Photon solid phases in driven arrays of nonlinearly coupled cavities

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Quantum Simulations

Benasque, Spain, 1st October 2013

Photon solid phases in driven arrays of nonlinearly coupled cavities

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@ TUM, Münich

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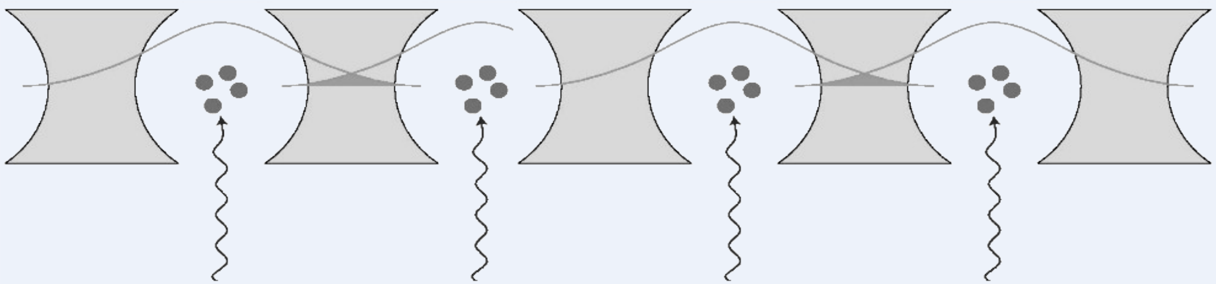
Arrays of coupled QED cavities



– M. Hartmann, F. Brandão, M. Plenio, *Nature Phys.* **2**, 849 (2006)
– A. Greentree, C. Tahan, J. Cole, L. Hollenberg, *Nature Phys.* **2**, 856 (2006)
– D. Angelakis, M. Santos, S. Bose, *PRA* **76** 031805(R) (2007)

Many body physics ?

Two competing phenomena

- Photon hopping between adjacent cavities
- Effective on-site nonlinearity

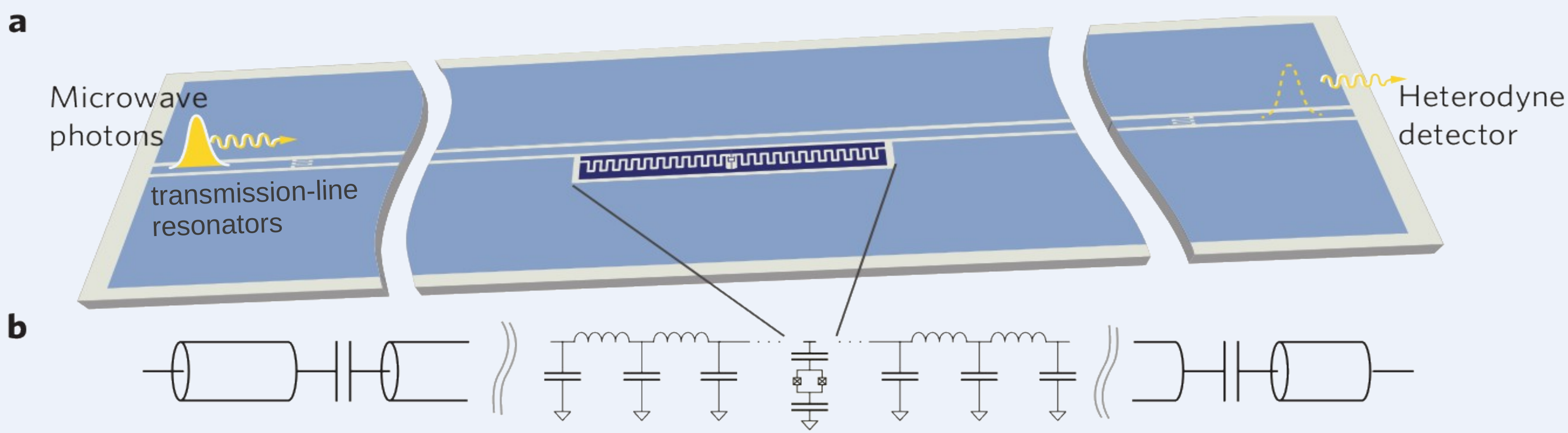
$$\mathcal{H} \sim -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + U \sum_i n_i^2$$

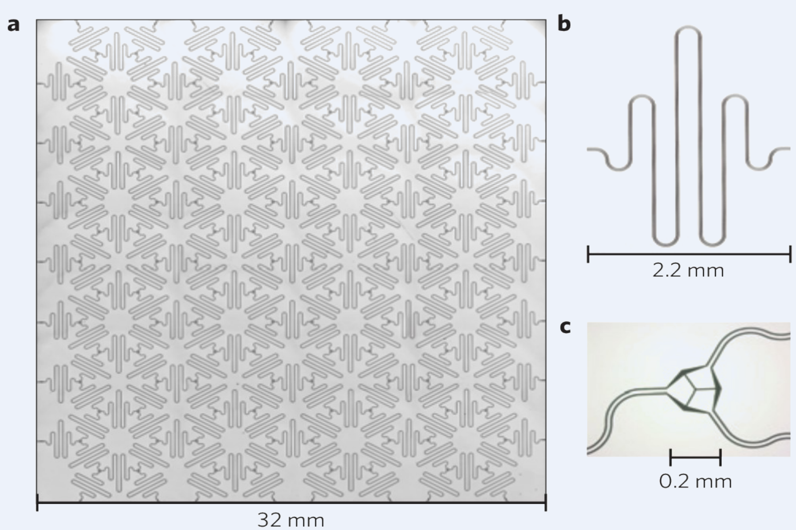
effective *Bose-Hubbard model*
for ***dressed photons***

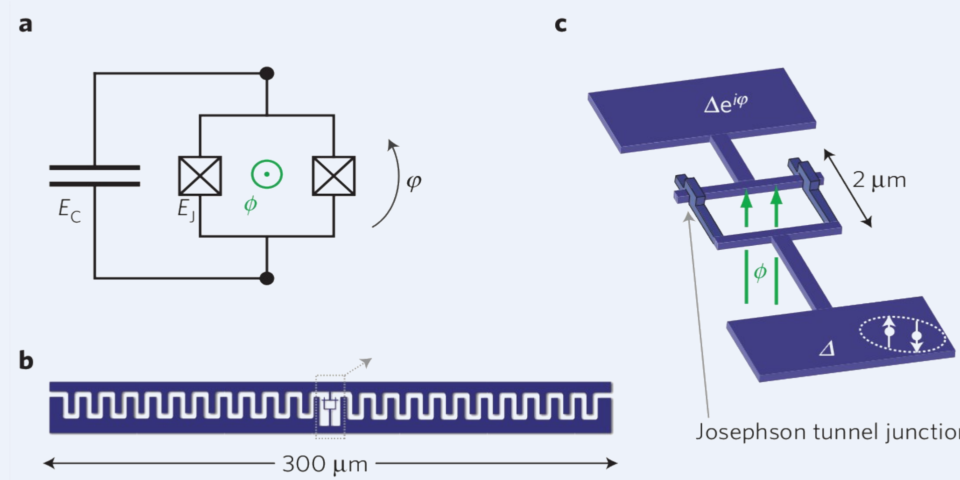
Strongly-coupled QED cavities

e.g. with superconducting circuits

particles ⟷ circuit excitations

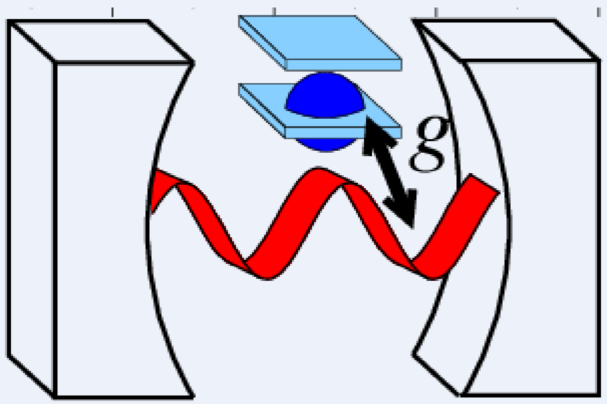






A. Houck, H. Türeci, J. Koch, *Nature Phys.* **8**, 292 (2012)

Effective on-site nonlinearity (*photon blockade*)

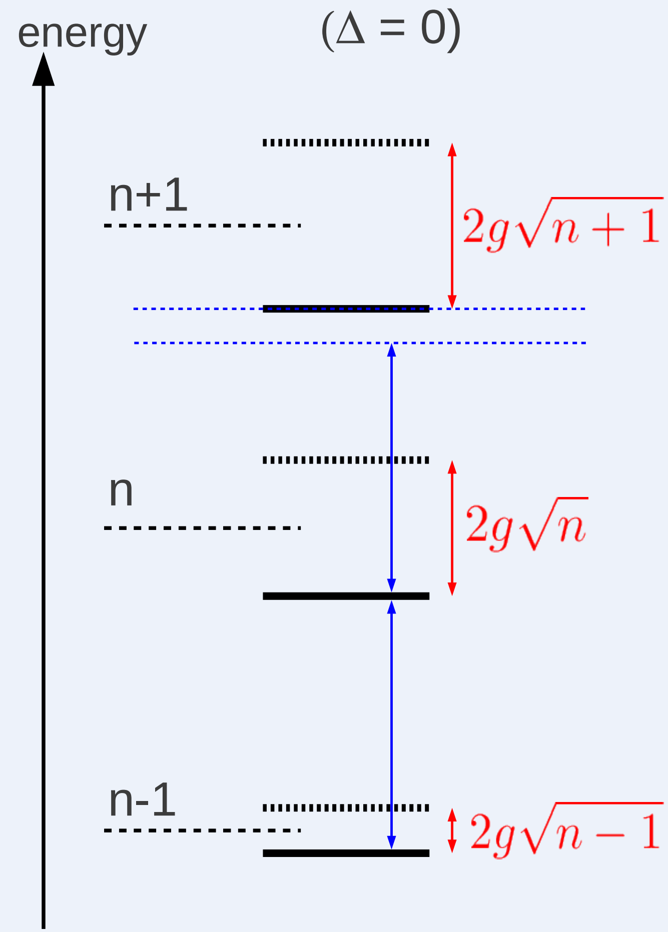


The Jaynes-Cummings model in a cavity

$$\mathcal{H}_{\mathrm{JC}} = \omega a^\dagger a + \varepsilon S^z + g\left(S^+ a + S^- a^\dagger\right)$$

$$E_n^{\pm} = n\omega + \Delta/2 \pm \sqrt{ng^2 + \Delta^2/4}$$

$$\Delta = \epsilon - \omega$$



$$U_{\mathrm{eff}} \sim \partial_n^2 E(n) > 0$$

A. Imamoğlu, H. Schmidt, G. Woods, M. Deutsch, *PRL* **79**, 1467 (1997)
S. Rebić, S. Tan, A. Parkins, D. Walls, *J. Opt. B* **1**, 490 (1999)

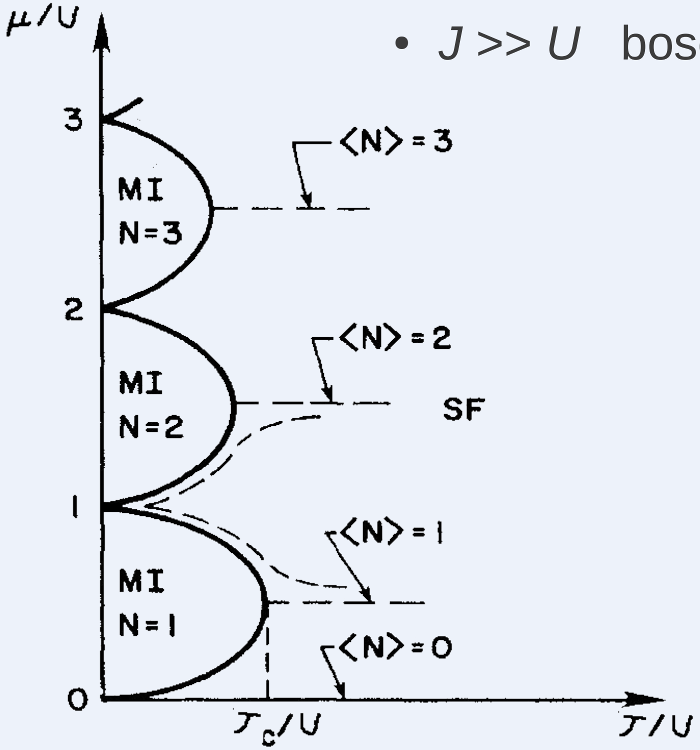
Bose-Hubbard model

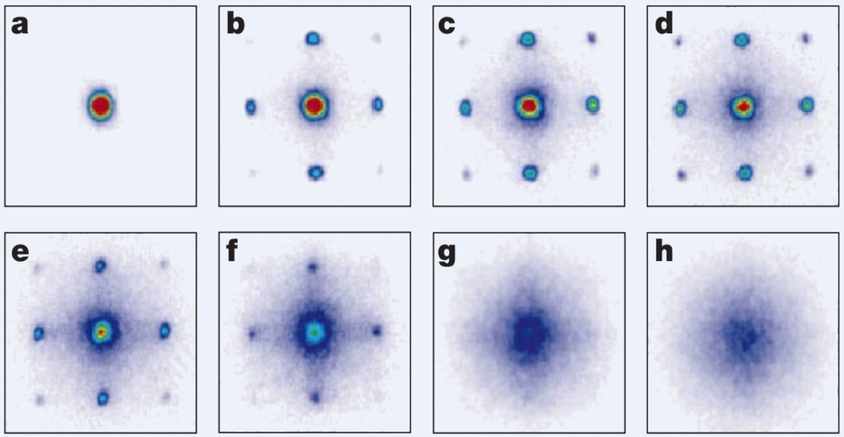
$$\mathcal{H}_{\mathrm{BH}} = -J \sum_{\langle i,j \rangle} (b_i^\dagger b_j + b_j^\dagger b_i) + \frac{U}{2} \sum_i n_i (n_i - 1)$$

- $J << U$ boson localization (*Mott Insulating* phase)
  gapped, zero superfluid density
- $J >> U$ boson delocalization (*superfluid* phase)
  gapless, non-zero superfluid density

2nd order QPT

$$\Psi = \langle b \rangle$$



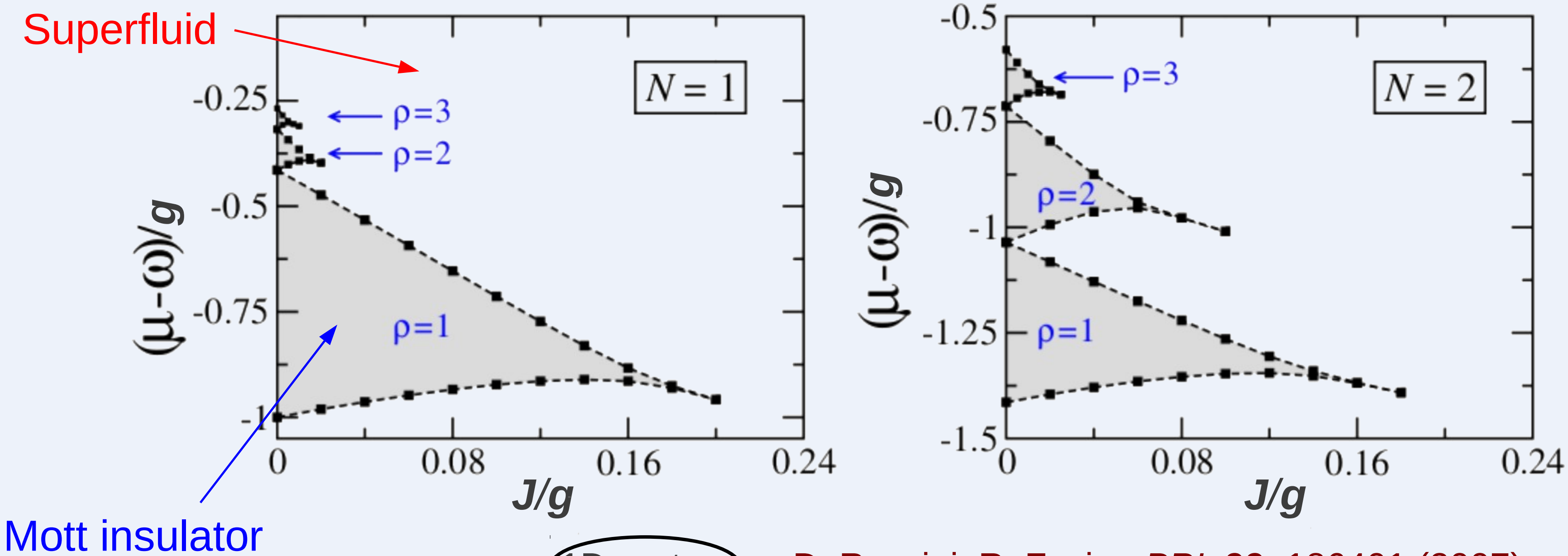


M. Greiner, O. Mandel, T. Esslinger, T. Hänsch, I. Bloch, *Nature* **415**, 39 (2002)

M. Fisher, P. Weichman, G. Grinstein, D. Fisher, *PRB* **40**, 546 (1989)
D. Jaksch, C. Bruder, J. Cirac, C. Gardiner, P. Zoller, *PRL* **81**, 3108 (1998)

Arrays of coupled QED cavities

$$\mathcal{H}_{\mathrm{JCH}} = -J \sum_{\langle i,j \rangle} \left(a_i^\dagger a_j + a_j^\dagger a_i \right) + \sum_i \mathcal{H}_{\mathrm{JC}}^{(i)}$$



1D system D. Rossini, R. Fazio, *PRL* **99**, 186401 (2007)

– M. Aichhorn, M. Hohenadler, C. Tahan, P. Littlewood, *PRL* **100**, 216401 (2008)
– S. Schmidt, G. Blatter, *PRL* **103**, 086403 (2009), *PRL* **104**, 216402 (2010)
– M. Schirò, M. Bordyuh, B. Oztop, H. Türeci, *PRL* **109**, 053601 (2012)

BH & JCH models belong to the same universality class

Polariton/photon fluctuations
in a sub-block with M cavities

$$\delta n^2 = \left\langle \left(\sum_{i \in M} n_i \right)^2 \right\rangle - \left\langle \sum_{i \in M} n_i \right\rangle^2$$

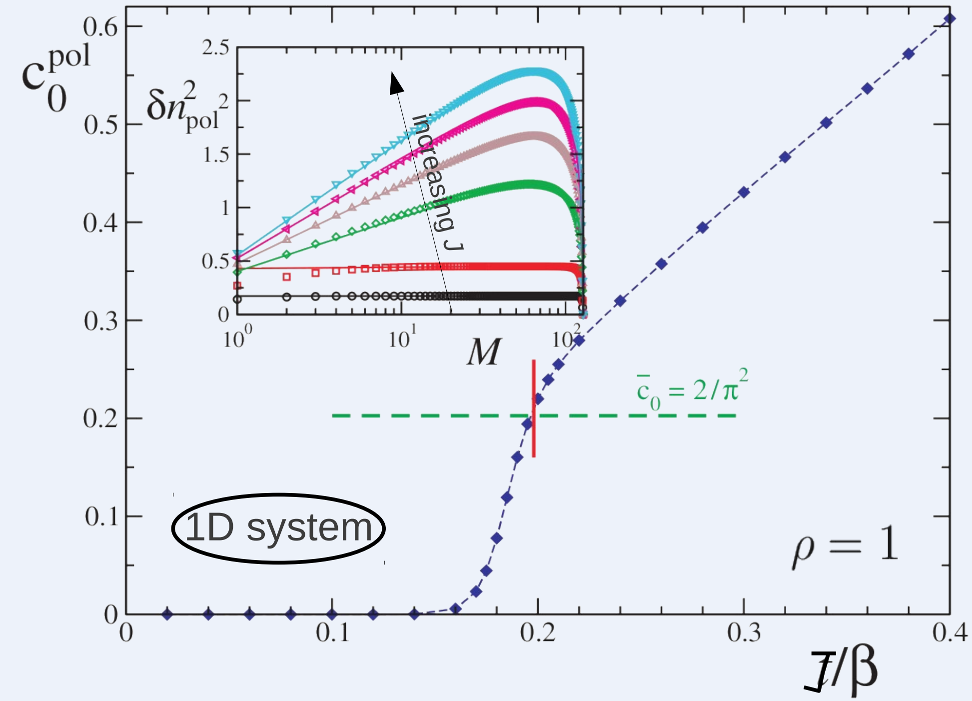
For a 1D critical system (LL):

$$C_r = \langle n_{i+r} n_i \rangle - \langle n_{i+r} \rangle \langle n_i \rangle \sim \frac{2}{K} (2\pi\rho r)^{-2}$$

$$\delta n^2 \sim \frac{1}{K\pi^2\rho^2} \ln M$$

K = ½ at the BKT transition

D. Haldane, *PRL* **47**, 1840 (1981)

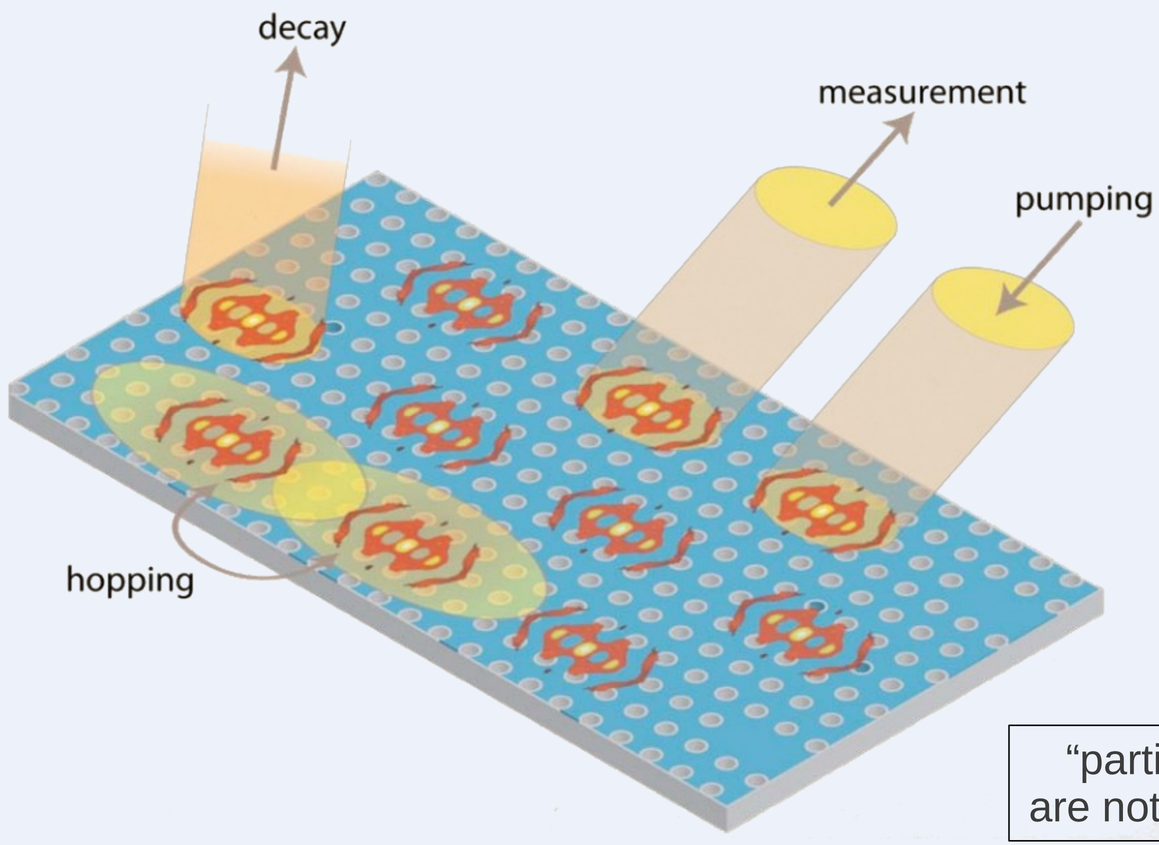


D. Rossini, R. Fazio, G. Santoro, *EPL* **83**, 47011 (2008)

See also:

– J. Koch, K. Le Hur, *PRA* **80**, 023801 (2009)
– M. Hohenadler, M. Aichhorn, S. Schmidt, L. Pollet, *PRA* **84** 041608(R) (2011); *PRA* **85**, 013810 (2012)

BUT: Realistic conditions would also include cavity losses:



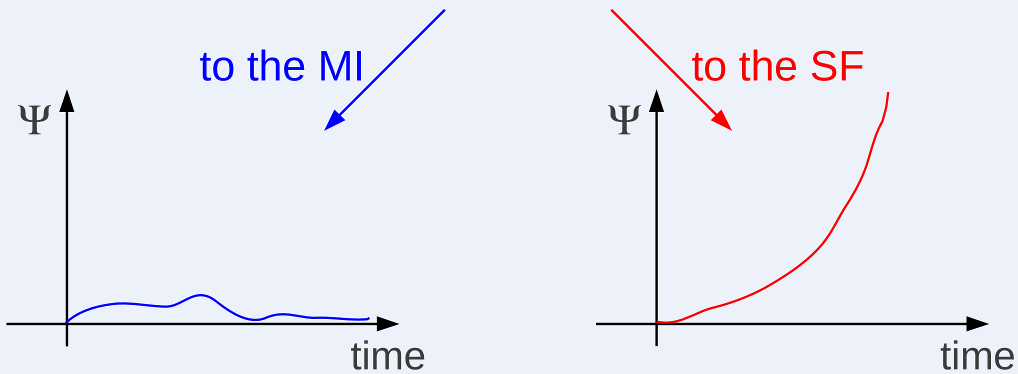
"particles" (*i.e.* polaritons)
are not necessarily conserved

- Realize & observe the steady-state phase diagram
- Study the non-equilibrium phases in ***open*** quantum many-body systems

See *e.g.* I. Carusotto, C. Ciuti, *PRL* (2004); M. Wouters, I. Carusotto, *PRL* (2007); …

First attempts to characterize the SF-MI transition with dissipation:

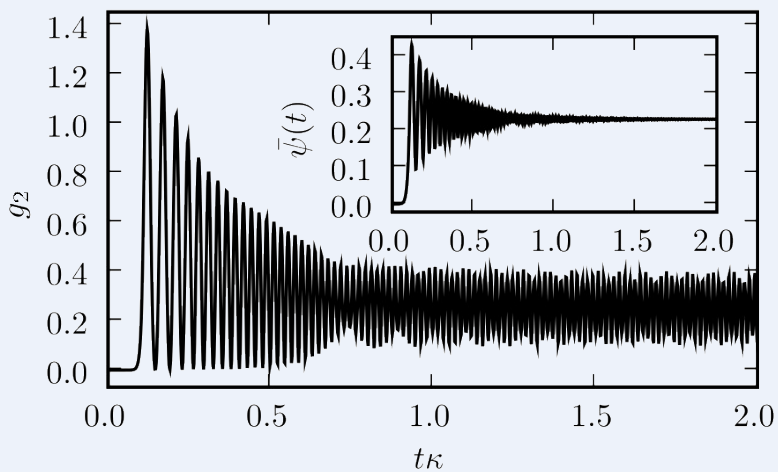
Perform a ***sudden quench*** from the MI

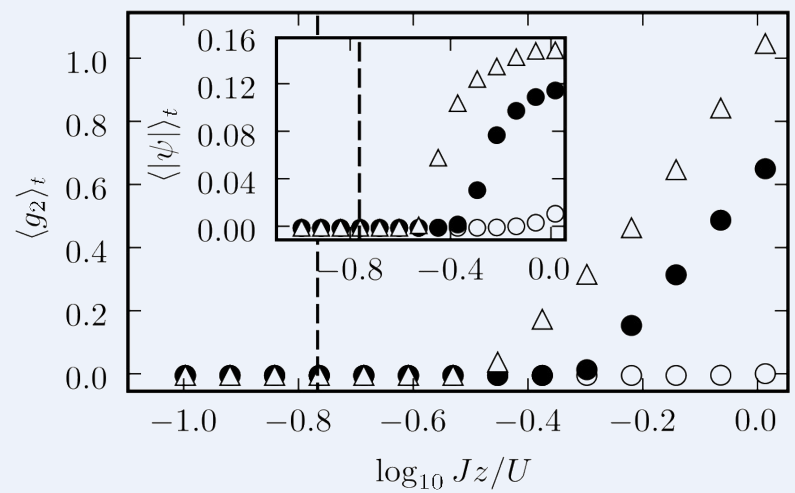


Behavior in closed systems:

E. Altman, A. Auerbach
PRL **89**, 250404 (2002)

Behavior in open cavities:





A. Tomadin, V. Giovannetti, R. Fazio, D. Gerace, I. Carusotto, H. Türeci, A. Imamoğlu, *PRA* **81**, 061801 (2010)

Many-body physics in dissipative systems

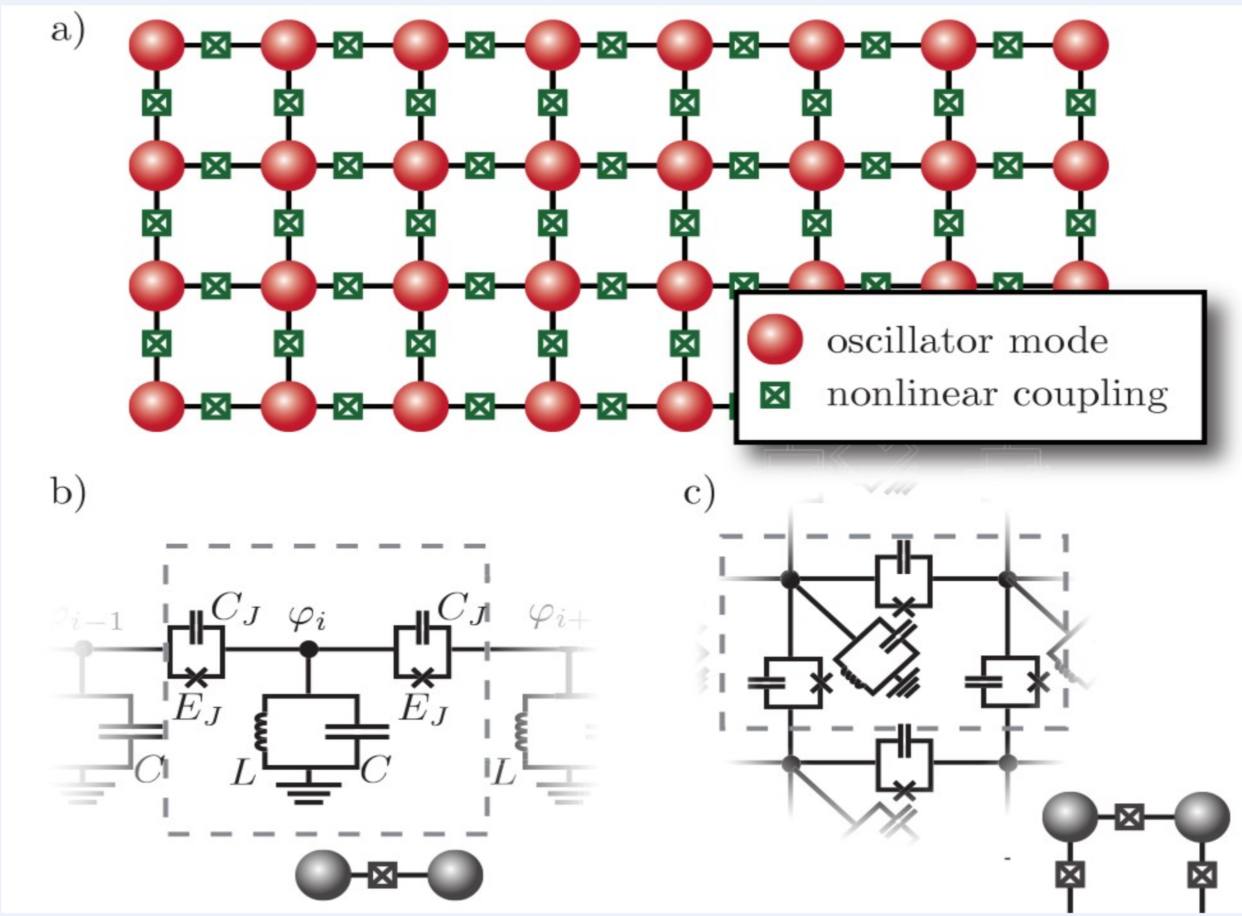
Novel quantum phases of matter?

J. Jin, D. Rossini, R. Fazio, M. Leib, M. Hartmann, *Phys. Rev. Lett.* **110**, 163605 (2013)

Our model: on-site + cross-Kerr non-linearities between cavities

e.g. in superconducting circuits

cavities are represented by LC circuits, mutually coupled through Josephson nanocircuits



Cross-Kerr non-linearity
(capacitive & inductive coupling)

For another implementation of cross-Kerr non-linearities in circuit QED, see also:
Y. Hu, G.-Q. Ge, S. Chen, X.-F. Yang, Y.-L. Chen, *PRA* **84**, 012329 (2011)

Effective Hamiltonian:

$$\mathcal{H} = -J \sum_{\langle i,j \rangle} \left(b_i^\dagger b_j + b_j^\dagger b_i \right)$$

A) *photon hopping*

$$-\delta \sum_i b_i^\dagger b_i + \Omega \sum_i (b_i^\dagger + b_i)$$

B) *laser driving*

$$+ \boxed{U \sum_i n_i(n_i - 1)} + \boxed{V \sum_{\langle i,j \rangle} n_i n_j}$$

C) *non-linearities*

on-site — cross-Kerr

Dynamics ruled by the master equation:

$$\boxed{\partial_t \rho = -i[\mathcal{H}, \rho] + \mathcal{L}[\rho]}$$

with $\mathcal{L}[\rho] = \frac{\kappa}{2} \sum_i \left(2 b_i \rho b_i^\dagger - n_i \rho - \rho n_i \right)$

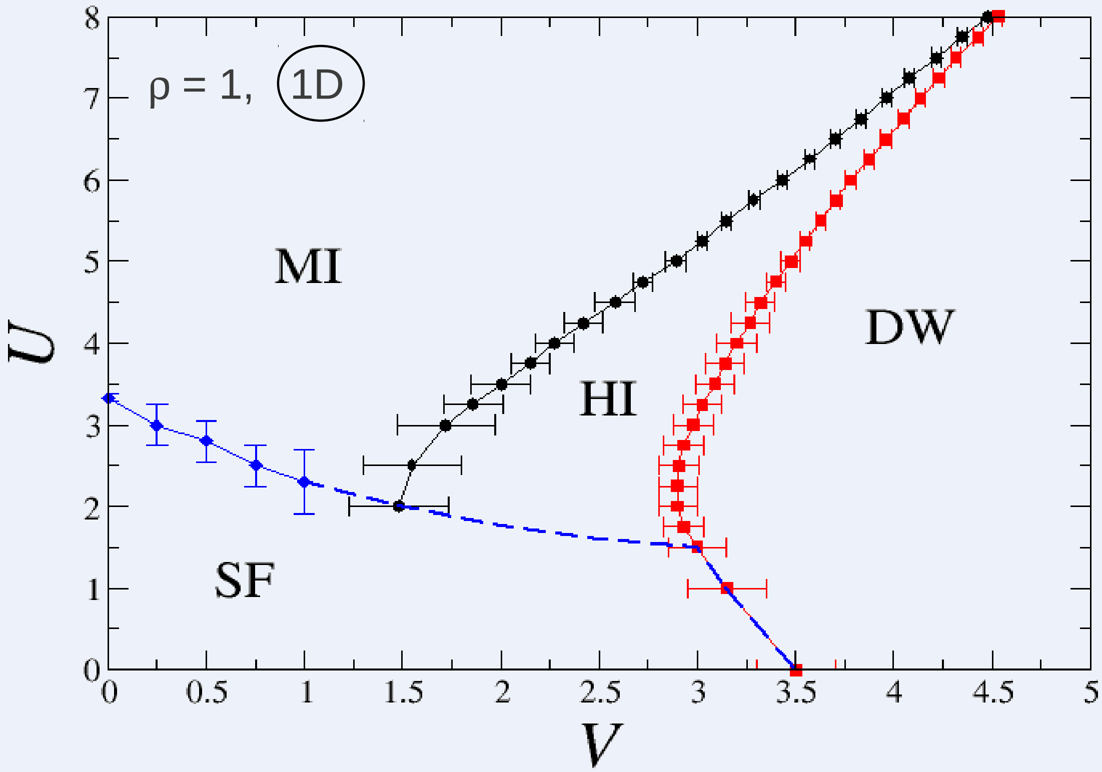
D) *photon losses*

Digression on what happens without dissipation …

$$\mathcal{H}_{\rm EBH} = -J \sum_{\langle i,j \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i n_i(n_i - 1) + V \sum_{\langle i,j \rangle} n_i n_j$$

Extended Bose-Hubbard

E. Dalla Torre, E. Berg, T. Giamarchi, E. Altman, *PRL* **97**, 260401 (2006): *PRB* **77**, 245119 (2008)



SF: superfluid
MI: Mott Insulator
HI: Haldane insulator
DW: density wave

– D. Rossini, R. Fazio, New J. Phys. **14**, 065012 (2012)

– Sowinski, Dutta, Hauke, Tagliacozzo, Lewenstein, *PRL* **108**, 115301 (2012) → + correlated hopping

– Batrouni, Scalettar, Rousseau, Grémaud, *PRL* **110**, 265303 (2013) → supersolid phases at different filling

…

… Let us now go back to our dissipative effective Hamiltonian

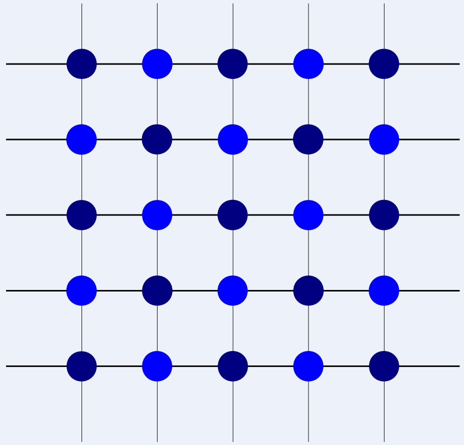
A) Mean-field approximation:

$$z^{-1}\sum_{\langle i,j\rangle} b_i^\dagger b_j \longrightarrow \langle b_A^\dagger\rangle \sum_{i\in B} b_i + \langle b_B^\dagger\rangle \sum_{j\in A} b_j$$

$$z^{-1}\sum_{\langle i,j\rangle} n_i n_j \longrightarrow \langle n_A\rangle \sum_{i\in B} n_i + \langle n_B\rangle \sum_{j\in A} n_j$$

we decouple the lattice sites in a minimal structure
a bipartite lattice (A-B) is required to take care of the n.n. interactions

z: coordination number



$$\begin{cases} \partial_t \rho_A = -i[\mathcal{H}_A, \rho_A] + \mathcal{L}[\rho_A] \\ \partial_t \rho_B = -i[\mathcal{H}_B, \rho_B] + \mathcal{L}[\rho_B] \end{cases}$$

$$\mathcal{H}_A = -zJ\left[b_A^\dagger\langle b_B\rangle + b_A\langle b_B^\dagger\rangle\right] + \left[\Omega(b_A^\dagger + b_A) - \delta n_A\right] + \left[U n_A(n_A-1) + zV\langle n_B\rangle n_A\right]$$

$$\mathcal{H}_B = -zJ\left[b_B^\dagger\langle b_A\rangle + b_B\langle b_A^\dagger\rangle\right] + \left[\Omega(b_B^\dagger + b_B) - \delta n_B\right] + \left[U n_B(n_B-1) + zV\langle n_A\rangle n_B\right]$$

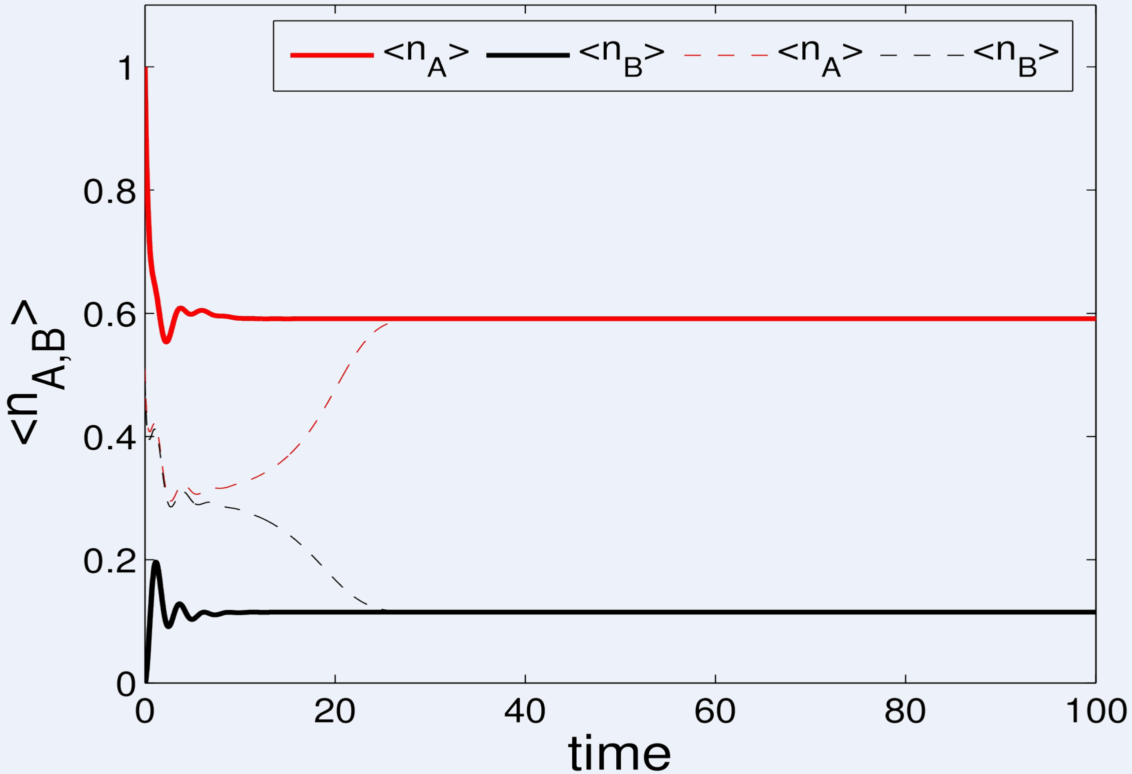
with $\langle \mathcal{O}_A\rangle = \mathrm{Tr}\big(\mathcal{O}_A\rho_A\big)$ and $\langle \mathcal{O}_B\rangle = \mathrm{Tr}\big(\mathcal{O}_B\rho_B\big)$

Emergence of photon crystal

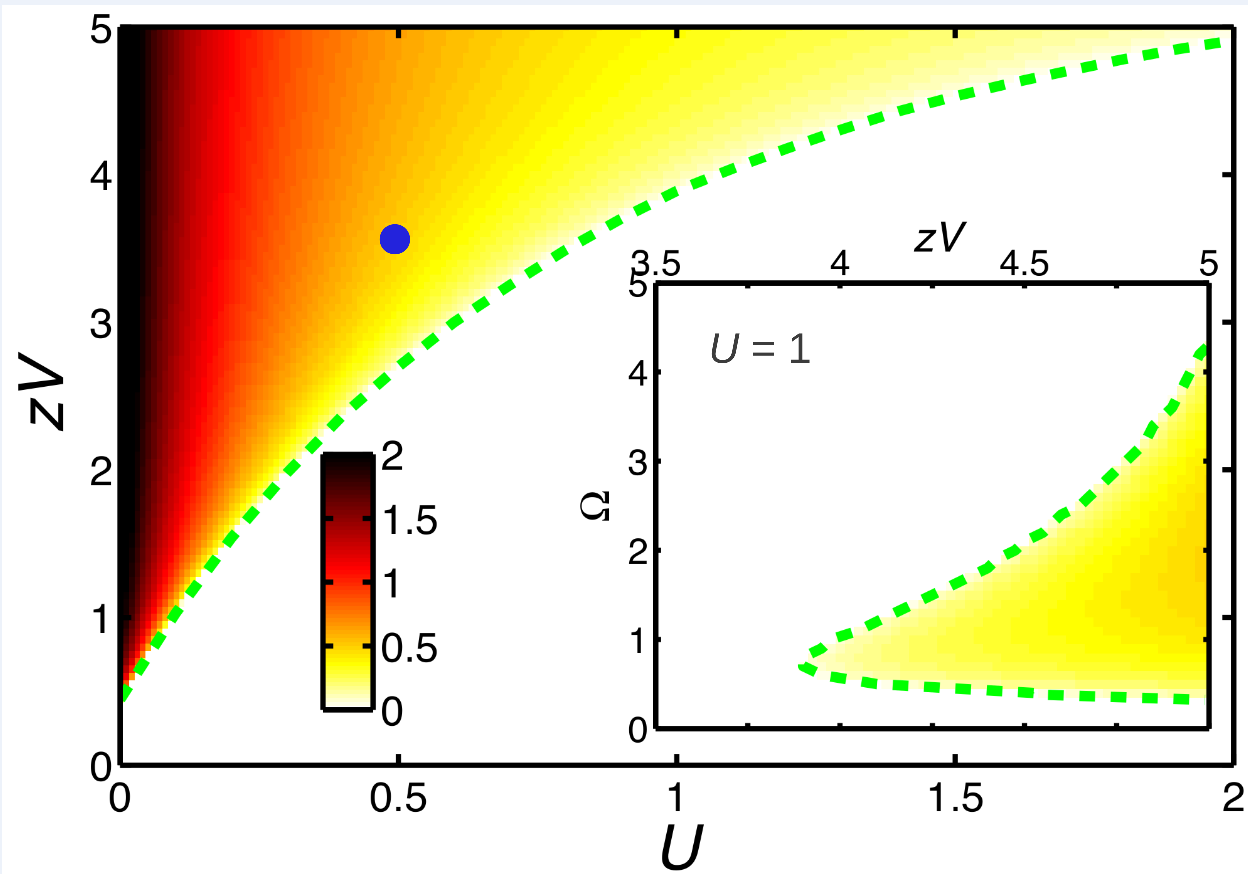
Periodic (anti-ferromagnetic) modulation of the photon blockade

$$\Delta n \equiv \left| \langle n_A \rangle - \langle n_B \rangle \right| \overset{?}{>} 0$$

order parameter for the crystalline phase



Phase diagram (without hopping: $J = 0$)



U → ∞

T. Lee, H. Häffner, M. Cross, *PRA* **84**, 031402(R) (2011)

For our choice of parameters in the main plot [$\Omega = 0.75$, $\delta = 0$]

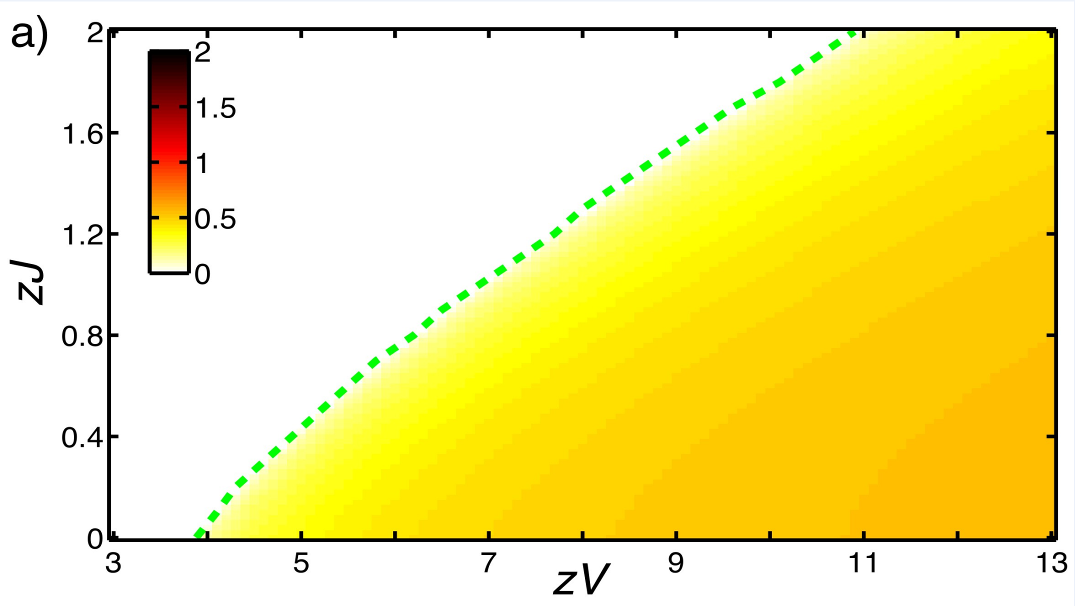
$$zV_C \overset{U\to\infty}{\approx} 5.73$$

$$zV_C \overset{U=0}{\approx} 0.44$$

Reentrant behavior as a function of the drive Ω:

– too small density at small pumping
– homogeneous arrangement favored by the pumping

Phase diagram (with hopping $J \neq 0$)



[$U = 1$, $\Omega = 0.75$, $\delta = 0$]

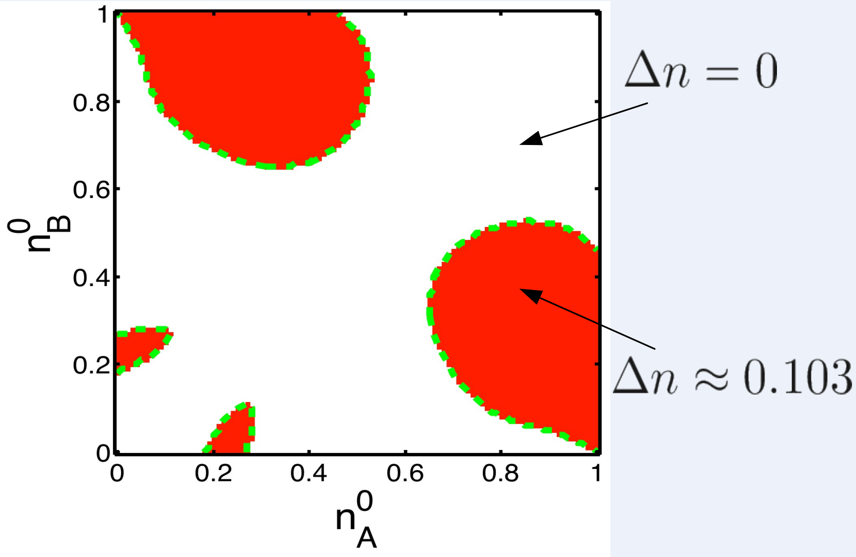
photon delocalization suppresses the solid phase

- reentrance observed at smaller U
- what happens at very large J ?
- beyond mean-field ?

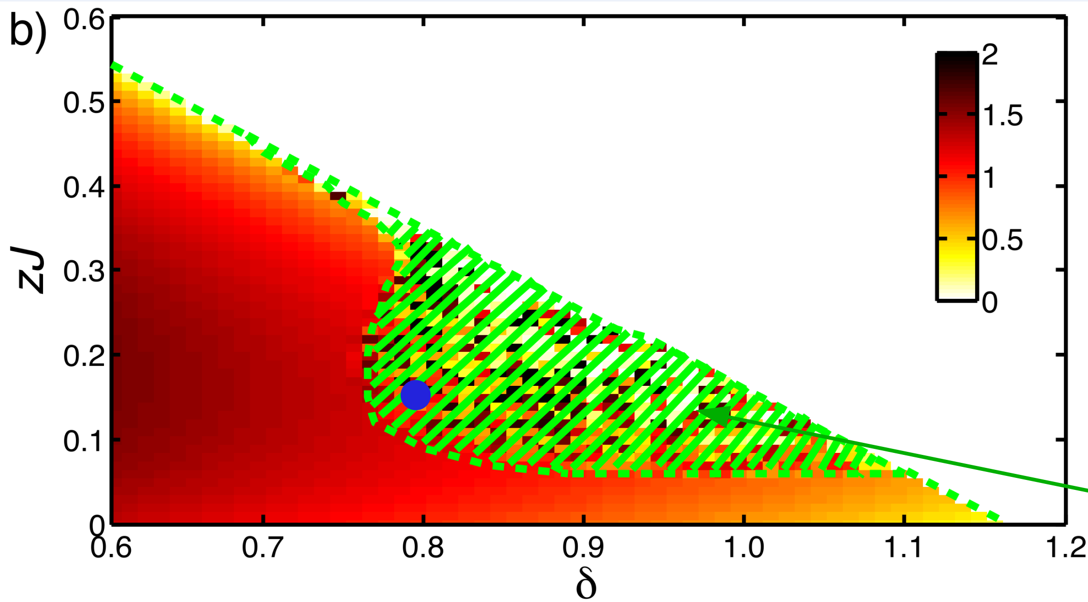
Photon crystal also for $V = 0$?

Depends on initial conditions…
(only under non-equilibrium!)

[$U = \infty$, $zJ = 0.6$, $\Omega = 2$, $\delta = 0.8$]

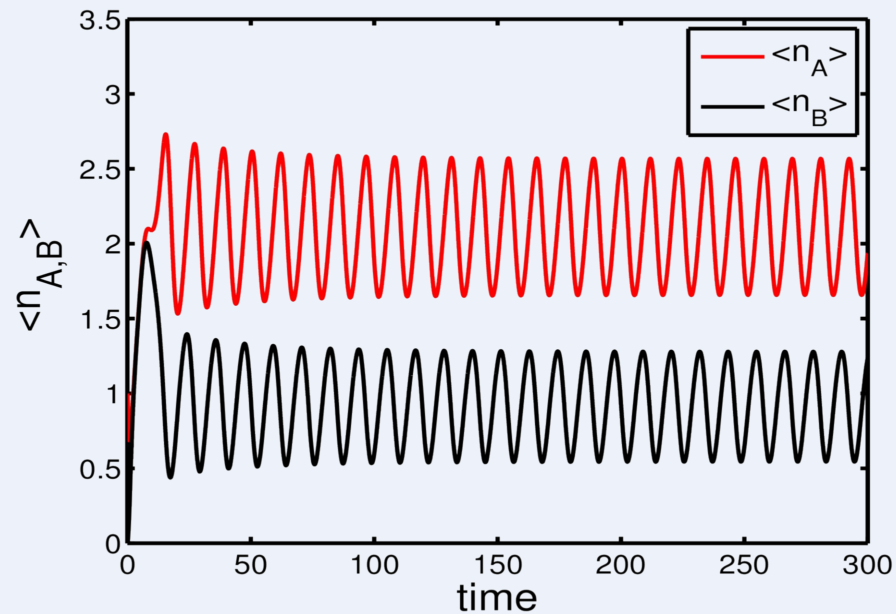


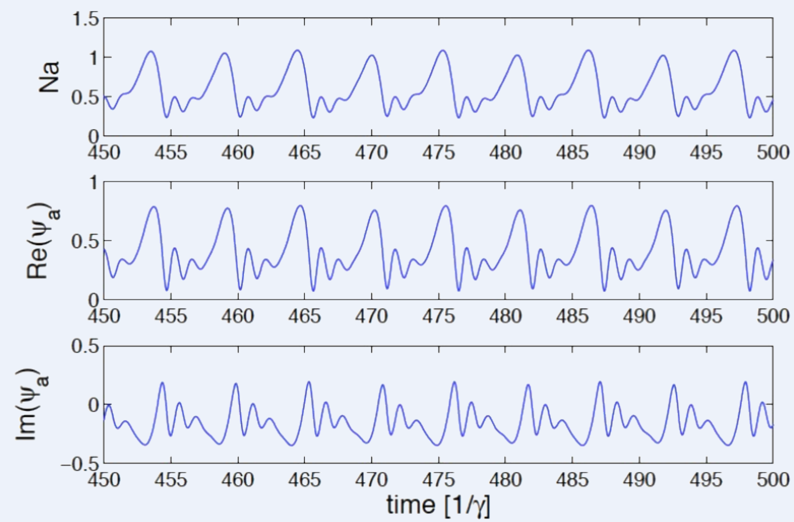
Phase diagram (with hopping $J \neq 0$)



$[U = 0, \; zV = 0.6, \; \Omega = 0.75]$

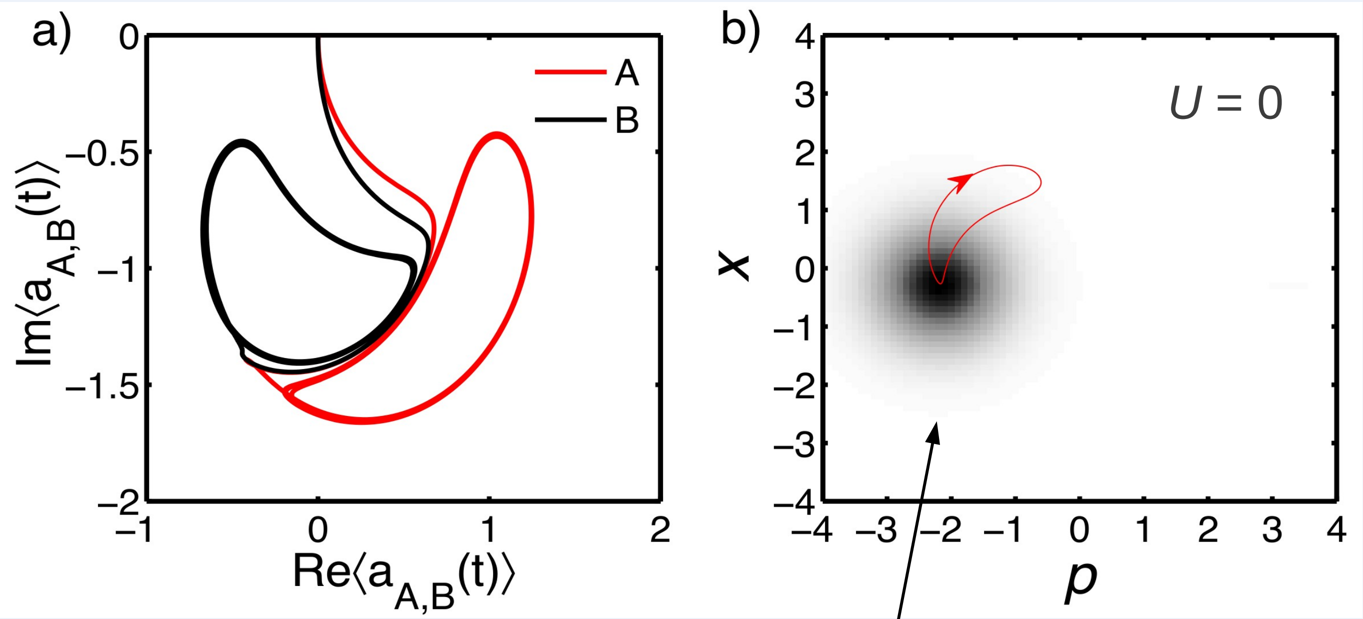
qualitatively new behavior in the shaded green area





Emergence of superfluidity

Sublattice phase synchronization



single-site density matrix evolves periodically in time

See also: M. Ludwig, F. Marquardt, *PRL* **111**, 073603 (2013)

A) Checkerboard solid ordering   crystal

+

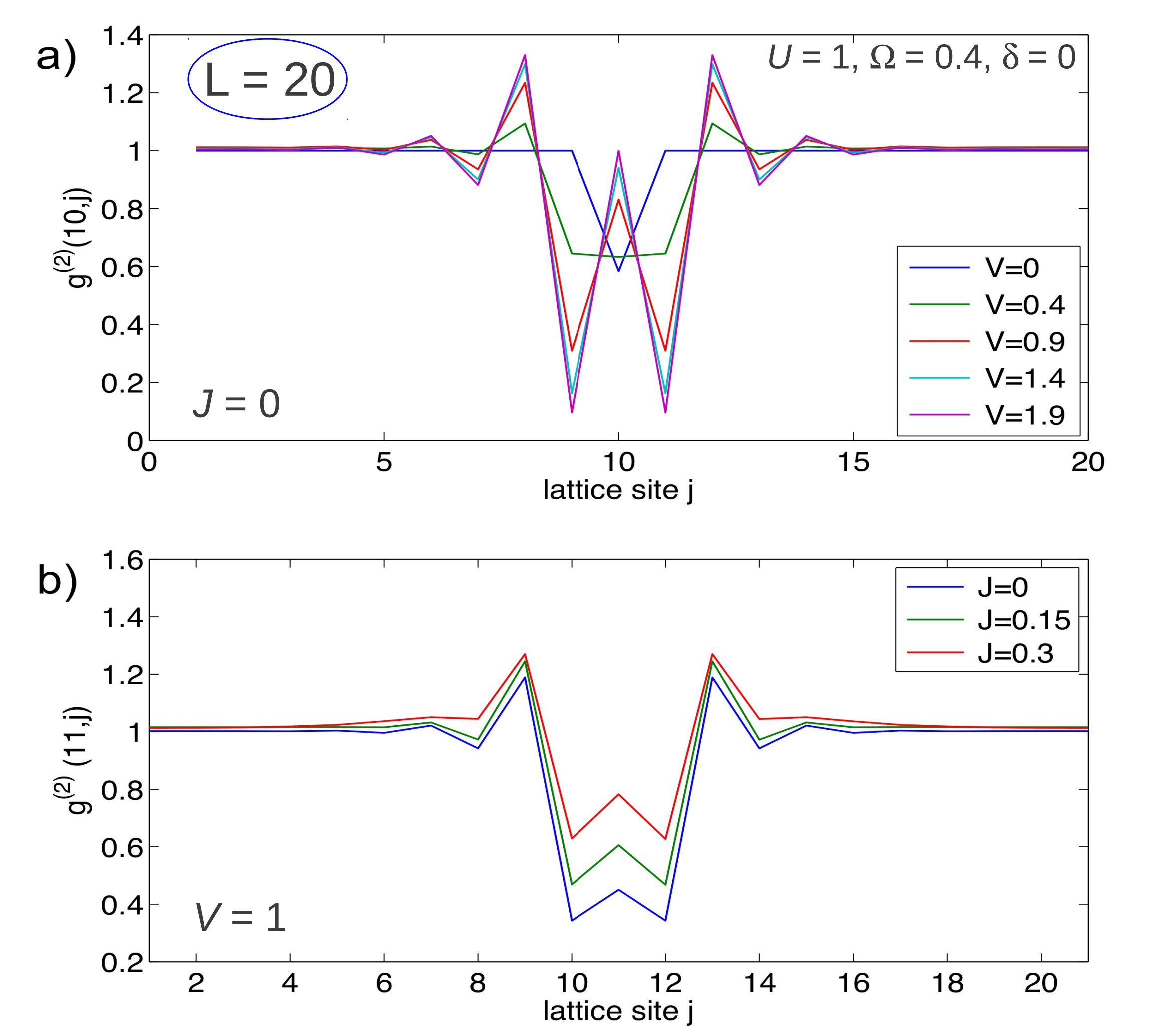
B) global dynamical phase coherence   superfluid

=

non-equilibrium supersolid phase ?

M. Boninsegni, N.V. Prokof'ev, *Rev. Mod. Phys.* **84**, 759 (2012)

B) Preliminary DMRG results (1D):



$$g^{(2)}(i,j) = \frac{\langle b_i^\dagger b_j^\dagger b_j b_i \rangle}{\langle n_i \rangle \langle n_j \rangle}$$

Staggered behavior for V > 0
(strong density-density correl.)

Spatial range of correlations
decreases with increasing *J*

True ordering in steady state
maybe only in D > 1

MPO technique for dissipative coupled cavities: M. Hartmann, *PRL* **104**, 113601 (2010)

Summary

- Arrays of coupled cavities

  a quantum simulator of many-body states of light

- Non-dissipative phase diagram

  photon hopping *vs.* blockade – the JCH model

- Steady-state phase diagram with on-site + cross-Kerr nonlinearities

  → Emergence of a photon solid

  → Quantum phase synchronization

  steady-state *supersolid* ?