A modified gravity from metric quantum fluctuations

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Spanish-Portuguese Relativity Meeting 2013

As we know gravity is highly non-linear theory. The non-linearities appear in front of derivative terms:

$$g^{\cdots} g''_{\cdots}$$

One of the most neglected of approaches to the quantization of a non-linear theory is nonperturbative (NP) quantization by Heisenberg.

Heisenberg has applied NP quantization for non-linear spinor field.

The essence of NP quantization is to write operator equation

$$\hat{R}_{\mu\nu} - \frac{1}{2}\hat{g}_{\mu\nu}\hat{R} = \varkappa \hat{T}_{\mu\nu}, \qquad (1)$$

Heisenberg offer to write an infinite equations set for all Green functions as follows

$$
\begin{aligned}
\langle Q \,|\ \text{Eq. (1)} |\, Q\rangle &= 0, \\
\langle Q \,|\hat{g}(x_1)\cdot\ \text{Eq. (1)} |\, Q\rangle &= 0, \\
\langle Q \,|\hat{g}(x_1)\hat{g}(x_2)\cdot\ \text{Eq. (1)}\ |\, Q\rangle &= 0, \\
\cdots &= 0, \\
\langle Q \,|\text{prod. of } g \text{ at different points } (x_1, \cdots, x_n)\cdot\ \text{Eq. (1)} |\, Q\rangle &= 0
\end{aligned}
$$

The first way is to cut off equations set by using some decomposition $G_{m+n} \approx G_m G_n$ and taking into account the first $p < m + n$ equations. The second one is to write some functional (for example, action) and average it using some assumptions about expectation value of Green functions.

Strategy for calculations

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + \widehat{\delta g}_{\mu\nu}$$

$$\downarrow$$

$$\left\langle \mathcal{L}(g + \widehat{\delta g}) \right\rangle \approx$$

$$\mathcal{L}(g) + \mathcal{L}' \, \widehat{\delta g^{\mu\nu}} + \mathcal{L}'' \, \widehat{\delta g^{\mu\nu}} \widehat{\delta g^{\rho\sigma}}$$

$$\downarrow$$

$$\left\langle \widehat{\delta g_{\mu\nu}}(x_1) \cdot \widehat{\delta g_{\rho\sigma}}(x_2) \right\rangle \approx$$

$$P_{\mu\nu}(x_1) P_{\rho\sigma}(x_2)$$

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Assumption of 2-point Green function

$$\left\langle \widehat{\delta g_{\mu\nu}}(x_1) \cdot \widehat{\delta g_{\rho\sigma}}(x_2) \right\rangle \approx P_{\mu\nu}(x_1) P_{\rho\sigma}(x_2)$$

It is necessary to emphasize that even for x_1, x_2 spacelike points this expression is not zero !

For perturbative quantization the situation is opposite.

- $P_{\mu\nu}$ is proportional to the metric tensor

$$P_{\mu\nu} \propto g_{\mu\nu};$$

- $P_{\mu\nu}$ is proportional to the Ricci tensor

$$P_{\mu\nu} \propto \frac{R_{\mu\nu}}{R};$$

The proportionality coefficient should be some invariant. Consequently it has to be $\tilde{F}(R, R_{\mu\nu}R^{\mu\nu}, \cdots)$.

For the ansatz $P_{\mu\nu} = g_{\mu\nu}$ we have

$$\langle \mathcal{L}(g + \delta g) \rangle \approx \sqrt{-g} \left(-\frac{c}{2\varkappa} R - 2R\tilde{F}(R, \cdots) \right) = \sqrt{-g} F(R, \cdots)$$

Thus we have obtained a *modified gravity theory*.

For the choice $\tilde{F} = \frac{\Lambda}{2R}$ we obtain

$$\langle \mathcal{L}(g + \delta g) \rangle \approx \sqrt{-g} \left(-\frac{c}{2\varkappa} R - \Lambda \right)$$

Thus we have obtained Einstein gravity with $\Lambda-$term.

For the choice $\tilde{F} = const$ we obtain

$$\langle \mathcal{L}(g + \delta g) \rangle \approx \sqrt{-g}(-R)\left(\frac{c}{2\varkappa} + \text{const}\right)$$

Thus we have obtained Einstein gravity with *modified gravitational constant*.

The calculation of the expectation value of $\left\langle \mathcal{L} + \delta^2 \mathcal{L} \right\rangle$ gives us

$$\left\langle \mathcal{L} + \delta^2 \mathcal{L} \right\rangle = \sqrt{-g} \left\{ \left[\frac{1}{2} - F\left(R, \cdots\right) \right] \nabla^\mu \phi \nabla_\mu \phi - \left(\left[1 + 2F\left(R, \cdots\right) \right] V(\phi) \right\}. \tag{2}$$

We see that: (a) for big enough metric quantum fluctuations the function F becomes $F > 1/2$ and the scalar field can be a phantom one; (b) non-minimal coupling between scalar field and gravity appears.

Quantum fluctuations of metric give rise to $F(R)$ modified gravitational theories (with the applications to the explanation of modern Universe acceleration and so on).