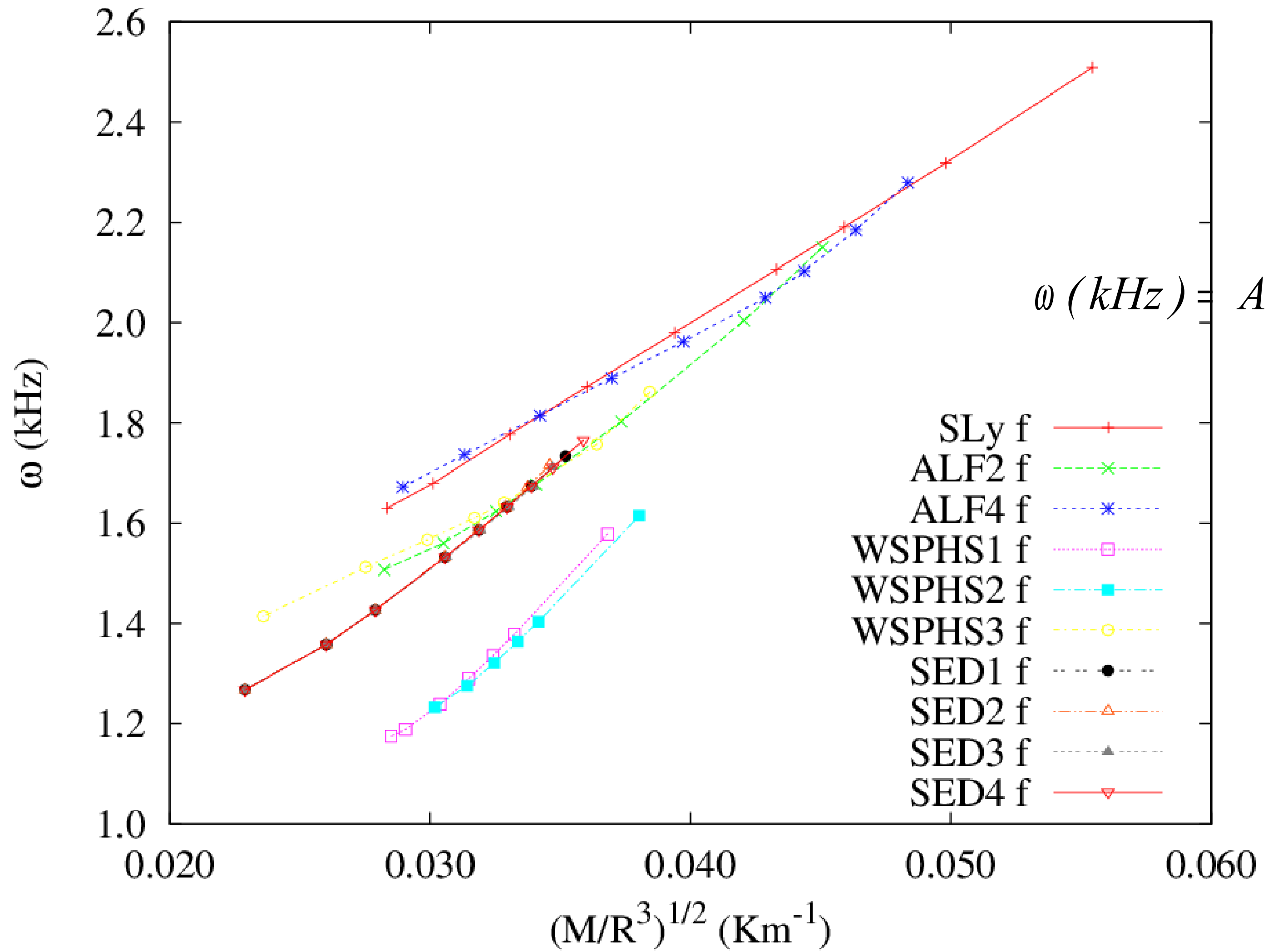
Polar f mode: Hyperon matter



$$\omega\,(kHz) = A + B\sqrt{\frac{M}{R^3}}$$

3. Results: c) f modes

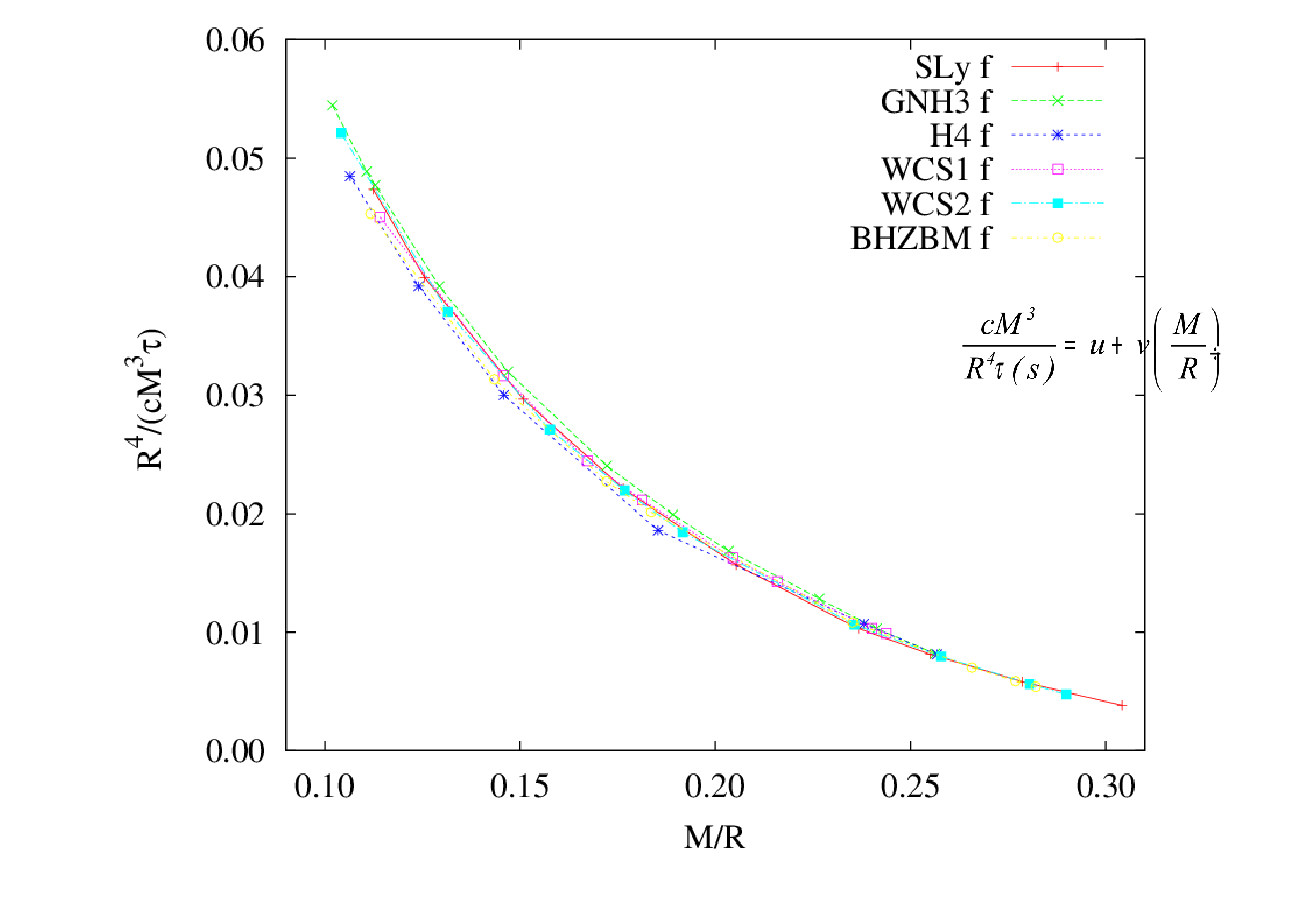
Polar f mode: Hybrid – quark matter



$$\omega\,(kHz) = A + B\sqrt{\frac{M}{R^3}}$$

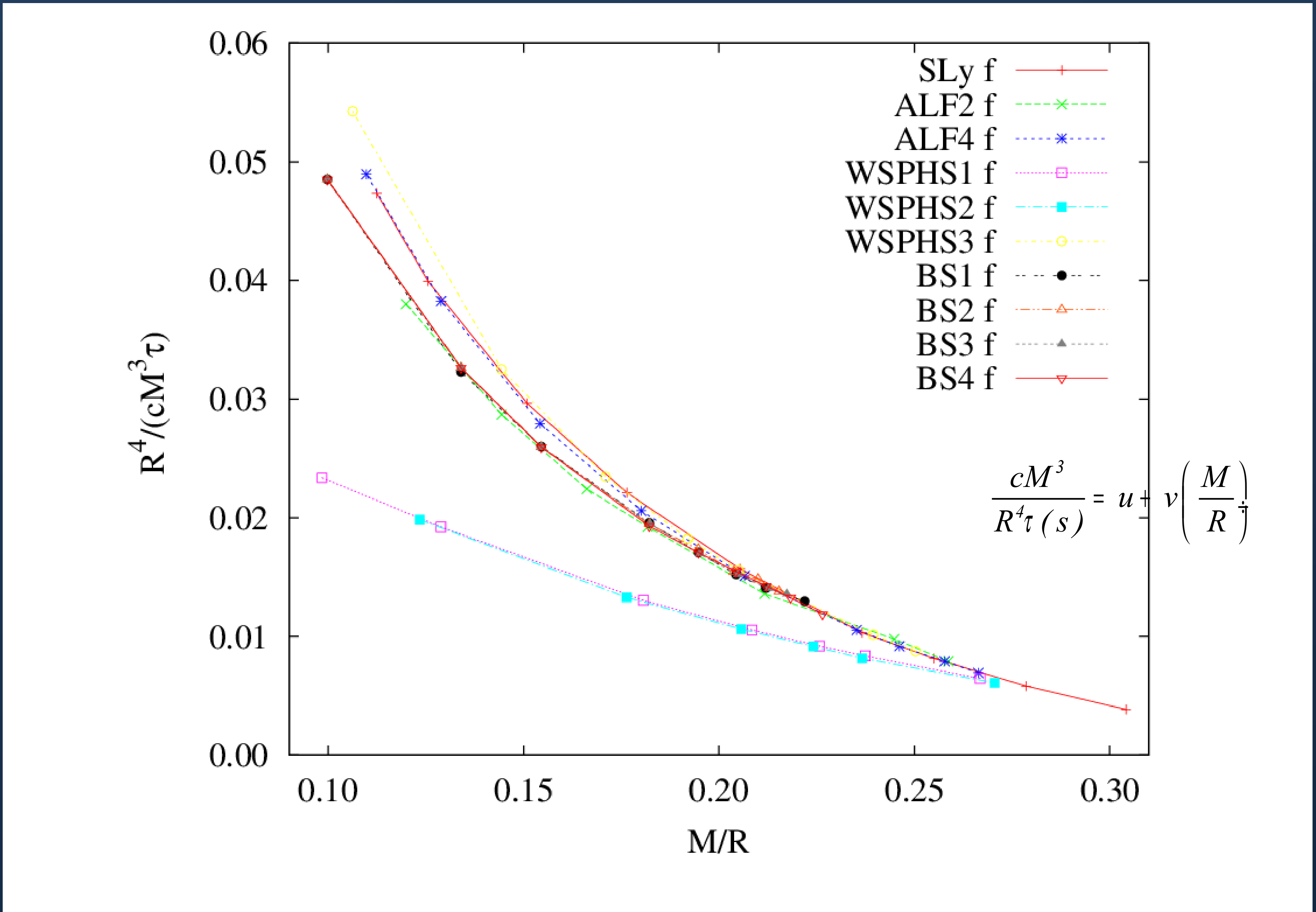
3. Results: c) f modes

Polar f mode: Hyperon matter



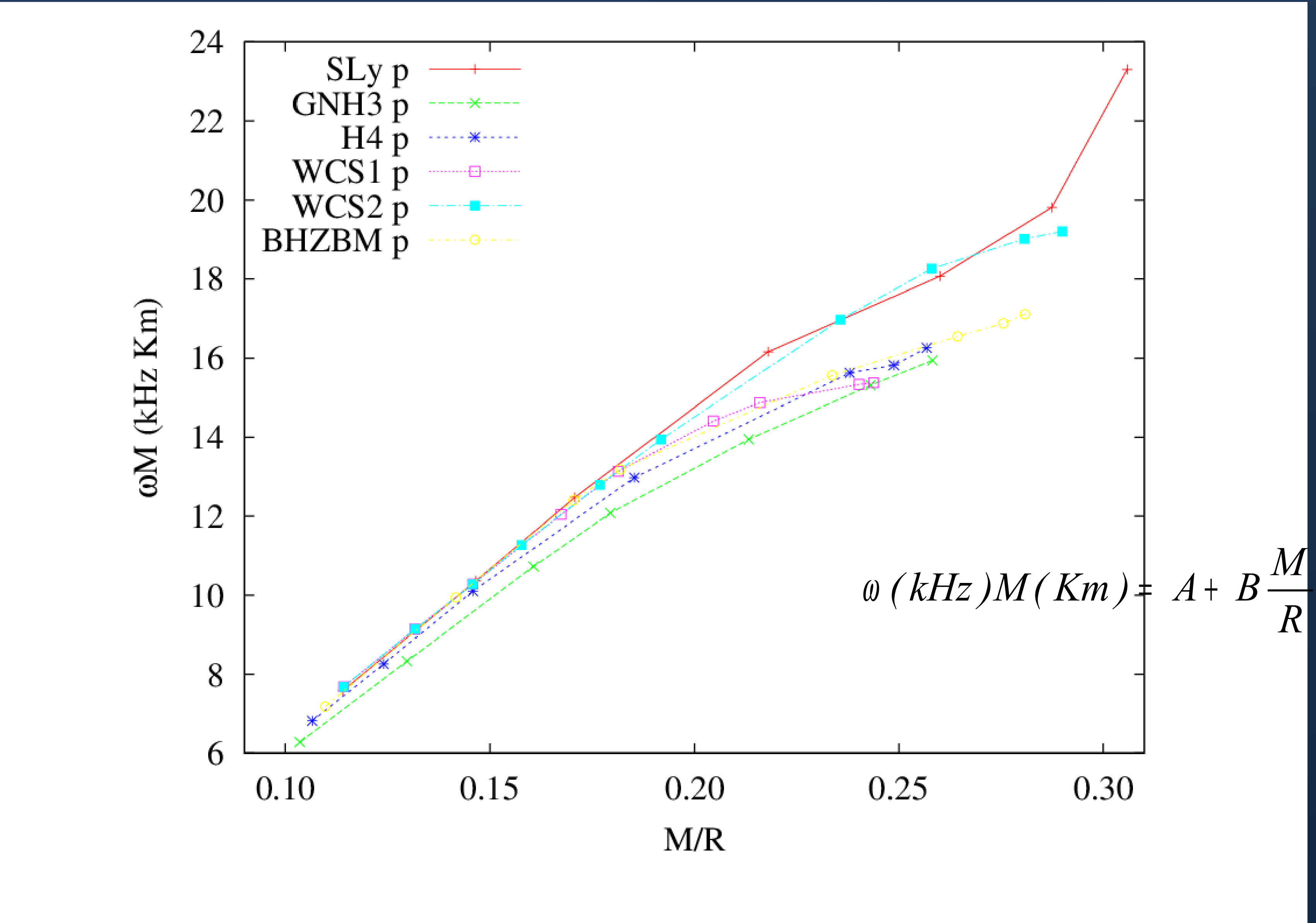
3. Results: c) f modes

Polar f mode: Hybrid – quark matter



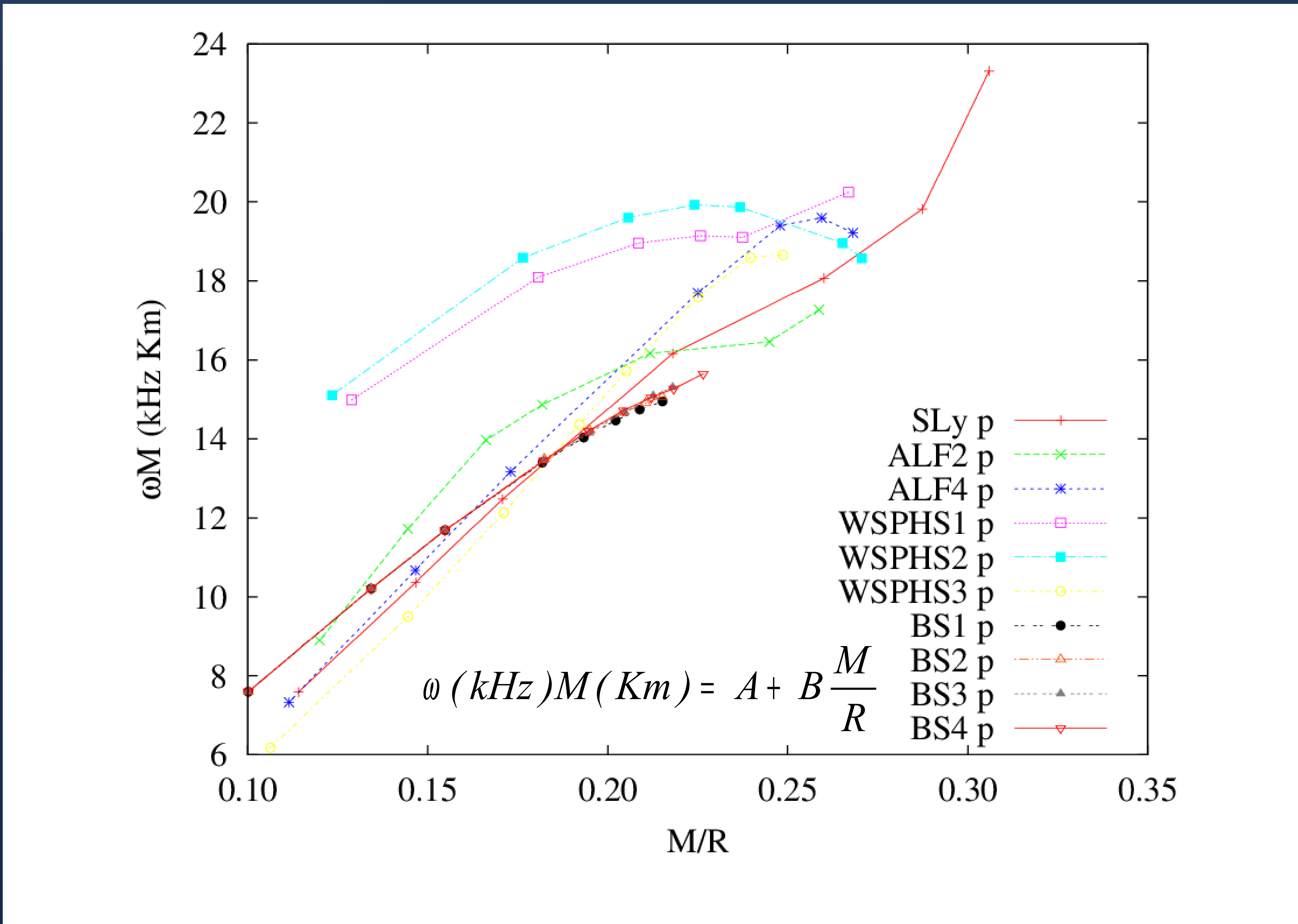
d) Polar p mode

Polar fundamental p mode: Hyperon matter



3. Results: d) p modes

Polar fundamental p mode: Hybrid – quark matter



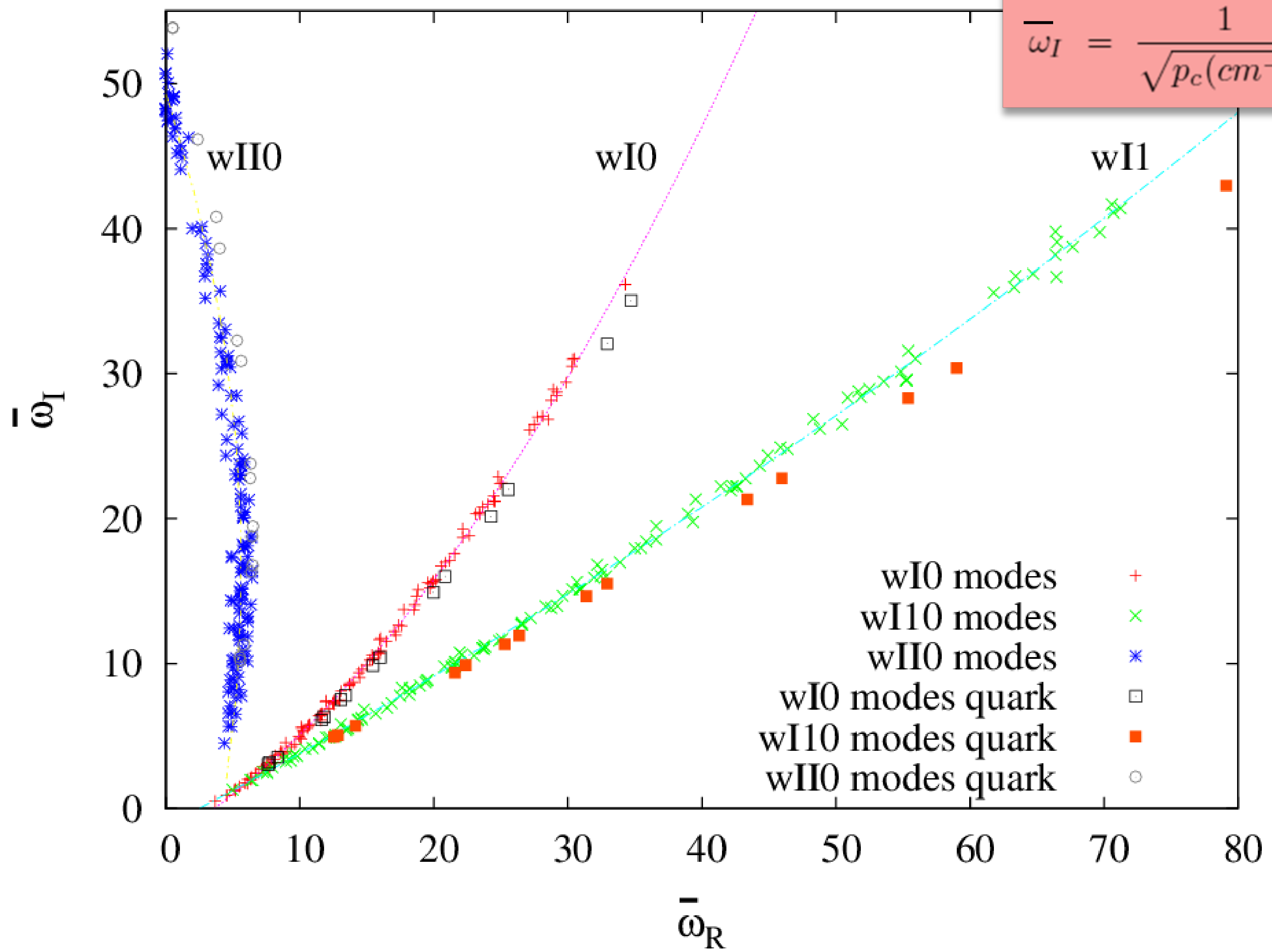
e) Empirical relations

3. Results: e) Empirical relations

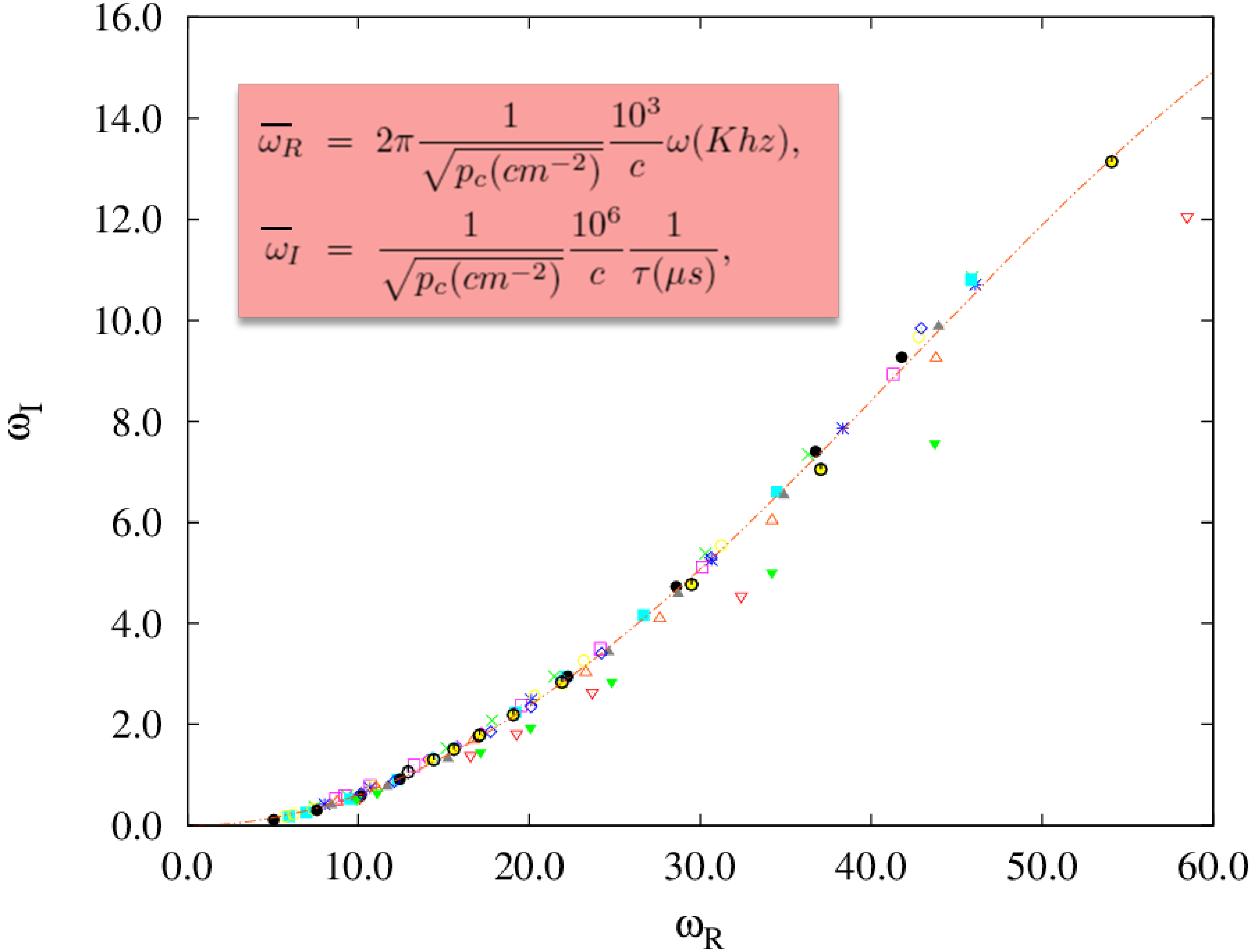
Axial modes: scaling in units of the central pressure

$$\bar{\omega}_R = 2\pi \frac{1}{\sqrt{p_c(cm^{-2})}} \frac{10^3}{c} \omega(Khz),$$

$$\bar{\omega}_I = \frac{1}{\sqrt{p_c(cm^{-2})}} \frac{10^6}{c} \frac{1}{\tau(\mu s)},$$



Similar por Polar wl0 modes:
scaling in units of the central pressure



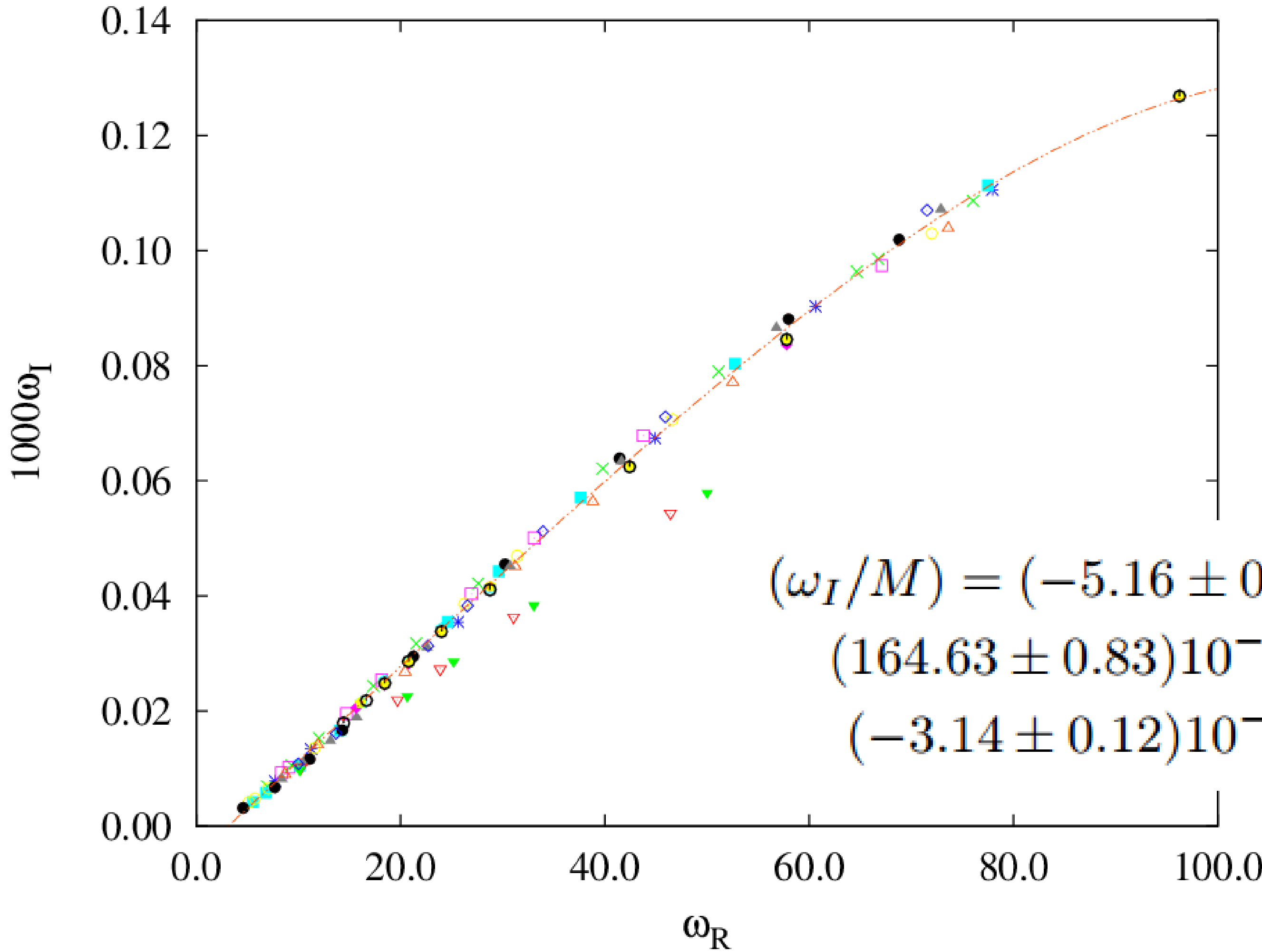
Axial fundamental wI modes

$$\bar{\omega}_I = (-2.30 \pm 0.13) + (0.58 \pm 0.16)\bar{\omega}_R + (0.0165 \pm 0.0004)\bar{\omega}_R^2$$

Polar fundamental wI modes

$$\bar{\omega}_I = (6.146 \pm 0.039)10^{-3}\bar{\omega}_R^2 + (-5.57 \pm 0.18)10^{-7}\bar{\omega}_R^4$$

Polar f modes: scaling in units of Mass / Radius respectively



$$(\omega_I/M) = (-5.16 \pm 0.24)10^{-3} + (164.63 \pm 0.83)10^{-5}(\omega_R/R) + (-3.14 \pm 0.12)10^{-10}(\omega_R/R)^4$$

4. Conclusions

4. Conclusions

- *We have studied axial and polar modes for 16 realistic equations of state*
- *These equations satisfy the 2 Msun condition*
- *Phenomenological relations useful for neutron star asteroseismology have been obtained*
- *New empirical relations have been obtained, valid for every equation of state, which allow to estimate the central pressure and radius*

Thank you for your attention!