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Stability of creativeness of Schwarzschild metric

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1.- INTRODUCTION

- After selecting a suitable energy-momentum complex, one can derive some expression for the linear, P^α, and angular $J^{\alpha\beta}$, 4-momenta for a given space-time in General Relativity, $\alpha, \beta, ... = 0, 1, 2, 3$.

    - S. Weinberg, *Gravitation and Cosmology* (John Wiley and Sons, 1972)

- We select the Weinberg complex by reasons explained in

    - R. Lapiedra and J. A. Morales, *Gen. Relativ. Gravit.* (2013) **45** 1145

- These 4-momenta are dramatically dependent on the coordinates used.

- Then, we define intrinsic P^α and $J^{\alpha\beta}$ values as the ones calculated in intrinsic coordinates

    - R. Lapiedra and J. A. Morales, *Gen. Relativ. Gravit.* (2013) **45** 1145

- By definition, intrinsic coordinates (t, x^i) for an asymptotically Minkowski space-time are the ones satisfying the three following properties:
  - They are Gauss coordinates (their metric components $g_{0\alpha}$ are $g_{00} = -1$, $g_{0i} = 0$), asymptotically fast enough for a given $t = t_0$.
  - Their associated linear and angular 3-momenta vanish: $P^i = J^{ij} = 0$, J^{ij} vanishing irrespective of the momentum origin.
  - They are asymptotically fast enough rectilinear coordinates.
- Intrinsic coordinates are not unique but they can be proved to exist for any given constant time t_0.

  - J. J. Ferrando, *et al.*, *Phys. Rev. D* **75** (2007) 124003

2.- CREATABLE UNIVERSES

- People have speculated about the possibility that our Universe come from a vacuum quantum fluctuation.

  - M. G. Albrow, *Nature* **241** (1973) 56; E. P. Tryon, *Nature* **246** (1973) 396

- The idea has been developed lately

  - A. Vilenkin, *Phys. Rev. D* **32** (1985) 2511

- We could then conjecture that the corresponding space-time should have vanishing intrinsic 4-momenta.

- We have called classically *self-creatable* (creatable for short) the space-times with vanishing 4-momenta

$$P^{\alpha} = 0, \quad J^{\alpha\beta} = 0,$$

J^{ij} components vanishing irrespective of its origin.

  - R. Lapiedra and J. A. Morales, *Gen. Relativ. Gravit.* (2012) **44** 367

3.- THE SCHWARZSCHILD METRIC IN INTRINSIC COORDINATES

- The Schwarzschild (S) metric can be written as

$$ds^2 = -dT^2 + \frac{\rho}{r} d\rho^2 + r^2(d\theta^2 + \sin^2\theta \, d\phi^2),$$

  - R. Lapiedra and J. A. Morales, *Gen. Relativ. Gravit.* (2013) **45** 1145

- The coordinates (T, ρ) are the following functions of the standard stationary ones (t, r):

$$T = t + 2\sqrt{r_0 r} + r_0 \ln \left| \frac{\sqrt{r} - \sqrt{r_0}}{\sqrt{r} + \sqrt{r_0}} \right|, \quad \rho = \left(r^{3/2} + \frac{3}{2}\sqrt{r_0}\, T + C\right)^{2/3}$$

  with r_0 the S radius and C an arbitrary constant.

- It can be easily seen that the coordinates (T, ρ, θ, ϕ) are a particular example of intrinsic ones everywhere except for the essential singularity $r = 0$.

- They are asymptotically at rest coordinates and they are adapted to the spherical symmetry of the S metric.

4.- THE ADM ENERGY EXPRESSION

- For a regular enough space-time, we can write the energy P^0 derived from the Weinberg complex as the ADM energy, i.e., the following 2-surface integral

$$P^0 = \frac{1}{16\pi} \int (\partial_j g_{ij} - \partial_i g) d\Sigma_{2i}$$

  - R. Arnowitt, S. Deser and C. W. Misner, in *Gravitation: an introduction to current research* (John Wiley, 1962).

  - Gravitational constant $G =$ speed of light $c = 1$,
  - g_{ij}: 3-space metric components $i, j, ... = 1, 2, 3,$
  - repeated indices are summed,
  - $g \equiv \delta_{ij} g_{ij}$, δ_{ij} is the Kronecker symbol,
  - Σ_2 is the boundary of the 3-space,
  - $d\Sigma_{2i}$ is the 2-surface element of integration.

5.- VANISHING INTRINSIC VALUES OF P^α and $J^{\alpha\beta}$ FOR A SCHWARZSCHILD (S) METRIC

- The value of P^0 in the particular intrinsic coordinates (T, ρ, θ, ϕ) vanishes if the radius, r_1, of the corresponding ideal star is larger than r_0.
- It also vanishes in a natural sense if $r_1 < r_0$ (a black hole).
- $P^i = J^{0i} = 0$, $J^{ij} = 0$, merely because the intrinsic coordinates (T, ρ, θ, ϕ) are adapted to the spherical symmetry of the S metric.
- Thus, the S metric is a creatable space-time
  - R. Lapiedra and J. A. Morales, *Gen. Relativ. Gravit.* (2013) **45** 1145
- This is not in contradiction with the fact that $P^0 = m$ (m being the star mass) when calculated in standard stationary coordinates.

6.- IS THE CREATIVENESS OF THE SCHWARZSCHILD METRIC STABLE?

- Is this creatable character a mere artifact of the extreme symmetry of the S metric deprived of any physical meaning?
- To test this: we confer a slow rotation to our non rotating star.
- That is, we consider the corresponding Lense-Thirring metric $\equiv$ Kerr metric, linearized in the a parameter (the angular momentum per unit mass), far away enough from the center.
- In a given family of intrinsic coordinates, (T, ρ, θ, Φ), this linearized metric can be written as

$$ds^2 = -dT^2 + \frac{\rho}{r} d\rho^2 + r^2 (d\theta^2 + \sin^2\theta \, d\Phi^2) - 2a \frac{\sqrt{r_0 \rho}}{r} [1 + r^3 h'(R)] \sin^2\theta \, d\rho \, d\Phi$$

with

$$\Phi = \phi + \frac{a}{r_0} \Big[2\sqrt{\frac{r}{r_0}} + \ln \Big| \frac{\sqrt{r} - \sqrt{r_0}}{\sqrt{r} + \sqrt{r_0}} \Big| \Big] + a \, r_0 \, h(R),$$

h and arbitrary function, $R \equiv \frac{2}{3\sqrt{r_0}} (\rho^{3/2} + C)$, and $h' \equiv dh/dR$.

7.- TESTING THE STABILITY OF THE SCHWARZSCHILD CREATIVENESS

- Some calculations allow us to prove that we can choose the arbitrary function $\Psi(\rho) \equiv h'(R)$ such that $P^{\alpha} = 0$ and $J^{\alpha\beta} = 0$ for our linearized Kerr metric in the (T, ρ, θ, Φ) intrinsic coordinates.
- Thus, the linearized Kerr metric is creatable.
- Consequently, the creativeness of the S metric is a stable property, and we say that the S metric is actually creatable.

8.- FUTURE WORK: ADDRESSING THE QUANTUM CREATION QUESTION

- In order to estimate the probability of creation of a Schwarzschild metric, in our Gauss coordinates, through quantum tunnelling (see for example A. Vilenkin, PRD 32, 2511 (1985)), we look forward to calculate the action value of the corresponding instanton.

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