Stationary Lifshitz black holes of R^2-corrected gravity theory

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Lifshitz spacetimes:

• emerged in applications of the AdS/CFT duality to non-relativistic, specifically, condensed matter systems.

• thought of as gravity duals of theories w/ nontrivial scaling properties.

Lifshitz black holes:

• BH solutions to some gravity (+ matter) theory that are asymptotically Lifshitz.

• needed for describing "finite temperature aspects" of these non-relativistic systems.

• nice review on all related story on AdS/CMT by S.A. Hartnoll, Class. Quant. Grav. **26**, 224002 (2009) [arXiv:0903.3246 [hep-th]].

Observation: Few exact analytic Lifshitz BHs exist, and all known ones are <u>static</u>.

Advertisement: the first work to present stationary D-dimensional exact analytic Lifshitz spacetimes and (similarly) black "objects".

A closer look at *static* Lifshitz spacetimes:

• a typical feature in condensed matter systems is the "dynamical scaling" property:

$t \mapsto \lambda^z t\,, \quad \vec{x} \mapsto \lambda \vec{x}$, where $z \neq 1$ is called the "dynamical exponent",

instead of the more familiar conformal scalings:

$$t \mapsto \lambda t\,, \quad \vec{x} \mapsto \lambda \vec{x}\,.$$

• plus the following usual additional symmetries:
spatial translations + temporal translations + spatial rotations + parity (P) symmetry + time reversal (T) symmetry

• distinguish one spatial coordinate, call it r ($0 \leq r < \infty$), and let the dynamical scaling transformations act as

$$t \mapsto \lambda^z t\,, \quad \vec{x} \mapsto \lambda \vec{x}\,, \quad r \mapsto r/\lambda\,,$$

where $z \neq 1$ again and $\vec{x}$ denotes a $(D-2)$-dimensional vector.
• Then one ends up with the D-dimensional static Lifshitz spacetime suitable for AdS/CFT games:

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}\,dt^2 + \frac{\ell^2}{r^2}\,dr^2 + \frac{r^2}{\ell^2}\left(\sum_{i=1}^{D-2} dx_i^2\right), \tag{1}$$

• When $z = 1$, usual AdS$_D$ metric with $SO(D-1,2)$ symmetry.
• the length scale set by $\ell > 0$.

Stationary Lifshitz spacetimes:

• Replace the two separate "P symmetry" + "T symmetry" requirements with the weaker "PT symmetry" invariance, i.e. ask for the following symmetries:
dynamical scaling transformations + spatial translations + temporal translations + spatial rotations + PT symmetry

• Then one ends up with the D-dimensional stationary Lifshitz spacetime:

$$ds^2 = -\frac{r^{2z}}{\ell^{2z}}\, dt^2 + 2\omega\, \frac{r^{z+1}}{\ell^{z+1}}\, dt\, d\phi + \frac{r^2}{\ell^2}\, d\phi^2 + \frac{\ell^2}{r^2}\, dr^2 + \frac{r^2}{\ell^2}\left(\sum_{i=1}^{D-3} dx_i^2\right), \tag{2}$$

• For later convenience, $x_{D-2} \equiv \phi$ distinguished from the remaining x_i $(1 \leq i \leq D-3)$.

• the static Lifshitz spacetime is obtained when ω, the dimensionless "rotation parameter", is set to 0.

R^2-corrected gravity theory:

- The action:

$$I = \int d^D x \sqrt{-g} \left(R + 2\Lambda + \alpha R^2 \right), \tag{3}$$

where Λ is the cosmological constant and α is a coupling constant .

The first result:

Stationary Lifshitz spacetime (2) solves the field equations following from the action (3) for generic values of the parameters z and ω in any $D \geq 3$, provided that α and Λ are tuned as

$$\alpha = \frac{1}{8\Lambda}, \tag{4}$$

$$\Lambda = \frac{2D^2 + 3(z-1)^2 + 2D(2z-3)}{8\ell^2} + \frac{(z-1)^2}{8\ell^2(1+\omega^2)}. \tag{5}$$

The main result:

Provided that α and Λ are precisely as in (4) and (5), the metric

$$
\begin{aligned}
ds^2 &= -\frac{r^{2z}}{\ell^{2z}}\,h(r)\,dt^2 + \frac{r^2}{\ell^2}\Big(d\phi + \omega\,\frac{\ell^2}{r^2}\,dt\Big)^2 \\
&\quad + \frac{\ell^2}{r^2}\,\frac{dr^2}{h(r)} + \frac{r^2}{\ell^2}\left(\sum_{i=1}^{D-3} dx_i^2\right), \text{ where} \qquad (6) \\
h(r) &\equiv c + k\,\frac{\ell^{2(1+z)}}{r^{2(1+z)}} + M^-\,\frac{\ell^{p_-}}{r^{p_-}} + M^+\,\frac{\ell^{p_+}}{r^{p_+}}, \text{ with} \qquad (7) \\
c &\equiv \frac{4\,\ell^2\,\Lambda}{2z^2 + (D-2)(2z+D-1)}, \qquad (8) \\
k &\equiv \frac{2\,\omega^2}{D^2 - 7D + 14 - 2z(D-3)}, \text{ and} \qquad (9) \\
p_\pm &\equiv \frac{1}{2}\Big(3z + 2(D-2) \pm \sqrt{z^2 + 4(D-2)(z-1)}\Big), \quad (10)
\end{aligned}
$$

solves the field equations of the action (3) for any $D \geq 3$.

Remarks:

- Note that the coefficients c and k are completely determined by z and ω, whereas the integration constants $M^{\pm}$ are left as free parameters.
- (1) and the static version of the metric (6) [i.e. the one with $\omega = 0$ for which $c = 1$, $k = 0$, the relations (4) and (5) for α and Λ, and the metric function $h(r)$ in (7) are simplified accordingly] were first presented by E. Ayon-Beato, *et al.* in JHEP **1004**, 030 (2010) [arXiv:1001.2361 [hep-th]].
- However with ω on, (2) and the stationary metric (6) (with the accompanying equations (4), (5) and (7)-(10), respectively) are clearly more general.
- In the conformal limit $z = 1$ with $D = 3$, the metric (6) becomes identical to the BTZ metric when one sets $M^{+} = 0$, $M^{-} = -M < 0$ and $\omega = -j/2$.

• The curvature scalars of the metrics (2) and (6) are both given by $R = -4\Lambda$ precisely. This allows for the casting of the action (3) into the form

$$I = \frac{1}{8\Lambda} \int d^D x \sqrt{-g}\, (R + 4\Lambda)^2 \,,$$

and this theory cannot be mapped into a scalar-tensor theory by a conformal transformation of the metric.

• To have $h(r)$ real, one needs

$$\begin{aligned} z < z_- &\equiv 4 - 2D - 2\sqrt{(D-1)(D-2)} < 0 \quad \text{or} \\ z > z_+ &\equiv 4 - 2D + 2\sqrt{(D-1)(D-2)} > 0 \,. \end{aligned}$$

For the metric (6) to describe a black hole, a careful analysis further chooses the branch $z > z_+ > 0$.

• One can construct analogous solution(s) of the form (6) for the critical value(s) of the dynamical exponent $z = z_\pm$ with logarithmic $h(r)$ function(s).
• In fact the region $z \in (z_-, z_+)$ is not excluded either! However, let me skip the details of it here!
• A careful consideration shows that both the energy E and the entropy S, as well as the angular momentum J, vanish for this class of solutions: $E = S = J = 0$. Perhaps this is not so surprising after all, since the action $I = 0$ at the first place for these solutions! (Recall my previous remark following $R = -4\Lambda$.)

Conclusions and open problems:

• As put forward by E. Ayon-Beato, *et al.*, one may think of these solutions (or their Euclidean counterparts) as some kind of "gravitational instantons".

• Stability of these solutions has not been analyzed yet.

• The implications of these solutions on the CMT side has not been studied yet.