Dipartimento di Matematica e Fisica "Ennio De Giorgi"

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INFN Istituto Nazionale di Fisica Nucleare

Bound states and resonances in Effective Field Theories and Lattice QCD calculations

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Centro de Ciencias de Benasque Pedro Pascual

Determining the three-nucleon force from three-nucleon data

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work in progress

in collaboration with Alejandro Kievsky and Michele Viviani (INFN Pisa)

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providing a realistic three-nucleon force

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and other problems in the 3N continuum

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we explore the second option in particular, since A_y is a puzzle only at very low-energy, we focus on the subleading contact 3NF

A_y puzzles

- notice that $p{-}^3$He A_y is almost solved by chiral 3NF at N2LO (or by AV18+IL7)
  [Viviani et al. PRL111 (2013) 172302]

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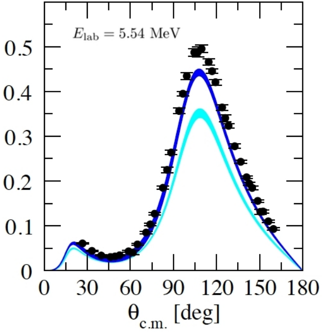

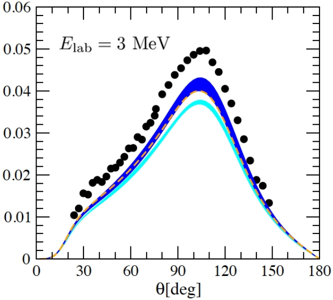

- for $p-d$ the discrepancy remains at the $\sim 20\%$ level

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we start with the AV18 NN potential, in the absence of further 3NF

Contact terms - leading order

At leading order (no derivatives) the most general effective Lagrangian satisfying rotational, parity, time-reversal and isospin symmetry is

$$
\begin{aligned}
\mathcal{L} \quad \equiv \quad & -\sum_i^6 E_i O_i = -E_1 N^\dagger N N^\dagger N N^\dagger N - E_2 N^\dagger \sigma^i N N^\dagger \sigma^i N N^\dagger N \\
& -E_3 N^\dagger \tau^a N N^\dagger \tau^a N N^\dagger N - E_4 N^\dagger \sigma^i \tau^a N N^\dagger \sigma^i \tau^a N N^\dagger N \\
& -E_5 N^\dagger \sigma^i N N^\dagger \sigma^i \tau^a N N^\dagger \tau^a N - E_6 \epsilon^{ijk} \epsilon^{abc} N^\dagger \sigma^i \tau^a N N^\dagger \sigma^j \tau^b N N^\dagger \sigma^k \tau^c N
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but the 6 operators are redundant. Ultimately, this is due to the anticommuting nature of the nucleon fields $N(x)$ and to Fierz-like identities

$$\begin{aligned}&(\mathbf{1})[\mathbf{1}] = \tfrac{1}{2}(\mathbf{1}][\mathbf{1}) + \tfrac{1}{2}(\boldsymbol{\sigma}]\cdot[\boldsymbol{\sigma}) \\ &(\sigma^i)[\mathbf{1}] = \tfrac{1}{2}(\sigma^i][\mathbf{1}) + \tfrac{1}{2}(\mathbf{1}][\sigma^i) - \tfrac{i}{2}\epsilon^{ijk}(\sigma^j][\sigma^k) \\ &(\sigma^i)[\sigma^j] = \tfrac{1}{2}\left\{\delta^{ij}(\mathbf{1}][\mathbf{1}) - \delta^{ij}(\boldsymbol{\sigma}]\cdot[\boldsymbol{\sigma}) + (\sigma^i][\sigma^j) + (\sigma^j][\sigma^i) + i\epsilon^{ijk}(\sigma^k][\mathbf{1}) - i\epsilon^{ijk}(\mathbf{1}][\sigma^k)\right\}\end{aligned}$$

Redundancies of contact operators

Simultaneous Fierz rearrangements of spin and isospin indeces of a given pair of nucleon fields allow to derive linear relations, e.g.

$$
\begin{aligned}
O_1 &= -\tfrac{1}{4}\left(O_1 + O_2 + O_3 + O_4\right)\\
O_2 &= -\tfrac{1}{2}\left(O_2 + O_5\right)\\
O_3 &= -\tfrac{1}{2}\left(O_3 + O_5\right)\\
O_4 &= -\tfrac{1}{4}\left(2O_4 + 2O_5 - O_6\right)\\
O_5 &= -\tfrac{1}{2}\left(3O_2 - O_5\right)\\
O_6 &= 2\left(O_4 - O_5\right)
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$$V \sim \sum_{ijk} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j Z_0(r_{ik}) Z_0(r_{jk})$$

which introduces a difference between the choices of the operator.

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which introduces a difference between the choices of the operator. Such difference is a cutoff effect, beyond the reach of a leading order description, to be regarded as a theoretical uncertainty

Contact terms - subleading order

Parity invariance requires that the subleading $3N$ contact Lagrangian contain 2 gradients
Using translational invariance the possible space-structures are

$$
\begin{aligned}
X^{+}_{A,ij} &= (N^\dagger \overleftrightarrow{\nabla}_i N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\
X^{+}_{B,ij} &= \nabla_i(N^\dagger N)\nabla_j(N^\dagger N)(N^\dagger N) \\
X^{-}_{C,ij} &= i\nabla_i(N^\dagger N)(N^\dagger \overleftrightarrow{\nabla}_j N)(N^\dagger N) \\
X^{+}_{D,ij} &= (N^\dagger \overleftrightarrow{\nabla}_i \overleftrightarrow{\nabla}_j N)(N^\dagger N)(N^\dagger N),
\end{aligned}
$$

to be combined with all possible isospin invariant structures

$$
T^+ = \mathbf{1}, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \quad \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3, \quad \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3, \quad T^- = \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3
$$

and contracted in all possible time-reversal invariant ways with spin matrices

As a result, we get a list of 146 operators

$$\overleftrightarrow{\nabla}_1 \cdot \overleftrightarrow{\nabla}_2 [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3]$$
$$\overleftrightarrow{\nabla}_1 \cdot \vec{\sigma}_1 \, \overleftrightarrow{\nabla}_2 \cdot \vec{\sigma}_2 [\mathbf{1}, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3]$$
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same as before with $\overleftrightarrow{\nabla} \to \overrightarrow{\nabla}$

$$i \, \overleftrightarrow{\nabla}_1 \cdot \overrightarrow{\nabla}_2 [\boldsymbol{\tau}_1 \times \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3]$$
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Fierz constraints

As before, a set of linear relations among the 146 operators can be found by using Fierz's reshuffling, which in this case also involves the fields' derivatives: e.g. under exchange of nucleons 1–2

$$\overleftrightarrow{\nabla}_1 \to \frac{1}{2}(\overrightarrow{\nabla}_2 + \overleftrightarrow{\nabla}_2 - \overrightarrow{\nabla}_1 + \overleftrightarrow{\nabla}_1), \quad \overrightarrow{\nabla}_1 \to \frac{1}{2}(\overrightarrow{\nabla}_2 + \overleftrightarrow{\nabla}_2 + \overrightarrow{\nabla}_1 - \overleftrightarrow{\nabla}_1)$$

Out of the 3×146 relations, 132 are linearly independent
$\Longrightarrow$ we are left with 14 independent operators

Constraints from relativity

We still have to impose the requirements of Poincaré covariance
They can be implemented order by order in the low-energy expansion
As a result, the subleading $3N$ effective Hamiltonian consists of

- fixed terms (relativistic corrections to the lower order terms)
- free terms, which have to commute with the lowest order boost operator $\mathbf{K}_0$

with the choice $N(x) = \int \frac{d\mathbf{p}}{(2\pi)^3}\, b_s(\mathbf{p})\, \chi_s\, \mathrm{e}^{-ip\cdot x}$ $\mathbf{K}_0$ acts as

$$[\mathbf{K}_0\, ,\, b_s(\mathbf{p})] = -i\, m\, \boldsymbol{\nabla}_{\mathbf{p}}\, b_s(\mathbf{p})$$

and only 10 independent combinations of the 14 operators can be found to commute with $\mathbf{K}_0$

Subleading contact potential

Choosing a momentum cutoff depending only on momentum transfers the potential is local incoordinate space

$$
\begin{aligned}
V \quad = & \sum_{i\neq j\neq k} (E_1 + E_2 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3 \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_5 + E_6 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j) S_{ij}\left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right] Z_0(r_{ik}) \\
& + (E_7 + E_8 \boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\mathbf{S})_{ij}\frac{Z_0'(r_{ij})}{r_{ij}} Z_0(r_{ik}) \\
& + (E_9 + E_{10} \boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik} Z_0'(r_{ij}) Z_0'(r_{ik})
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Subleading contact potential

Choosing a momentum cutoff depending only on momentum transfers the potential is local incoordinate space

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Most terms are ordinary 2-body interactions between particles ij with a further dependence on the coordinate of particle k
Spin-orbit terms suitable for the A_y puzzle [Kievsky PRC60 (1999) 034001]

Numerical implementation

The N-d scattering wave function is written as

$$\Psi_{LSJJ_z} = \Psi_C + \Psi_A$$

with Ψ_C expanded in the HH basis

$$|\Psi_C\rangle = \sum_\mu c_\mu |\Phi_\mu\rangle$$

and Ψ_A describing the asymptotic relative motion

$$\Psi_A \sim \Omega^R_{LS}(k,r) + \sum_{L'S'} R_{LS,L'S'}(k)\Omega^I_{L'S'}(k,r)$$

with the unknown c_μ and R-matrix elements (related to the S-matrix) to be determined so that the Kohn functional is stationary

$$[R_{LS,L'S'}] = R_{LS,L'S'} - \langle \Psi_C + \Psi_A | H - E | \Psi_C + \Psi_A \rangle$$

imposing the Kohn functional to be stationary leads to a *linear* system

$$\sum_{L''S''} R_{LS,L''S''} X_{L'S',L''S''} = Y_{LS,L'S'}$$

with the matrices

$$X_{LS,L'S'} = \langle \Omega^I_{LS} + \Psi^I_C | H - E | \Omega^I_{L'S'} \rangle \quad Y_{LS,L'S'} = -\langle \Omega^R_{LS} + \Psi^R_C | H - E | \Omega^I_{L'S'} \rangle$$

and the $\Psi_C^{R/I}$ solutions of

$$\sum_{\mu'} c_\mu \langle \Phi_\mu | H - E | \Phi_{\mu'} \rangle = -D^{R/I}_{LS}(\mu)$$

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$$D^{R/I}_{LS}(\mu) = \langle \Phi_\mu | H - E | \Omega^{R/I}_{LS} \rangle$$

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11 set of matrices are calculated once for all, and only linear systems are solved for each choice of E_i's

Fit strategy

we thus have 11 LECs, $E = \frac{c_E}{F_\pi^4 \Lambda}$ (LO) and $E_{i=1,\dots,10} = \frac{e_i^{NN}}{F_\pi^4 \Lambda^3}$ (NLO) to be fitted to $B(^3H)$, $^2a_{nd}$, $^4a_{nd}$ and the p-d phaseshifts for different values of Λ

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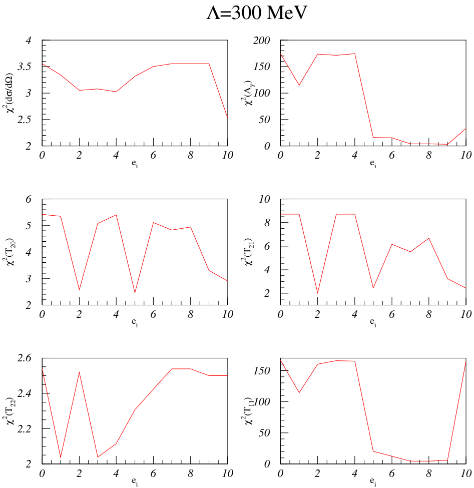
- we then add 1 among the other 10 LECs, and fix it to those observables

Λ(MeV)	200	300	400	500
c_E/χ^2	1.269/13	0.525/40	0.410/114	0.45/170
c_E/e_1	X	1.335/-0.822	1.09/-0.99	0.894/-1.45
c_E/e_2	2.382/0.844	X	1.701/2.016	0.896/2.02
c_E/e_3	0.389/-0.954	0.888/0.511	0.807/0.828	0.654/1.27
c_E/e_4	X	1.450/0.331	1.202/0.400	0.965/0.541
c_E/e_5	1.519/1.237	0.152/-0.898	-0.491/-1.255	-1.47/-1.53
c_E/e_6	1.704/-0.648	-0.028/0.384	-0.785/0.470	-1.911/0.5238
c_E/e_7	X	X	X	X
c_E/e_8	0.647/986	0.925/-7.948	0.715/-10.944	X
c_E/e_9	X	X	1.365/-7.292	1.262/-6.937
c_E/e_{10}	X	-0.029/-4.599	X	-1.293/-3.032

Sensitivity to e_i

- χ^2 from 2-parameter fit with (c_E, e_i)
- strong sensitivity of A_y and iT_{11} to E_7, E_8 and E_9



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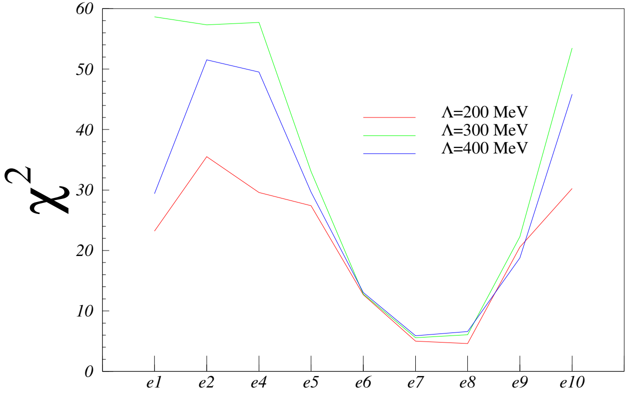
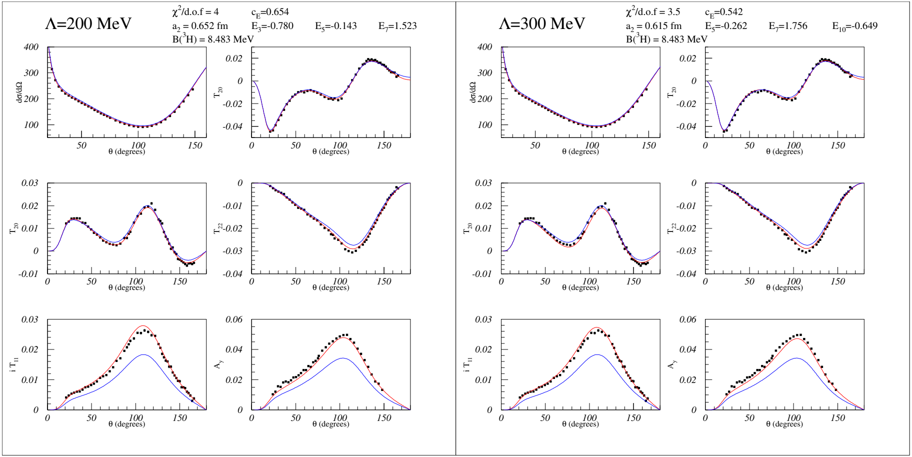
- use c_E and E_3 to account for $B(^3\text{H})$ and $^2a_{nd}$
- use another one of the E_i to fit scattering observables at 3 MeV



4-parameter fits

- the χ^2 can be reduced to 3-4 per d.o.f.



Comparison with the pionful theory

$$E_1 = \frac{755 g_A^6}{24576\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^4 C_T}{64\pi F_\pi^4 M_\pi} - \frac{g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \qquad \sim 0.10$$

$$E_2 = \frac{601 g_A^6}{36864\pi F_\pi^6 M_\pi} + \frac{23 g_A^4 C_T}{384\pi F_\pi^4 M_\pi} - \frac{5 g_A^2 C_T}{192\pi F_\pi^4 M_\pi} - \frac{g_A^2(5C_T+2C_S)}{48m F_\pi^2 M_\pi^2} \qquad \sim 0.06$$

$$E_3 = -\frac{3 g_A^6}{2048\pi F_\pi^6 M_\pi} + \frac{3 g_A^4 C_T}{64\pi F_\pi^4 M_\pi} + \frac{9 g_A^2 C_T}{16m F_\pi^2 M_\pi^2} \qquad \sim 0.00$$

$$E_4 = -\frac{g_A^6}{1024\pi F_\pi^6 M_\pi} - \frac{3 g_A^2 C_T}{16m F_\pi^2 M_\pi^2} \qquad \sim 0.00$$

$$E_5 = \frac{79 g_A^6}{12288\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^4 C_T}{64\pi F_\pi^4 M_\pi} - \frac{g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \qquad \sim 0.02$$

$$E_6 = \frac{319 g_A^6}{36864\pi F_\pi^6 M_\pi} + \frac{g_A^4}{256\pi F_\pi^6 M_\pi} - \frac{g_A^2(C_S-2C_T)}{24m\pi F_\pi^2 M_\pi^2} \qquad \sim 0.04$$

$$E_7 = -\frac{83 g_A^6}{6144\pi F_\pi^6 M_\pi} - \frac{3 g_A^4}{128\pi F_\pi^6 M_\pi} + \frac{3 g_A^2 C_T}{4m F_\pi^2 M_\pi^2} \qquad \sim -0.08$$

$$E_8 = -\frac{7 g_A^6}{3072\pi F_\pi^6 M_\pi} - \frac{g_A^4}{128\pi F_\pi^6 M_\pi} + \frac{g_A^2 C_T}{4m F_\pi^2 M_\pi^2} \qquad \sim -0.02$$

$$E_9 = \frac{193 g_A^6}{4096\pi F_\pi^6 M_\pi} - \frac{3 g_A^2 C_T}{8m F_\pi^2 M_\pi^2} \qquad \sim 0.14$$

$$E_{10} = \frac{c_1 g_A^2}{2F_\pi^4 M_\pi^2} + \frac{g_A D}{8F_\pi^2 M_\pi^2} + \frac{427 g_A^6}{12288\pi F_\pi^6 M_\pi} + \frac{9 g_A^4}{512\pi F_\pi^6 M_\pi} - \frac{g_A^2(C_S+C_T)}{8m F_\pi^2 M_\pi^2}$$
$$- \frac{2 g_A^2 \bar{e}_{14}}{F_\pi^4} + \frac{g_A^2(2c_1-c_3)}{128\pi^2 F_\pi^6} = -0.05 + 0.15 \qquad \sim 0.09$$

numerical values are in units of $F_\pi^4 M_\pi^3$

Insight from the large-N_c limit

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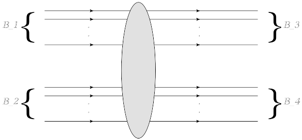

$$T \sim N_c \sum \left(\frac{S}{N_c}\right)^n_S \left(\frac{I}{N_c}\right)^n_I \left(\frac{G}{N_c}\right)^n_G \qquad S \sim \sigma, I \sim \tau, G \sim \sigma\tau$$

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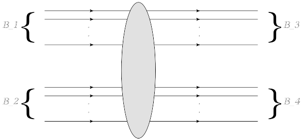

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- as a result, one finds e.g.

$$\mathbf{1} \sim \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(N_c)$$

while

$$\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \sim \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \sim O(1/N_c)$$

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reobtaining the well-established fact that $C_S >> C_T$

3NF and large-N_c

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large-N_c constraints on subleading 3N contact interaction

- applying Phillips and Schat counting to our redundant operators we get 13 leading structures
- using Fierz identities we find 7 leading operators, out of 10
- we thus have predictions for some of the E_i

$$E_2 = 0 + O(1/N_c)$$

$$E_3 = E_5 + O(1/N_c)$$

$$E_9 = 3E_3 + O(1/N_c)$$

Summary and outlook

- We advocate a pragmatic approach, in which the subleading $3N$ contact interaction is treated as a sort of remainder, to fine-tune existing realistic models
- We are in the middle of the fitting procedure to $p-d$ elastic scattering data
- We have started by adopting the AV18 NN interaction. The χ^2 is drastically reduced, until 3-4 per d.o.f., but the exploration of the parameter space is not complete yet.

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It will be interesting, in the near future

- to repeat the analysis using a realistic pionless NN potential;
- to extend the analysis to other energies, and to include the breakup channel
- to implement the constraints from the large-N_c analysis