A controllability problem for a non convex conservation law

Andrea Marson
University of Padova

Joint work with:

Boris Andreianov (Université de Besançon, moving to Tours)
Carlotta Donadello (Université de Besançon)

Controllability-stabilizability

- Scalar conservation law, convex flux: F. Ancona & A.M. (1998), T. Horsin (1998), M. Chapouly (2009), V. Perrollaz (2012, 2013), Adimurthi-Sundar Ghoshal-Veerappa Gowda (2014), M. Corghi & A.M. (2015, to appear)
- Scalar conservation law, non convex flux: Leautaud (2012)
- Scalar conservation law, non local flux: Coron-Wang (2012)
- Genuinely nonlinear Temple systems: F. Ancona & G. Coclite (2005)
- Genuinely nonlinear systems: A. Bressan & G.M. Coclite (2002), F. Ancona & A.M. (2007), O. Glass (2007, 2013), O. Glass & S. Guerrero (2007), Dick-Gugat-Leugering (2010, 2011)
- Smooth solutions: Li. T.T. & collaborators (book in 2010)

Optimal control

- A. Bressan & A.M. (1995), S. Ulbrich (2003), Coron-Kawski-Wang (2010), Coron-Shang-Wang (2011), Colombo-Herty-Mercier (2011), Leugering-Pfaff-Ulbrich (2014), S. Ulbrich & S. Pfaff (2015)

And others that for sure I am forgetting...

Necessary conditions

Theorem

1. *Let* $\bar{x} \in \mathbb{R}$ *be given and assume that* $v(\bar{x}+) \neq 0$*. Then*

$$\bar{x} - f'(v(\bar{x}+))\, T < b\,. \tag{1}$$

2. *Assume that* $v(\alpha+) \neq 0$*. Then*

$$\alpha - f'\big(v(\alpha+)\big)\, T \geq a\,. \tag{2}$$

3. *Assume that* $v(\alpha+) = 0$*. Then there exists a sequence* $\{x_n\}_{n\in\mathbb{N}}$ *such that*

$$x_n \downarrow \alpha \quad \text{and} \quad x_n - f'(x_n+)\, T > a\,. \tag{3}$$

4. *At points of jumps admissibility conditions are fulfilled.*

Sufficient conditions

Theorem

1. *v fulfills the necessary conditions.*
2. *Let I be a maximal interval where v is continuous. Then either*

$$D_x f'(v(x)) \leq 1/T \quad \forall x \in I \,, \tag{4}$$

or, if

$$J = \left\{ x \in I : D_x f'(v(x)) > 1/T \right\} , \tag{5}$$

then J is a subinterval of I and $\sup J = \sup I = \bar{x}$, point of jump of v.

Sufficient conditions (continued)

Theorem (continued)

Moreover, the following conditions hold.

- *For any* $x \in J$

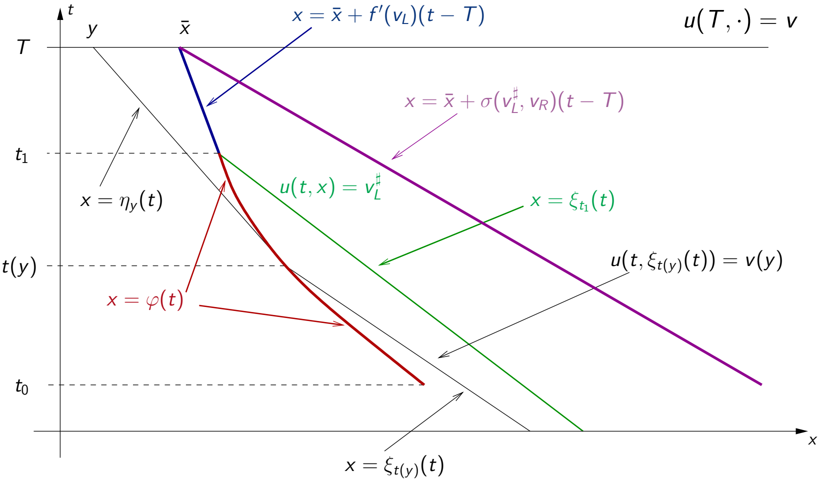
$$D(D_x f'(v(x))) > 0; \tag{6}$$

$$D\left[x + \frac{f'\left(v^\sharp(x)\right) - f'\left(v(x)\right)}{D_x f'(v(x))} - f'\left(v^\sharp(x)\right) T\right] \geq 0\,. \tag{7}$$

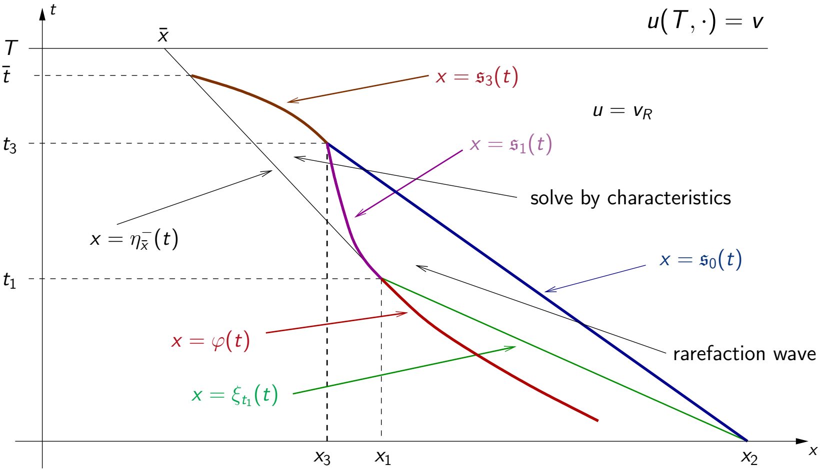
- *If* $0 < |v_L| < |v_R| < \left|(v_L^\sharp)^\sharp\right|$, *there holds*

$$\frac{(v_R - v_L^\sharp)\left[f'(v_L) - f'(v_L^\sharp)\right]}{f(v_R) - f(v_L) - (v_R - v_L)f'(v_L)}\left[\frac{1}{D_x^- f'(v(\bar{x}))} - T\right] \leq T\,. \tag{8}$$

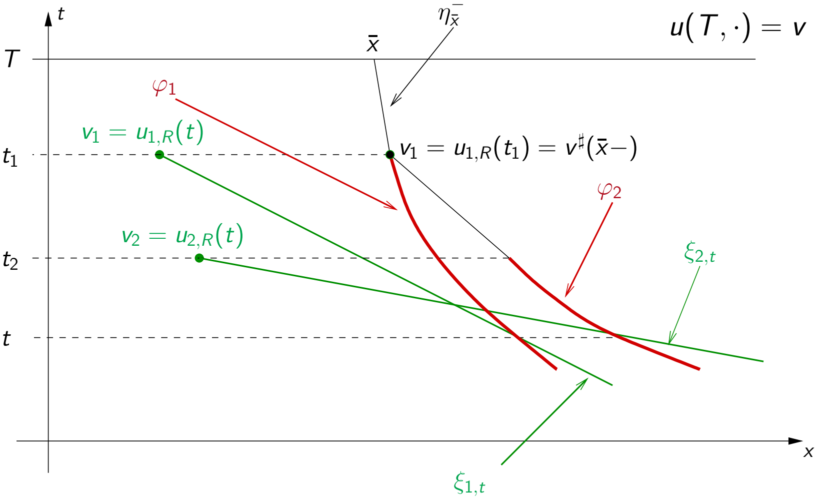
The second building block



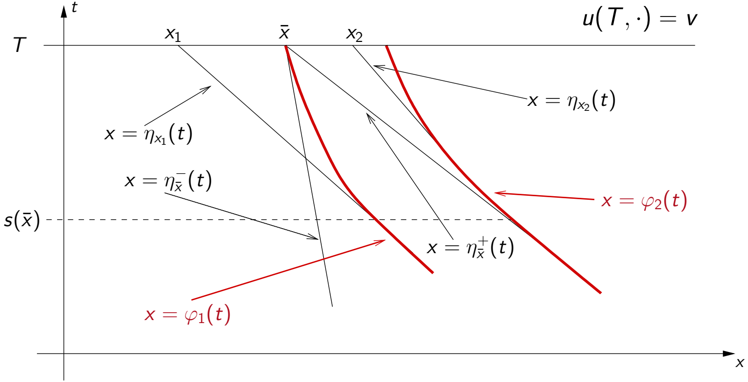
The third building block



A recursive procedure

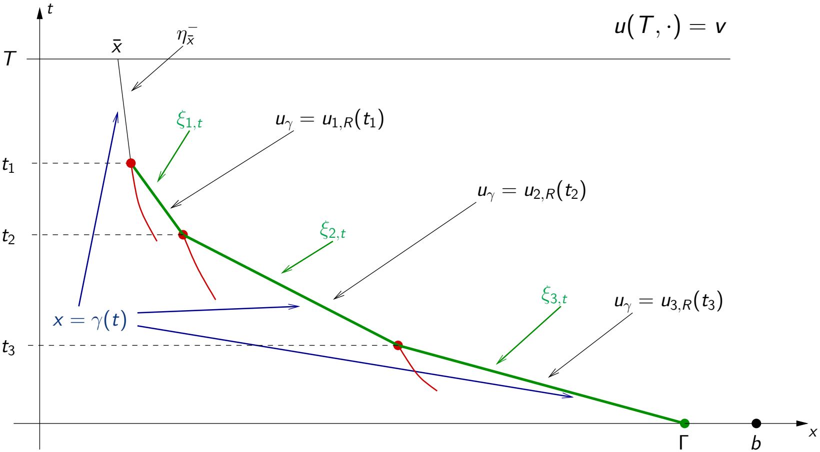


Two consecutive contact discontinuities: a new necessary condition on jumps



$$\lim_{x \to \bar{x}+} D_x f'(v(x)) = \lim_{x \to \bar{x}-} \frac{\big[D_x f'(v(x))\big]^2 D_x f'(v^\sharp(x))}{D^2_{xx} f'(v(x))\big[f'(v_L) - f'(v_R)\big]}$$

An open problem



Thank you for your attention!