Non-perturbative modeling of neutron stars

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based on work in collaboration with

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General idea:

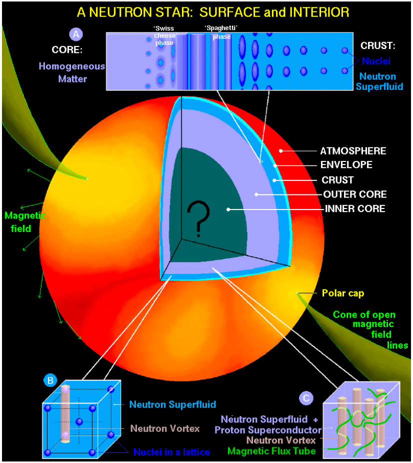
- Use a solitonic model of nuclear matter (the "BPS Skyrme model") to
  - model properties of (nuclei and) neutron stars (NS)
  - study the difference in NS properties between mean field (MF) theory and full field theory calculations

References: arXiv:1503.03095
Phys. Lett. B742 (2015) 136
Phys. Rev. D90 (2014) 045003

Contents

Neutron Stars

- Neutron Star Structure from Nucl. Phys.
- Dany Page, http://www.astroscu.unam.mx/neutrones/NS-Picture/NStar/



Neutron Stars: theoretical description

- Self-gravitating nuclear (etc.) matter $\Rightarrow$ Einstein equations
  - Tolman-Oppenheimer-Volkoff (TOV) approach
  - Perfect fluid em-tensor

$$T^{\rho\sigma} = (p+\rho)u^\rho u^\sigma - pg^{\rho\sigma}$$

  - Spherically sym. metric

$$ds^2 = \mathbf{A}(r)dt^2 - \mathbf{B}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

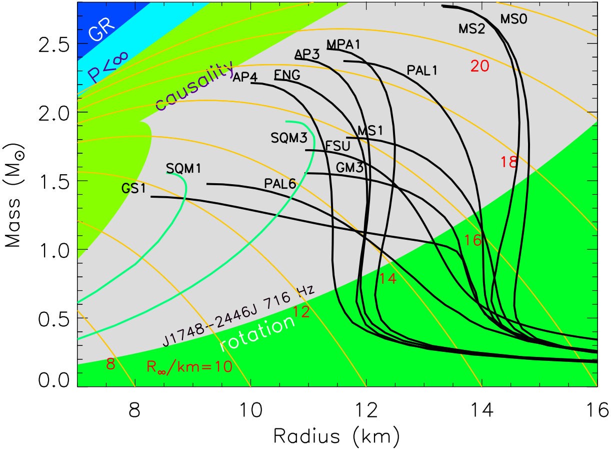
  - into Einstein eqs. $G_{\rho\sigma} = \frac{\kappa^2}{2}T_{\rho\sigma}$ $\Rightarrow$ TOV equations

$$\begin{aligned}
\text{TOV1}: \quad M' &= 4\pi r^2\rho, \quad \mathbf{B}(r) \equiv \left(1 - \frac{\kappa^2}{8\pi}\frac{M(r)}{r}\right)^{-1}, \\
\text{TOV2}: \quad rp' &= (\rho+p)\left(\frac{1}{2}(1-\mathbf{B}) - \frac{\kappa^2}{4}r^2\mathbf{B}p\right. \\
\left(\frac{\mathbf{A}'}{\mathbf{A}}\right. &= \left.\left.\frac{1}{r}(\mathbf{B}-1) + \frac{\kappa^2}{2}r\mathbf{B}p\right)\right)
\end{aligned}$$

- To close system: equation of state (EoS) $\rho = \rho(p)$
- Then: integrate TOV & $\rho(p)$ from $r = 0$, $\rho(r = 0) \equiv \rho_0$ to $r = R$ where $p(R) = 0$; $M = M(R) \to$ Fig.

- EoS: from some model of nuclear matter
  - Problem: normally neither perfect fluid nor algebraic EoS
  - $\Rightarrow$ approximation ... mean field (MF) theory
  - Example: Walecka model (QHD): EFT of nucleons Ψ_n, mesons (nucl. forces) σ, ω^μ etc.
  - MF: $\sigma, \omega^\mu \to$ const.; thermal PI over Ψ_n $\Rightarrow$ partition function $Z(V, T, c.c) \Rightarrow$ EoS ($T \to 0$)
  - How "good" is MF theory? ... difficult. How to go beyond?
  - Worse with gravity? (metric couples to derivatives)

- *M* vs. *R* from various Nucl. Phys. EoS
- J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485



Skyrme models

- Degrees of freedom of QCD:
  - high energy: quarks and gluons
  - low energy: hadrons
- $\Rightarrow$ low energy effective field theory (EFT) of hadrons (currently unknown)
- Supported by large N_c: QCD = EFT of mesons
- One proposal: Skyrme model
  - primary fields: mesons
  - baryons and nuclei realized as top. solitons ("vortices" in "meson fluid")
  - simplest case (two flavors): target space = SU(2) (isospin) matrix U (three pions)
  - topological charge = baryon number B

Original Skyrme model

$$\mathcal{L} = \underbrace{\mathcal{L}_2 + \mathcal{L}_4}_{\mathcal{L}_{skyrme}} + \mathcal{L}_0 \ , \qquad \mathcal{L}_0 = -\lambda_0 \mathcal{U}(\mathrm{Tr}\, U) \ldots \text{potential}$$

$$\text{e.g. } \mathcal{U}_\pi = \tfrac{1}{2}\mathrm{Tr}(1-U) \qquad \text{vac}$$

$$\mathcal{L}_2 = -\lambda_2 \ \mathrm{Tr}\ (L_\mu L^\mu), \quad \mathcal{L}_4 = \lambda_4 \ \mathrm{Tr}\ ([L_\mu, L_\nu]^2), \quad L_\mu = U^\dagger \partial_\mu U$$

- Description of nuclei:
  Some successes: (iso-) spin excitational spectra
- Main problems:
  - too large binding energies: $\exists$ topological energy bound $E \geq cB$, but not saturated (non-BPS theory)
    $\Rightarrow$ generalize to (near) BPS theory
  - Large B: crystals (not liquid)

Generalizations

- Poincare invariance & standard Hamiltonian (quadratic in time derivatives): quite restrictive

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_0 + \mathcal{L}_6$$

$$\mathcal{L}_6 = -(24\pi^2)^2 \lambda_6 \mathcal{B}_\mu \mathcal{B}^\mu, \quad \mathcal{B}^\mu = \frac{1}{24\pi^2} \mathrm{Tr}\, (\epsilon^{\mu\nu\rho\sigma} L_\nu L_\rho L_\sigma)$$

$\mathbb{B}_\mu \ldots$ baryon current with baryon number $B = \int d^3 x \mathcal{B}_0$

- $\exists$ BPS? YES: $\mathcal{L}_0 + \mathcal{L}_6$

BPS Skyrme model

- Energy functional

$$E_{06} = \int d^3x(\mu^2\mathcal{U} + \lambda^2\pi^4\mathcal{B}_0^2) \equiv \int d^3x\mathcal{E}_{06}$$

- BPS bound and equation

$$E_{06} \geq 2\pi^2\lambda\mu|B|\langle\sqrt{\mathcal{U}}\rangle, \quad \mathcal{B}_0 = \pm\frac{\mu}{\pi^2\lambda}\sqrt{\mathcal{U}}$$

  - BPS solutions $\forall B$ comp
  - static energy: ∞ many base space sym. (SDiff($\mathbb{R}^3$))
    $\Rightarrow$ solutions with arbitrary shapes, same vol
  - e-m-tensor of a perfect fluid. Static case:

$$T^{00} = \rho, \qquad \rho = \lambda^2\pi^4\mathcal{B}_0^2 + \mu^2\mathcal{U},$$
$$T^{ij} = p\delta^{ij}, \qquad p = \lambda^2\pi^4\mathcal{B}_0^2 - \mu^2\mathcal{U}$$

Thermodynamics and MF

- $\partial_\mu T^{\mu\nu} = \delta^{ij}\partial_i p = 0 \;\Rightarrow\; p \equiv \lambda^2\pi^4\mathcal{B}_0^2 - \mu^2\mathcal{U} = P = \text{const.}$

$$\boxed{\mathcal{B}_0 = \pm\frac{\mu}{\pi^2\lambda}\sqrt{\mathcal{U} + P/\mu}\,,\;\; \rho| = 2\mu^2\mathcal{U}| + P}$$

- Energy ($\forall$ solutions)

$$\boxed{E(P) = BE_1(P) = B\pi^2\lambda\mu\left\langle\frac{2\mathcal{U} + P/\mu}{\sqrt{\mathcal{U} + P/\mu}}\right\rangle}$$

- Volume (EoS)

$$\boxed{V(P) = BV_1(P) = B\pi^2\frac{\lambda}{\mu}\left\langle\frac{1}{\sqrt{\mathcal{U} + P/\mu}}\right\rangle}$$

- Standard thermodyn. relation

$$\boxed{P = -\frac{\partial E}{\partial V}}$$

- $\rho| = \rho(x, P)$ NOT (algebraic) EoS (except for $\mathcal{U}_\Theta = \Theta(\mathrm{Tr}(1 - U)))$
- MF: average (constant) density

$$\boxed{\bar{\rho} = \bar{\rho}(P) = \frac{E(P)}{V(P)}}$$

- MF-EoS $\bar{\rho}(P)$:
  large P: $\bar{\rho} \sim P + \bar{\rho}_\infty$ (maximally stiff)
  small P: $\bar{\rho} \sim \bar{\rho}_0 + f(P)$ where $f(P) \sim P^{\frac{1}{2}}, \sim P \ln P$ (soft)
- average baryon density

$$\boxed{\bar{n}_b(P) = \frac{B}{V(P)}}$$

- $\bar{\rho}$ to be used in MF TOV calculations
- Limiting case $\mathcal{U}_\Theta$: $\rho| = \bar{\rho}(P) = \text{const.} \Rightarrow$ MF = exact

BPS Skyrme model and nuclear matter

- perfect fluid e-m tensor & SDiff ... field theory realization of liquid droplet model of nuclear matter
- (classical) BPS skyrmions: zero binding energies; small corrections (e.g. Semiclass. quantization & Coulomb energies: good description of nuclei)
- $\forall B$: finite V (finite ρ) solutions with $P = 0$ (BPS sol.) ... nuclear saturation
- Macroscopic (thermodyn) properties encoded in microscopic (field theory) properties: no thermodyn limit needed
- Further improvement: Near BPS Skyrme model

$$\mathcal{L} = \epsilon(\mathcal{L}_2 + \mathcal{L}_4 + \tilde{\mathcal{L}}_0) + \underbrace{\mathcal{L}_0 + \mathcal{L}_6}_{\mathcal{L}_{BPS}}$$

complicated numerics

BPS Skyrmions as neutron stars

- Mean field theory
  - EoS $\bar{\rho}(p)$ & TOV1 & TOV2
- Exact: BPS Skyrme model coupled to gravity
  - Matter action

$$S_{06} = \int d^4x |g|^{\frac{1}{2}} \left(-\lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma - \mu^2 \mathcal{U} \right)$$

  - Energy-momentum tensor (perfect fluid)

$$T^{\rho\sigma} = -2|g|^{-\frac{1}{2}} \frac{\delta}{\delta g_{\rho\sigma}} S_{06} = (p+\rho) u^\rho u^\sigma - p g^{\rho\sigma}$$

4-velocity

$$u^\rho = \mathcal{B}^\rho / \sqrt{g_{\sigma\pi} \mathcal{B}^\sigma \mathcal{B}^\pi}$$

energy density and pressure

$$\begin{aligned} \rho &= \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma + \mu^2 \mathcal{U} \\ p &= \lambda^2 \pi^4 |g|^{-1} g_{\rho\sigma} \mathcal{B}^\rho \mathcal{B}^\sigma - \mu^2 \mathcal{U} \end{aligned}$$

- In general $\rho(x)$, $p(x)$ ... $p \neq p(\rho)$ ... NO universal EoS.
- Axially sym. ansatz for baryon number B Skyrme field $U(x) = e^{i\xi(x)\vec{n}(x)\cdot\vec{\tau}} \quad (h \equiv (1/2)(1 - \cos\xi))$

$$h = h(r), \quad \vec{n} = (\sin\theta\cos B\phi, \sin\theta\sin B\phi, \cos\theta)$$

$\Rightarrow \rho(r)$, $p(r)$, $\mathcal{B}^0(r)$ & radially symmetric metric

- Compatible with Einstein eq. $G_{\rho\sigma} = \frac{\kappa^2}{2} T_{\rho\sigma}$
  $\Rightarrow$ TOV1 & TOV2 where now

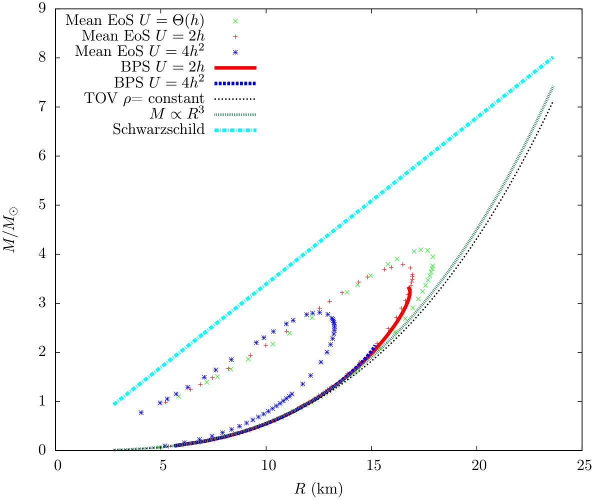
$$\begin{matrix}\rho \\ p\end{matrix} = \frac{4B^2\lambda^2}{\mathbf{B}r^4} h(1-h)h'^2 \pm \mu^2\mathcal{U}(h)$$

  are NOT independent
- Two eq. for **B** and h ... no need for EoS
- Now: fit λ, μ to infinite nuclear matter:
  $E/B = E_{\mathrm{n}} - E_{\mathrm{b}} = (939.6 - 16.3)\,\mathrm{MeV} = 923.3\,\mathrm{MeV}$
  saturation density $B/V = n_0 = 0.153\ \mathrm{fm}^{-3}$

- Integrate: "shooting from center" from $r = 0$ to $r = R$
  $r = 0$: $h(0) = 1$ (anti-vac), $\mathbf{B}(0) = 1$ (no enclosed matter), one free constant $\rho(0)$
  $r = R$: $h(R) = 0$ (vacuum) and $p'(R) \equiv 0$ (one condition)
  $\Rightarrow$ different solutions ... different B
- Solutions exist up to maximum value of B or $n = B/B_\odot$

- Potentials: $\mathcal{U} = \Theta(h), \mathcal{U}_\pi = 2h$ and $\mathcal{U}_\pi^2 = 4h^2$
- Compare exact and MF results $\Rightarrow$ Figs.
- E.g., $M_{\mathrm{max,MF}} \geq M_{\mathrm{max,exact}}$
- Big differences for local quantities $(\rho(r), \mathbf{B}(r), \ldots)$
- Compare results and observations
- E.g., $M_{\mathrm{max}} \sim 2M_\odot$ established, indications for $M_{\mathrm{max}} \sim 2.5M_\odot$;
  10 km $< R_{\mathrm{max}} < 20$ km

- Neutron star mass vs. neutron star radius

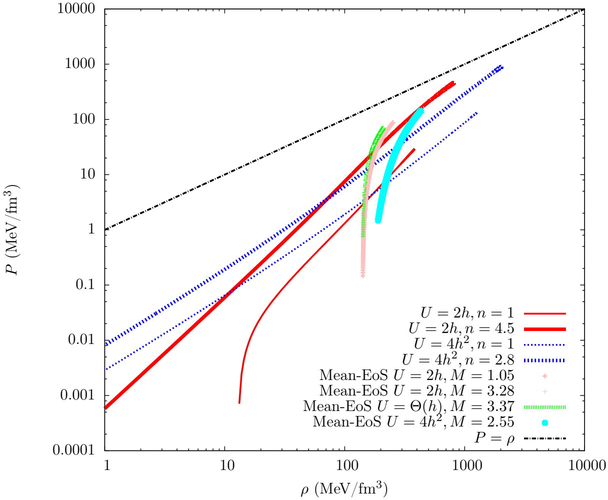


- EoS
  - typically $\partial M/\partial R > 0 \ldots$ stiff EoS
- On-shell EoS: solution $\rho(r), p(r) \;\Rightarrow\; \rho(p)$
  - MF: on-shell EoS $\equiv$ MF-EoS
  - exact: NO (algebraic) off-shell EoS
  - On-shell EoS NOT universal (solution-dependent):

$$\text{numerically} \qquad p(\rho) = a(B)\rho^{b(B)}$$

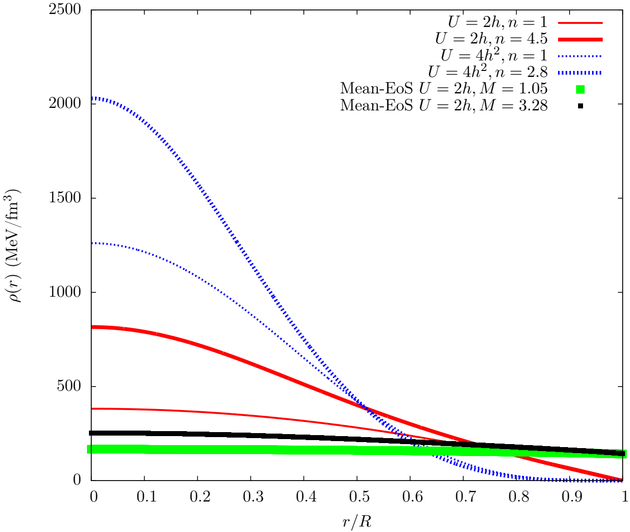
    EoS of polytrope but $a = a(B)$, $b = b(B)$
  - Concretely, stiffer for larger B (larger M, i.e., stronger gravity)

- On-shell EoS

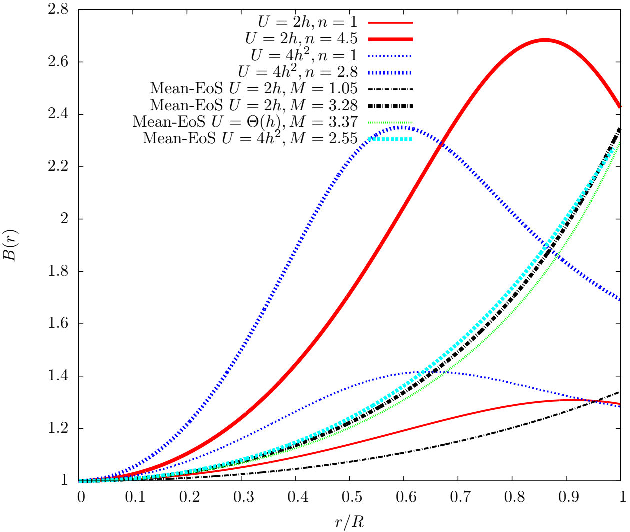


10000
1000
100
10
1
0.1
0.01
0.001
0.0001
1
10
100
1000
10000
P (MeV/fm^3)
ρ (MeV/fm^3)
$U = 2h, n = 1$
$U = 2h, n = 4.5$
$U = 4h^2, n = 1$
$U = 4h^2, n = 2.8$
Mean-EoS $U = 2h, M = 1.05$
Mean-EoS $U = 2h, M = 3.28$
Mean-EoS $U = \Theta(h), M = 3.37$
Mean-EoS $U = 4h^2, M = 2.55$
$P = \rho$

- $\rho(r)$ and $\bar{\rho}(r)$



- **B**(r)


$U = 2h, n = 1$
$U = 2h, n = 4.5$
$U = 4h^2, n = 1$
$U = 4h^2, n = 2.8$
Mean-EoS $U = 2h, M = 1.05$
Mean-EoS $U = 2h, M = 3.28$
Mean-EoS $U = \Theta(h), M = 3.37$
Mean-EoS $U = 4h^2, M = 2.55$
$B(r)$
r/R
2.8
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

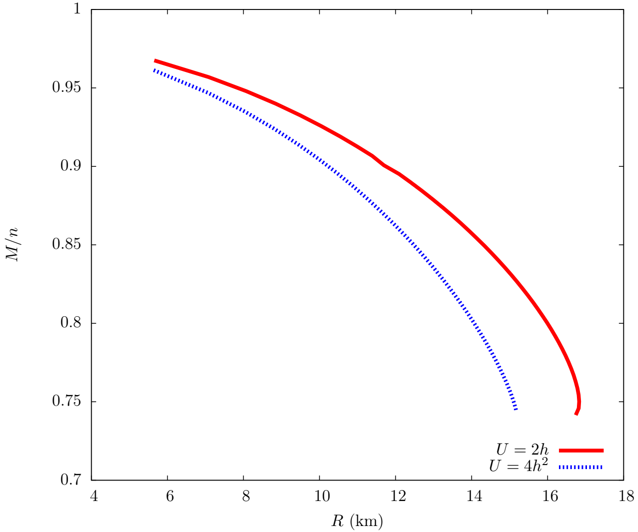
Summary

- Exact vs. MF
  - BPS Skyrme model allows to study this difference
  - Differences can be considerable (e.g., $M_{\max,\mathrm{MF}} \geq M_{\max,\mathrm{exact}}$)
  - Big differences for local quantities
  - ¿Big difference e.g. for mom. of inertia?
  - ¿Necessary to go beyond MF also in other models of NS?
    . . . Difficult

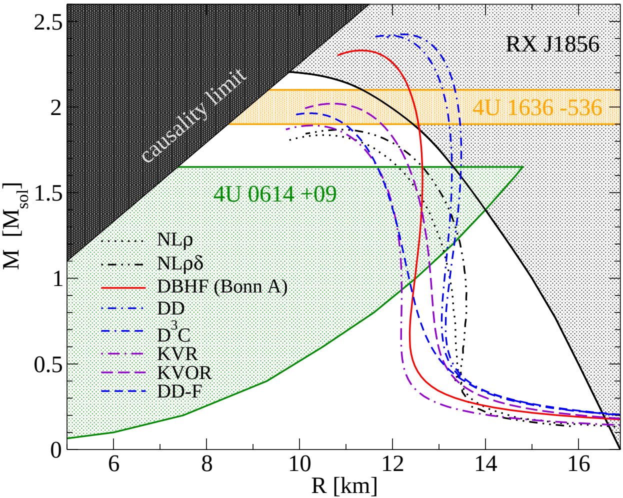
- BPS Skyrmions as NS
  - BPS Skyrme model produces very good results also for neutron stars (e.g., $M_{\max}$ and $R_{\max}$)
  - Rather stiff (effective) EoS $\Rightarrow M(R)$ is monotonously growing function
  - Different from most Nucl. Phys. descriptions with fixed EoS
  - But completely compatible with (not very precise) observational data
  - $dM/dR > 0$ one observational "smoking gun"
  - Precise quantitative predictions still premature:
    ¿which potential? ¿precision of fit values?
    ¿more terms from full near-BPS Skyrme model?

Backup

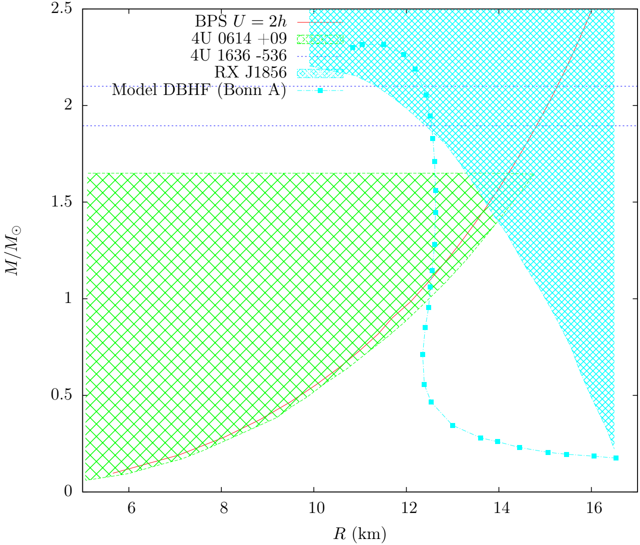
- Gravitational (and binding) mass loss



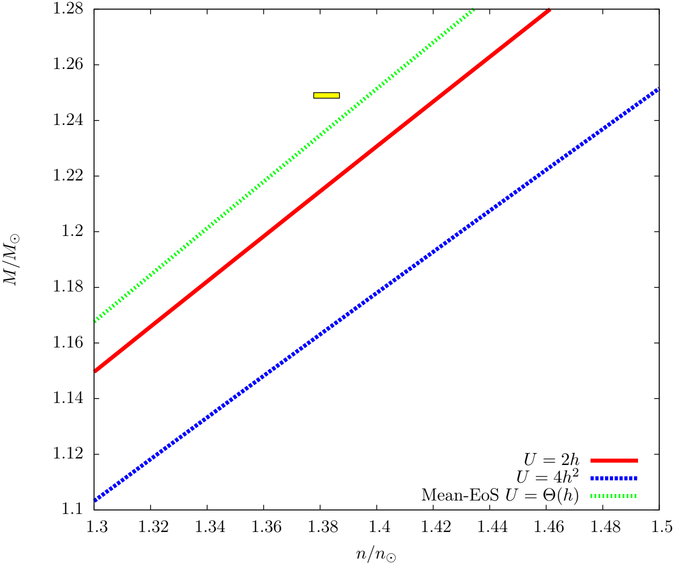
- T. Klähn et al, Phys. Rev. C**74** (2006) 035802.



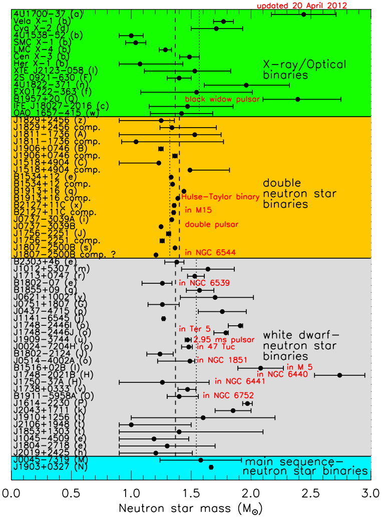
- T. Klähn et al vs. BPS Skyrme model



- T. Klähn et al, Phys. Rev. C**74** (2006) 035802.



- Observed Neutron Star Masses
- J. M. Lattimer, Ann. Rev. Nucl. Part. Sci. 62 (2012) 485



- *M* and *R* from Photospheric Radius Expansion (PRE) bursts
- J. M. Lattimer, A. W. Steiner, Eur. Phys. J. A50 (2014) 40

Table 3. PRE X-ray burster solutions resulting from Monte Carlo trials with parameters taken from their uncertainty intervals. Only solutions with real values of R are accepted; the fraction of Monte Carlo acceptances is shown in the last column.

PRE Source	α	γ km	R_∞ km	R km	M $M_\odot$	acceptance %
			$z_{\rm ph} = z$			
EXO 1745-248	0.117 ± 0.006	109.0 ± 14.2	12.77 ± 1.62	9.11 ± 1.55	1.45 ± 0.28	4.87
4U 1608-522	0.115 ± 0.010	110.8 ± 16.4	12.73 ± 2.22	9.21 ± 1.74	1.41 ± 0.38	0.861
4U 1820-30	0.121 ± 0.004	103.4 ± 7.5	12.48 ± 0.96	8.81 ± 1.04	1.46 ± 0.19	0.0311
KS 1731-260	0.121 ± 0.004	124.5 ± 9.0	15.01 ± 1.03	10.58 ± 1.23	1.76 ± 0.21	1.01
SAX J1748.9-2021	0.116 ± 0.008	132.9 ± 17.4	15.27 ± 1.65	11.05 ± 1.86	1.69 ± 0.33	9.67
			$z_{\rm ph} = 0$			
EXO 1745-248	0.158 ± 0.021	85.35 ± 15.55	13.25 ± 1.67	10.00 ± 1.45	1.42 ± 0.27	66.3
4U 1608-522	0.167 ± 0.020	103.5 ± 16.2	17.20 ± 3.08	12.41 ± 1.98	1.96 ± 0.49	20.7
4U 1820-30	0.173 ± 0.014	87.04 ± 10.39	15.03 ± 1.58	10.63 ± 1.25	1.77 ± 0.25	24.5
KS 1731-260	0.163 ± 0.018	92.29 ± 13.68	14.87 ± 1.21	11.01 ± 1.28	1.64 ± 0.22	59.2
SAX J1748.9-2021	0.154 ± 0.023	102.1 ± 20.7	15.26 ± 1.64	11.70 ± 1.61	1.58 ± 0.30	72.9

z . . . redshift at R; $z_{\rm ph}$ . . . redshift at photosphere

BPS Skyrme model: limit of generalized Skyrme model

$$L_{06} = L_6 + L_0$$

- ∞ many symmetries, ∞ many conservation laws
- BPS (Bogomolny) bound
- ∞ many exact solutions saturating the BPS bound
- Parametrization for U

$$U = e^{i\xi\vec{n}\cdot\vec{\sigma}} = \cos\xi + i\sin\xi\vec{n}\cdot\vec{\sigma} \qquad \vec{n}^2 = 1$$

and stereographic projection

$$\vec{n} = \frac{1}{1+|u|^2}\left(u+\bar{u}, -i(u-\bar{u}), 1-|u|^2\right)$$

$$\Rightarrow \quad L_{06} = -\frac{\lambda^2 \sin^4\xi}{(1+|u|^2)^4}\left(\epsilon^{\mu\nu\rho\sigma}\xi_\nu u_\rho \bar{u}_\sigma\right)^2 - \mu^2 V(\xi)$$

Symmetries

- ∞ many target space diffeomorphisms
  - L_6 is square of pullback of the volume form in target space SU(2) $\sim S^3$,

$$d\Omega = -2i \frac{\sin^2 \xi}{(1+|u|^2)^2} d\xi du d\bar{u}$$

    $\Rightarrow$ has all volume-preserving diffeomorphisms (VPDs) on target space S^3 as symmetries.
  - $L_0 = -\mu^2 V(\xi)$ respects some of them: the ones that act nontrivially only on u, $\bar{u} \Rightarrow$ area-preserving diffeos on target space S^2 spanned by u, but may depend on ξ as a parameter:

$$\xi \to \xi \,, \quad u \to \tilde{u}(u, \bar{u}, \xi) \,, \quad (1+|\tilde{u}|^2)^{-2} d\xi d\tilde{u} d\bar{\tilde{u}} = (1+|u|^2)^{-2} d\xi u d\bar{u}$$

  - ∞ many conservation laws ... integrable

- Poincare Symmetries
- Energy functional for static fields: base space VPDs

$$
\begin{aligned}
E &= \int d^3x \left(\frac{\lambda^2 \sin^4 \xi}{(1+|u|^2)^4} (\epsilon^{mnl} i\xi_m u_n \bar{u}_l)^2 + \mu^2 V(\xi) \right) \\
&\equiv \int d^3x \left(\frac{\lambda^2}{4} \left(M(\xi^a) \epsilon^{mnl} \xi^1_m \xi^2_n \xi^3_l \right)^2 + \mu^2 V(\xi^3) \right)
\end{aligned}
$$

$\xi \equiv \xi^3$, $u \equiv \xi^1 + i\xi^2$, $\quad M \equiv 2\sin^2(\xi^3)\left(1 + (\xi^1)^2 + (\xi^2)^2\right)^{-2}$

Both d^3x and $\epsilon^{ijk}\partial_i\xi\partial_j u\partial_k\bar{u}$ invariant under VPDs on base space $\mathbb{R}^3$. NOT a Noether symmetry.

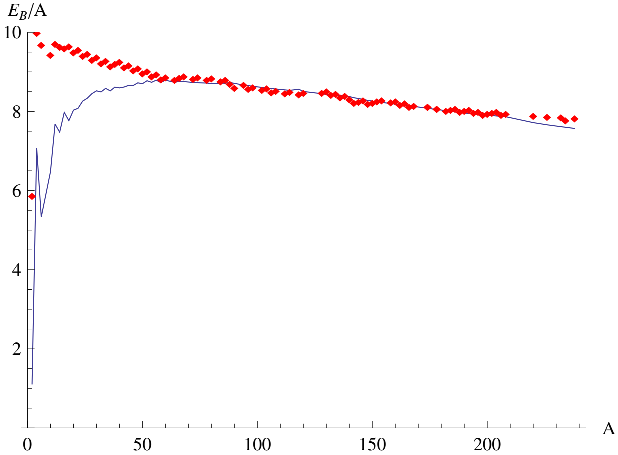
M target volume density: $\quad d\Omega = M d^3\xi$

Bogomolny (BPS) bound

- BPS bound B ... top. charge (baryon number)

$$
\begin{aligned}
E &= \int d^3x \left(\frac{\lambda^2}{4} M^2 (\epsilon^{jkl} \xi^1_j \xi^2_k \xi^3_l)^2 + \mu^2 V(\xi) \right) \\
&= \int d^3x \left(\frac{\lambda}{2} M \epsilon^{jkl} \xi^1_j \xi^2_k \xi^3_l \pm \mu \sqrt{V} \right)^2 \\
&\mp \int d^3x \mu \lambda M \sqrt{V} \epsilon^{jkl} \xi^1_j \xi^2_k \xi^3_l \\
&\geq \pm (2\lambda \mu \pi^2) \left[\frac{1}{2\pi^2} \int d^3x M \sqrt{V} \epsilon^{jkl} \xi^1_j \xi^2_k \xi^3_l \right] \\
&= 2\lambda\mu\pi^2 |B| \frac{1}{2\pi^2} \int_{S^3} d\Omega \sqrt{V(\xi)} \equiv 2\lambda\mu\pi^2 < \sqrt{V} >_{S^3} |B|
\end{aligned}
$$

BPS eq.: $\frac{\lambda}{2\mu} M \epsilon^{jkl} \xi^1_j \xi^2_k \xi^3_l = \mp \sqrt{V}$



Binding energy per atomic mass number A in MeV vs. A, for the most abundant nucleus for each A