

$$\mathrm{rank}_{\mathrm{puri}}(\rho') \leq O(D \log(1/\epsilon)^2)$$