Asymptotic Analysis of Periodic Optimal Control Problem

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PARTIAL DIFFERENTIAL EQUATIONS, OPTIMAL DESIGN AND NUMERICS
CENTRO DE CIENCIAS DE BENASQUE PEDRO PASCAL
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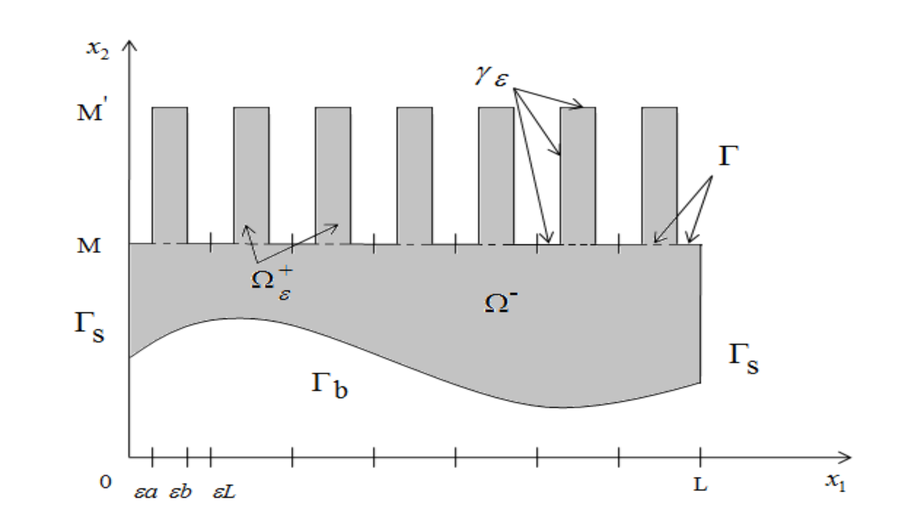
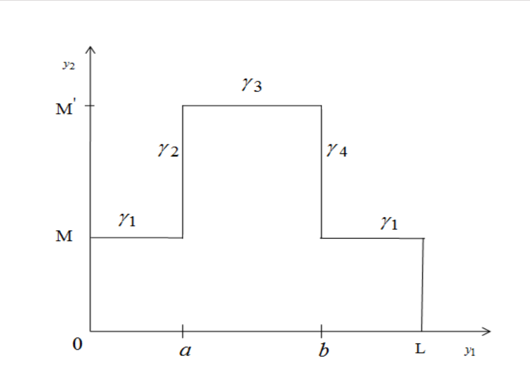

x2
M'
M
Γs
γε
Γ
Ω+ε
Ω-
Γb
Γs
0
εa
εb
εL
L
x1

Reference Boundary



For $\theta \in L^2_{per}(\gamma)$,

$$\theta^\epsilon = (\chi_{\gamma_1} + \epsilon\chi_{\gamma_2} + \chi_{\gamma_3} + \epsilon\chi_{\gamma_4})\,\theta.$$

Define

$$\widetilde{\theta^\epsilon}(x_1, x_2) = \theta^\epsilon\left(\frac{x_1}{\epsilon}, x_2\right).$$

$$\begin{cases} -\Delta u_\epsilon + u_\epsilon = f \quad \text{in } \Omega_\epsilon, \\ \dfrac{\partial u_\epsilon}{\partial \nu} = \widetilde{\theta^\epsilon} \quad \text{on } \gamma_\epsilon. \end{cases} \tag{0.1}$$

Model Periodic Control Problem in Highly Oscillating Boundary Domain

$$\inf\{J_\epsilon(u_\epsilon,\theta) | \theta \in L^2(\gamma), (u_\epsilon, \widetilde{\theta}^\epsilon) \text{ obeys } (0.1)\}$$

where

$$J_\epsilon(u_\epsilon,\theta) = \frac{1}{2}\int_{\Omega_\epsilon} |u_\epsilon - u_d|^2 + \frac{\beta}{2}\int_\gamma |\theta|^2.$$

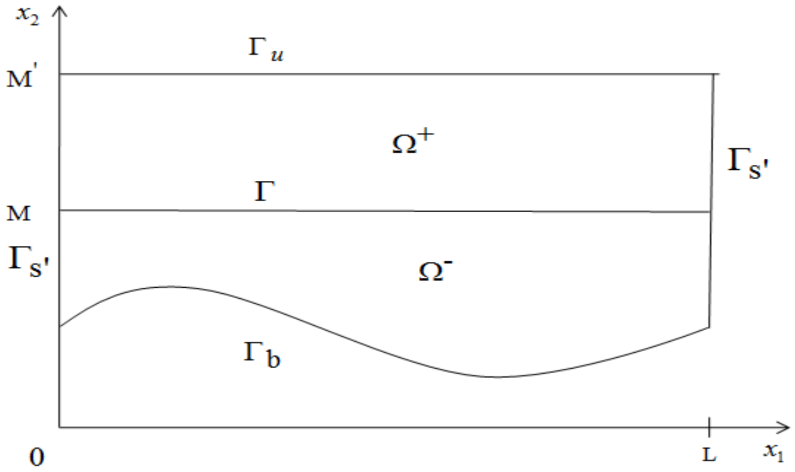
Questions

- Will the optimal control be scaled-periodic?
- Characterization?

Characterization

$$
\begin{aligned}
\overline{\theta}_\epsilon(y_1, y_2) \quad = \quad & -\frac{1}{\beta}\left[\chi_{\gamma_1}\left\{\frac{1}{L}\int_0^L T_1^\epsilon(\overline{v}_\epsilon)(x_1, M, y_1)\, dx_1\right\}\right. \\
& +\chi_{\gamma_2}\left\{\frac{1}{L}\int_0^L T^\epsilon \overline{v}_\epsilon(x_1, y_2, a) dx_1\right\} \\
& +\chi_{\gamma_3}\left\{\frac{1}{L}\int_0^L T_2^\epsilon \overline{v}_\epsilon(x_1, M', y_1) dx_1\right\} \\
& \left. + \chi_{\gamma_4}\left\{\frac{1}{L}\int_0^L T^\epsilon(\overline{v}_\epsilon)(x_1, y_2, b)\, dx_1\right\}\right]
\end{aligned}
$$

Limit Domain



For $\theta \in L^2(M, M')$, C_1 and C_2 in $\mathbb{R}$, consider the system

$$\begin{cases} -\dfrac{\partial^2 u^+}{\partial x_2^2} + u^+ = f + \theta\chi_{\Omega^+} \ \text{ in } \Omega^+, \\ -\Delta u^- + u^- = f \ \text{ in } \Omega^-, \\ \dfrac{\partial u^+}{\partial \nu} = C_1\chi_{(a,b)} \ \text{ on } \Gamma_u, \\ u^+ = u^-, \ \dfrac{\partial u^-}{\partial x_2} - \dfrac{b-a}{L}\dfrac{\partial u^+}{\partial x_2} = C_2\chi_{(0,a)\cup(b,L)} \ \text{ on } \Gamma, \\ u^- = h \ \text{ on } \Gamma_b \end{cases} \tag{0.2}$$

with

$$\inf\{J(u,\theta,C_1,C_2) : (u,\theta,C_1,C_2) \text{ obeys } (0.2)\}$$

and

$$J(u,\theta,C_1,C_2) = \frac{1}{2}\int_{\Omega}\left((b-a)\chi_{\Omega^+}+\chi_{\Omega^-}\right)|u-u_d|^2 + w_1\int_M^{M'}\theta^2 + w_2|C_1|^2 + w_3|C_2|^2\chi_{(0,a)\cup(b,L)}.$$

Possible Questions

Correctors

- Uncontrolled
- Controlled

Thank You!