Drude weight, cyclotron resonance and the Dicke model of graphene cavity QED

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Superradiant Phase Transition

$$\mathcal{H}[\mathbf{A}] = \mathcal{H}_{\rm mat}[\mathbf{A}] + \mathcal{H}_{\rm em}[\mathbf{A}]$$

Superradiant GS when minimum energy for finite static **A**

$$\Delta E_{\rm mat} = \int d^D\mathbf{r}\; \delta\mathbf{A}\cdot\langle \hat{\mathbf{J}}_{\rm phys}(\mathbf{r})\rangle$$

$$\langle \hat{J}^\mu_{\rm phys}(\mathbf{r})\rangle = (e/c)V^{-1}\sum_{\mathbf{q}} \Xi^{\mu\nu}(q)\delta A_\nu(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}} + {\rm c.c.}$$

$$\Xi^{\mu\nu}(q) = \chi^{\mu\nu}(q) + K^{\mu\nu}(q)$$

paramagnetic diamagnetic

Gauge invariance

gauge invariance of the EM field $\mathbf{A}(\mathbf{r}) \to \mathbf{A}(\mathbf{r}) + \nabla\Lambda(\mathbf{r})$

If **gauge invariance** is unbroken the system **cannot respond** to a static uniform **A**

diamagnetic contribution always cancels the paramagnetic contribution

$$\lim_{q\to 0} K^{\mu\nu}(q) = -\lim_{q\to 0}\chi^{\mu\nu}(q)$$

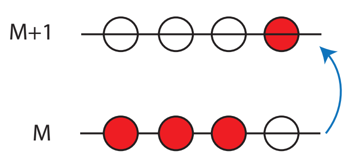
Superradiant phase transition cannot occur at equilibrium in systems in which gauge invariance is unbroken

Graphene cyclotron resonance

$$\mathcal{H}_0 = \sum_{\lambda,n,k} \lambda\hbar\omega_c\sqrt{n}|\lambda,n,k\rangle\langle\lambda,n,k|$$

$$\mathcal{H}_{\rm int} = v_D(e/c)\boldsymbol{\sigma}\cdot\mathbf{A}_{\rm em}(\mathbf{r}) \qquad \mathbf{A}_{\rm em}(\mathbf{r}) \to \mathbf{A}_{\rm em}$$

$$(\mathcal{H}_{\rm int})_{\lambda',n';\lambda,n} \propto \lambda\delta_{n',n+1}A^-_{\rm em} + \lambda'\delta_{n',n-1}A^+_{\rm em}$$



$$\mathcal{H}_{\rm eff} = \sum_{k=1}^{\mathcal{N}}\left(-\frac{\Omega_M}{2}\tau^z_k + \epsilon_{\rm em}\tau^+_k + \epsilon^*_{\rm em}\tau^-_k\right)$$

Does Superradiance occur?

Quantum phase transition of second order $\langle A_{\rm em}\rangle$ order parameter

origin of the transition

$$\Delta E^{\rm intra}_M = -\mathcal{N}\left(\frac{ev_D}{2c}\right)^2\frac{\mathbf{A}^2_{\rm em}}{\hbar\omega_c}(\sqrt{M+1}+\sqrt{M})$$

negative correction to matter energy at second order in the field A

not compatible with gauge invarinace:
need to compensates for the negative energy shift

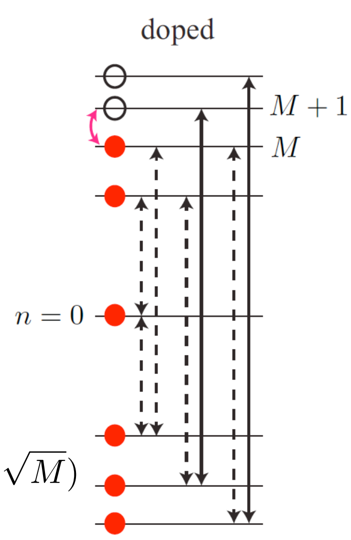
Role of valence band

Energy correction at second order in A

$$|-,n,k\rangle \to |+,n-1,k\rangle,\quad |+,n+1,k\rangle$$

dynamically-generated inter-band diamagnetic contribution

$$\Delta E^{\rm (inter)}_M = \mathcal{N}\left(\frac{ev_D}{2c}\right)^2\frac{\mathbf{A}^2_{\rm em}}{\hbar\omega_c}(\sqrt{M+1}+\sqrt{M})$$



Dicke model of Graphene cavity QED

$$\mathcal{H}_{\rm eff} \to \mathcal{H}_{\rm eff} + S\frac{\mathcal{D}_M}{2\pi c^2}\mathbf{A}^2_{\rm em} \qquad \text{Drude weight}$$

$$\mathcal{H}_{\rm Dicke} = \hbar a^\dagger a - \frac{\Omega_M}{2}\sum_{k=1}^{\mathcal{N}}\tau^z_k + \frac{g}{\sqrt{\mathcal{N}}}\sum_{k=1}^{\mathcal{N}}\tau^x_k(a+a^\dagger) + \kappa(a+a^\dagger)^2$$

QPT if $\omega\Omega_M(1+4\kappa/\omega)/(4g^2) < 1$

SPT forbidden due to $g^2 = \kappa\Omega_M$

(in parabolic band f-sum rule)

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