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GOQ QCD



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Towards Exploring Parity Violation with Lattice QCD

- Hadronic Weak Interactions
- Lattice QCD calculations
- Hadronic Parity Violation
 isovector and isotensor



Goal: provide a sense of what challenges lattice QCD computations must confront

Quark Interactions to Hadronic Couplings

- **Textbook**: gauge theories defined in perturbation theory
- **QCD**: short distance perturbative, long distance non-perturbative

 $\overline{\psi} \left(D + m_q \right) \psi + \frac{1}{4} G_{\mu\nu} G_{\mu\nu}$ Many Technicalities

Wilson Lattice Action Wilson Fermions

Non-perturbative definition of asymptotically free gauge theories' $\delta_{NN}(k)$

 \mathbf{Q}

Spectrum Interactions

 $M_N \quad \epsilon_b(D)$

Strong interaction observables

Quarks couple to other fundamental interactions: e.g. weak interaction

 $J(x)D(x,0)J(0) = \sum C_i(\mu)\mathcal{O}_i(x,\mu)$ Wilson Operator Product Expansion, Wilson Coefficients, Wilson Renormalization Group

Hadronic weak (& BSM) interactions require all the Wilson brand names



1936-2013

Weak Interactions

• Leptonic weak interaction

• Semi-leptonic weak interaction





• Non-leptonic (hadronic) weak interaction



Example: $K \rightarrow \pi\pi$ and $\Delta I = 1/2$ Rule



- Old Puzzle: I = 0 weak decay channel experimentally observed ~500x over I = 2
- Amplitude level: A0 / A2 ~ 22.5 pQCD contributes a factor of ~2 Rest non-perturbative?

PRL 110, 152001 (2013)

• Almost There? C. Lehner,⁵ Q ${\cal A}_0/{\cal A}_2(m_\pi=330\,{
m MeV})=12.0(1.7)$

$\mathcal{A} = \sum_{i} C_{i}(\mu) \langle \pi \pi | \mathcal{O}_{i}(\mu) | K \rangle_{\text{Lattice}}$

Emerging understanding of the $\Delta I=1/2$ Rule from Lattice QCD

P.A. Boyle,¹ N.H. Christ,² N. Garron,³ E.J. Goode,⁴ T. Janowski,⁴ C. Lehner,⁵ Q. Liu,² A.T. Lytle,⁴ C.T. Sachrajda,⁴ A. Soni,⁶ and D. Zhang² (1 7) (The RBC and UKQCD Collaborations)

Theoretical Challenges ΔS = 1 Processes

Usual Suspects: pion ma	ass, lattice spacing, lattice volume	underway
Additional Challenges:	Physical Kinematics	underway
	Multi-Hadron States and Normalization	\checkmark
	Operator Renormalization & Scale Invarian	ce 🗸
	Statistically Noisy Operator Self-Contractions	\checkmark
Can such success	carry over to weak nuclear processes?	

Dirtiest Corner of Standard Model



Example: $N \rightarrow (N\pi)_s$ and $\Delta I = 1$ Parity Violation

• Old Problem: hadronic neutral weak interaction is the least constrained SM current



• Theoretical Challenges $\Delta I = 1$ Processes

Usual Suspects: pion ma	to be done	
Additional Challenges:	Physical Kinematics	largely solved
	Multi-Hadron States and Normalization	to be done
	to be done	
	Statistically Noisy Operator Self-Contractions	to be done

How many lattice advances carry over to weak nuclear processes?

Particle Physics (B=0) vs. Nuclear Physics (B>0)

Statistical nature of lattice QCD two-point correlation functions (*Parisi, Lepage*)

Pion Correlation Function

Signal

Signal
$$\sum_{\{A_{\mu}\}} \langle q\overline{q}(t)q\overline{q}(0) \rangle \sim e^{-m_{\pi}t}$$
Signal/NoiseNoise^2
$$\sum_{\{A_{\mu}\}} \langle q\overline{q}(t)q\overline{q}(t)q\overline{q}(0)q\overline{q}(0) \rangle \sim e^{-2m_{\pi}t}$$
 $\sim \text{const}$

Nucleon Correlation Function

Signal
$$\sum_{\{A_{\mu}\}} \langle qqq(t) \overline{qqq}(0) \rangle \sim e^{-Mt}$$

Noise^2
$$\sum_{\{A_{\mu}\}} \langle qqq(t) \overline{qqq}(t) qqq(0) \overline{qqq}(0) \rangle \sim e^{-3m_{\pi}t}$$

Baryons are statistically noisy.... nuclear physics has an extra hurdle

Higher statistics **Optimal operators**

 $\sim e^{-(M-\frac{3}{2}m_{\pi})t}$

Signal/Noise

(Un)Physical Kinematics in $N \rightarrow (N\pi)_s$

Lattice states are created on-shell

$$G(\tau) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle N(\vec{x},\tau)N^{\dagger}(0,0) \rangle = Z e^{-\sqrt{\vec{p}^2 + M_N^2} \tau} + \cdots \text{ ground-state saturation}$$

Hadronic transition matrix elements have energy insertion

$$E_N = M_N$$

$$E_{(\pi N)_s} = M_N + m_{\pi}$$

$$\langle (\pi N)_s | \mathcal{O}_i(\mu) | N \rangle_{\text{Lattice}} = h_{\pi NN}^1(\Delta E)$$

• Partial solution implemented (due to Beane, Bedaque, Parreno, Savage, NUPHA:747, 55 (2005))

Consequence: remove via chiral extrapolation but then only can determine chiral limit coupling Likely small: only~10% at 400 MeV pion mass. Precision demands in nuclear physics typically not as great as particle physics

• Full solution: determine energy dependence, extrapolate to zero, e.g. TwBCs

Example: $N \rightarrow (N\pi)_s$ and $\Delta I = 1$ Parity Violation

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How many lattice advances carry over to weak nuclear processes?

Multi-Hadron States and Normalization



• Finite volume and infinite volume states have different normalizations

Lellouch, Lüscher, Commun. Math. Phys. 291, 31 (2001)

Computed

Lellouch-Lüscher factor requires two-particle energy

Not Computed

Lellouch-Lüscher Factor

• Single Particle Energy Quantization: $E = \sqrt{\vec{p}^2 + M^2}$ $\vec{p} = \frac{2\pi}{T}\vec{n}$ $E_{\text{total}} = \sqrt{k^2 + M^2} + \sqrt{k^2 + m^2}$ $\vec{P} = 0$ **Two Particle Energy Quantization:** $n\pi - \delta_0(k) = \phi(k)$ $\rho_V(E) = \frac{dn}{dE} = \frac{\phi'(k) + \delta'(k)}{4\pi k} E$ (known function for a torus) $|2\rangle_{\infty} = 4\pi \sqrt{\frac{V E \rho_V(E)}{l}} \,|2\rangle_V$ **One-to-Two Particle Amplitude:** $|\mathcal{M}_{\infty}|^{2} = \frac{8\pi V^{2} M E_{\text{total}}^{2}}{k^{2}} \left[\delta'(k) + \phi'(k)\right] |\mathcal{M}_{V}|^{2}$ Computed $|(h_{\pi NN}^1)_V|^2$ **Not Computed**

Generalization for energy insertion:

Lin, Martinelli, Pallante, Sachrajda, Villadoro **NuPhB:**650, 301 (2003) Kim, Sachrajda, Sharpe **NuPhB:**727, 218 (2005)

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Tree Level





Tree Level

$$\mathcal{L}_{\mathrm{PV}}^{I=1} = \sum_{i} C_i(\mu) \mathcal{O}_i(\mu)$$

 $\sin^2 \theta_W$ Non-Strange

1 vs. Strange

One Loop Results

 $C_i(\mu = 1 \, \text{GeV}) \, / \, C_1^{\text{Tree}}$

(Fierz)

LO

0.264

0.981

-0.592

0

5.97

-2.30

5.12

-3.29

TO

	l	LO
$O_1 = (\bar{u}u - dd)_A (\bar{u}u + dd)_V,$ $O_1 = (\bar{u}u - \bar{d}d) [\bar{u}u + \bar{d}d]$	1	0.403
$O_2 = (uu - aa]_A [uu + aa)_V,$ $O_2 = (\bar{u}u - \bar{d}d)_2 (\bar{u}u + \bar{d}d)_3$	2	0.765
$O_3 = (uu - du)_V (uu + du)_A,$ $O_4 = (\bar{u}u - \bar{d}d)_V [\bar{u}u + \bar{d}d)_A,$	3	-0.463
$O_4 = (uu uu_{JV} [uu + uu)_A,$	4	0
$O_5 = (\overline{u}u - \overline{d}d)_A(\overline{s}s)_V$	5	5.61
$O_6 = (\overline{u}u - \overline{d}d)_A [\overline{s}s)_V$	6	-1.90
$O_7 = (\overline{u}u - \overline{d}d)_V(\overline{s}s)_A$	7	4.74
$O_8 = (\overline{u}u - \overline{d}d]_V[\overline{s}s)_A$	8	-2.67

•

• Discrepancies

DSLS provide only ratios $\alpha_s(m_c)/\alpha_s(m_b) = 1.44$

Using their ratios, I get their values

No heavy quark masses quoted in 1990 **PDG**

Dia, Savage, Liu, Springer PLB **271**, 403 (1991)

Tiburzi, PRD 85 054020 (2012)

Tree Level

$$\mathcal{L}_{\mathrm{PV}}^{I=1} = \sum_{i} C_i(\mu) \mathcal{O}_i(\mu)$$

 $\sin^2 \theta_W$ Non-Strange

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One Loop Results

 $C_i(\mu = 1 \, \mathrm{GeV}) \, / \, C_1^{\mathrm{Tree}}$

	ı	LO	LO	LO. 1992 PDG
$O_1 = (\bar{u}u - \bar{d}d)_A(\bar{u}u + \bar{d}d)_V,$	1	0.403	0.264	0.54(4)
$O_2 = (\bar{u}u - dd]_A [\bar{u}u + dd)_V,$	2	0.765	0.981	0.55(6)
$O_3 = (uu - dd)_V (uu + dd)_A,$ $O_4 = (\bar{u}u - \bar{d}d) [\bar{u}u + \bar{d}d)$	3	-0.463	-0.592	-0.35(3)
$O_4 = (uu - aa]_V [uu + aa)_A,$	4	0 (Fie	erz) O	0
$O_{5} = (\overline{u}u - \overline{d}d) \sqrt{(\overline{s}s)}_{V}$	5	5.61	5.97	5.35(7)
$O_6 = (\overline{u}u - \overline{d}d)_A (\overline{s}s)_V$	6	-1.90	-2.30	-1.57(10)
$O_7 = (\overline{u}u - \overline{d}d)_V (\overline{s}s)_A$	7	4.74	5.12	4.45(8)
$O_8 = (\overline{u}u - \overline{d}d]_V [\overline{s}s)_A$	8	-2.67	-3.29	-2.12(15)

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Tiburzi, PRD 85 054020 (2012)

$\mathcal{A} = \sum C_i(\mu) \langle (\pi N)_s | \mathcal{O}_i(\mu) | N \rangle$



(11)

(12)

(13)

QCD Renormalization of Isovector Parity Violation

Results ('t Hooft-Veltman scheme)

$$\mathcal{L}_{\mathrm{PV}}^{I=1} = \sum_{i} C_i(\mu) \mathcal{O}_i(\mu)$$

 $\Delta I = 1$

Alleged: 95% probe of hadronic neutral current			$C_i(\mu=1{ m GeV})$	$) / C_1^{\text{Tree}}$		
		i	LO	LO	NLO (Z)	NLO $(Z + W)$
		1	0.403	0.264	-0.054	-0.055
$\sin^2 heta_W$	Non-Strange	2	0.765	0.981	0.803	0.810
		3	-0.463	-0.592	-0.629	-0.627
		4	0 (Fierz) 0	(Fierz) (Fierz)	0
	VS.	5	5.61	5.97	4.85	5.09
		6	-1.90	-2.30	-2.14	-2.55
1	Strange	7	4.74	5.12	4.27	4.51
80 - 100%		8	-2.67	-3.29	-2.94	-3.36
Dynamical	Question!					

Tiburzi, PRD 85 054020 (2012)

QCD Renormalization of Isovector Parity Violation

Results ('t H	Hooft-Veltmai	n sc	heme)	Ĺ	$\mathcal{L}_{\mathrm{PV}}^{I=1} = \sum_{i} \mathcal{L}_{i}$	$C_i(\mu)\mathcal{O}_i(\mu)$
	$\Delta I = 1$		Additi chiral (conse	onal finding: basis reveals o quence of nor	only 5 independent n-singlet chiral sym	operators metry)
						$L\otimes L-R\otimes R$
Alleged: 95%	6 probe of		$C_i(\mu = 1$	$\operatorname{\tt GeV})/C_1^{\operatorname{Tre}}$	e	$L\otimes R-R\otimes L$
hadronic net	utral current	i	LO	LO	NLO (Z	NLO (Z + W)
		1	0.403	0.26	4 -0.054	4 -0.055
$\sin^2 heta_W$	Non-Strange	2	0.765	0.98	1 0.803	0.810
		3	-0.463	-0.59	-0.629	-0.627
	VS	4	0	(Fierz) 0	(Fierz) 0	(Fierz) 0
	v 3 .	5	5.61	5.97	4.85	5.09
		6	-1.90	-2.3	0 -2.14	-2.55
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80 - 100%		8	-2.67	-3.2	.9 -2.94	-3.36
Dynamical Question!				Tibu	ırzi, PRD 85 054020	(2012)



computable in pQCD at high scale

computable on lattice at low scale

• Scale Invariance: requires same renormalization scheme

pQCD 't Hooft-Veltman scheme

5 independent PV operators in chiral basis

Anisotropic Lattice Regularization + Wilson Fermions

14 independent PV operators

Unphysical + unphysical chiral mixing

• Matching calculation required...

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Statistically Noisy Operator Self-Contractions

 $G(\tau',\tau) = \langle 0|N(\tau')\mathcal{O}_i(\tau)N^{*\dagger}(0)|0\rangle$





Another notorious difficulty



quark disconnected diagrams

Vector and Axial-Vector self-contractions

Wilson coeffs.

Flavor dependence? ~ \mathcal{M}_q Extend to SU(3) + chiral corrections?

Utilize Fierz redundancy?

 $\overline{s}s$ $\overline{s}\gamma_{\mu}s$ small nucleon strangeness

$$\langle \overline{s}\gamma_{\mu}s \rangle \ll \langle \overline{q}\gamma_{\mu}q \rangle?$$

0.16 from Adelaide

Isotensor Parity Violation $\mathcal{O} = (\overline{q}\tau^3 q)_A (\overline{q}\tau^3 q)_V - \frac{1}{3} (\overline{q}\vec{\tau} q)_A \cdot (\overline{q}\vec{\tau} q)_V$

• Only one operator & without self-contractions

$$\mathcal{L}_{PV}^{\Delta I=2} = \frac{G_F}{\sqrt{2}} C(\mu) \mathcal{O}(\mu)$$

Operator Renormalization

Tiburzi, PRD86: 097501 (2012)

LO	$C(1{ m GeV})/C^{(0)}$]
LO [15]	0.79	1992 PDG
LO	0.70	0.78(1)
NLO	$C(1{ m GeV})/C^{(0)}$]
't Hooft-Veltman	0.58	
Naïve Dim. Reg.	0.74	
RI/MOM	0.77	
$\operatorname{RI}/\operatorname{SMOM}(\gamma_{\mu}, q)$	0.67	
$\operatorname{RI/SMOM}(\gamma_{\mu}, \gamma_{\mu})$	0.75	
RI/SMOM(q, q)	0.73	
$RI/SMOM(q, \gamma_{\mu})$	0.81	

[15] Kaplan Savage, NuPhA 556 (1993)

Wilson fermions still to do...

Better proving ground for Lattice QCD?

$$\mathcal{L}_{NN} = [\vec{\nabla} p^{\dagger} \cdot \vec{\sigma} \, \sigma_2 \, p^*] \cdot [n^T \sigma_2 \, n] + \dots$$

s- to p-wave NN interaction

Operator matrix element between 2 hadrons (one step beyond multi-hadron calculations done)

 πN interactions

 $\mathcal{L}_{\pi\pi N} + \mathcal{L}_{\pi\gamma N}$ External fields could ``substitute'' for pions

πPV

Isotensor 3 pion interaction exists

Easier for lattice compute parameters in DDH model?

Summary

- Lattice QCD: Wilsonian machinery turns high-scale interactions (both SM & *Beyond*) into QCD-scale hadronic couplings
- After decades of dedicated work, trustworthy results emerging e.g. $K \rightarrow \pi\pi$
- Some of this success will carry over to weak nuclear processes!

Challenges = Opportunity

• Hadronic Parity Violation:

 π N-coupling more or less challenging than K $\rightarrow \pi \pi$? Use external axial fields for coupling to pions? Develop technology for isotensor NN-interaction? Isovector parity-violating lattices from auxiliary fields?