Black holes in scalar-tensor gravity

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INTRODUCTION

- In GR spacetime singularities are generic (Hawking & Penrose) and they are usually cloacked by horizons (Cosmic Censorship).
- GR: **stationary** black holes (endpoint of grav. collapse) must be **axisymmetric** (Hawking '72). Asympt. flat black holes in GR are simple.
- Non-asympt. flat black holes can be very complicated: "cosmological" black holes have appearing/disappearing apparent horizons (McVittie, generalized McVittie, LTB, Husain-Martinez-Nuñez, Fonarev, ...). Interaction between black hole and cosmic "background".
- Scalar-tensor, $f(R)$ gravity, higher order gravity, low-energy effective actions for quantum gravity, etc.: Birkhoff's theorem is lost.

Prototype: Brans-Dicke theory (Jordan frame)

$$S_{BD} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega_0}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

- Hawking '72: endpoint of axisymmetric collapse in this theory must be GR black holes. Result generalized *for spherical symmetry only* by Bekenstein + Mayo '96, Bekenstein '96, + bits and pieces of proofs.
- What about more general theories?

$$S_{ST} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega(\varphi)}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi - V(\varphi) + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

This action includes metric and Palatini $f(R)$ gravity inportant for cosmology.

A SIMPLE PROOF

This work (T.P. Sotiriou & VF 2012, *Phys. Rev. Lett.* 108, 081103): extend result to *general* scalar-tensor theory

$$S_{ST} = \int d^4x \sqrt{-\hat{g}} \left[\varphi \hat{R} - \frac{\omega(\varphi)}{\varphi} \hat{\nabla}^\mu \varphi \hat{\nabla}_\mu \varphi - V(\varphi) + L_m(\hat{g}_{\mu\nu}, \psi) \right]$$

we require

- **asymptotic flatness** (collapse on scales $\ll H_0^{-1}$): $\varphi \to \varphi_0$ as $r \to +\infty$, $V(\varphi_0) = 0$, $\varphi_0 \, V'(\varphi_0) = 2\, V(\varphi_0)$
- **stationarity** (endpoint of collapse).

Use Einstein frame $\hat{g}_{\mu\nu} \to g_{\mu\nu} = \varphi \, \hat{g}_{\mu\nu}$, $\varphi \to \phi$ with

$$d\phi = \sqrt{\frac{2\omega(\varphi) + 3}{16\pi}} \, \frac{d\varphi}{\varphi} \qquad (\omega \neq -3/2)$$

brings the action to

$$S_{ST} = \int d^4x \sqrt{-g} \Big[\frac{R}{16\pi} - \frac{1}{2} \nabla^\mu \phi \nabla_\mu \phi - U(\phi) + L_m(\hat{g}_{\mu\nu}, \psi) \Big]$$

where $U(\phi) = V(\varphi)/\varphi^2$. Field eqs. are

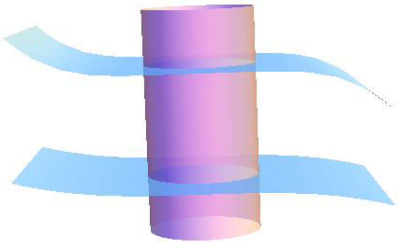
$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{R}\hat{g}_{\mu\nu} = \frac{\omega(\varphi)}{\varphi^2}\left(\hat{\nabla}_\mu\varphi\hat{\nabla}_\nu\varphi - \frac{1}{2}\hat{g}_{\mu\nu}\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi\right) + \frac{1}{\varphi}\left(\hat{\nabla}_\mu\hat{\nabla}_\nu\varphi - \hat{g}_{\mu\nu}\hat{\Box}\varphi\right) - \frac{V(\varphi)}{2\varphi}\hat{g}_{\mu\nu} \,,$$

$$(2\omega + 3)\,\hat{\Box}\varphi = -\omega'\,\hat{\nabla}^\lambda\varphi\hat{\nabla}_\lambda\varphi + \varphi\,V' - 2V \,,$$

$\Omega = \Omega(\varphi) \longrightarrow$ same symmetries as in the J. frame:

- ξ^μ timelike Killing vector (stationarity)
- ζ^μ spacelike at spatial infinity (axial symmetry).

Consider, in vacuo, a 4-volume $\mathcal{V}$ bounded by the horizon H, two Cauchy hypersurfaces $\mathcal{S}_1$, $\mathcal{S}_2$, and a timelike 3-surface at infinity



multiply $\Box\phi = U'(\phi)$ by U', integrate over $\mathcal{V} \longrightarrow$

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\, U'(\phi)\Box\phi = \int_{\mathcal{V}} d^4x\sqrt{-g}\, U'^2(\phi)$$

rewrite as

$$\int_{\mathcal{V}} d^4x\sqrt{-g}\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right] = \int_{\partial\mathcal{V}} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi$$

where n^μ =normal to the boundary, h =determinant of the induced metric $h_{\mu\nu}$ on this boundary. Split the boundary into its constituents $\int_{\mathcal{V}} = \int_{\mathcal{S}_1} + \int_{\mathcal{S}_2} + \int_{horizon} + \int_{r=\infty}$ Now, $\int_{\mathcal{S}_1} = -\int_{\mathcal{S}_2}$, $\int_{r=\infty} = 0$, $\int_{horizon} d^3x\sqrt{|h|}\, U'(\phi)n^\mu\nabla_\mu\phi = 0$ because of the symmetries.

$$\longrightarrow \int_{\mathcal{V}} d^4x\sqrt{-g}\left[U''(\phi)\nabla^\mu\phi\nabla_\mu\phi + U'^2(\phi)\right] = 0\,.$$

Since $U'^2 \geq 0$, $\nabla^\mu \phi$ (orthogonal to both ξ^μ, ζ^μ on H) is spacelike or zero, and $U''(\phi) \geq 0$ for stability (black hole is the endpoint of collapse!), it must be $\nabla_\mu \phi \equiv 0$ in $\mathcal{V}$ and $U'(\phi_0) = 0$. For ϕ =const., **theory reduces to GR, black holes must be Kerr**.

- Metric $f(R)$ gravity is a special case of BD theory with $\omega = 0$ and $V \neq 0$.
- for $\omega = -3/2$, vacuum theory reduces to GR, Hawking's theorem applies (Palatini $f(R)$ gravity is a special BD theory with $\omega = -3/2$ and $V \neq 0$).

Exceptions not covered by our proof:

- theories in which $\omega \to \infty$ somewhere
- theories in which φ diverges (at ∞ or on the horizon) ex: maverick solution of Bocharova *et al.* '80 (unstable).
- Proof extends immediately to electrovacuum/conformal matter ($T = 0$).

CONCLUSIONS

- Even though Birkhoff's theorem is lost, black holes which are the endpoint of axisymmetric gravitational collapse (and asympt. flat) in *general* scalar-tensor gravity are the same as in GR (*i.e.*, Kerr-Newman). Proof extends to electrovacuum.
- Exceptions (exact solutions) are unphysical or unstable solutions which cannot be the endpoint of collapse, or do not satisfy the Weak/Null Energy Condition.
- Proof is simple!
- Asymptotic flatness is a technical assumption, but can't eliminate it at the moment. Excludes "large" primordial black holes in a "small" universe.
- What about more general theories with other degrees of freedom?