Deformations of Cosmological Solutions of D=11 Supergravity

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Plan:

- D=11 Supergravity

- Lunin- Maldacena Deformations

- Deformations of some cosmological solutions

- Compactification to D=4

- Conclusions

Deformations of Cosmological Solutions of D=11 Supergravity
N.S.D., Ali Kaya
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D=11 Supergravity

The bosonic action of the 11-dimensional supergravity can be written as

$$S = \int d^{11}x(\sqrt{-g}R - \frac{1}{2}F \wedge *F + \frac{1}{6}F \wedge F \wedge A)\,,$$

where the last term is the Chern-Simons term. The equations of motion are given by

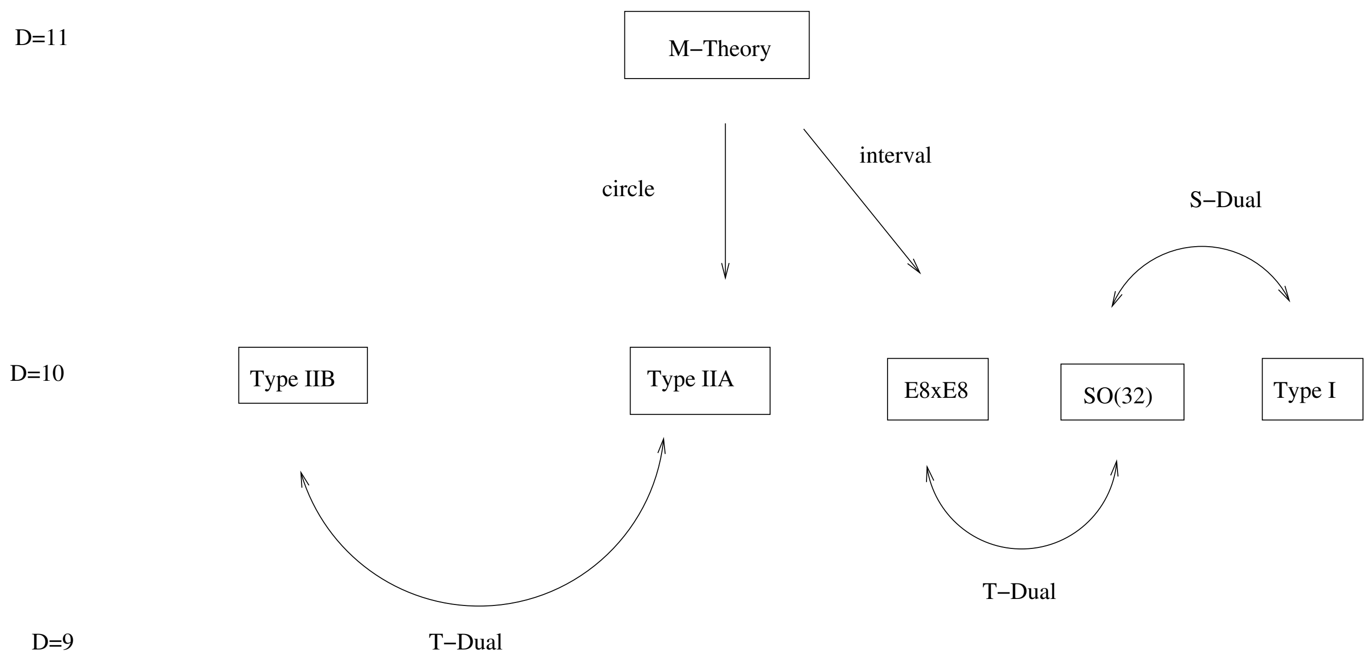
$$\begin{aligned} R_{AB} &= \frac{1}{2.3!}F_{ACDE}F_B{}^{CDE} - \frac{1}{6.4!}g_{AB}F^2, \\ d*F &= \frac{1}{2}F \wedge F\,. \end{aligned}$$

We also have the Bianchi identity $dF = 0$.

This is quite a simple theory in terms of number of fields and for a solution it is enough to specify its metric and the 4-form field strength only.

Now we want to find cosmological (that is, time dependent) solutions of this theory using solution generating techniques.

Using symmetries to find new solutions is quite an old and powerful idea. A string theory background which is independent of d-toroidal coordinates has an $O(d,d)$ T-duality symmetry. Exploiting this makes it possible to obtain new solutions starting from an existing one.



T-Duality: The energy spectrum of closed string states compactified on a circle of radius R is:

$$E = \frac{n}{R} + mR$$

where m (winding number) and n are integers. This is invariant under: $n \leftrightarrow m\,, \quad R \leftrightarrow \frac{1}{R}$. The Type IIA theory compactified on a circle of radius R is equivalent to the Type IIB string theory compactified on a circle of radius $1/R$.

We have $T^2 = Identity$ but if we do something non-trivial in between two T-dualities we may generate new solutions.

Deforming field theories with $U(1) \times U(1)$ *global symmetry and their gravity duals*
Oleg Lunin, Juan Maldacena
JHEP 0505 (2005) 033, hep-th/0502086.

Here they found the gravity dual of a marginal deformation (called β-deformation) of the dual CFT where the superconformal symmetry is preserved. Their primary example was the Type IIB $AdS^5 \times S^5$ background whose dual CFT is the $D = 4, N = 4$ super Yang-Mills theory.

Assuming that the ten dimensional gravity dual of the original, undeformed CFT has two $U(1)$ isometries, then on the gravity side this corresponds to a sequence of **TsT** transformation. Parametrizing the two torus by (x^1, x^2):

* T-duality along x^1

* A shift along x^2:($x^2 \to x^2 + \gamma x^1$), (γ is a real parameter)

* Another T-duality along x^1

- This is a particular solution generating method.

- If the initial geometry is non-singular, so is the new solution.

- If the original ten dimensional background is supersymmetric which is invariant under this $U(1) \times U(1)$, then the deformed background will also be supersymmetric (possibly with a lower amount).

- If the initial 10-dimensional solution has more than two $U(1)$'s we can repeat this procedure several times to obtain multiparameter solutions.

(*Lax Pair for Strings in Lunin-Maldacena Background*, Sergey Frolov, JHEP 0505:069,2005, hep-th/0503201.)

- To generalize this to D=11 backgrounds, a third $U(1)$ is necessary in the original solution since we have to first perform a dimensional reduction on a circle. After that, the TsT transformation is applied using the remaining two directions and the result is lifted back to 11-dimensions.

Deformations og D=11 Backgrounds

In

Beta, Dipole and Noncommutative Deformations of M-theory Backgrounds with One or More Parameters,
A. Çatal-Özer, N.S.D.
Class. Quantum Grav. 26 (2009) 245015
arXiv:0904.0629

general formulas for these deformed solutions in D=11 were obtained. With the help of these, one can write down deformed solutions directly in D=11 without going through details of this rather lengthy calculation. To use these formulas, it does not matter where these $U(1)$ directions lie in the geometry however the initial solution should satisfy certain conditions which are not too.

Assume that we have a solution of D=11 field equations with the following two properties:

(i) Its metric contains $I \geq 3$ $U(1)$ isometries, which do not mix with other coordinates.

(ii) Its 4-form field strength has at most one overlapping with these I directions.

$\{x^1, x^2, x^3\}$: 3 $U(1)$ directions which possibly mix among themselves but not with any other coordinate in the metric.

T: Denotes the 3×3 torus matrix that corresponds to $\{x^1, x^2, x^3\}$ coordinates. The entries of this matrix are read from the metric of the original solution, i.e. $T_{mn} = g_{mn}$.

Then, starting with a solution $\{F_4, g_{AB}\}$ where each term in F_4 has at most one common direction with $\{x^1, x^2, x^3\}$, after the deformation we find:

$$
\begin{aligned}
\tilde{F}_4 &= F_4 - \gamma i_1 i_2 i_3 \star_{11} F_4 + \gamma d\left(K detT\, dx^1 \wedge dx^2 \wedge dx^3\right), \\
d\tilde{s}^2_{11} &= K^{-1/3} g_{\mu\nu} dx^\mu dx^\nu + K^{2/3} g_{mn} dx^m dx^n \,, \\
K &= [1 + \gamma^2 detT]^{-1} \,,
\end{aligned}
$$

where $m, n =\{1, 2, 3\}$ and μ, ν denote the remaining directions. The new solution is given by $\{\tilde{F}_4, \tilde{g}_{AB}\}$. There is no need to check field equations again. The Hodge dual $\star_{11}$ is taken in the 11-dimensions, with respect to the undeformed metric and i_m is the contraction with respect to the isometry direction $\partial/\partial x^m$, i.e. $i_m \equiv i_{\partial/\partial x^m}$.

Here γ is a real deformation parameter and when $\gamma = 0$ we go back to the original solution.

When $I > 3$ one can repeat this process with different choices of 3 $U(1)$'s consistent with the above conditions and obtain a multiparameter deformation.

Observation:

$$\tilde{F}_4 = F_4 - \gamma i_1 i_2 i_3 \star_{11} F_4 + \gamma d \left(\frac{detT}{[1+\gamma^2 detT]} \, dx^1 \wedge dx^2 \wedge dx^3 \right)$$

Note that such a deformation always generates a 3-form potential along the deformation directions even when the original $F_4 = 0$, provided that $detT$ is not constant. For time-dependent solutions this term corresponds to the flux of a generalized SM2-brane lying along the $\{x^1, x^2, x^3\}$ directions. Its charge is proportional to the deformation parameter γ.

This is an interesting result since, in particular it means that if we have a cosmological solution of pure D=11 supergravity with the geometry $\mathbb{R}^{1+3} \times \mathcal{M}_7$, we can easily generalize this to a solution which has a nonzero 4-form flux along $\mathbb{R}^{1+3}$ using $U(1)$ directions of $\mathbb{R}^3$ which can be obtained by periodic identifications. The new solution is electrically charged and can be interpreted as a generalized SM2-brane with an arbitrary transverse space $\mathcal{M}_7$.

Example 1: S-Branes

Our starting point is the vacuum S-brane solution of D=11 supergravity which is given as:

$$
\begin{aligned}
ds^2 &= e^{2\lambda(t-t_1)/3}(dx_1^2+dx_2^2+dx_3^2)+e^{-\lambda(t-t_1)/3}\sum_{i=1}^{k} e^{2(b_i t-c_i)}d\theta_i^2 \\
&+ e^{-\lambda(t-t_1)/3}e^{2(b_0 t-c_0)}G_{n,\sigma}^{-\frac{n}{n-1}}(-dt^2+G_{n,\sigma}d\Sigma_{n,\sigma}^2)\,, \\
F_4 &= 0\,,
\end{aligned}
$$

where $d\Sigma_{n,\sigma}^2$ is the metric on the n-dimensional unit sphere, unit hyperbola or flat space and

$$
G_{n,\sigma}=\begin{cases} m^{-2}\sinh^2[(n-1)m\,(t-t_0)]\,, & \sigma=-1 \quad \text{(hyperbola)}, \\ m^{-2}\cosh^2[(n-1)m\,(t-t_0)]\,, & \sigma=1 \quad \text{(sphere)}, \\ \exp[2(n-1)m\,(t-t_0)], & \sigma=0 \quad \text{(flat)}, \end{cases}
$$

with $k+n=7$ and $n\geq 2$. Constants satisfy

$$
\begin{aligned}
b_0 t-c_0 &= -\frac{1}{n-1}\sum_{i=1}^{k} b_i t+\frac{1}{n-1}\sum_{i=1}^{k} c_i\,, \\
2n(n-1)m^2 &= \frac{2}{n-1}\left(\sum_{i=1}^{k} b_i\right)^2+2\sum_{i=1}^{k} b_i^2+\lambda^2\,.
\end{aligned}
$$

Here, we took the exponentials multiplying $\{x_1, x_2, x_3\}$ directions the same since after the deformation we want to have a homogeneous 3-dimensional space which will correspond to the worldvolume of an SM2.

We can set one of the constants $\{\lambda, b_1, ..., b_k\}$ to 1 by a rescaling and one of the integration constants $\{t_0, t_1, c_1, ..., c_k\}$ to zero by a shift in the time coordinate.

Since the metric is diagonal and the 4-form is zero, we can use any 3 from x or θ coordinates for the deformation by assuming that they are periodic. Choosing deformation directions as $\{x_1, x_2, x_3\}$, we see that the 3×3 torus matrix T is also diagonal with $detT = e^{2\lambda(t-t_1)}$ and applying our deformation formula to the vacuum S-brane solution we find:

$$
\begin{aligned}
d\tilde{s}^2 &= e^{2\lambda(t-t_1)/3}(1+\gamma^2 e^{2\lambda(t-t_1)})^{-2/3}(dx_1^2+dx_2^2+dx_3^2) \\
&+ (1+\gamma^2 e^{2\lambda(t-t_1)})^{1/3} e^{-\lambda(t-t_1)/3}\{\sum_{i=1}^{k} e^{2(b_i t - c_i)} d\theta_i^2 \\
&+ e^{2(b_0 t - c_0)} G_{n,\sigma}^{-\frac{n}{n-1}}(-dt^2 + G_{n,\sigma} d\Sigma_{n,\sigma}^2)\} \\
\tilde{F}_4 &= \gamma d[e^{2\lambda(t-t_1)}(1+\gamma^2 e^{2\lambda(t-t_1)})^{-1} dx_1 \wedge dx_2 \wedge dx_3]\,,
\end{aligned}
$$

where γ is the deformation parameter and when $\gamma = 0$ we go back to the vacuum S-brane solution. Note also that even though we started with a solution with no 4-form field, after the deformation we have a solution with $\tilde{F}_4 \neq 0$.

This is an SM2-brane solution located at $\{x_1, x_2, x_3\}$, however its metric is not in the familiar form which contains hyperbolic functions. To understand the relation, we scale $\{x_1, x_2, x_3\}$ coordinates with $\gamma^{-1/3}$ and all other coordinates and constants $\{m, \lambda, b_1, ..., b_k\}$ with $\gamma^{1/6}$ in the vacuum solution before performing the deformation, which makes γ disappear in K. Then, deforming this rescaled metric using $\{x_1, x_2, x_3\}$ directions we get:

$$
\begin{aligned}
d\tilde{s}'^2 &= H^{-1/3}(dx_1^2 + dx_2^2 + dx_3^2) + H^{1/6}[\sum_{i=1}^{k} e^{2(b_i t - c_i)} d\theta_i^2 \\
&+ e^{2(b_0 t - c_0)} G_{n,\sigma}^{-\frac{n}{n-1}} (G_{n,\sigma} d\Sigma_{n,\sigma}^2 - dt^2)] \\
\tilde{F}'_4 &= q\lambda H^{-1} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \,, \\
H &\equiv q^2 \cosh^2 \lambda(t - t_1) \quad , \quad q \equiv 2\gamma \,,
\end{aligned}
$$

which is the standard SM2 solution with k-smearings whose transverse space is of the form $\mathbb{R}_1 \times ... \times \mathbb{R}_k \times \Sigma_{n,\sigma}$.

Note that $\gamma \to 0$ is not well-defined anymore, i.e., this solution is valid only when $q \neq 0$. This analysis clarifies the passage from the SM2-brane solution to the vacuum S-brane solution.

From this point it is possible to continue with more deformations since there were $(k+3)$ appropriate coordinates in the initial vacuum solution and we used only 3. We have two options which are consistent with our rules: either we choose one $U(1)$ direction from the worldvolume of SM2 and two from outside or we choose all of them transverse to the SM2. For the first choice, if we take $\{x_3, \theta_1, \theta_2\}$ directions for deformation, then this adds another SM2 along these directions and we get the standard SM2⊥SM2(0) intersection found in:

Intersecting S-Brane Solutions of D=11 Supergravity
N.S.D., Ali Kaya
JHEP 0207 (2002) 038, hep-th/0206057.

where the worldvolume of the second SM2 is inhomogeneous with some exponentials of time which is due to the choice that we made for homogeneous directions in our initial vacuum.

For the latter, without loss of generality let us use $\{\theta_1, \theta_2, \theta_3\}$ coordinates to deform the SM2 solution. Since these do not overlap with any of the directions of its 4-form field strength and they do not mix with any coordinate in the metric, we are allowed to use the deformation formula. Again the 3×3 torus matrix T is diagonal and after the deformation of the SM2 solution with the parameter γ_1 we find:

$$
\begin{aligned}
d\widehat{s}^2_{11} &= K^{2/3}H^{1/6}\sum_{i=1}^{3} e^{2(b_it-c_i)}d\theta_i^2 + K^{-1/3}H^{1/6}\sum_{i=4}^{k} e^{2(b_it-c_i)}d\theta_i^2 \\
&+ K^{-1/3}H^{-1/3}(dx_1^2+dx_2^2+dx_3^2) \\
&+ K^{-1/3}H^{1/6}G_{n,\sigma}^{-\frac{n}{n-1}}e^{2(b_0t-c_0)}\left[-dt^2+G_{n,\sigma}d\Sigma^2_{n,\sigma}\right]\,, \\
K &= [1+\gamma_1^2 q\cosh\lambda(t-t_1)e^{2(bt-c)}]^{-1}\,, \\
b &= (b_1+b_2+b_3)\,,\; c=(c_1+c_2+c_3)\,, \\
\widehat{F}_4 &= \widetilde{F}'_4 - \gamma_1 q\lambda \mathrm{Vol}(\theta_k)\wedge\mathrm{Vol}(\Sigma_{n,\sigma}) \\
&+ \frac{\gamma_1 q\cosh\lambda(t-t_1)[\lambda\tanh\lambda(t-t_1)+2b]e^{2(bt-c)}}{[1+\gamma_1^2 q\cosh\lambda(t-t_1)e^{2(bt-c)}]^2}dt d\theta_1 d\theta_2 d\theta_3
\end{aligned}
$$

This is nothing but a slight generalization of the solution given in:

Chern-Simons S-Brane Solutions in M-theory and Accelerating Cosmologies
N.S.D., Ali Kaya
JHEP 0904:109,2009, arXiv: 0903.1186.

which was previously obtained by directly solving the field equations. It is more general because exponentials in front of the $\{\theta_1,\theta_2,\theta_3\}$ coordinates in the metric are not all equal and it is possible to have two smearings instead of one. Moreover, in the previous solution the constant γ_1 does not appear explicitly, and hence it does not reduce to the single SM2-brane by setting a constant to zero unlike the solution above.

Looking at $\widehat{F}_4$ we see that this new solution describes two SM2-branes located at $\{x_1, x_2, x_3\}$ and $\{\theta_1, \theta_2, \theta_3\}$ and an SM5-brane at $\{x_1, x_2, x_3, \theta_1, \theta_2, \theta_3\}$. Note that $\widehat{F}_4 \wedge \widehat{F}_4 \neq 0$ and therefore the contribution of the Chern-Simons term to the 4-form field equations is non-zero:

$$d * F = \frac{1}{2} F \wedge F$$

From the deformation formula:

$$\tilde{F}_4 = F_4 - \gamma i_1 i_2 i_3 \star_{11} F_4 + \gamma d \left(\frac{detT}{[1 + \gamma^2 detT]} \, dx^1 \wedge dx^2 \wedge dx^3 \right)$$

we see that there is no way to obtain the SM5-brane solution from any vacuum solution. However, we can start directly with the SM5-brane solution. Since the 4-form field of an SM5-brane lies along the transverse space there are two options for deformations that are compatible with our rules: Either all 3 are chosen from the worldvolume of SM5 or two are chosen from the worldvolume and one from the outside. In the first case, after deformation one gets again the above Chern-Simons system. Whereas, from the latter starting from an SM5 with inhomogeneous worldvolume and 1-smearing one obtains the standard SM2$\perp$SM5(1) intersection. Thus, we have all the standard double intersections between SM2 and SM5 branes which have supersymmetric p-brane analogs except SM5$\perp$SM5(3).

Now, a large number of S-brane configurations can be constructed by applying more deformations that are compatible with our conditions. To increase this number one can use SM2, SM5 and SM5$\perp$SM5(3) as a basis and systematically perform deformations. In finding the list of resulting configurations it is enough to remember the following set of rules about positions of available deformation directions:

$$\text{SM2} \xrightarrow{\text{2 transverse}} \quad \text{SM2}\perp\text{SM2(0)}$$

$$\text{SM2} \xrightarrow{\text{3 transverse}} \quad \text{CSS}$$

$$\text{SM5} \xrightarrow{\text{2 worldvolume}} \text{SM2}\perp\text{SM5(1)}$$

$$\text{SM5} \xrightarrow{\text{3 worldvolume}} \text{CSS}$$

where CSS stands for the Chern-Simons S-brane system in which there are 2 non-intersecting SM2-branes inside an SM5. Of course, when there are more than one brane in the initial system these rules should be used simultaneously. In this way, we can get all the standard S-brane intersections found earlier which have static supersymmetric analogs. There are also intersections where each S-brane pair makes a standard intersection but overall intersection has no supersymmetric analog, however their construction still follows intersection rules found in the past. Besides these, overlappings between CSS systems and CSS systems with extra S-branes are allowed which are new in the S-brane literature.

Example 2: Power Law Solution

Of course, it is straightforward to generate additional new solutions using this mechanism. For example, deforming the power-law solution given in:

Kaluza-Klein Cosmologies
P.C.O. Freund
Nucl.Phys.B209:146,1982.

we arrive at:

$$\begin{aligned}
d\tilde{s}^2 &= (1+\gamma^2\alpha_1^3 t^{-2/7})^{-2/3}\alpha_1 t^{-2/21}(dx_1^2+dx_2^2+dx_3^2) \\
&+ (1+\gamma^2\alpha_1^3 t^{-2/7})^{1/3}[\alpha_1 t^{-2/21}(dx_4^2+...+dx_7^2) \\
&+ \alpha_2 t^{-2/3} d\Sigma_{3,-1}^2 - \alpha_3 t^{-8/3} dt^2]\,, \\
\tilde{F}_4 &= \frac{\lambda t^{-2}}{2} dt\wedge \mathrm{Vol}(\Sigma_{3,-1}) + \gamma d\left[\frac{\alpha_1^3 t^{-2/7}}{1+\gamma^2\alpha_1^3 t^{-2/7}} dx_1\wedge dx_2\wedge dx_3\right] \\
&- \frac{\gamma\lambda\alpha_2^{-3}}{2} dx_4\wedge\cdots\wedge dx_7\,,
\end{aligned}$$

where constants are fixed as $(\alpha_1)^{21} = 27\lambda^4/224$, $(\alpha_2)^3 = 2/(7\lambda^2)$ and $\alpha_3 = (\alpha_1)^7(\alpha_2)^3$.

This is another SM2-SM2-SM5 Chern-Simons system for which $\tilde{F}_4\wedge\tilde{F}_4 \neq 0$. The initial SM2-brane has hyperbolic worldvolume $\Sigma_{3,-1}$. The other SM2 is located at $\{x_1, x_2, x_3\}$ and SM5 worldvolume contains both of them.

Example 3: A New SM2

Deforming the vacuum solution found recently we get:

$$\begin{aligned} d\tilde{s}^2 &= e^{2\lambda t/3}(1+\gamma^2 e^{2\lambda t})^{-2/3}(dx_1^2+dx_2^2+dx_3^2) \\ &+ (1+\gamma^2 e^{2\lambda t})^{1/3} e^{-\lambda t/3} G_{7,\sigma}^{-7/6}\left(G_{7,\sigma}\sum_{i=1}^{n} e^{\beta_i/3} d\Sigma_{m_i,\sigma}^2 - e^{\beta/3} dt^2\right) \\ \tilde{F}_4 &= \frac{2\gamma\lambda e^{2\lambda t}}{(1+\gamma^2 e^{2\lambda t})^2} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \,, \end{aligned}$$

where the $\Sigma_{m_i,\sigma}$'s are $m_i \geq 2$ dimensional spaces with the same type of constant curvature specified by σ and $\sum_{i=1}^{n} m_i = 7$.

This represents an SM2-brane located at $\{x_1, x_2, x_3\}$ with a transverse space of the form $M_{m_1,\sigma} \times \cdots \times M_{m_n,\sigma}$. The warping constants β_i and β are determined as

$$\begin{aligned} \beta_i &= \frac{1}{2}\ln\left[\frac{6}{(m_i-1)}\prod_{j=1}^{n}\left(\frac{m_i-1}{m_j-1}\right)^{m_j}\right] \\ \beta &= \sum_{i=1}^{n} m_i\beta_i = \frac{7}{2}\ln\left[6\prod_{i=1}^{n}(m_i-1)^{-m_i/7}\right] \end{aligned}$$

When the transverse space is only one piece ($n=1$), all β_i's and β become zero and the above solution reduces to the usual SM2-brane with no smearings.

Compactification to $D = 4$ and Accelerating Cosmologies

It is well-known that compactified SM2-branes can yield accelerating cosmologies in $D = 4$. Unfortunately, in all the examples studied so far the amount of e-foldings is only of order 1 and hence these are not useful for explaining early time inflation. Yet, such solutions might be relevant for the presently observed acceleration of our universe.

After compactification from D=11 to (1+3)-dimensions, the 4-dimensional part of all the above S-brane solutions in the Einstein frame has the form:

$$ds_E^2 = -S^6\,dt^2 + S^2\,ds_{M_3}^2,$$

where S is some function of time that depends on the solution and M_3 is a three dimensional homogeneous space.

This universal structure is due to a particular property of these solutions. Namely, the function in front of the time coordinate in the metric is given as multiplication of powers of other functions appearing in the metric where powers are the dimensions of spaces that these functions multiply.

Now, the proper time is given by $d\tau = S^3 dt$ and the expansion and acceleration parameters can be found respectively as

$$H = S^{-1}\frac{dS}{d\tau} = S^{-4}\frac{dS}{dt}, \qquad a = \frac{d^2S}{d\tau^2} = -\frac{1}{2}S^{-3}\frac{d^2}{dt^2}S^{-2}.$$

An accelerating phase requires $H > 0$ and $a > 0$.

Looking at the deformation formula again:

$$d\tilde{s}^2_{11} = K^{-1/3} g_{\mu\nu} dx^\mu dx^\nu + K^{2/3} g_{mn} dx^m dx^n \,, K = [1+\gamma^2 detT]^{-1}$$

we see that effect of the deformation on a metric is to bring factors of K.

If after the deformation we compactify on a (1+3)-dimensional space whose spatial part M_3 was not used for the deformation, then it is easy to see that in the Einstein frame the factor K does not appear in the function S. Hence, the cosmology of compactication on (t, M_3) will be the same before and after the deformation.

On the other hand, if we use three coordinates of M_3 for the deformation, then we get a factor of $K^{-1/4}$ in the S function and the cosmology will now be different.

This is actually not a surprise, since the main effect of such a deformation is to produce a 3-form potential along the deformation directions (possibly with some additional fluxes) which will change the 4-dimensional cosmology only if we compactify on these coordinates.

This argument, together with the fact that we want M_3 to be a homogeneous space imply that increasing the number of standard intersections will not improve the amount of e-foldings as was explicitly checked for double intersections previously.

The above discussion shows that for the usual SM2 solution only compactification on $\{t, x_1, x_2, x_3\}$ may lead to a result different than the vacuum. In this case the S function is given as

$$S = e^{-\lambda t/4}(1+\gamma^2 e^{2\lambda t})^{1/4} e^{-\frac{(b_1+\ldots+b_k)t}{2(n-1)}} e^{\frac{(c_1+\ldots+c_k)}{2(n-1)}} G_{n,\sigma}^{-\frac{n}{4(n-1)}},$$

where $k+n=7$ and $k \leq 5$.

The enhancement of the 4-form flux on acceleration can be clearly seen in this example. For the vacuum case ($\gamma = 0$) an accelerating phase happens only when $\sigma = -1$ whereas when $\gamma \neq 0$ the expansion factor gets slightly bigger for $\sigma = -1$ and acceleration exists also for $\sigma = 0$ and $\sigma = 1$. For $\gamma \neq 0$ previously only $k = 0$ and $k = 1$ cases were analyzed explicitly. We found that increasing the number of flat product spaces does not lead to a significant modification and still the acceleration is of order 1.

For the Chern-Simons S-brane system compactification on $\{t, x_1, x_2, x_3\}$ will give the same answer like SM2-brane as was explicitly observed before. However, compactification on $\{t, \theta_1, \theta_2, \theta_3\}$ will give different S functions and hence different cosmologies. For this case, choosing $b_1 = b_2 = b_3 \equiv d/6$ and $c_1 = c_2 = c_3 \equiv t_2/6$ and we find

$$S = [1+\gamma_1^2 q \cosh \lambda(t-t_1) e^{dt-t_2}]^{1/4} e^{-\frac{(n+2)(dt-t_2)}{12(n-1)}} e^{-\frac{(b_4+b_5)t-(c_4+c_5)}{2(n-1)}} G_{n,\sigma}^{-\frac{n}{4(n-1)}},$$

where $2 \leq n \leq 4$. Previously this compactification with no smearings ($n = 4$) were studied and an accelerating interval was found for each σ . However, the amount of e-folding was again of order unity. When $n = 3$ and $n = 2$ there is a short period of acceleration too, however there is no major change in the expansion factor.

For the new Chern-Simons S-brane system that we constructed above, after the compactification we get a different cosmology from the original one only if we compactify on $\{t, x_1, x_2, x_3\}$. In this case we have

$$S = (1 + \gamma^2 t^{-2/7})^{1/4} \, t^{-9/14} \, .$$

However, we find that acceleration is always negative with or without deformation. Even though, we have flux along $\{t, x_1, x_2, x_3\}$ and some part of the transverse space is hyperbolic we do not get an accelerating phase.

Finally, for the new SM2 solution with product transverse space, if we compactify on $\{t, x_1, x_2, x_3\}$ the S function is given as

$$S = e^{\beta/12} e^{-\lambda t/4} (1 + \gamma^2 \alpha_1^3 e^{2\lambda t})^{1/4} G_{7,\sigma}^{-7/24},$$

whose form is almost identical with the SM2-brane case. Hence, it immediately follows that when $\gamma = 0$ there is acceleration only for $\sigma = -1$. However, it also occurs for $\sigma = 1$ and $\sigma = 0$ after the 4-form becomes nonvanishing, alas only of order 1.

Conclusions

• Here we looked at applications of Lunin-Maldacena deformations to cosmological solutions of D=11 supergravity. The method becomes especially useful if we have a solution of pure Einstein equations which has an $\mathbb{R}^3$ part in its geometry. Then, using the deformation this can be generalized to a solution with a 3-form potential along these directions. To realize inflation is a big challenge for String/M-theory and compactifications with different transverse space geometries and fluxes is a promising way to attack this puzzle. Furthermore, this also shows that to construct flat SM2-brane solutions with general transverse spaces, it is enough to concentrate only on Einstein equations with $F_4 = 0$. We hope that with this simplification it will be easier to construct such solutions which may have better cosmological features.

• As we saw, using the deformation repeatedly it is possible to obtain configurations with several S-branes some of which are new solutions. We showed that all known SM2-brane solutions and their intersections among each other can be ontained via this deformation. If we extend our basis of initial solutions to include SM2⊥SM2(-1) and SM2⊥SM5(0) intersections found earlier which have no supersymmetric analogs, then we will obtain intersections between these, standard S-branes and Chern-Simons S-brane systems too. Cosmological aspects of such solutions need further examination.

• This method also makes transparent why there is no acceleration in certain compactifications.

• It would be interesting to find generalization of this which covers also the SM5-brane.

• It may also be interesting to apply this method to intersections of S-branes with p-branes.

• We can of course apply our method to static solutions as well. However, deformation of a static vacuum does not give an M2-brane but a static SM2 whose world-volume is Euclidean. The connection between one of the static versions of our Chern-Simons S-brane system and dyonic M-brane solution was already noted before. We expect our approach to be useful in construction of Poincaré symmetric versions of these black brane solutions.