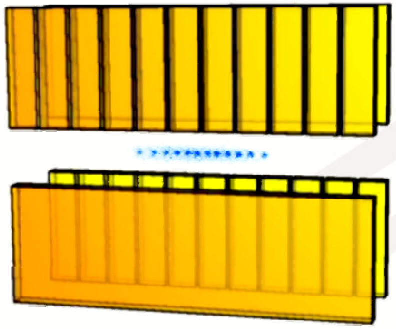
Tailoring Coupling Constants to Simulate Spin Models in a Segmented Trap



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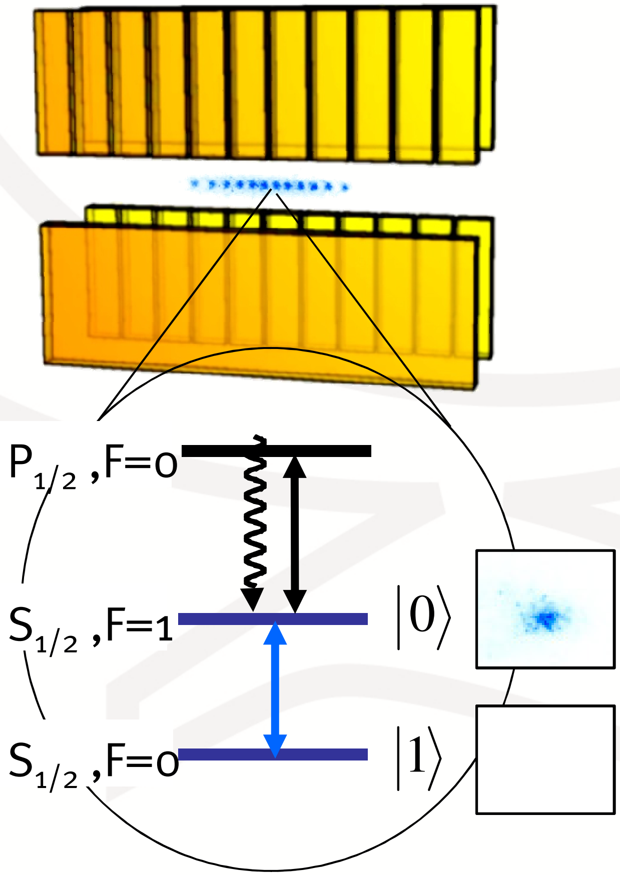
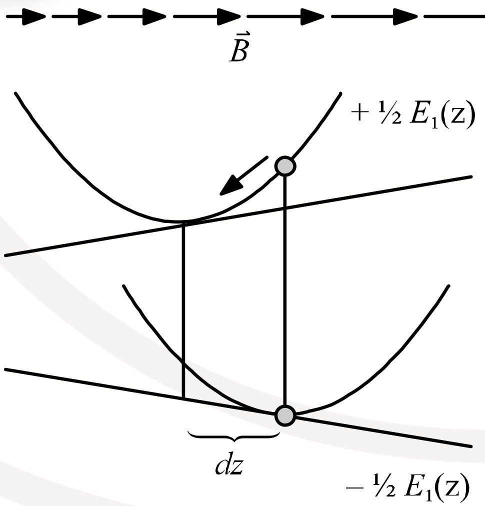
Microwaves & Hyperfine Qubit

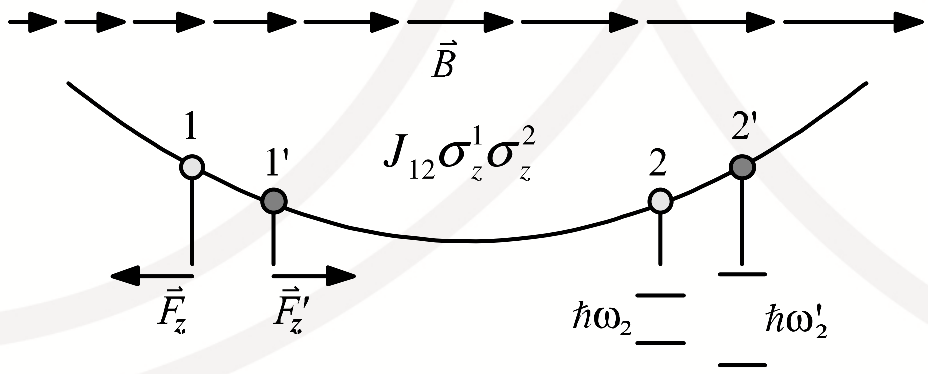


- use microwaves and rf for coherent state manipulation of $^{171}Yb^{+}$
- the magnetic gradient makes Zeeman splitting position dependent
- the resonance frequency is unique for each ion

F. Mintert & Ch. Wunderlich, *Phys. Rev. Lett.* 87, 4 (2001); 91, 029902 (2003).
Ch. Wunderlich, *Laser Physics at the Limit,* Springer Berlin;
C. Ospelkaus et al., Phys. Rev: Lett 101, 090502 (2008)

Spin-Spin Coupling



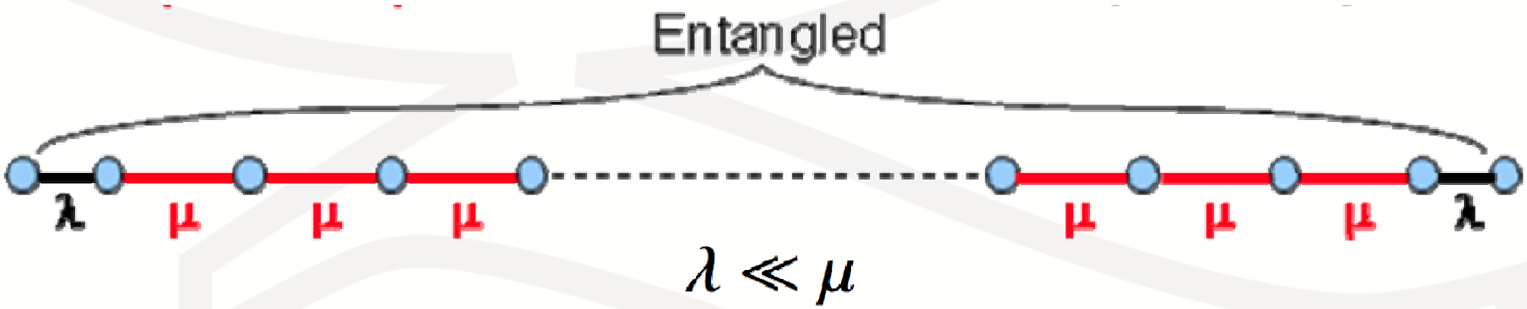


- total energy: sum of Zeeman shift and the trapping potential
- equilibrium positions becomes state dependent, mimicks momentum transfer; MAgnetic Gradient Induced Coupling: MAGIC
- via Coulomb repulsion each ion affects the equilibrium positions of all other ions, changing their eigen values

F. Mintert & Ch. Wunderlich, *Phys. Rev. Lett.* 87, 4 (2001); 91, 029902 (2003).
Ch. Wunderlich, *Laser Physics at the Limit,* Springer Berlin;
Ch. Wunderlich & Ch. Balzer, *Adv. in At. Mol. Opt. Phys.* 49, 293–372 (2003).

Long distance entanglement (LDE)

- ground-state, indirect, end-to-end entanglement in spin-chains, useful for quantum bus
- loosely coupled end/messenger ions



L. Campos Venuti et al., Phys. Rev. Lett., 96, 247206 (2006);
L. Campos Venuti et al., Phys. Rev. A 76, 052328 (2007);
S. M. Giampaolo et al., New J. of Phys. 12, 025019 (2010).

LDE Conditions

- weak coupling between the two external spins and the remaining spins
- non-degenerate ground state, no B field
- examples: models with competing interactions along orthogonal axes as XY, XYZ, Heisenberg...

$$H_{XY}=\sum_{j,k}\left(J_x\sigma_j^x\sigma_k^x+J_y\sigma_j^y\sigma_k^y\right)$$

$$H_{XYZ}=\sum_{j,k}\left(J_x\sigma_j^x\sigma_k^x+J_y\sigma_j^y\sigma_k^y+J_z\sigma_j^z\sigma_k^z\right)$$

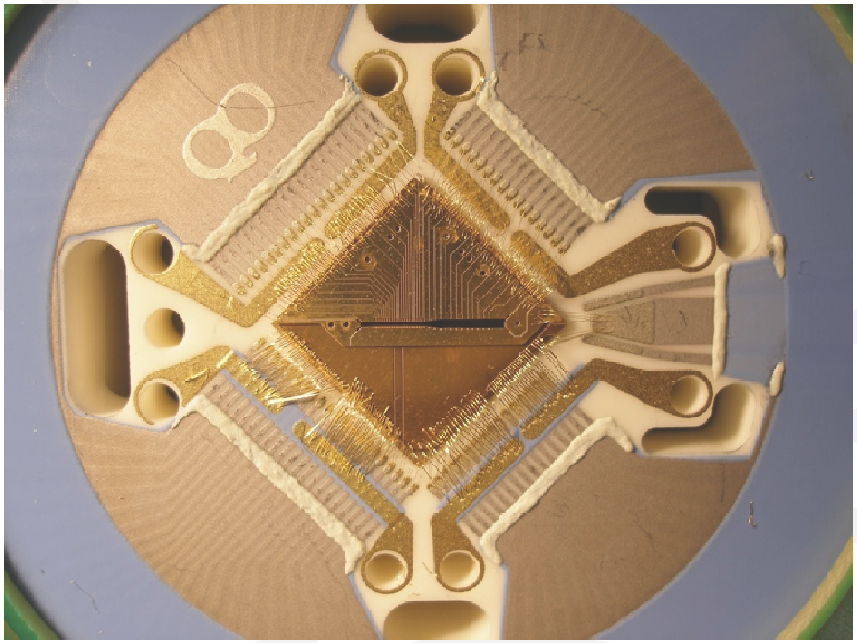
$$H_{Heisenberg}=J\sum_{j,k}\left(\sigma_j^x\sigma_k^x+\sigma_j^y\sigma_k^y+\sigma_j^z\sigma_k^z\right)$$

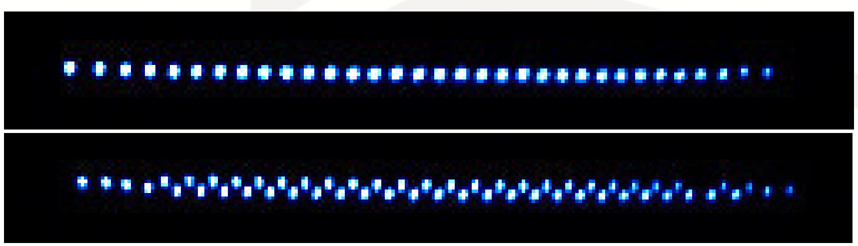
LDE Work Plan

1. Manipulation of the spin-spin couplings:
   - weakly coupled end-spins
2. Engineering of Hamiltonian:
   - interaction along z and along another axis

   ...
3. Preparation of the ground state:
   - adiabatic variation of some Hamiltonian parameter

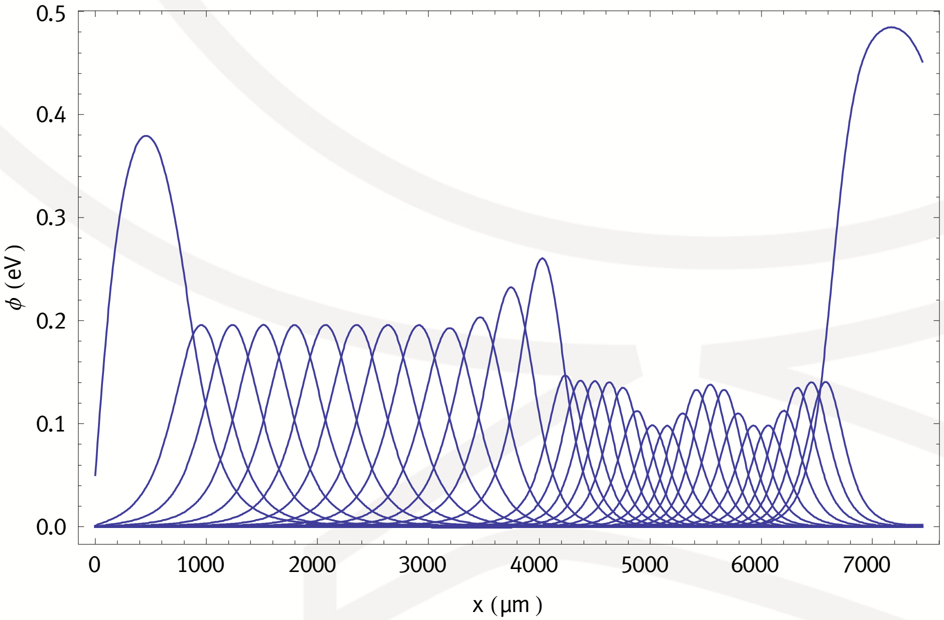
Segmented µtrap

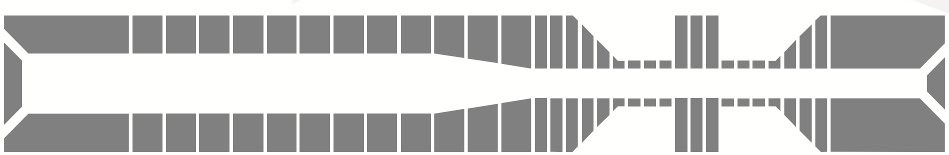




- linear Paul trap, three layer structure
- basic design from Mainz group (Schmidt-Kaler), altered by our group
- segments allow flexible axial trapping potentials and multiple trapping zones
- carrier acts as vacuum interface (!)

Potential Simulation





- boundary element method [1]
- structures reflected in potentials simulation:
  - width of electrodes
  - electrode separation
  - middle layer coils
- helpful for shuttling, tayloring of potentials

- [1] K. Singer et al., Rev. Mod. Phys., arXiv:0912.0196v2

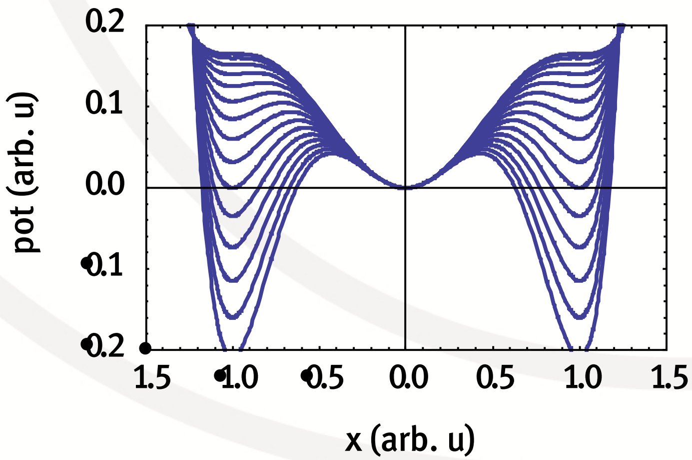
Extension: Tikhonov Regularization

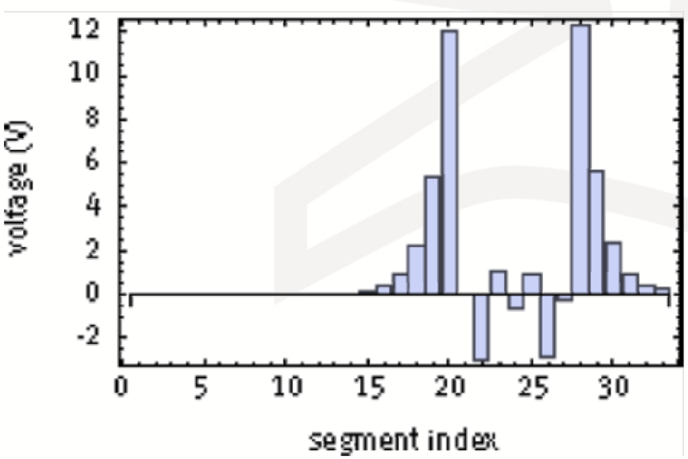
- we want to get a specific potential $\vec{\phi}$
- we need to solve vor the voltages V_i:

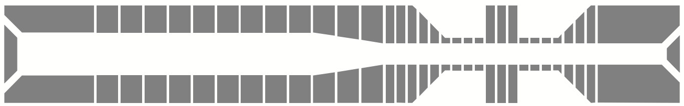
$$\hat{A}\vec{V} = \vec{\phi}$$

- often ill conditioned, instead weminimize $\left\|\hat{A}\vec{V} - \vec{\phi}\right\|^2$ (least squares)
- boundary condition $\left\|\hat{A}\vec{V} - \vec{\phi}\right\|^2 - \left\|\hat{\Gamma}\vec{V}\right\|^2$
- if Tikhonov matrix $\vec{\Gamma}$ equals identitiy, algorithm prefers solution with smaller norm

Designing the Coupling

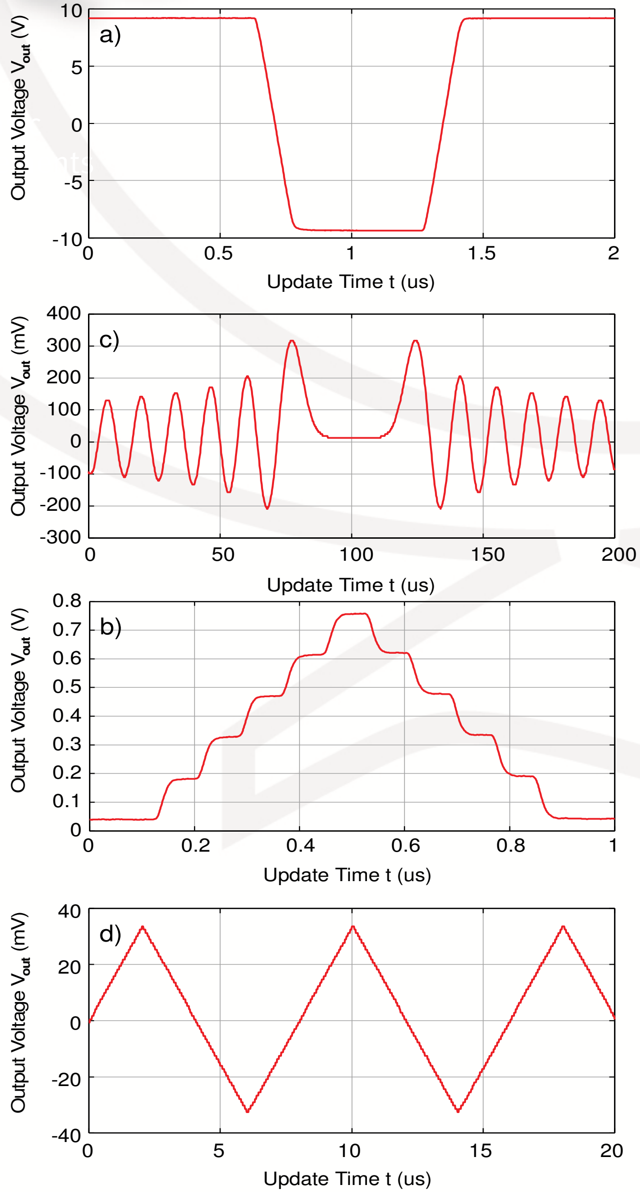






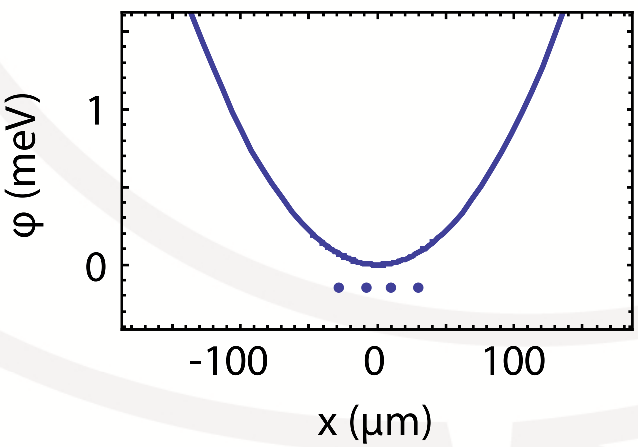
- triple well potential defines couplings with three parameters
- find appropriate triple well
- minima can be much closer than 2 segment widths (!)
- find voltages

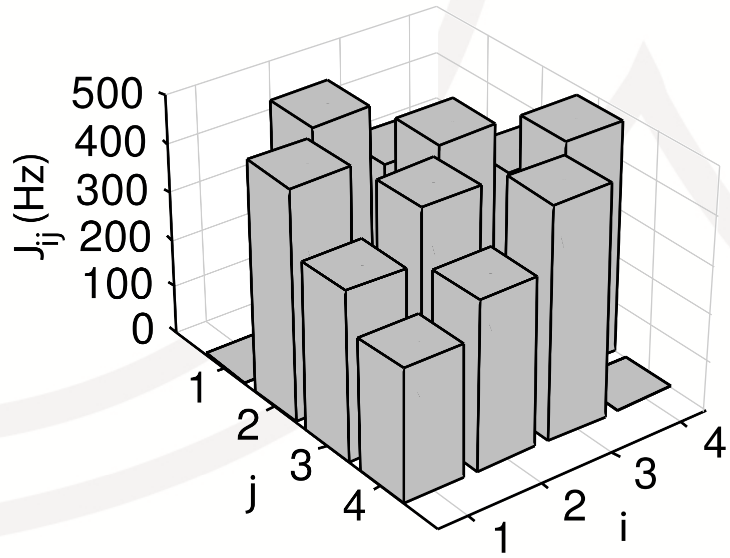
Multi Channel AWG



- 24 independent channels
- synchronous, arbitrary sequences
- amplitude ±10 V
- update rate 20 MHz
- 16 bit resolution
- low noise & drift
- loss free transmission up to 2 m
- freely programmable via USB

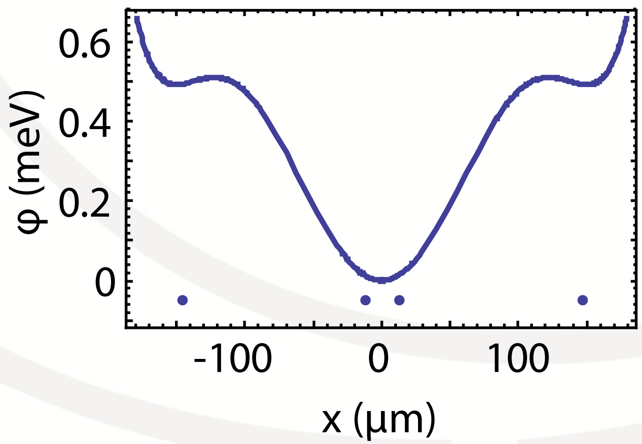
1. Tailoring Coupling Constants

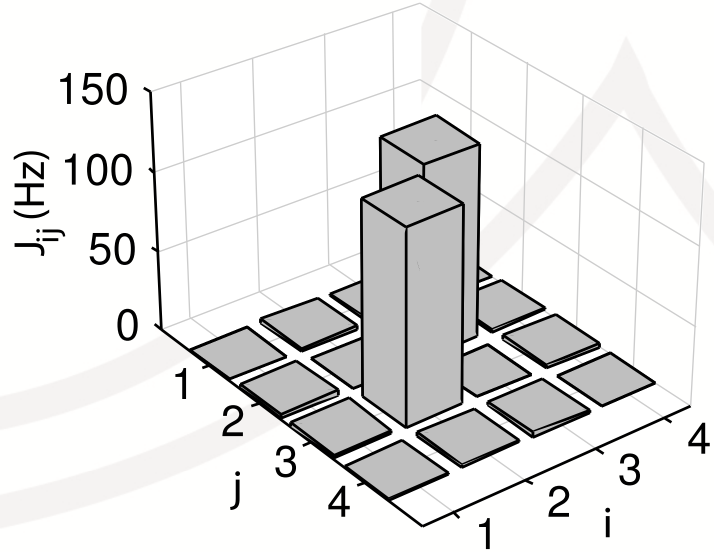




- harmonic coupling:
  - delocalized normal modes
  - all ions are strongly coupled, long range
  - global scaling by trap frequency and gradient

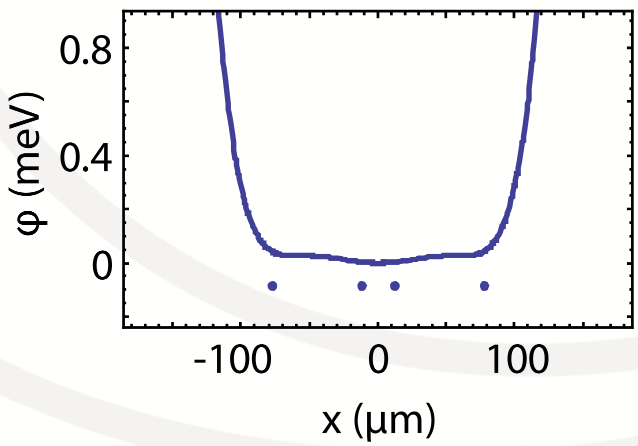
1. Tailoring Coupling Constants

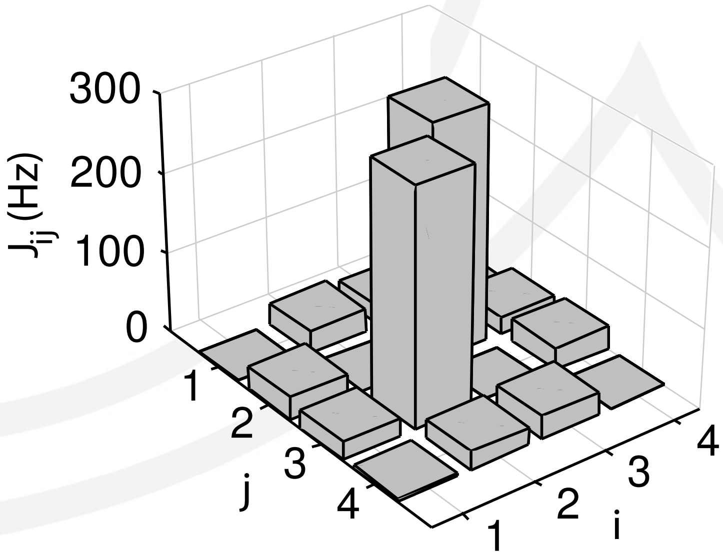




- triple well:
  - localized modes within wells
  - ions within the same well are strongly couples
  - inter-well coupling strongly reduced
  - local scaling by curvature and gradient

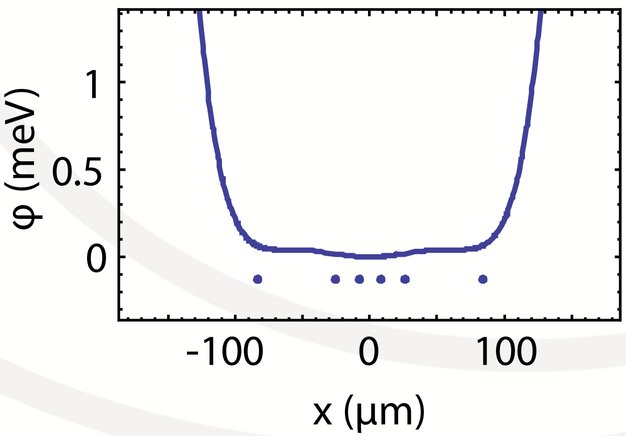
1. Tailoring Coupling Constants

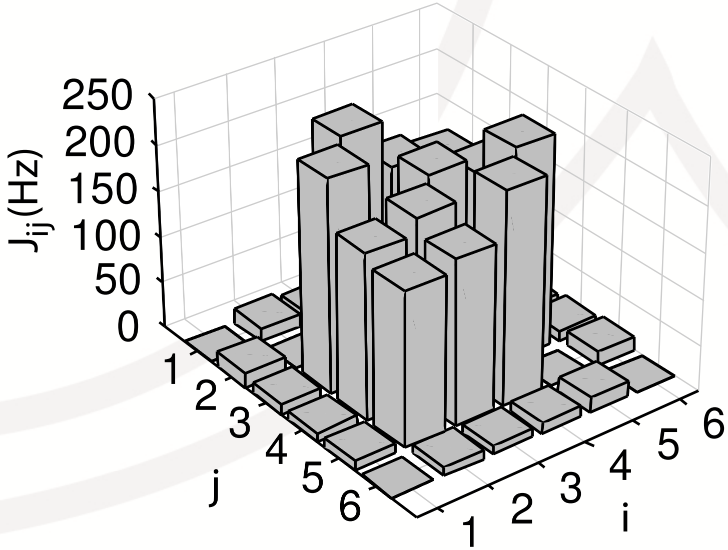




- anharmonic trap:
  - some modes localized, localization scalable
  - scalable inter-well coupling

1. Tailoring Coupling Constants





- anharmonic trap
  - some modes localized, localization scalable
  - scalable inter-well coupling
  - similar conditions for more ions in centre well

2. Mimic Other Hamiltonians

- engineer Hamiltonian dynamics using microwave fields which sequentially drive the atomic spins

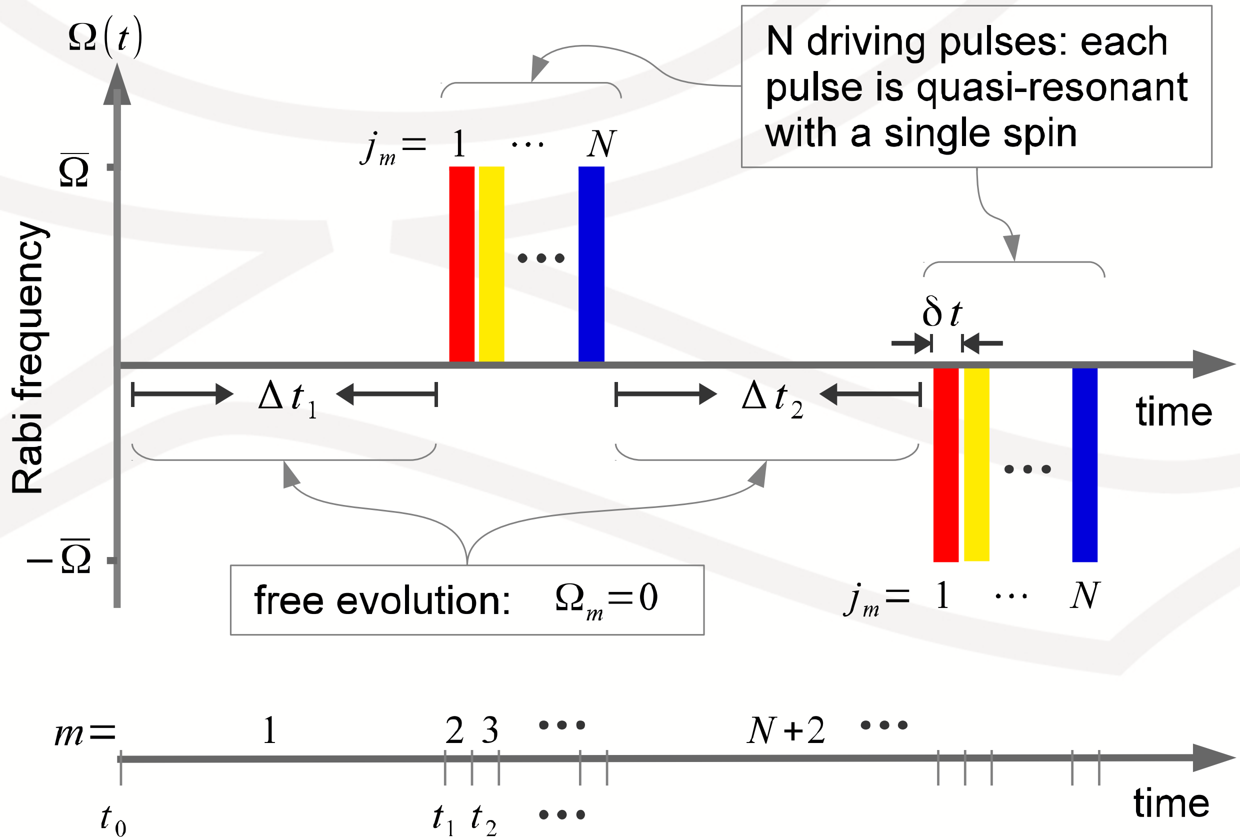
$$H = \sum_{j=1}^{N} \omega_j \sigma_j^z + \sum_{j,k} J_{j,k} \sigma_j^z \sigma_k^z - \mathrm{i} \sum_m \Omega_m(t) \left[\sigma_{j_m}^+ \mathrm{e}^{\mathrm{i}\nu_{jm} t} - \sigma_{j_m}^- \mathrm{e}^{-\mathrm{i}\nu_{jm} t} \right]$$

- every time-step: driving field is quasi-resonant with a single spin resonance; effect on the other off-resonant spins is negligible
- bracketing a free evolution between µwave pulses mimics coupling along a different axis:

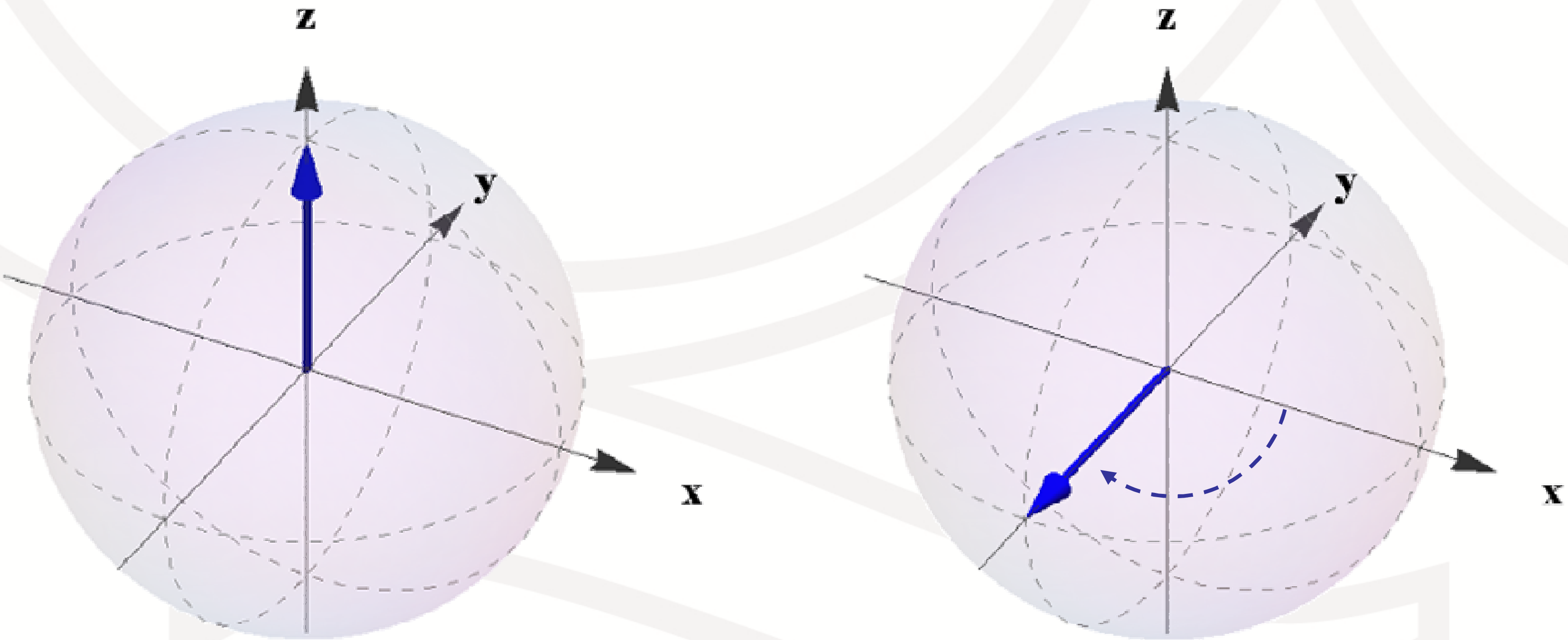
$$e^{-i\pi\sigma_j^y/4} \sigma_j^z e^{i\pi\sigma_j^y/4} = \sigma_j^x$$

Sequential Forth and Back

$$e^{-iHt} \simeq \left(\prod_j e^{-\frac{i\pi}{4}\sigma_j^y}\right) e^{-iH_I^{(z)}\Delta t} \left(\prod_j e^{\frac{i\pi}{4}\sigma_j^y}\right) = e^{-iH_I^{(x)}\Delta t}$$

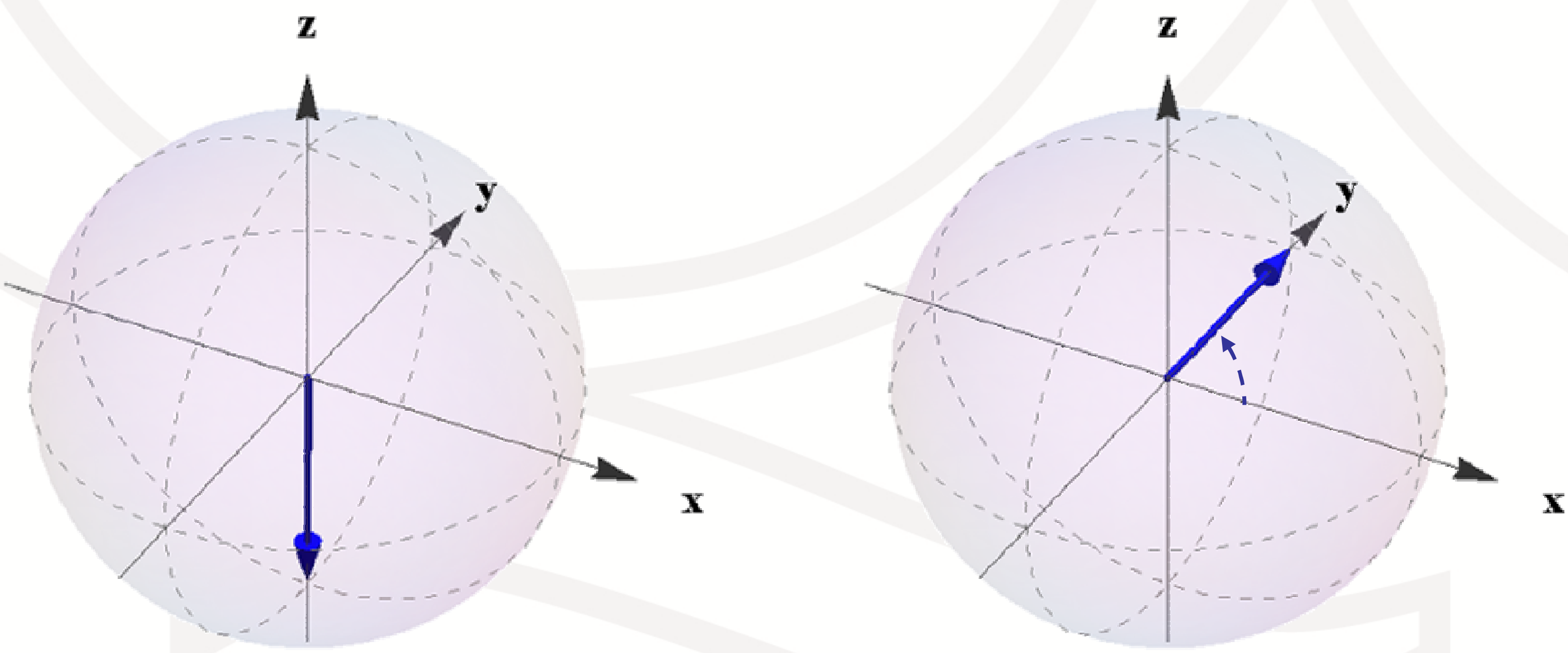


$\sigma^z\sigma^z$-Coupling



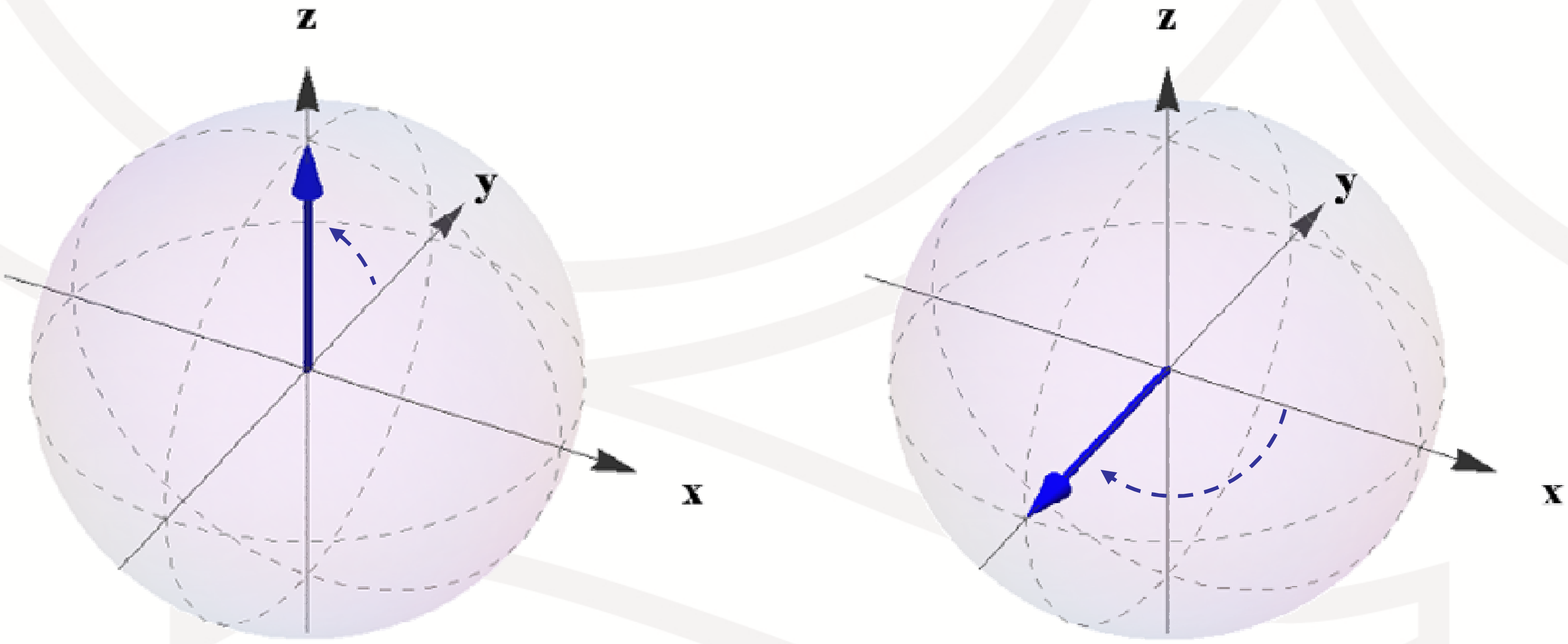
cw precession

$\sigma^z\sigma^z$-Coupling



ccw precession

$\sigma^y\sigma^z$-Coupling



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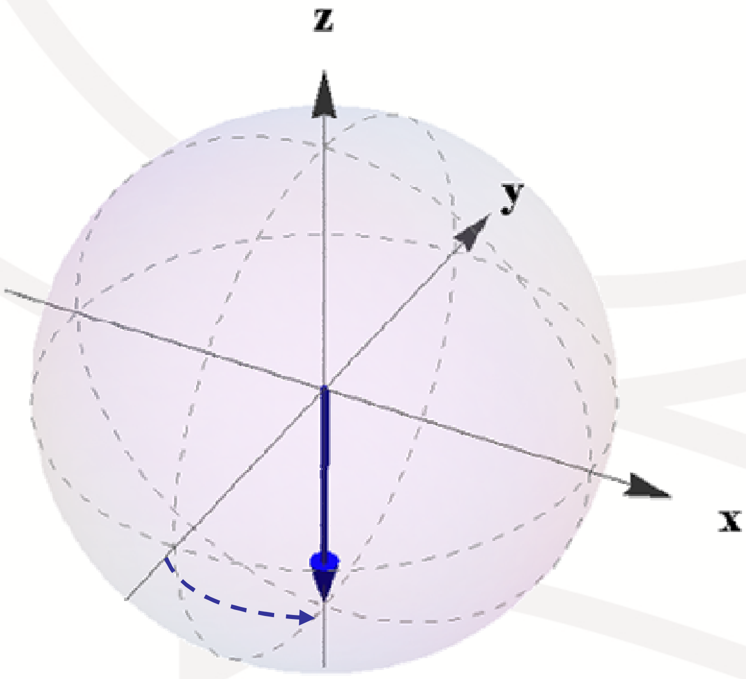
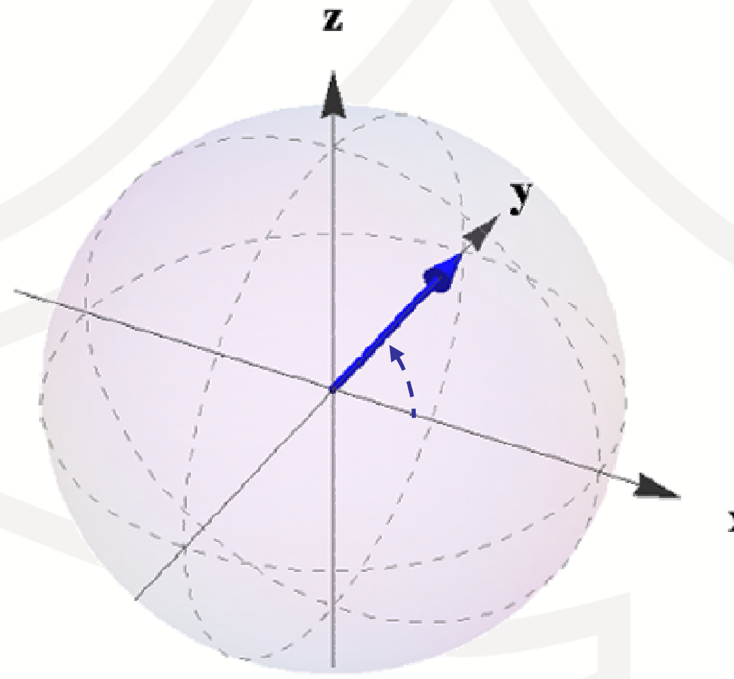




ccw precession

or: $e^{-i\pi\sigma_j^y/4}\sigma_j^z e^{i\pi\sigma_j^y/4} = \sigma_j^x$

3. Adiabatic Prep. of Ground State

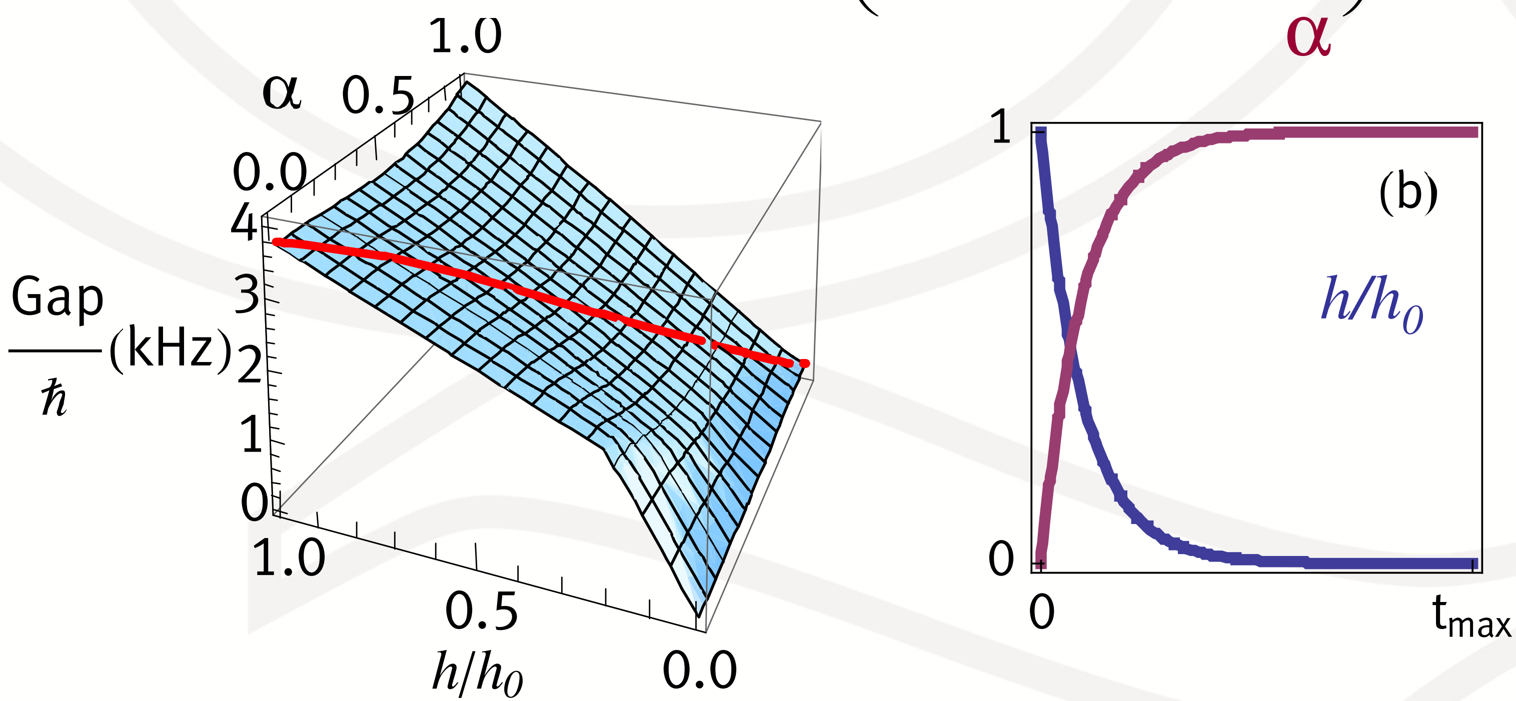
Under a slow variation of some Hamiltonian parameter, a system initially in an eigenstate will follow the instantaneous eigenstate

$$H_{\mathrm{eff}}(h,\alpha) = H_I^{(z)}(h) + \alpha H_I^{(x)}(h)$$

- $\begin{cases} h(t_0) &= h_0 \\ \alpha(t_0) &= 0 \end{cases} \rightarrow \begin{cases} h(t_f) &= 0 \\ \alpha(t_f) &= 1 \end{cases}$
- $H(h_0,0) = H_I^{(z)}(h_0) \rightarrow H(0,1) = \sum_{j,k} J_{j,k}\left(\sigma_j^x\sigma_k^x + \sigma_j^z\sigma_k^z\right)$
- $|\uparrow,\uparrow,\cdots\uparrow,\uparrow\rangle \rightarrow$ Long distance entanglement

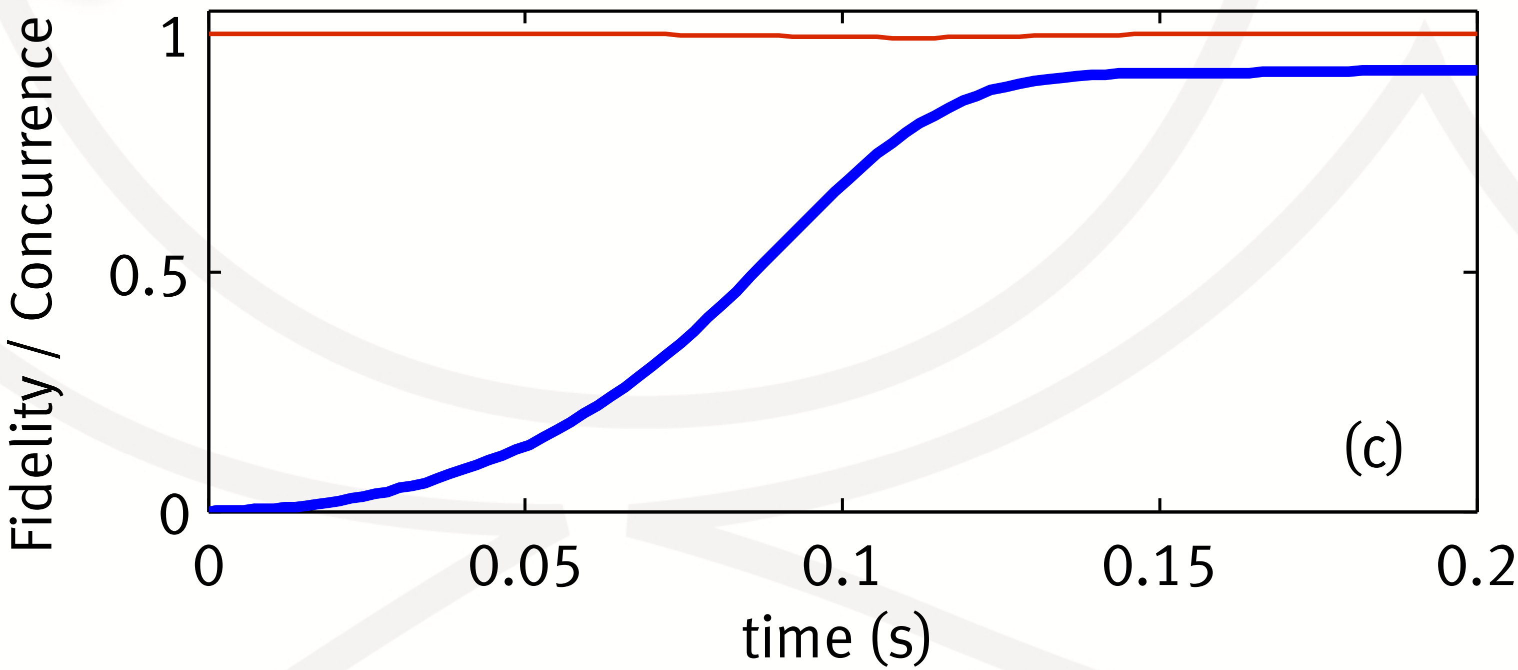
Parameters: Time Dependence

$$e^{-i(H^{x}_{\mathrm{Ising}}+H^{z}_{\mathrm{Ising}})\tau} \approx \left(e^{-iH^{x}_{\mathrm{Ising}}\frac{\tau}{n}} e^{-iH^{z}_{\mathrm{Ising}}\frac{\tau}{n}} \right)^{n}$$



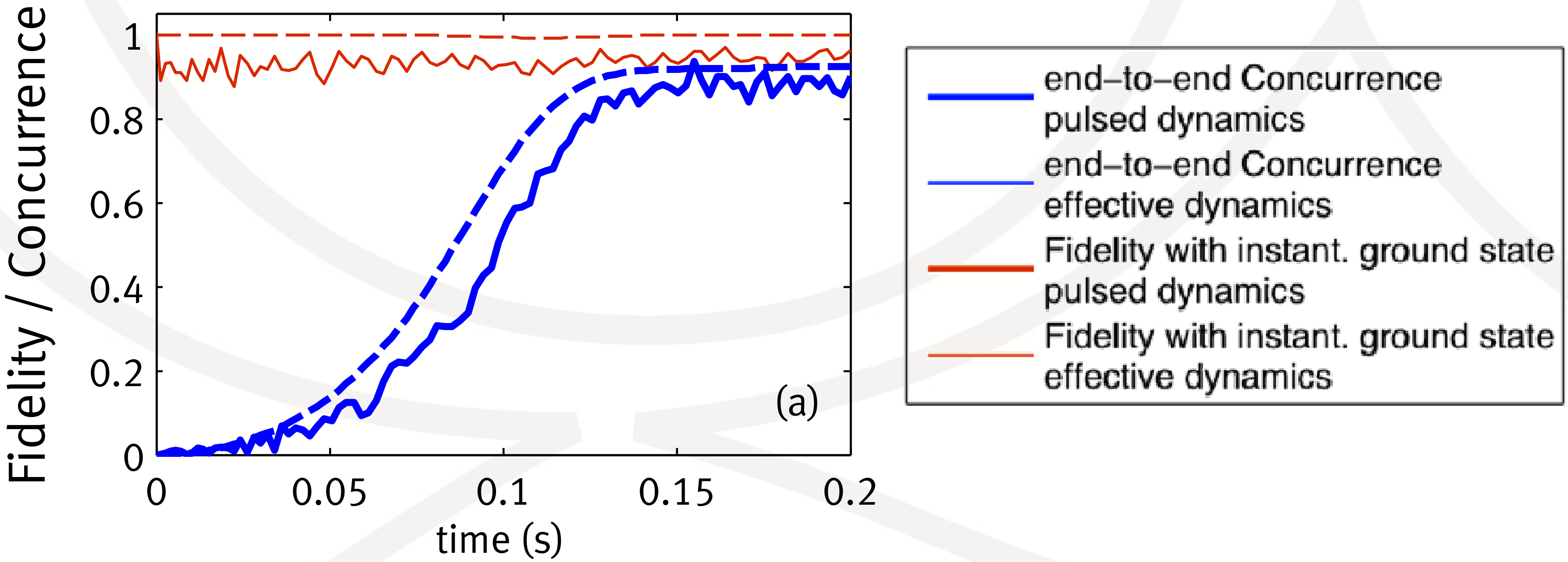
- initially the large gap allows for fast sweeping
- slow down towards smaller gaps to remain adiabatic

Adiabatic transformation



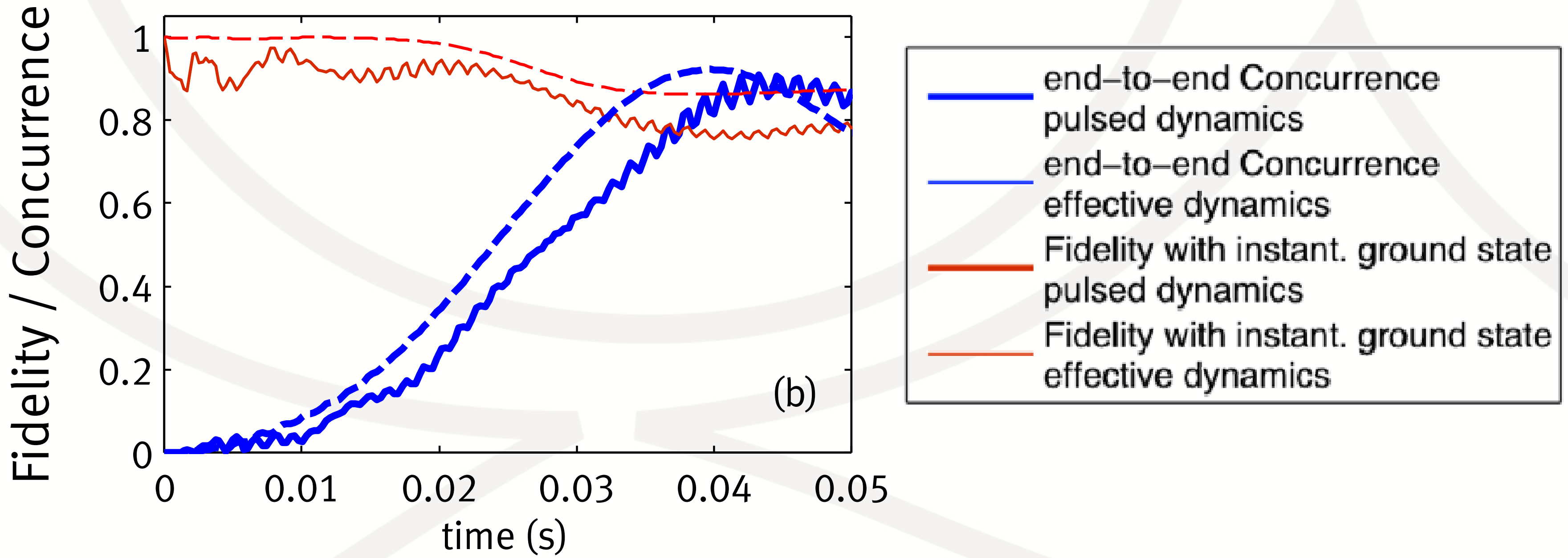
- adiabaticity well fulfilled
- final state: high fidelity with instantaneous ground state (red curve)
- large concurrence / entanglement (blue curve)

Adiabatic Transformation + Pulsed Dynamics



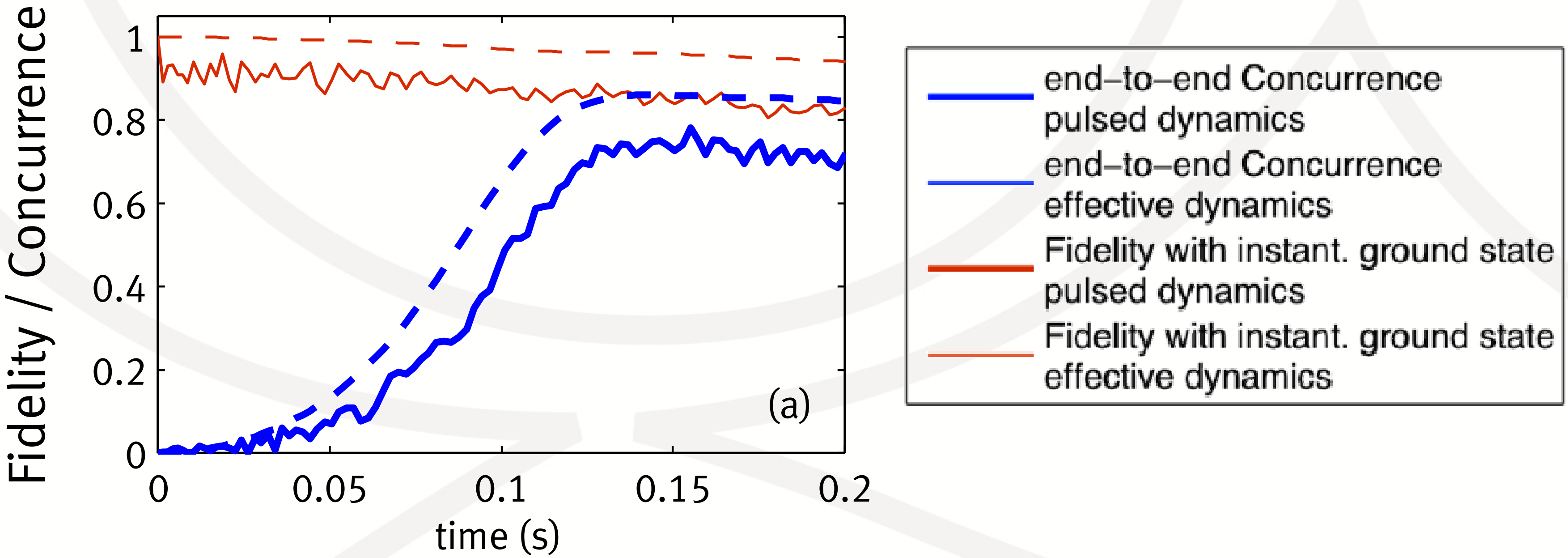
- reasonable agreement with effective Hamiltonian
- quantum simulation
- ground state entanglement

Increase Ramp Speed



- lower fidelity
- oscillatory behavior
- still good concurrence

Adiabatic Transformation + Pulsed Dynamics + Dephasing



- fidelity decreases over time
- still good concurrence

LDE Outlook

- tailoring the axial trapping potential can generate coupling patterns useful for LDE
- an effective XZ Hamiltonian by bracketing the evolution time with resonant microwave pulses
- changing the relative evolution times allows to sweep from Ising to XZ Hamiltonian
- simulations show ground state population, concurrence
- challenges: large gradients, coherence

Microtrap Witches And Wizards

funding:

Bundesministerium
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und Forschung

Deutsche
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DFG

PICC

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