Chiral Projected Entangled Pair States

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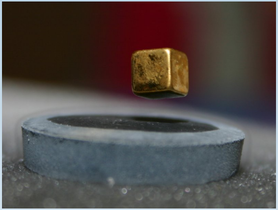
Max-Planck Institute of Quantum Optics, Garching, Germany

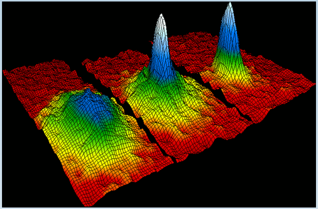
- QCCC Elitenetzwerk Bayern
- Alexander von Humboldt Foundation
- EU Integrated Project SIQS

H.-H. Tu, N. Schuch, S. Haßler, S. Yang, J. I. Cirac

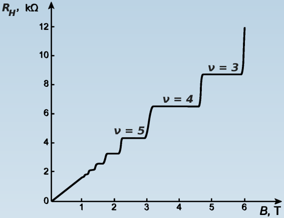
source: Wikipedia



Superconductivity

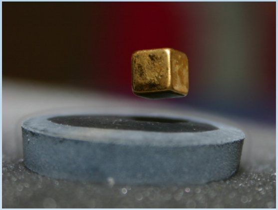


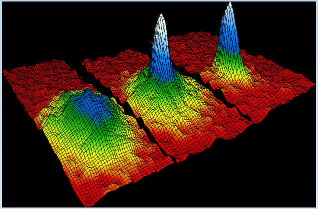
Bose-Einstein condensate in Rb-87

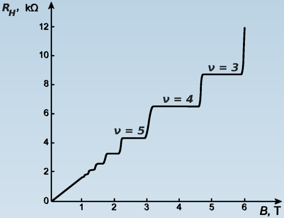


Quantum Hall Effect

source: Wikipedia







Superconductivity

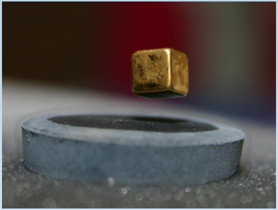
Bose-Einstein condensate in Rb-87

Quantum Hall Effect

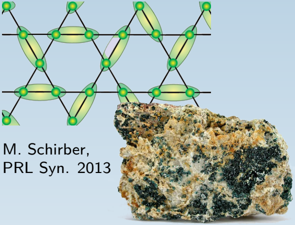
Approxiation Schemes

- Mean Field Theory / Hartree-Fock

source: Wikipedia

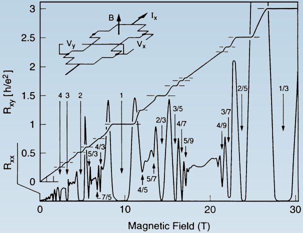


High temperature Superconductivity



M. Schirber, PRL Syn. 2013

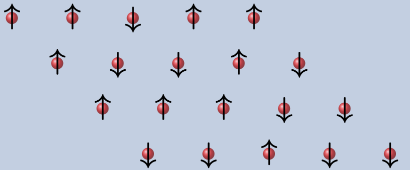
Quantum Spin Liquids



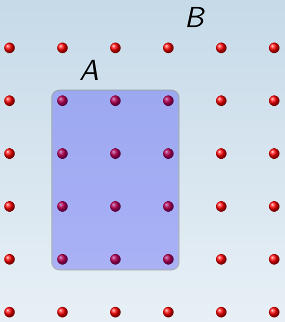
fractional QHE
fract. Chern insulators

Approxiation Schemes

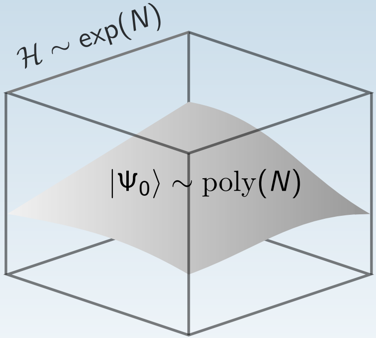
- Mean Field Theory / Hartree-Fock
- Quantum Monte Carlo
- Tensor Network States
- ...



The Area Law of Entanglement



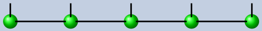
$$S_{AB} := S(\rho_A) \propto \partial A$$



Tensor Network States

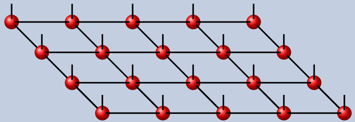
1D: Matrix Product States (MPS)

- efficient approximation of local Hamiltonians in 1D
  F. Verstraete, and J. I. Cirac, Phys. Rev. B 73, 094423 (2006)
- classification of all phases in 1D
  F. Pollmann, A. M. Turner, E. Berg, and M. Oshikawa, Phys. Rev. B 81, 064439 (2010)
  X. Chen, Z.-C. Gu, and X.-G. Wen, Phys. Rev. B 83, 035107 (2011)
  N. Schuch, D. Pérez-García, and J. I. Cirac, Phys. Rev. B 84, 165139 (2011)



2D: Projected Entangled-Pair States (PEPS)

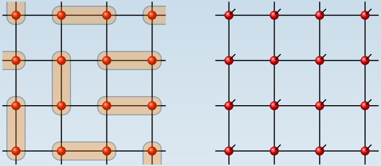
- presumably efficient approximation in 2D, proven for finite temperatures
  M. B. Hastings, Phys. Rev. B 76, 035114 (2007)
- no sign problem



Previous topological PEPS

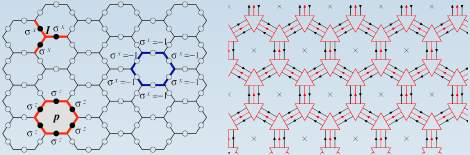
Resonating valence bond states

P. W. Anderson, Mater. Res. Bull. 8, 153 (1973)



String net models

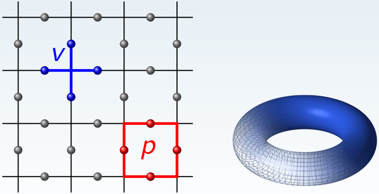
M. A. Levin, and X.-G. Wen, Phys. Rev. B 71, 045110 (2005)



O. Buerschaper, M. Aguado, and G. Vidal, Phys. Rev. B 79, 085119 (2009)

Toric code

A. Kitaev, Ann. Phys. 303, 2 (2003)



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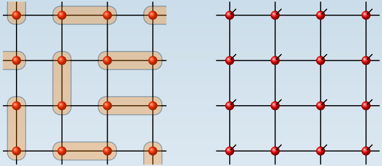

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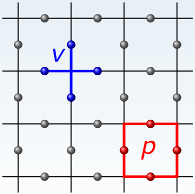

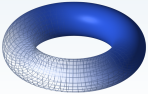

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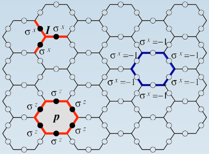

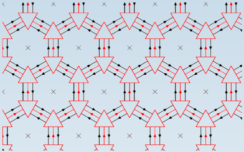

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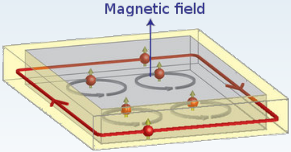

D. Kong, and Y. Cui, Nature Chemistry 3, 845 (2011)

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Tensor Network States - Overview

$$|\Psi\rangle = |\Psi(A^{i}_{v_1 v_2 \dots v_z})\rangle$$

- in 1D: Matrix Product State (MPS): $A^{i}_{lr} \quad (l, r = 1, \dots, D)$

$$|\Psi\rangle = \sum_{i_1,\dots,i_N} \underbrace{\mathrm{tr}\left(A^{i_1} \dots A^{i_N}\right)}_{c_{i_1 \dots i_N}} |i_1 \dots i_N\rangle$$

- in 2D: Projected Entangled Pair States (PEPS): A^{i}_{lrud}

MPS construction

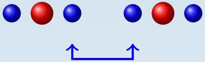
$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i l r\rangle$$

$|\Psi_1\rangle$

MPS construction

$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i\, l\, r\rangle$$

$|\Psi_1\rangle$ $|\Psi_1\rangle$



$$\sum_{\alpha=1}^{D} \langle \alpha\, \alpha|$$

MPS construction

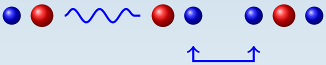
$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i l r\rangle$$

$$|\Psi_2\rangle$$

MPS construction

$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i\, l r\rangle$$

$|\Psi_2\rangle$

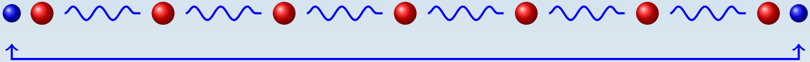


MPS construction

$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i\,l\,r\rangle$$

MPS construction

$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i l r\rangle$$



MPS construction

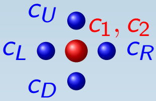
$$|\Psi_1\rangle = \sum_{i=1}^{d} \sum_{l,r=1}^{D} A^{i}{}_{lr} |i\, l r\rangle$$

$$|\Psi_{\rm MPS}\rangle = \sum_{i_1 \ldots i_N} \mathrm{tr}\left(A^{i_1} A^{i_2} \ldots A^{i_N}\right) |i_1 i_2 \ldots i_N\rangle$$

D. Pérez-García, F. Verstraete, M. M. Wolf, and J. I. Cirac, Quantum Inf. Comput. 7, 401 (2007)

PEPS construction

$|\Psi_1\rangle =$



Majorana modes: $c^\dagger = c$

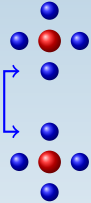
$\frac{1}{2}(c_1 - ic_2) = a$

$\frac{1}{2}(c_1 + ic_2) = a^\dagger$

PEPS construction

$|\Psi_1\rangle =$

$\langle 1|_{D1,U2}$



Majorana modes: $c^\dagger = c$

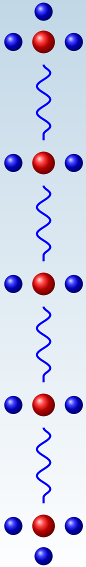
$\frac{1}{2}(c_1 - ic_2) = a$

$\frac{1}{2}(c_1 + ic_2) = a^\dagger$

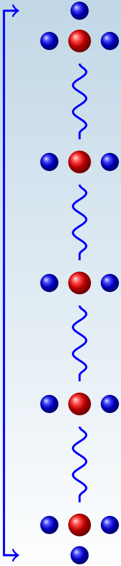
PEPS construction

$|\Psi_2\rangle =$

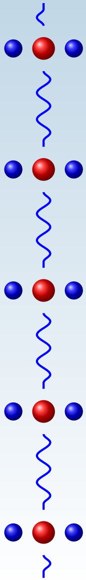
PEPS construction



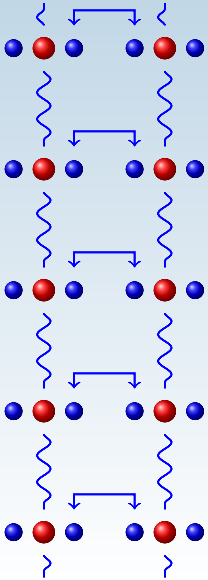
PEPS construction



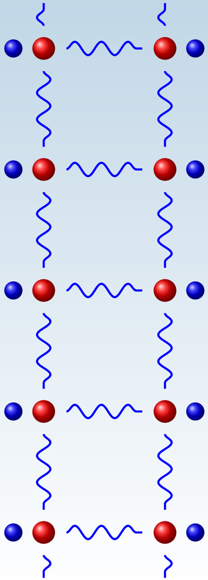
PEPS construction



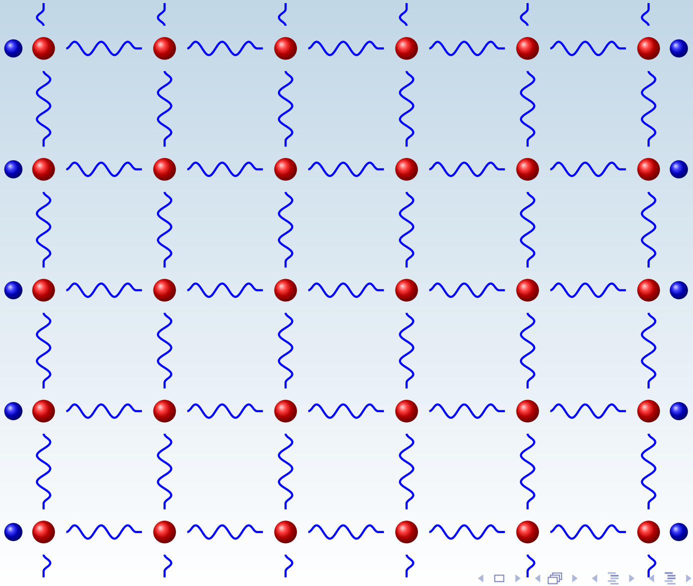
PEPS construction



PEPS construction



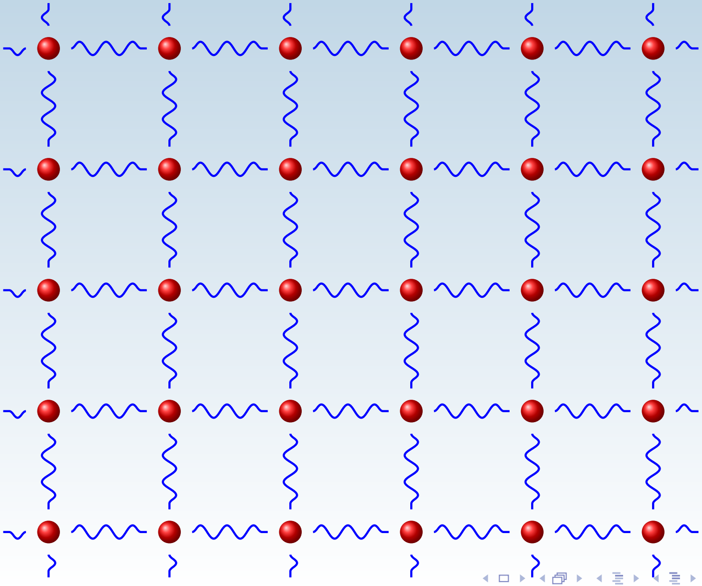
PEPS construction



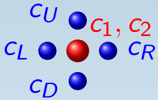
PEPS construction

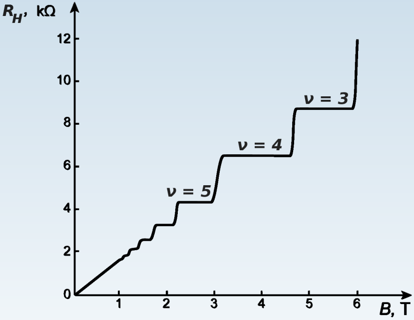
$|\Psi_{\rm PEPS}\rangle =$

F. Verstraete and J. I. Cirac, Phys. Rev. A 70, 060302(R) (2004)



PEPS vs. Quantum Hall Effect

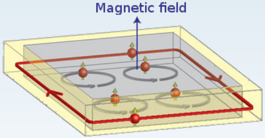
$A^{p}_{lrud} \longrightarrow |\Psi_1\rangle =$ 



$$R_H = \frac{h}{\nu\, e^2}$$

Chern number

$$\nu = C = 0, \pm 1, \pm 2, \ldots$$



D. Kong, and Y. Cui, Nature Chemistry 3, 845 (2011)

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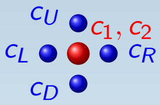
$$H = \sum_{k,j} T_{kj} a_k^\dagger a_j + \Delta_{kj} a_k^\dagger a_j^\dagger + \overline{\Delta}_{kj} a_j a_k$$

Free fermionic chiral PEPS

- all chiral PEPS with one Majorana mode
- general properties for more Majorana modes

The simplest chiral PEPS

$|\Psi_1\rangle =$ 

Parameterization of simplest chiral PEPS

$$|\Psi_1\rangle = \left(1 + a^\dagger b^\dagger\right)|\text{vac}\rangle$$

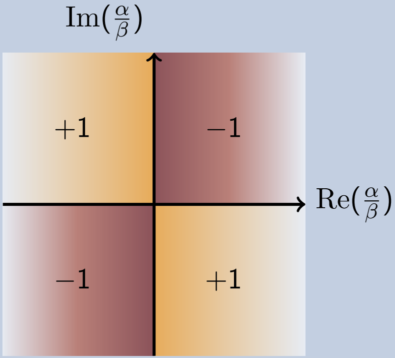
$$b = \alpha(c_L \pm i c_R) + \beta(c_U \pm i c_D)$$

Virtual Symmetry:

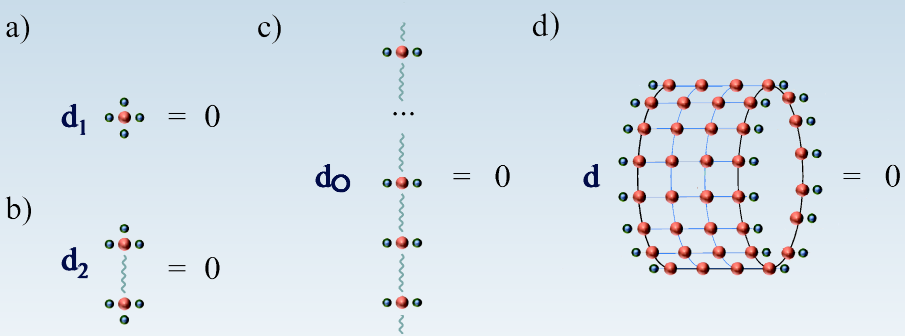
$$d_1|\Psi_1\rangle = 0$$

$$d_1 = -\overline{\beta}(c_L \pm i c_R) + \overline{\alpha}(c_U \pm i c_D)$$

T. B. Wahl, H.-H. Tu, N. Schuch, J. I. Cirac, PRL '13
T. B. Wahl, S. T. Haßler, H.-H. Tu, J. I. Cirac, N. Schuch, PRB '14
see also: J. Dubail and N. Read, PRB '15



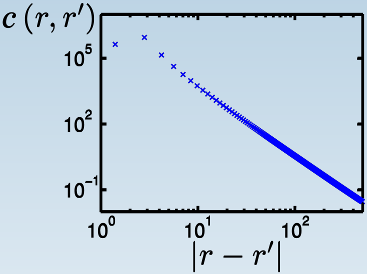
Local vs. global Symmetries



Free fermionic chiral PEPS

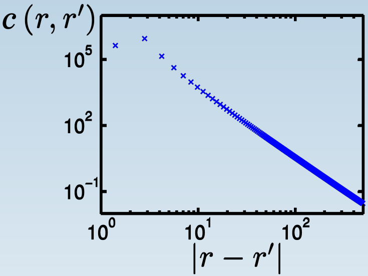
- general properties for more Majorana modes

General properties of chiral free fermionic PEPS



Rigorously shown in:
J. Dubail and N. Read, PRB 92, 205307 (2015)

General properties of chiral free fermionic PEPS



Rigorously shown in:
J. Dubail and N. Read,
PRB 92, 205307 (2015)

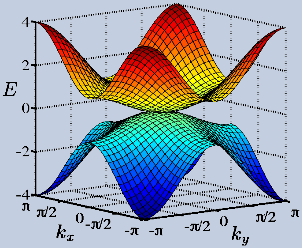
Flat band Hamiltonian

- long-range
- stable to perturbations

Frustration free Hamiltonian

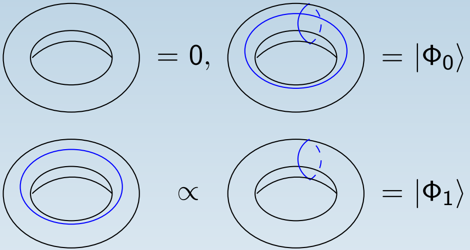
$$\mathcal{H} = \sum_{\mathbf{r}} h_{\mathbf{r}}$$

- local
- gapless



General properties of chiral free fermionic PEPS

insert string operators $\sum_x c_x$:



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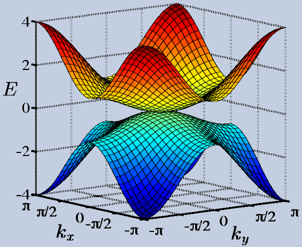
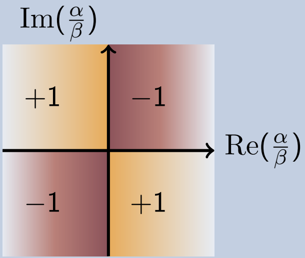


Summary: chiral free fermionic PEPS

PEPS with one Majorana mode

- chiral cases: $|\Psi_1(\alpha, \beta)\rangle$
- virtual symmetry $d_1|\Psi_1\rangle = 0$



any chiral free fermionic PEPS

- polynomially decaying correlations
- critical local Hamiltonians
- long range gapped topological Hamiltonians

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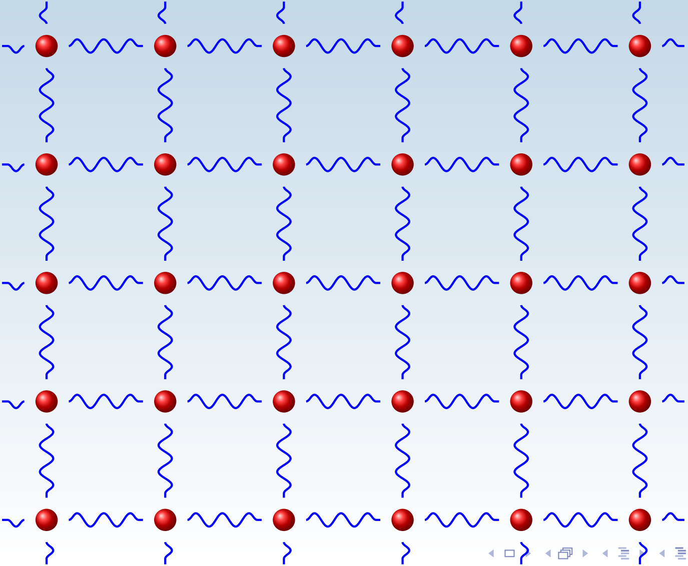
Gutzwiller projection

non-interacting chiral PEPS



Gutzwiller projection

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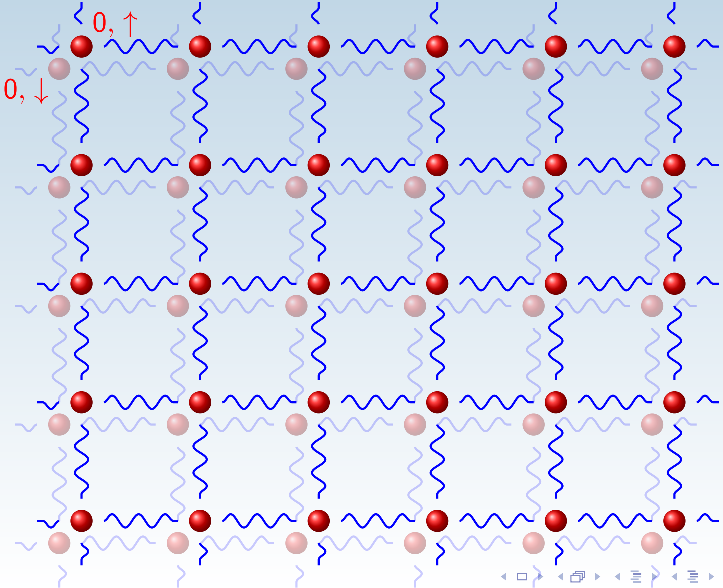
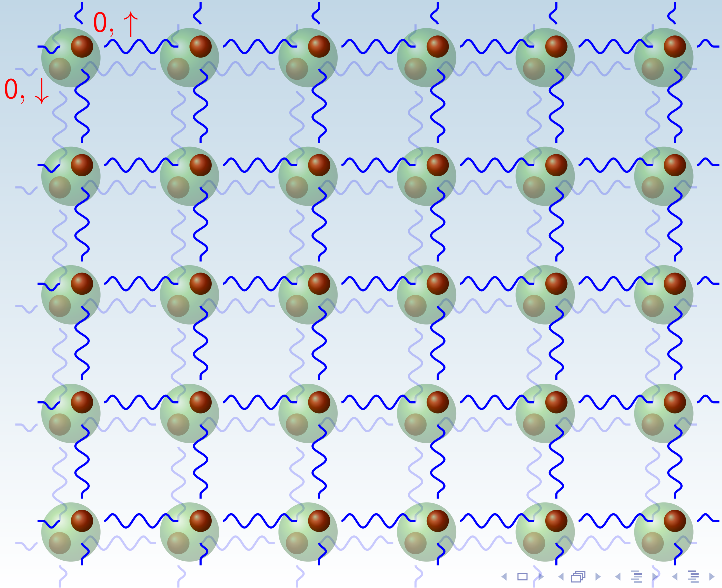


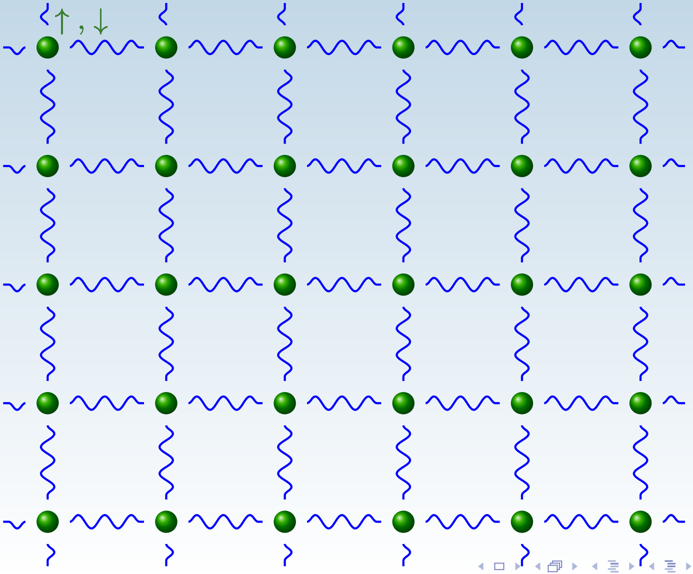
Gutzwiller projection

non-interacting chiral PEPS



Gutzwiller projection

interacting chiral PEPS

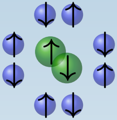


Topologically ordered chiral PEPS

$$|\Phi_1\rangle = \left(a_\uparrow^\dagger b_\uparrow^\dagger + a_\downarrow^\dagger b_\downarrow^\dagger\right)|\mathrm{vac}\rangle$$

$$d_{\uparrow,\downarrow}|\Phi_1\rangle = 0$$

$$\sigma_L^z\,\sigma_R^z\,\sigma_U^z\,\sigma_D^z|\Phi_1\rangle = -|\Phi_1\rangle$$



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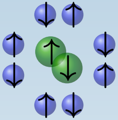
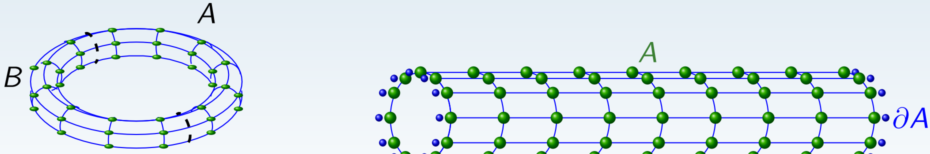
$$d_{\uparrow,\downarrow}|\Phi_1\rangle = 0$$

$$\sigma_L^z\,\sigma_R^z\,\sigma_U^z\,\sigma_D^z|\Phi_1\rangle = -|\Phi_1\rangle$$



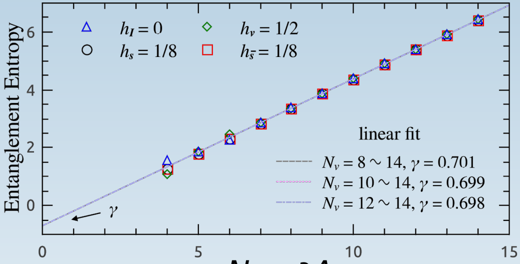
Properties:

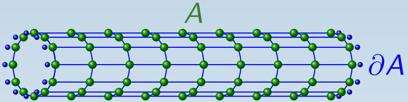
- algebraically decaying correlations
- $\mathcal{H} = \sum_\mathbf{r} h_\mathbf{r}$ has 5 ground states → **Topological degeneracy?**



$$\rho_A = \mathrm{tr}_B(|\Phi\rangle\langle\Phi|) \propto e^{-H_{\mathrm{ent}}}$$

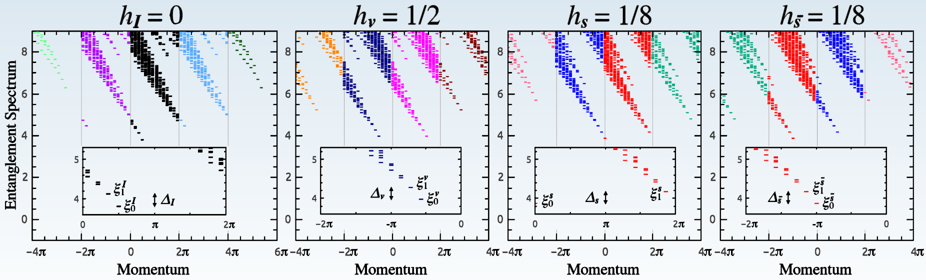
Area Law and Entanglement Spectrum





$$S(A) = c\,\partial A - \ln(2)$$

$$N_v = \partial A$$



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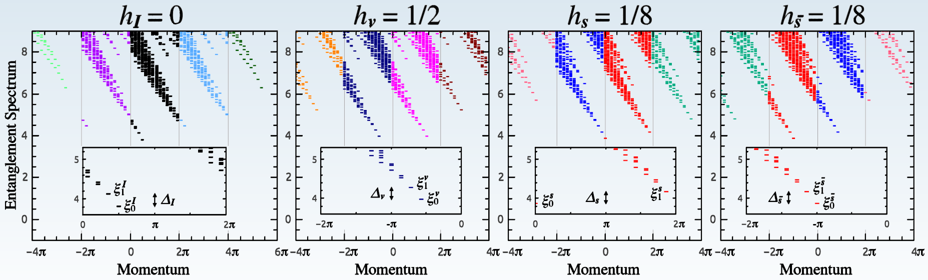
Area Law and Entanglement Spectrum

Properties:

- algebraically decaying correlations
- $\mathcal{H} = \sum_{\mathbf{r}} h_{\mathbf{r}}$ has 5 ground states $\rightarrow$ (4 topological, 1 gapless)
- $SO(2)_1$ CFT with $c = 1$
  topological correction to area law $S(A) = c\,\partial A - \gamma$; $\gamma = \ln(2)$
  entanglement specta with the primary fields:



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Conclusions

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- PEPS can represent Quantum Hall systems
- non-interacting chiral PEPS have long range correlations (gapless local Hamiltonians, long range flat band Hamiltonians)
- also true for example of interacting chiral PEPS

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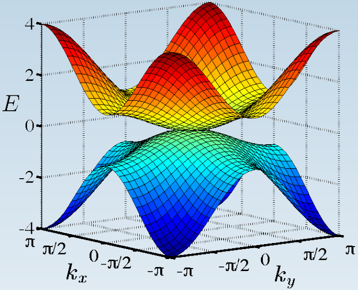
- also true for example of interacting chiral PEPS

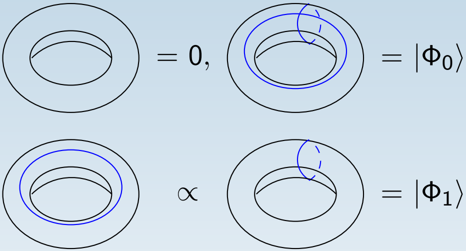
Outlook:

- disprove existence of short range chiral PEPS
- numerical simulations of chiral systems

Frustration free Hamiltonian



insert string operators $\sum_x c_x$:



$$\mathcal{H} = \mathcal{H}_{\mathrm{ff}} + \mu_0 \mathcal{H}_{\mathrm{on-site}} + \nu_0 \mathcal{H}_{\mathrm{hopping}}$$