Many-Body Localization & Entanglement

Nicolas Laflorencie
CNRS - LPT Toulouse

Acknowledgements

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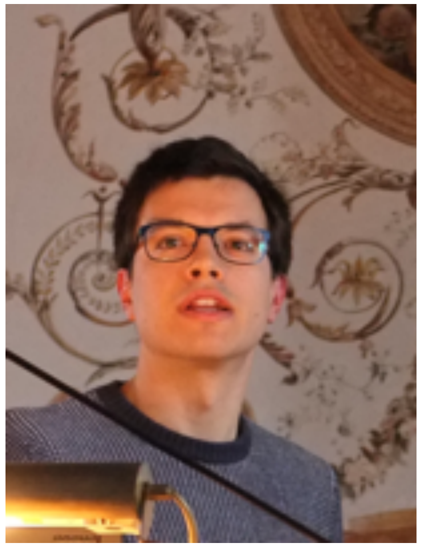
now at Urbana-Champaign



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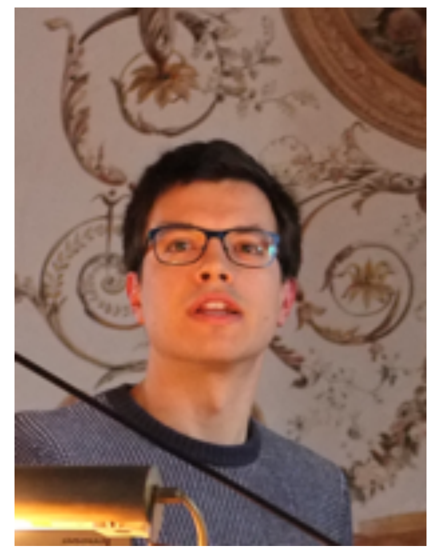

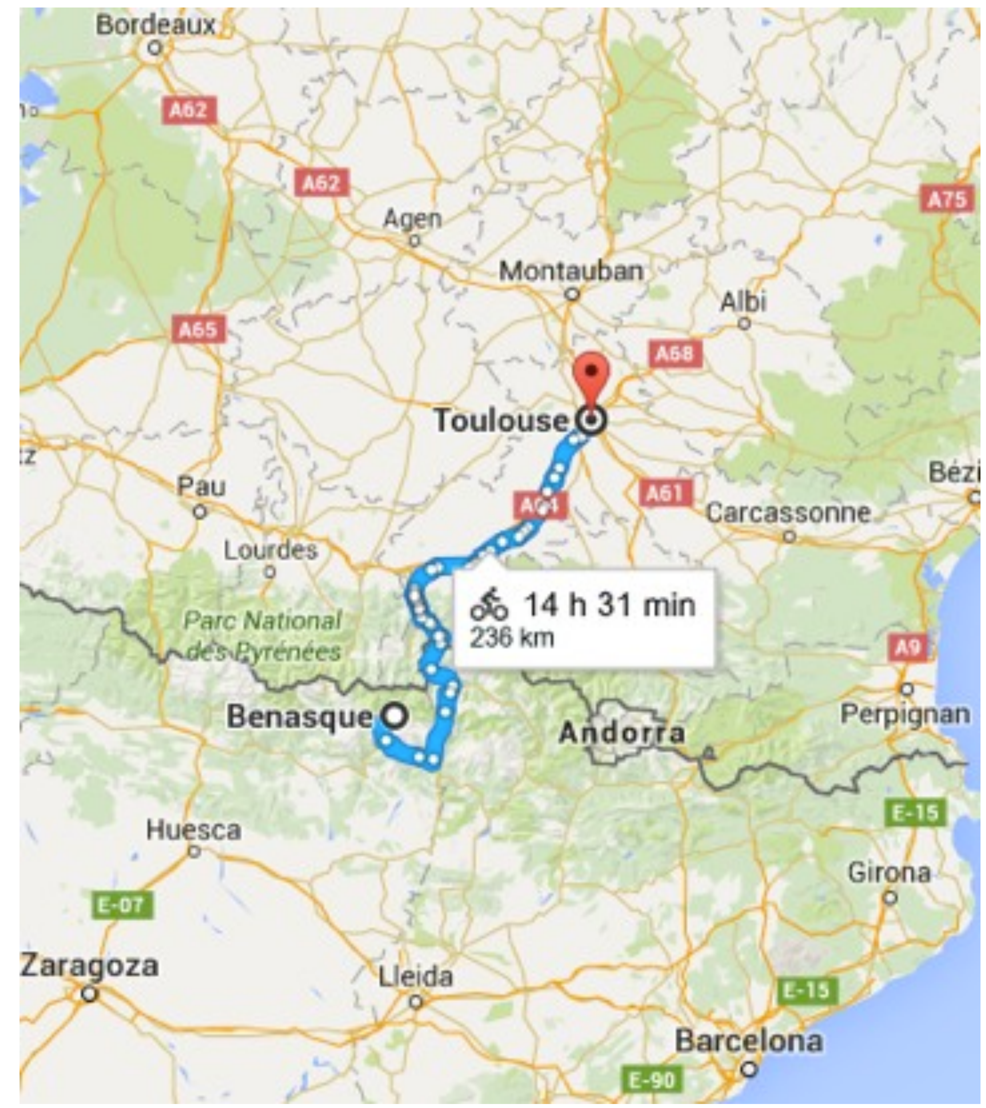

PROJET FINANCÉ PAR L'ANR - PROJECT FUNDED BY THE ANR

ANR

UNIVERSITÉ TOULOUSE III PAUL SABATIER

Who / Why care of MBL?

- ***Many many people...***

Who / Why care of MBL?

- *Many many people...*

(including experimentalists)

RESEARCH ARTICLE

QUANTUM GASES

Observation of many-body localization of interacting fermions in a quasirandom optical lattice

Michael Schreiber,[1,2] Sean S. Hodgman,[1,2] Pranjal Bordia,[1,2] Henrik P. Lüschen,[1,2] Mark H. Fischer,[3] Ronen Vosk,[3] Ehud Altman,[3] Ulrich Schneider,[1,2,4] Immanuel Bloch[1,2]*

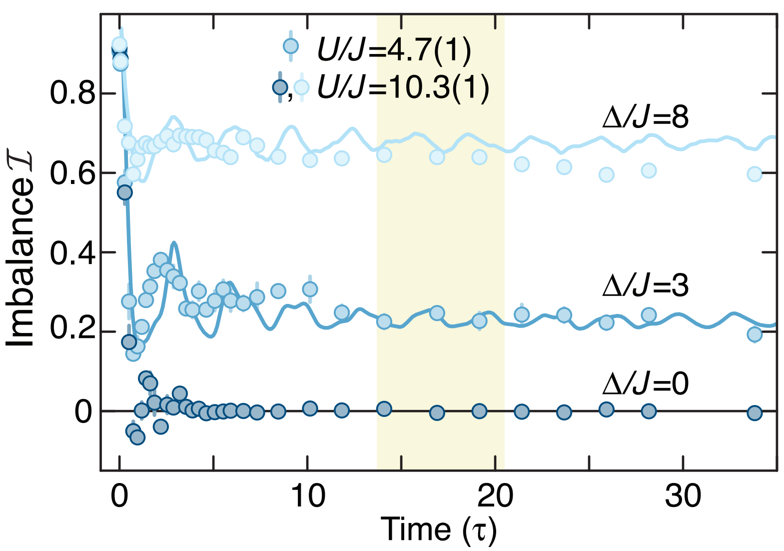

Fig. 2. Time evolution of an initial CDW.

SCIENTIFIC REPORTS

OPEN

Evidence for a Finite-Temperature Insulator

M. Ovadia[1,†], D. Kalok[1], I. Tamir[1], S. Mitra[1], B. Sacépé[1,2,3] & D. Shahar[1]

Received: 10 May 2015
Accepted: 29 July 2015
Published: 27 August 2015

In superconductors the zero-resistance current-flow is protected from dissipation at finite temperatures (T) by virtue of the short-circuit condition maintained by the electrons that remain in the condensed state. The recently suggested finite-T insulator and the "superinsulating" phase are different because any residual mechanism of conduction will eventually become dominant as the finite-T insulator sets-in. If the residual conduction is small it may be possible to observe the transition to these intriguing states. We show that the conductivity of the high magnetic-field insulator terminating superconductivity in amorphous indium-oxide exhibits an abrupt drop, and seem to approach a zero conductance at $T < 0.04$ K. We discuss our results in the light of theories that lead to a finite-T insulator.

arXiv.org > quant-ph > arXiv:1508.07026

Quantum Physics

Many-body localization in a quantum simulator with programmable random disorder

Jacob Smith, Aaron Lee, Philip Richerme, Brian Neyenhuis, Paul W. Hess, Philipp Hauke, Markus Heyl, David A. Huse, Christopher Monroe

(Submitted on 27 Aug 2015)

Who / Why care of MBL?

- ***Many many people... (including experimentalists)***

- ***Probably because it is interesting!...***
  - ➡ Go beyond single-particle Anderson localization
  - ➡ Address the question of "thermalization" in closed systems
  - ➡ Eigenstates "dynamical" transition
  - ➡ Unusual entanglement properties
  - ➡ Very peculiar out-of-equilibrium properties
  - ➡ MBL can help to store quantum information at long time

➡ ***Good theoretical challenge***

For recent reviews, see: Nandkishore and Huse,
Many-Body Localization and Thermalization in Quantum Statistical Mechanics
Annual Review of Condensed Matter Physics (2015)

Where are we now...?

- Anderson localization survives interactions
- Dynamical transition for high-energy eigenstates at finite disorder strength

Metallic/Ergodic

- GOE level statistics
- Thermalization (ETH)
- Volume-law entanglement
- Ballistic entanglement growth(?)

Insulator/MBL

- Poisson level statistics
- No thermalization
- Area-law entanglement
- logarithmic entanglement growth
- quasi-local integrals of motion
- Eigenstates ~ MPS

disorder strength

working plan

- Energy-resolved eigenstates properties (=> mobility edge?)
- MBL transition: Finite size scaling and critical exponents(?)
- Dynamical properties after a high-energy quench

entanglement growth? spreading of correlations?

Editors' Suggestion   Rapid Communication

Many-body localization edge in the random-field Heisenberg chain

David J. Luitz, Nicolas Laflorencie, and Fabien Alet
Phys. Rev. B **91**, 081103(R) – Published 9 February 2015

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **00**, 000200(R) (2016)

Extended slow dynamical regime close to the many-body localization transition

David J. Luitz,[1,2,*] Nicolas Laflorencie,[2,†] and Fabien Alet[2,‡]
[1]*Department of Physics and Institute for Condensed Matter Theory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[2]*Laboratoire de Physique Théorique, IRSAMC, Université de Toulouse, CNRS, 31062 Toulouse, France*
(Received 30 November 2015; published xxxxxx)

«Standard model» for MBL

$$\mathcal{H} = \sum_{i=1}^{L} \vec{S}_i \cdot \vec{S}_{i+1} - h_i S_i^z \qquad h_i \in [-h, h]$$

Numerical methods for excited states? No groundstate methods (Lanczos, DMRG(?), QMC), no finite temperature methods.

Exact diagonalization?

[*Pal 2010*]

- restrict to $S_z = 0$ Hilbert space with dimension $\binom{L}{\frac{L}{2}}$
- $L_{\mathrm{max}} = 16.$
- **Localized phase** at strong disorder, **Ergodic phase** at weak disorder.
- $h_c \approx 3.5.$

Spectral statistics: Gap ratio

- Level statistics: natural tool to check for localization
  - Thermal (ETH) phase: expect Random Matrix Theory (in particular GOE) to correctly capture highly-excited eigenvalues
  - MBL phase: expect Poisson statistics (no correlation, no level repulsion)

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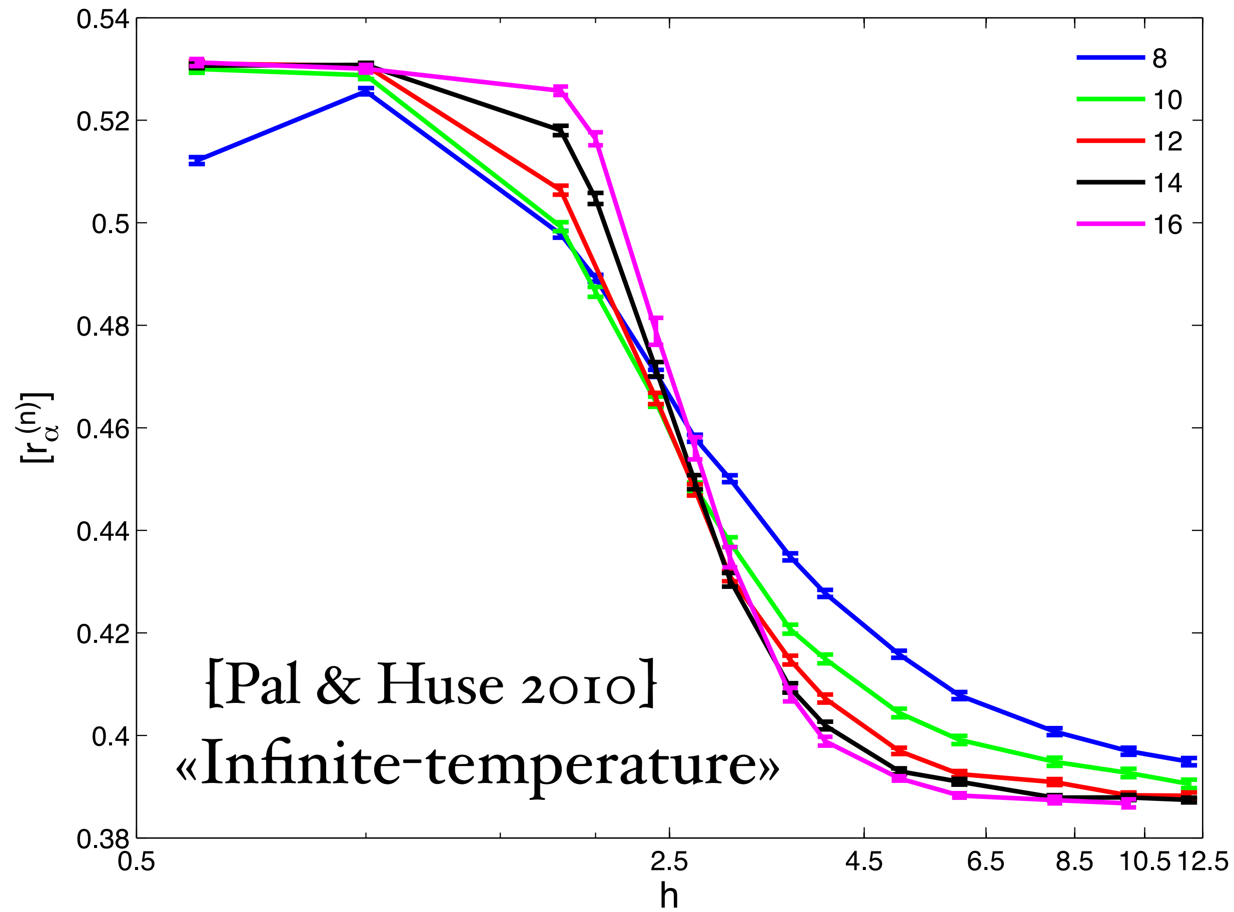
- <u>**Gap ratio**</u> [Oganesyan, Huse]

$$g_n = |E_n - E_{n-1}| \qquad r = \min(g_n, g_{n+1}) / \max(g_n, g_{n+1})$$

$$\langle r \rangle_{\text{GOE}} \simeq 0.5307 \qquad \langle r \rangle_{\text{Poisson}} \simeq 0.3863$$

- Extensively used in the many-body context



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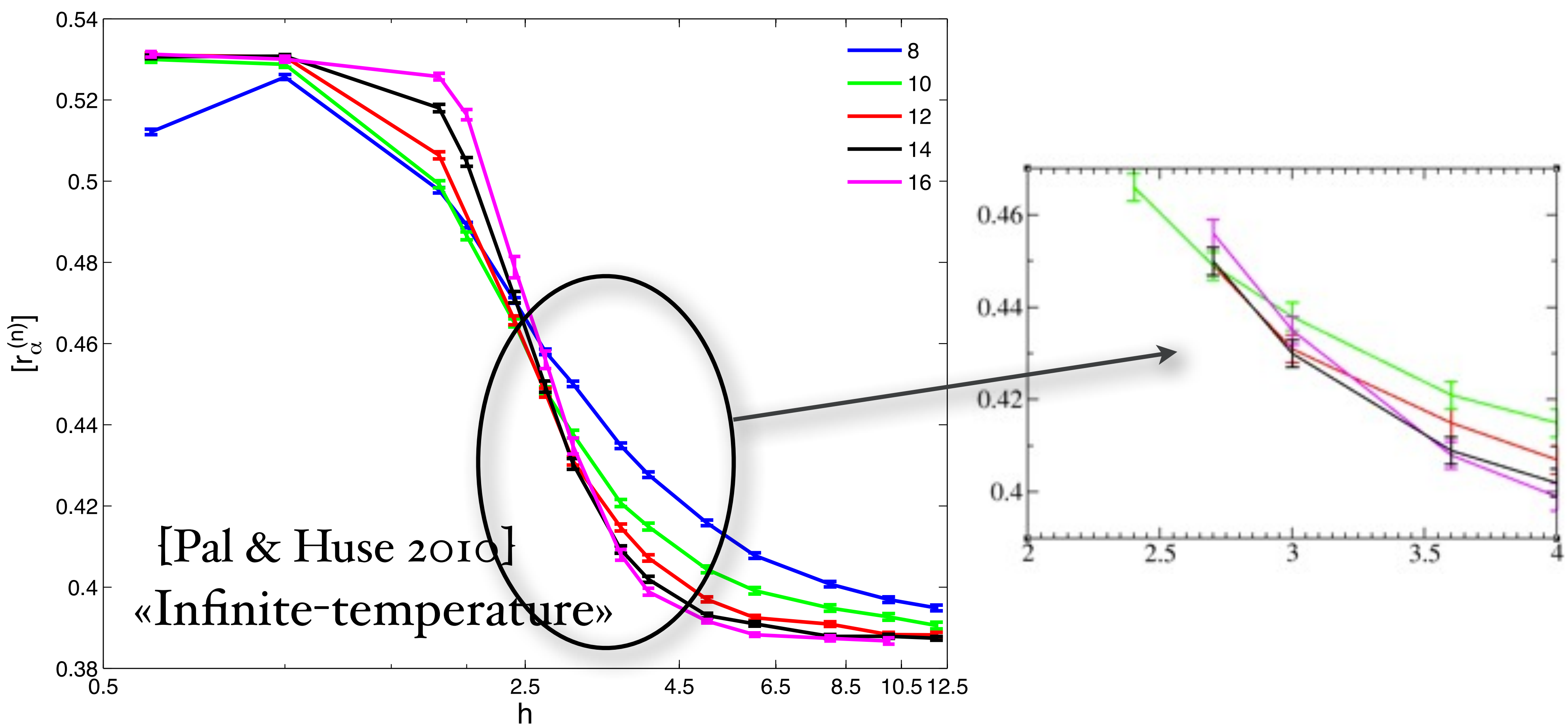
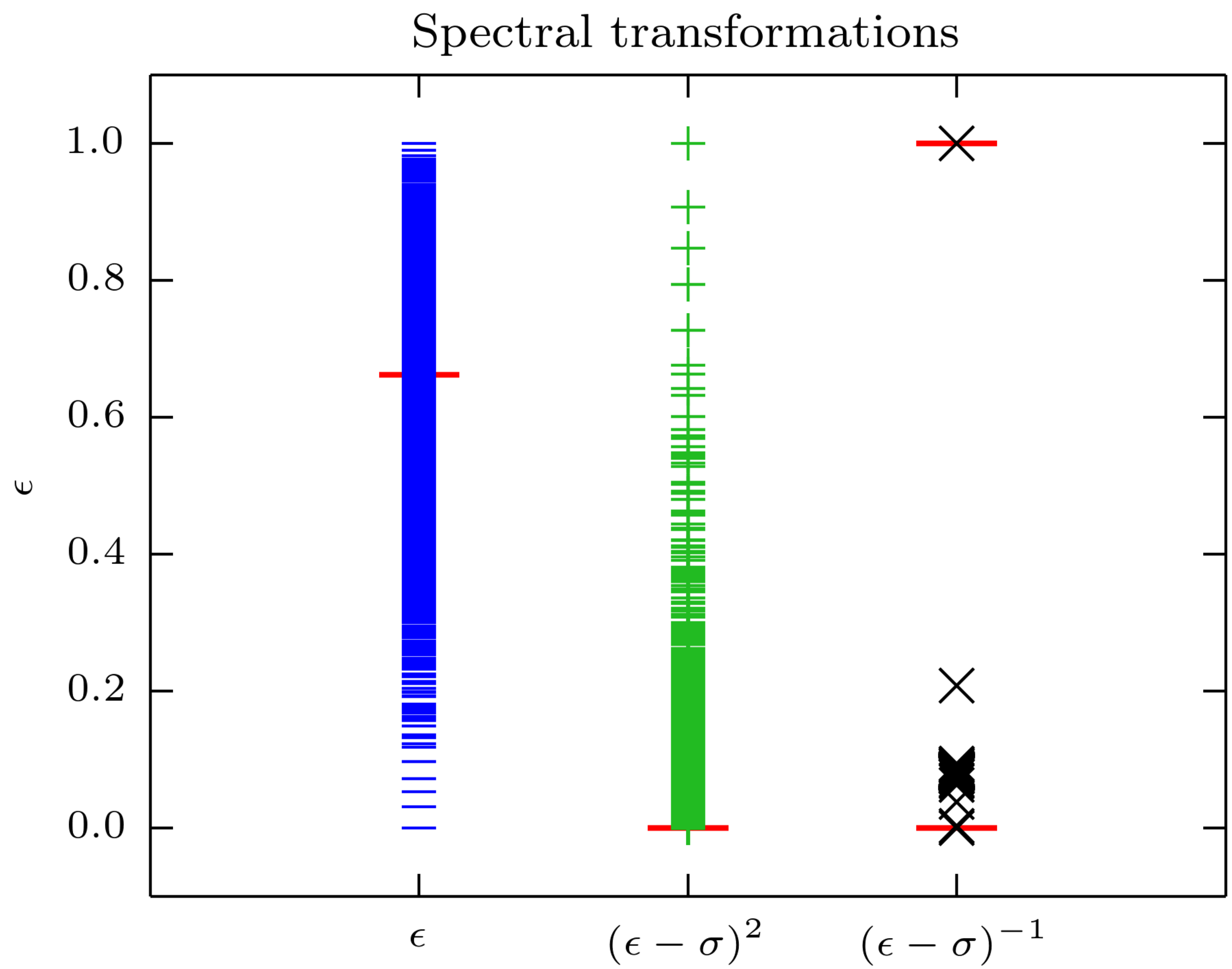


Can we do better?
Exact Diag. for high-energy eigenstates

- Hilbert space dimension for L=22: 705432, $\sim$ 2TiB for all eigenvectors in single precision.
- Krylov space methods only good for extrema in spectrum.
- Use Krylov space method for transformed spectrum.



Best transformation: $(\mathbf{H}-\sigma\mathbf{1})^{-1}$.
Don't calculate full inverse but $(\mathbf{H}-\sigma\mathbf{1})=\mathbf{LU}$.
Iterative solver only needs $\mathbf{v}'$ as solution of $\mathbf{LUv}'=\mathbf{v}$.
Use MUMPS for sparse LU decomposition. (MUltifrontal Massively Parallel sparse direct Solver)

Exact Diag. for high-energy eigenstates

- Calculate E_{min} and E_{max}.
- Normalized energy

$$\epsilon = \frac{E - E_{\min}}{E_{\max} - E_{\min}} \in [0, 1]$$

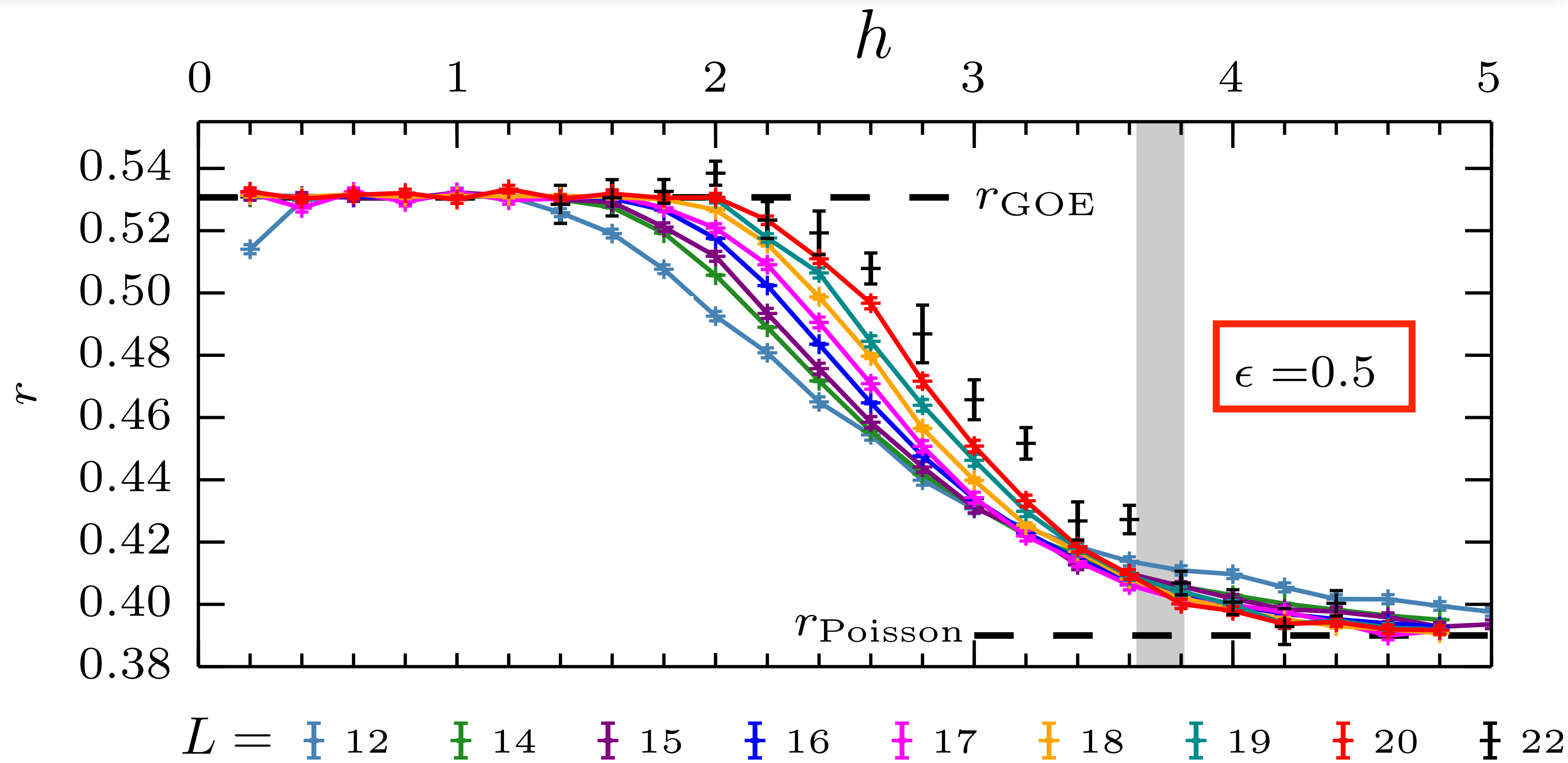
- Calculate 50 eigenpairs close to target $\sigma = 0.05, 0.1, \ldots, 0.95$.
- For each target: Average observables over 50 eigenpairs and ~ 1000 disorder realizations for every L and h.

Store wave functions on disk:
Total of $> 4 \cdot 10^6$ CPUh and ≈ 50TB of disk storage.

EOS machine in Toulouse

Spectral statistics: Gap ratio

- Energy-resolved data



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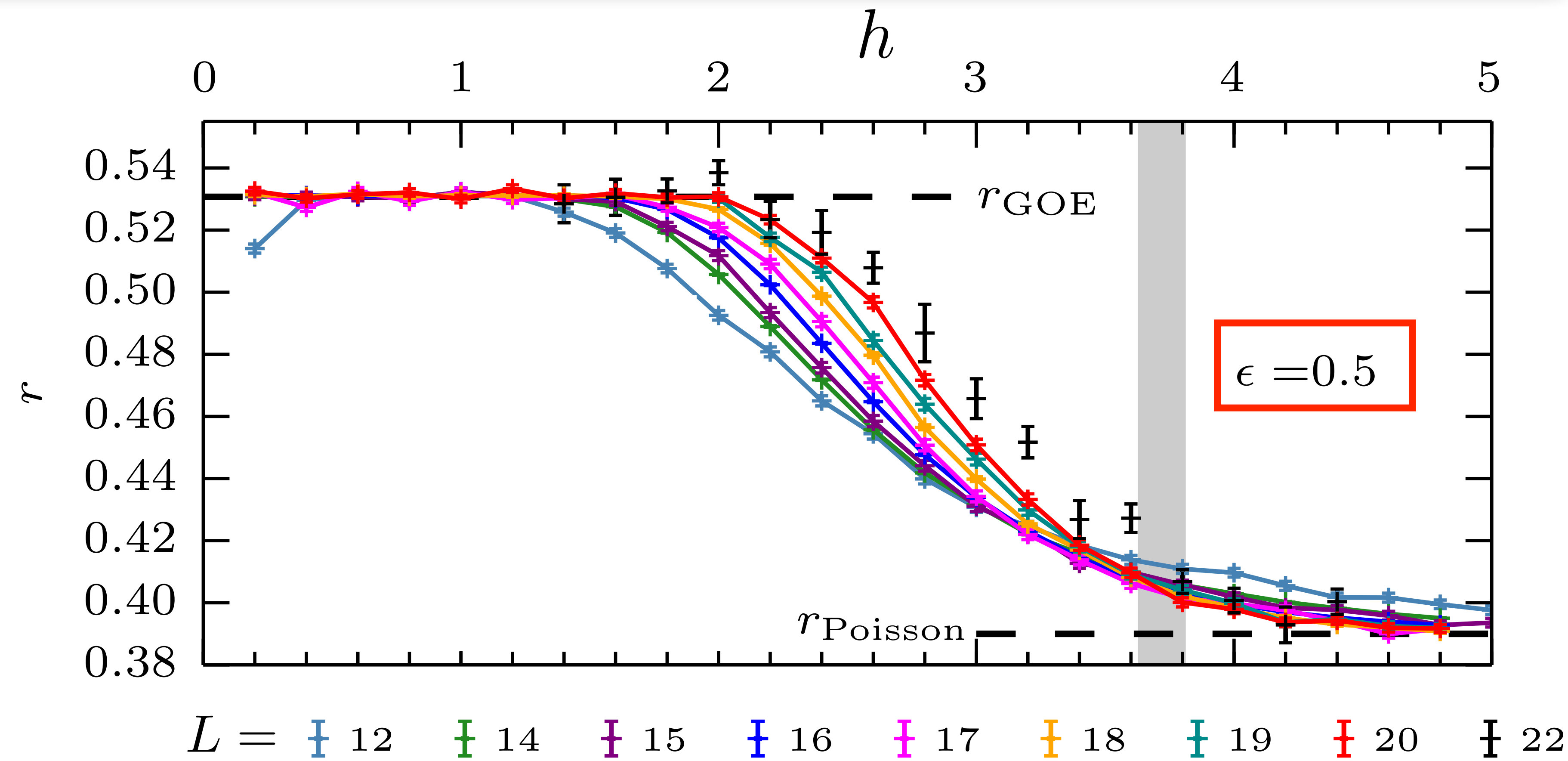
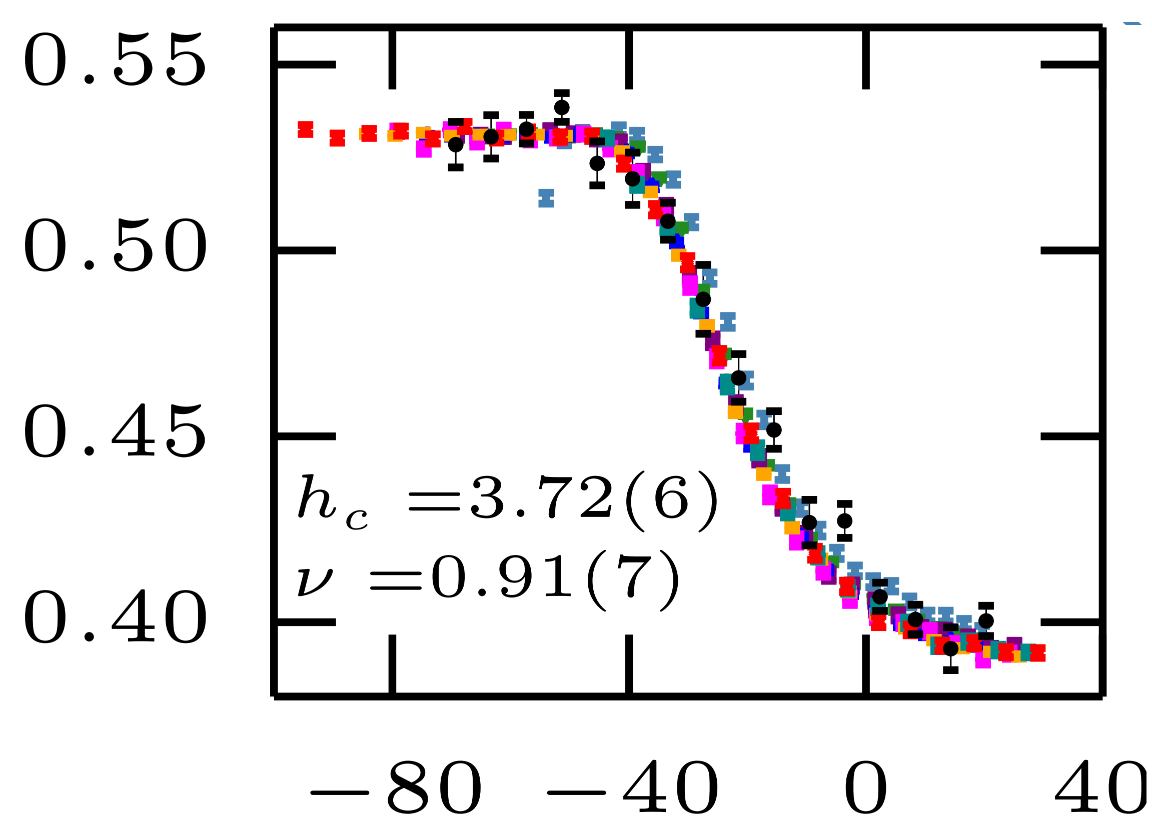
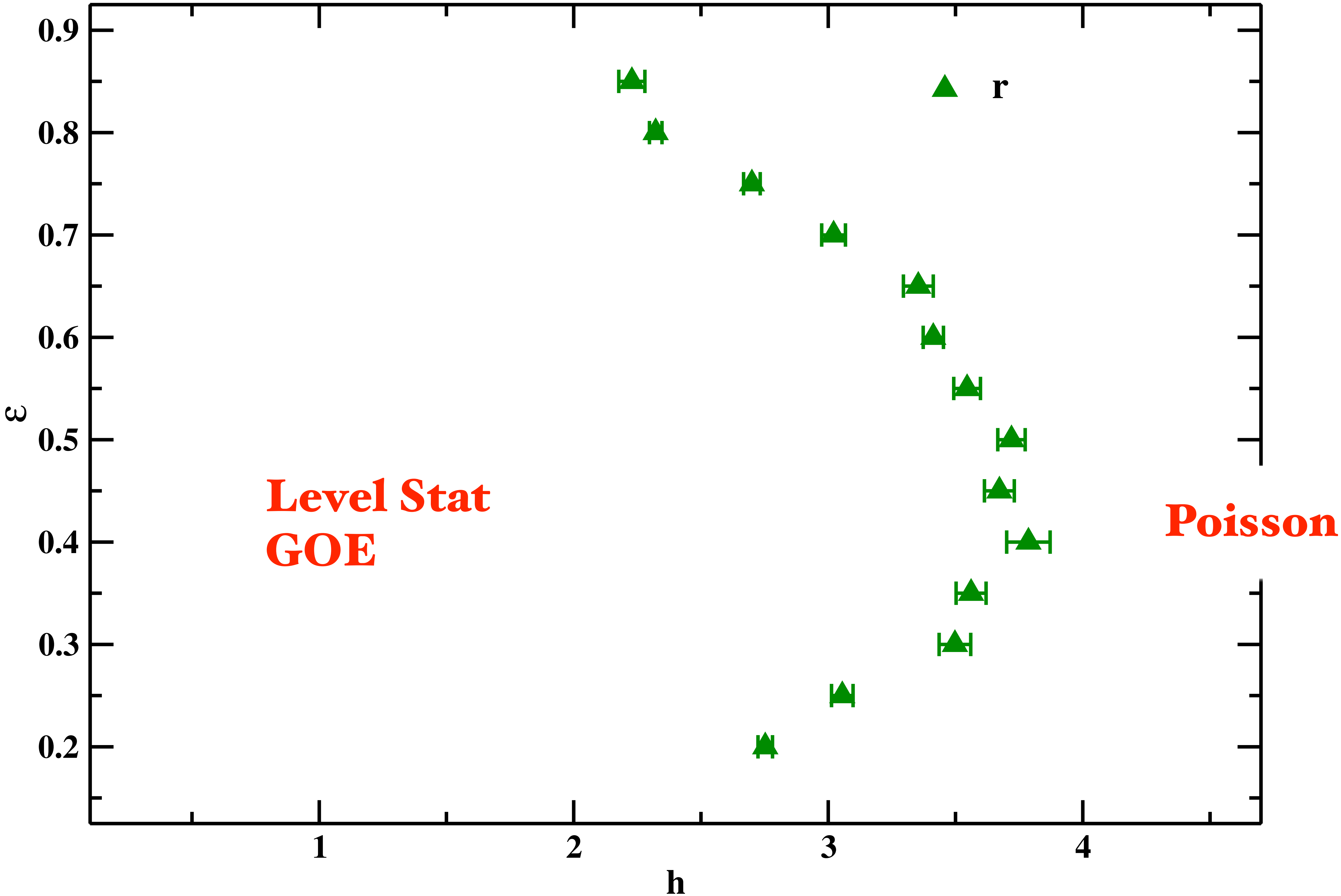


- Finite-size scaling ansatz

$$r = f_r(|h - h_c|.L^{1/\nu})$$



Energy-resolved phase diagram (I)



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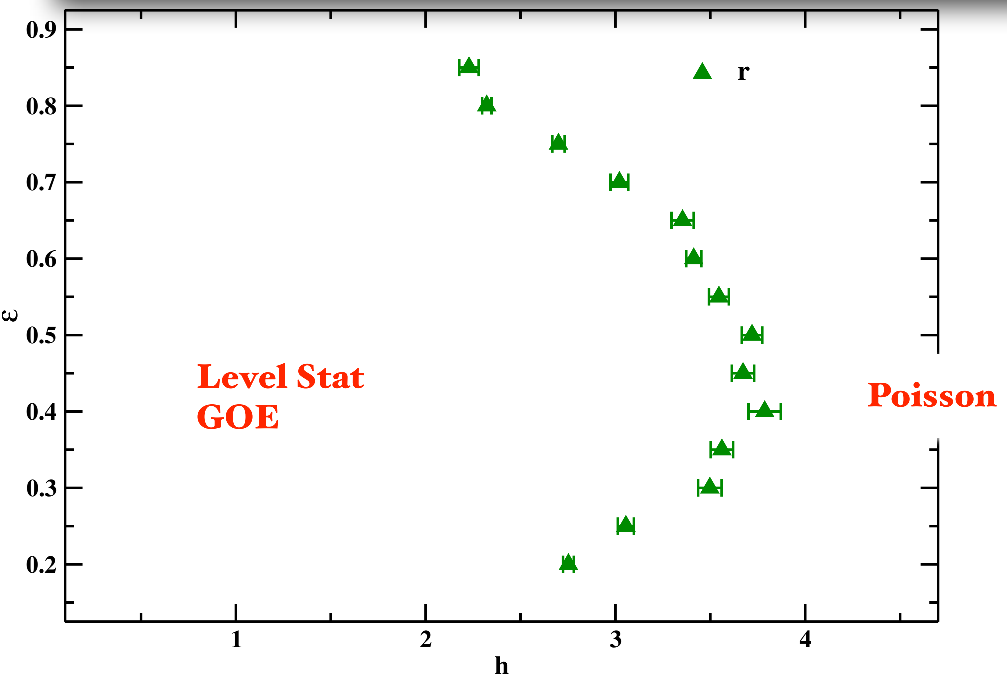
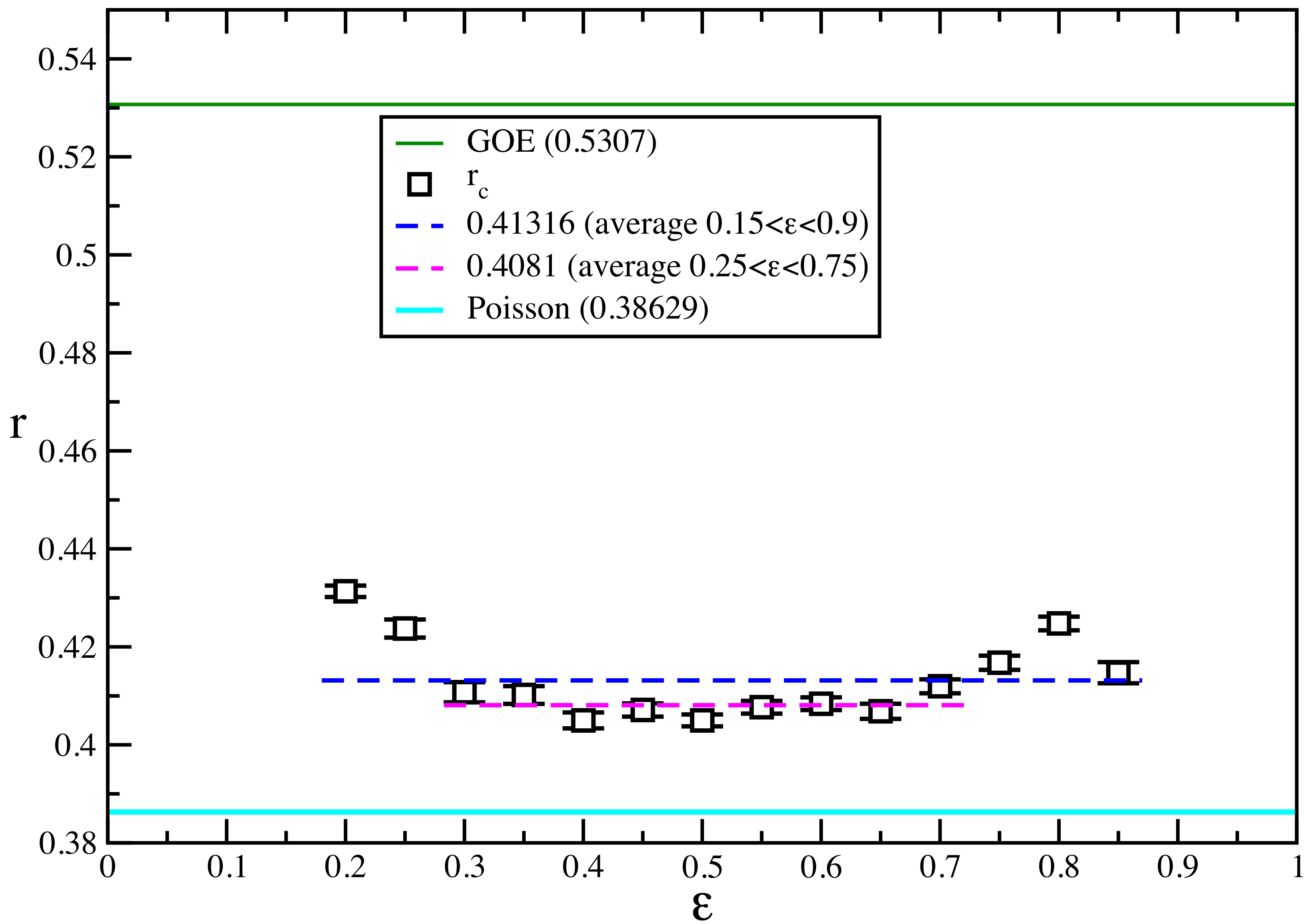


Critical ratio?



- Quasi-flat $r_c \;\; vs. \;\epsilon$
- **<u>Closer to Poisson</u>**
- also discussed in [Lauman, Pal, Scardicchio 2014]

Entanglement entropy

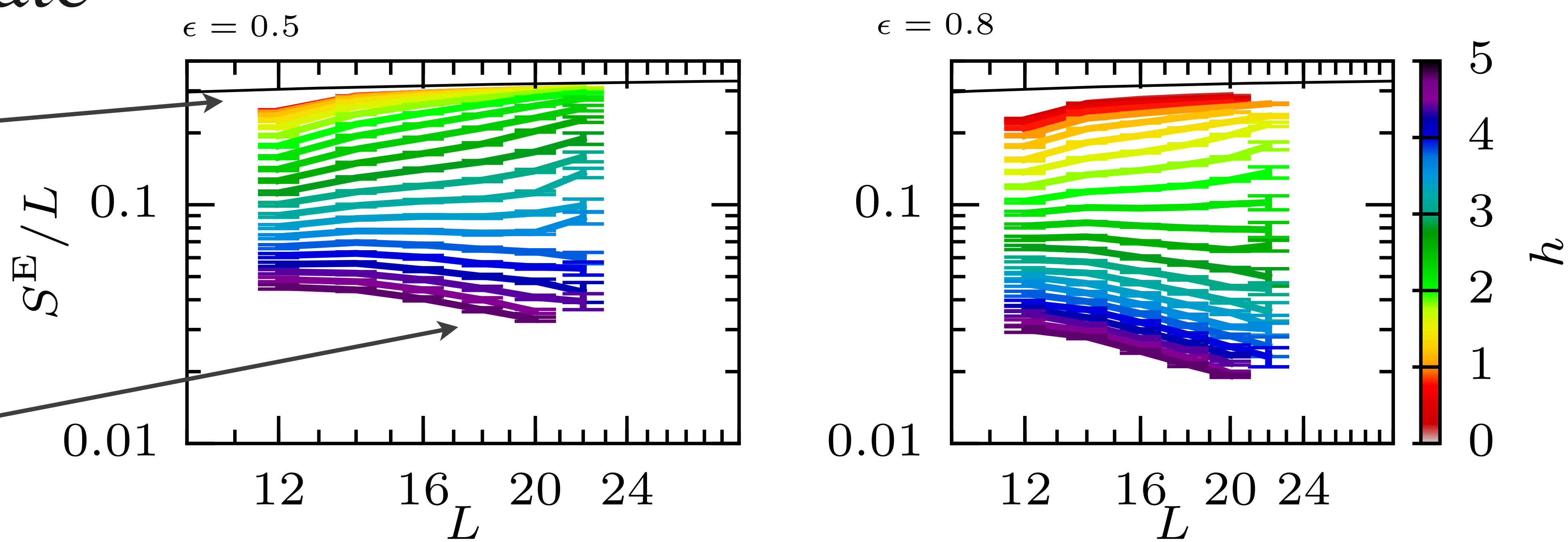
Area *vs.* volume law scaling of entanglement entropy distinguishes the two phases

See also [Bauer & Nayak; Kjall, Bardarson, Pollmann]

ETH: random state

$$S^{\mathrm{E}}(L) \sim L$$

MBL: area-law



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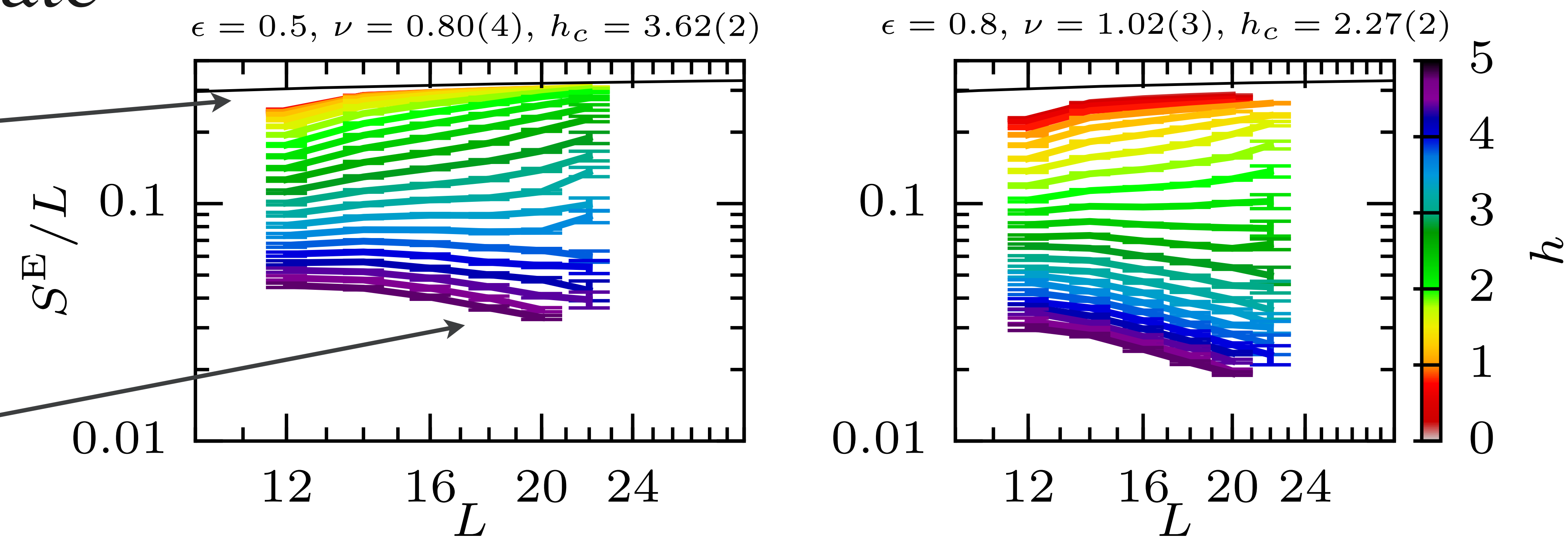
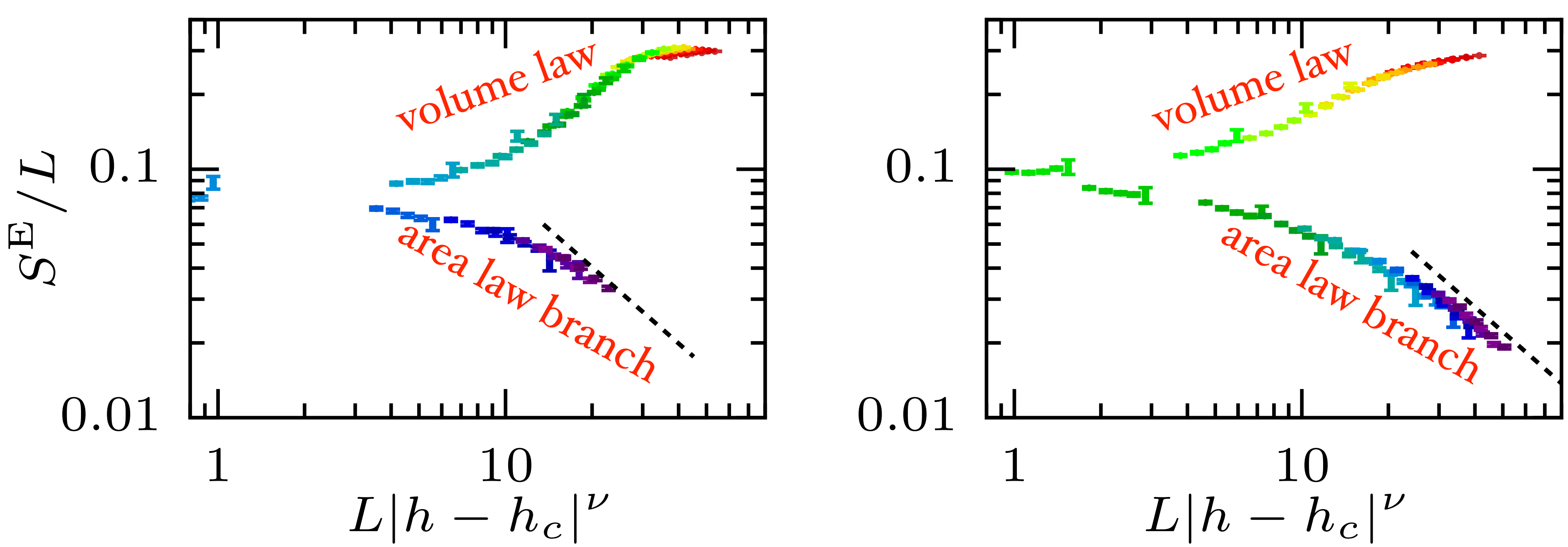
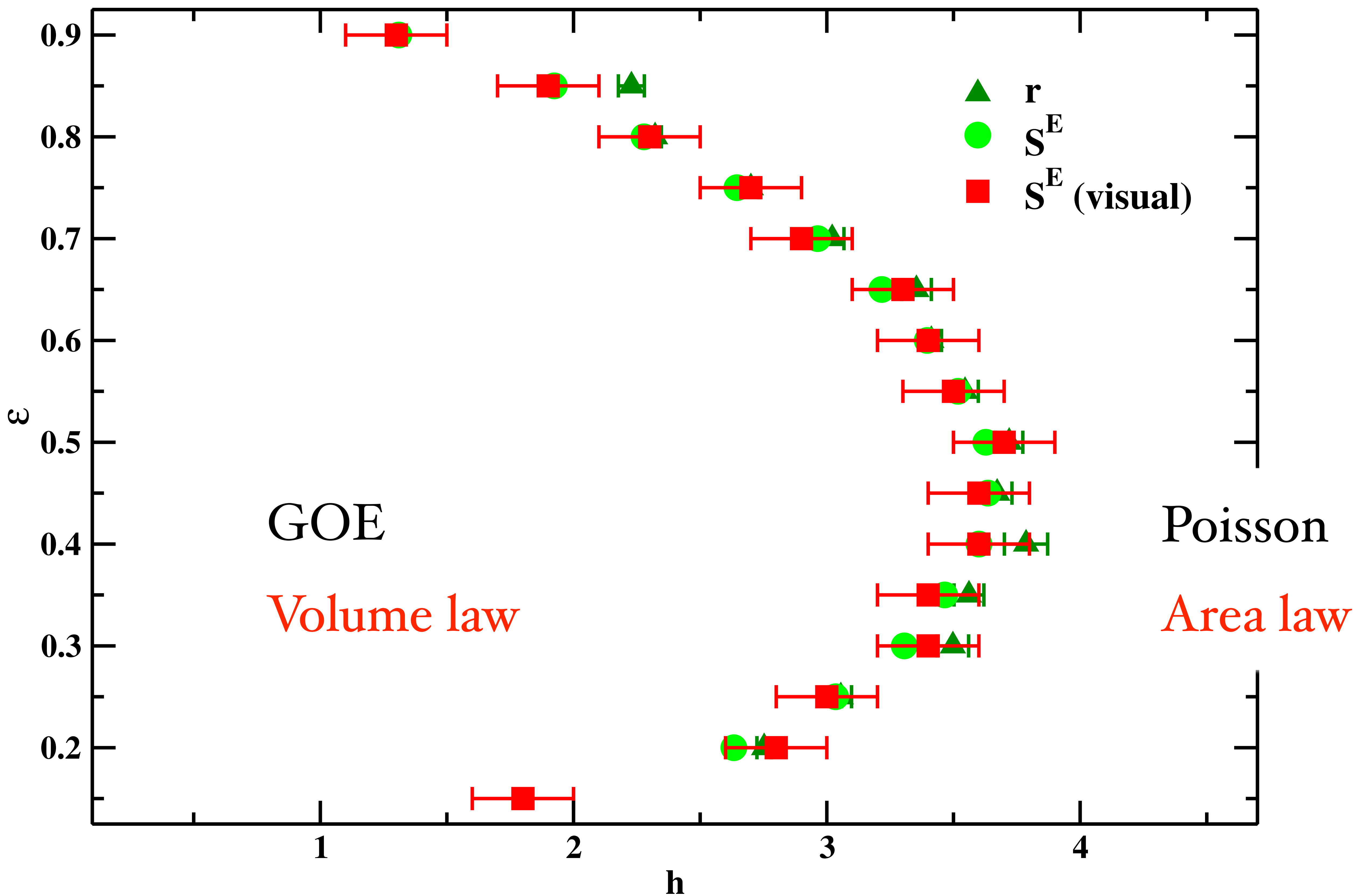


Collapse to finite-size scaling form: $S^{\mathrm{E}}/L = f_S(|h - h_c|.L^{1/\nu})$

Transition
sub-thermal
but volume-law



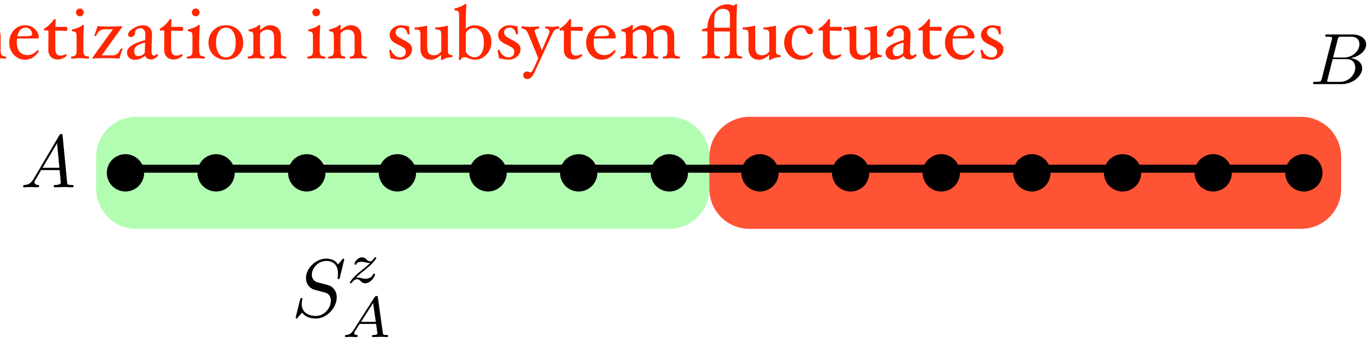
Energy-resolved phase diagram (II)



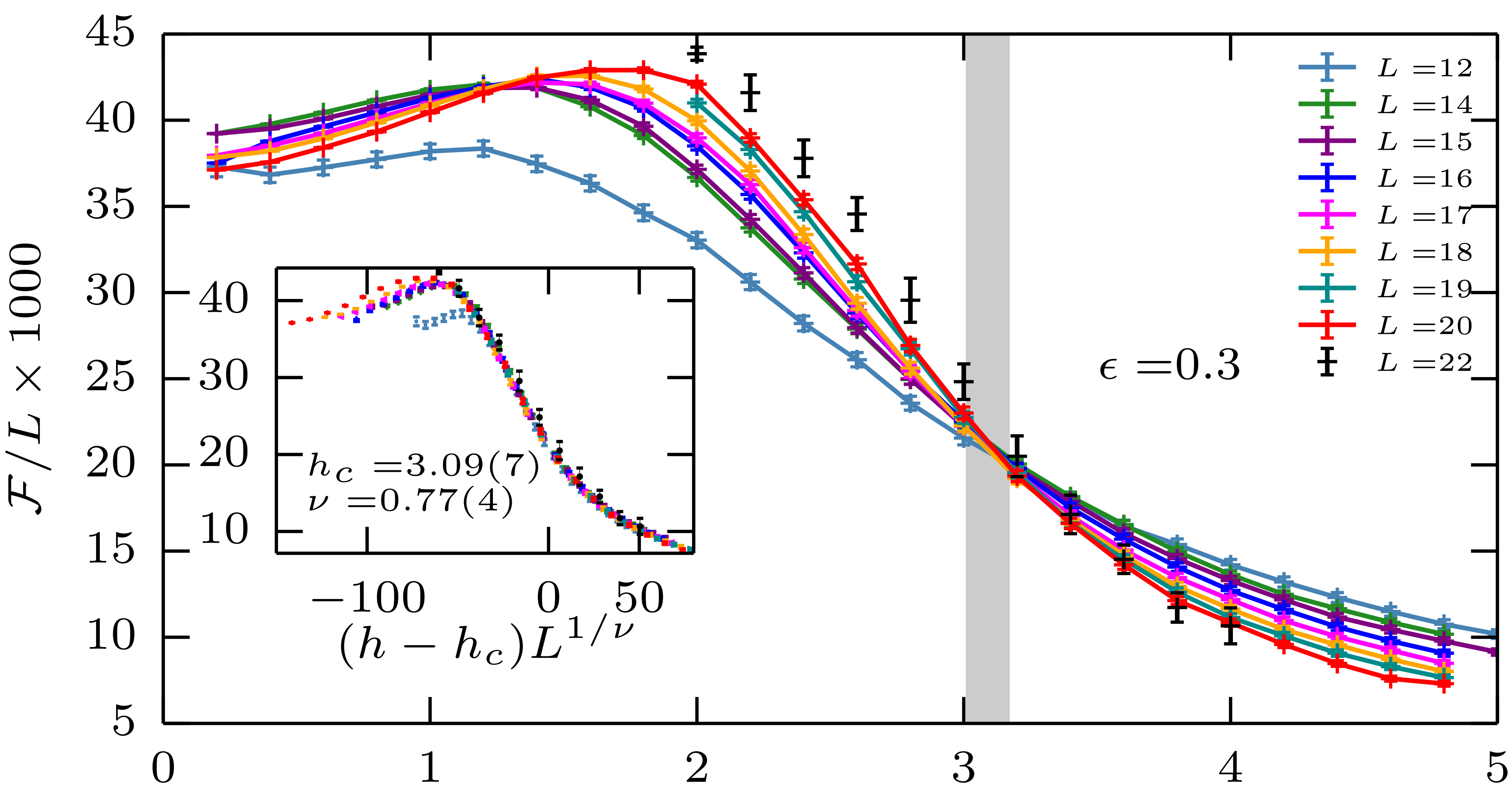
Bipartite fluctuations

- Total magnetization is conserved; Magnetization in subsytem fluctuates

$$\mathcal{F} = \langle (S_A^z)^2 \rangle - \langle S_A^z \rangle^2$$

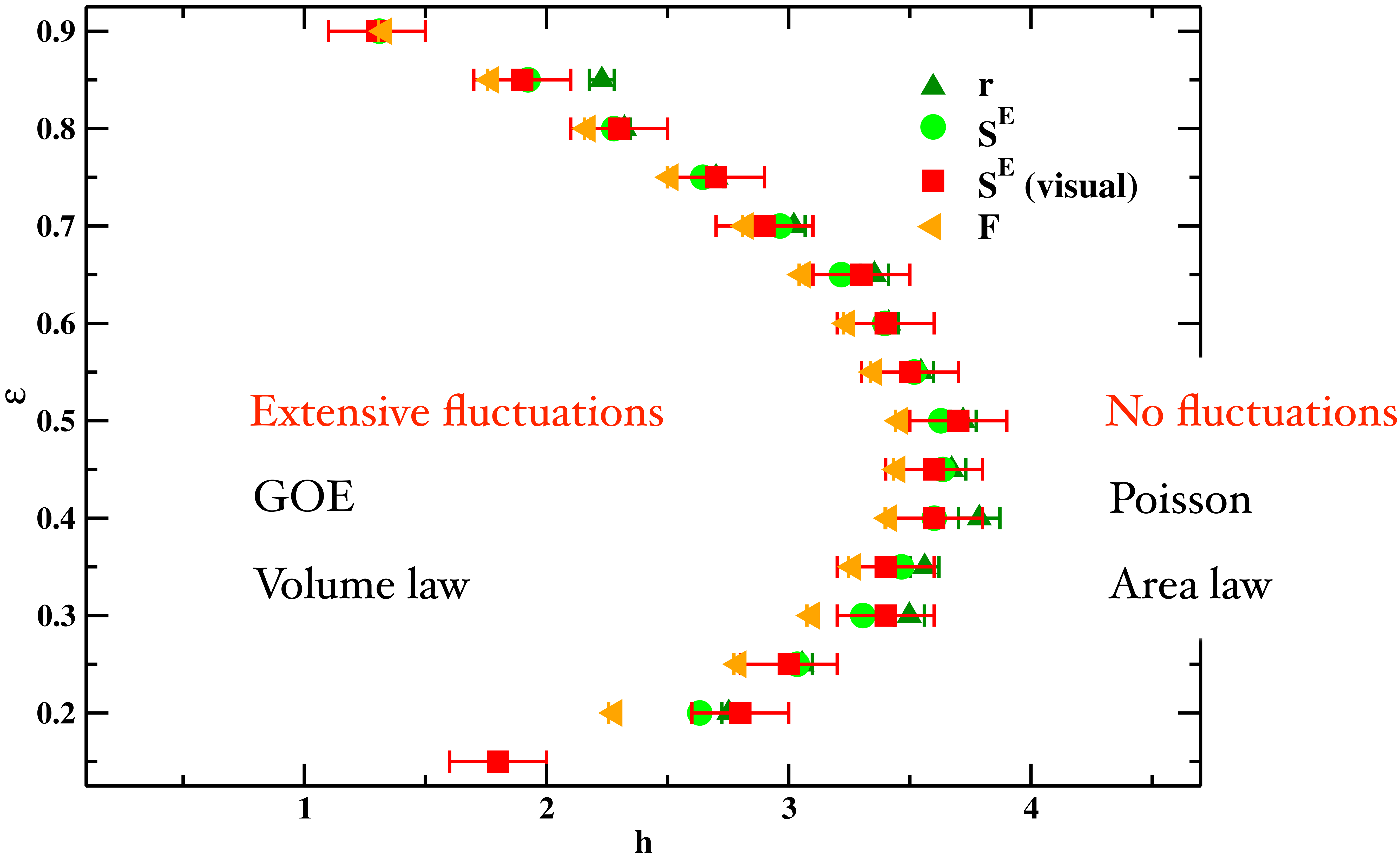


- Bipartite Fluctuations are a good probe of entanglement [Klich & Levitov, Song et al]
- Expect extensive fluctuations in thermal phase; no fluctuations in MBL



- Finite-size scaling $\mathcal{F}/L = f_{\mathcal{F}}(|h - h_c|.L^{1/\nu})$

Energy-resolved phase diagram (III)



Localization in configuration space?

Rewrite the many-body problem as an Anderson model in the configuration space

$$\mathcal{H} = -\frac{1}{2}\sum_{j,k} |j\rangle\langle k| - \sum_j \mu_j |j\rangle\langle j| \qquad \mu_j = \sum_{i=1}^{L} \langle j| h_i S_i^z - S_i^z S_{i+1}^z |j\rangle$$

Complex network with $\mathcal{N} \sim 2^L$ sites
Large connectivity $z \sim L$
On-site disordered potential μ: Gaussian $\sigma \sim h\sqrt{L}$, but highly correlated
AL on Bethe lattice $\sigma_c \sim z \ln z$ [Biroli et al.]

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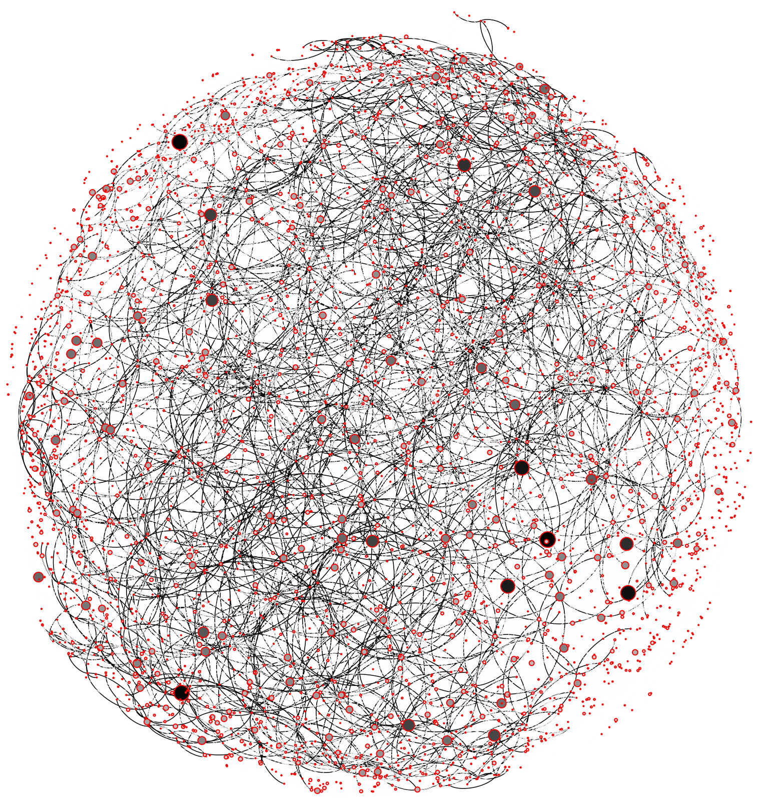
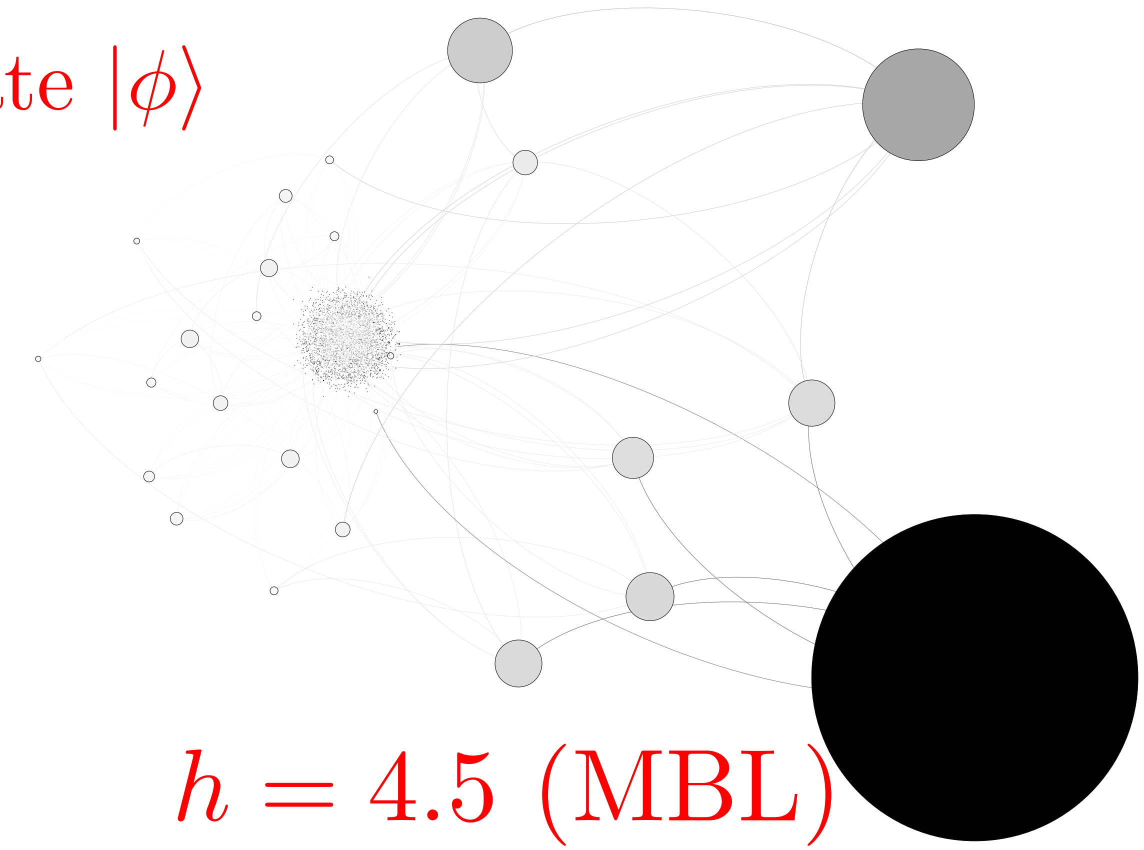


High-energy state $|\phi\rangle$

$p_i = |\langle\phi|i\rangle|^2$



$h = 0.4$ (delocalized) $\qquad\qquad$ $h = 4.5$ (MBL)

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Localization in configuration space?

[See also De Luca & Scardicchio]

- How much a many-body wave-function is localized in a given basis?

$$|\phi\rangle = \sum_i a_i |i\rangle \qquad p_i = |\langle\phi|i\rangle|^2 \qquad \{|i\rangle\} = \{S^z\}\ \text{basis}$$

Participation entropies $\quad S_1^p = -\sum_i p_i \ln(p_i) \qquad S_q^p = \frac{1}{1-q} \ln \sum_i p_i^q$

= ln (IPR)

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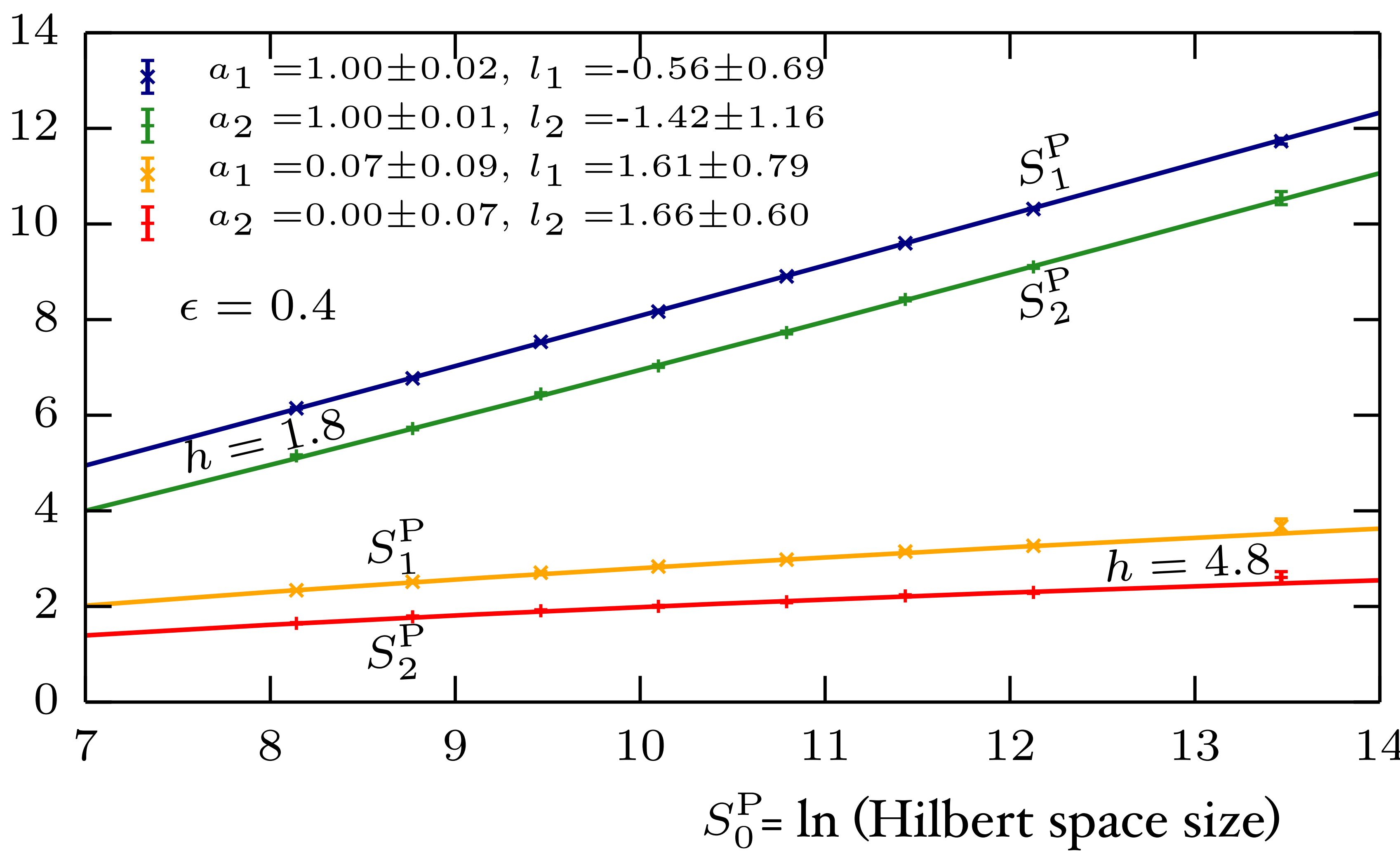
- Scaling of participation entropy



Thermal:

$$S_q^p = a_q S_0^p - \ldots$$

$$a_q = 1$$

MBL: $a_q \ll 1$

or

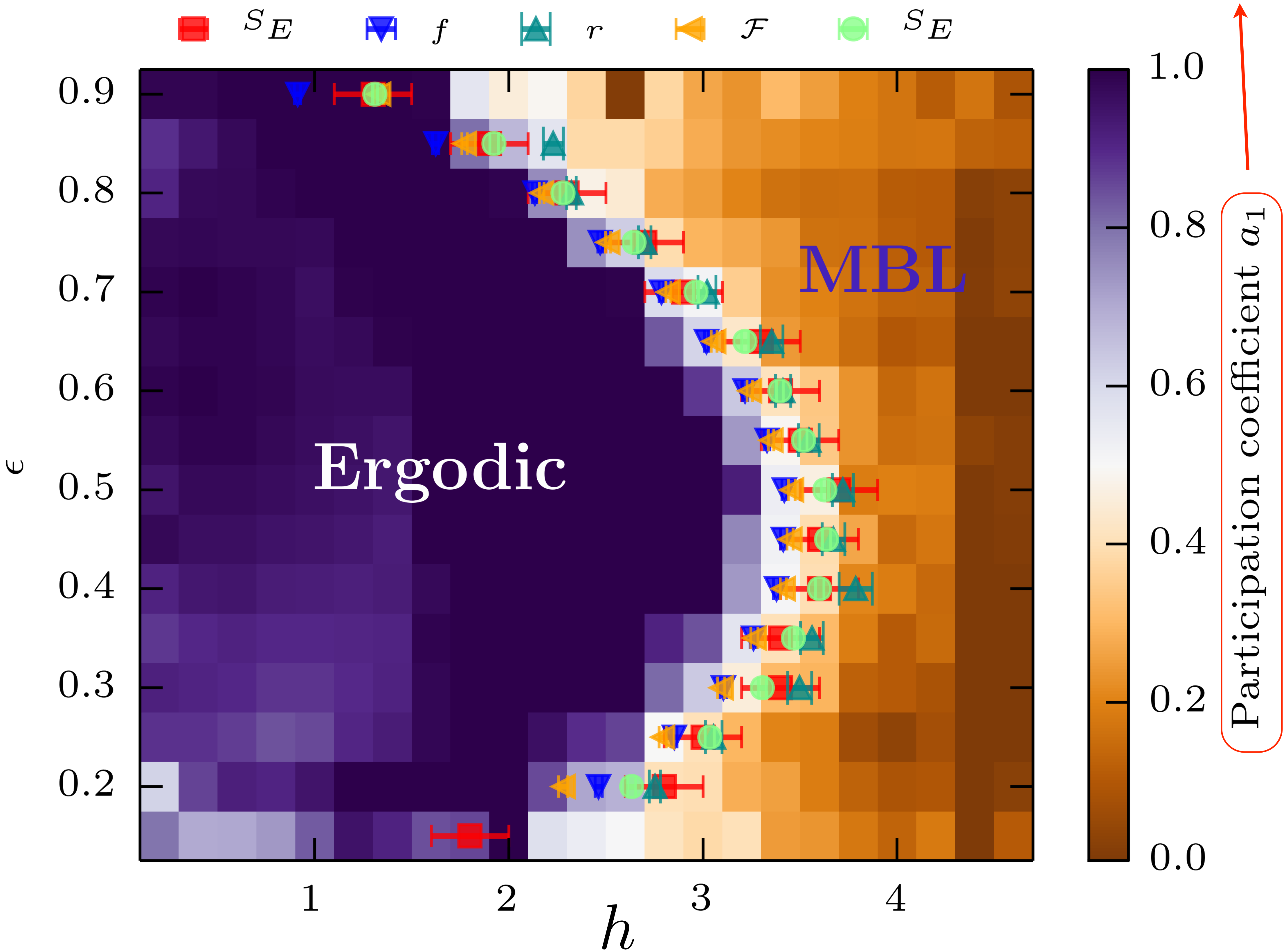
$$S_q^p = l_q \ln(S_o^p) + \ldots$$

No Hilbert space localization but strongly reduced entropy

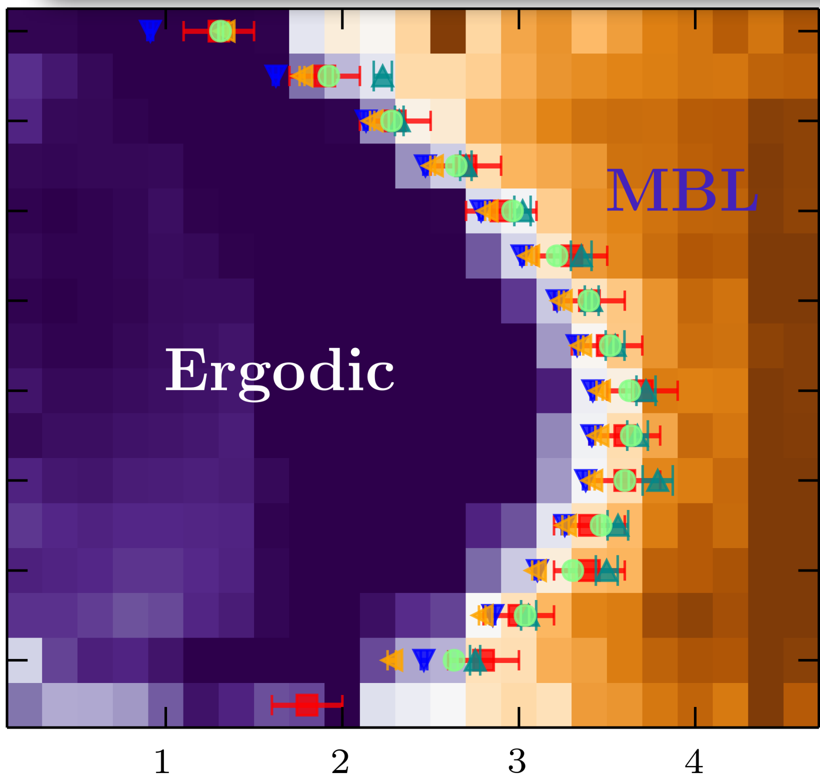
Energy-resolved phase diagram

- Evidence for a many-body localization edge

$$S_1^p = a_1 S_0^p + \ldots$$



end of the story... ?



- **Some analytical arguments claim for**

doi:10.1103/PhysRevB.93.014203

Absence of many-body mobility edges

Wojciech De Roeck,[1,2] Francois Huveneers,[2,3] Markus Müller,[2,4,5,6] and Mauro Schiulaz[7]

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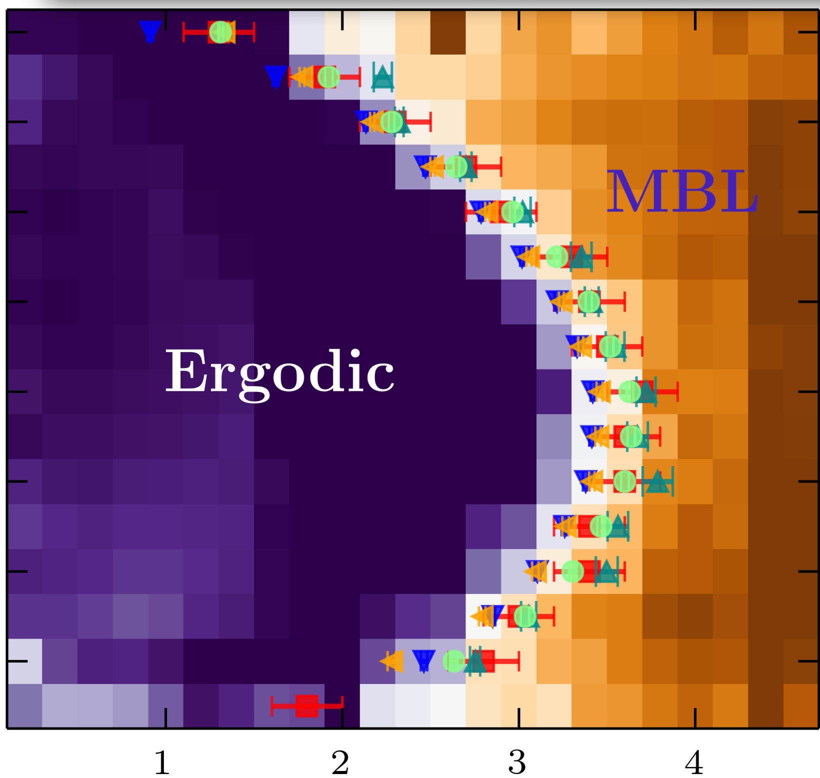

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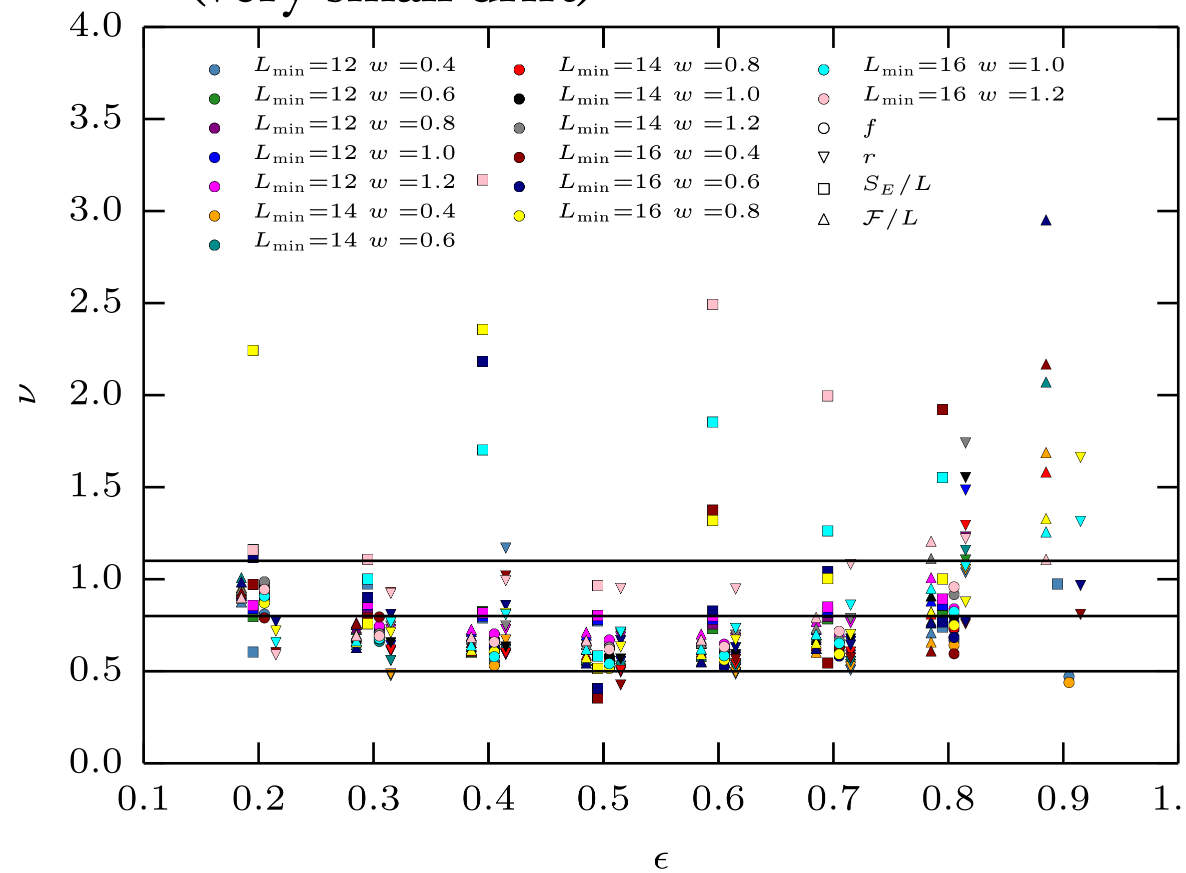
- **Critical exponents ?** $\nu = 0.8(3)$ violates Harris $\nu > 2/d$

Systematic study of fit qualities to finite-size scaling ansätze

- Different quantities
- Different fit windows
- Starting from minimal size $L_{\min}$
- Including or not corrections to scaling … (very small drift)



Note also $\nu \sim 1$ [Kjall, Bardarson and Pollmann (2014)]

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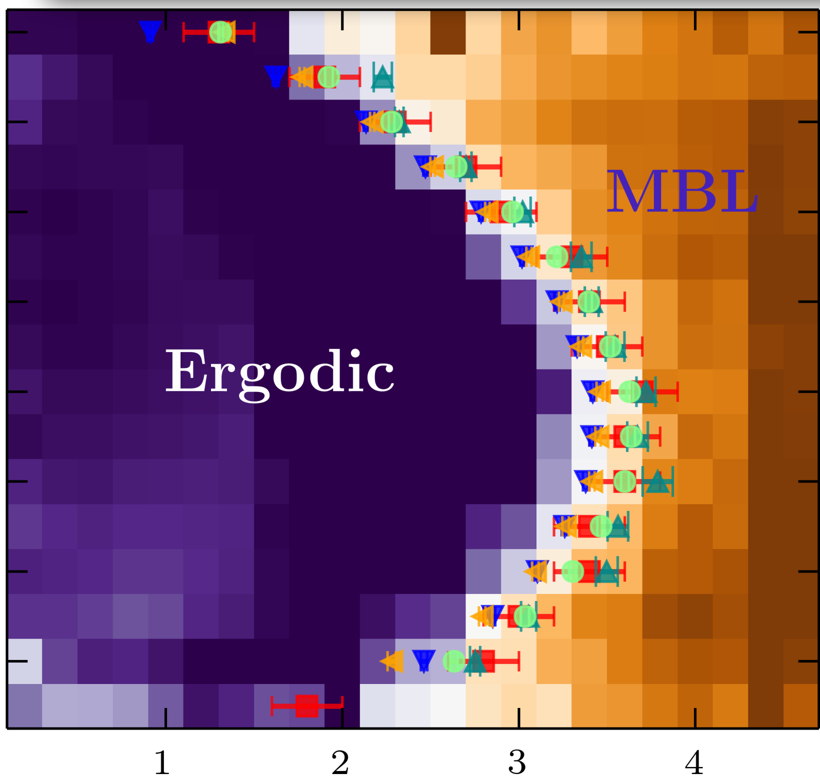

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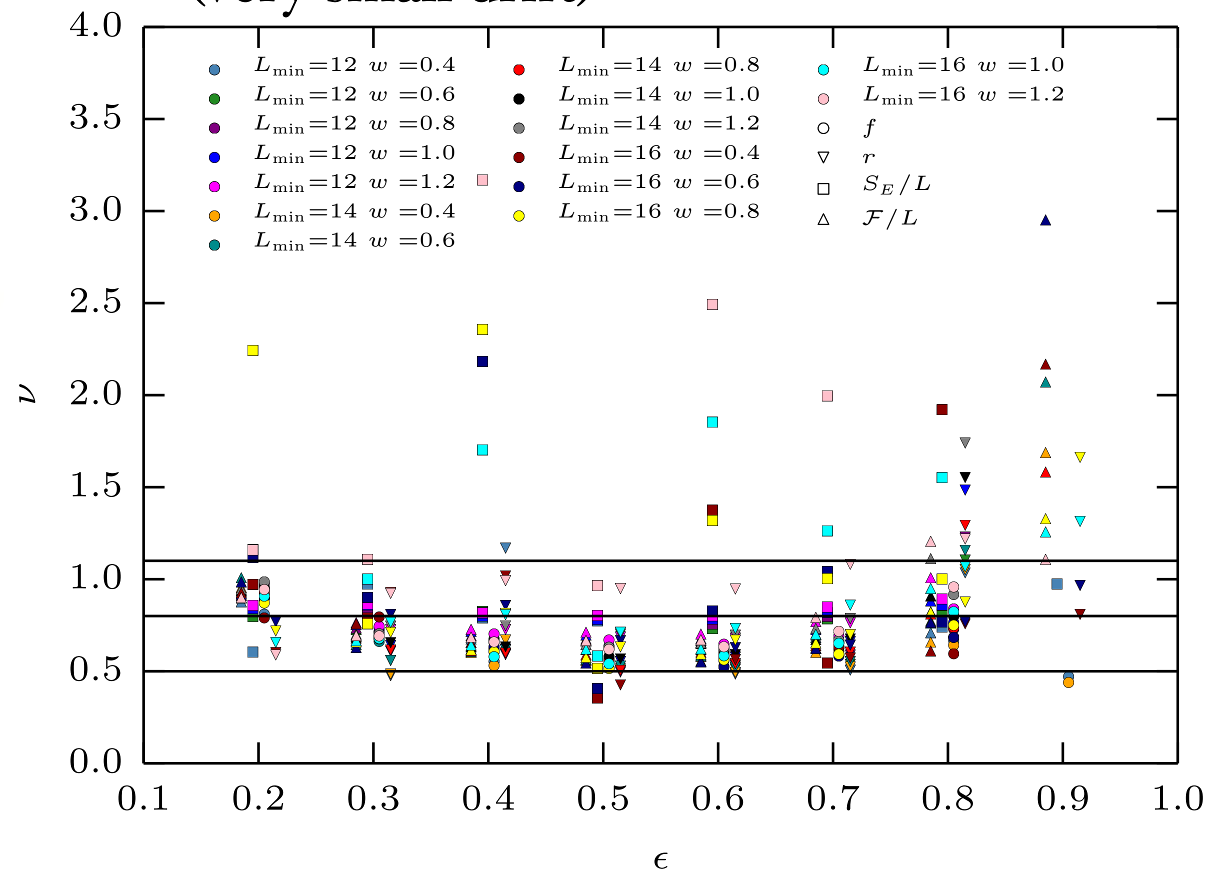
Many Body Localization Transition in the strong disorder limit : entanglement entropy from the statistics of rare extensive resonances

Cécile Monthus

Institut de Physique Théorique, Université Paris Saclay, CNRS, CEA, 91191 Gif-sur-Yvette, France

The space of one-dimensional disordered interacting quantum models displaying a Many-Body-Localization Transition seems sufficiently rich to produce critical points with level statistics interpolating continuously between the Poisson statistics of the Localized phase and the Wigner-Dyson statistics of the Delocalized Phase. In this paper, we consider the strong disorder limit of the MBL transition, where the critical level statistics is close to the Poisson statistics. We analyse a one-dimensional quantum spin model, in order to determine the statistical properties of the rare extensive resonances that are needed to destabilize the MBL phase. At criticality, we find that the entanglement entropy can grow with an exponent $0 < \alpha < 1$ anywhere between the area law $\alpha = 0$ and the volume law $\alpha = 1$, as a function of the resonances properties. In the MBL phase near criticality, we obtain the simple value $\nu = 1$ for the correlation length exponent. Independently of the strong disorder limit, we explain why for the Many-Body-Localization transition concerning individual eigenstates, the correlation length exponent ν is not constrained by the usual Harris inequality $\nu \geq 2/d$, so that there is no theoretical inconsistency with the best numerical measure $\nu = 0.8(3)$ obtained by D. J. Luitz, N. Laflorencie and F. Alet, Phys. Rev. B 91, 081103 (2015).

arXiv:1510.03711



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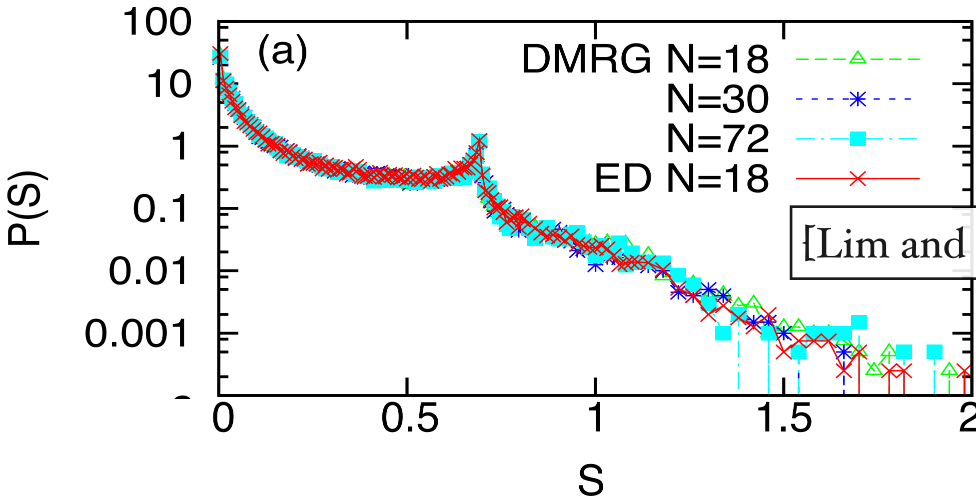
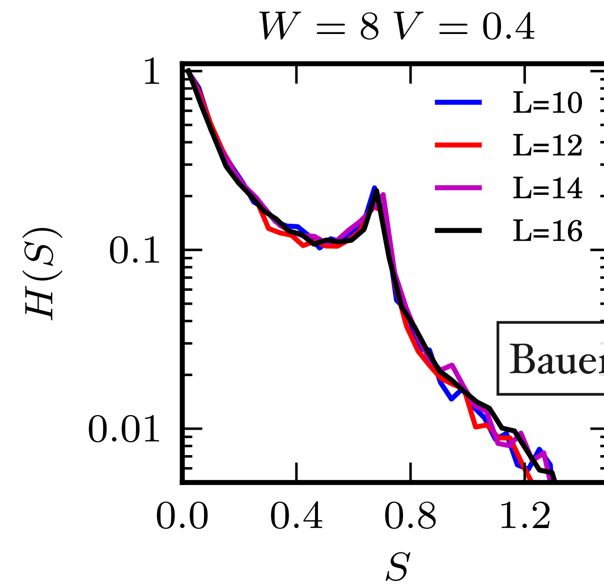
Distribution of entropies, Griffiths regions?

- **Deep in the MBL regime**



[Lim and Sheng 2016]



Bauer and Nayak 2013]

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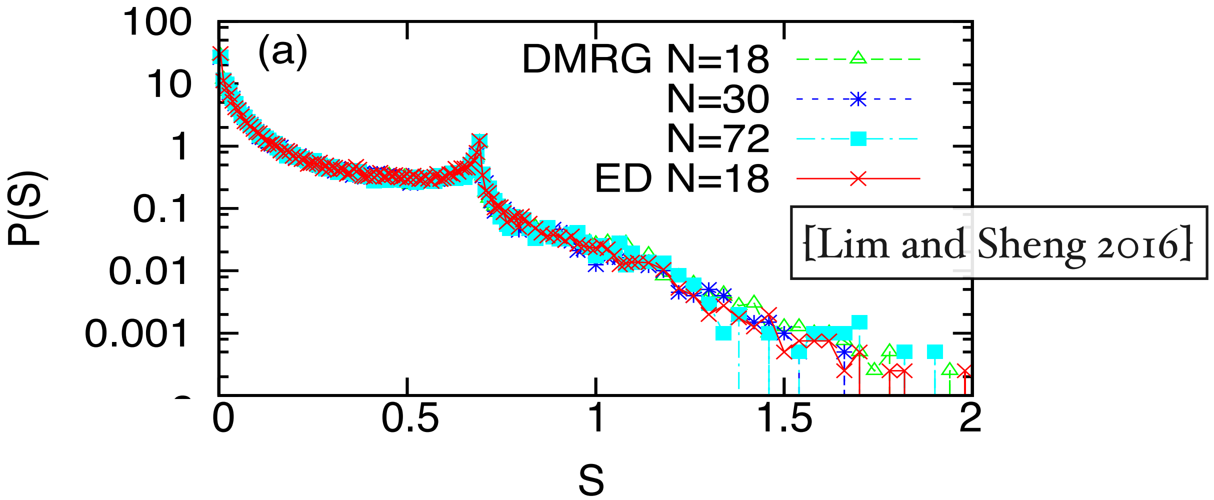
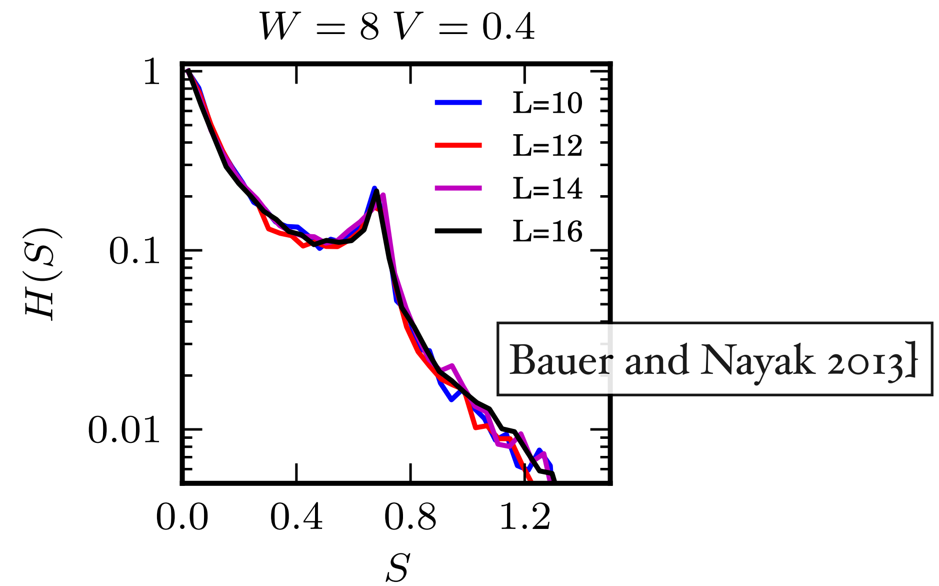
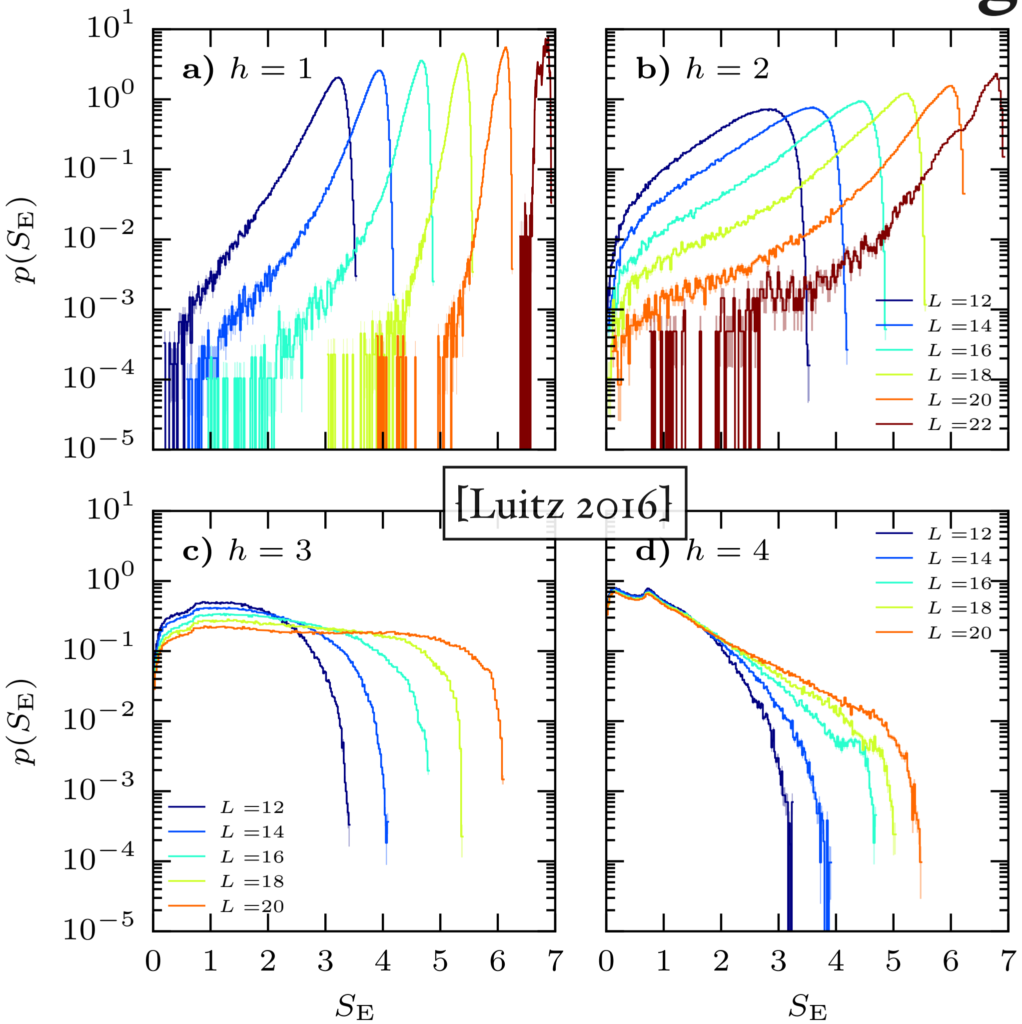




- **ETH and critical-MBL regime**



Distribution of entropies, Griffiths regions?

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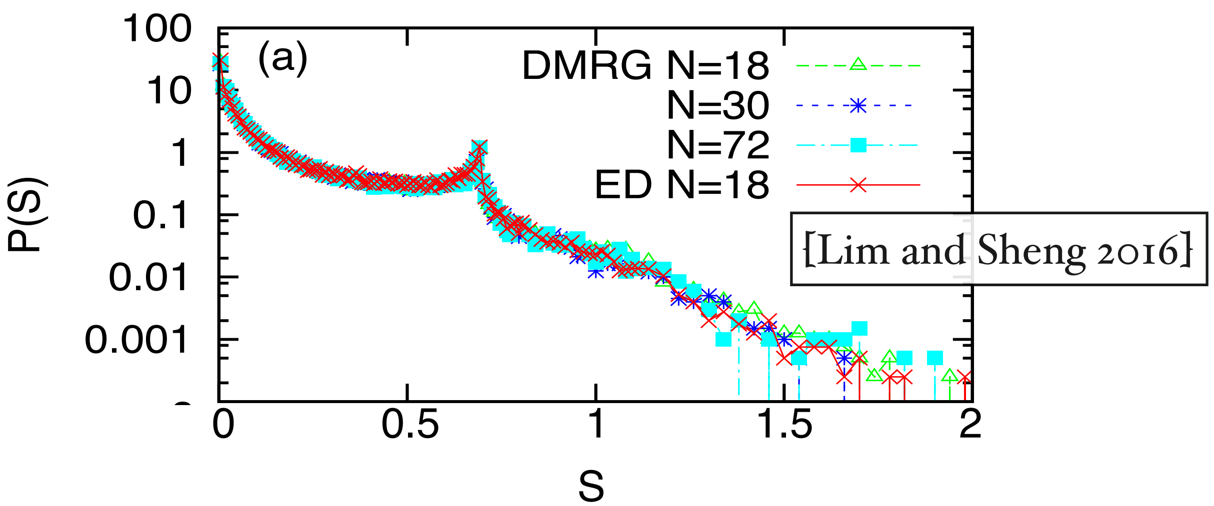
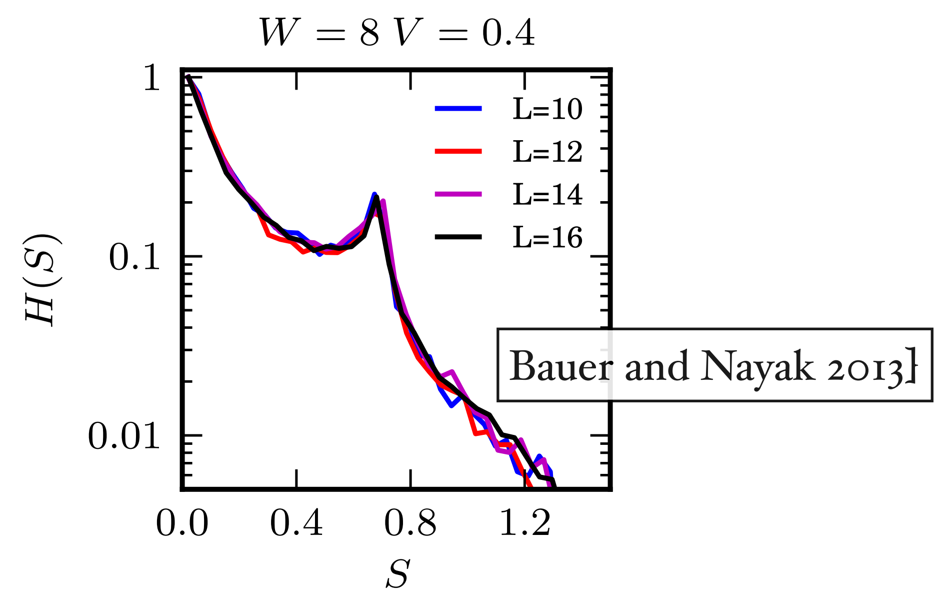
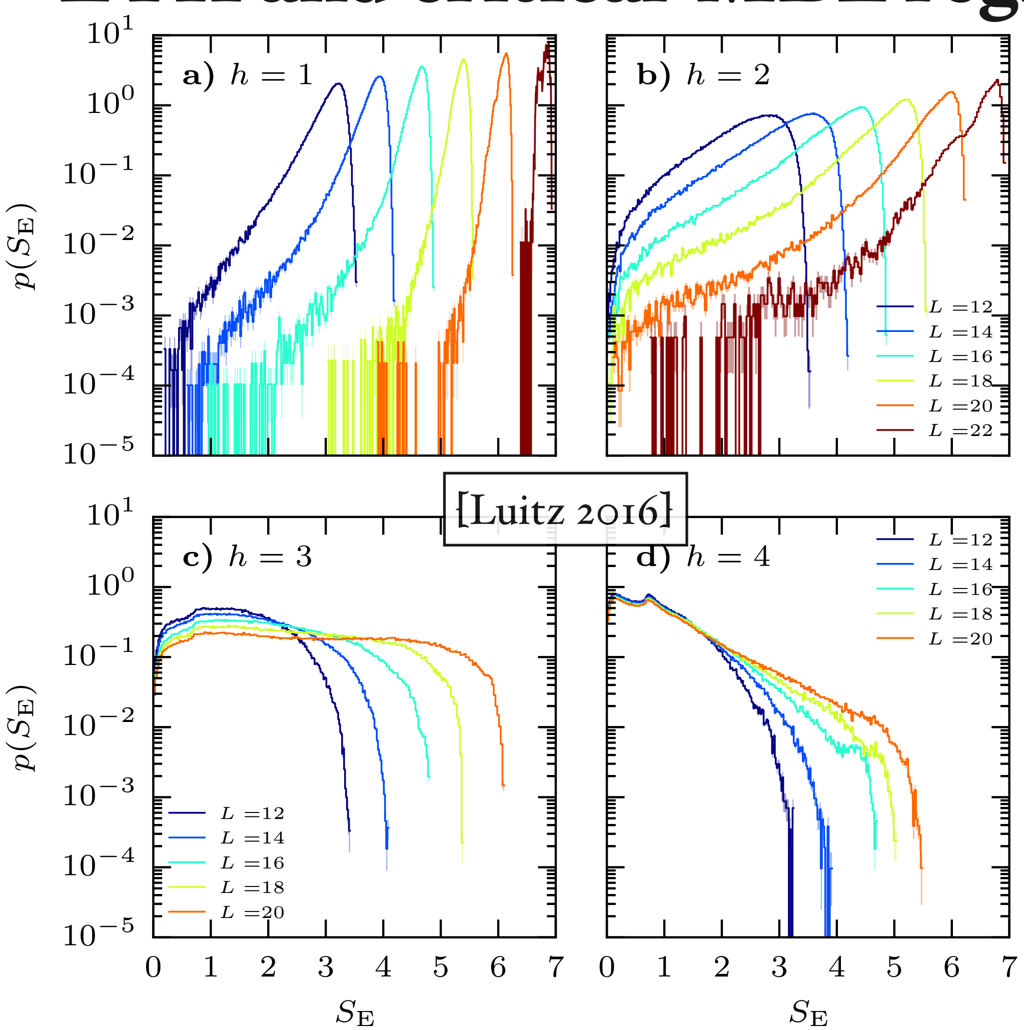
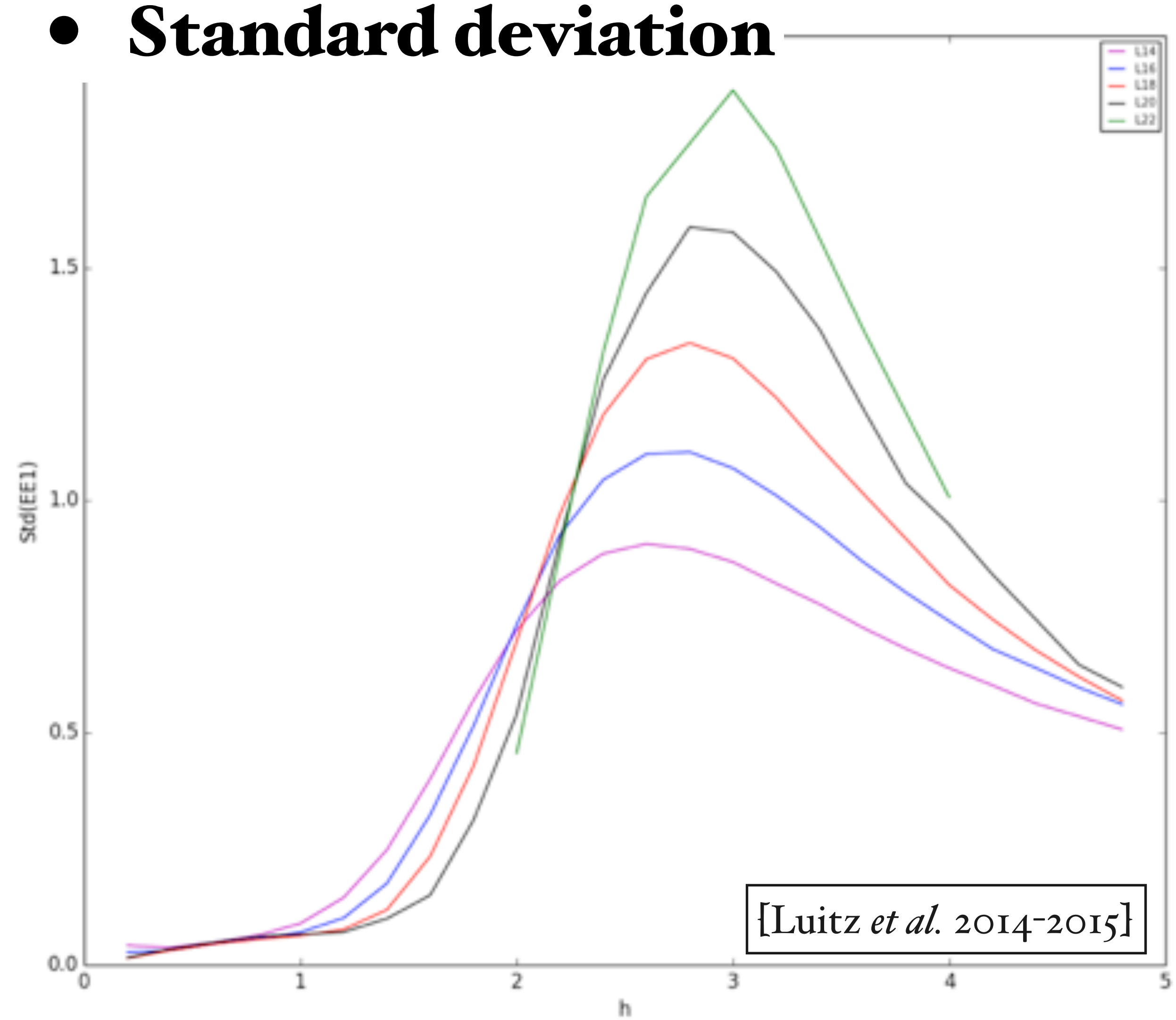




- **ETH and critical-MBL regime**

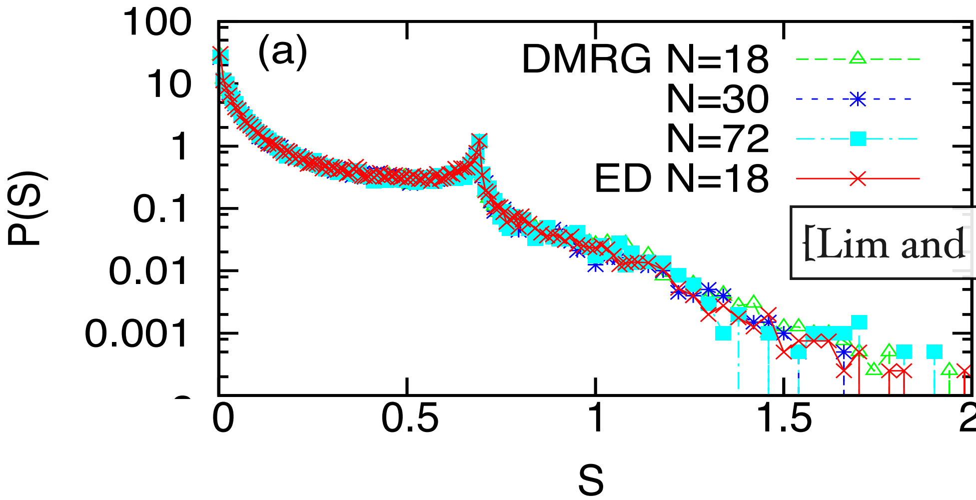


- **Standard deviation**

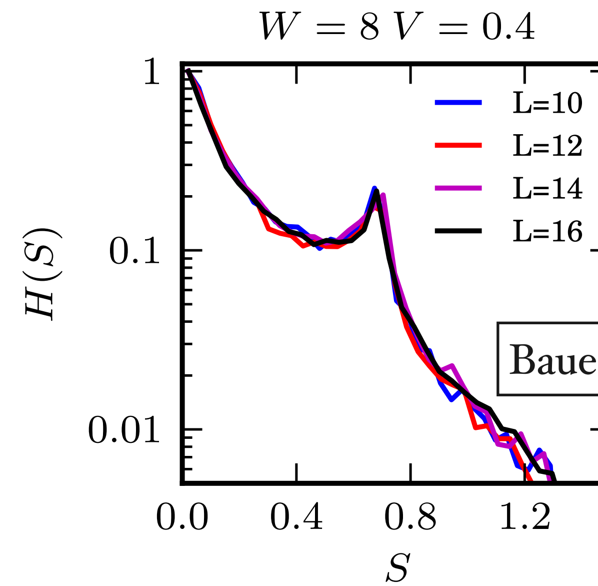


Distribution of entropies, Griffiths regions?

- **Deep in the MBL regime**

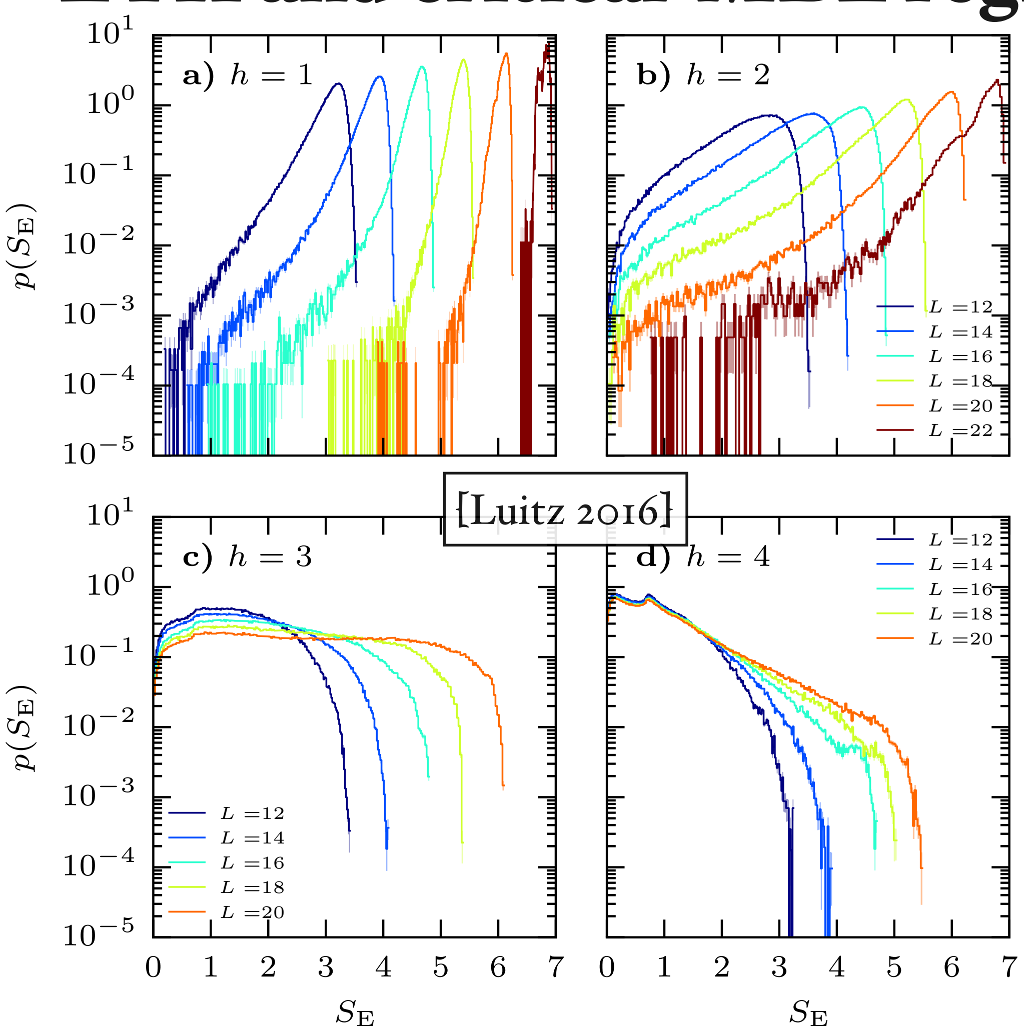


[Lim and Sheng 2016]



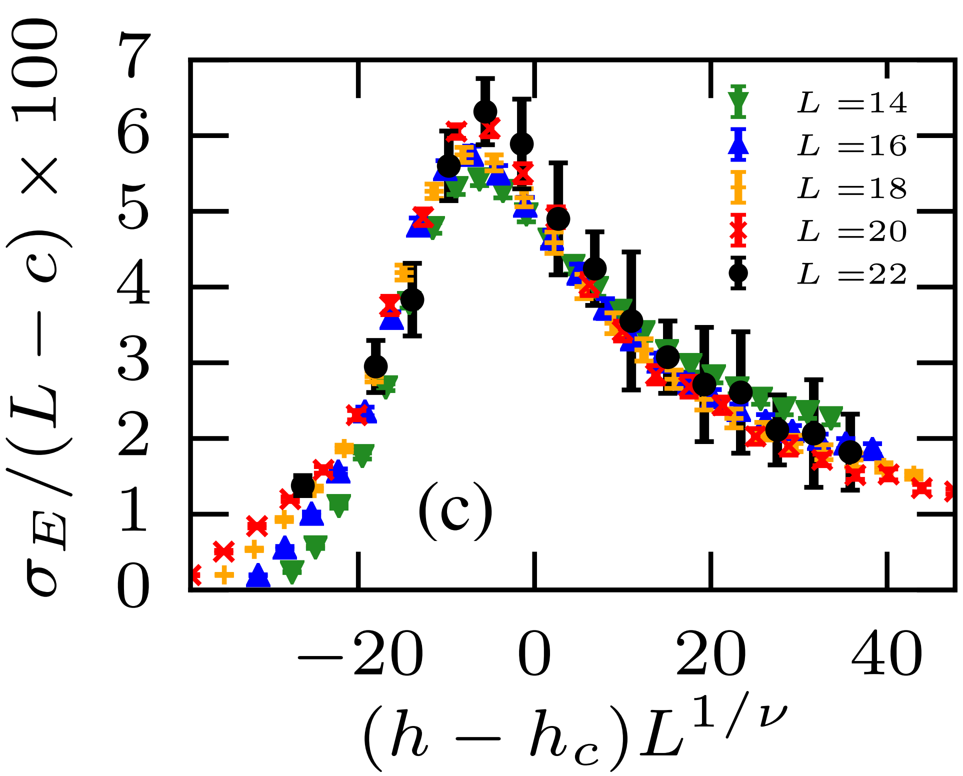
[Bauer and Nayak 2013]

- **ETH and critical-MBL regime**



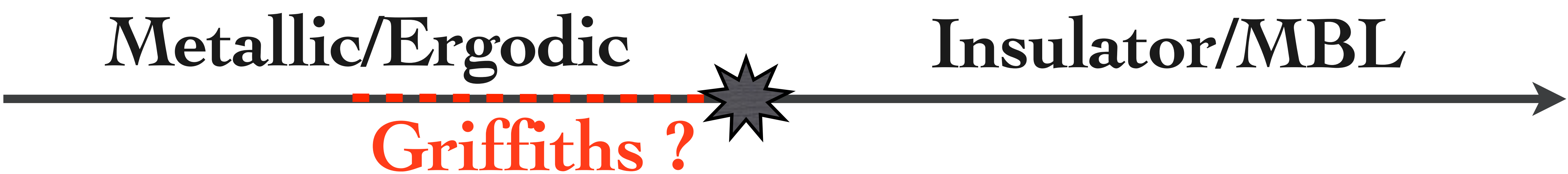
[Luitz 2016]

- **Standard deviation**



[Luitz *et al.* 2014-2015]

	Delocalized	Transition	MBL
Spectral statistics	GOE	?	Poisson
Entanglement entropy $S^{\mathrm{E}}(L)$	volume-law	volume-law[15]	area law
Entanglement variance $\sigma_{\mathrm{E}}^2(L)$	vanishes	diverges	finite



PHYSICAL REVIEW X **5,** 031032 (2015)

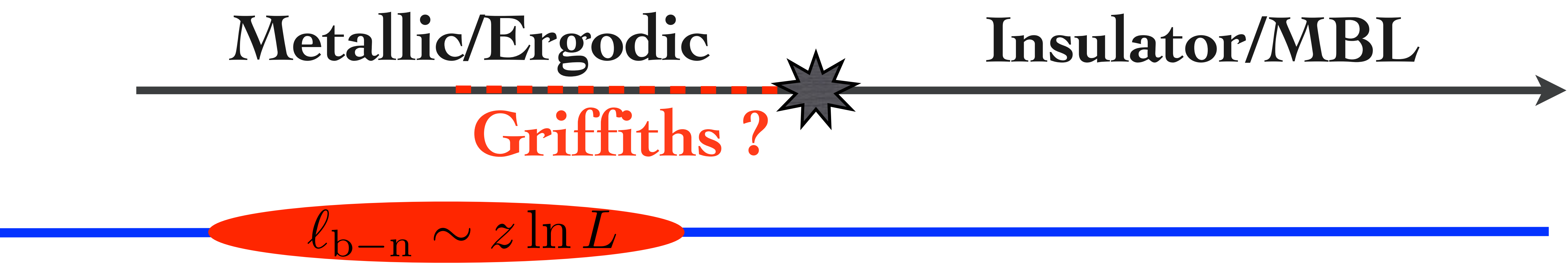
Theory of the Many-Body Localization Transition in One-Dimensional Systen

Ronen Vosk,[1] David A. Huse,[2] and Ehud Altman[1]

[1]*Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[2]*Physics Department, Princeton University, Princeton, New Jersey 08544, USA*
(Received 31 December 2014; revised manuscript received 29 May 2015; published 14 September 2015)

We formulate a theory of the many-body localization transition based on a novel real-space renormalization group (RG) approach. The results of this theory are corroborated and intuitively explained with a phenomenological effective description of the critical point and of the "badly conducting" state found near the critical point on the delocalized side. The theory leads to the following sharp predictions: (i) The delocalized state established near the transition is a Griffiths phase, which exhibits subdiffusive transport of conserved quantities and sub-ballistic spreading of entanglement. The anomalous diffusion exponent $\alpha < 1/2$ vanishes continuously at the critical point. The system does thermalize in this Griffiths phase.

	Delocalized	Transition	MBL
Spectral statistics	GOE	?	Poisson
Entanglement entropy $S^{\mathrm{E}}(L)$	volume-law	volume-law[15]	area law
Entanglement variance $\sigma^2_{\mathrm{E}}(L)$	vanishes	diverges	finite



How to probe this?
Out of Equilibrium response

RAPID COMMUNICATIONS

PHYSICAL REVIEW B **00**, 000200(R) (2016)

Extended slow dynamical regime close to the many-body localization transition

David J. Luitz,[1,2,*] Nicolas Laflorencie,[2,†] and Fabien Alet[2,‡]
[1]*Department of Physics and Institute for Condensed Matter Theory, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801, USA*
[2]*Laboratoire de Physique Théorique, IRSAMC, Université de Toulouse, CNRS, 31062 Toulouse, France*
(Received 30 November 2015; published xxxxxx)

$$t = 0$$

$$\uparrow\downarrow\uparrow\uparrow\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\downarrow\uparrow\uparrow\uparrow\downarrow\uparrow\downarrow\downarrow\downarrow$$

$$\exp(-i\mathcal{H}t)$$

Random field Heisenberg

- $\mathrm{diag}(\mathrm{e}^{-iE_n t})$ Limited to very small systems $L = 16$
- Work in Krylov space $\mathcal{K} = \mathrm{span}\left(|\psi_0\rangle, H|\psi_0\rangle, \ldots H^n|\psi_0\rangle\right)$ $\boxed{L = 28}$

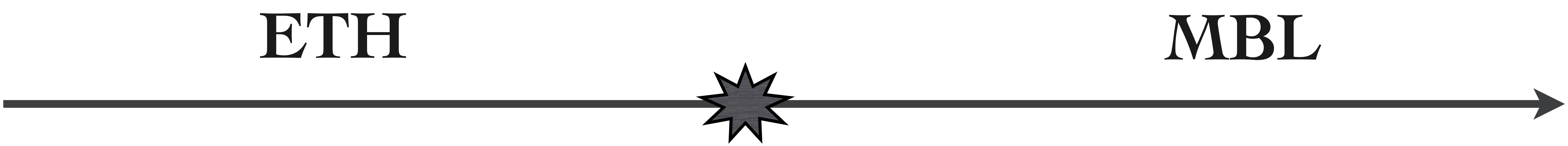
André Nauts and Robert E. Wyatt, "New Approach to Many-State Quantum Dynamics: The Recursive-Residue-Generation Method," Phys. Rev. Lett. **51**, 2238–2241 (1983).

Y. Saad, "Analysis of Some Krylov Subspace Approximations to the Matrix Exponential Operator," SIAM J. Numer. Anal. **29**, 209–228 (1992).

Roger B. Sidje, "Expokit: A Software Package for Computing Matrix Exponentials," ACM Trans. Math. Softw. **24**, 130–156 (1998).

Vicente Hernandez, Jose E. Roman, and Vicente Vidal, "SLEPc: A Scalable and Flexible Toolkit for the Solution of Eigenvalue Problems," ACM Trans. Math. Softw. **31**, 351–362 (2005).

Entanglement growth after a global quench



Entanglement growth after a global quench

ETH **MBL**

- Ballistic

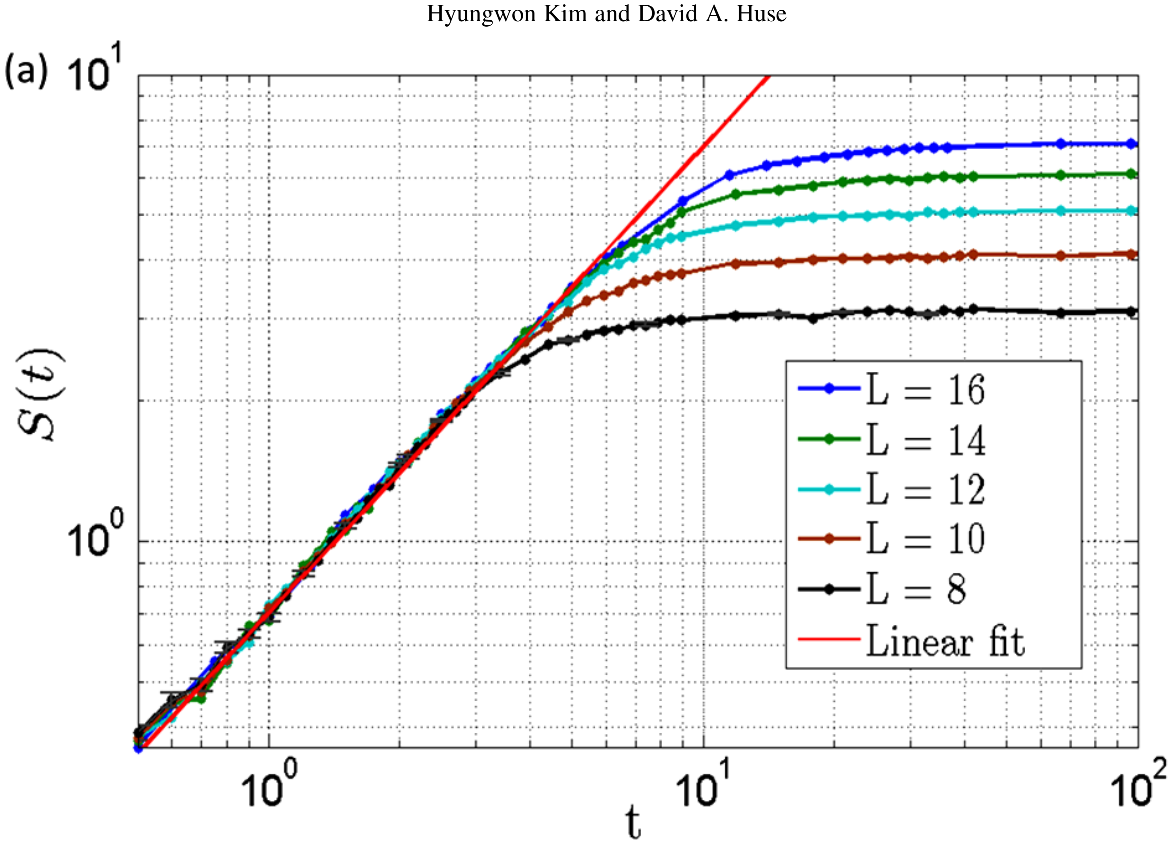
[Calabrese-Cardy 2005

De Chiara 2006]

PRL **111,** 127205 (2013) PHYSICAL REVIEW LETTERS week ending 20 SEPTEMBER 2013

Ballistic Spreading of Entanglement in a Diffusive Nonintegrable System

Hyungwon Kim and David A. Huse



Entanglement growth after a global quench

ETH

MBL

- Ballistic

[Calabrese-Cardy 2005

De Chiara 2006]

- Logarithmic

[De Chiara 2006, Znidaric 2008, Serbyn 2013, Vosk, ...

PRL **111,** 127205 (2013) PHYSICAL REVIEW LETTERS week ending 20 SEPTEMBER 2013

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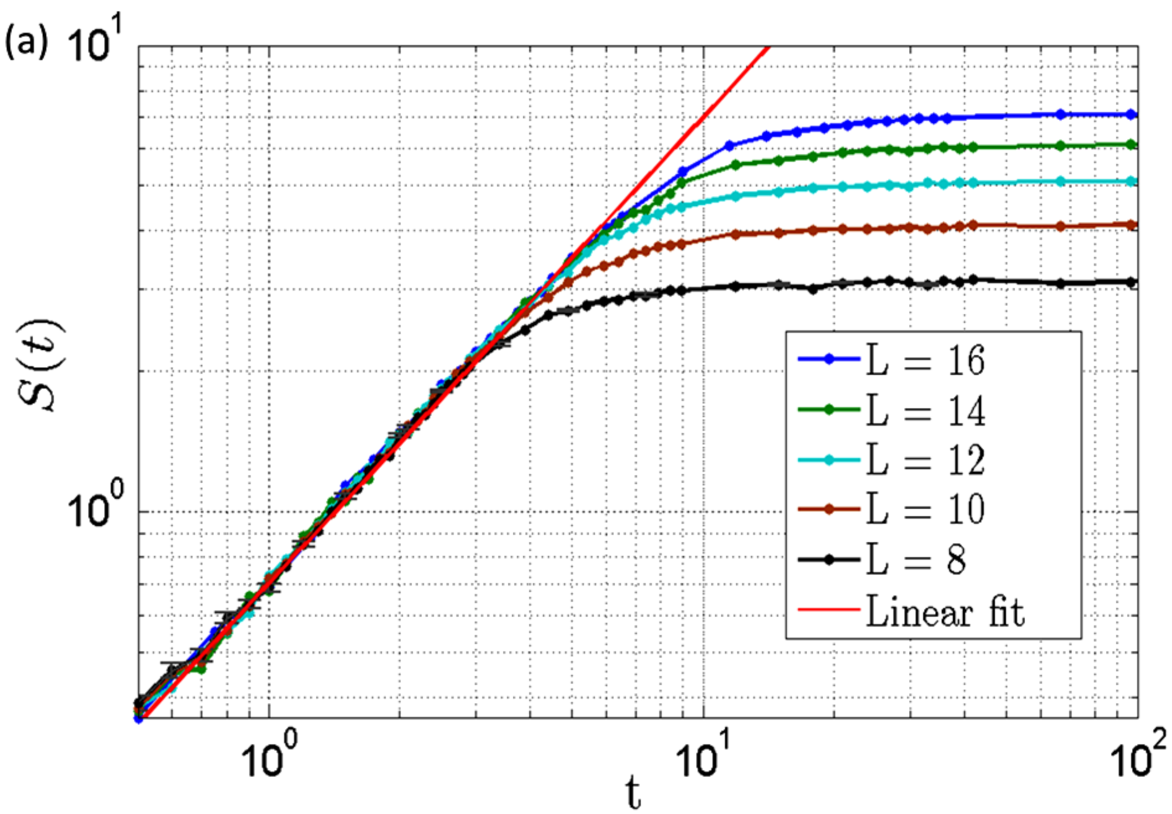
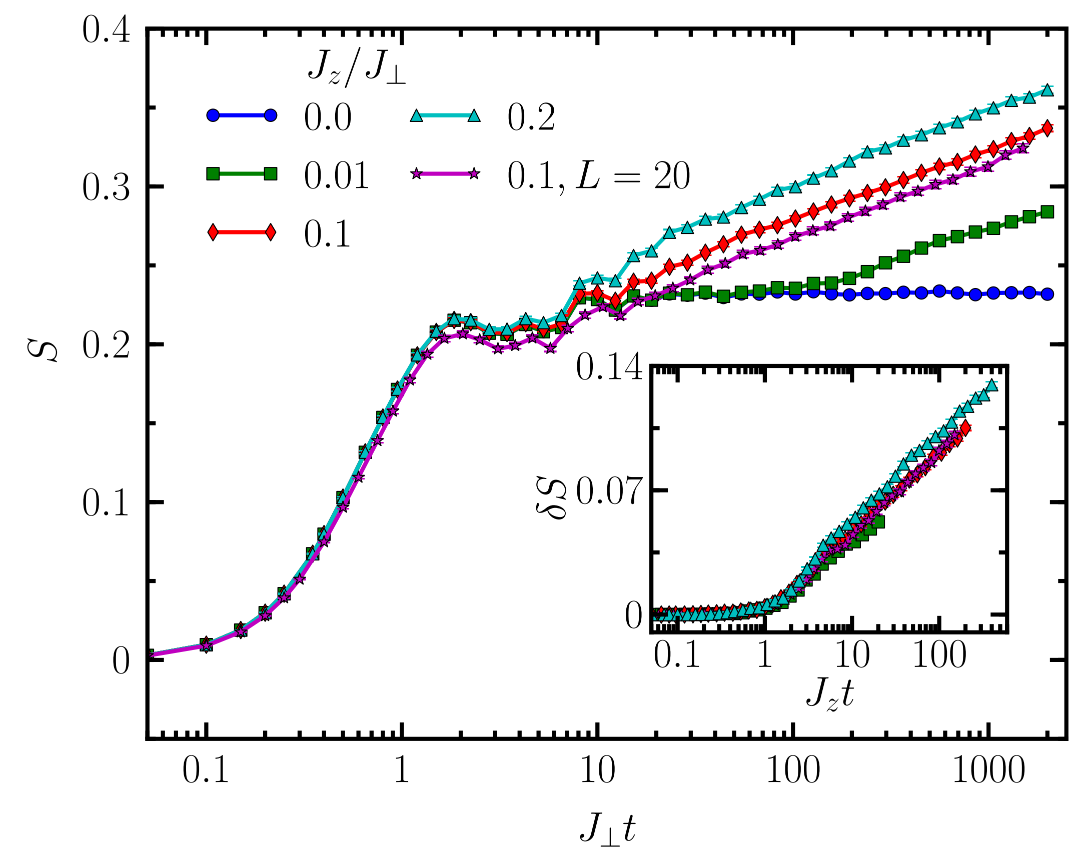


PRL **109,** 017202 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

Unbounded Growth of Entanglement in Models of Many-Body Localization

Jens H. Bardarson,[1,2] Frank Pollmann,[3] and Joel E. Moore[1,2]



Entanglement growth after a global quench

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Sub-ballistic?

[Vosk, Huse, Altman]
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Logarithmic

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PRL **111,** 127205 (2013) PHYSICAL REVIEW LETTERS week ending 20 SEPTEMBER 2013

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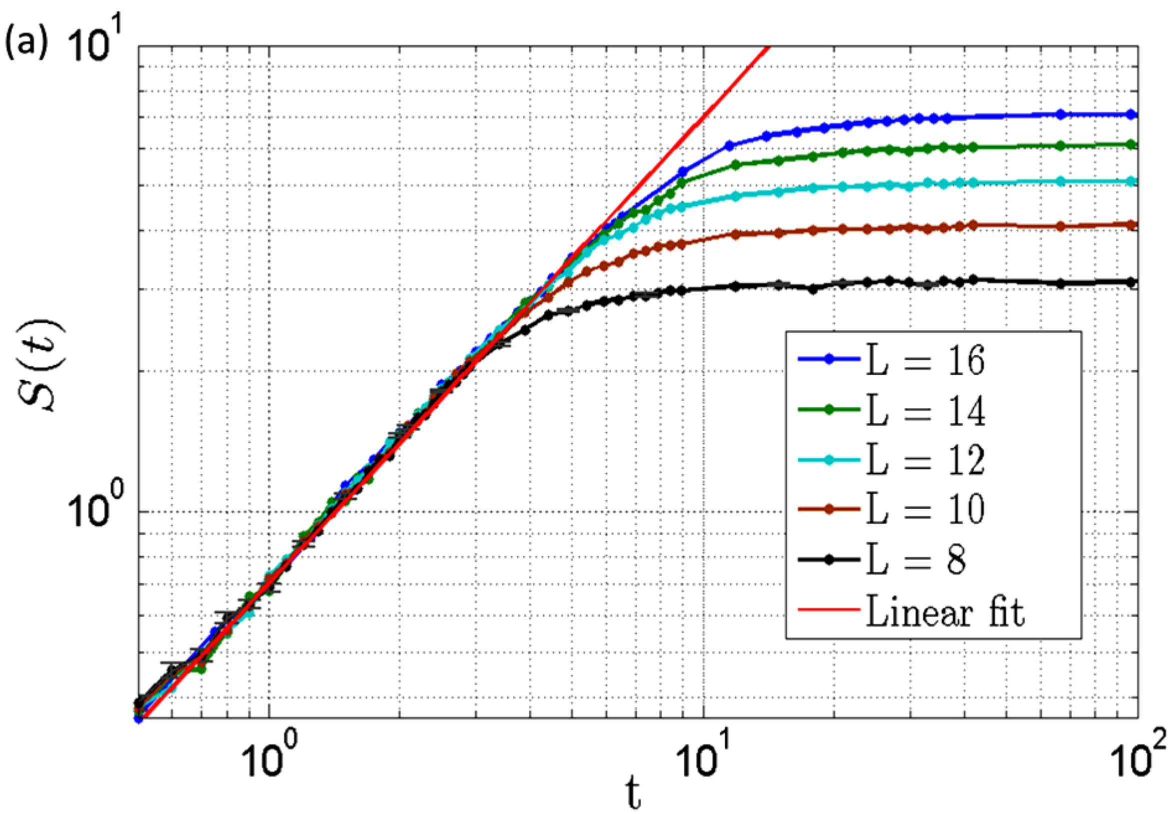

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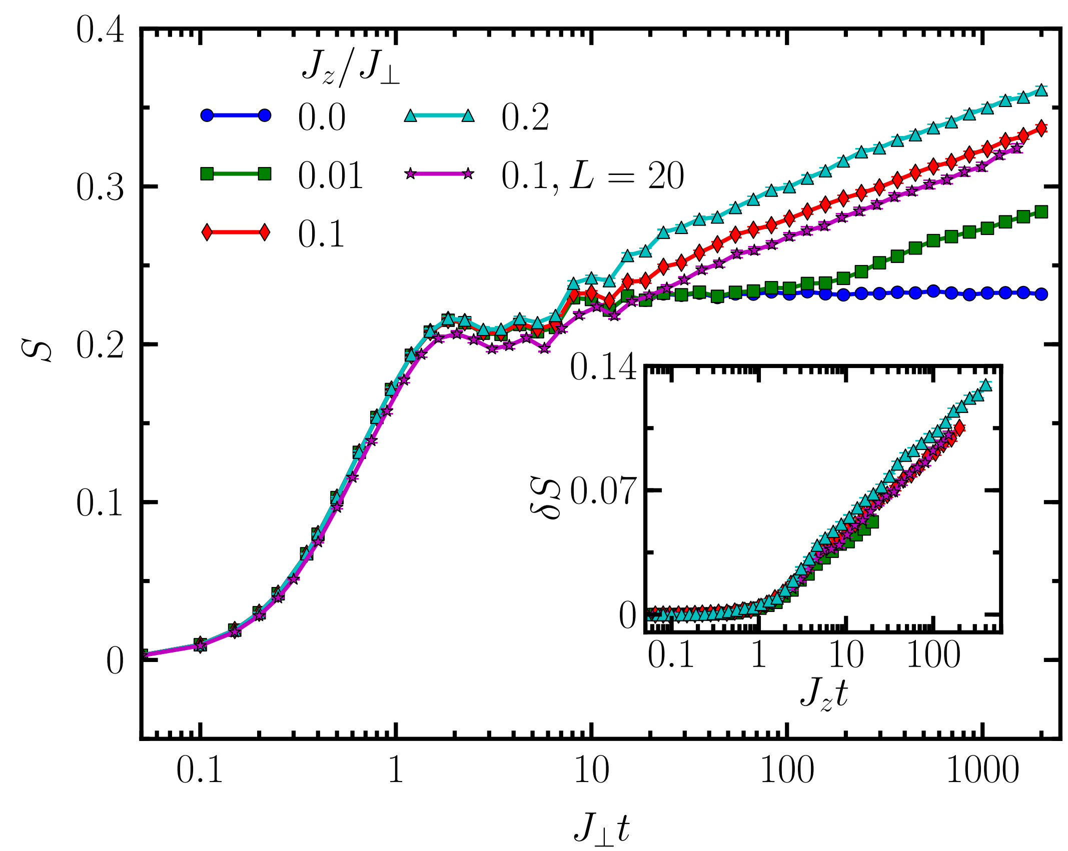

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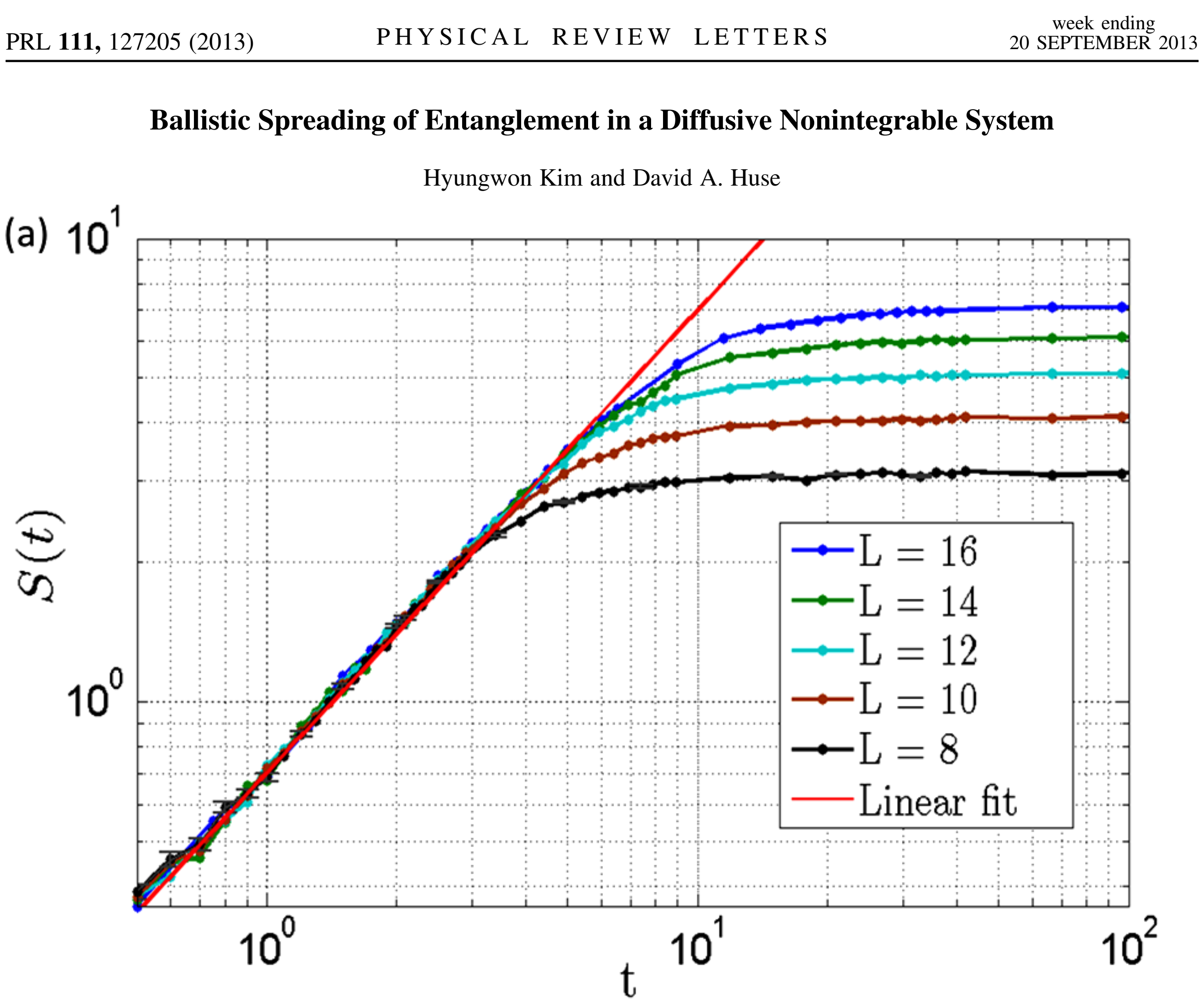
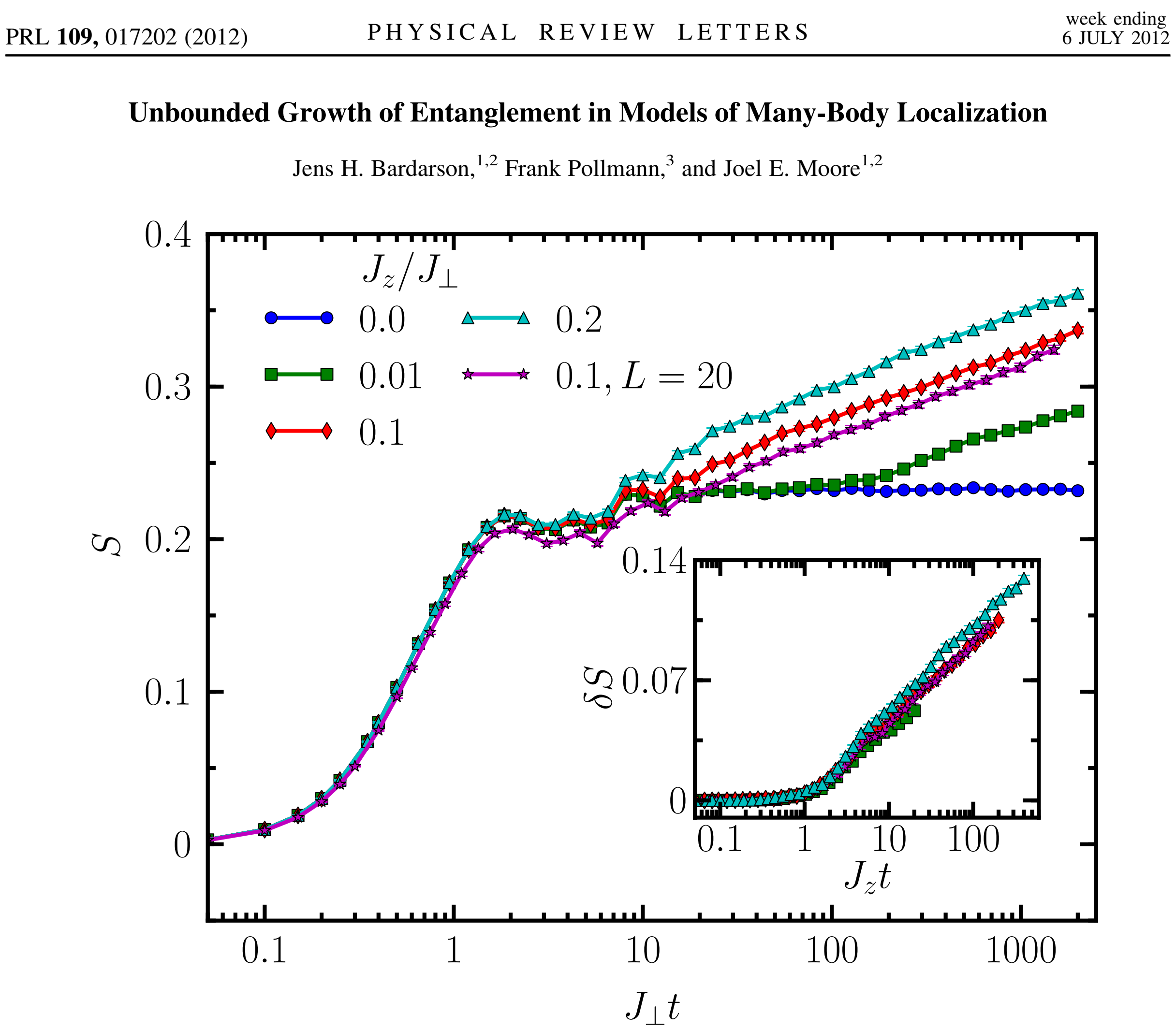
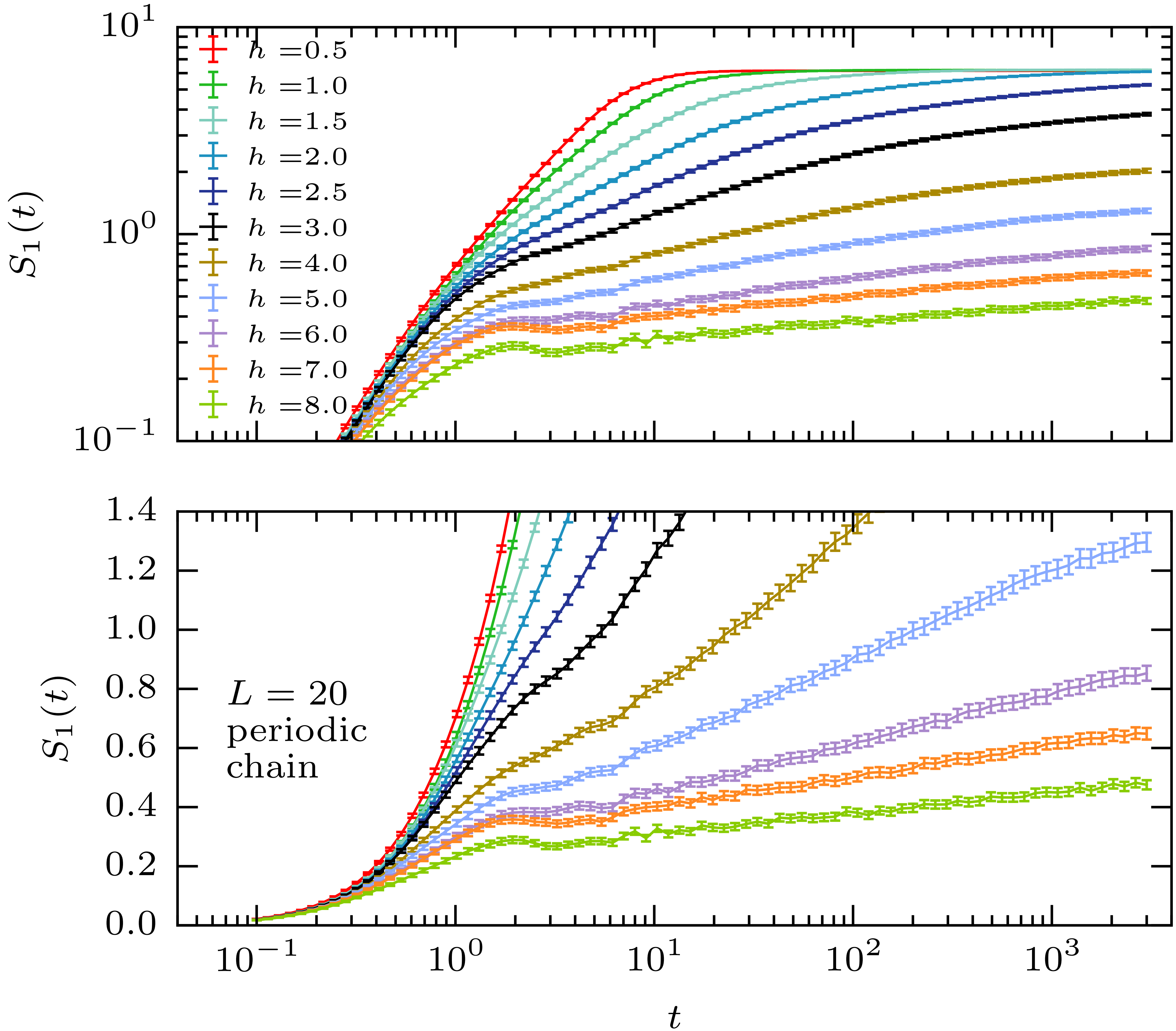


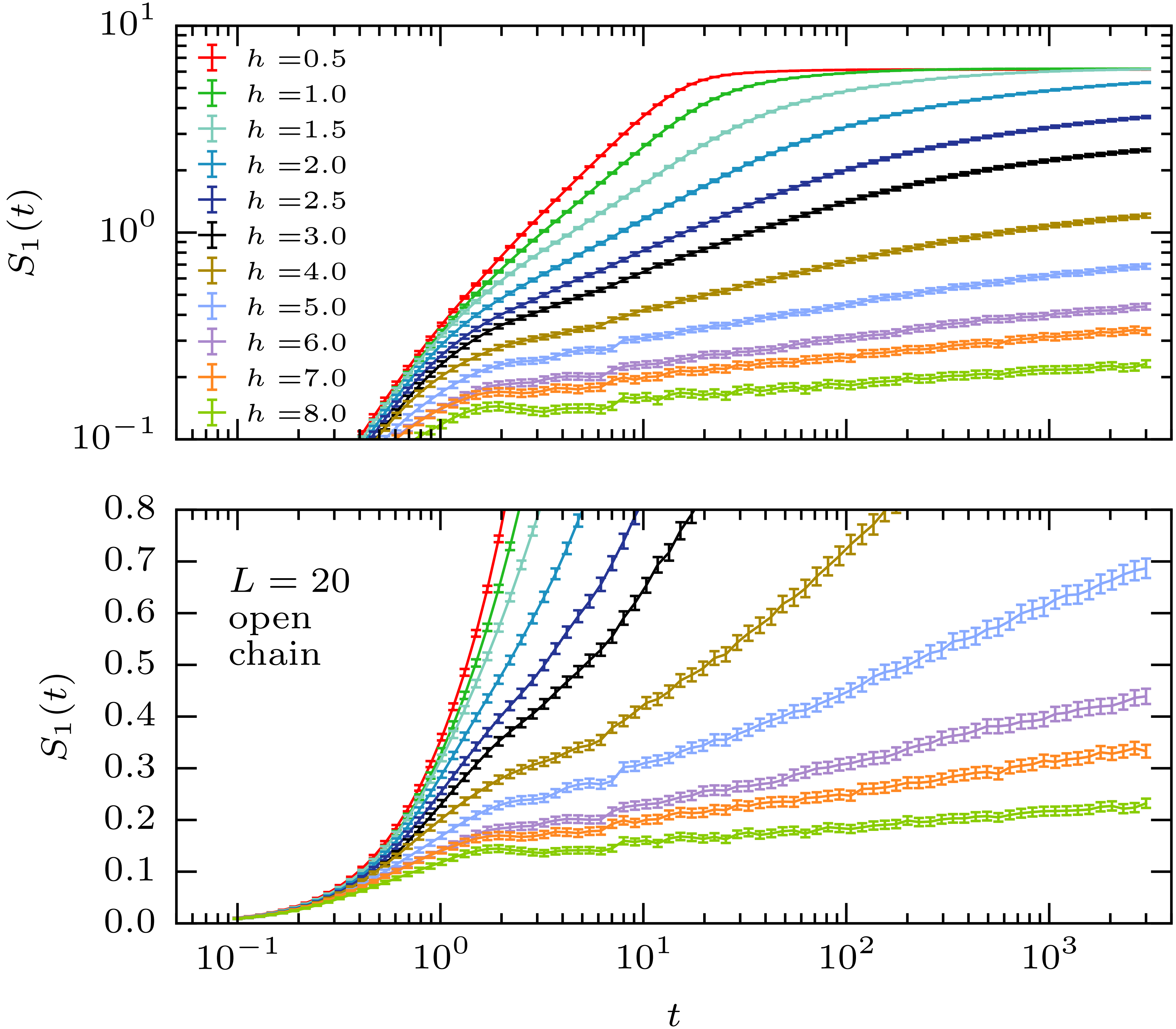


- Let's try some «large scale» ED at energy $\epsilon = 0.5$

Better to use OBC

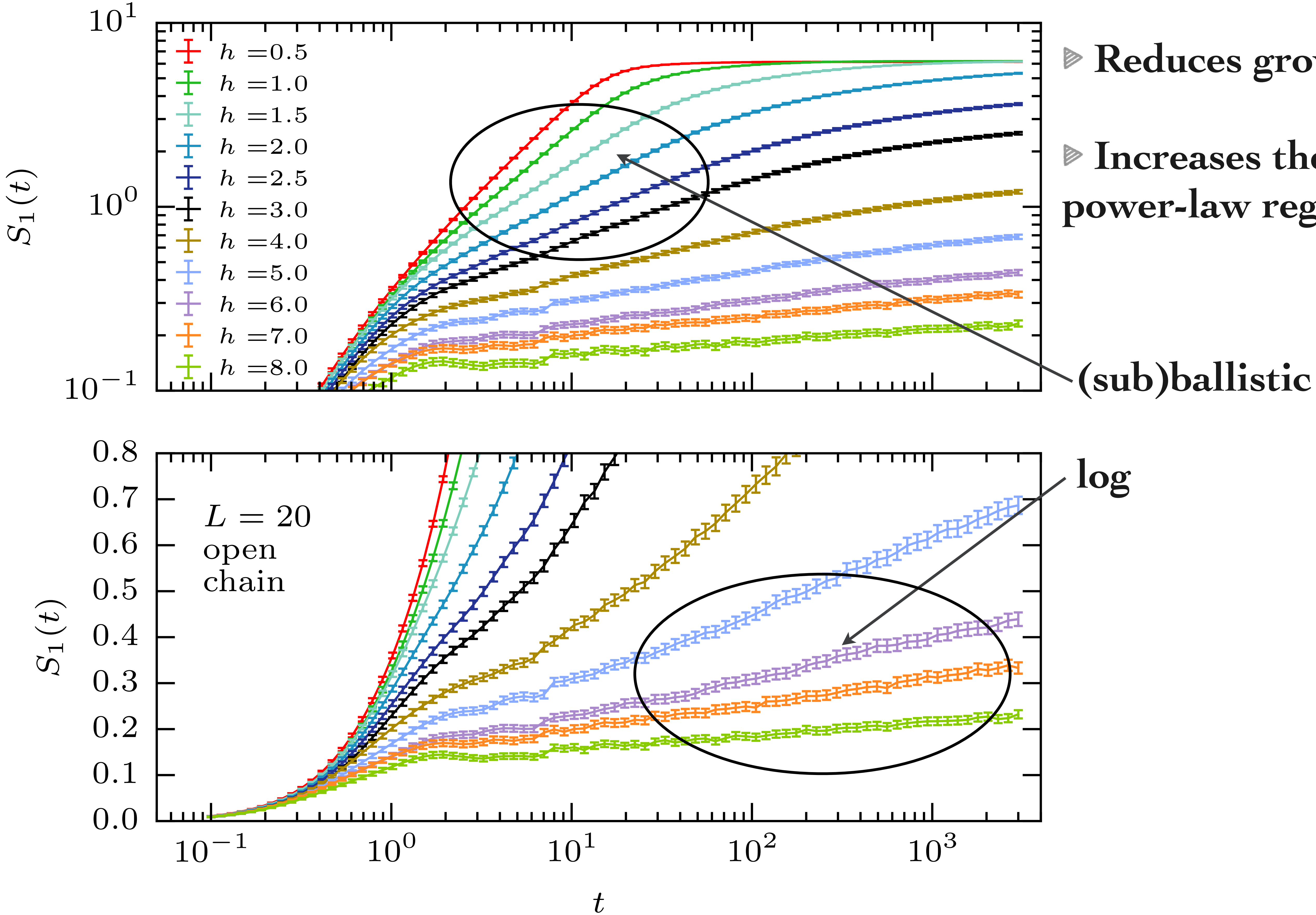


Better to use OBC



- Reduces growth
- Increases the power-law regime

Better to use OBC



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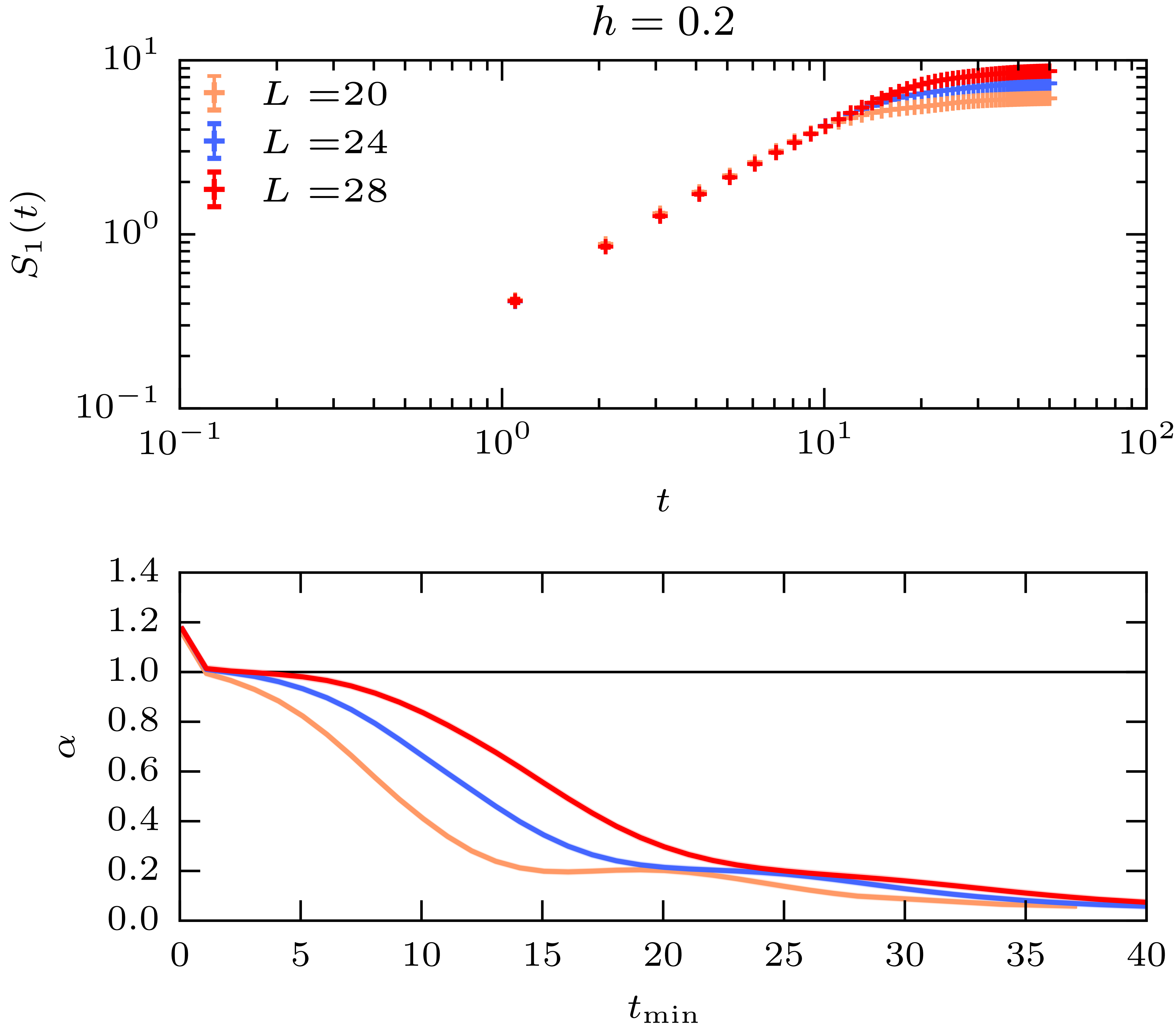
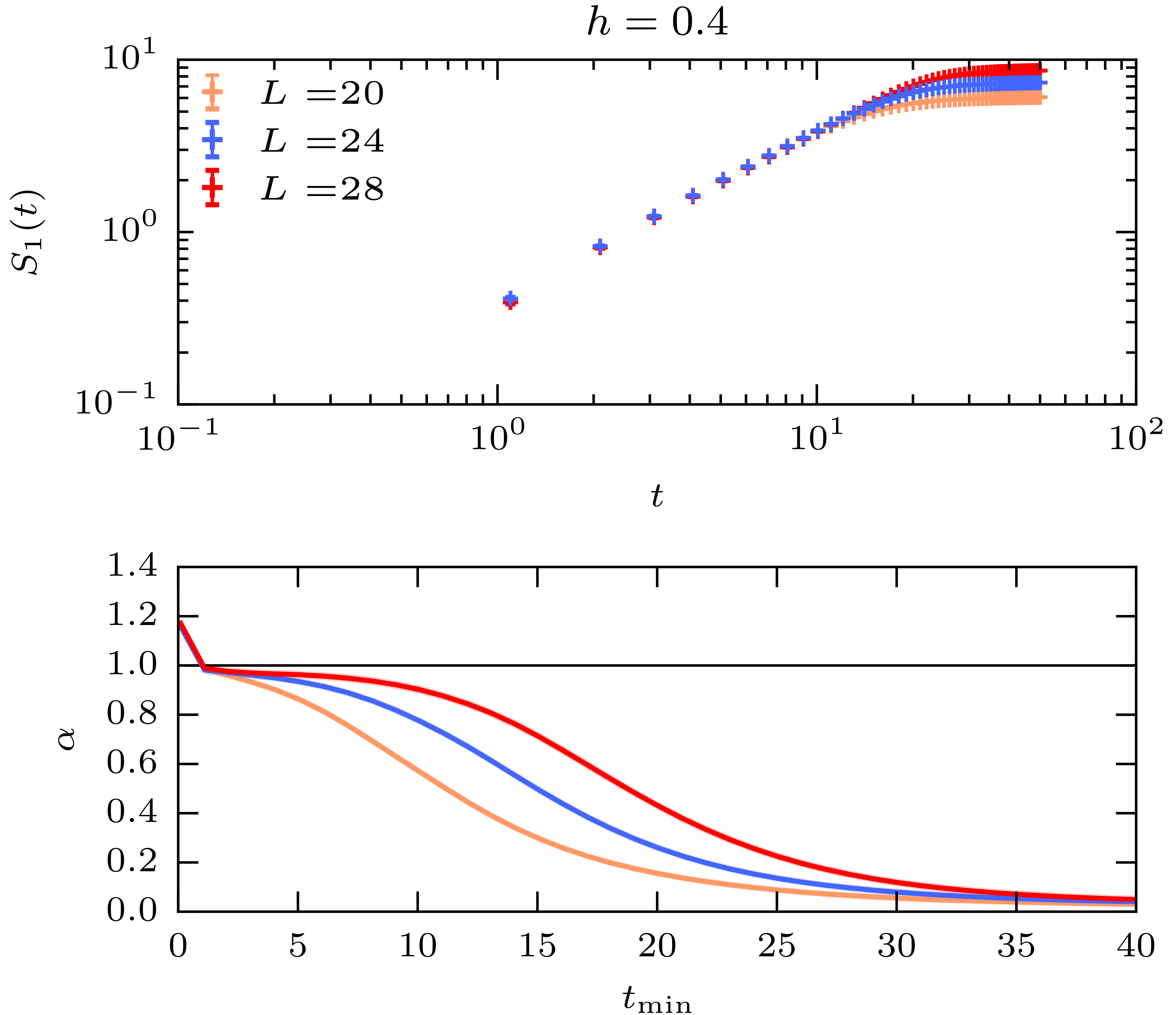
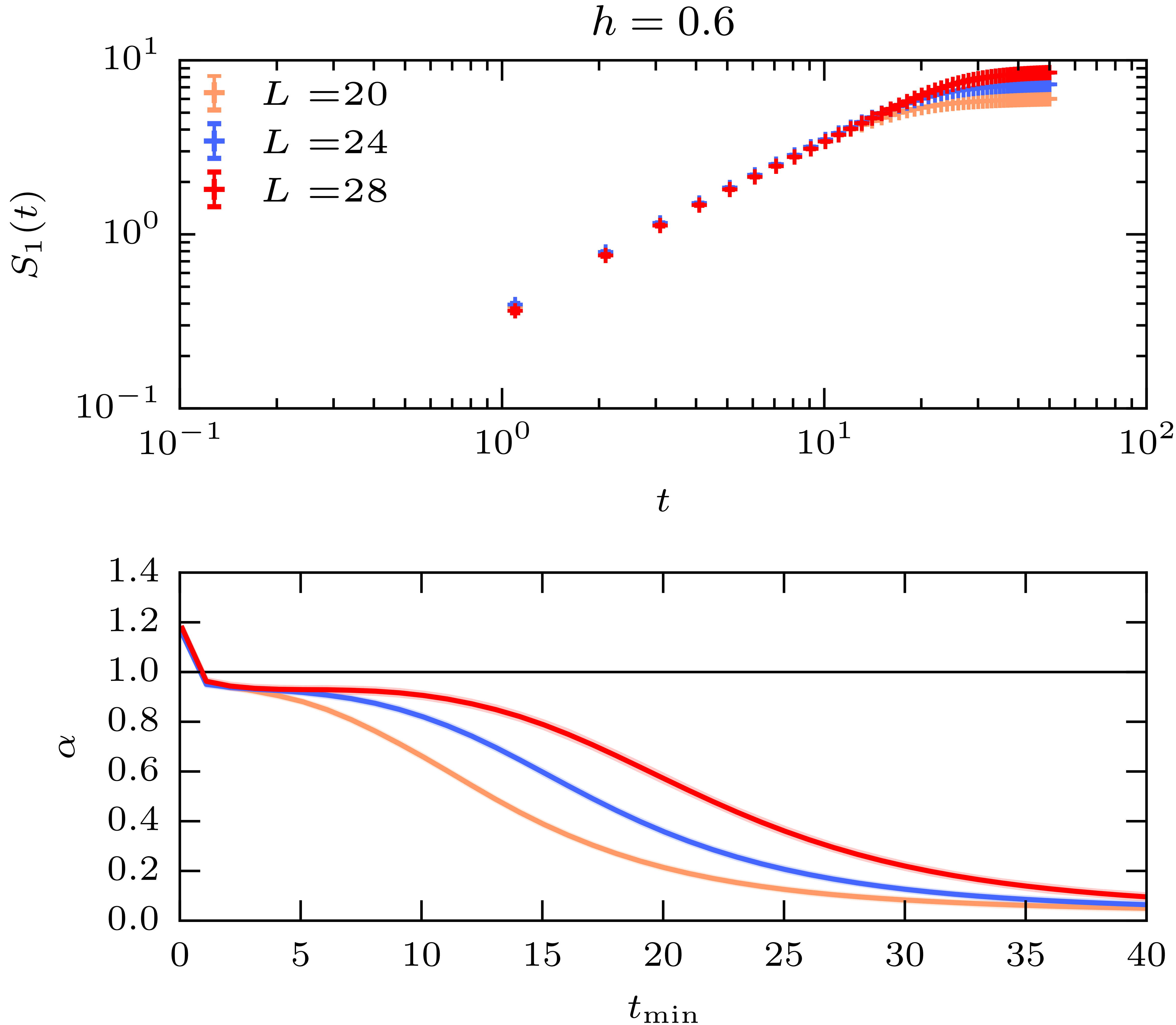


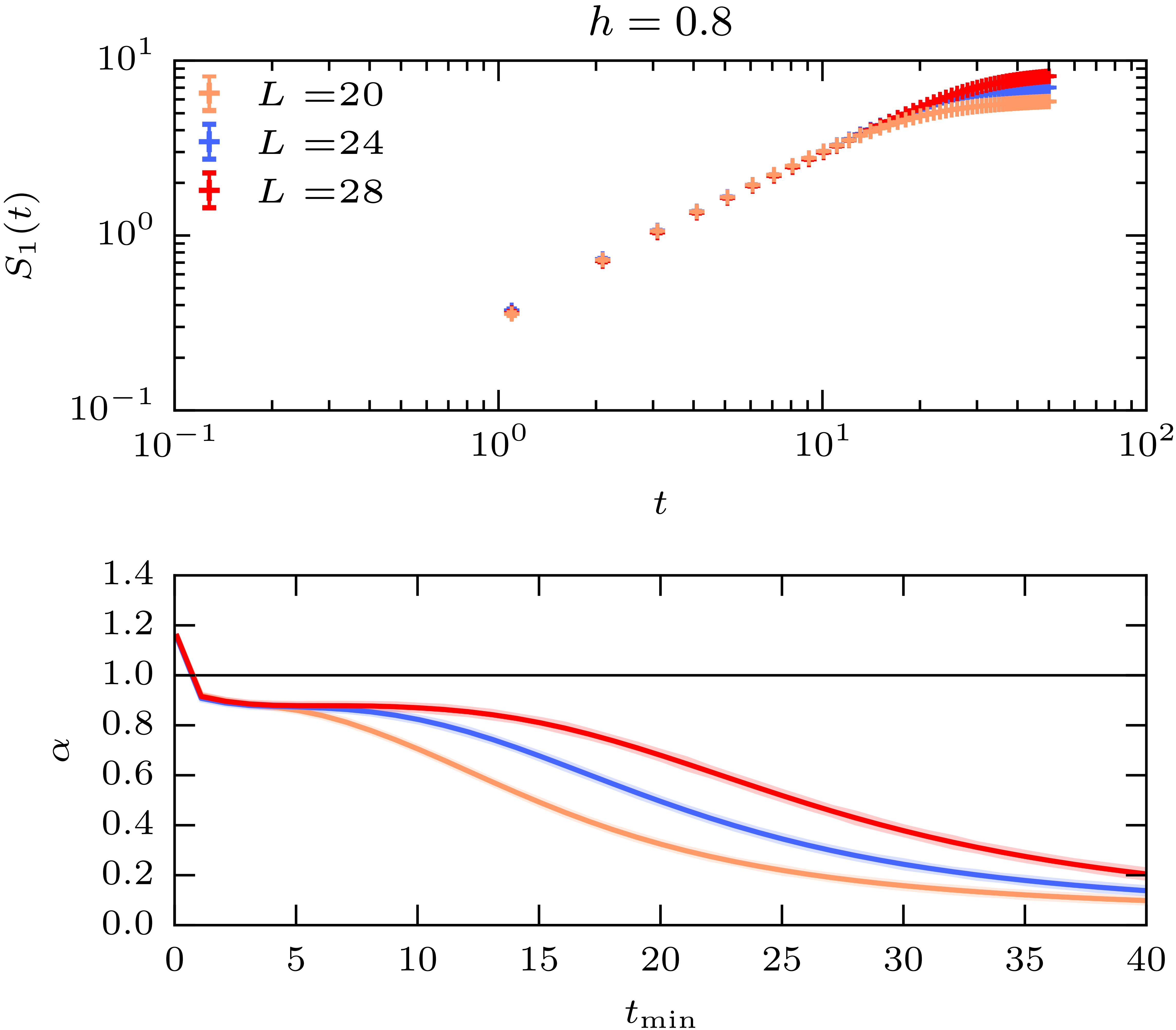
$h = 0.2$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
α
$t_{\min}$



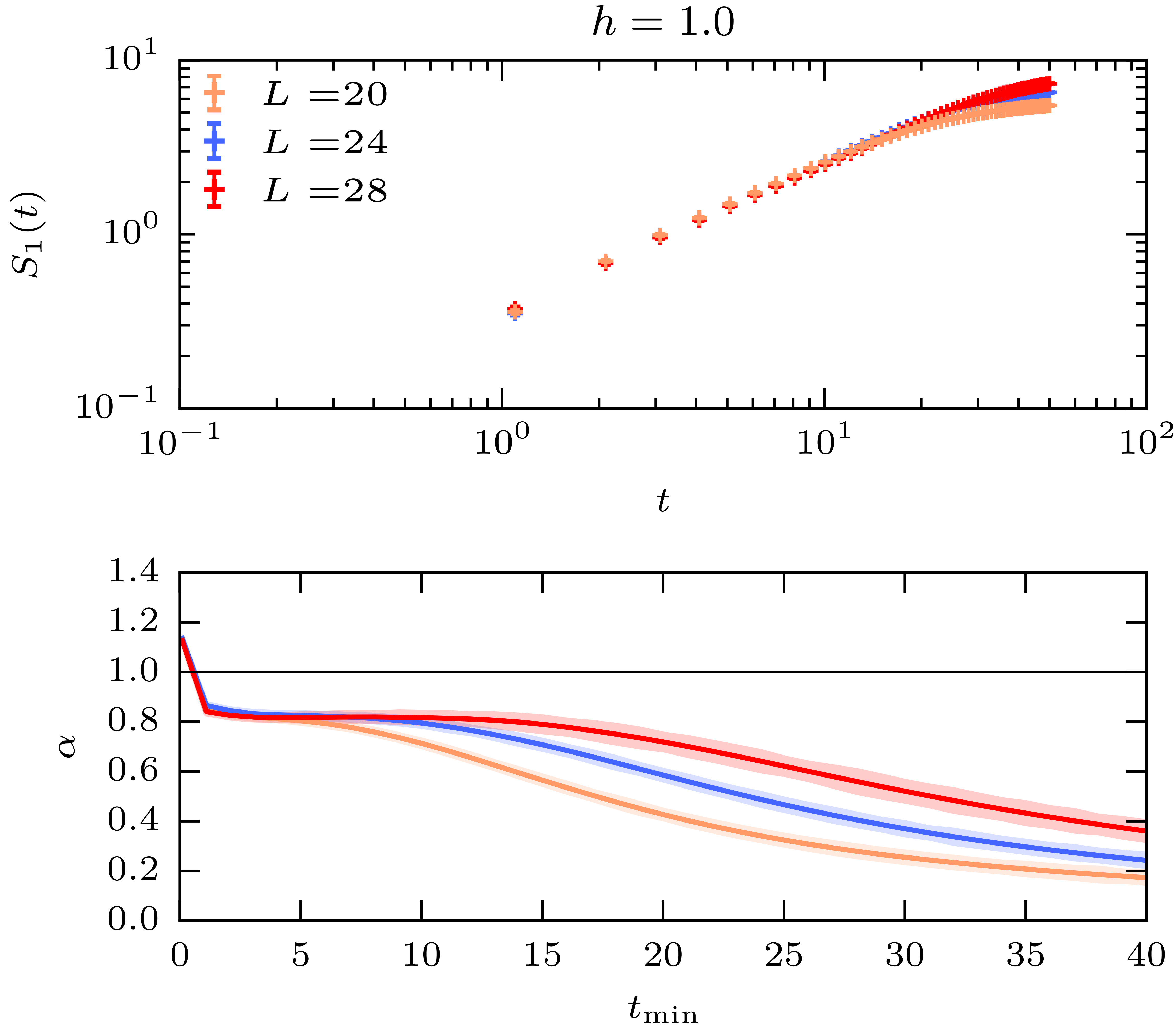
$h = 0.4$
L =20
L =24
L =28
$S_1(t)$
10^1
10^0
10^{-1}
10^{-1}
10^0
10^1
10^2
t
α
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
40
$t_{\min}$



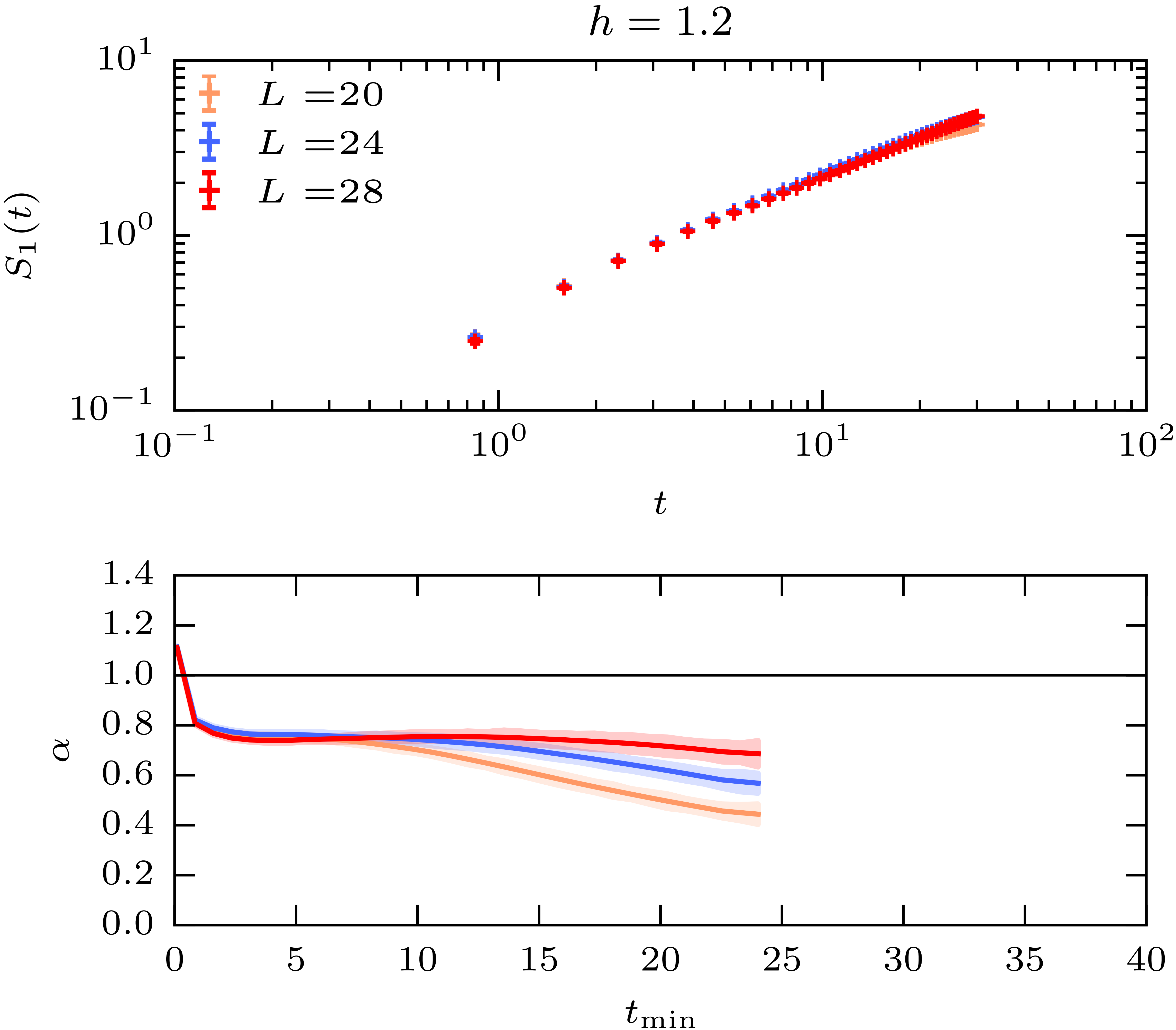
$h = 0.6$
10^{1}
10^{0}
10^{-1}
$S_1(t)$
$L = 20$
$L = 24$
$L = 28$
10^{-1}
10^{0}
10^{1}
10^{2}
t
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
α
0
5
10
15
20
25
30
35
40
$t_{\min}$

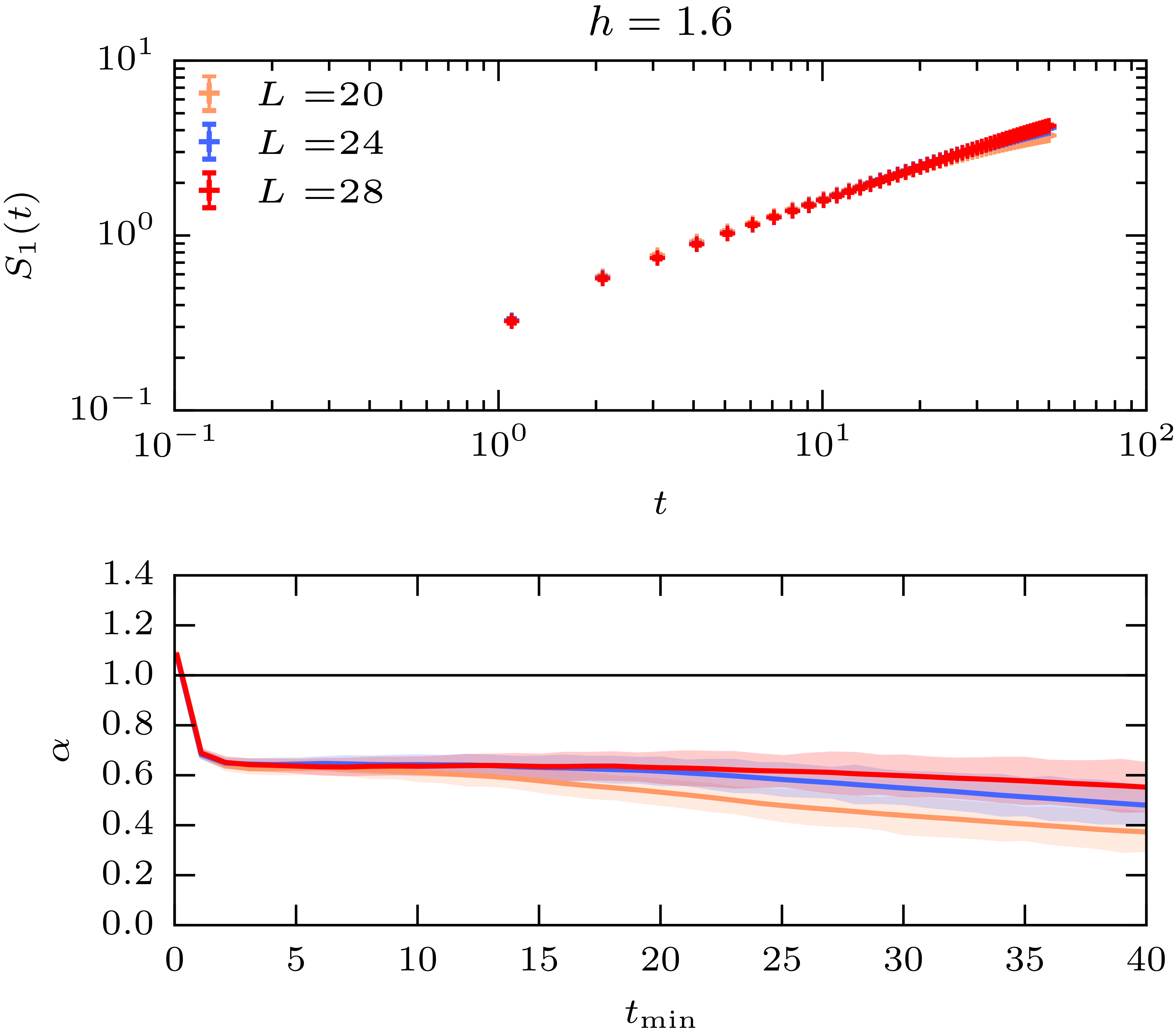


$h = 0.8$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
α
$t_{\min}$

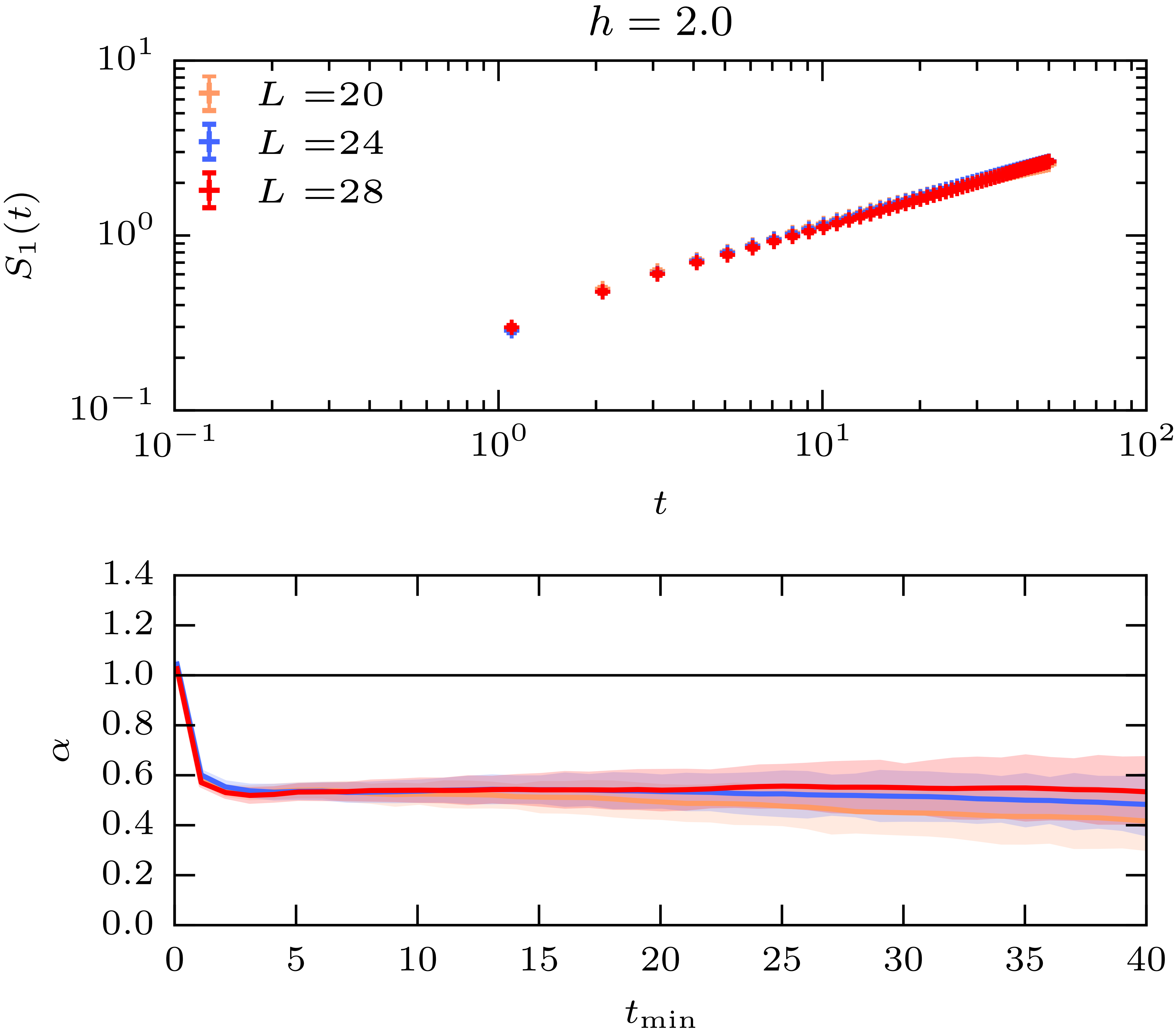


$h = 1.0$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
α
t_{min}

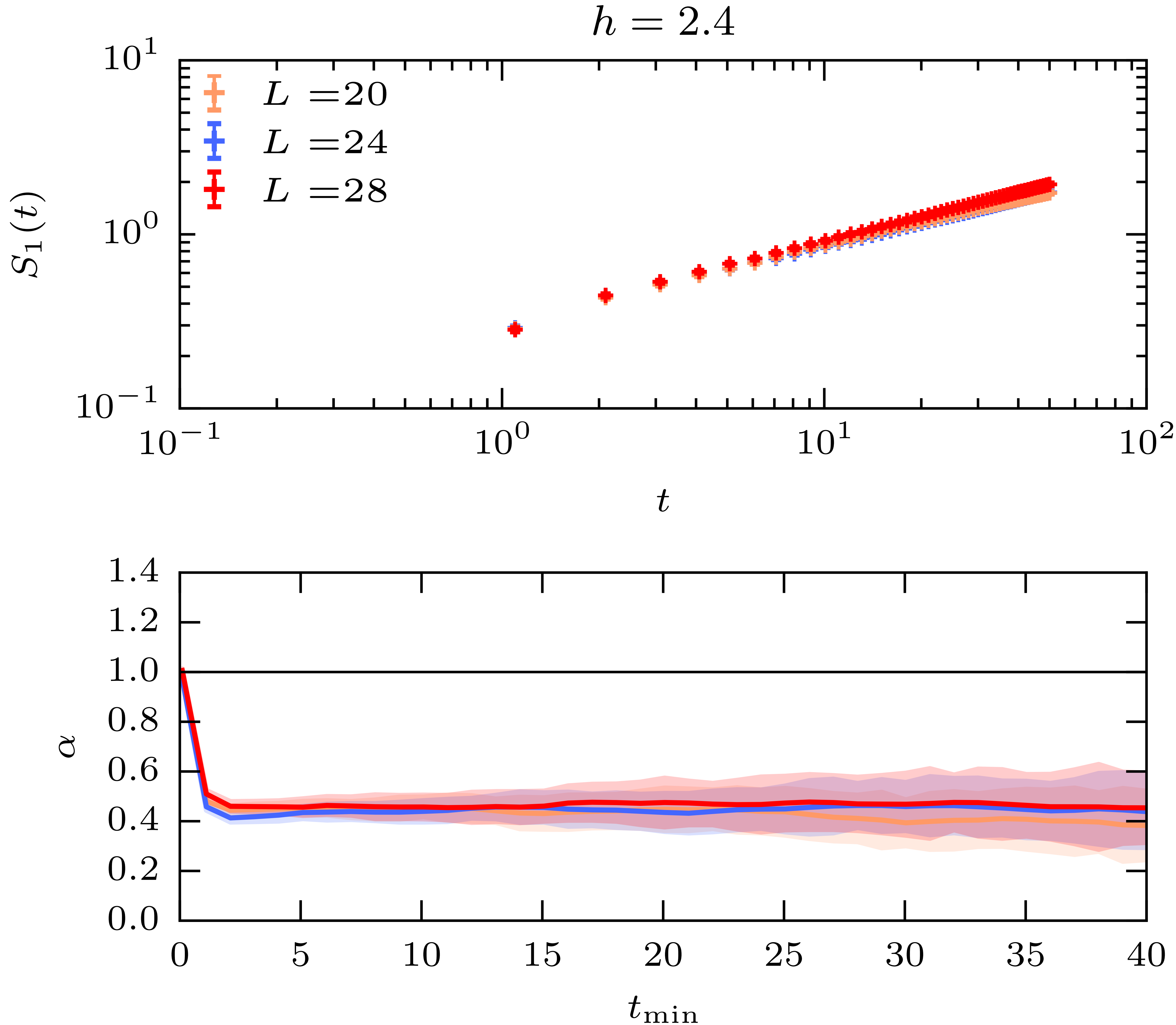

$h = 1.2$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
α
$t_{\min}$



$h = 1.6$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
10^1
10^0
10^{-1}
10^{-1}
10^0
10^1
10^2
t
α
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0
5
10
15
20
25
30
35
40
$t_{\min}$



$h = 2.0$
10^{1}
10^{0}
10^{-1}
$S_1(t)$
$L = 20$
$L = 24$
$L = 28$
10^{-1}
10^{0}
10^{1}
10^{2}
t
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
α
0
5
10
15
20
25
30
35
40
$t_{\min}$



$h = 2.4$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
10^{-1}
10^{0}
10^{1}
10^{2}
α
$t_{\min}$
1.4
1.2
1.0
0.8
0.6
0.4
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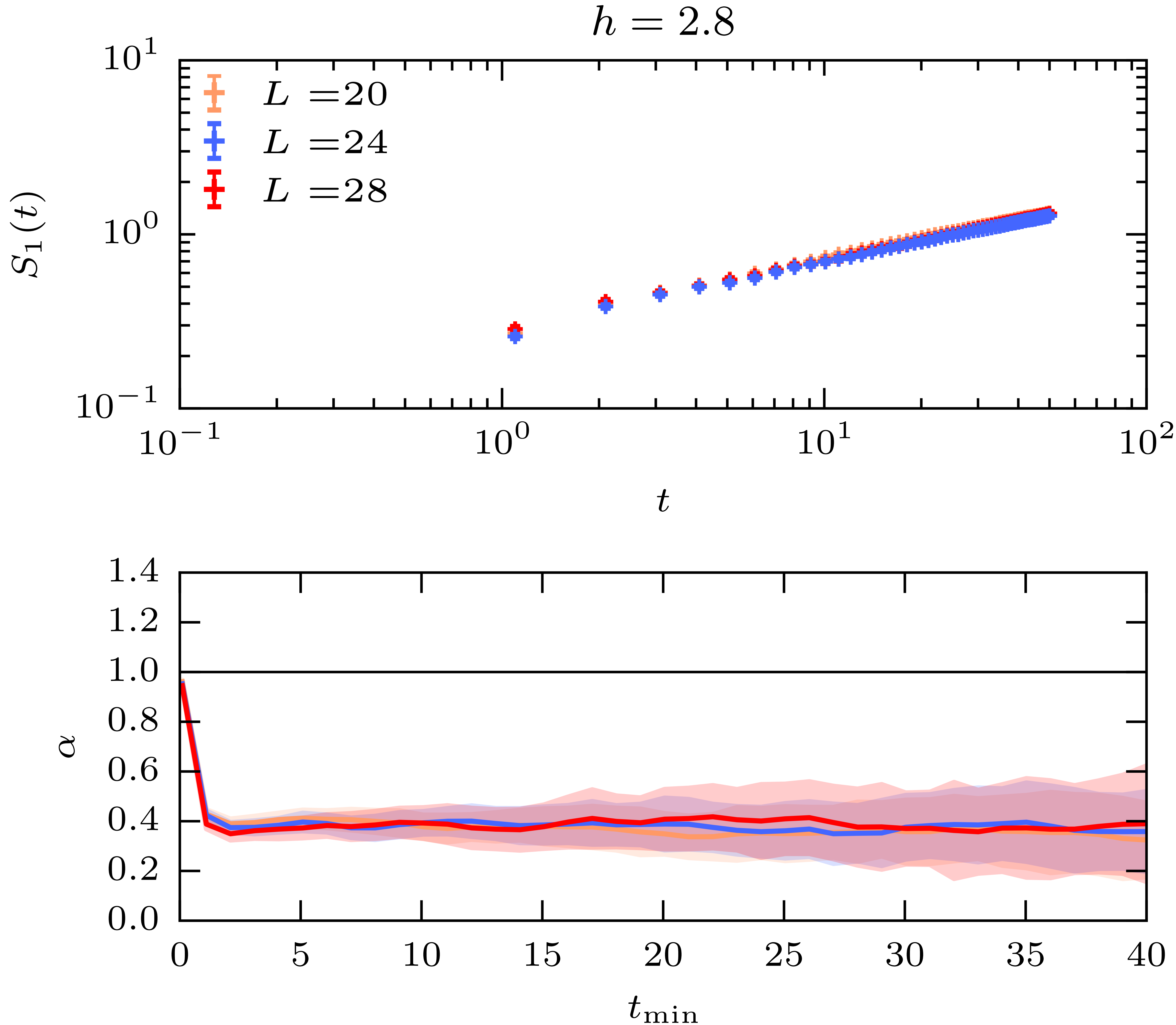
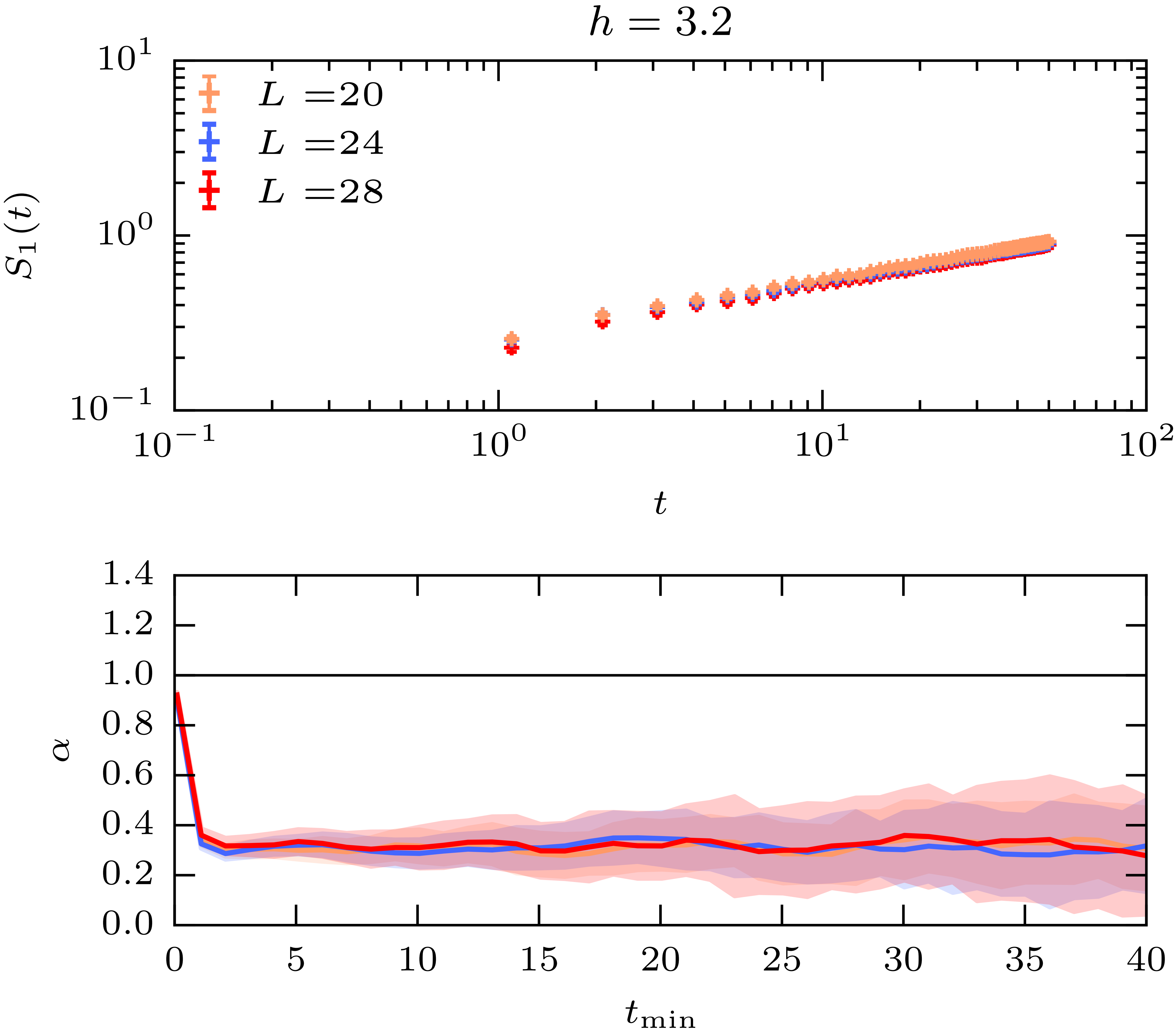
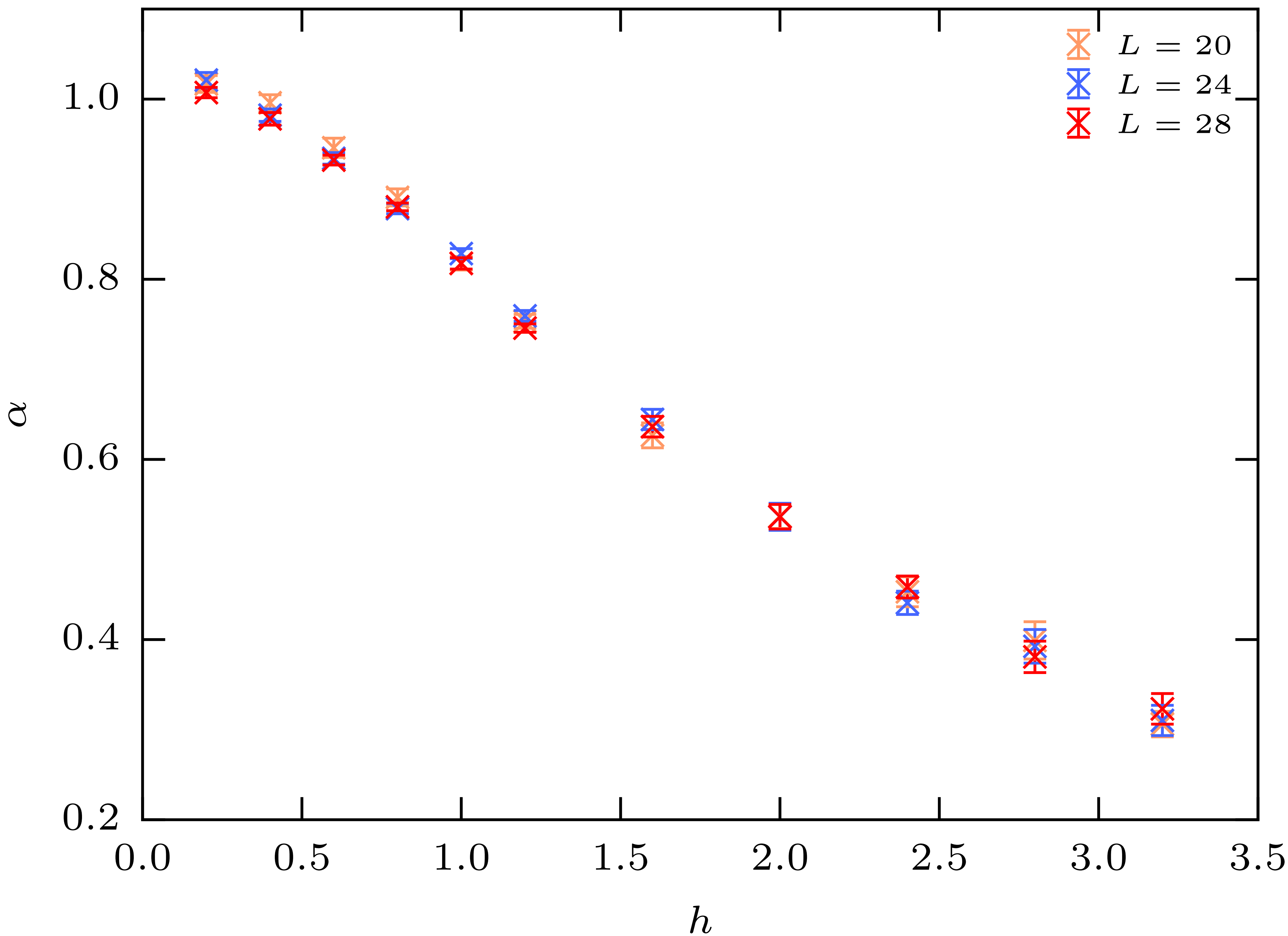


$h = 2.8$
$L = 20$
$L = 24$
$L = 28$
$S_1(t)$
t
10^{-1}
10^{0}
10^{1}
10^{2}
α
$t_{\min}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25
30
35
40



$h = 3.2$
10^1
10^0
10^{-1}
$S_1(t)$
$L = 20$
$L = 24$
$L = 28$
10^{-1}
10^0
10^1
10^2
t
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
α
0
5
10
15
20
25
30
35
40
$t_{\min}$

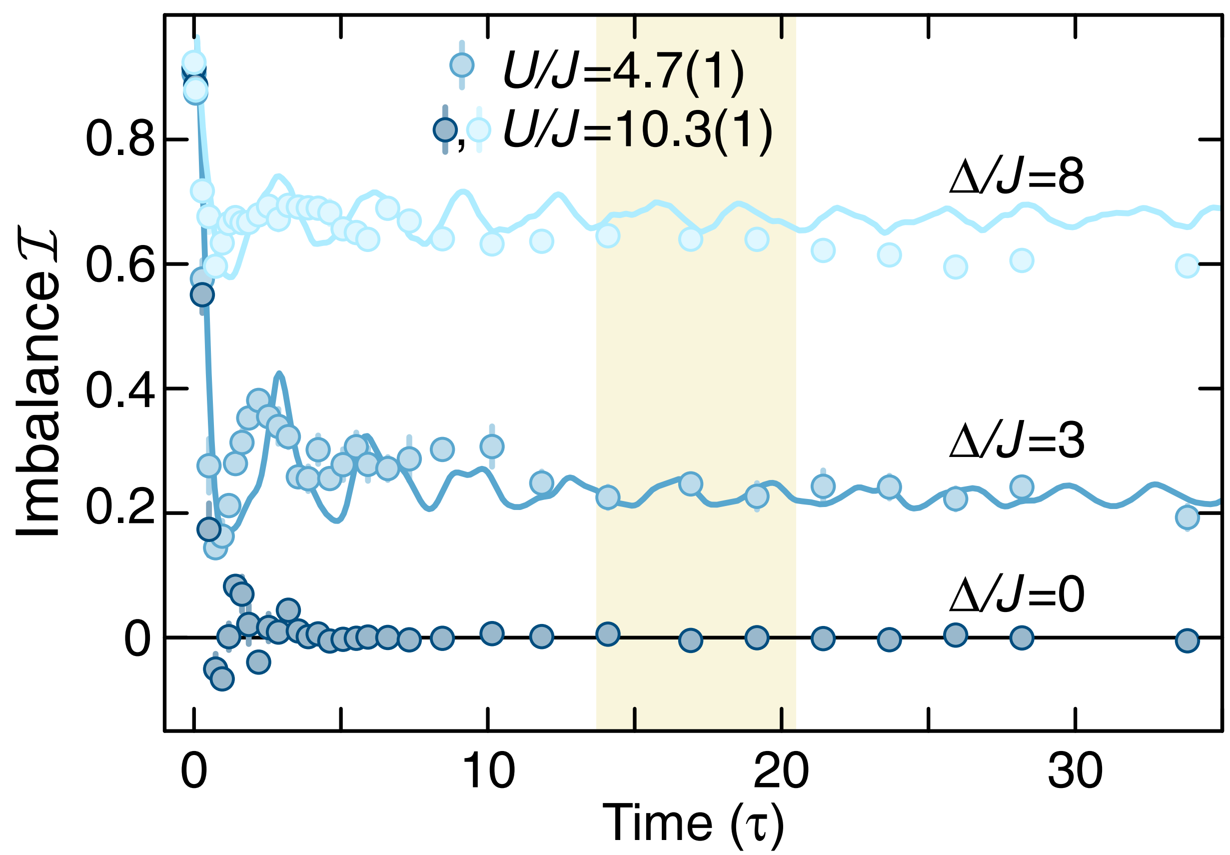
Sub-ballistic growth $S \sim t^{\alpha}$ in the most of the delocalized regime



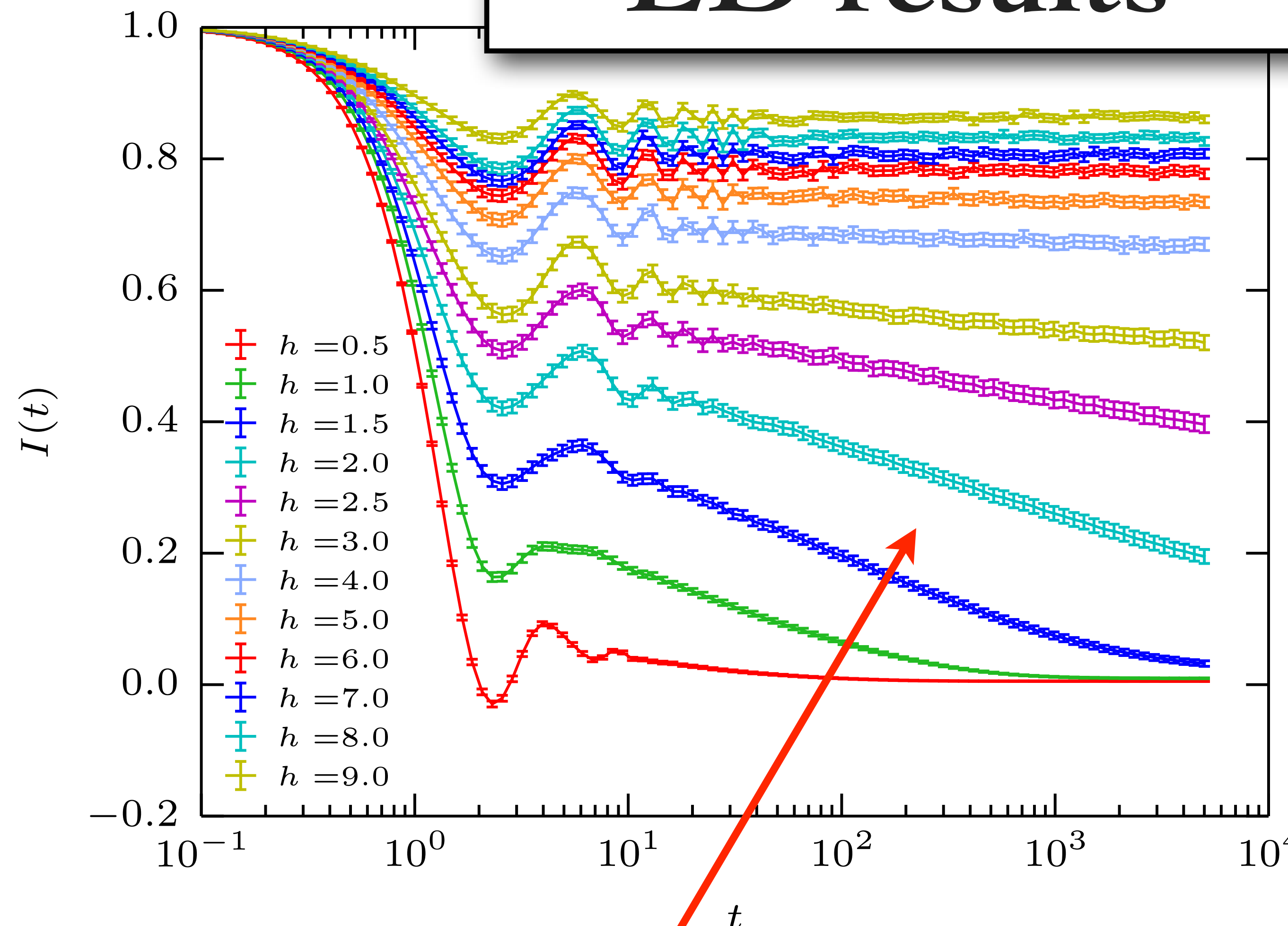
spin density imbalance

Observation of many-body localization of interacting fermions in a quasi-random optical lattice

Michael Schreiber[1,2], Sean S. Hodgman[1,2], Pranjal Bordia[1,2], Henrik P. Lüschen[1,2], Mark H. Fischer[3], Ronen Vosk[3], Ehud Altman[3], Ulrich Schneider[1,2] and Immanuel Bloch[1,2]



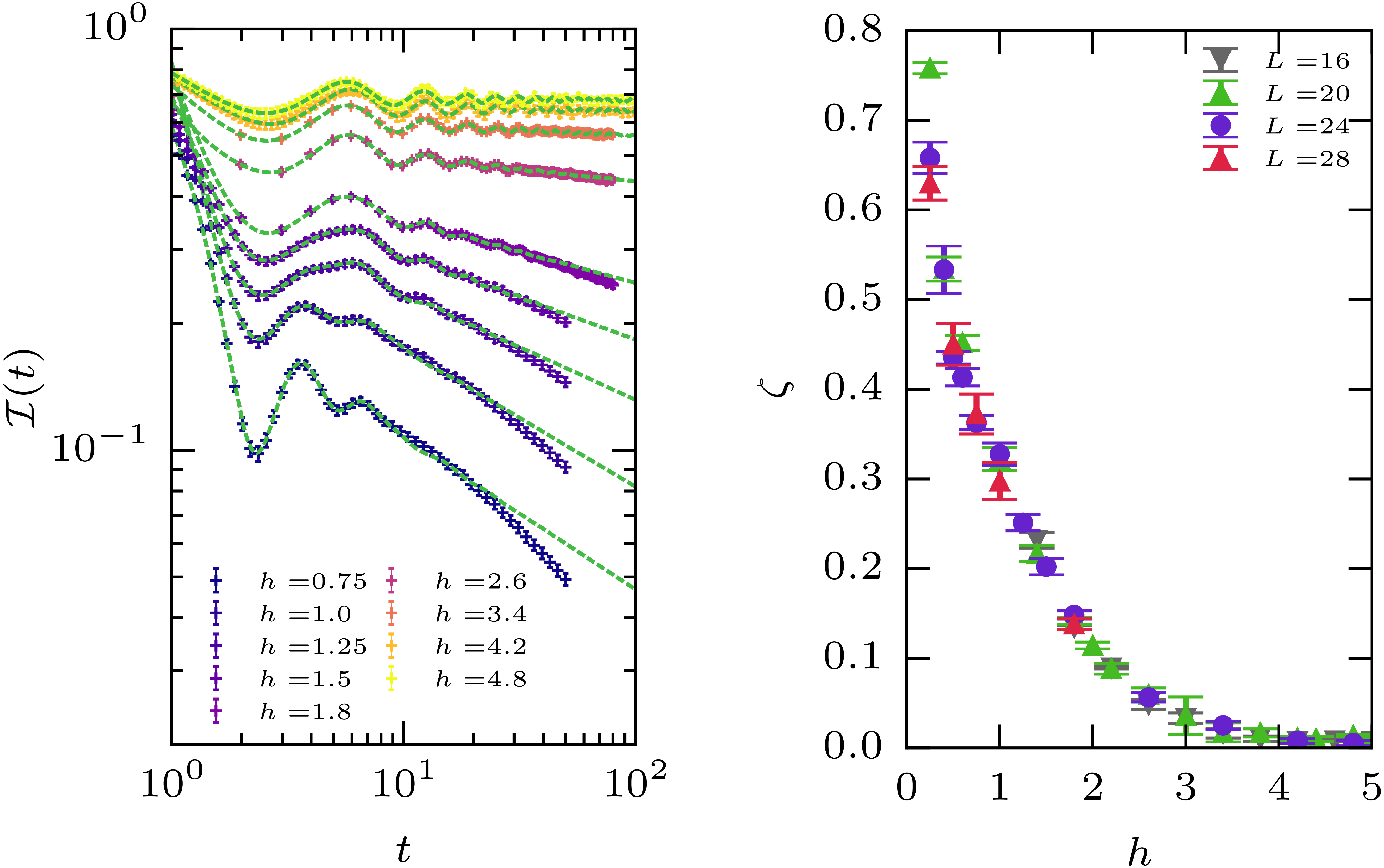
ED results



- We want to explore the sub-diffusive regime

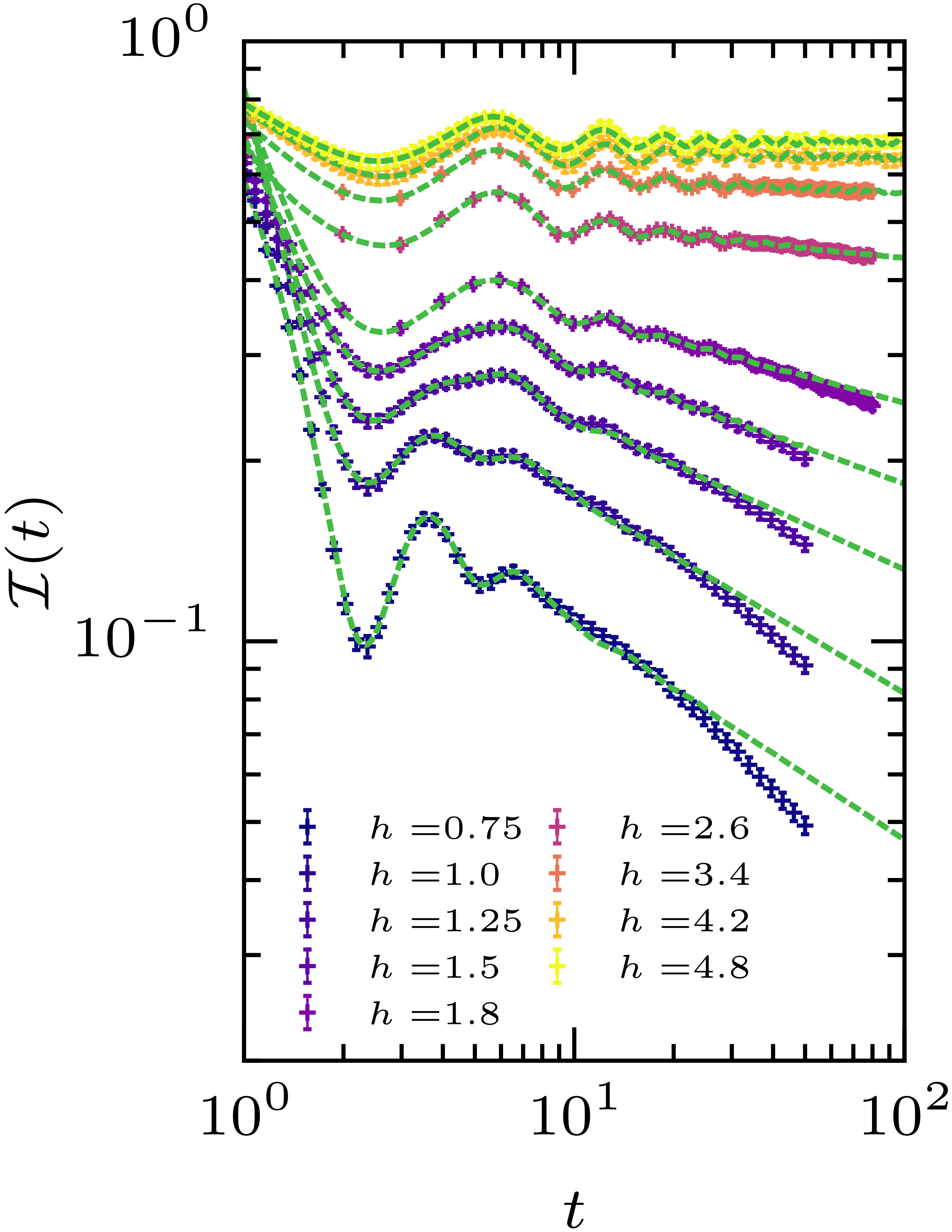
Imbalance

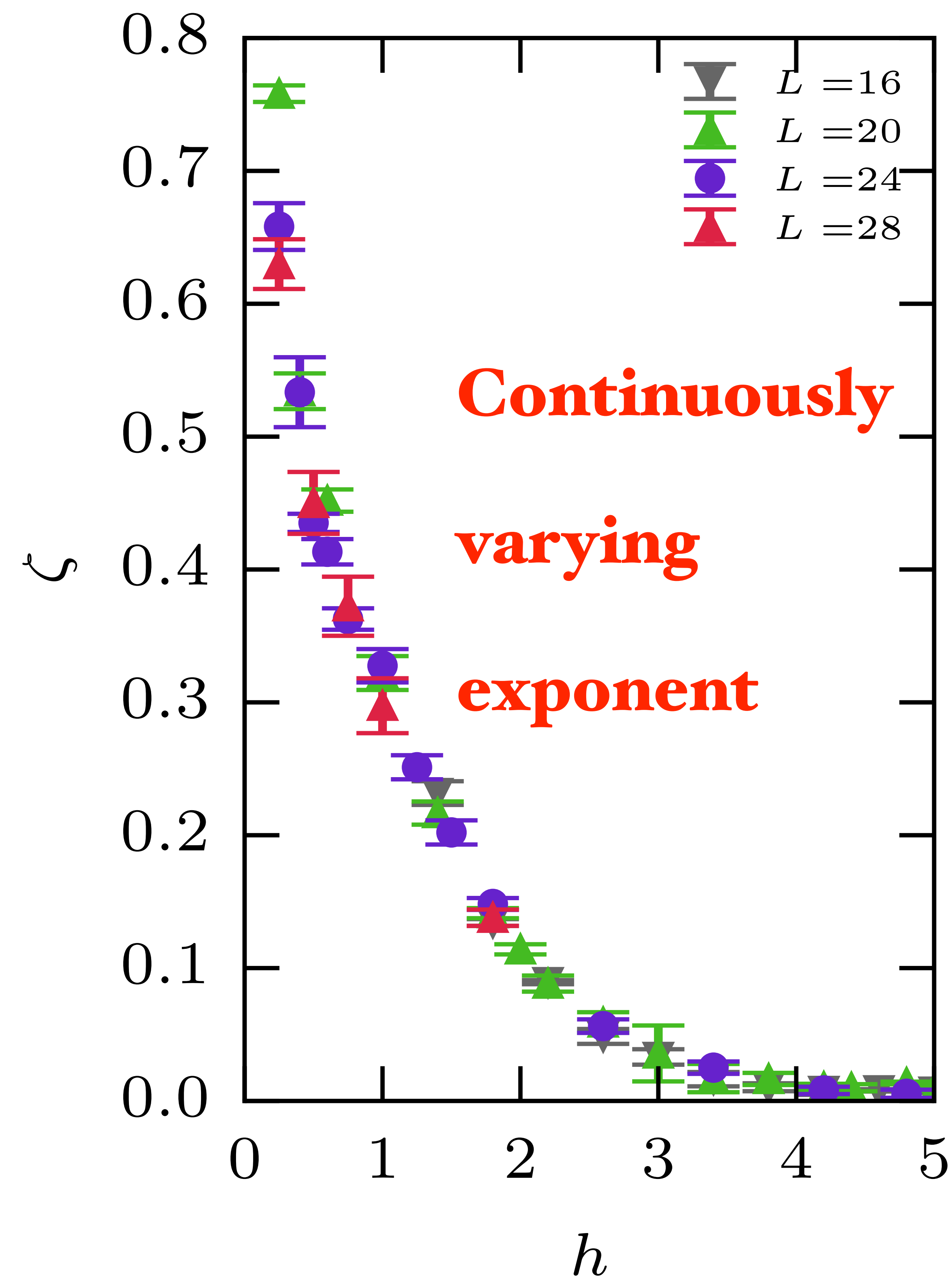
$$\mathcal{I}(t) = a\mathrm{e}^{-\frac{t}{\tau}}\cos(\omega_1 t+\varphi)+bt^{-\zeta}[1+ct^{-\eta}\sin(\omega_2 t+\varphi)]$$



Imbalance

$$\mathcal{I}(t) = a e^{-\frac{t}{\tau}} \cos(\omega_1 t + \varphi) + b t^{-\zeta}[1 + c t^{-\eta} \sin(\omega_2 t + \varphi)]$$





Rare regions: bottlenecks for transport in the delocalized regime

$$\ell_{\mathrm{b-n}} \sim z \ln L$$

[Vosk, Huse, Altman, PRX 2015]
[Potter, Vasseur, Parameswaran, PRX 2015]

$$z \propto \xi \qquad t^{\mathrm{ent.}}_{\mathrm{waiting}} \sim L^z \Rightarrow S(t) \sim t^{1/z}$$

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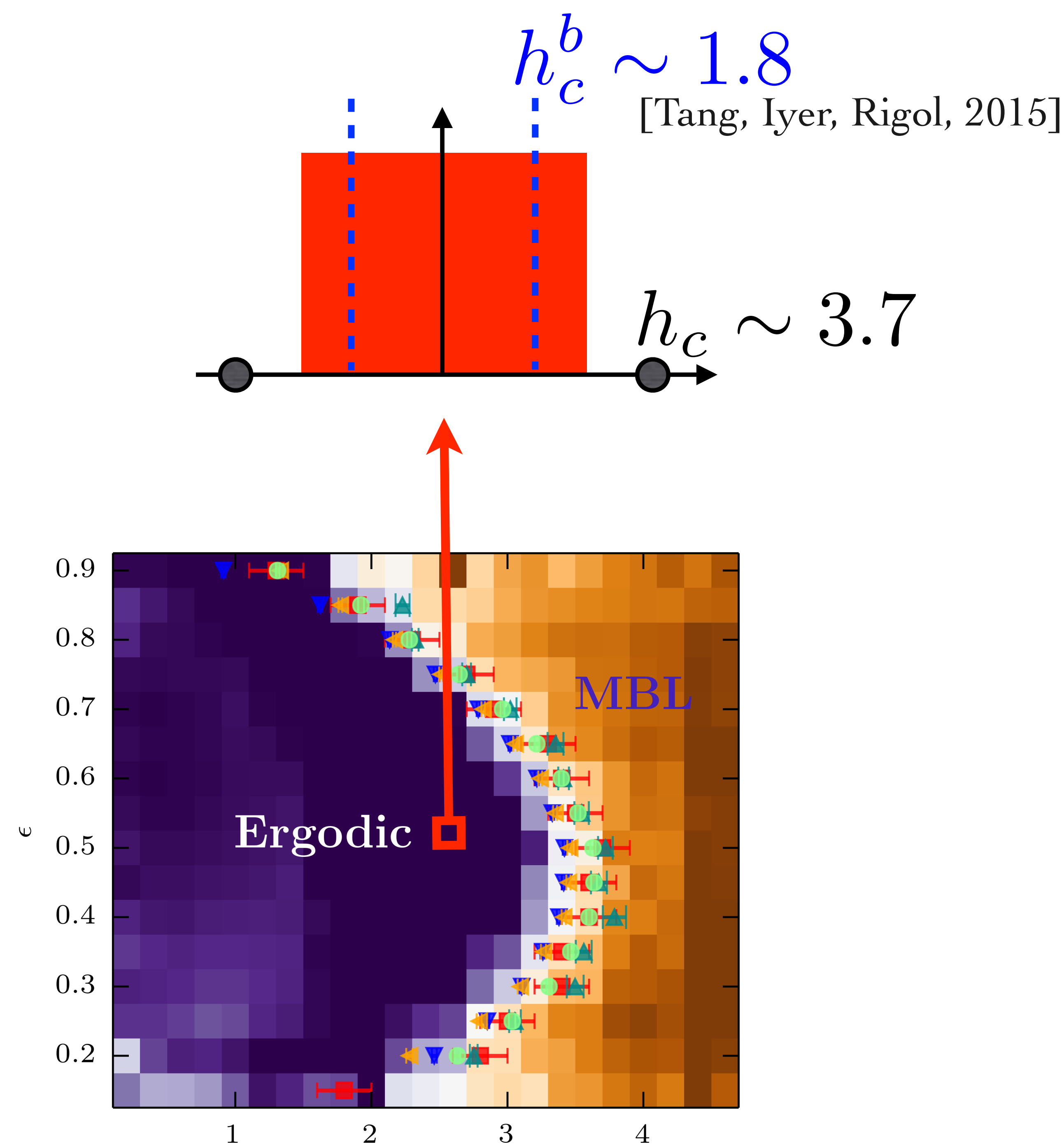
$$z \propto \xi \qquad t^{\mathrm{ent.}}_{\mathrm{waiting}} \sim L^z \Rightarrow S(t) \sim t^{1/z}$$

Rare regions ? two sources

1) local disorder configuration in the other (MBL) phase or critical



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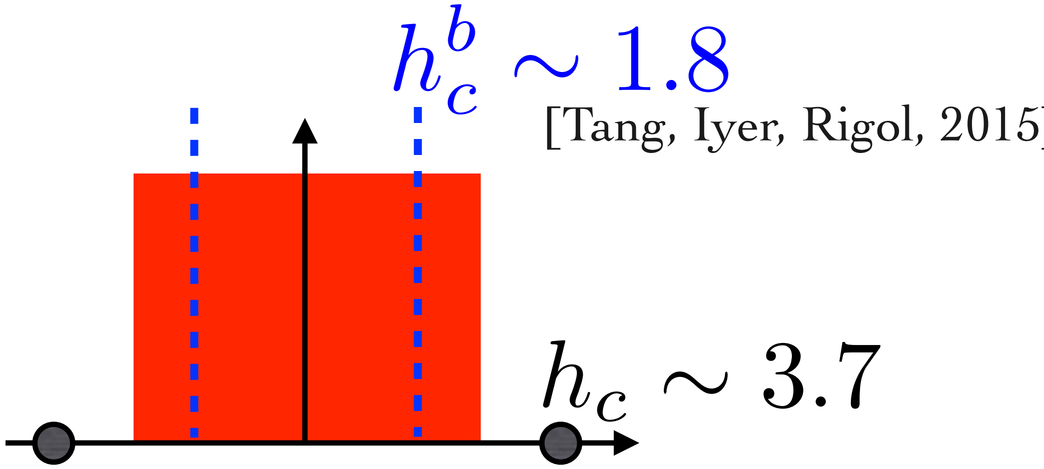
$$z \propto \xi \qquad t^{\mathrm{ent.}}_{\mathrm{waiting}} \sim L^z \Rightarrow S(t) \sim t^{1/z}$$

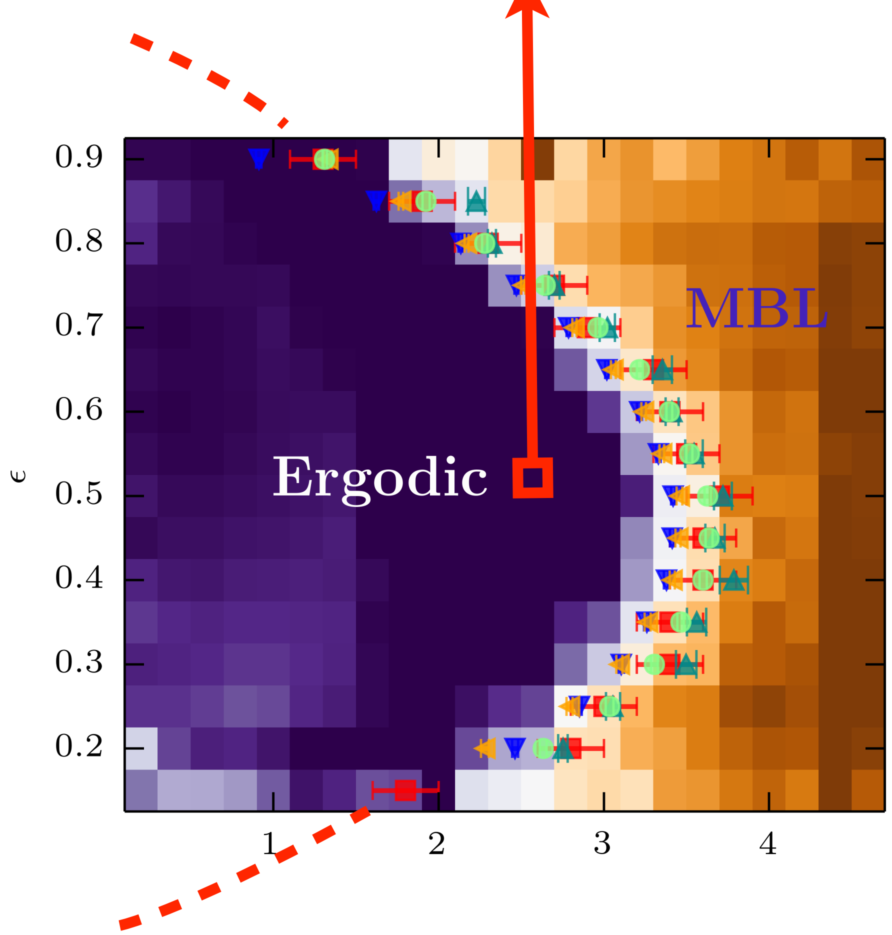
Rare regions ? two sources

1) local disorder configuration in the other (MBL) phase or critical

2) ϵ_i fluctuate: Anomalously hot or cold regions are possible in the random initial state

➡ **Signature of the edge**

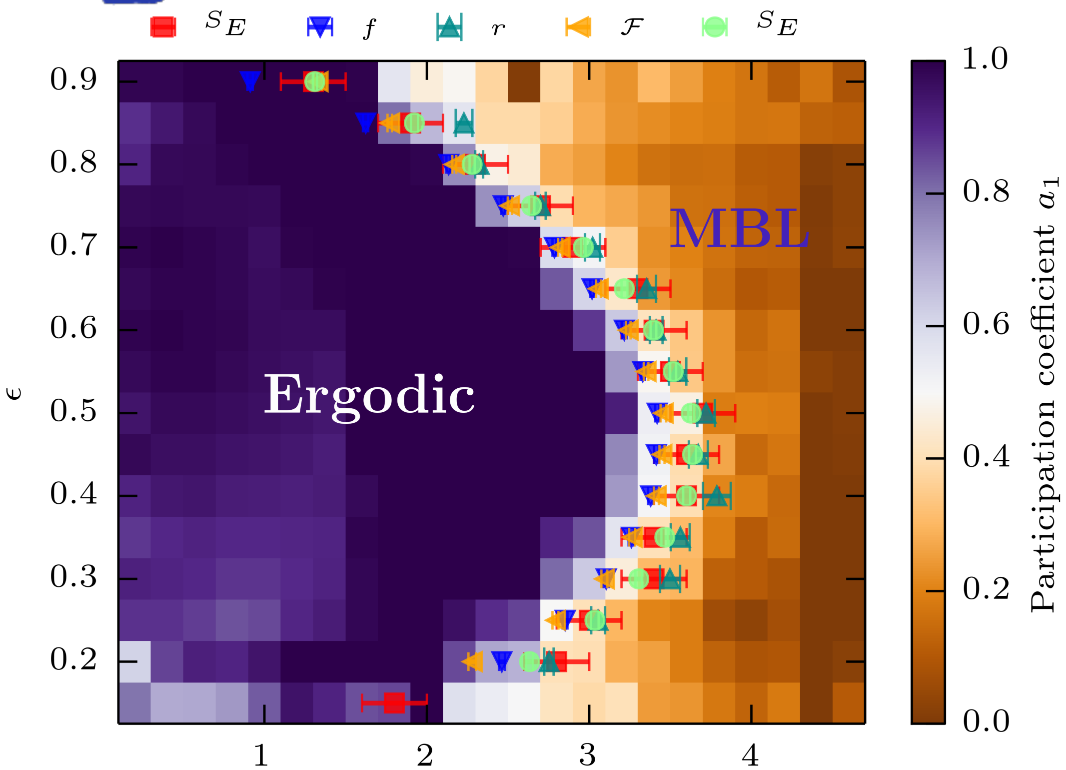




Conclusions

- Heisenberg chain with Random field **ED results for excited states L=12-22**

  ☑ Consistent with a mobility edge



➡ GOE - Poisson statistics

➡ Volume - area law entanglement/fluctuations

➡ Hilbert space participation changes abruptly

➡ All estimates are consistent with $h_c(\epsilon)$

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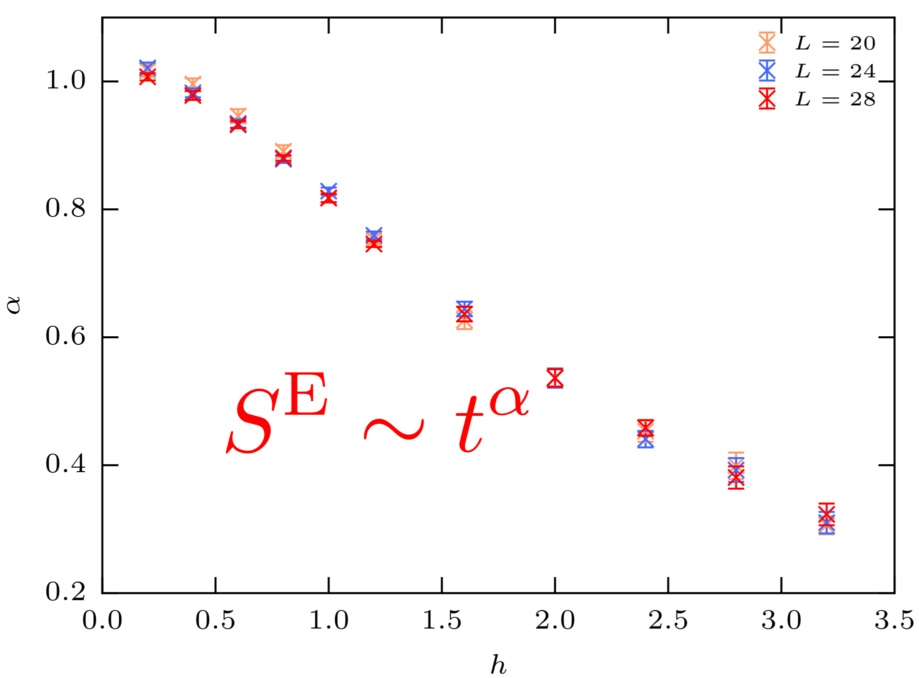
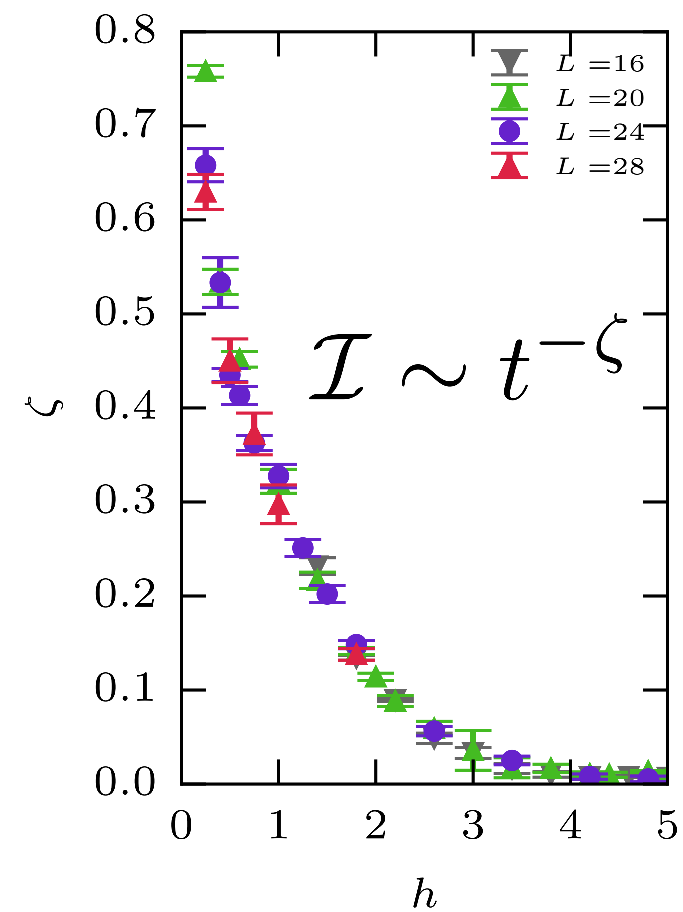
- Dynamics: **ED results for L=20-28**

  ☑ Quench: sub-ballistic entanglement $\forall h > 0$

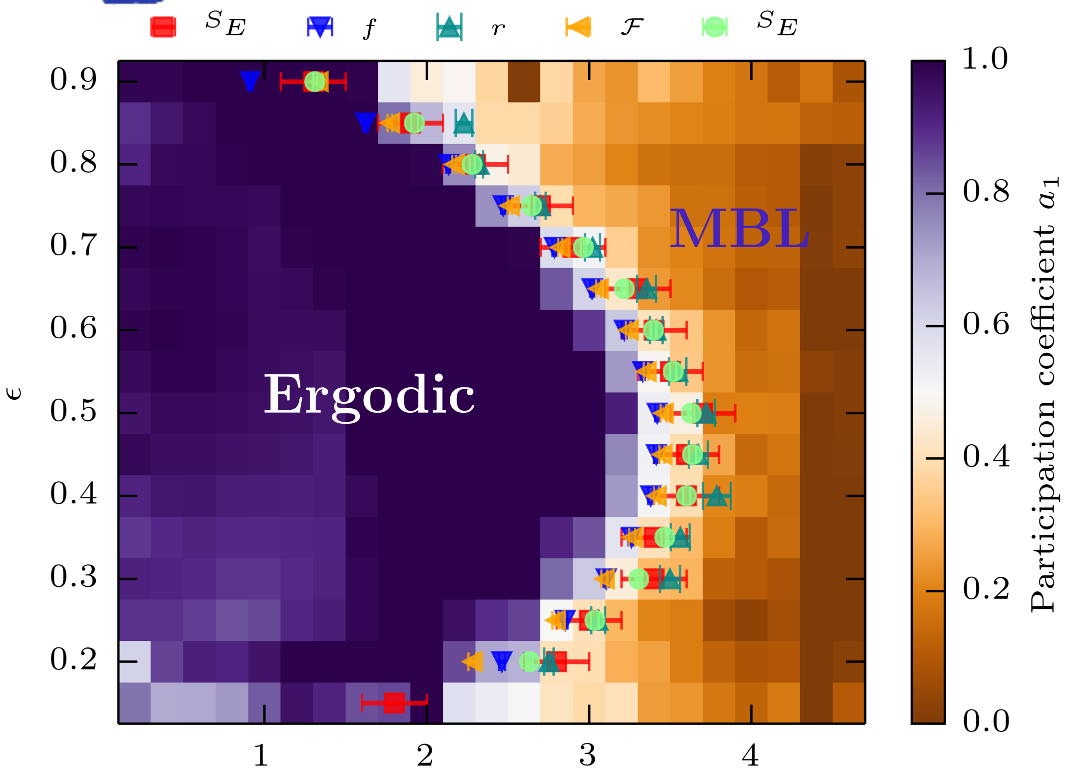


consistent with an extended slow dynamical regime



Conclusions

- Heisenberg chain with Random field **ED results for excited states L=12-22**
  - ☑ Consistent with a mobility edge



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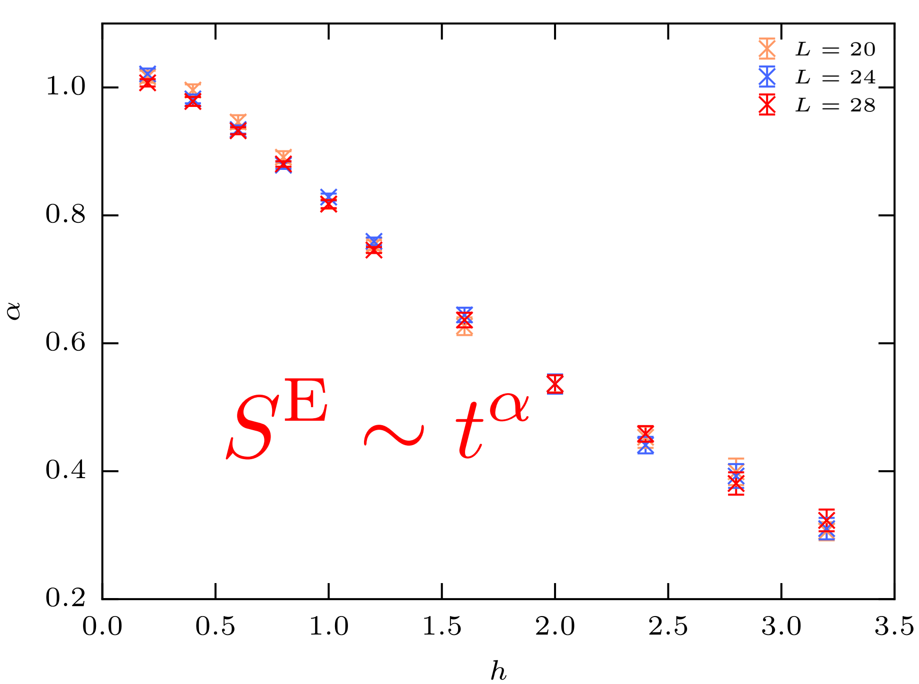

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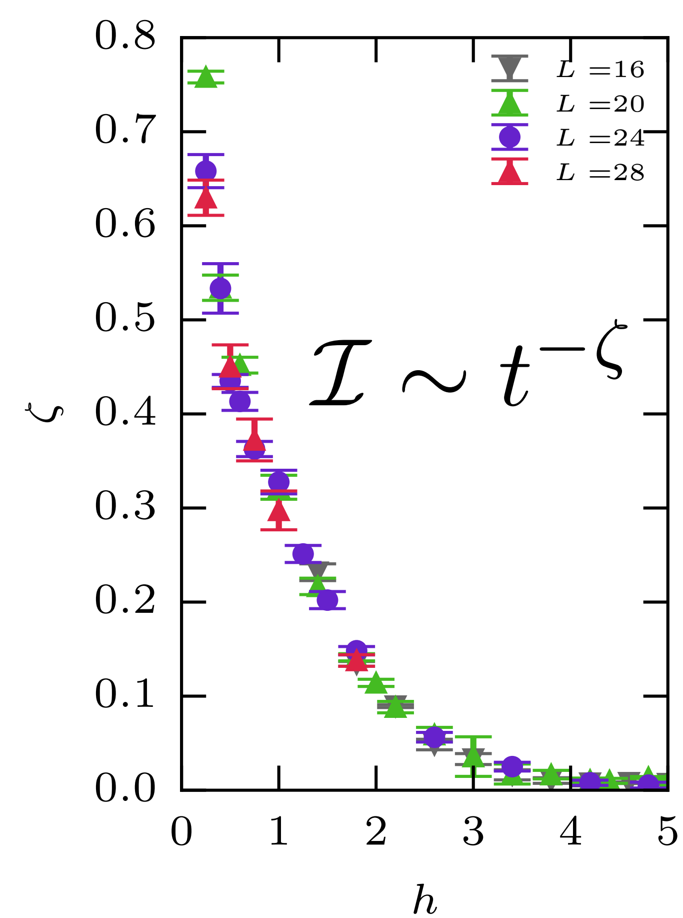
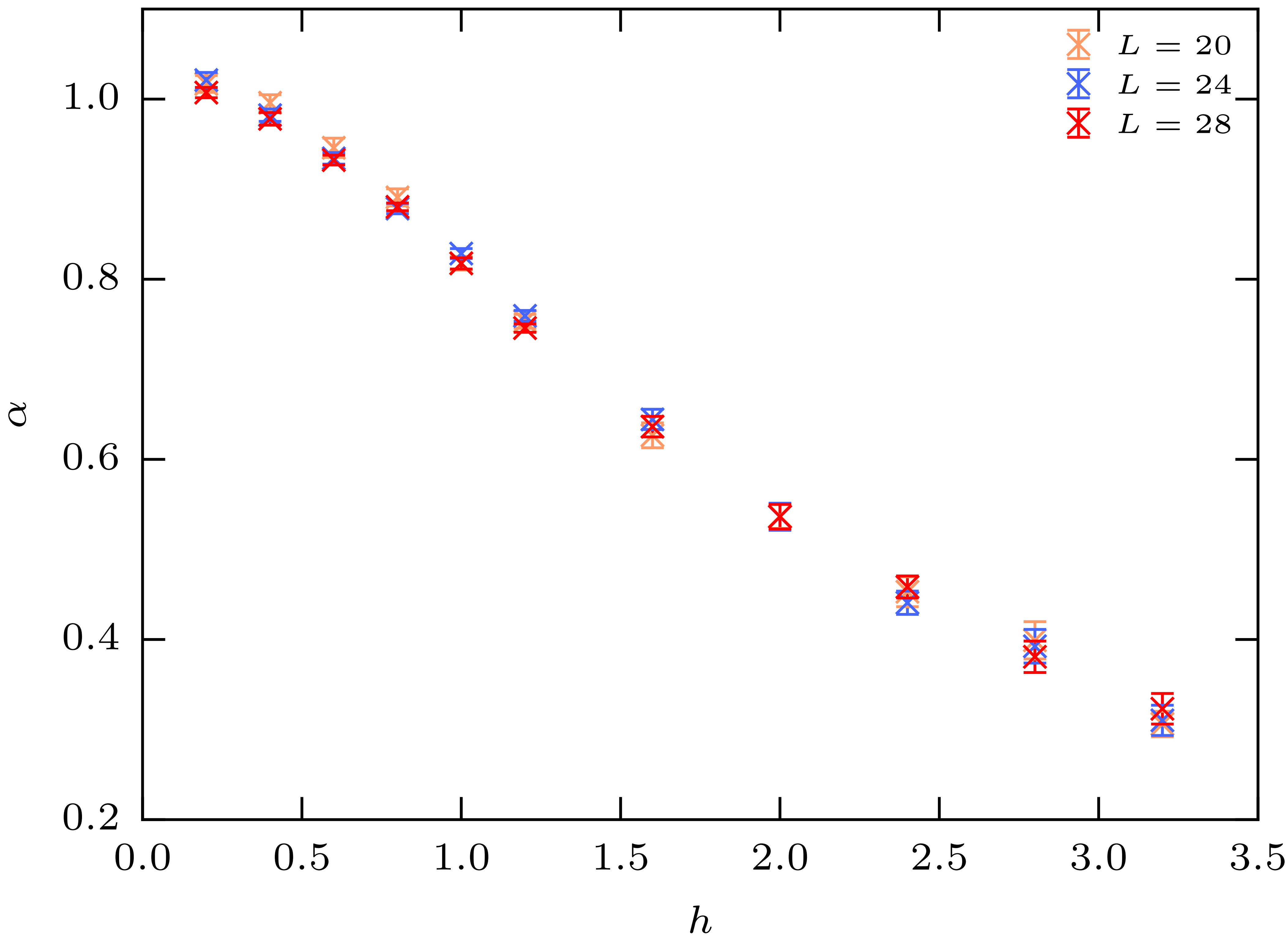
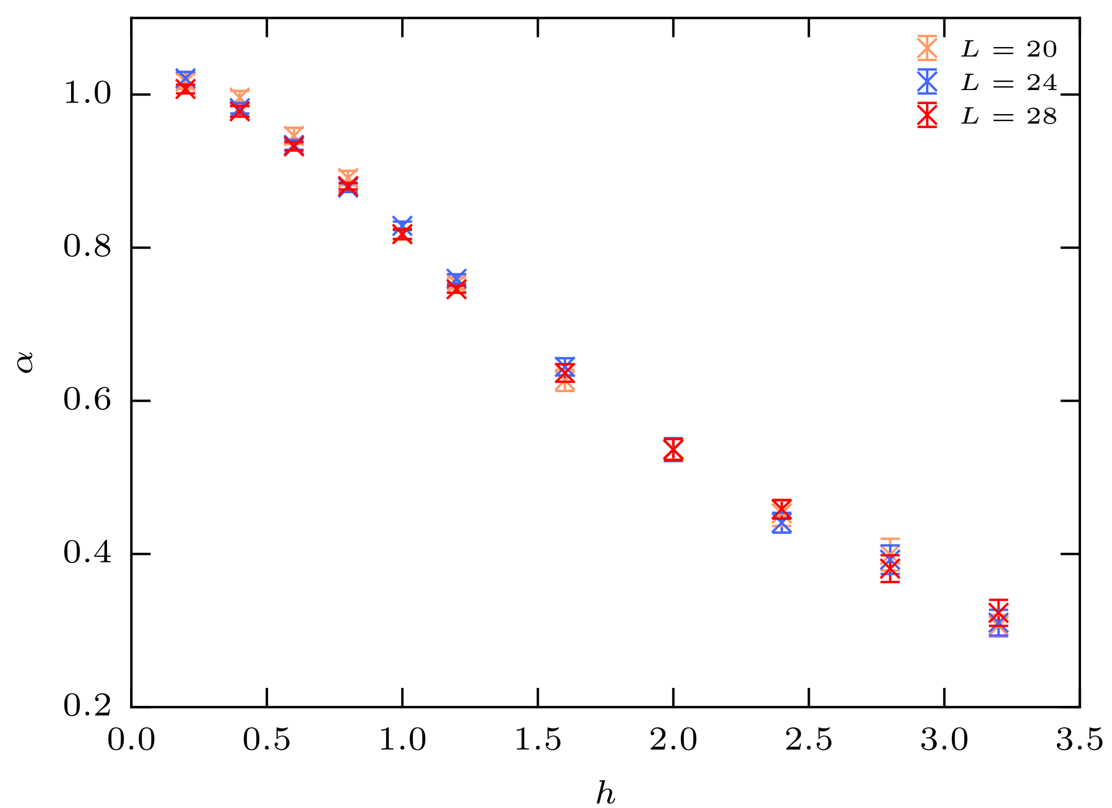




$L = 20$
$L = 24$
$L = 28$
1.0
0.8
0.6
0.4
0.2
α
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
h



Good agreement

[Lerose, Varma, Pietracaprina, Goold, Scardicchio, arXiv:1511.09144]

and also

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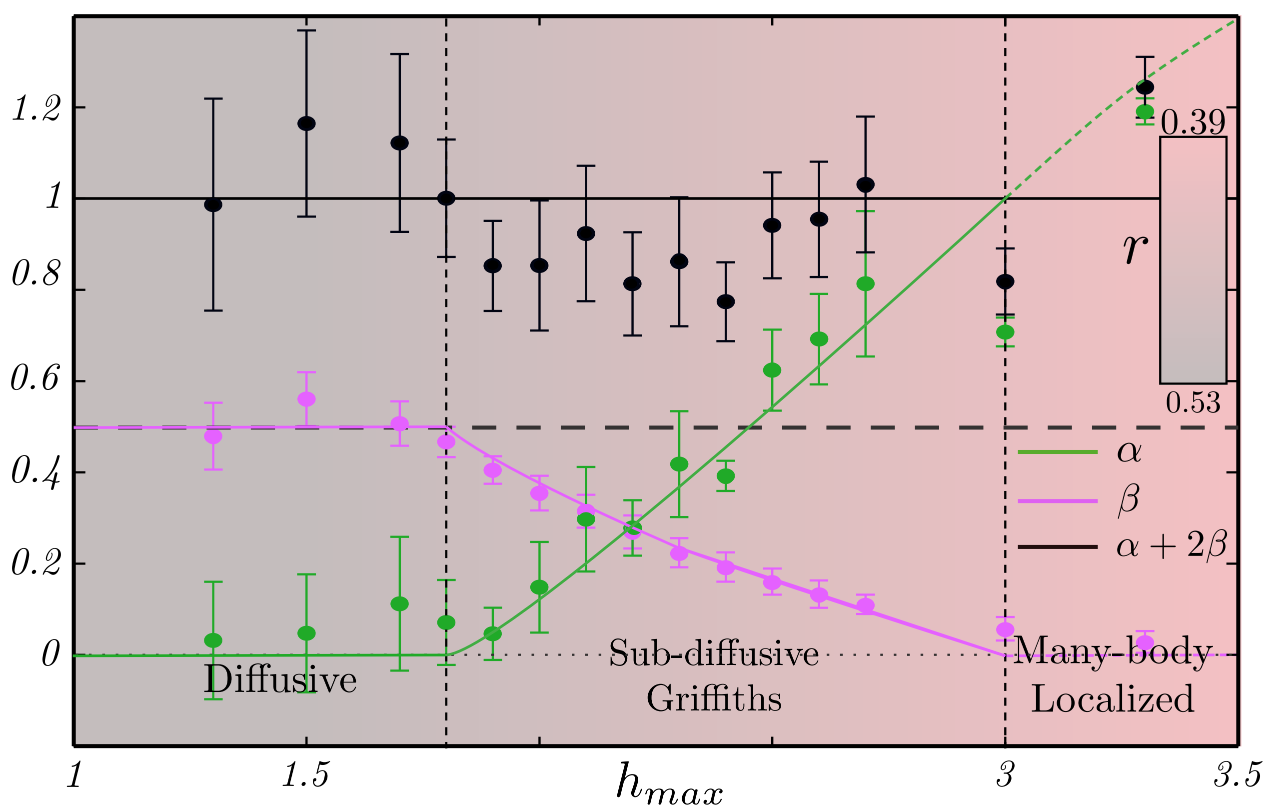
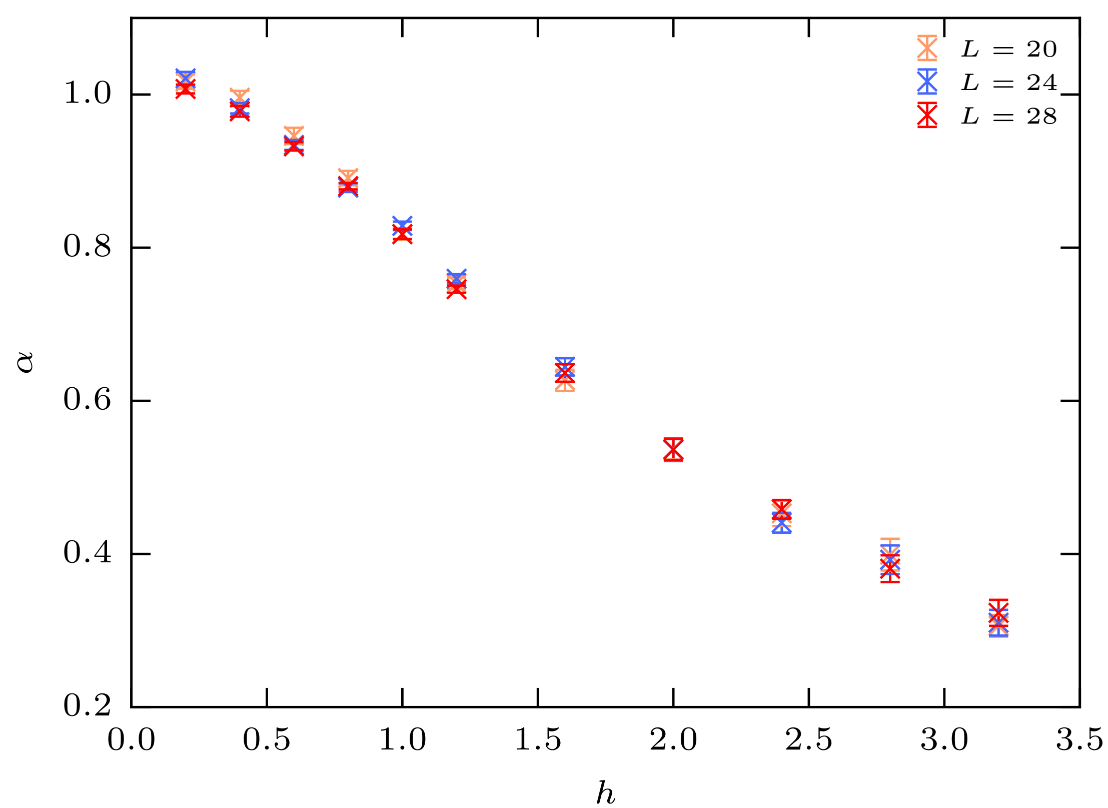
PRL **114,** 160401 (2015) PHYSICAL REVIEW LETTERS week ending 24 APRIL 2015

Anomalous Diffusion and Griffiths Effects Near the Many-Body Localization Transition

Kartiek Agarwal,[1,*] Sarang Gopalakrishnan,[1] Michael Knap,[1,2] Markus Müller,[3,4] and Eugene Demler[1]



Disagreement



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Spectral statistics 2: Eigenstates correlations

- Beyond level statistics: correlations between eigenstates
    - Thermal (ETH) phase: expect eigenstates to be «similar»
    - MBL phase: expect eigenstates to be «very different»
- Kullback-Leibler divergence quantify similarity between eigenstates (in a given basis)

$$\mathrm{KL} = \sum_i p_i \ln(p_i/q_i) \qquad \begin{aligned} p_i &= |\langle n|i\rangle|^2 \\ q_i &= |\langle n'|i\rangle|^2 \end{aligned} \qquad \{|i\rangle\} = \{S^z\} \text{ basis}$$

- GOE: $\mathrm{KL} = 2$; MBL: diverges for very different states

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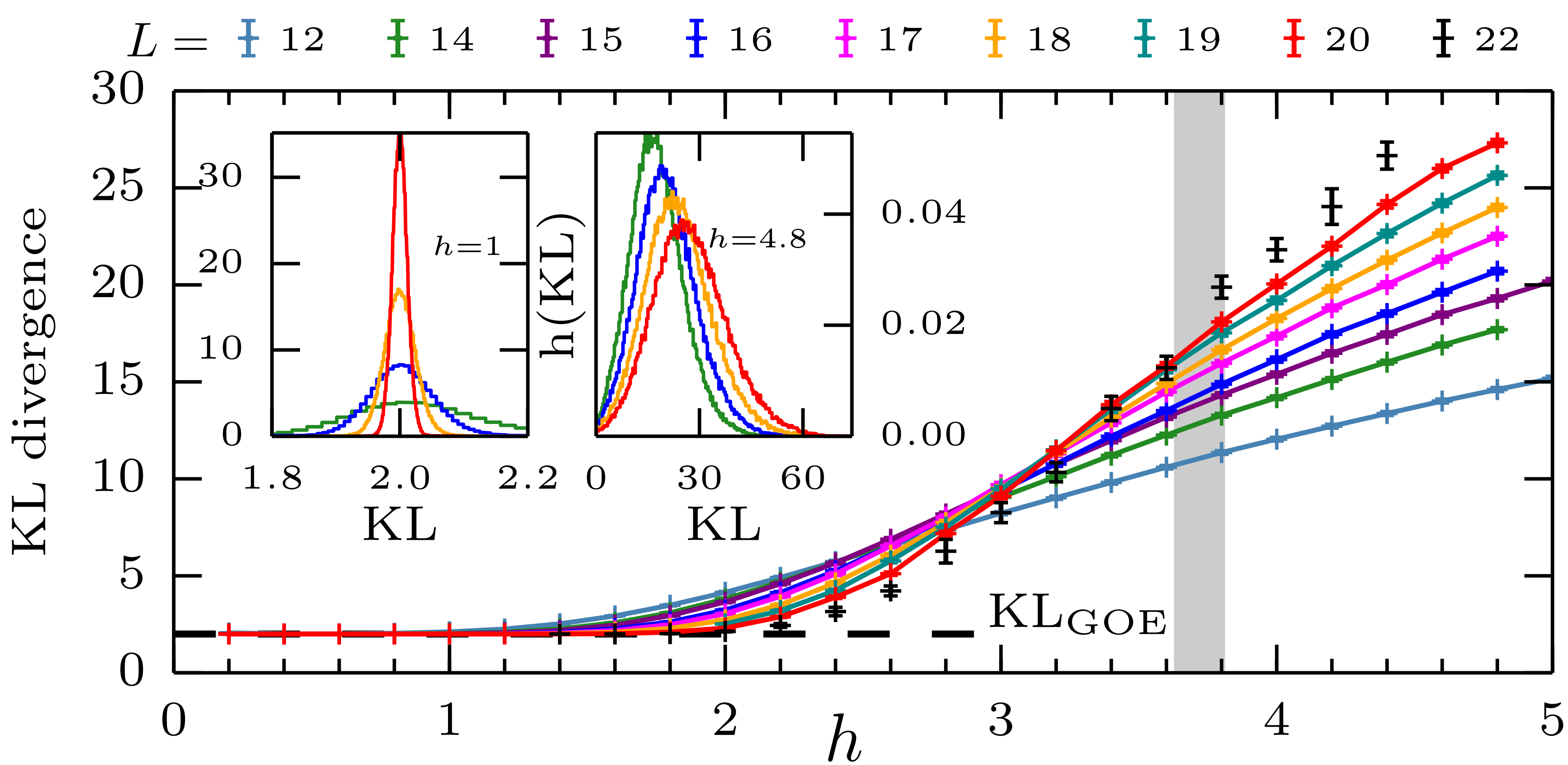

$\epsilon = 0.5$