





# Quantum Simulations of Fermions and Bosons with Trapped Ions

### Lucas Lamata University of the Basque Country UPV/EHU, Bilbao, Spain

Quantum Simulations, Benasque, 30 September 2013



#### Our group develops interdisciplinary research in

Quantum optics Quantum information Relativistic quantum mechanics Circuit quantum electrodynamics Quantum Biomimetics

Laura García-Álvarez (M. Sc. student) Urtzi Las Heras (M. Sc. student) Julen S. Pedernales (PhD student) Unai Alvarez-Rodriguez (PhD student) Antonio Mezzacapo (PhD, European SOLID grant) Simone Felicetti (PhD, CCQED Marie Curie grant) Roberto Di Candia (PhD, CCQED Marie Curie grant) Mikel Sanz (Postdoc, European PROMISCE grant) Jorge Casanova (Postdoc, European PROMISCE grant) Guillermo Romero (Postdoc, European PROMISCE grant) Lucas Lamata (Ramón y Cajal Fellow since 2014)

Enrique Solano (Group Leader and Ikerbasque Professor)







País Vasco

EUSKO JAURLARITZA GOBIERNO VASCO





QUANTUM ELECTRODYNAMIC

### Outline



#### **Overview of Bilbao Proposals**



Fermions + Bosons in lons  $\{b, b^{\dagger}\} = 1$   $[a, a^{\dagger}] = 1$ 



## Quantum Simulations

### Simulating efficiently quantum systems



Feynman '82

### Simulating unreproducible physics



Lloyd '96



Buluta and Nori Science 326, 108 (2009)

### Trapped lons



14-qubit entanglement and 140-gate simulator (Innsbruck), 300-ion simulator (NIST)

#### **Trapped Ions in Bilbao**

PRL 98, 253005 (2007)

PHYSICAL REVIEW LETTERS

#### week ending 22 JUNE 2007

Vol 463|7 January 2010|doi:10.1038/nature08688

#### ç Dirac Equation and Quantum Relativistic Effects in a Single Trapped Ion

L. Lamata,<sup>1</sup> J. León,<sup>1</sup> T. Schätz,<sup>2</sup> and E. Solano<sup>3,4</sup>

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We present a method of simulating the Dirac equation in 3 + 1 dimensions for a free spin-1/2 particle in a single trapped ion. The Dirac bispinor is represented by four ionic internal states, and position and momentum of the Dirac particle are associated with the respective ionic variables. We show also how to simulate the simplified 1 + 1 case, requiring the manipulation of only two internal levels and one motional degree of freedom. Moreover, we study relevant quantum-relativistic effects, like the Zitterbewegung and Klein's paradox, the transition from massless to massive fermions, and the relativistic and nonrelativistic limits, via the tuning of controllable experimental parameters.

#### LETTERS

nature

#### Quantum simulation of the Dirac equation

R. Gerritsma<sup>1,2</sup>, G. Kirchmair<sup>1,2</sup>, F. Zähringer<sup>1,2</sup>, E. Solano<sup>3,4</sup>, R. Blatt<sup>1,2</sup> & C. F. Roos<sup>1,2</sup>

The Dirac equation' successfully merges quantum mechanics with easily accessed experimentally, while allowing parameter tunability special relativity. It provides a natural description of the electron spin, predicts the existence of antimatter<sup>2</sup> and is able to reproduce accurately the spectrum of the hydrogen atom. The realm of the Dirac equation-relativistic quantum mechanics-is considered to be the natural transition to quantum field theory. However, the Dirac equation also predicts some peculiar effects, such as Klein's paradox<sup>3</sup> and "Zitterbeegung", an unexpected quivering motion of a free relativistic quantum particle<sup>4</sup>. These and other predicted phenomena are key fundamental examples for understanding

over a wide range. The difficulties in observing real quantum rela-tivistic effects have generated significant interest in the quantum simulation of their dynamics. Examples include black holes in Bose–Einstein condensates<sup>5</sup> and Zitterbewegung for massive fermions in solid-state physics<sup>6</sup>, neither of which have been experimentally realized so far. Also, graphene is studied widely in connection to the Dirac equation15

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- Klein (theory)
- Klein (experiment)
- Majorana equation
- Unphysical operations

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Klein (theory)

- Klein (experiment)
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- Unphysical operations

See J. Casanova's talk Tuesday

150

### Trapped lons in Bilbao

Quantum Field Theories, Casanova, Lamata, ..., Solano, PRL '11



Holstein Model and unbounded Hs

Mezzacapo, Casanova, Lamata, Solano, PRL '12



Fermion Lattice Models, Casanova, Mezzacapo, Lamata, Solano, PRL '12





QFT Scattering



Fermion Lattices

 $\{b, b^{\dagger}\} = \{d, d^{\dagger}\} = 1$   $[a, a^{\dagger}] = 1$ 

Interacting fermions+bosons Efficiently implementable!!

Fermionic modes: internal levels

**Bosonic modes:** motional d.o.f.

digital-analog simulator, polynomial time



#### Fermions and Bosons in Trapped lons Fermionic interactions:

i) Jordan Wigner  $b_i^{\dagger} = I \otimes I \otimes \ldots \otimes \sigma_i^{+} \otimes \sigma_{i-1}^{z} \otimes \ldots \otimes \sigma_1^{z}$   $\{b_i, b_j^{\dagger}\} = \delta_{i,j}$ 

ii) Trotter expansion

$$e^{-iHt} \simeq (e^{-iH_1t/n}e^{-iH_2t/n}\dots e^{-iH_mt/n})^n$$

iii) Efficient implementation spin operators

 $H = \sum_{i=1}^{m} H_i, H_i = g_i \sigma_{i_1} \otimes \sigma_{i_2} \otimes \ldots \otimes \sigma_{i_N}$ 

Casanova, Mezzacapo, Lamata, Solano, PRL '12 Mezzacapo, Casanova, Lamata, Solano, PRL '12

$$H = \sum_{n=2}^{\alpha} \left[ \sum_{i_1...i_n=1}^{N} g_{i_1...i_n} c_{i_1} \cdots c_{i_n} + \text{H.c.} \right],$$
$$\{c_{i_l}, c_{i_{l'}}^{\dagger}\} = \delta_{l,l'}$$

Fermion Hamiltonian

$$H = \sum_{n=2}^{\alpha} \left[ \sum_{i_{1}...i_{n}=1}^{N} g_{i_{1}...i_{n}} c_{i_{1}} \cdots c_{i_{n}} + \text{H.c.} \right], \quad \text{Fermion Hamiltonian}$$
$$\int_{m} \left\{ c_{i_{l}}, c_{i_{l'}}^{\dagger} \right\} = \delta_{l,l'}$$
$$H = \sum_{i=1}^{m} H_{i} \quad H_{i} = g_{i} \sigma_{i_{1}} \otimes \sigma_{i_{2}} \otimes \dots \otimes \sigma_{i_{N}}$$

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$$\int_{i_1...i_n=1}^{M} \{c_{i_l}, c_{i_{l'}}^{\dagger}\} = \delta_{l,l'}$$

$$H = \sum_{i=1}^{m} H_i \quad H_i = g_i \sigma_{i_1} \otimes \sigma_{i_2} \otimes \dots \otimes \sigma_{i_N}$$

Trapped-ion implementation: Mølmer-Sørensen gates (highly efficient)

Müller,...,Zoller, NJP'I I

Casanova, Mezzacapo, Lamata, Solano, PRL '12





Mølmer and Sørensen, PRL'99, PRA'00

F>99%

 $\mathcal{U} = \mathcal{U}_{\mathrm{MS}}(-\pi/2, 0)\mathcal{U}_{\sigma_{z}}(\phi)\mathcal{U}_{\mathrm{MS}}(\pi/2, 0)$   $= \exp\left[i\phi \ \sigma_{1}^{z} \otimes \sigma_{2}^{x} \otimes \sigma_{3}^{x} \otimes \cdots \sigma_{k}^{x}\right], \qquad \vdots$   $\mathcal{U}_{\mathrm{MS}}(\theta, \phi) = \exp[-i\theta(\cos\phi S_{x} + \sin\phi S_{y})^{2}/4] \qquad \vdots$   $S_{x,y} = \sum_{i=1}^{k} \sigma_{i}^{x,y} \qquad \mathcal{U}_{\sigma_{z}}(\phi) = \exp(i\phi'\sigma_{1}^{z}) \qquad \phi' = \pm\phi$ 



Arbitrary number Pauli matrices with 3 gates!!

Trotter errors

$$\begin{aligned} \mathcal{U} = \exp(-iHt), H &= \sum_{j=1}^{m} H_j \qquad \tilde{\mathcal{U}} = \prod_{l=1}^{N_e} e^{-iH_{j_l}t_l}, \\ &||\mathcal{U} - \tilde{\mathcal{U}}|| < \epsilon \\ N_e &\leq m 5^{2k} (m||H||t)^{1+1/2k} / \epsilon^{1/2k}, \end{aligned}$$

m

D. Berry, G. Ahokas, R. Cleve and B. Sanders, Commun. Math. Phys.'07

 $N_e$  polynomial in number of modes, error and time

**Trotter errors**  $\mathcal{U} = \exp(-iHt), H = \sum_{i=1}^{N_{e}} H_{j} \qquad \tilde{\mathcal{U}} = \prod_{i=1}^{N_{e}} e^{-iH_{j_{i}}t_{i}},$ j=1  $||\mathcal{U} - \tilde{\mathcal{U}}|| < \epsilon$  $N_{\rm e} \le m 5^{2k} (m ||H||t)^{1+1/2k} / \epsilon^{1/2k},$ 

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## $N_e$ polynomial in number of modes, error and time **Efficient implementation!!**



 $b_1^{\dagger}b_{10} + b_{10}^{\dagger}b_1 \qquad 2^{10} \times 2^{10} = 2^{20} \simeq 10^6 \text{ gates}$ 









#### Fermions and Bosons in Trapped Ions Possible to simulate:

i) Kondo

$$H = \sum_{p\sigma} \epsilon_p b_{p\sigma}^{\dagger} b_{p\sigma} - J \sum_{pp'j} e^{iR_j \cdot (p-p')} [(b_{p\uparrow}^{\dagger} b_{p'\uparrow} - b_{p\downarrow}^{\dagger} b_{p'\downarrow})\sigma_j^z + b_{p\uparrow}^{\dagger} b_{p'\downarrow}\sigma_j^- + b_{p\downarrow}^{\dagger} b_{p'\uparrow}\sigma_j^+].$$

ii) Hubbard

$$H = w \sum_{\delta i\sigma} b_{i\sigma}^{\dagger} b_{i+\delta\sigma} + U \sum_{j} b_{j\uparrow}^{\dagger} b_{j\uparrow} b_{j\downarrow}^{\dagger} b_{j\downarrow},$$

iii) Fröhlich

$$H = \sum_{p} \frac{p^2}{2m} b_p^{\dagger} b_p + \omega_0 \sum_{q} a_q^{\dagger} a_q + \sum_{qp} M(q) b_{p+q}^{\dagger} b_p (a_q + a_{-q}^{\dagger}),$$

#### Quantum Chemistry with lons arXiv 1307.4326

From transistor to trapped-ion computers for quantum chemistry

M.-H. Yung,<sup>1,2,\*</sup> J. Casanova,<sup>3,\*</sup> A. Mezzacapo,<sup>3</sup> J. McClean,<sup>1</sup> L. Lamata,<sup>3</sup> A. Aspuru-Guzik,<sup>1,†</sup> and E. Solano<sup>3,4,‡</sup>

<sup>1</sup>Department of Chemistry and Chemical Biology, Harvard University, Cambridge MA, 02138, USA <sup>2</sup>Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing, 100084, P. R. China <sup>3</sup>Department of Physical Chemistry, University of the Basque Country UPV/EHU, Apartado 644, 48080 Bilbao, Spain <sup>4</sup>IKERBASQUE, Basque Foundation for Science, Alameda Urquijo 36, 48011 Bilbao, Spain (Dated: July 17, 2013)

Over the last few decades, quantum chemistry has progressed through the development of computational methods based on modern digital computers. However, these methods can hardly fulfill the exponentially-growing resource requirements when applied to large quantum systems. As pointed out by Feynman, this restriction is intrinsic to all computational models based on classical physics. Recently, the rapid advancement of trapped-ion technologies has opened new possibilities for quantum control and quantum simulations. Here, we present an efficient toolkit that exploits both the internal and motional degrees of freedom of trapped ions for solving problems in quantum chemistry, including molecular electronic structure, molecular dynamics, and vibronic coupling. We focus on applications that go beyond the capacity of classical computers, but may be realizable on stateof-the-art trapped-ion systems. These results allow us to envision a new paradigm of quantum chemistry that shifts from the current transistor to a near-future trapped-ion-based technology.

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# 10 ions, 10 modes, 7 excitations/mode: overcome classical computers!

#### Quantum Chemistry with lons arXiv 1307.4326



#### Quantum Simulations in Bilbao

#### Circuit QED: Different context, different technology

Access to continuum

•Strong/Ultrastrong coupling regime

•Quantum propagating mw



 $\Delta \psi$ 

 $\psi(x_2)$ 

#### Future in the field!

### **Quantum Biomimetics**

### Our new contribution: Quantum Biomimetics

Study biological systems + mimic them in quantum models







Aim: reproduce biological behaviours in quantum controllable systems







### **QUTIS Posters**

- Roberto Di Candia: Embedding quantum simulators
- Antonio Mezzacapo: Digital Quantum Simulation of the Holstein Model
- Simone Felicetti: Entanglement via Dynamical Casimir effect
- Julen Pedernales: Quantum simulations in cQED
- Unai Alvarez-Rodriguez: Bio-Inspired Cloning of Quantum Information
- Laura García-Álvarez: Quantum field theories in cQED
- Urtzi Las Heras: Digital Quantum Simulation of Spin Models in cQED
- Paul Pfeiffer: Brain inspired Quantum Networks

# Thank you for your attention!