

$$\text{rank}(S_d) = 3$$