Topological phases with cold atoms in optical lattices

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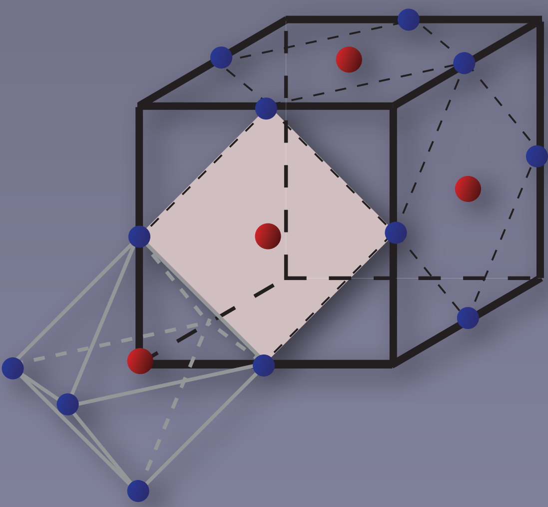


Atomic and molecular gases

Bose-Einstein condensation

- Gross-Pitaevskii equation
- non-linear dynamics

Quantum degenerate *dilute* atomic gases of fermions and bosons

control and tunability

Rotating condensates

- vortices
- fractional quantum Hall

Molecules

- Feshbach resonances
- BCS-BEC crossover
- polar molecules

Optical lattices

- Hubbard models
- strong correlations
- exotic phases

Quantum simulation

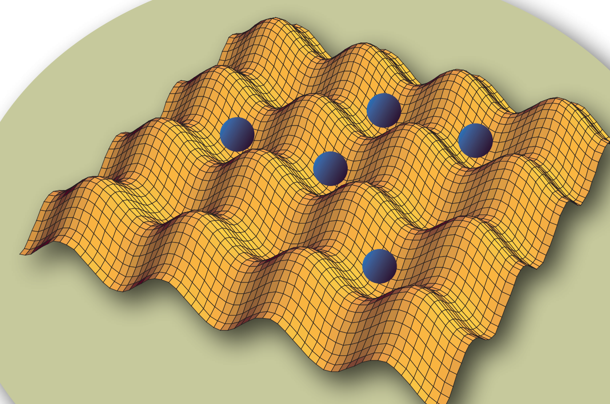
Feynman (1982):
Universal quantum simulator

- simulation of a quantum mechanical system with a well controlled system quantum system

- quantum systems are numerical hard problems
  - exponential grow of Hilbert space
  - sign problem in QMC

High-T_c superconductor

- understanding of quantum systems
  - Fermionic Hubbard model
  - ....
- search for novel states of matter:
  - topological phases
  - spin liquids
  - ...



atoms in an optical lattice

Why quantum simulations?

Why is it interesting?

- novel tool for strongly correlated states

- microscopic model for exotic and topological phases with many-body interaction terms

    - Pfaffian state: fractional quantum Hall state with a three-body interaction

    - toric code, color codes string nets

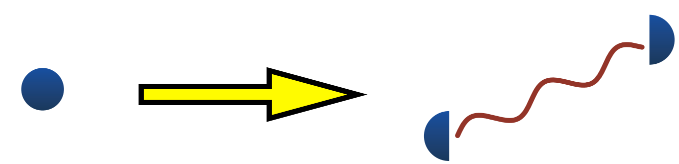
    - ring exchange interaction

Do exotic/topological phases exist in nature?

How often do they appear?

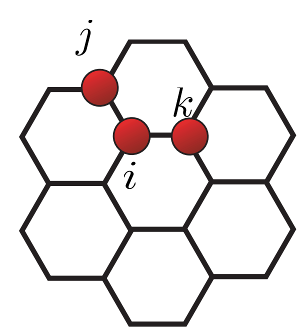
Exotic phases

- emergent symmetry
- gauge symmetry “QED”-like theories
- fractional excitations
- artificial light modes

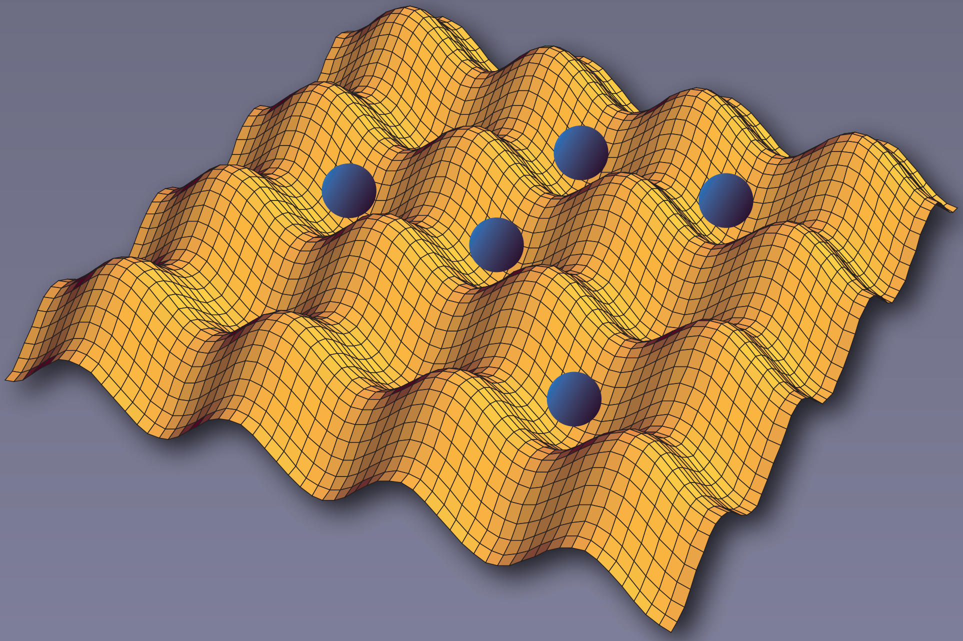


Topological phases

- fractional quantum Hall states
- (non)-abelian anyons
- application in topological quantum computing



Optical lattices



Interaction between light and atoms

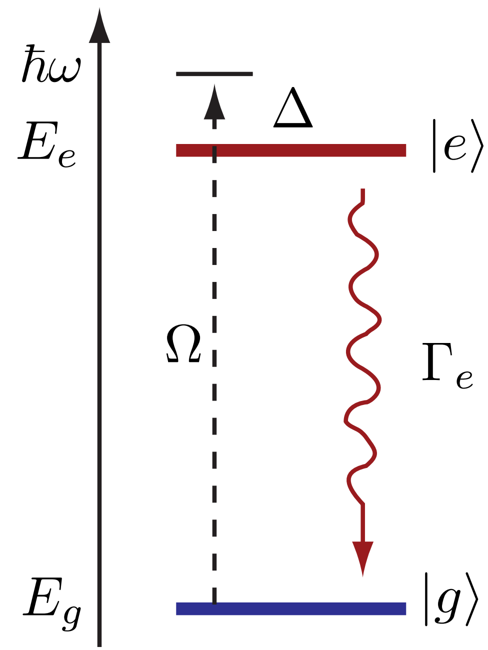
- Hamiltonian between atoms and light: dipole approximation

$$H = -\mathbf{d}\mathbf{E}(t, \mathbf{r})$$

- external laser field: $\mathbf{E}(t) = \mathbf{E}_\omega e^{-i\omega t} + \mathbf{E}_\omega^* e^{i\omega t}$

- rabi frequency: $\Omega = |\langle e|\mathbf{d}\mathbf{E}_\omega|g\rangle|/\hbar$

- detuning: $\Delta = \omega - (E_e - E_g)/\hbar$



- AC Stark shift (change in the grounds state energy due to coupling to excited state)

$$\Delta E_g = -\alpha(\omega)|E_\omega|^2$$

$$\alpha(\omega) \approx \frac{|\langle e|\mathbf{d}\epsilon|g\rangle|^2}{E_e - E_g - \hbar\omega}$$

dynamical polarizability

blue detuned:

- low field seeker

red detuned:

- high field seeker

Interaction between light and atoms

- spontaneous emission: Γ_e

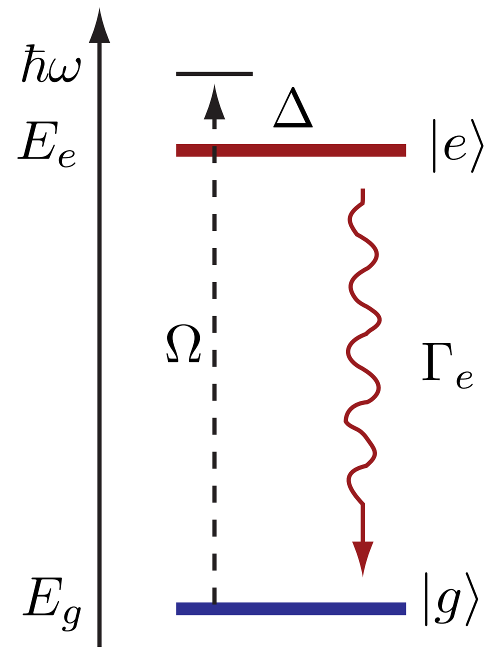
  excited state has a finite life time
  due to spontaneous emission

- AC Stark shift

$$\Delta E_g = \frac{\hbar \Omega^2 \Delta}{\Delta^2 + \Gamma_e^2/4}$$

- loss of atoms from the ground state

$$\Gamma_g = \frac{\Omega^2 \Gamma_e}{\Delta^2 + \Gamma_e^2/4}$$



- limits life-time of a BEC in an optical lattice

- requires large detuning

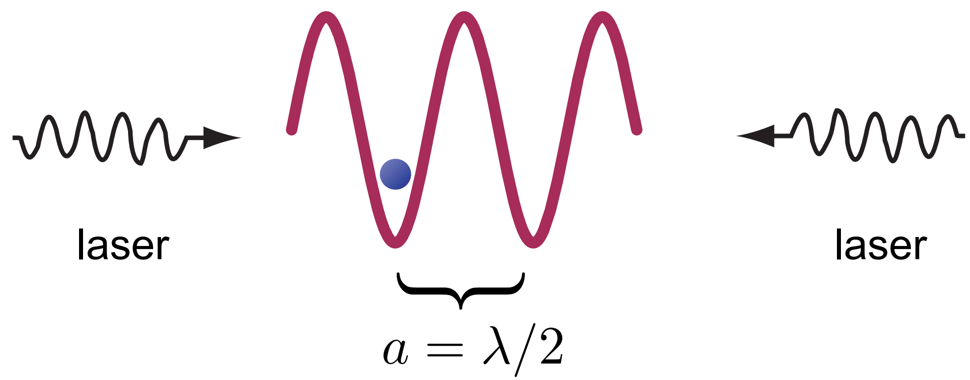
$$\Delta \gg \Gamma_e$$

- high laser power

Optical lattices

- a far-detuned standing laser wave provides
  a periodic potential for the particles

$$V(\mathbf{x}) = V_0 \cos(\mathbf{k}\mathbf{x})^2$$



- recoil energy: $E_r = \frac{\hbar^2 k^2}{2m}$

- structure in 3D

$$\mathbf{E}(t, \mathbf{r}) = \sum_i \mathbf{E}^i_{\omega_i} \cos(\mathbf{k}_i \mathbf{r}) e^{-i\omega_i t} + c.c.$$

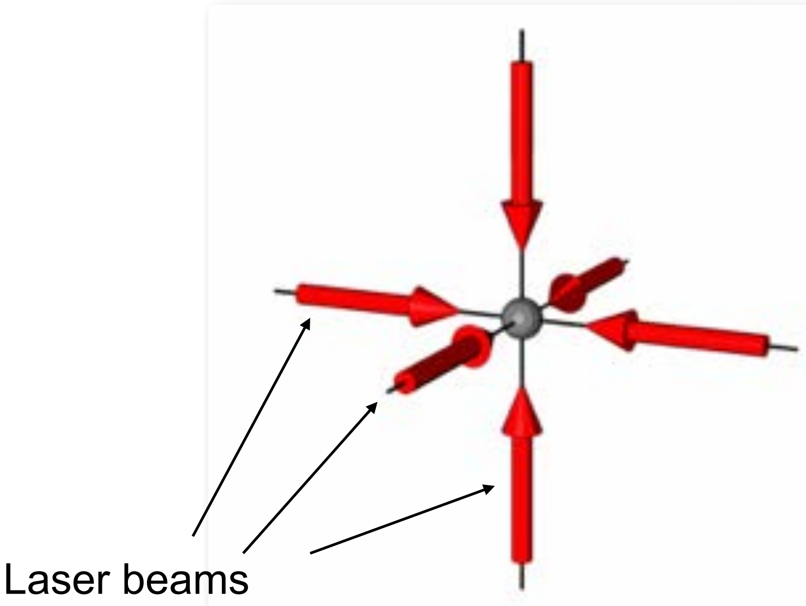
$\mathbf{k}_i$: wave length fixed by the atomic transition

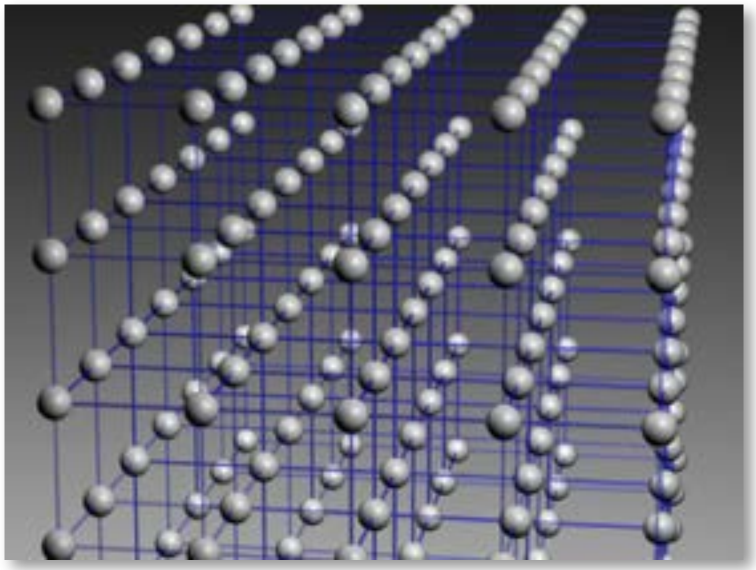
ω_i : slightly different frequencies to cancel cross terms

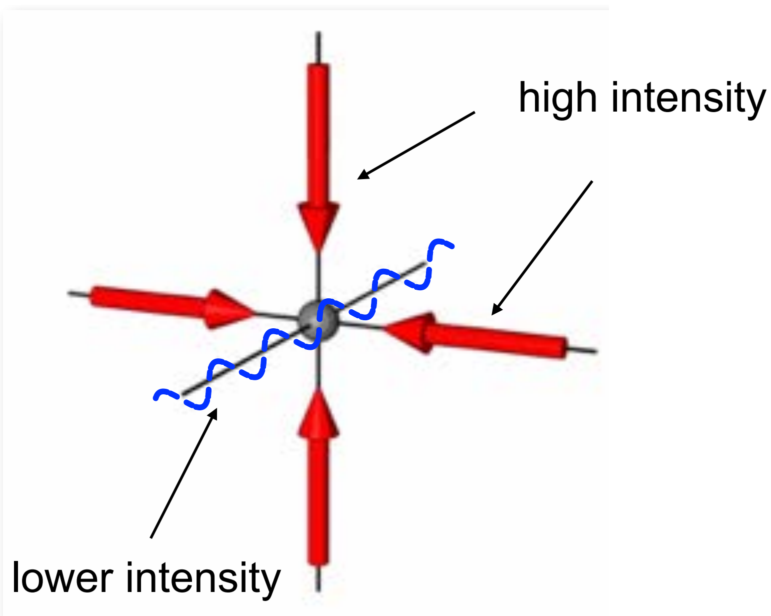
$\mathbf{E}^i_{\omega}$: polarization as additional degree of freedom

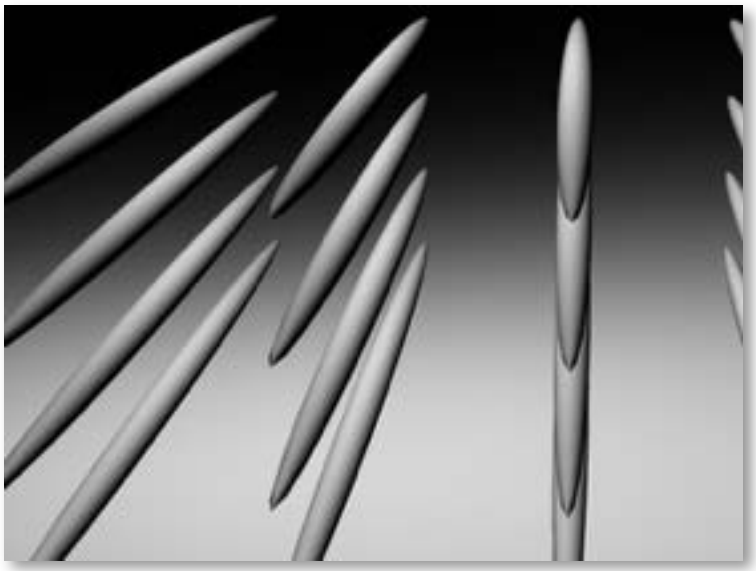
$$\Rightarrow \quad V(\mathbf{x}) = \sum_i V_i \cos(\mathbf{k}_i \mathbf{r})^2$$

Optical lattices





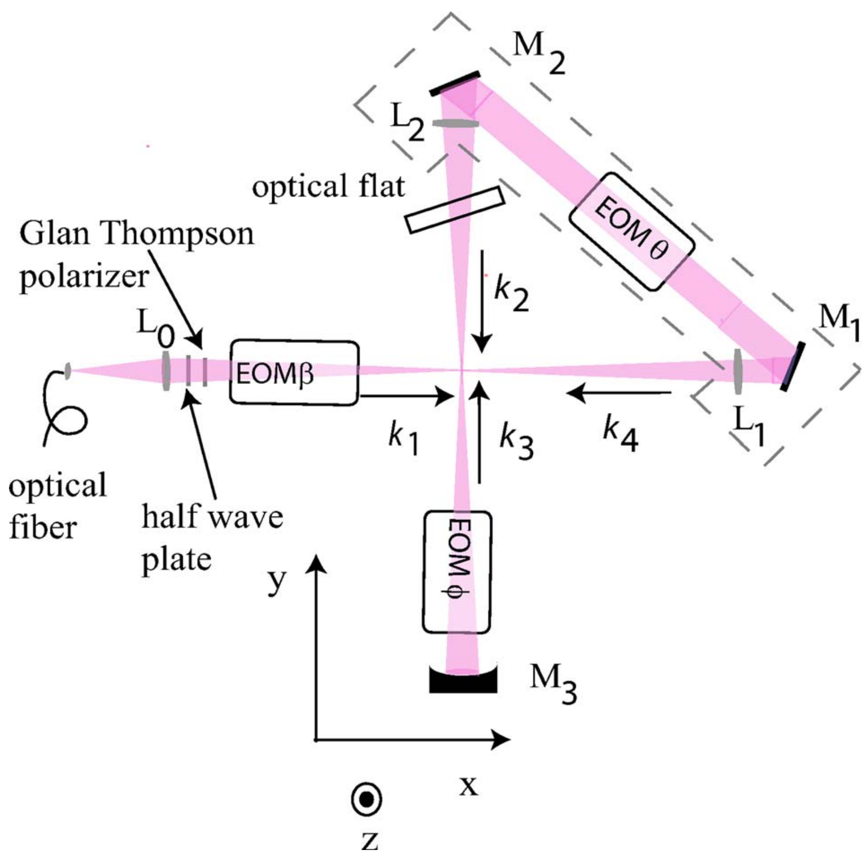


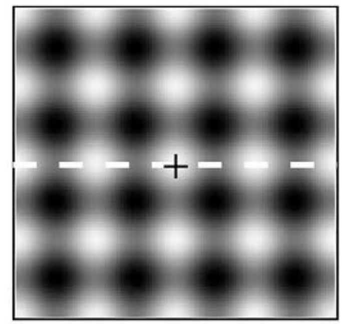


Optical lattices

Tricks with 2D optical lattice

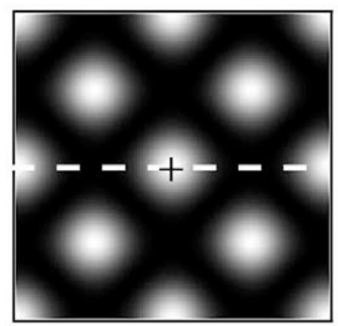
J. Sebby-Strabley, M. Anderlini, P.S. Jessen, and J.V. Porto, Phys Rev. A 73, 033605 (2006)





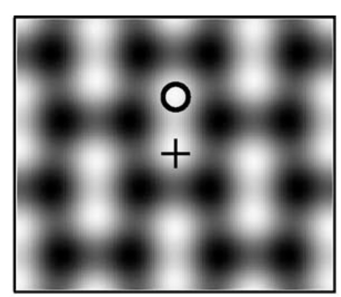
$$V(\mathbf{x}) = V_0\left[\cos(kx)^2 + \cos(ky)^2\right]$$

- in plane polarization
- cross terms disappear

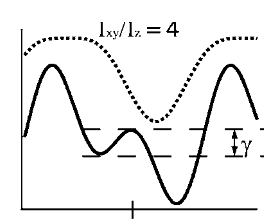


$$V(\mathbf{x}) = V_0\left[\cos(kx) + \cos(ky)\right]^2$$

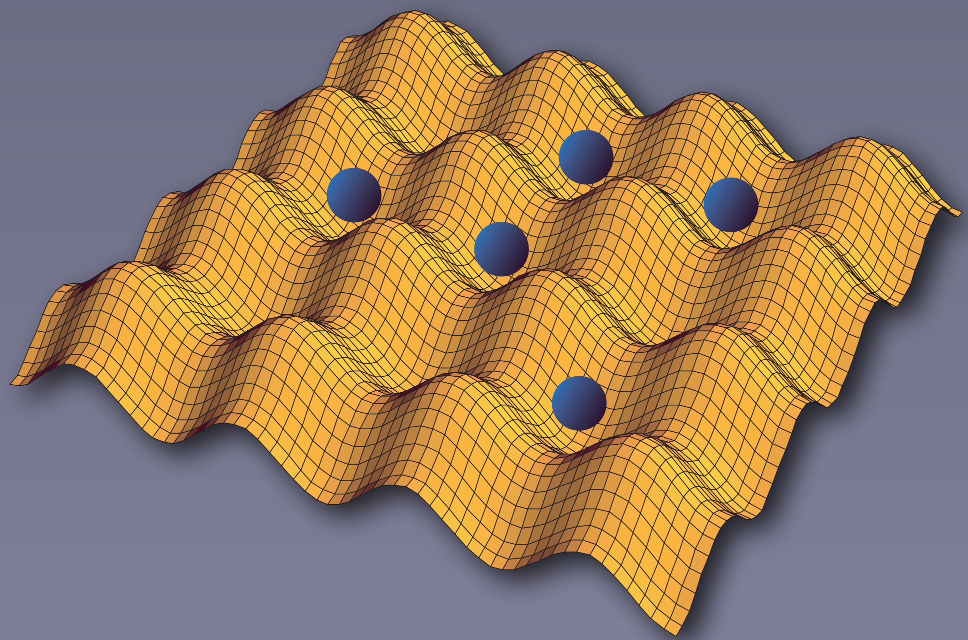
- polarization along z-axes
- lattice with cross terms



- combined lattice
- lattice of double wells



Many body Hamiltonian

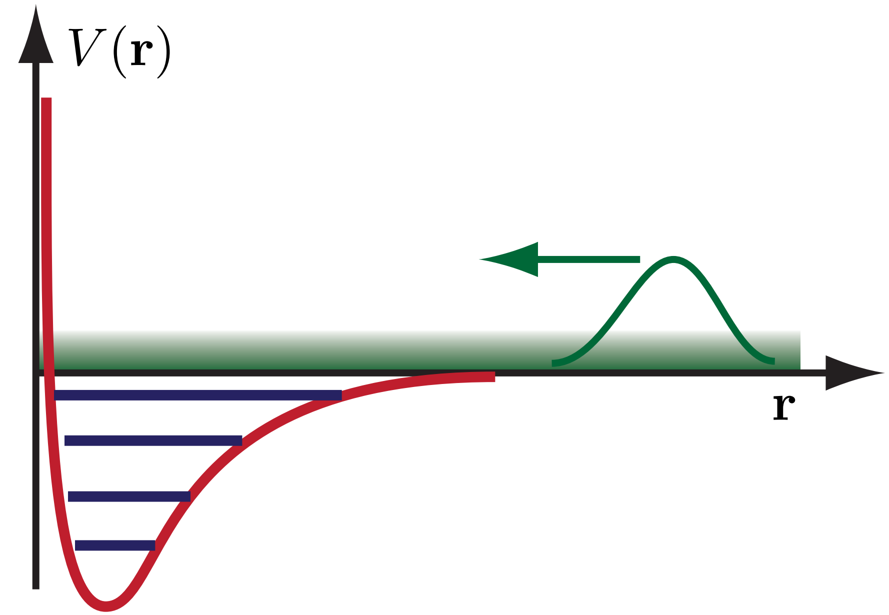


Pseudo-potential

- microscopic interaction potential

$V(\mathbf{r})$

- many bound states
- range $r_{\mathrm{vdW}} \sim 5\mathrm{nm}$



- Wigner threshold law:
  - low energy scattering dominated by s-wave
  - dilute system

$$n\, r_{\mathrm{vdW}}^3 \ll 1$$

- replace with a effective interaction reproducing the same scattering properties

$$V(\mathbf{r}) \rightarrow U(\mathbf{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\mathbf{r})\,\partial_r r$$

- 1. Born approximation produces exact scattering amplitude

- all higher terms in Born expansion vanish

Atoms in an optical lattice

Many-body Hamiltonian

- field operator $\psi^{\dagger}(\mathbf{x})$

$$H = \int d\mathbf{x}\psi^{\dagger}(\mathbf{x})\left[-\frac{\hbar^2}{2m}\Delta + V(\mathbf{x})\right]\psi(\mathbf{x}) + \frac{1}{2}\int d\mathbf{x}d\mathbf{y}U(\mathbf{x}-\mathbf{y})\psi^{\dagger}(\mathbf{x})\psi^{\dagger}(\mathbf{y})\psi(\mathbf{y})\psi(\mathbf{x})$$

optical lattice

Pseudo-potential: $U(\mathbf{r}) = \frac{4\pi\hbar^2 a_s}{m}\delta(\mathbf{r})\partial_r r$

Derivation of effective low energy theory:

Jaksch, Phys. Rev. Lett. 81, 3108 (1998)

(i) Solve the single particle problem in an optical lattice

(ii) Add the interaction as perturbation

→ Hubbard model for Fermions and bosons

Microscopic Hamiltonian

Wannier functions

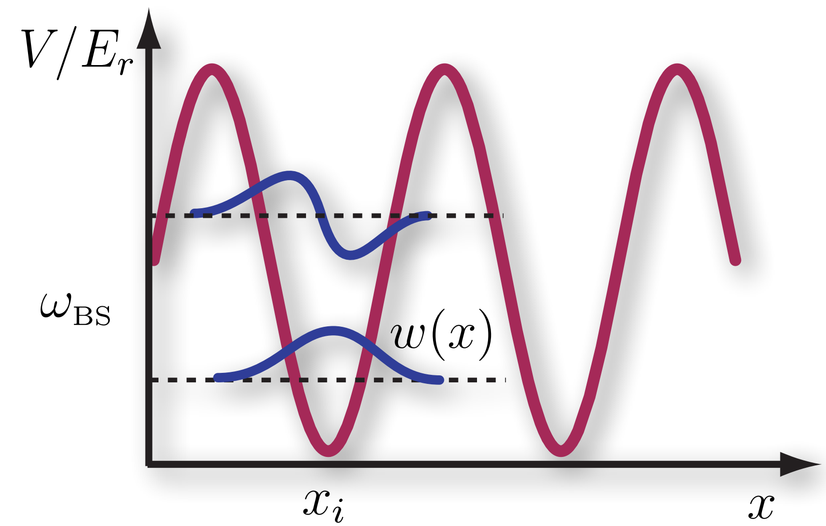
- localized wave function at each lattice site

Bloch wave function

$$w_n\left(\mathbf{x}-\mathbf{R}_i\right)=\int \frac{d\mathbf{k}}{v_0} e^{-i\mathbf{k}\mathbf{R}_i} \psi_{n,\mathbf{k}}(\mathbf{x})$$

- field operator

$$\psi(\mathbf{x})^{\dagger}=\sum_n b_{n,i}^{\dagger} w_n\left(\mathbf{x}-\mathbf{R}_i\right)$$



Interaction term

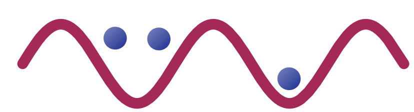
- replace pseudo potential

$$U(\mathbf{r}) \rightarrow \frac{4\pi\hbar^2 a_s}{m}\delta(\mathbf{r})$$

- wave function overlap

$$U_{ni;mj}^{n'i';m'j'}=\int d\mathbf{x}\, w_{n'i'}^{*} w_{m'j'}^{*} w_{mj} w_{ni}$$

onsite interactions dominate



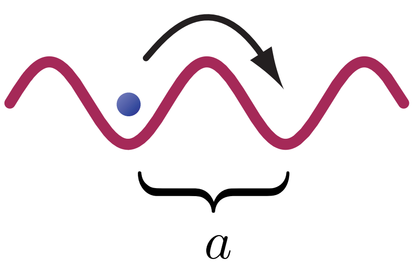
Hubbard model

Hubbard

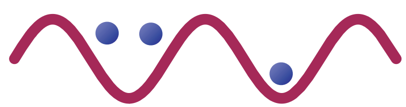
- restriction to the lowest Bloch band

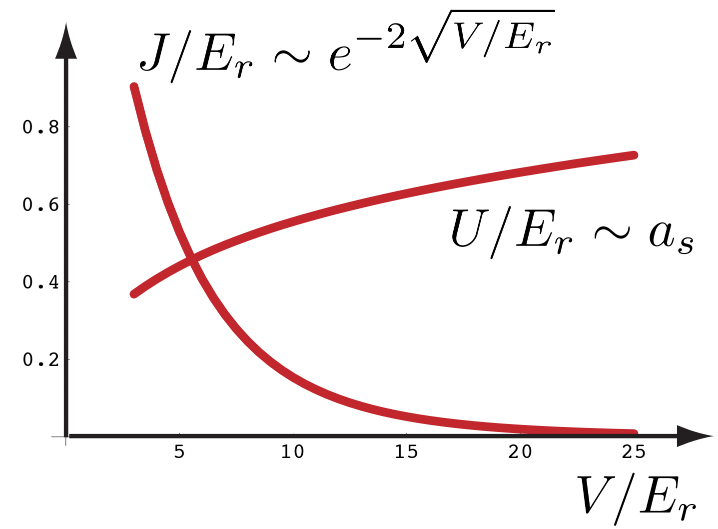
$$H = -J \sum_{\langle ij \rangle} b_i^\dagger b_j + \frac{U}{2} \sum_i b_i^\dagger b_i^\dagger b_i b_i$$

hopping energy



interaction energy





Two approximations

- δ - function interaction instead of pseudo potential

- restriction to lowest Bloch band

short distance cut-off with $\Lambda \sim a$

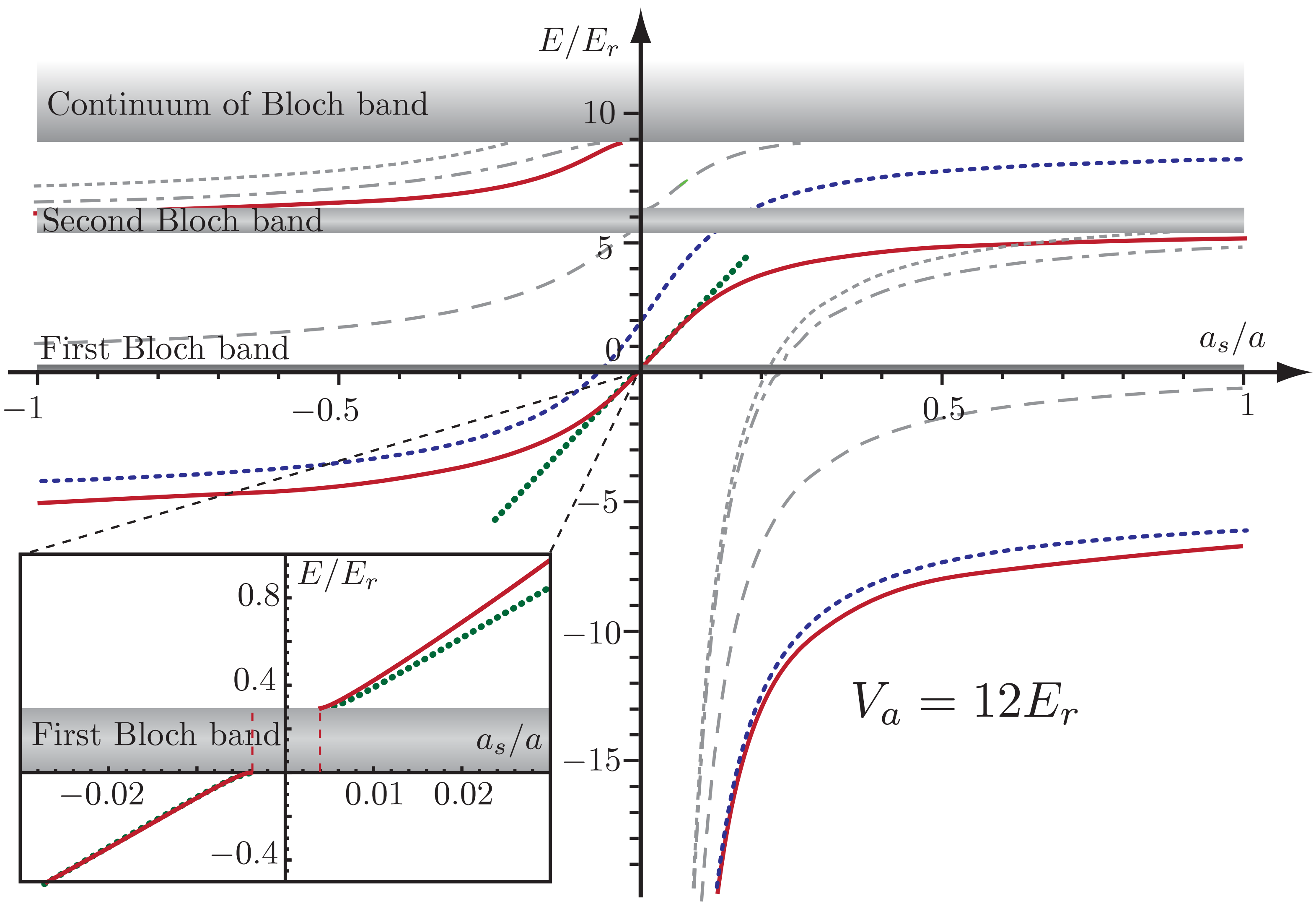
Valid for weak interactions

$$a_s/a_{\rm ho} \ll 1$$

for stronger interactions both approximations fail

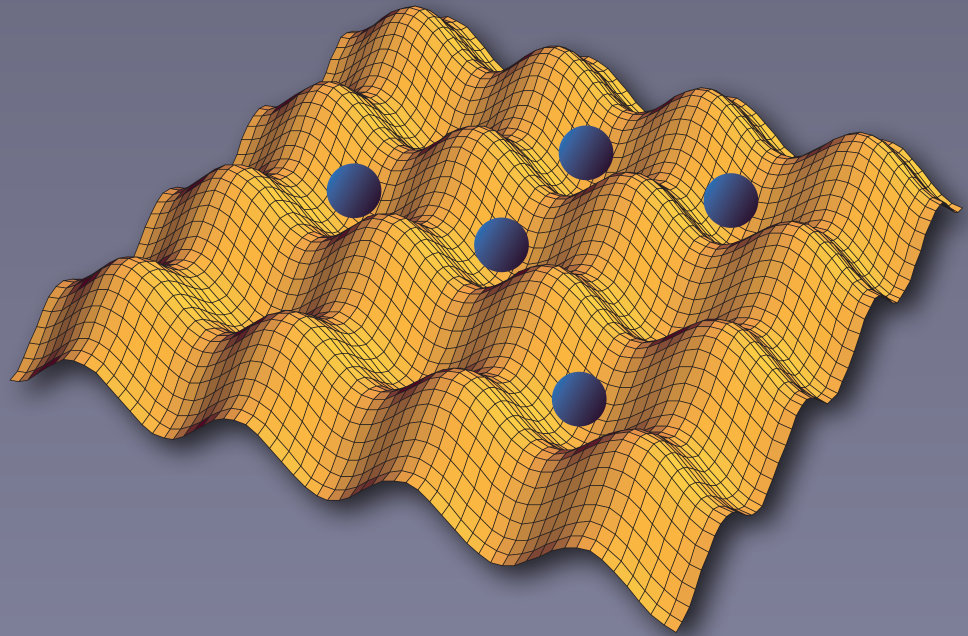
Bound states

- broad Feshbach resonance



Büchler, PRL (2010)

Topological phases for cold atomic gases in optical lattices



Spin exchange interaction

L.-M. Duan, E. Demler, and M. D. Lukin, PRL 91, 090402 (2003).

Hubbard model

- spin 1/2 system with spin dependent optical lattices

$$H = -\sum_{\langle ij \rangle} t_{\mu,\sigma} c^{\dagger}_{i,\sigma} c_{i,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

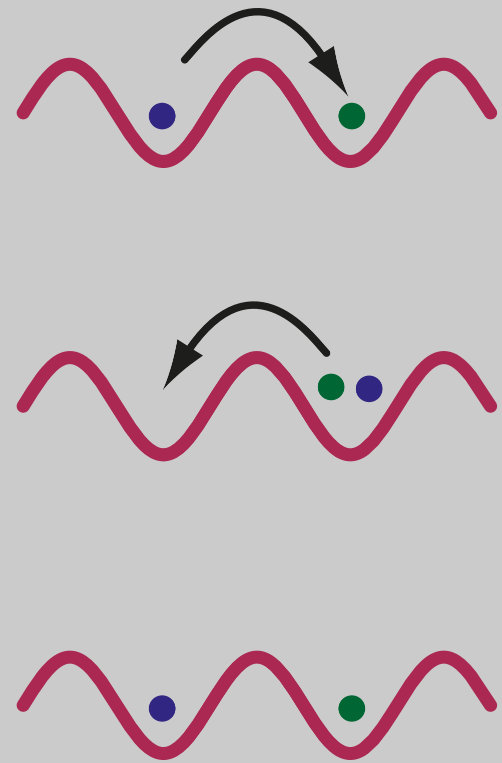
hopping dependent on spin and direction

- exchange interaction (XXZ model)

$$H = \sum_{\langle i,j \rangle} \left[\lambda_{\mu,z} \sigma_i^z \sigma_i^z + \lambda_{\mu,\perp} \left(\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y \right) \right]$$

$$\lambda_{\mu,z} = \frac{t^2_{\mu,\uparrow} + t^2_{\mu,\downarrow}}{2U} \qquad \lambda_{\mu,z} = \frac{t_{\mu,\uparrow} t_{\mu,\downarrow}}{U}$$

$$U \gg t_{\mu,\sigma}$$



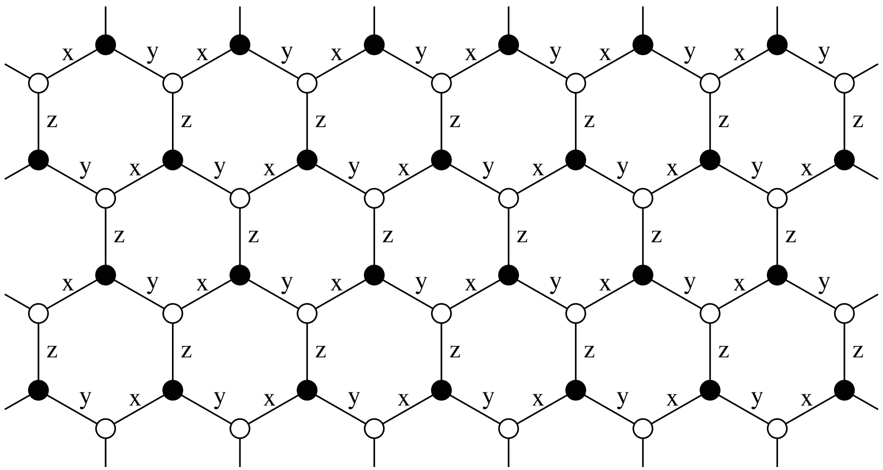
second order perturbation theory in the hopping

Spin exchange interaction

L.-M. Duan, E. Demler, and M. D. Lukin, PRL 91, 090402 (2003).

Kitaev model on hexagonal lattice

A. Kitaev, Annals of Physics, 321, 2 (2006)



- different interactions on the x,y,z -links

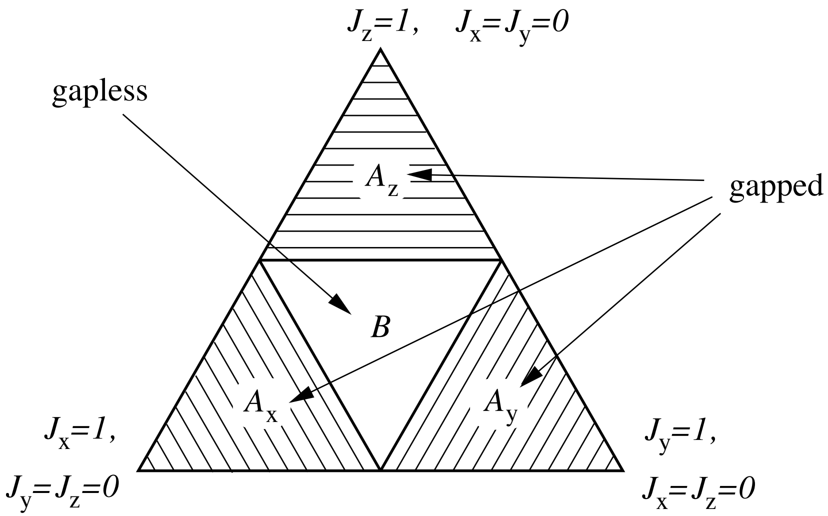
$$H = \sum_{\nu \in \{x,y,z\}} \lambda_\nu \sigma_i^\nu \sigma_j^\nu$$

Gapped phase (A):

- abelian anyonic excitations
- ground state degeneracy on torus
- string order parameter
- reduces to toric code

Gapless phase (B):

- in presence of a magnetic field: non-abelian anyons



String order parameter

E. G. Dalla Torre, E. Berg, and E. Altman, PRL 97, 260401 (2006)

Hubbard model with nearest-neighbor interactions

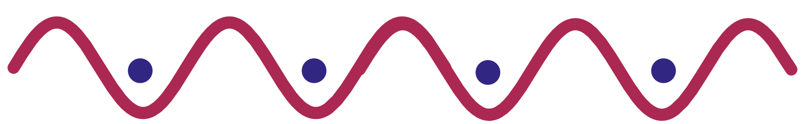
- one-dimensional setup
- bosonic particles

strong nearest-neighbor interaction

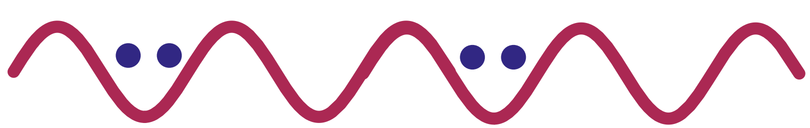
dipolar interactions?

$$H = -t \sum_i \left[b_i^\dagger b_{i+1} + \mathrm{H.c.} \right] + \frac{U}{2} \sum_i n_i (n_i - 1) + V \sum_i n_i n_{i+1}$$

- Mott insulator for $U \gg V, t$



- Density wave for $V \gg U, t$



New phase with string order parameter separating the Mott insulator and DW:

$$\langle \delta n_i e^{i\pi \sum_{k=i}^{j} \delta n_k} \delta n_j \rangle$$

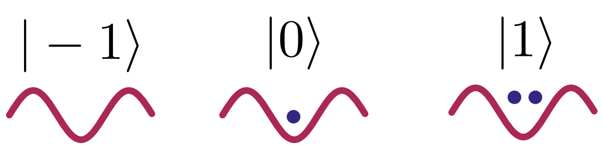
String order parameter

E. G. Dalla Torre, E. Berg, and E. Altman, PRL 97, 260401 (2006)

Hubbard model with nearest-neighbor interactions

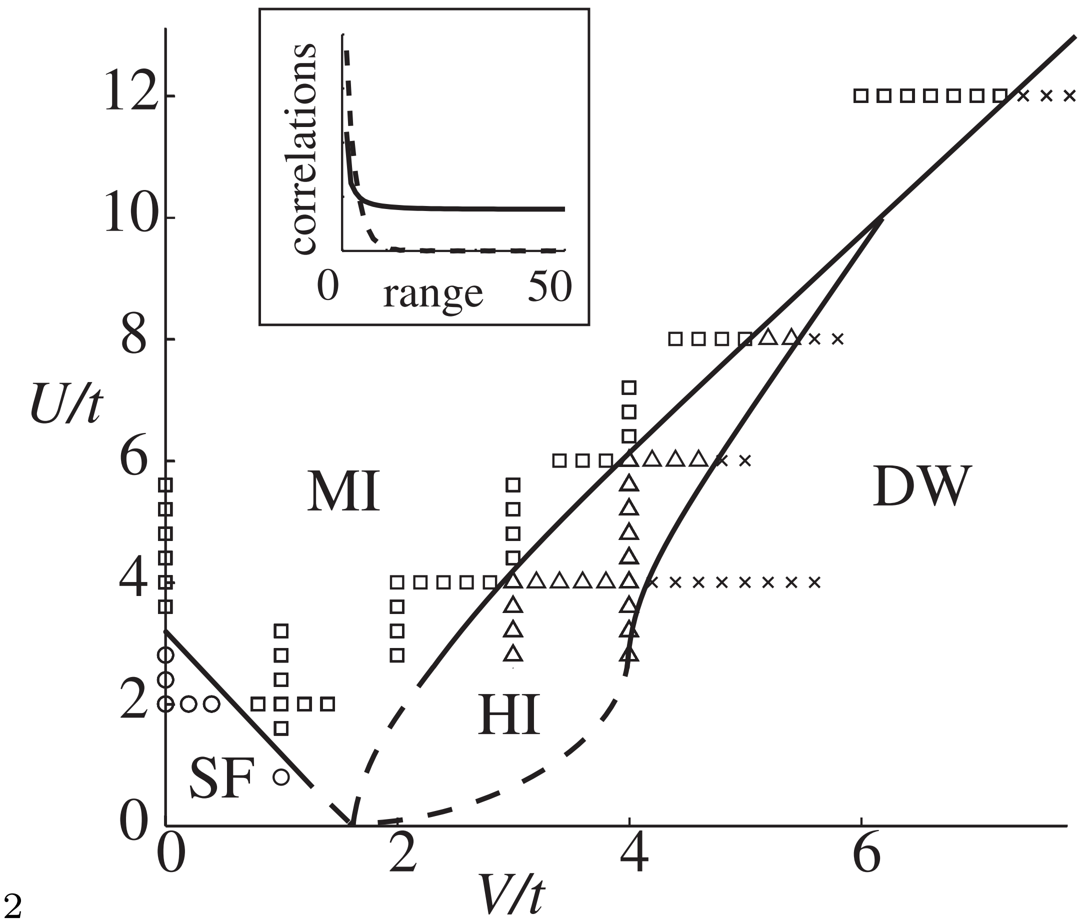
- novel phase with string order is in analogy to the Haldane gapped phase for integer spin

- mapping onto spin-1 Hamiltonian:



- effective spin Hamiltonian

$$H \approx t \sum_i \left(S_i^+ S_{i+1}^- + \mathrm{H.c.}\right) + \sum_i V S_i^z S_{i+1}^z + \frac{U}{2} \sum_i \left(S_i^z\right)^2$$



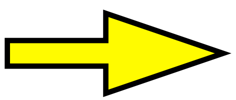
Majorana like edge states

Kraus, Dalmonte, Baranov, Läuchli, Zoller, PRL 111, 173004 (2013)

Double wire with pair hoping

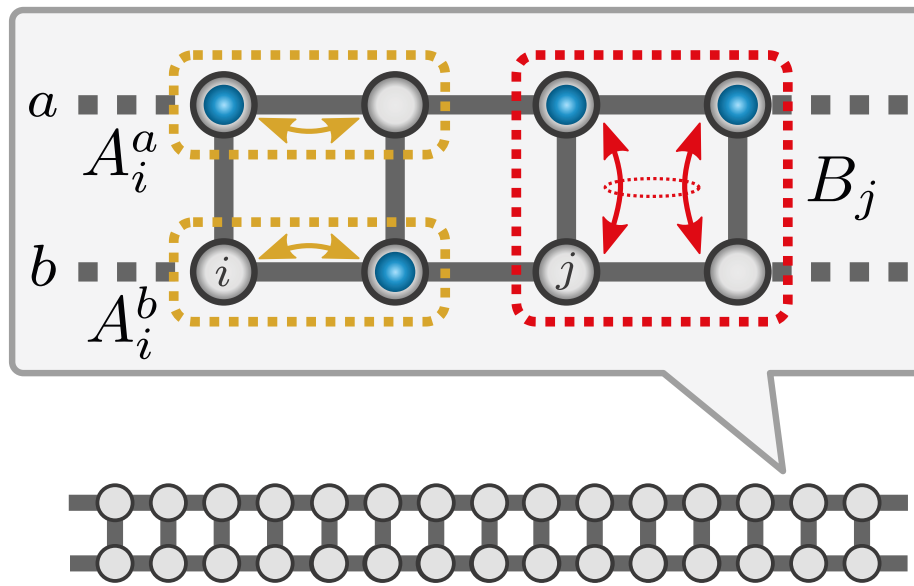
- one-dimensional setup of spineless fermions

- fixed total number of particles

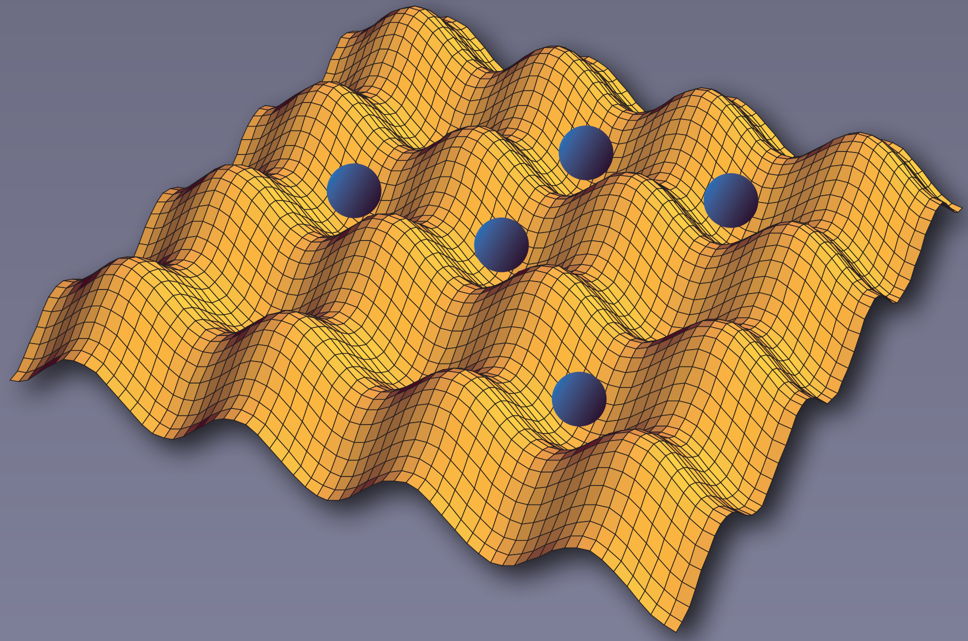


requires pair hopping between the chains

- DMRG is consistent with Majorana like edge states

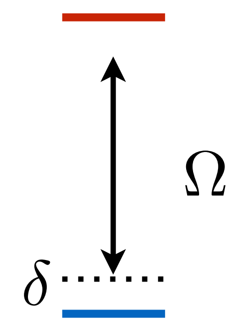


Artificial gauge fields



Artificial gauge fields

Two internal states



- coupling by laser fields
- free Hamiltonian

$$H = \frac{p^2}{2m} + V(\mathbf{r}) + U(\mathbf{r})$$

$$\frac{\Omega}{2}\begin{pmatrix} \cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & -\cos\theta \end{pmatrix}$$: internal coupling by the laser

- two dressed internal states $|\chi_1\rangle, |\chi_2\rangle$

$$|\chi_1\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}$$: spatial dependent

Reviews:

Goldman, Juzeliunas, Ohberg, Spielman, Rep. Prog. Phys. 77, 126401 (2014)
Dalibard et al, Rev. Mod. Phys. 83, 1523 (2011)

Artificial gauge fields

- general wave function:

$$|\psi\rangle = \sum_{j=1,2} \psi_j(r,t)|\chi_j(\mathbf{r})\rangle$$

- action of momentum operator

$$A_{jl} = i\hbar\langle\chi_j|\nabla|\chi_l\rangle$$

$$\mathbf{p}|\psi\rangle = \sum_{j,2}\left[\mathbf{p}\psi_j(r,t)\right]|\chi_j\rangle + \sum_{j,l=1,2} A_{jl}\psi_l|\chi_j\rangle$$

Adiabatic approximation

- strong couplings such that the atoms remain in the same dressed state

$$W = \frac{\hbar^2}{2m}|\langle\chi_2|\nabla|\chi_1\rangle|^2$$

$$i\partial_t\psi_1 = \left[\frac{(\mathbf{p}-\mathbf{A})^2}{2m} + V + \frac{\Omega}{2} + W\right]\psi_1$$

Reviews:

Goldman, Juzeliunas, Ohberg, Spielman, Rep. Prog. Phys. 77, 126401 (2014)
Dalibard et al, Rev. Mod. Phys. 83, 1523 (2011)

Artificial gauge fields

Limitations

- vector potential is limited by variation of the fields

$$|\mathbf{A}| \lesssim \hbar/\lambda$$

→

$$BL^2 = \oint d\mathbf{l}\mathbf{A} \lesssim L\hbar/\lambda$$

only weak magnetic fields

Optical flux lattices

Cooper, PRL 2011

- periodic variation of the dressing fields
- singularity in the vector potential

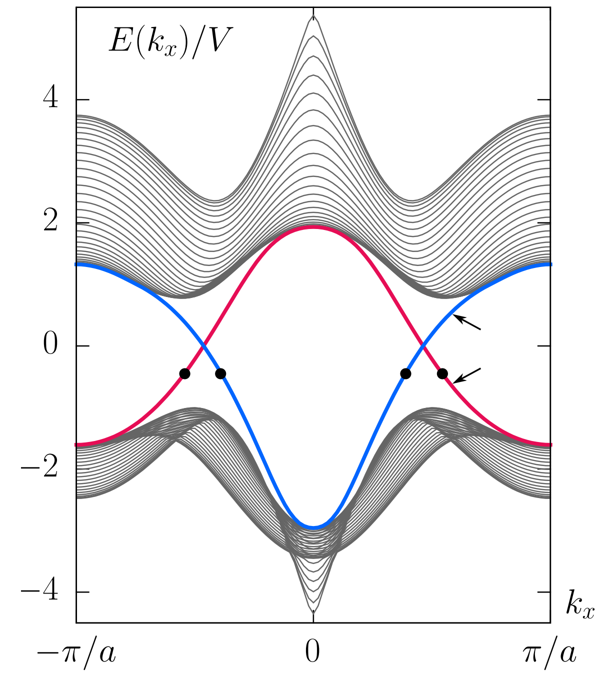
→ one flux per unit cell

Topological band structures

Topological band structure

singel particle band structure
independent on statistics of particles

- topological quantum numbers
- edge states



Requirements for topological states/phase

Fermions
- integer filling+ weak interactions → topological insulators
- flat bands + strong interactions → **fractional topological insulators**

Bosons
- flat bands + strong interactions → **bosonic fractional topological insulators**

Topological band structures using dipolar exchange interactions

Spin Hamiltonian

Spin Hamiltonian for polar molecules

- polar molecules trapped in an optical lattice

- suppressed tunneling

- one particle per lattice site

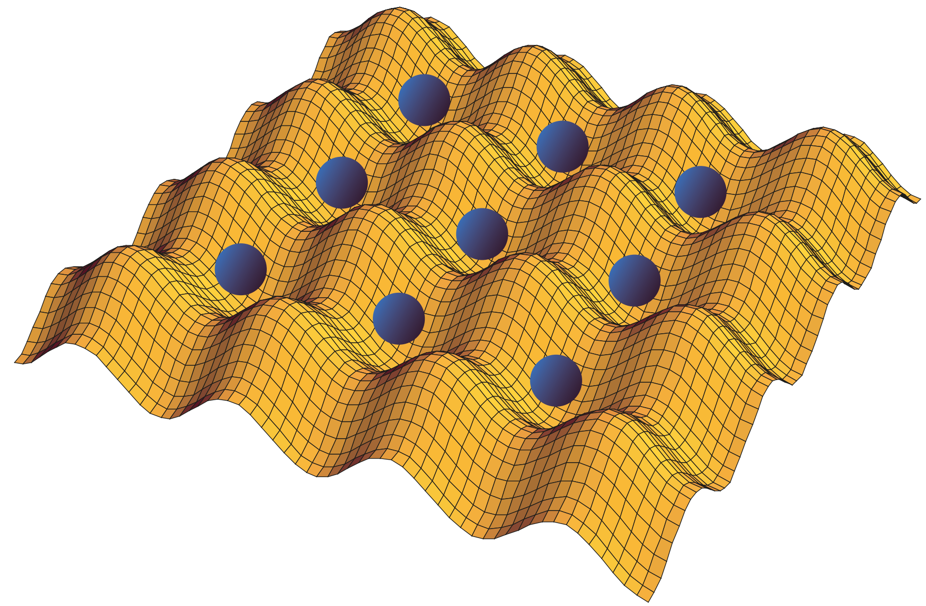
- electric field perpendicular to the plane splits rotational excitations

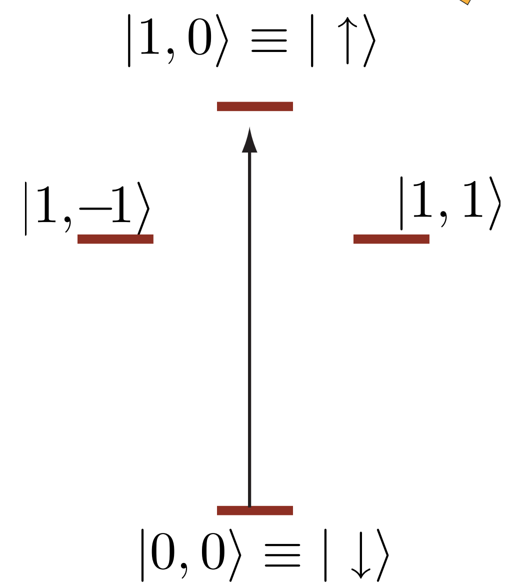
- two levels: spin 1/2 system

$$|\downarrow\rangle = |0,0\rangle$$

$$|\uparrow\rangle = |1,0\rangle$$

- dipole-dipole interaction gives rise to spin Hamiltonian





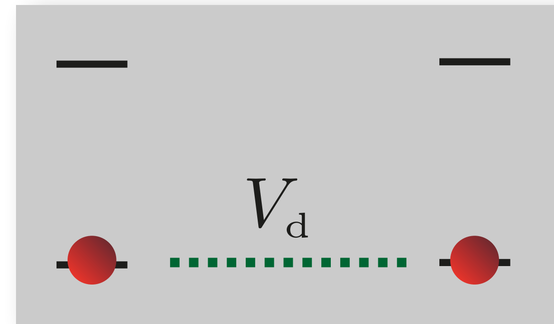
Spin Hamiltonian

Ising interactions

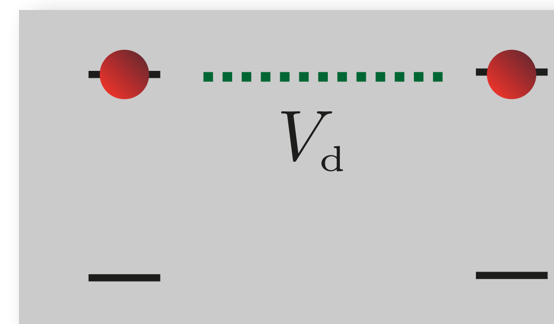
- static dipole moments within each rotational state

$$P_g^{(i)} = |g\rangle\langle g|_i$$

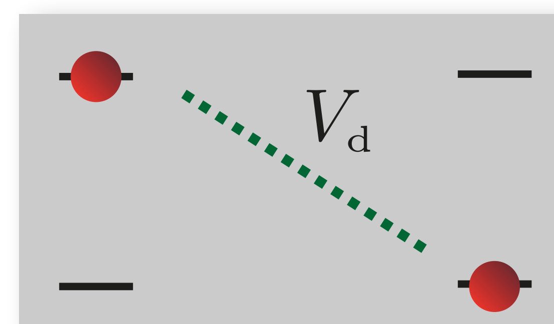
$$H = \frac{1}{2}\sum_{i\neq j}\frac{1}{|\mathbf{R}_i - \mathbf{R}_j|^3}\left[d_g^2 P_g^{(i)} P_g^{(j)}\right.$$



$$+d_e^2 P_e^{(i)} P_e^{(j)}$$



$$\left.+2d_e d_g P_e^{(i)} P_g^{(j)}\right]$$



$$= \frac{1}{2}\sum_{i\neq j}\frac{J_z}{|\mathbf{R}_i - \mathbf{R}_j|^3}\sigma_z^{(i)}\sigma_z^{(j)} + \sum_i h_{\mathrm{eff}}\sigma_z^{(i)} + E_0$$

Ising — magnetic field — energy shift

$$J_z = (d_g - d_e)^2$$

Spin Hamiltonian

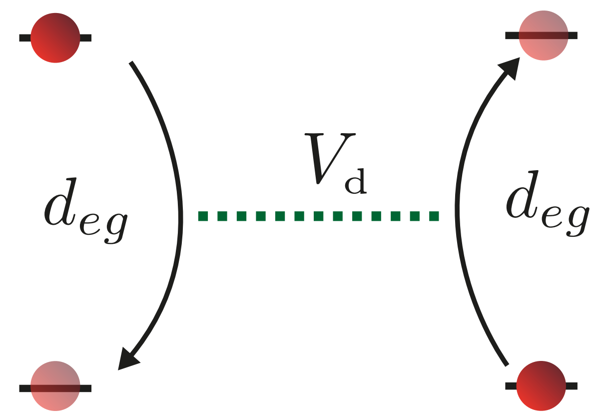
XY interactions

- resonant exchange interactions

$$\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+$$

$$H = \pm \frac{1}{2} \sum_{i \neq j} \frac{d_{eg}^2}{|\mathbf{R}_i - \mathbf{R}_j|^3} \left[\sigma_x^{(i)} \sigma_x^{(j)} + \sigma_y^{(i)} \sigma_y^{(j)} \right]$$

+ for m=0
- for m=1



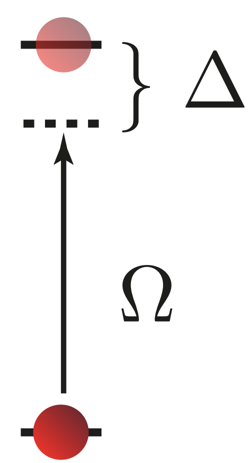
- ferro/ antiferro - magnetic interaction depending on excited rotational state

Magnetic field

- microwave field coupling ground and excited state

- rotating frame

$$H = \frac{\hbar}{2} \sum_i \left[\Delta \sigma_z^{(i)} + \Omega \sigma_x^{(i)} \right] = \sum_i \mathbf{h} \cdot \mathbf{S}^{(i)}$$



Spin Hamiltonian

XXZ model

- dipolar decay of the coupling parameters

- highly tunable from ferro- to anti-ferromagnetic coupling

$$H = J \sum_{i \neq j} \frac{1}{|\mathbf{R}_i - \mathbf{R}_j|^3} \left[\cos\theta \sigma_z^i \sigma_z^j + \sin\theta \left(\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j \right) \right]$$

$\theta = 0, \pi$

$\theta = \pm\pi/2$

Ising (anti-) ferromagnetic coupling

$$H = \mp J \sum_{i \neq j} \frac{\sigma_z^i \sigma_z^j}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

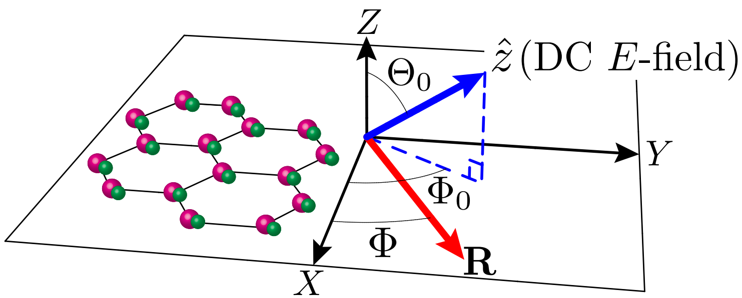
XY (anti-) ferromagnetic coupling

$$H = \mp J \sum_{i \neq j} \frac{\sigma_x^i \sigma_x^j + \sigma_y^i \sigma_y^j}{|\mathbf{R}_i - \mathbf{R}_j|^3}$$

Spin Hamiltonians for topological states

Kitaev Honeycomb lattice

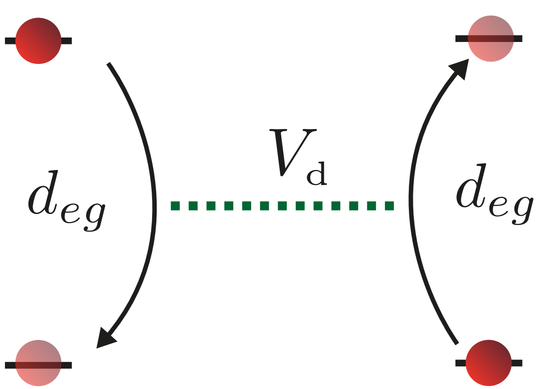
Gorshkov, Hazzard, and Rey, 2013
Micheli, Brennen, and Zoller, Nature Physics 2, 341 (2006)



- controlling the spin interactions with static electric and microwave fields

Topological flat bands

- resonant spin exchange interaction all hopping of spin excitations



- Hubbard model for hard-core bosons
- tunable hopping
- flat bands

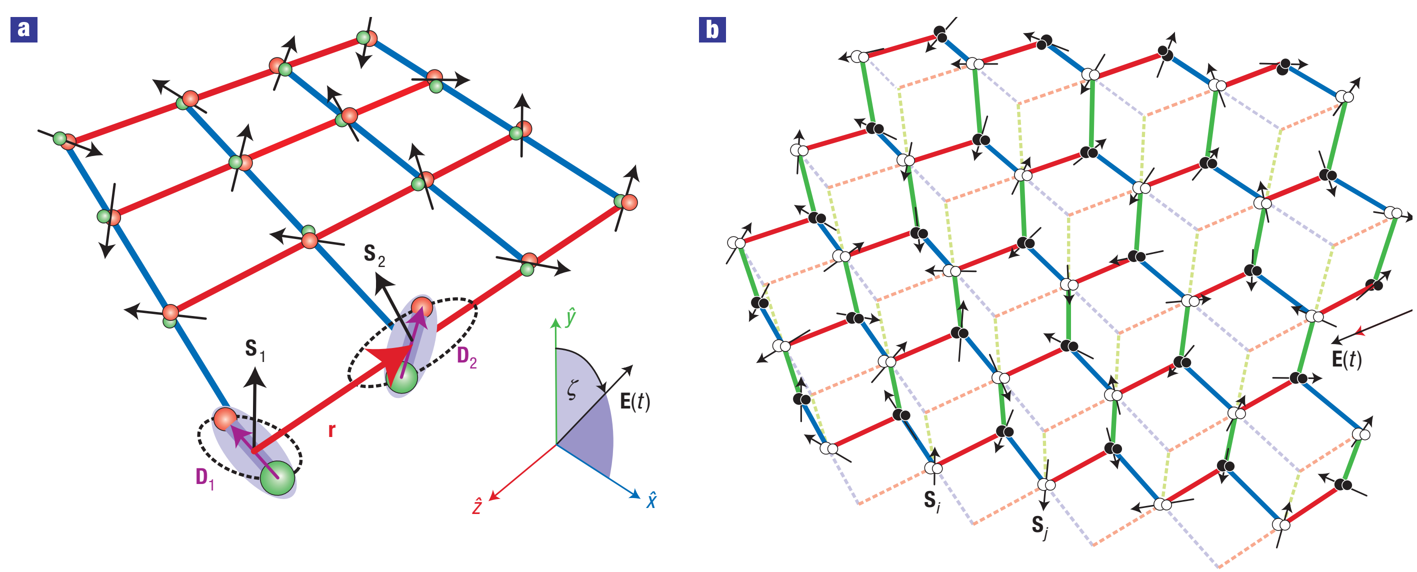
Spin tool box with polar molecules

A. Micheli, G. K. Brenne, and P. Zoller, Nature Physics (2006)

Polar molecules with additional spin degree of freedom

- integer filling of molecules in the lattice
- strong electric dipole moment with strong spin-rotational coupling

→ realization of Kitaev's Honeycomb lattice model



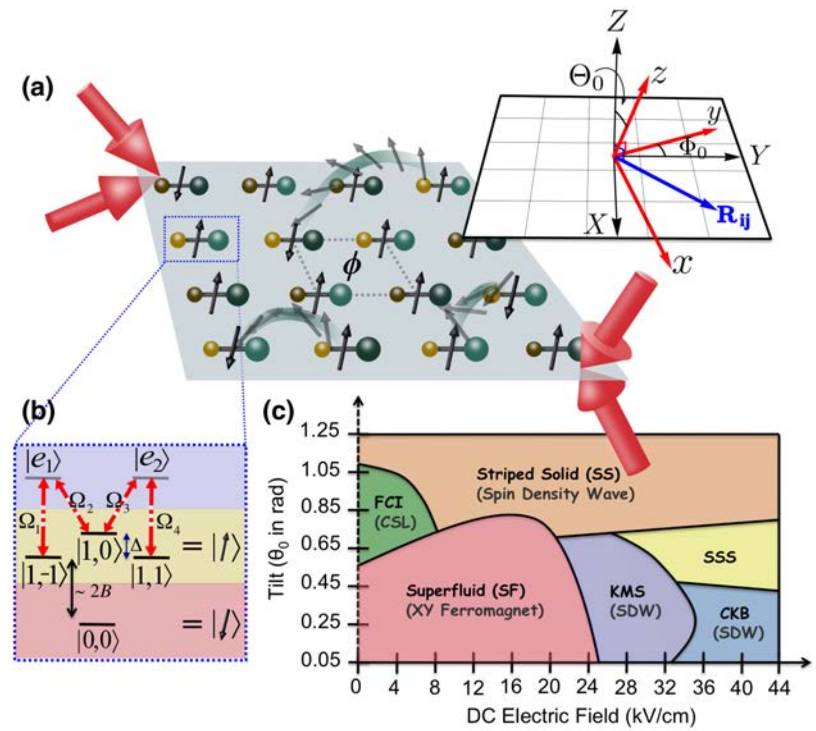
Bosonic Fractional Chern insulator

Yao, Gorshkov, Laumann, Läuchli, Ye, and M.D. Lukin, PRL 110, 185302 (2013)

Flat topological band

- combination of dipolar exchange interaction and artificial gauge fields

- effective bosonic particles with hard-core constraint and dipolar interaction



→ fractional bosonic Chern insulator at half filling

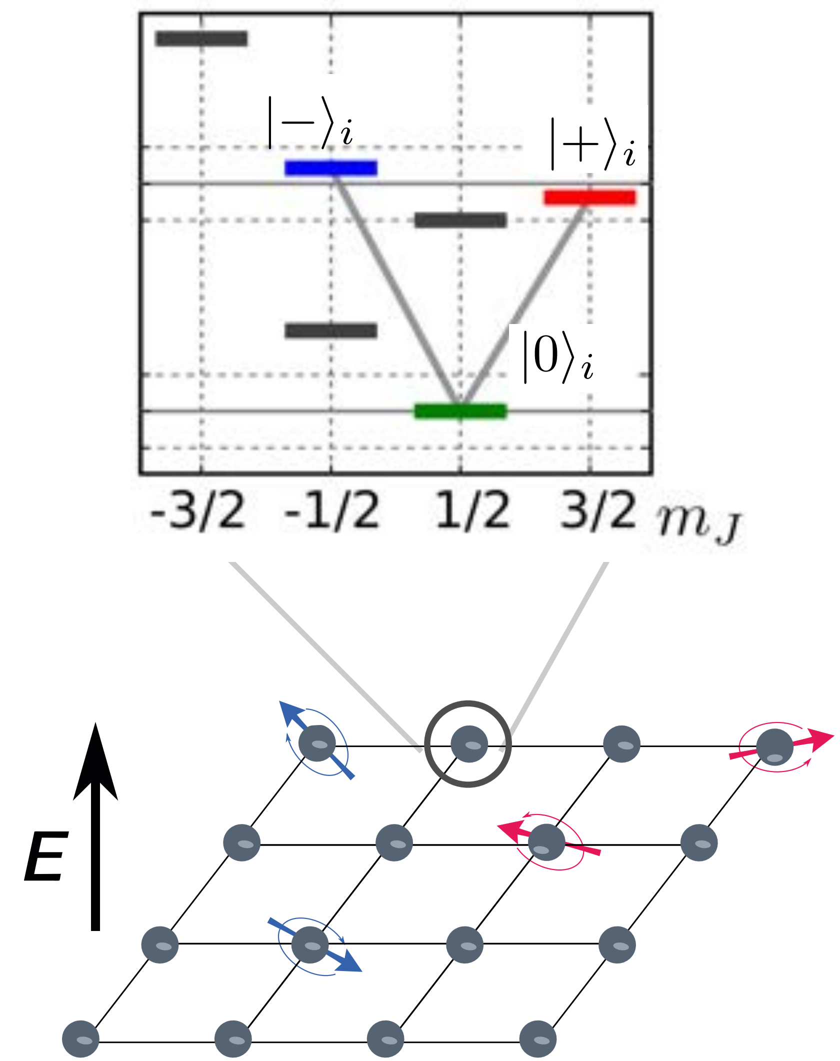
Rydberg atoms in an optical lattice

Setup

- one atom per lattice site with quenched tunneling
- static external electric field and magnetic field
- select three internal states

$|-\rangle_i \quad |+\rangle_i$: ground state

$|0\rangle_i$: two excited states



Mapping onto two hard-core bosons:

- bosonic creation operators for excitations

$$|+\rangle_i = b^\dagger_{i,+}|0\rangle$$

$$|-\rangle_i = b^\dagger_{i,-}|0\rangle$$

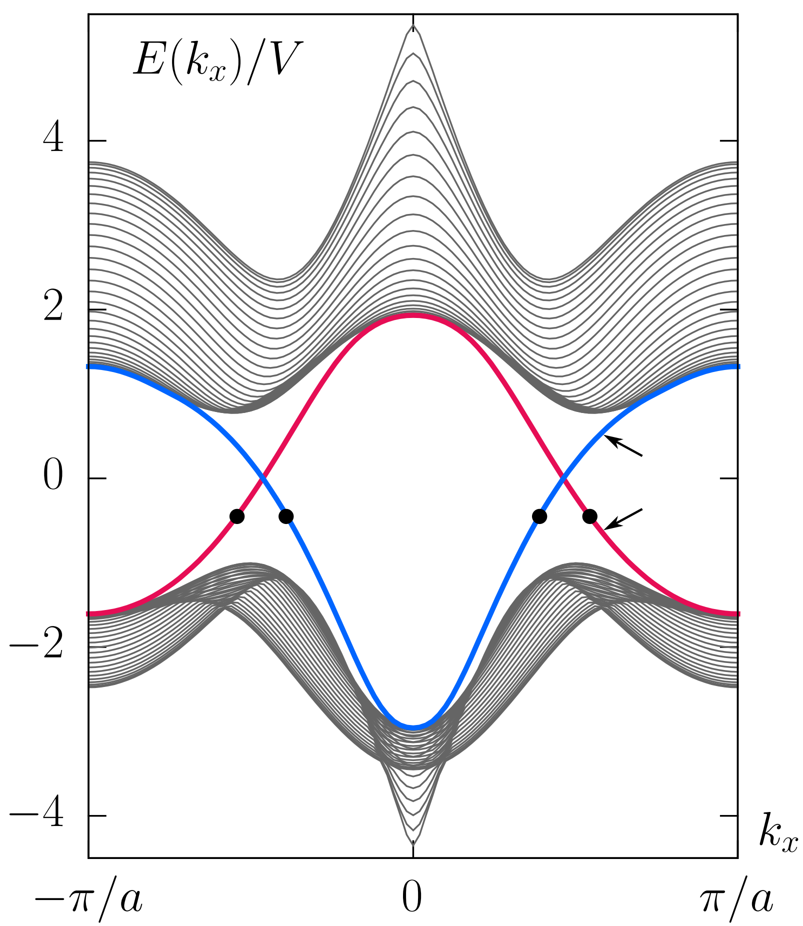
Edge states

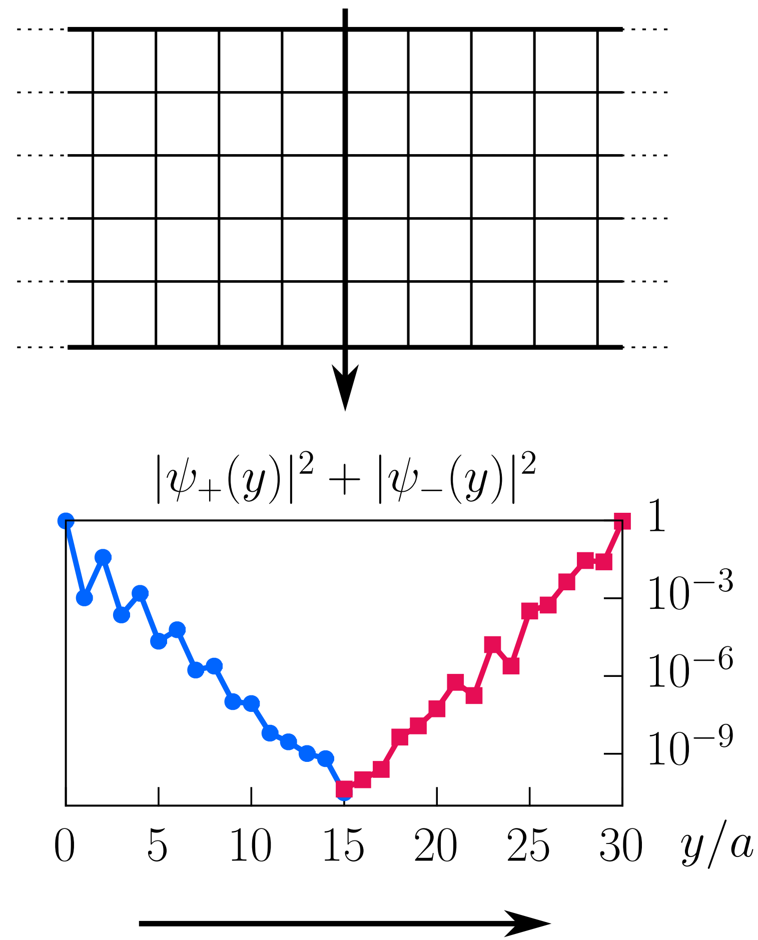
Finite system in y-direction

- bulk edge correspondence
  (Hatsugai PRL 1993)

C= 2 implies two edge states

- exponential localization in
  presence of long-range hopping





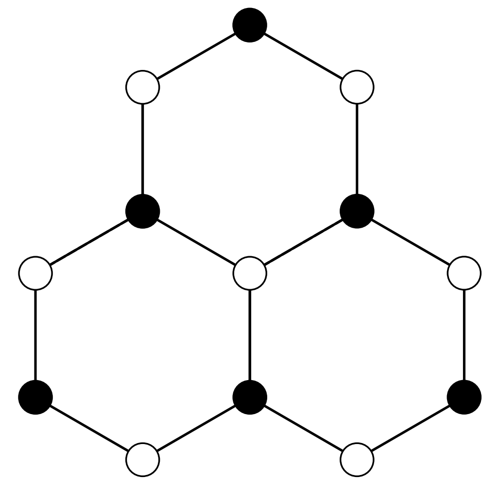
Flat topological bands

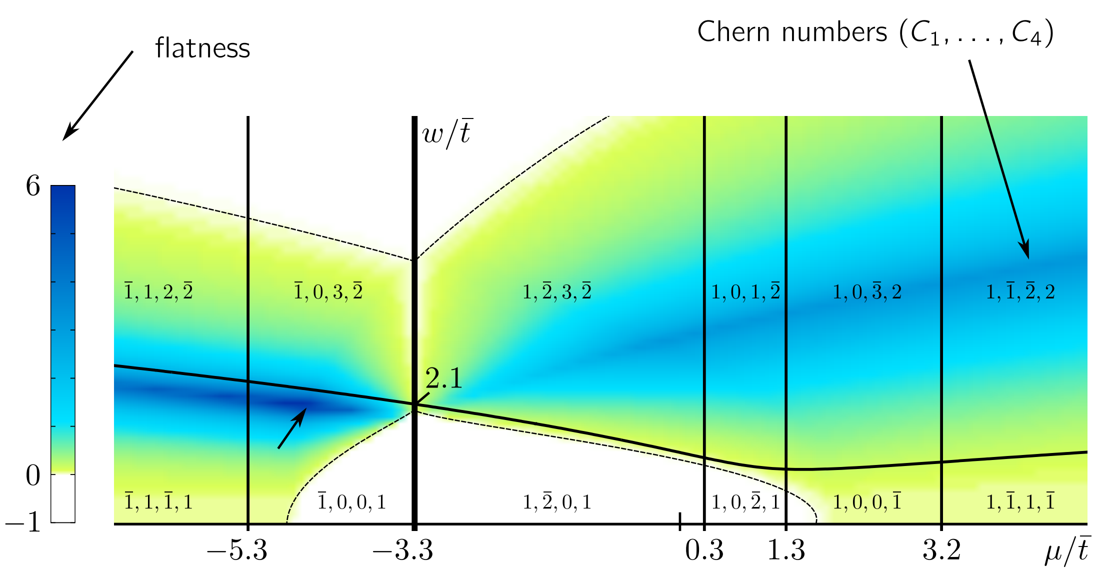
Honeycomb lattice

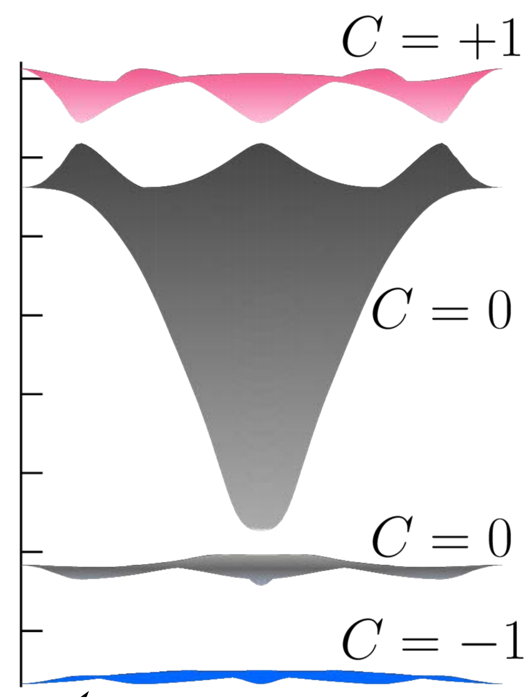
- much flatter bands accessible
- very rich topological structure

$$C \in \{0, \pm 1, \pm 2, \pm 3, \pm 4\}$$

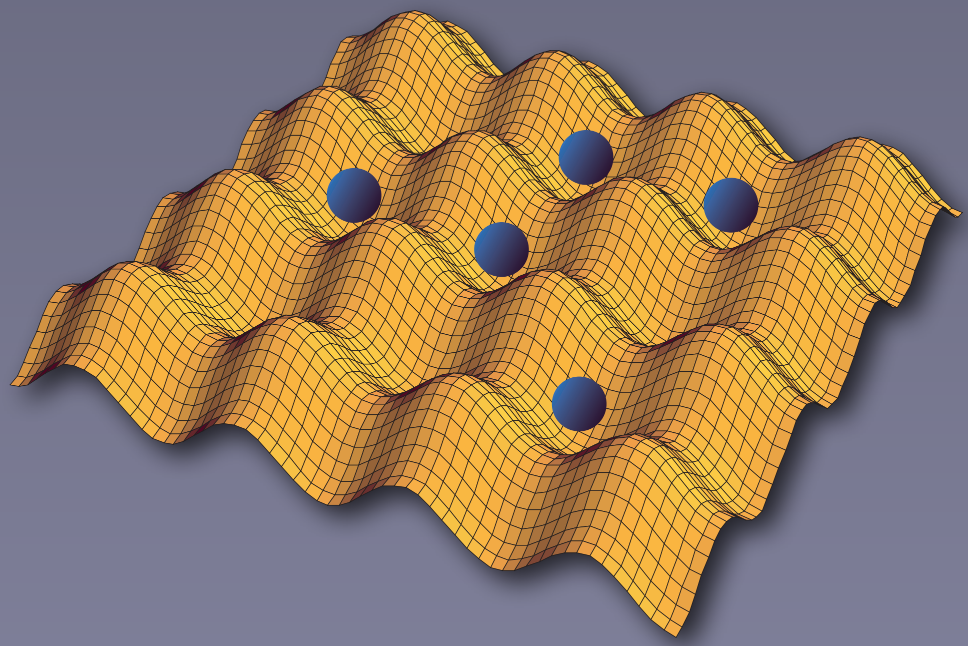
- even richer for Kagame lattice







Lattice shacking

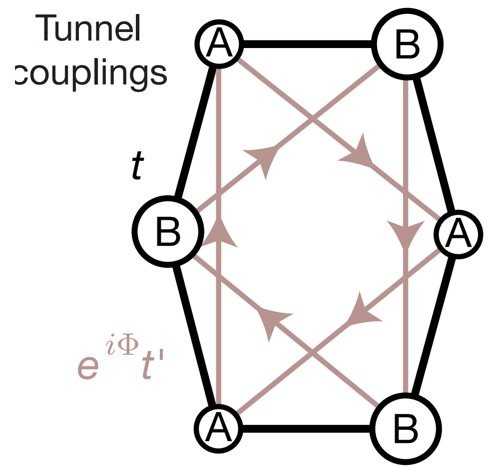


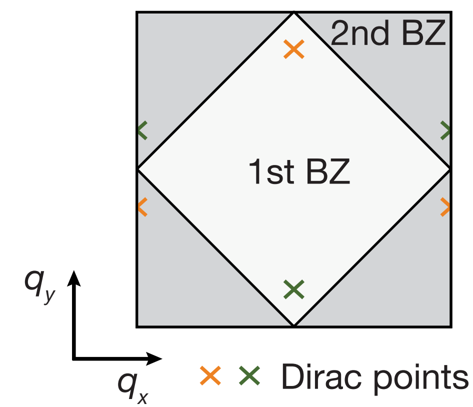
Haldane model by lattice shaking

Jotzu, Messer, Desbuqois, Lebrat, Uehlinger, Greif, Esslinger, Nature, (2014)

Distorted Honeycomb lattice

- non-interacting fermions
- nearest-neighbor hopping
- Dirac cones

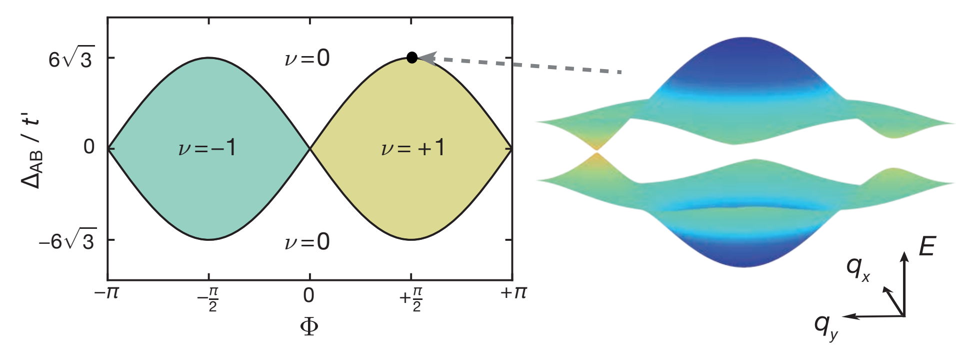




Haldane model

- complex tunneling on between the same sublattices

$$\hat{H} = \sum_{\langle ij \rangle} t_{ij} \hat{c}_i^\dagger \hat{c}_j + \sum_{\langle\langle ij \rangle\rangle} e^{i\Phi_{ij}} t'_{ij} \hat{c}_i^\dagger \hat{c}_j + \Delta_{\mathrm{AB}} \sum_{i \in A} \hat{c}_i^\dagger \hat{c}_i,$$



- topological phase transition: competition between terms breaking time reversal symmetry and inversion symmetry

Haldane model by lattice shaking

Jotzu, Messer, Desbuqois, Lebrat, Uehlinger, Greif, Esslinger, Nature, (2014)

Lattice shacking

- modifies tunneling strengths

- induces phases onto the tunneling

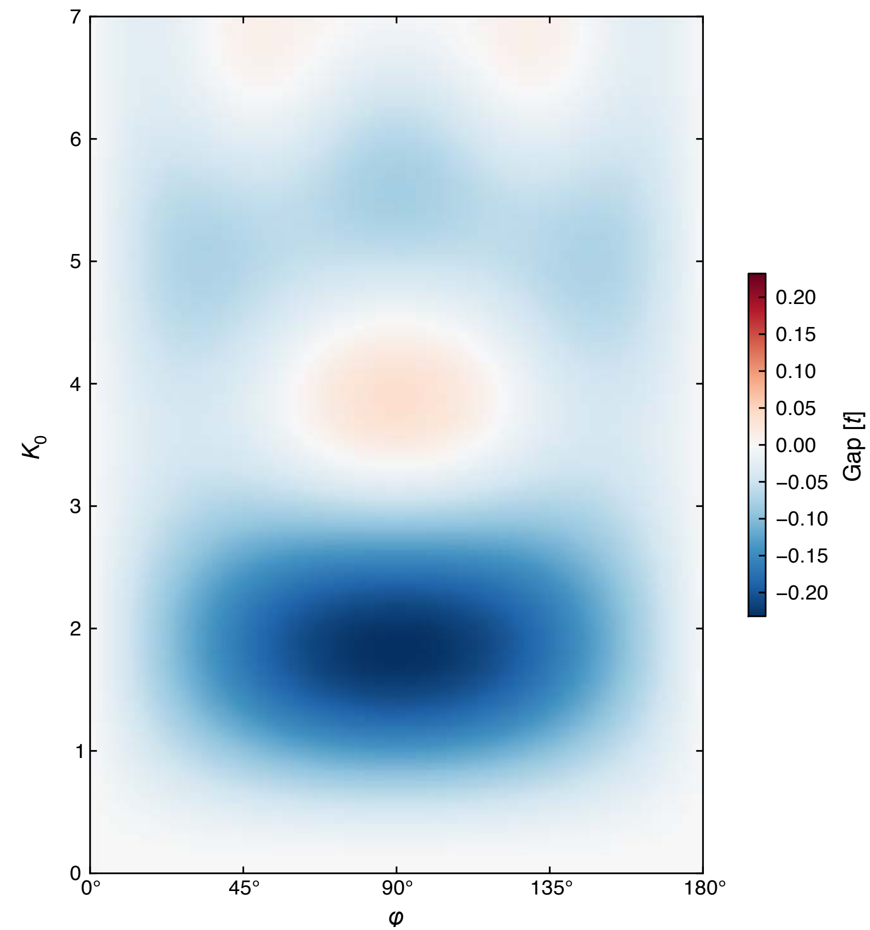
- also leads to longer-range hopping

Perturbation theory: ω/t

- leading order influence on hopping

$$t_{ij} \to J_0(z_{ij})t_{ij}$$

- first correction provides next-nearest hopping with a phase



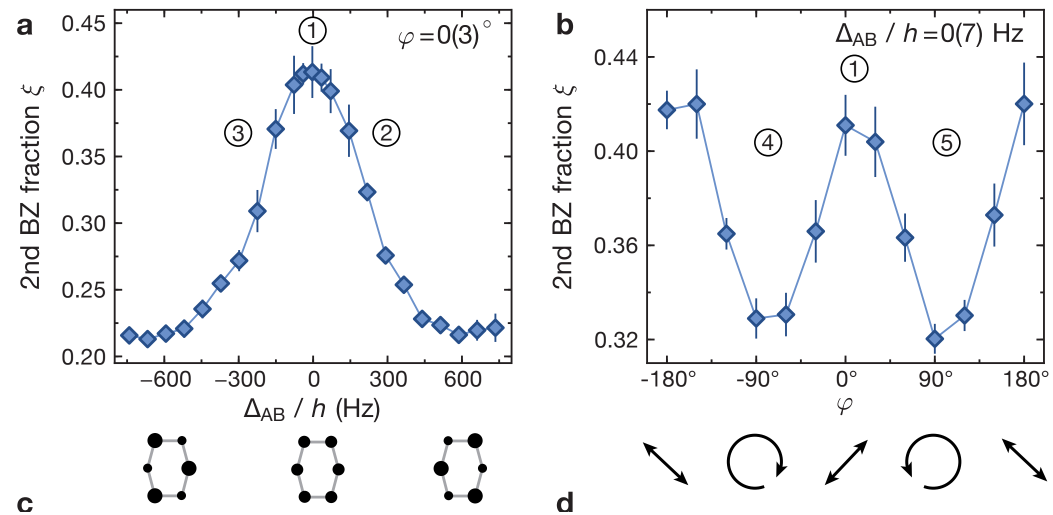
Full numerical calculation based on tight binding approximation

Haldane model by lattice shaking

Jotzu, Messer, Desbuqois, Lebrat, Uehlinger, Greif, Esslinger, Nature, (2014)

Probing the gap

- low filling of the band with fermions
- drive Bloch oscillations through the Dirac point
- measure diabatic transitions into higher band

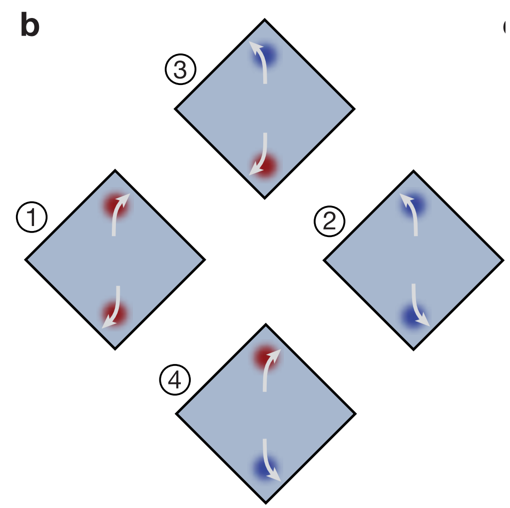


Probing the Berry curvature

- Berry curvature acts as a magnetic field in the semiclassical equation of motion

→ Lorentz force

- measure deflection for sign of Berry curvature

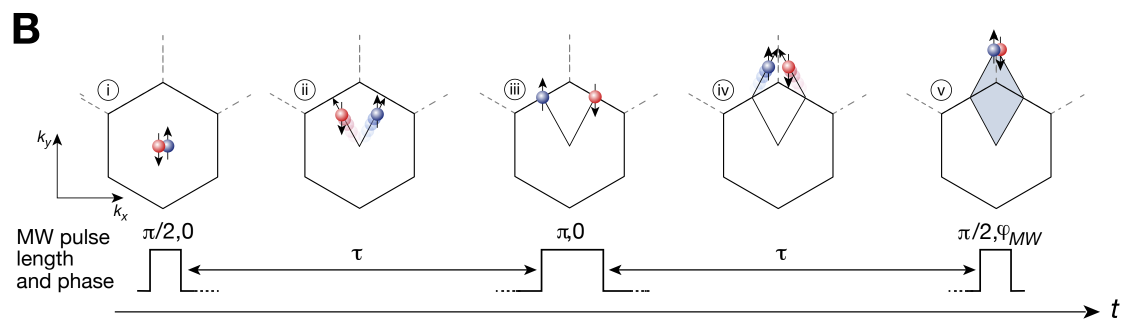


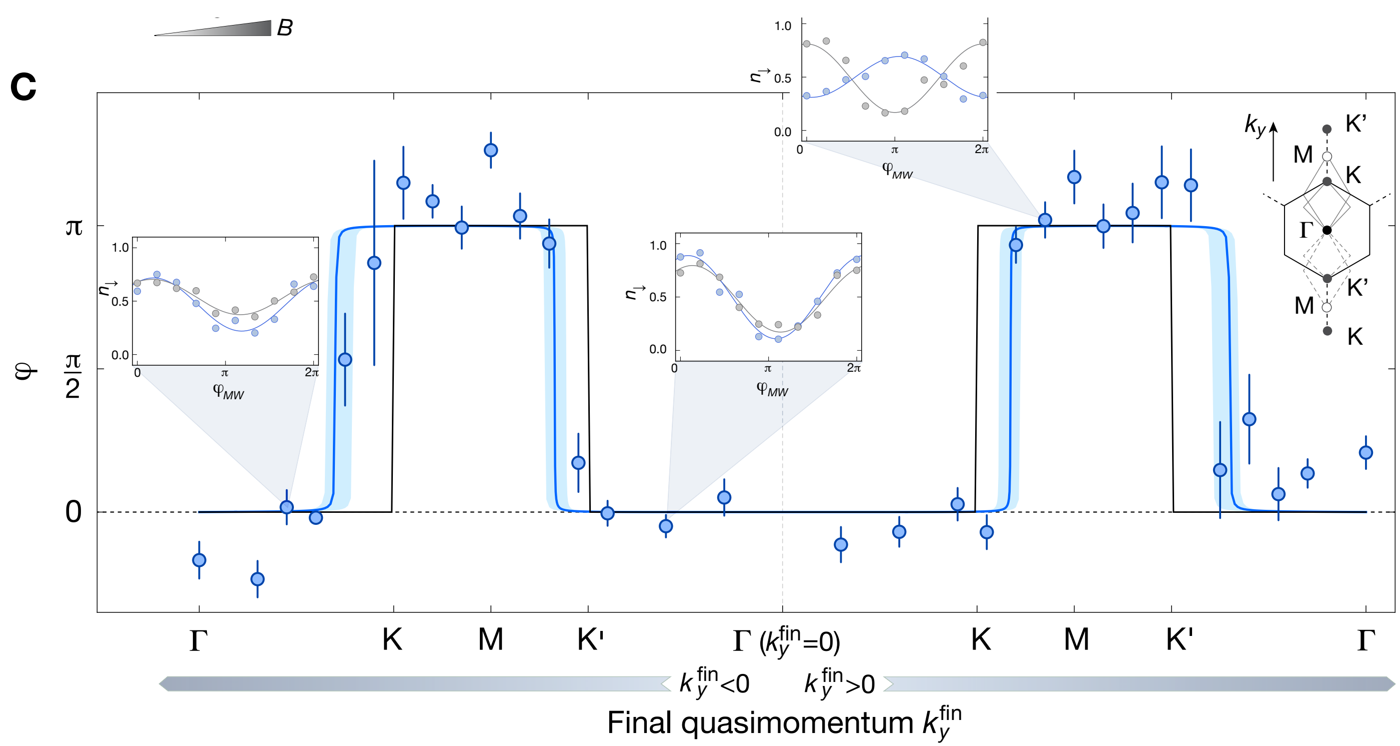
Interference Measurment

Duca, Li, Reitter, Bloch, Schleier-Smith, Schneider, Science (2015)

Quantitative probing of Berry flux

- interference with two different spin species (bosons)
- echo sequence
- direct probe of encircled Berry flux



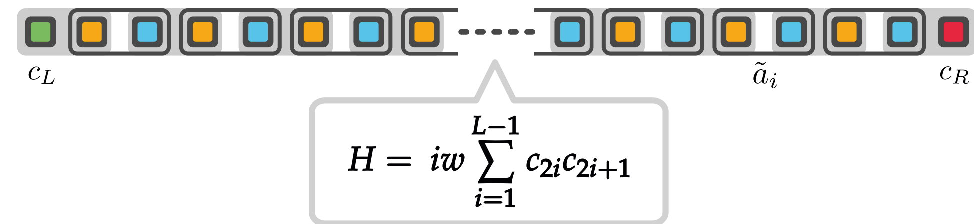


Topological states in a microscopic model of interacting one-dimensional fermions

Kitaev's Majorana chain

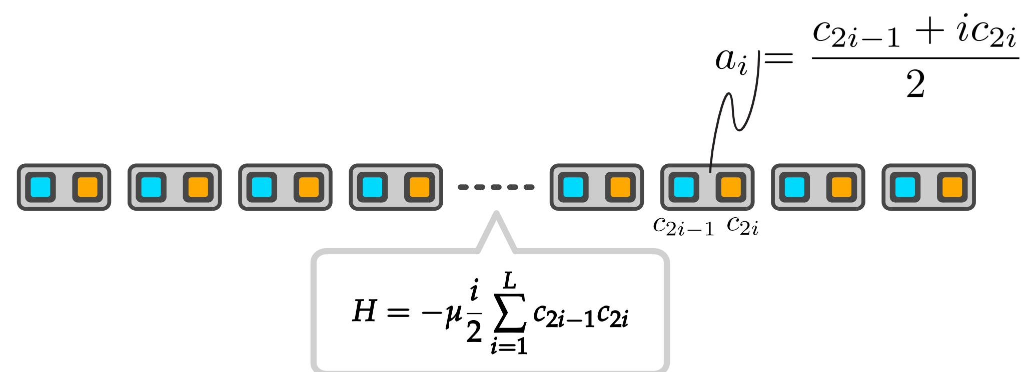
Kitaev's Majorana chain

- fermions on a 1D lattice
  with supefluid pairing term



$$H = -\sum_{i=1}^{L-1} \left[w a_i^\dagger a_{i+1} - \Delta a_i a_{i+1} + \text{h.c.} \right]$$

$$-\mu \sum_{i=1}^{L} a_i^\dagger a_i$$

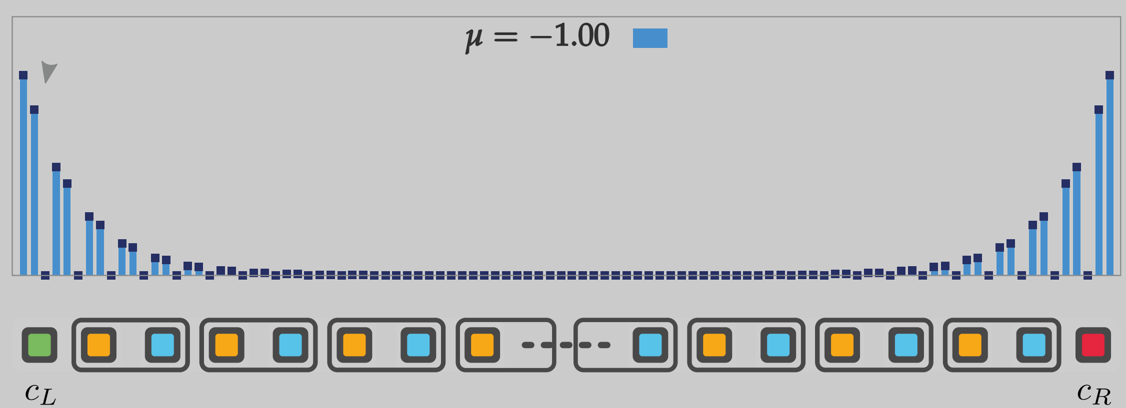


- exact solution by introducing
  Majorana operators

$$a_i = \frac{c_{2i-1} + i c_{2i}}{2}$$

Topological state

- robust ground state
  degeneracy
- non-local order
  parameter
- localized edge states

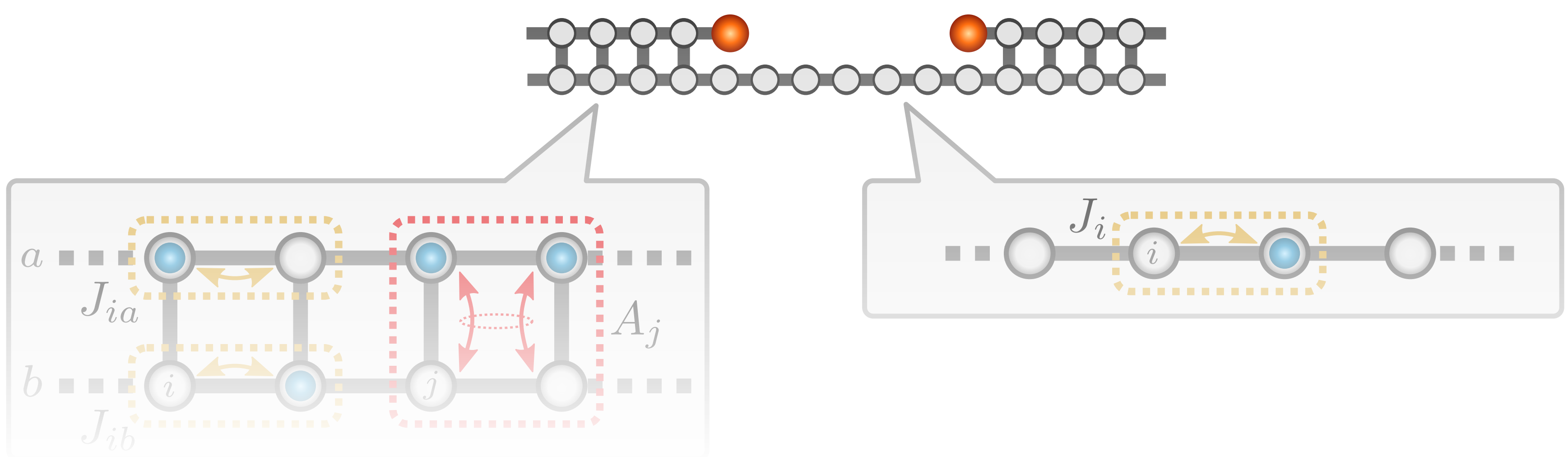


Beyond mean-field

$$H = -\sum_{i=1}^{L-1}\left[w a_i^\dagger a_{i+1} - \Delta a_i a_{i+1} + \text{h.c.}\right]$$

- mean-field coupling
- violates particle conservation

exist a particle conserving theory
with Majorana modes in one-dimension?



Beyond mean-field

→ M. Cheng and H.-H. Tu (2011). Physical Review B, 84(9), 094503.
Majorana edge states in interacting two-chain ladders of fermions.

→ J. D. Sau et al. (2011). Physical Review B, 84(14), 144509.
Number conserving theory for topologically protected degeneracy in one-dimensional fermions.

→ L. Fidkowski et al. (2011). Physical Review B, 84(19), 195436.
Majorana zero modes in one-dimensional quantum wires without long-ranged superconducting order.

→ J. Ruhman et al. (2014). arXiv:1412.3444
Topological States in a One-Dimensional Fermi Gas with Attractive Interactions.

Bosonization

→ C. V. Kraus et al. (2013). Physical Review Letters, 111(17), 173004.
Majorana Edge States in Atomic Wires Coupled by Pair Hopping.

Numerical

→ G. Ortiz et al. (2014). arXiv:1407.3793
Many-body characterization of topological superconductivity: The Richardson-Gaudin-Kitaev chain.

Long-range

Here:
- Sort-range interactions
- exact ground state
- "Majorana" like edge modes

N. Lang and H. P. Büchler, Phys. Rev. B 92, 041118 (2015).
F.Iemini,et al., Phys. Rev. Lett. 115, 156402 (2015).

Microscopic Hamiltonian

Hamiltonian

- double wire system

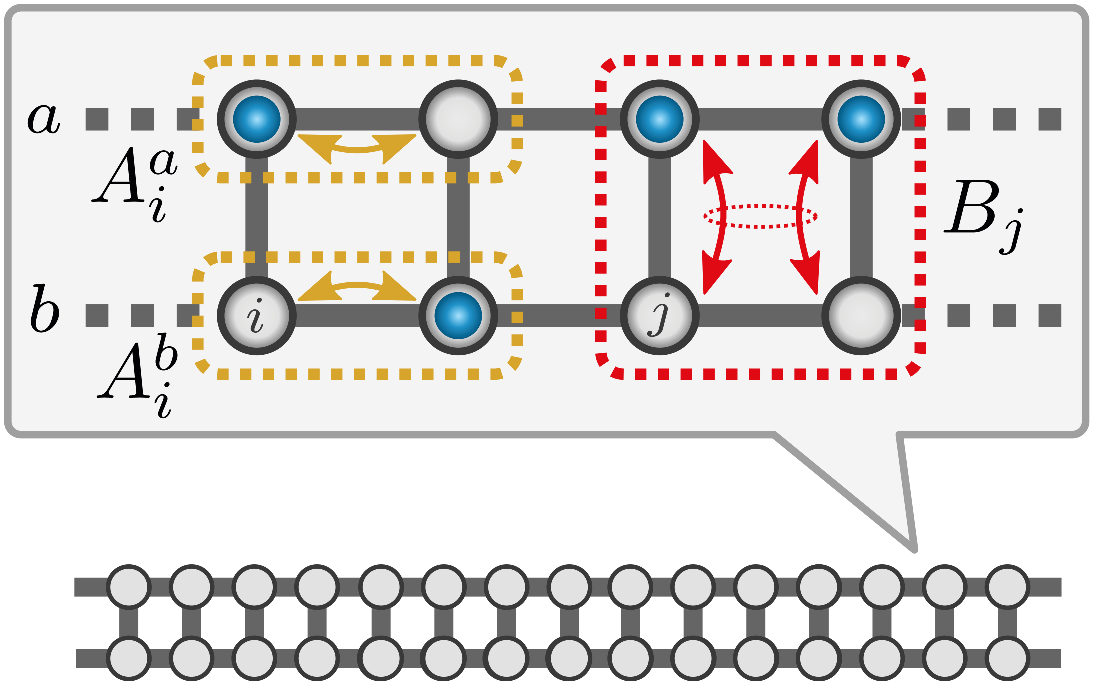
$$H = H_a + H_b + H_{ab}$$

- intra-chain contribution

$$H_a = \sum_i A_i^a \left(1 + A_i^a\right)$$

- inter-chain contribution

$$H_{ab} = \sum_i B_i \left(1 + B_i\right)$$



Symmetries

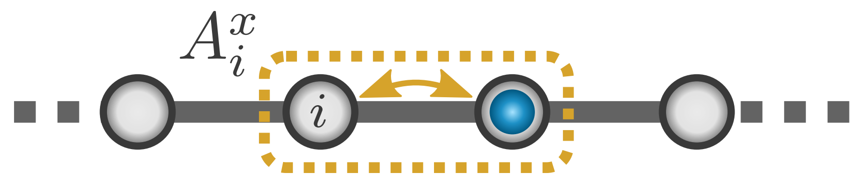
- total number of particles N
- time reversal symmetry T
- sub-chain parity P

Microscopic Hamiltonian

Inter-chain Hamiltonian

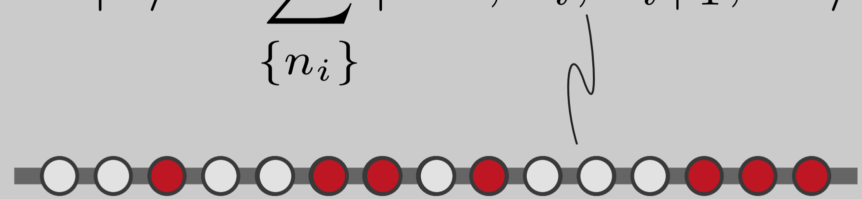
$$H_a = \sum_i A_i^a \left(1 + A_i^a\right)$$

$$A_i^a = a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i$$



- positive Hamiltonian
- zero-energy state is ground state

$$|n\rangle = \sum_{\{n_i\}} |\ldots, n_i, n_{i+1}, \ldots\rangle$$



equal weight superposition of all possible distribution of n fermions

Inter-chain Hamiltonian (expanded)

$$H_i^a = a_i a_{i+1}^\dagger + a_{i+1} a_i^\dagger + n_i^a \left(1 - n_{i+1}^a\right) + n_{i+1}^a \left(1 - n_i^a\right)$$

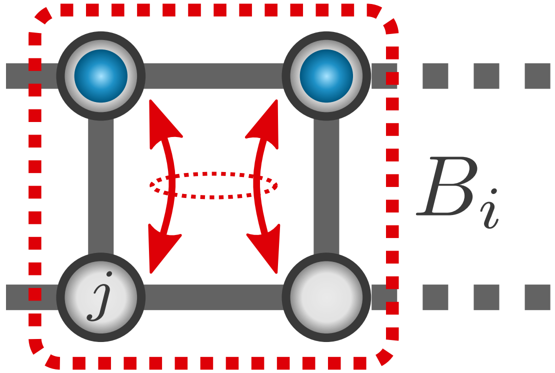
Microscopic Hamiltonian

Intra-chain Hamiltonian

$$H_{ab} = \sum_i B_i \left(1 + B_i\right)$$

$$B_i = a_i^\dagger a_{i+1}^\dagger b_i b_{i+1} + b_i^\dagger b_{i+1}^\dagger a_i a_{i+1}$$

pair-hopping between chains



- positive Hamiltonian
- zero-energy state is ground state
- fixed total number of particles

$$|\psi\rangle = \sum_n |n\rangle |N-n\rangle$$

equal weight superposition of all possible distribution of N fermions between the two wires

Intra-chain Hamiltonian (expanded)

$$H_{ab}^i = a_i^\dagger a_{i+1}^\dagger b_i b_{i+1} + b_i^\dagger b_{i+1}^\dagger a_i a_{i+1} + n_i^a n_{i+1}^a \left(1 - n_i^b\right)\left(1 - n_{i+1}^b\right) + n_i^b n_{i+1}^b \left(1 - n_i^a\right)\left(1 - n_{i+1}^a\right)$$

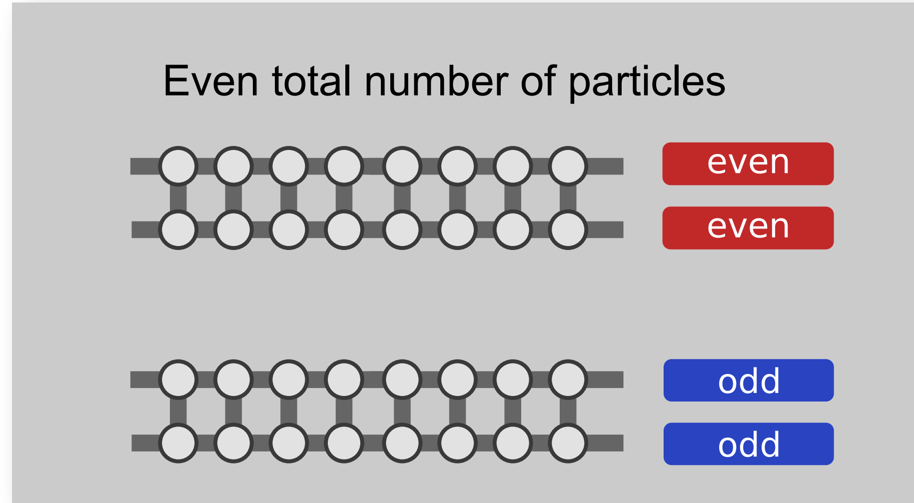
Ground state degeneracy

Two-open chains

- two-fold ground state degeneracy

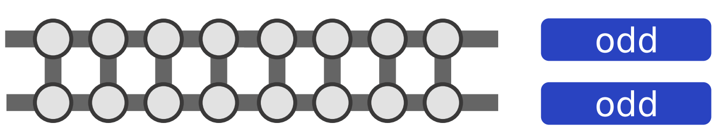
$$|\psi_{\mathrm{even}}\rangle = \sum_{n \in \mathrm{even}} |n\rangle |N-n\rangle$$

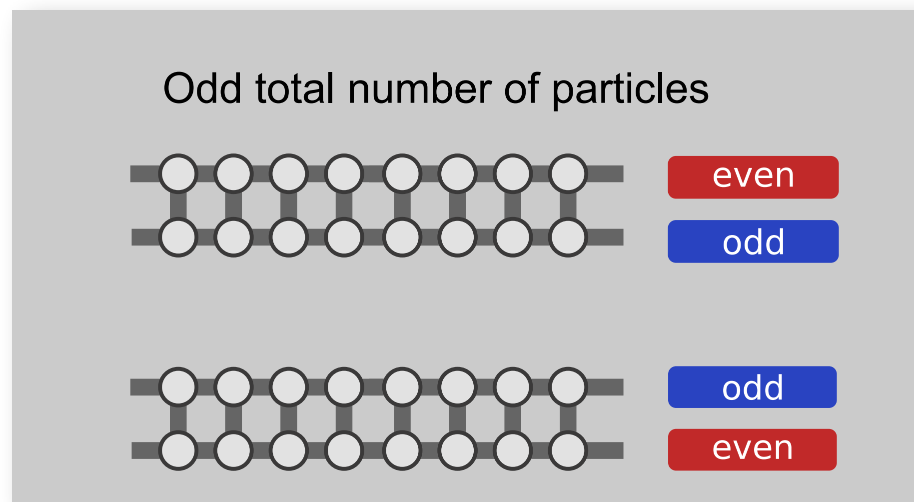
$$|\psi_{\mathrm{odd}}\rangle = \sum_{n \in \mathrm{odd}} |n\rangle |N-n\rangle$$



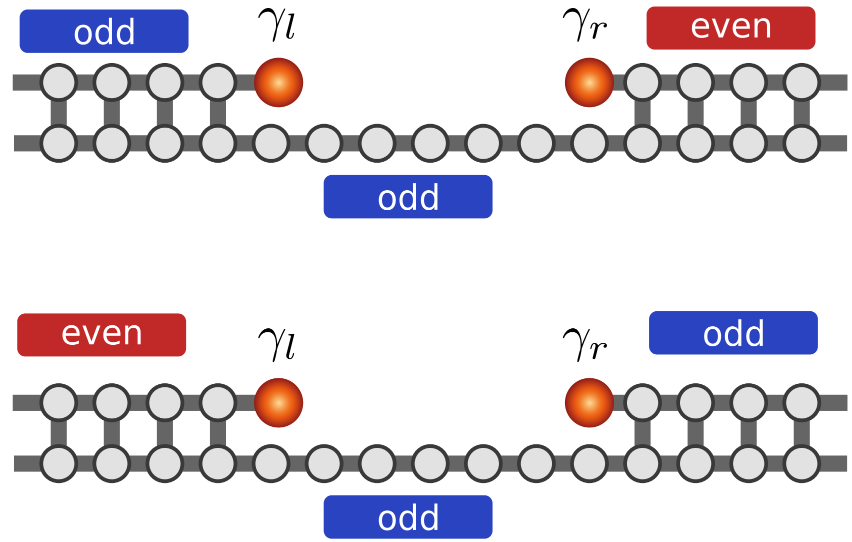
Two- closed chains

- only one zero energy state for total even number of particles





Wire networks



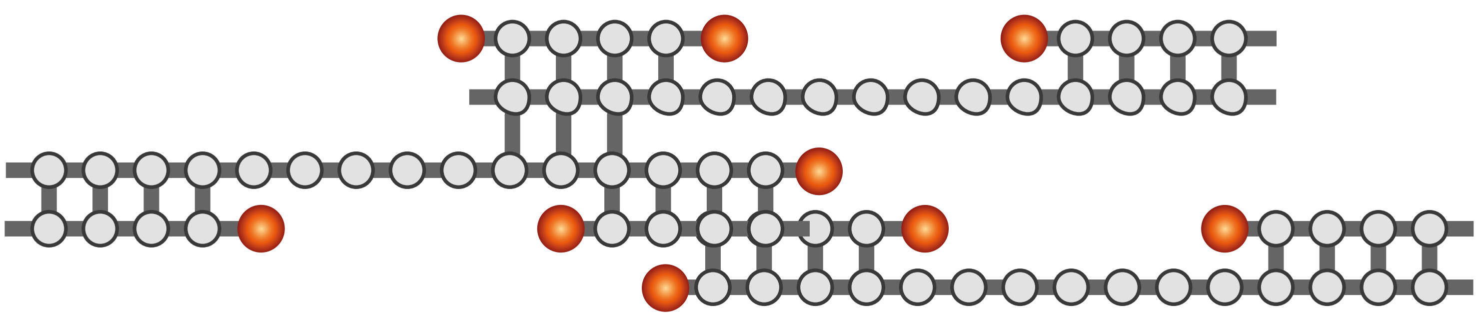
Networks of wires

- exact ground states for arbitrary networks

- degeneracy consistent with majorana modes at edges

$$2^{E/2-1}$$

number of edges

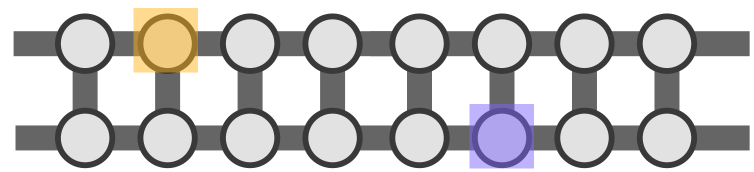


Ground state properties

Density-density correlations

- independent on ground state

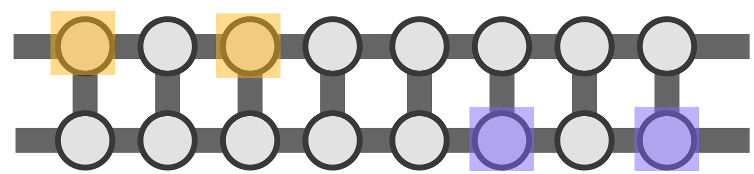
$$\langle n_i^{\sigma} n_j^{\sigma'} \rangle = \rho^2 \qquad i \neq j$$



Superfluid correlations

$$\langle a_i^\dagger a_{i+1}^\dagger a_j a_{j+1} \rangle = \rho(1-\rho)$$

- long-range superfluid p-wave pairing
- exponential decay

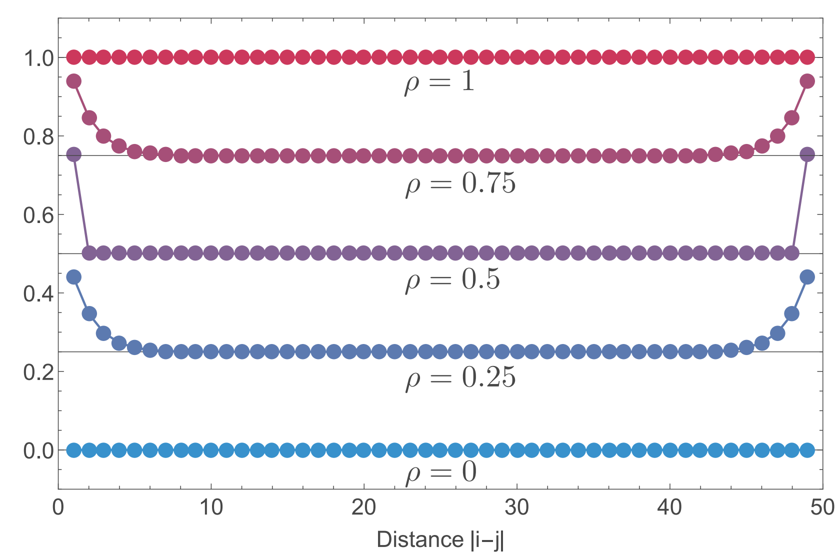


Green's function

$$\langle a_i^\dagger a_j \rangle$$

- exponential decay
- revival at the edge

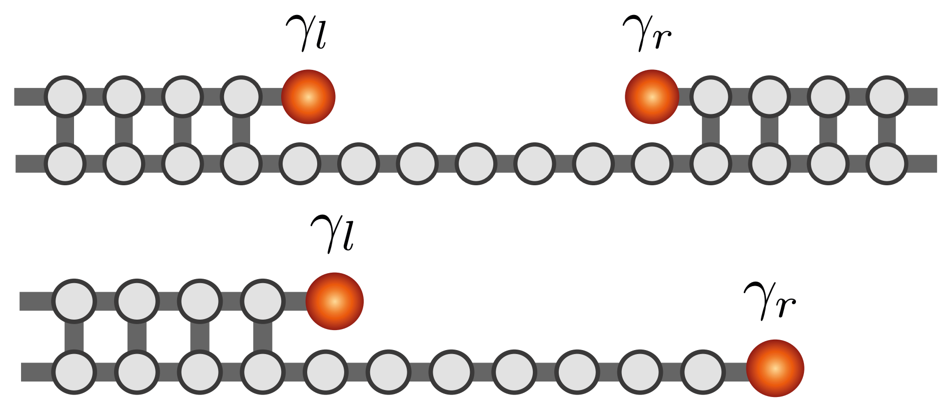
existence of edge modes



Ground state properties

Stability of ground state degeneracy of edge states

- stable under all local perturbations
- splitting decays exponentially

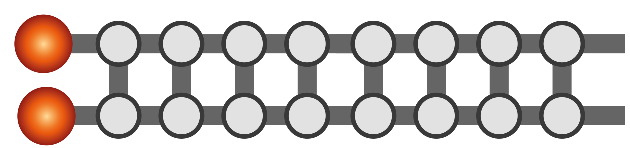


Stability of ground state degeneracy for open wires

- Protected by either time-reversal symmetry or subchain parity

$$a_i^\dagger b_i + b_i^\dagger a_i$$: stable under time reversal hopping

$$i a_i^\dagger b_i - i b_i^\dagger a_i$$: finite overlap between two ground states

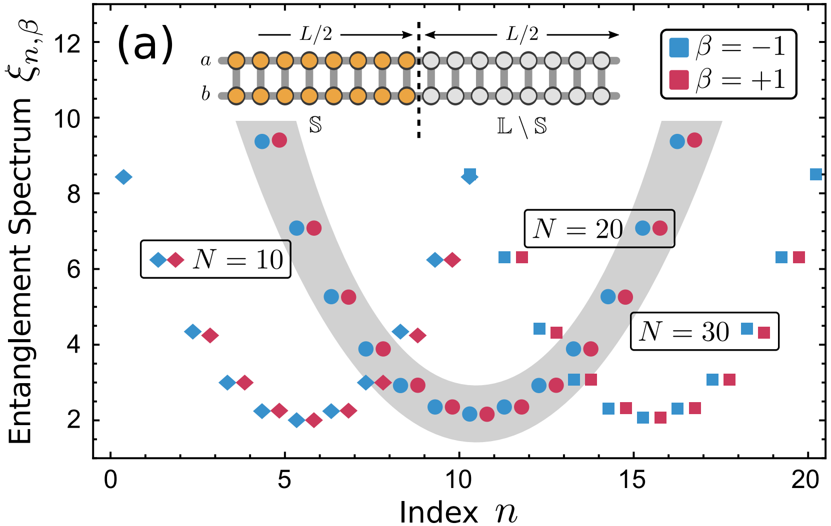


Entanglement spectrum

Entanglement spectrum

- two fold degenerate entanglement spectrum

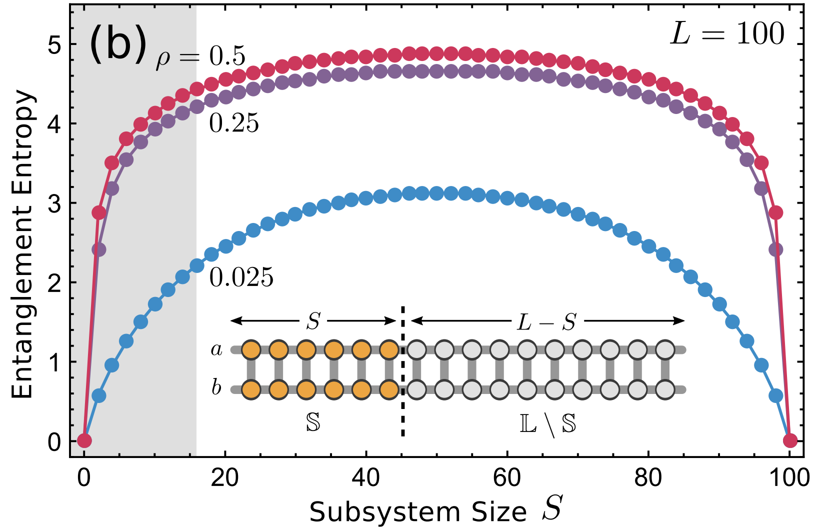
⇒ consistent with a topological state



Entanglement entropy

- area law with logarithmic correction

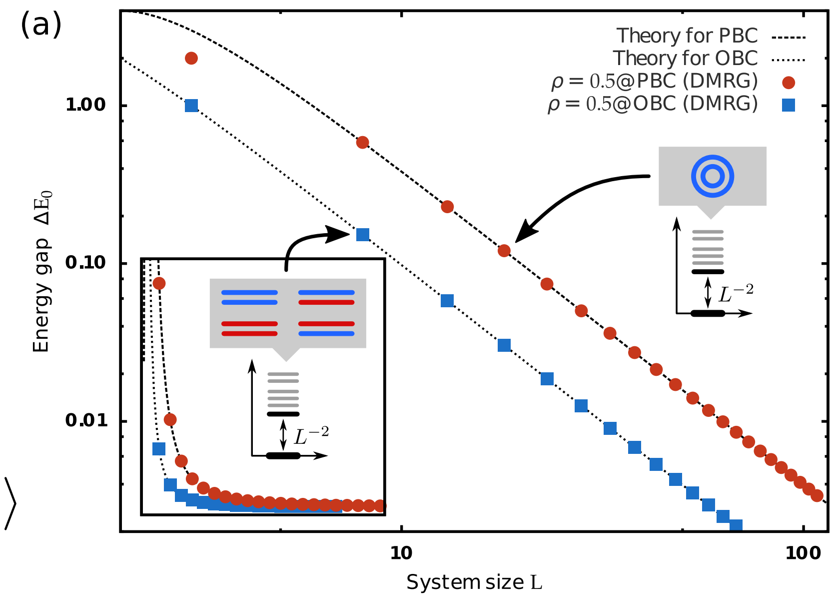
⇒ indication of a gapless state



Excitation spectrum

Low-energy excitations

- Goldstone mode due to broken U(1) symmetry
- exact wave function for single phase kink excitation



$$|k\rangle = \sum_j \cos\left[k\left(j-1/2\right)\right]\left[(-1)^{n_j^a} + (-1)^{n_j^b}\right]|\psi\rangle$$

Quadratic dispersion relation

$$\epsilon_k = 4\sin^2 k/2 \sim k^2$$

System is in a critical point

- vanishing compressibility
- Goldstone mode with quadratic dispersion

Excitation spectrum

Is there a single particle gap?

- expected from exponential decay of Green's function

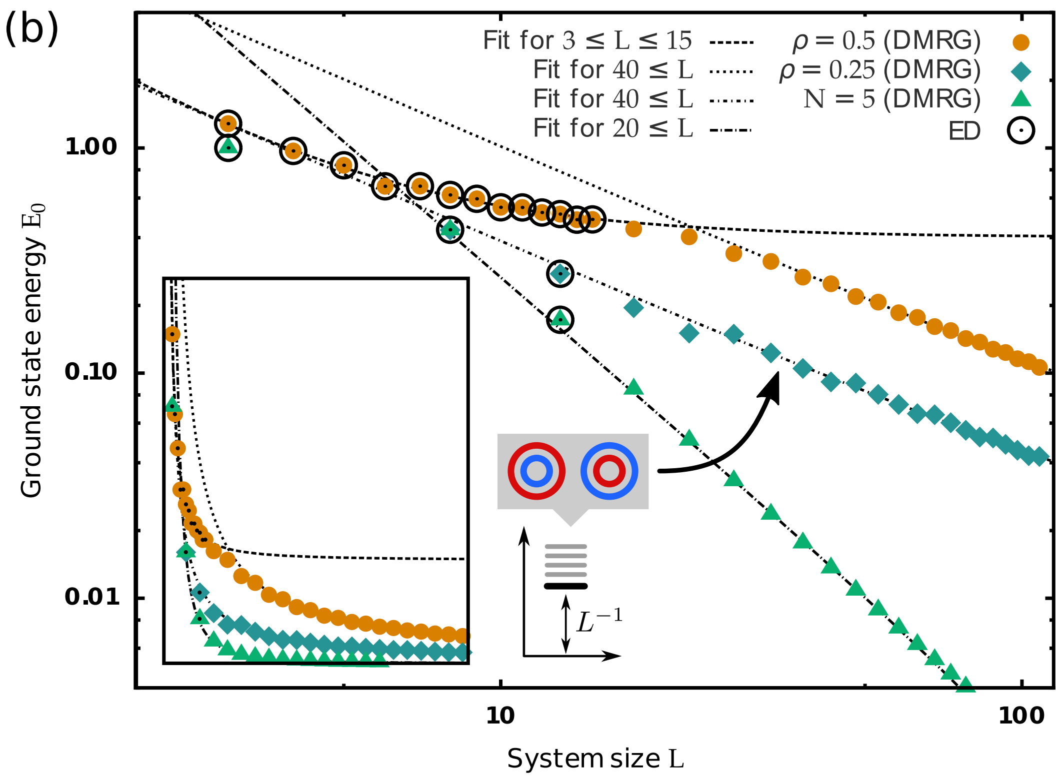
$\langle a_i^\dagger a_j \rangle$ property of the wave function and not the Hamiltonian

Proper definition

- ground state energy in a closed system for odd number of particles

- numerical
- analytical

Absence of single particle gap



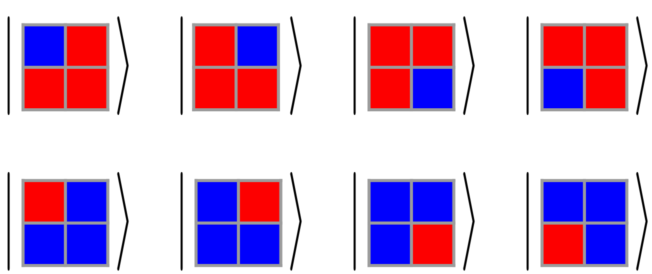
Non-abelian Braiding statistics

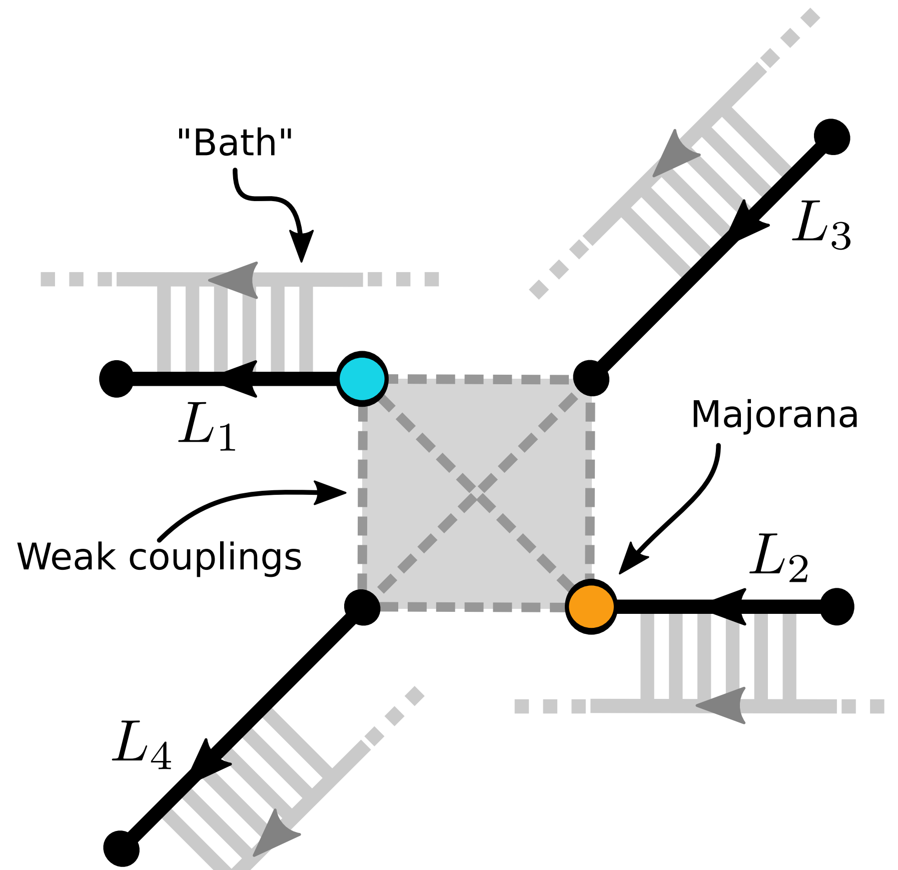
Setup for braiding of
two edge states

- wire network with two edges
- restriction to the low energy sector
- very weak coupling terms: adiabatic switching between them

8 relevant states

- characterized by subchain parity

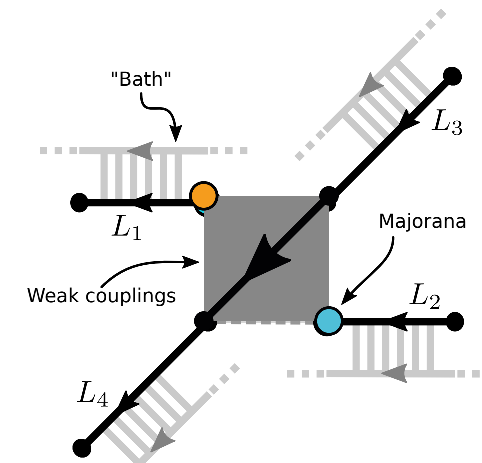


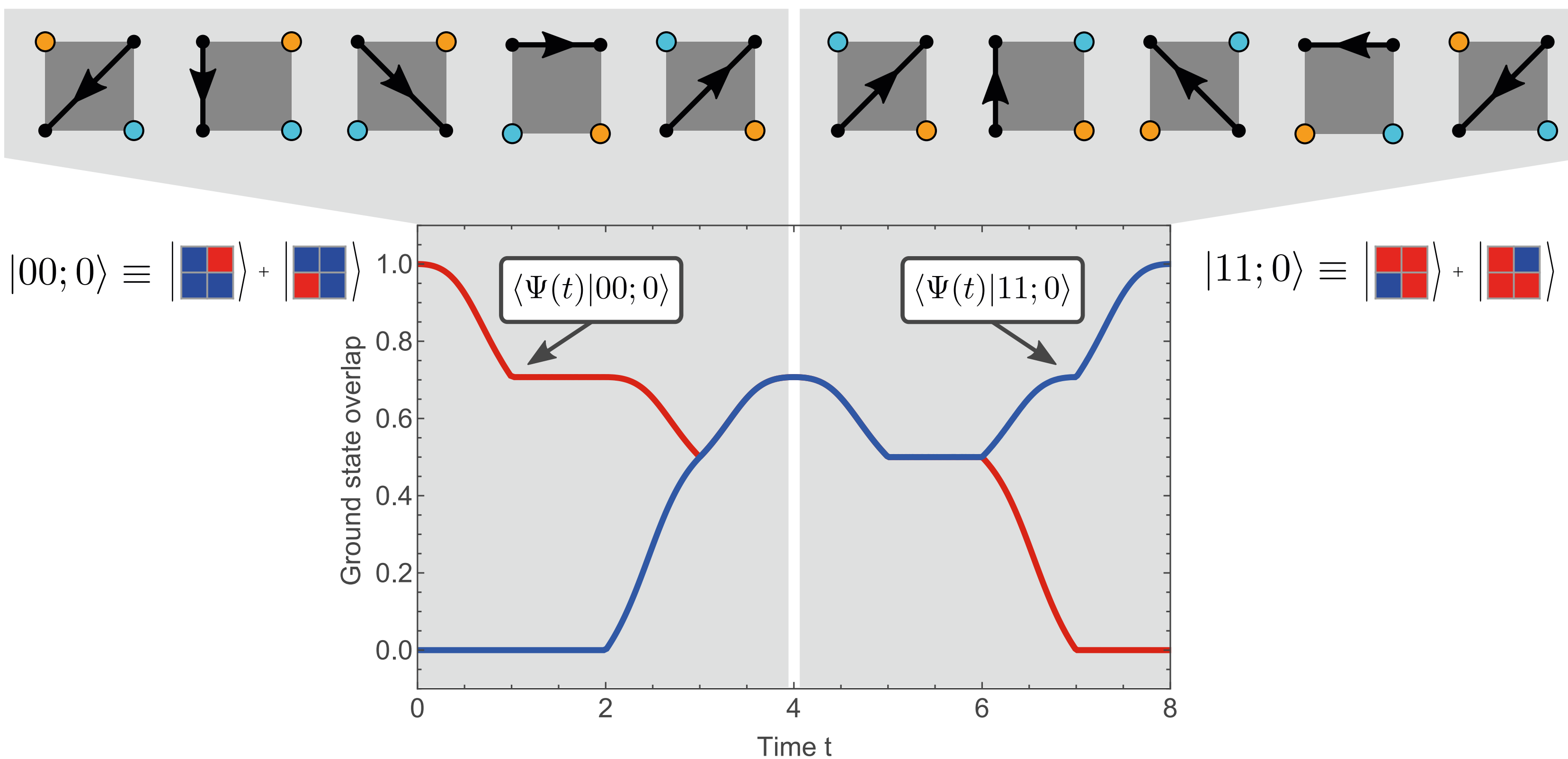


Non-abelian Braiding statistics

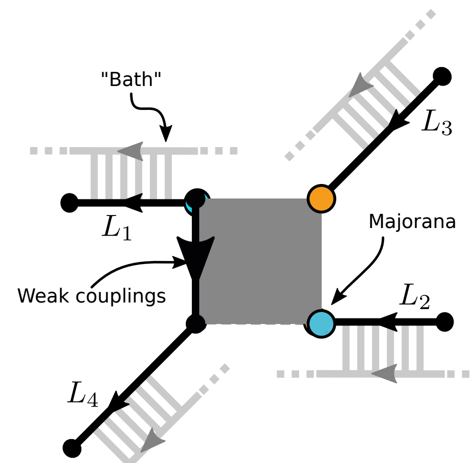
Adiabatic switching of coupling

- transformation of the ground state according to the non-abelian statistic of Majorana modes





Non-abelian Braiding statistics



Adiabatic switching of coupling

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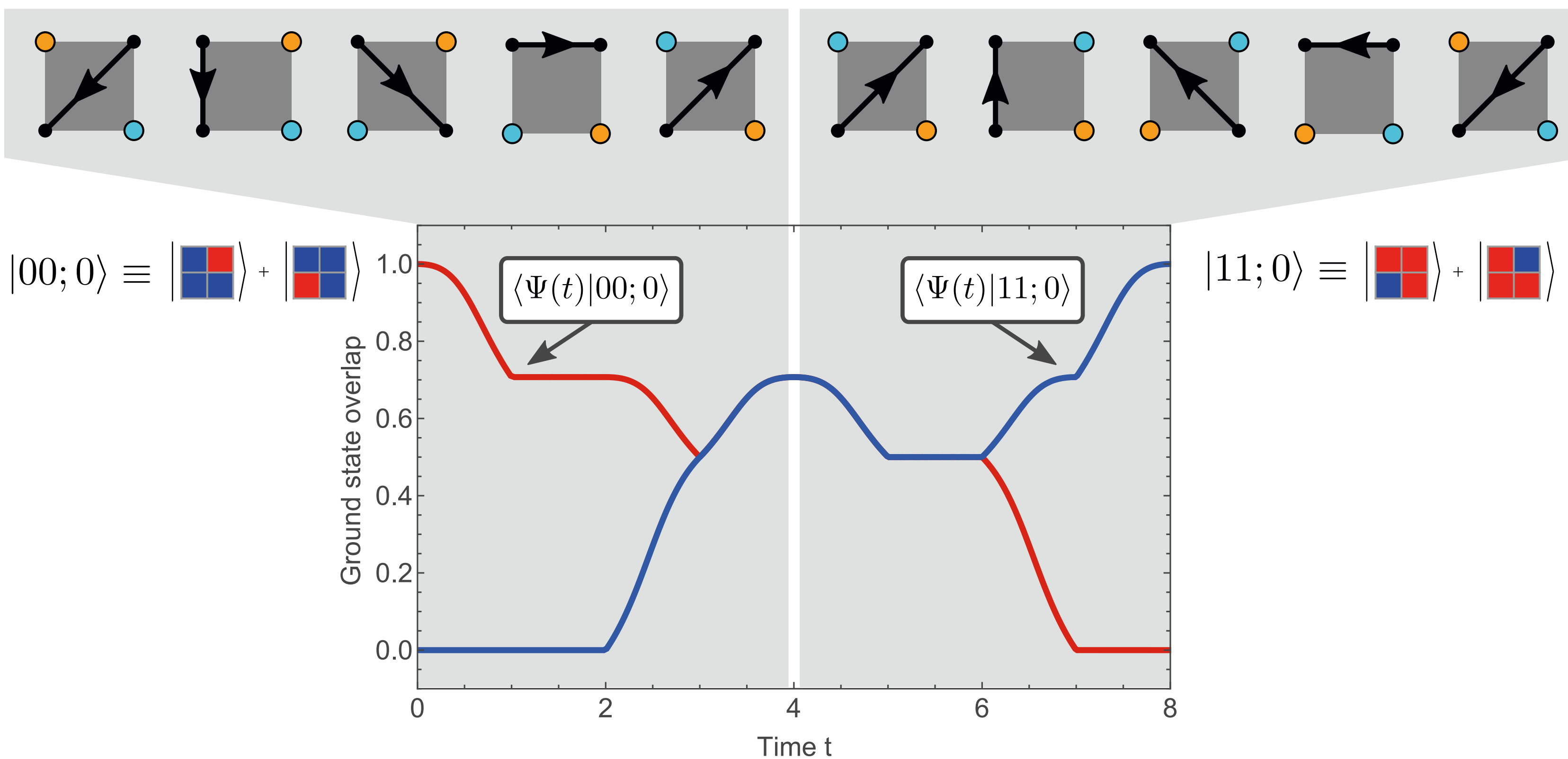
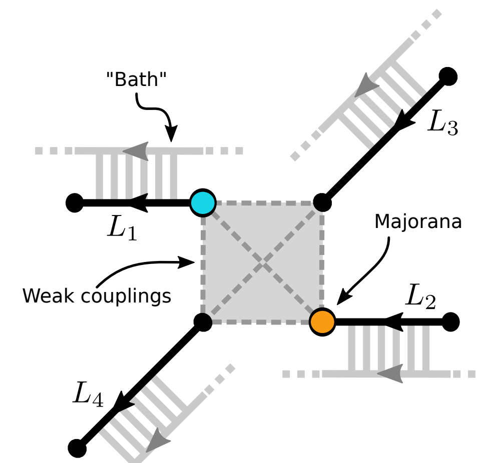
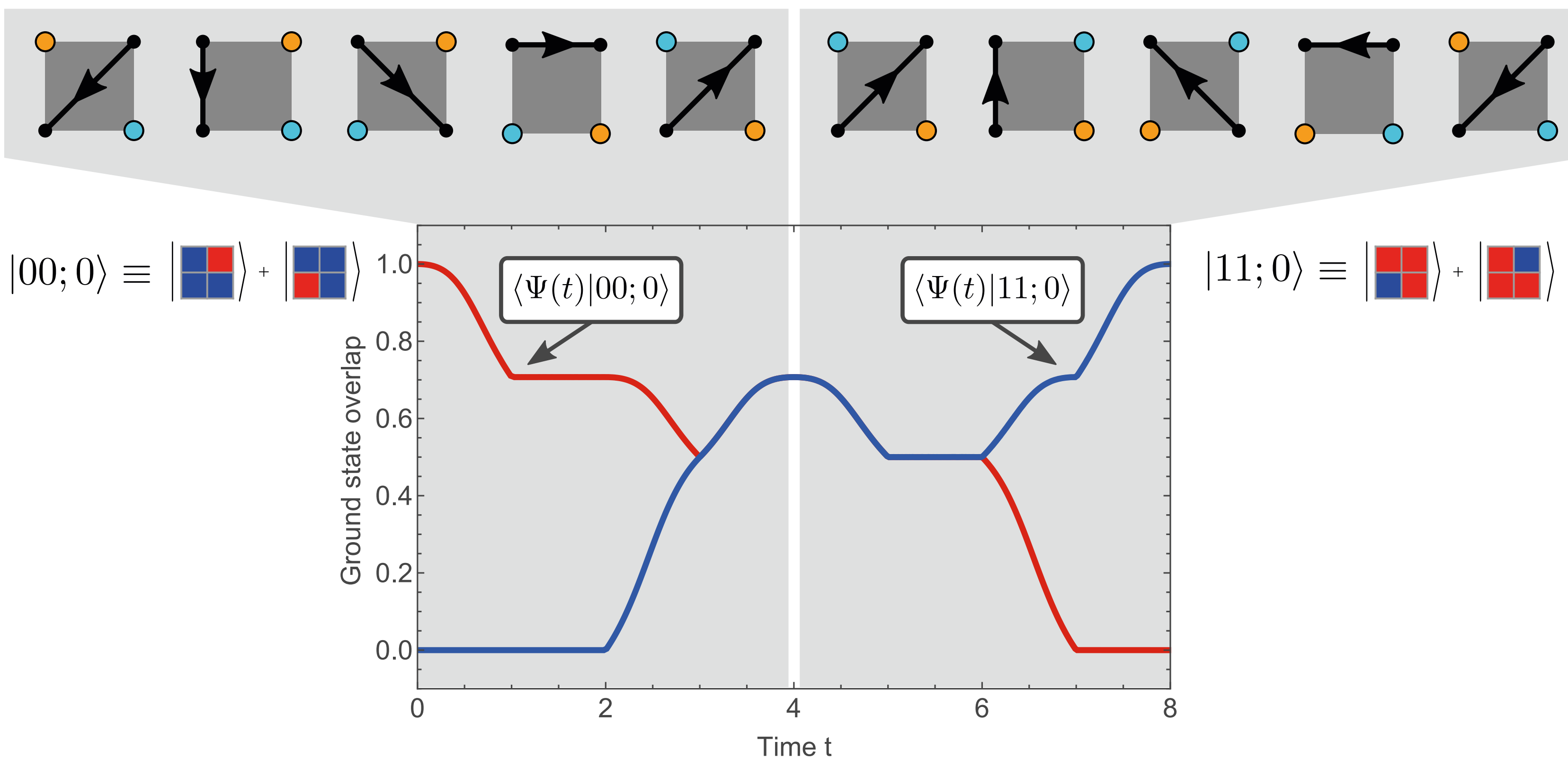


Non-abelian Braiding statistics

Adiabatic switching of coupling

- transformation of the ground state according to the non-abelian statistic of Majorana modes





Conclusion and outlook

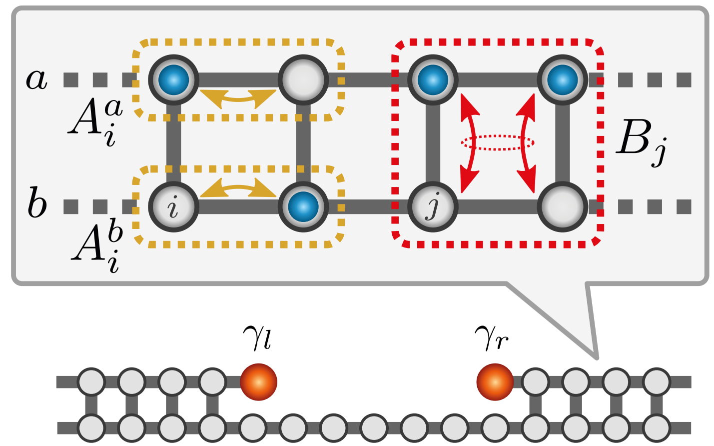
Majorana like Edge modes

- exact solvable system with
  fixed particle number

- analytical demonstration of
  Majorana edge modes

- toy model for understanding
  gapless topological states

- stable topological state expected
  for decreasing the attractive interaction



$$H_{ab} = \sum_i \left[B_i + \lambda B_i^2 \right]$$