Quantum interacting particles on a ring: binary Tree Tensor Networks studies

Matteo Rizzi

Johannes Gutenberg-Universität Mainz

Benasque - 19.02.2016

OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

Periodic Boundaries: numerical tool

- generally easier for analytical treatments (Fourier transform & co.)

Periodic Boundaries: numerical tool

- generally easier for analytical treatments
  (Fourier transform & co.)
- avoid undesired edge effects in observables
  (e.g., better and longer reliable correlations)
- faster and cleaner scaling to thermodynamic limit
  (useful to study phase transitions) Fisher, Barber, PRL **28,** 1516 (1972)

Periodic Boundaries: numerical tool

- generally easier for analytical treatments
  (Fourier transform & co.)

- avoid undesired edge effects in observables
  (e.g., better and longer reliable correlations)

- faster and cleaner scaling to thermodynamic limit
  (useful to study phase transitions) Fisher, Barber, PRL **28,** 1516 (1972)

- access phase stiffnesses and topological invariants
  via twisting the boundaries $\Psi(0) = e^{i\phi}\Psi(L)$

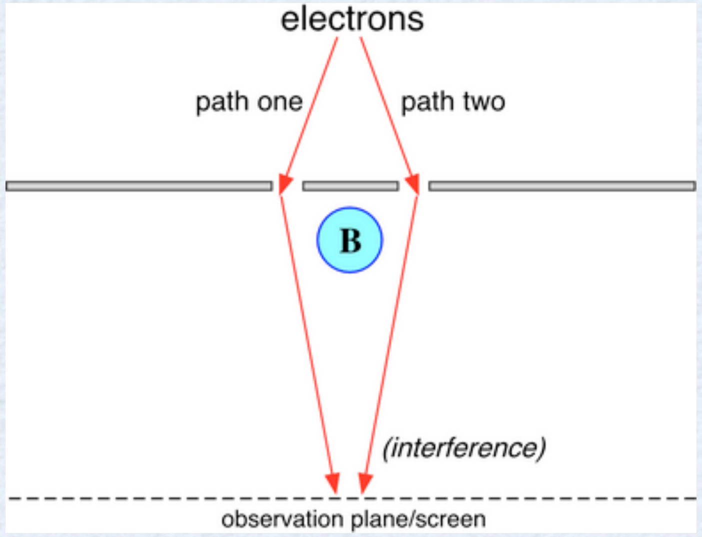
$I \propto \partial E/\partial\phi$ $\quad\quad$ $\rho_s \propto \partial^2 E/\partial\phi^2$ $\quad\quad$ $\phi_{\text{Zak}} = \int_{-\pi}^{\pi} \langle\Psi|\partial_\phi|\Psi\rangle$

Periodic Boundaries: physical consequences

- In a doubly-connected system, pierced by a flux Φ, all quantities are periodic in the flux by a period $\Phi_0 = h/q$ (q = effective particle charge)

Byers, Yang, PRL **7,** 46 (1961)

- AHARANOV-BOHM EFFECT



$$\vec{\nabla} \times \vec{A} = \vec{B} \qquad \Phi = \oint \vec{A} \cdot \mathrm{d}\vec{l}$$

$$\vec{p} \longrightarrow \vec{p} - q\vec{A} \quad \Rightarrow \quad \phi_{L/R} = \int_{L/R} \frac{q\vec{A}}{\hbar} \cdot \mathrm{d}\vec{l}$$

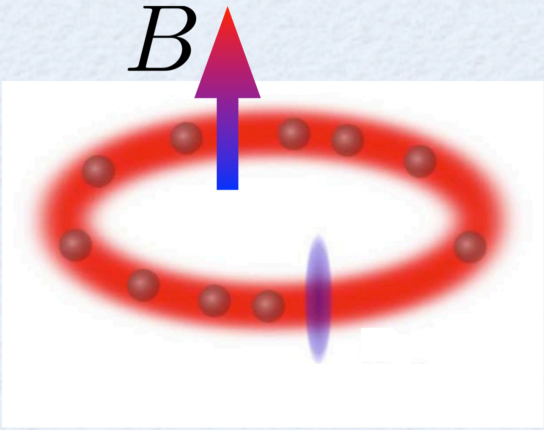
$$\boxed{\phi = \phi_L - \phi_R = 2\pi\Phi/\Phi_0}$$

Periodic Boundaries: physical consequences

- In a doubly-connected system, pierced by a flux Φ, all quantities are periodic in the flux by a period $\Phi_0 = h/q$ (q = effective particle charge)

Byers, Yang, PRL **7,** 46 (1961)

- PERSISTENT CURRENTS



$$\frac{dF}{dt} = I\,V \qquad V = -\frac{\partial \Phi}{\partial t}$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial \Phi}\frac{\partial \Phi}{\partial t}$$

Bloch, PRB **2,** 109 (1970)

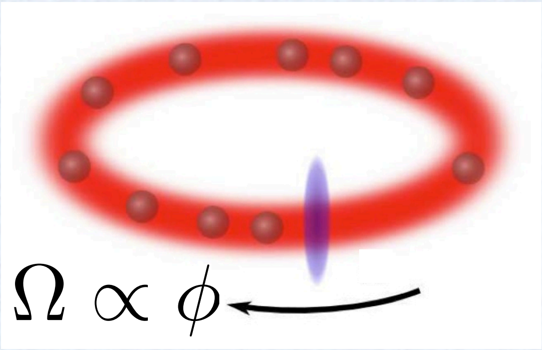
$$I(\phi) = -\frac{q}{\hbar}\frac{\partial F(\phi)}{\partial \phi}$$

Periodic Boundaries: physical consequences

- In a doubly-connected system, pierced by a flux Φ, all quantities are periodic in the flux by a period $\Phi_0 = h/q$ (q = effective particle charge)

Byers, Yang, PRL **7,** 46 (1961)

- PERSISTENT CURRENTS



$$\frac{dF}{dt} = I\,V \qquad V = -\frac{\partial \Phi}{\partial t}$$

$$\frac{dF}{dt} = \frac{\partial F}{\partial \Phi}\frac{\partial \Phi}{\partial t}$$

Bloch, PRB **2,** 109 (1970)

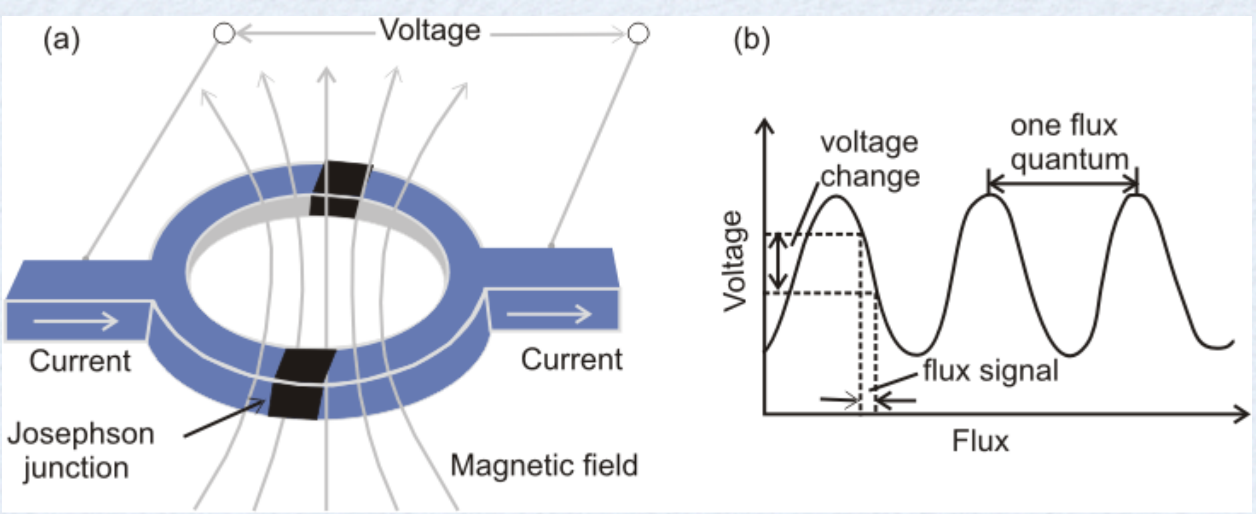
$$I(\phi) = -\frac{q}{\hbar}\frac{\partial F(\phi)}{\partial \phi}$$

Periodic Boundaries: physical systems

- bulk superconductors

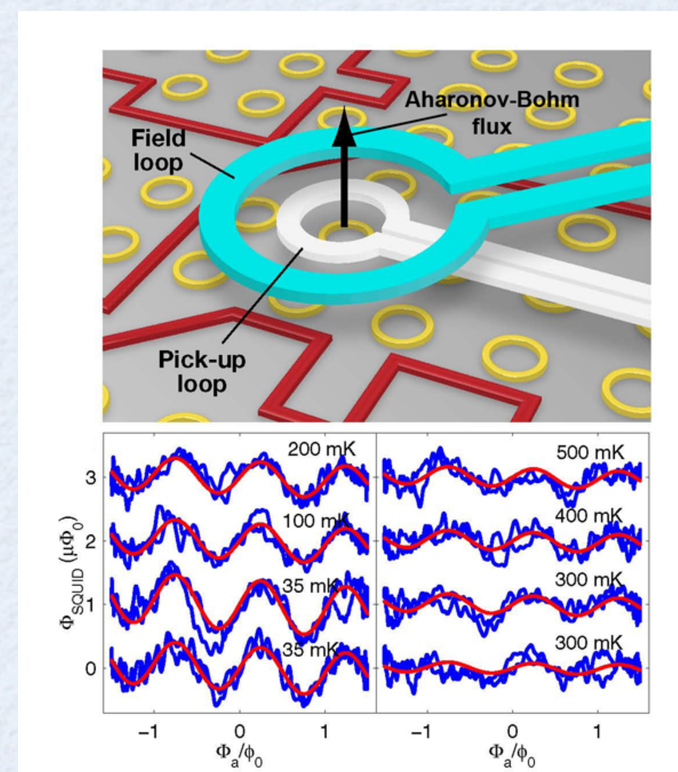
B. S. Deaver and W. M. Fairbank, PRL 7, 43 (1961)
N. Byers and C. N. Yang, PRL 7, 46 (1961)
L. Onsager, PRL 7, 50 (1961)

- SQUID = superconducting quantum interference device

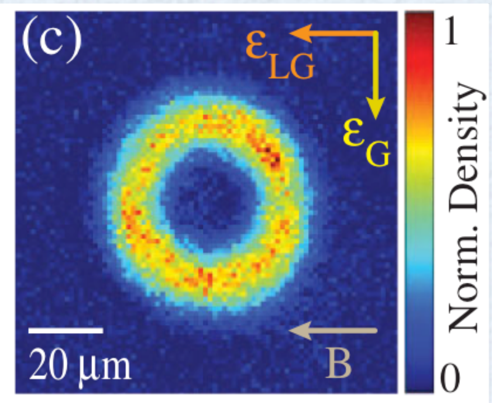


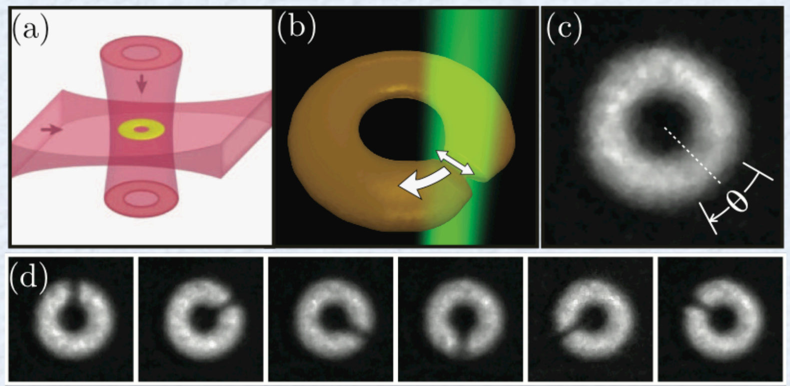
- normal metallic rings

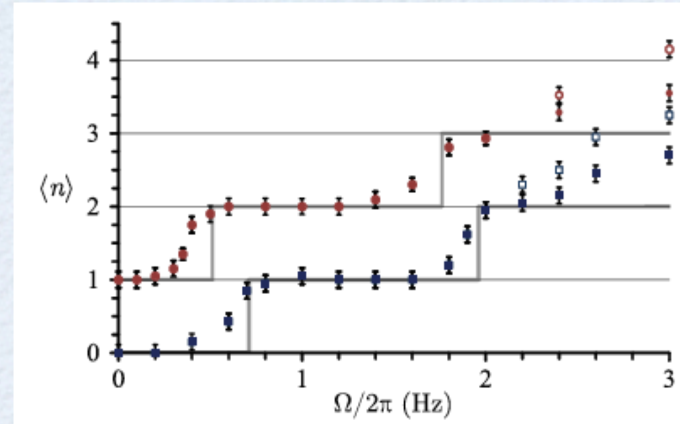
L. P. Levy, et al., PRL 64, 2074 (1990)
D. Mailly, et al., PRL 70, 2020 (1993)
H. Bluhm et al., PRL 102, 136802 (2009)
A. C. Bleszynski-Jayich, et al., Science 326, 272 (2009)

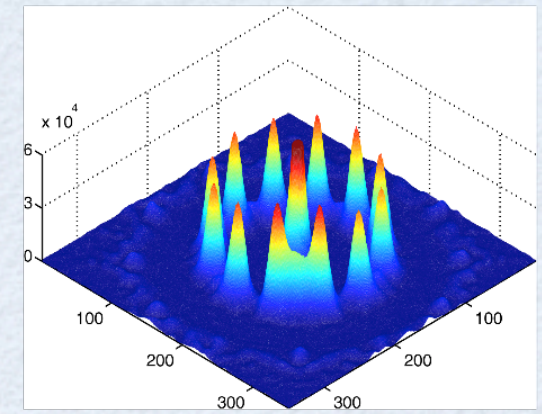


Periodic Boundaries: physical systems







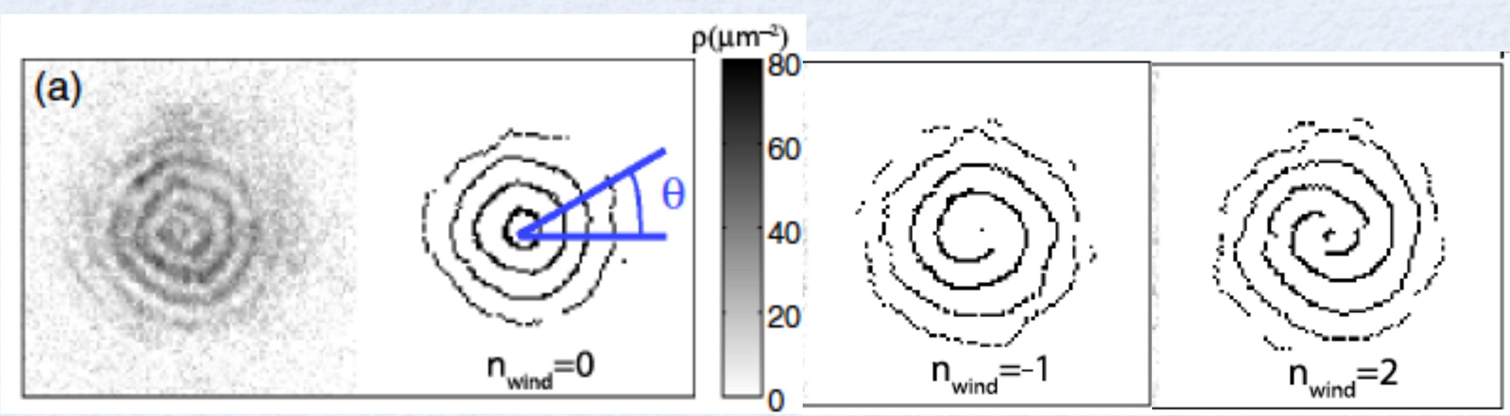


Ramanathan et al., PRL 106, 130401 (2011); Wright et al., PRL 110, 025302 (2013); Moulder et al., PRA 86, 013629 (2012); Beattie, et al., PRL 110, 025301 (2013); C. Ryu, et al., PRL 111, 205301 (2013); and many more

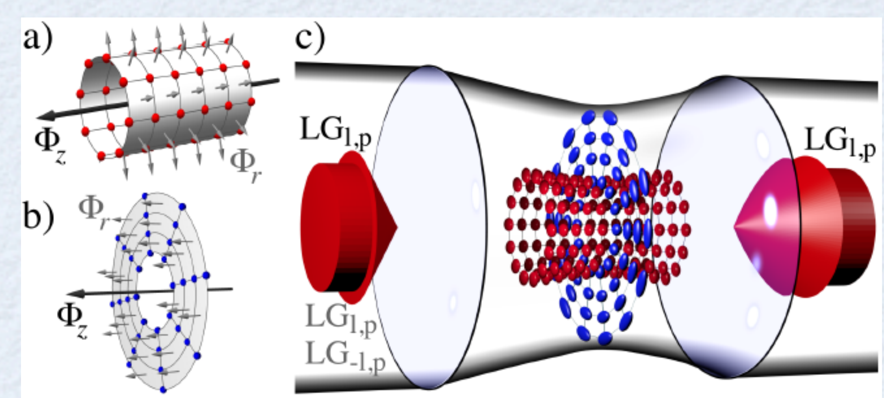
Amico et al., Sci. Rep. 4, 4298 (2014)

- persistent currents flowing > 2 mins !
- diverse starting & measuring protocols
- multi-species setups (i.e., spin currents)

★ long coherence qubits
★ precise interferometry
★ fractional QHE states



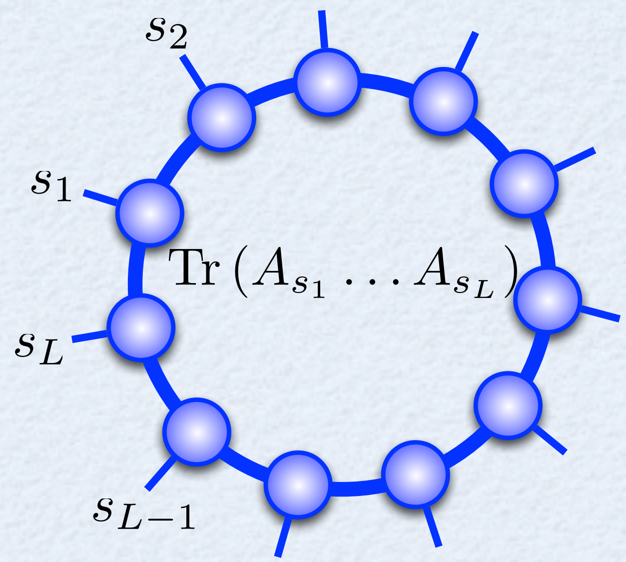
L. Corman, et al., PRL 113, 135302 (2014)

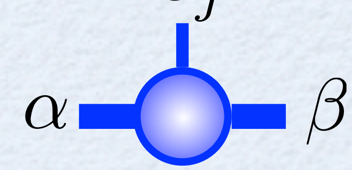


M. Łącki, et al., arXiv:1507.00030

Periodic Boundaries: tensor networks

$$|\psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1} \dots A_{s_L}\right) |s_1 s_2 \dots s_L\rangle$$





$$A^{[j]}_{s_j \alpha\beta}$$

$$s_j \in \{0, \dots, n_j^{\max}\}$$

$$\alpha, \beta = 1 \dots m$$

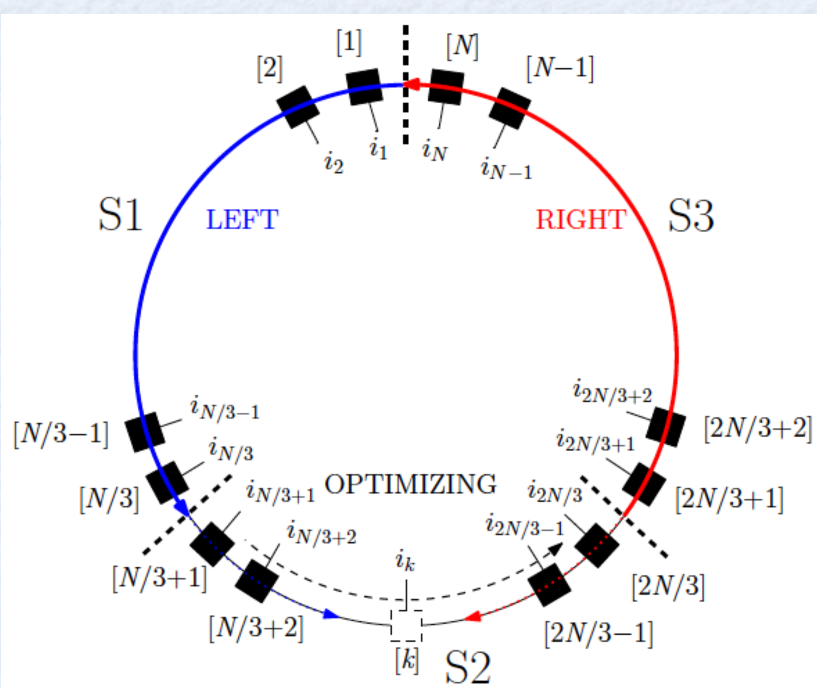
Verstraete, et al, PRL 93, 227205 (2004);

Pippan, et al. PRB 81, 081103(R) (2010);

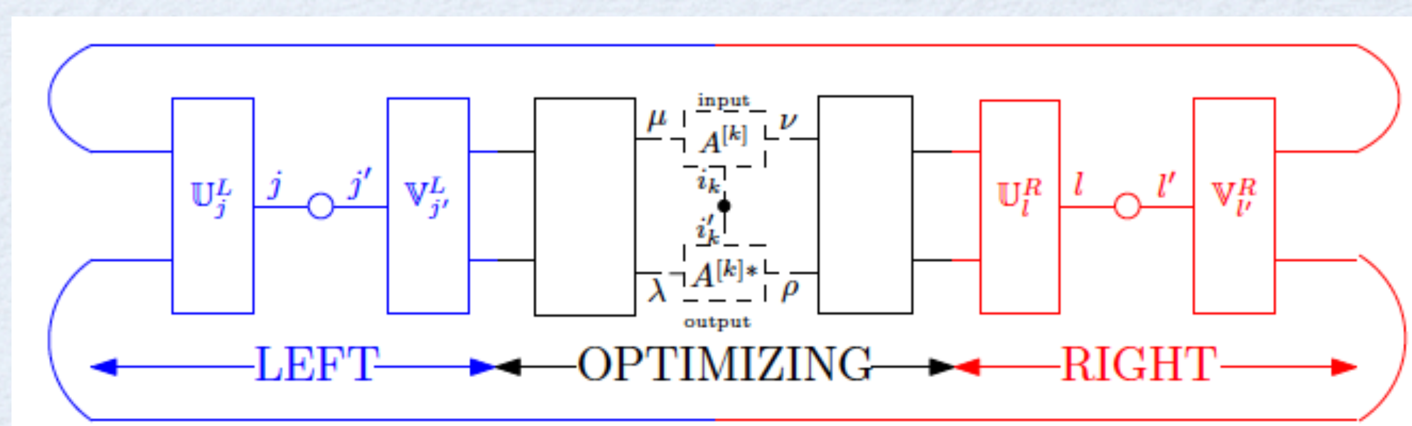
Pirvu, et al., PRB 83, 125104 (2011)

Weyrauch, et al., arXiv:1303.1333

- Periodic MPS suffer from: - higher contraction costs O(m5) vs. O(m3)
  - absence of a isometric gauge



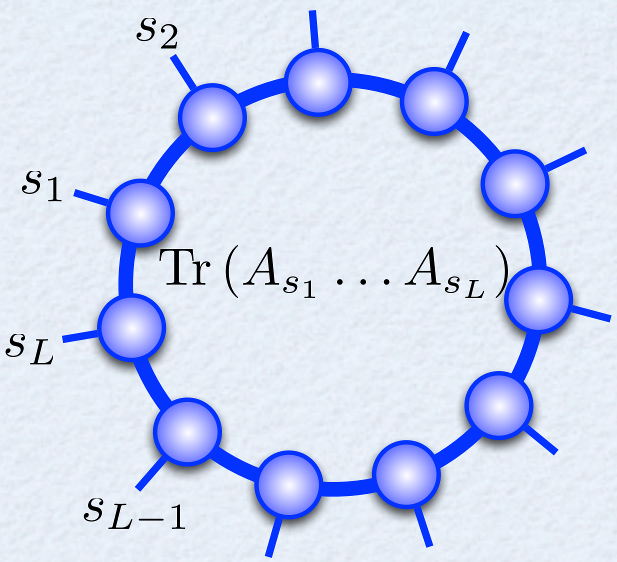
- tricks based on transfer matrices of long homogeneous chains may help

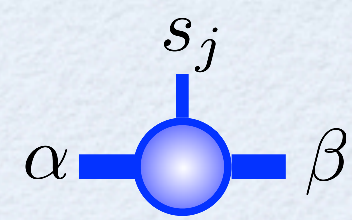


Rossini, et al., J. Stat. Mech. (2011) P05021

Periodic Boundaries: tensor networks

$$|\psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left(A_{s_1} \ldots A_{s_L}\right) |s_1 s_2 \ldots s_L\rangle$$





$$A^{[j]}_{s_j \alpha\beta}$$

$$s_j \in \{0, \ldots, n_j^{\max}\}$$

$$\alpha, \beta = 1 \ldots m$$

Verstraete, et al, PRL 93, 227205 (2004);

Pippan, et al. PRB 81, 081103(R) (2010);

Pirvu, et al., PRB 83, 125104 (2011)

Weyrauch, et al., arXiv:1303.1333

- Periodic MPS suffer from: - higher contraction costs O(m5) vs. O(m3)
  - absence of a isometric gauge

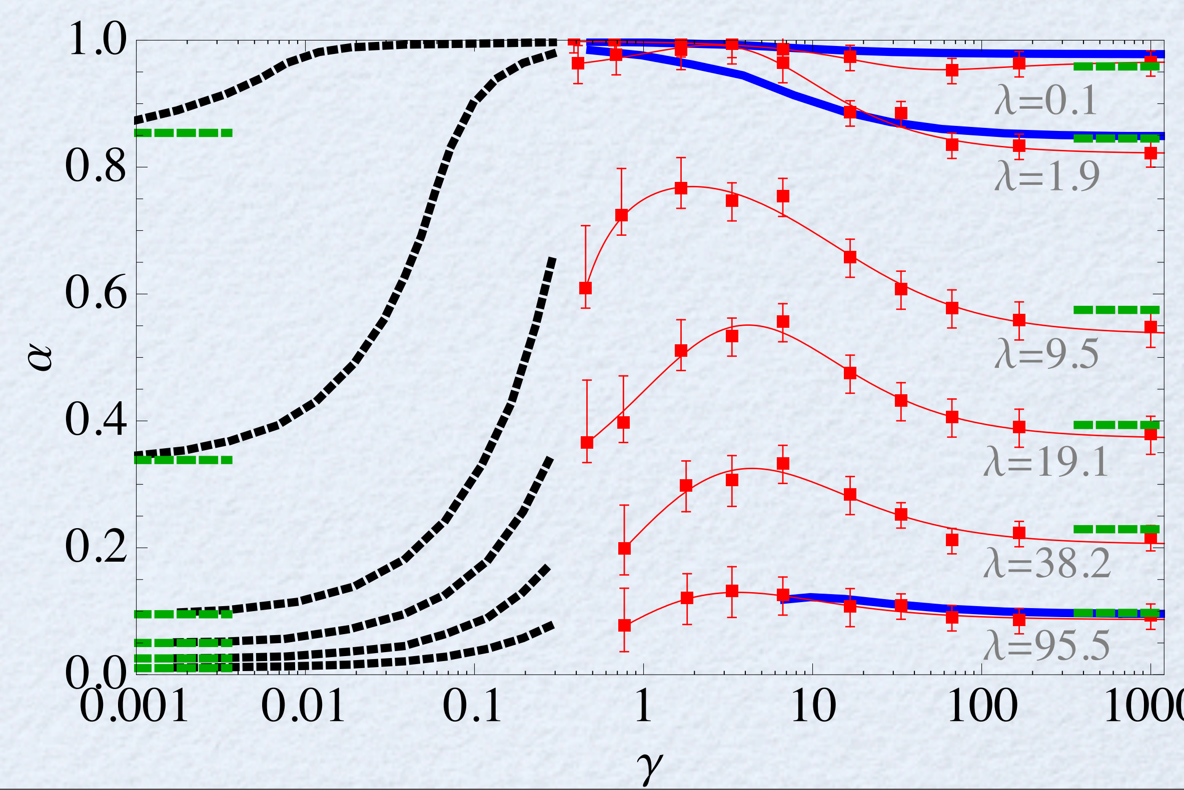
- EXAMPLE:

  Bose-Hubbard model at low filling + Peierls substitution + impurity

  Cominotti, MR, et al., PRL 113, 025301 (2014)

  alternative: cMPS for Lieb-Liniger

  ongoing with D. Draxler, J. Haegeman



OUTLINE

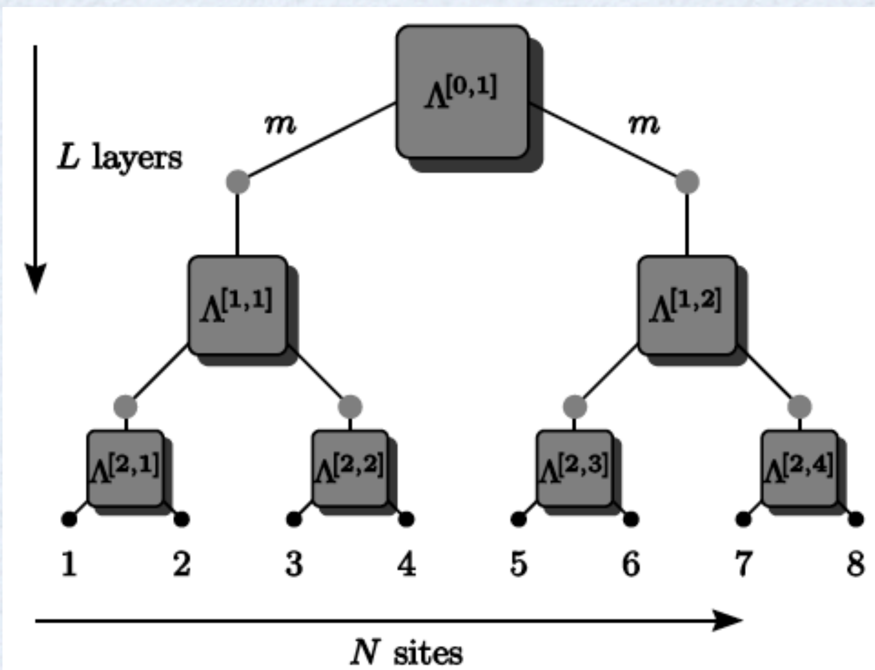
- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

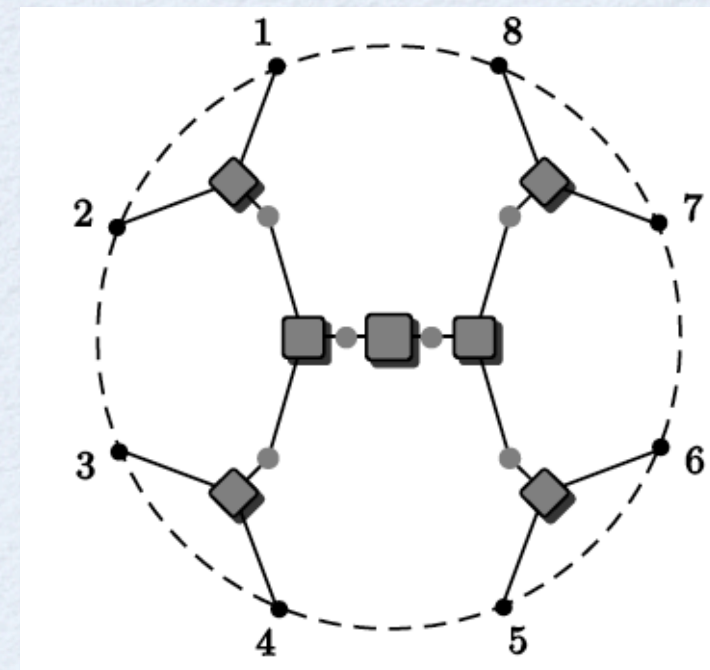
OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

Tree Tensor Networks (TTN)

- loop-free, hierarchical, structure treating PBC on the same footing of OBC

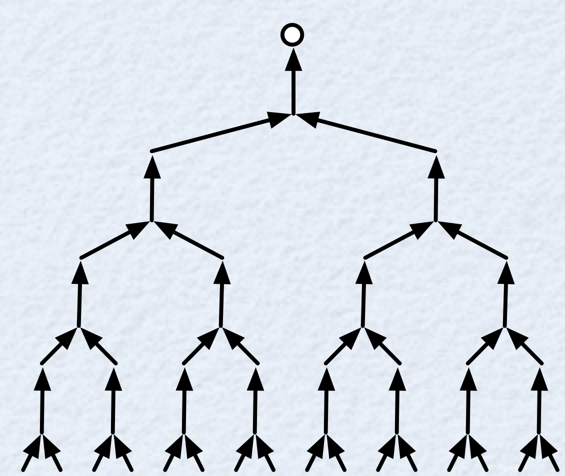




- set of (N-1) tensors $\left\{ \Lambda^{[l,n]} \right\}_{n=1,\ldots,2^l}^{l=0\ldots\log_2(N/2)}$ bond dimension **m**

- storage $O(Nm^3)$; computation $O(Nm^4)$

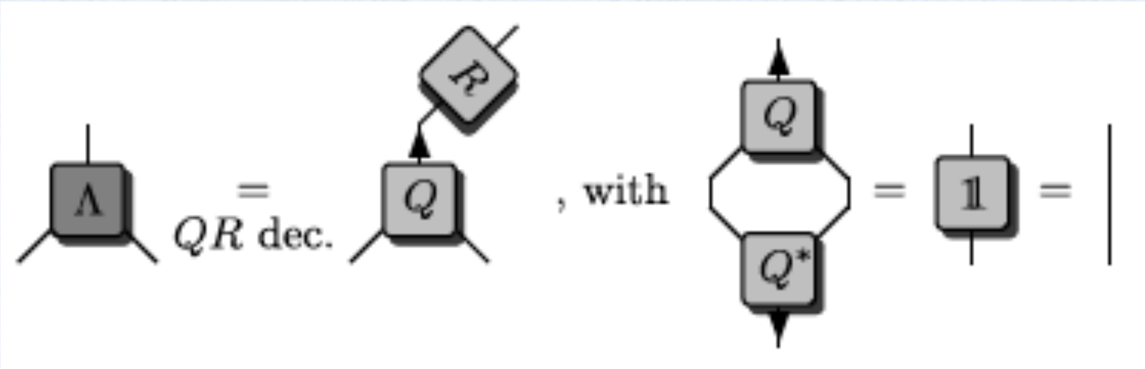
- standard interpretation: RG flow from L towards 0 [i.e., a MERA without disentanglers ...]



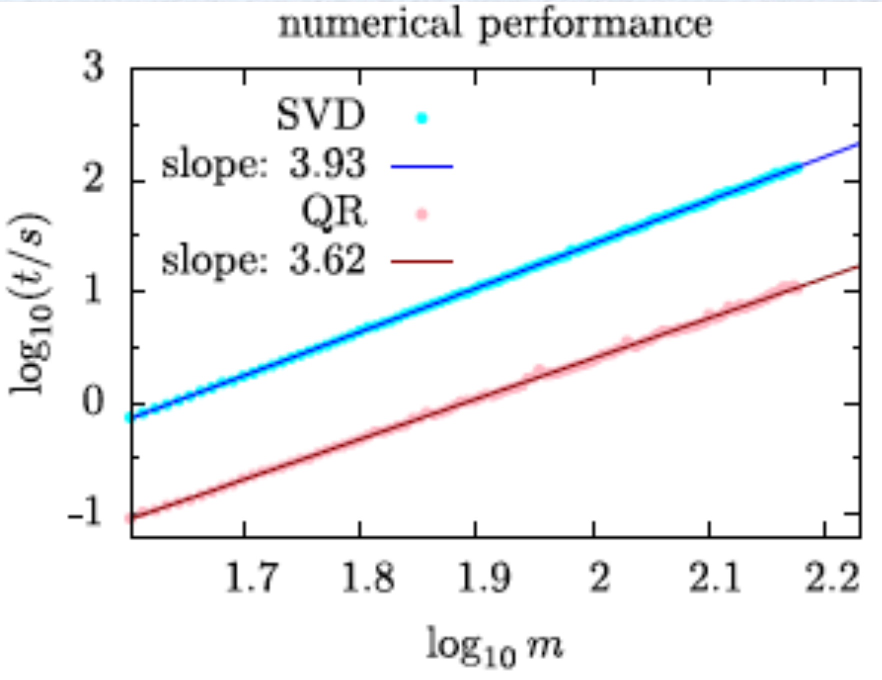
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Adaptive gauging of a TTN

- **loop-free structure** allows for more ! ***Central gauge*** towards <u>any</u> tensor !
- INGREDIENT: Isometrization of a single tensor via QR decomposition



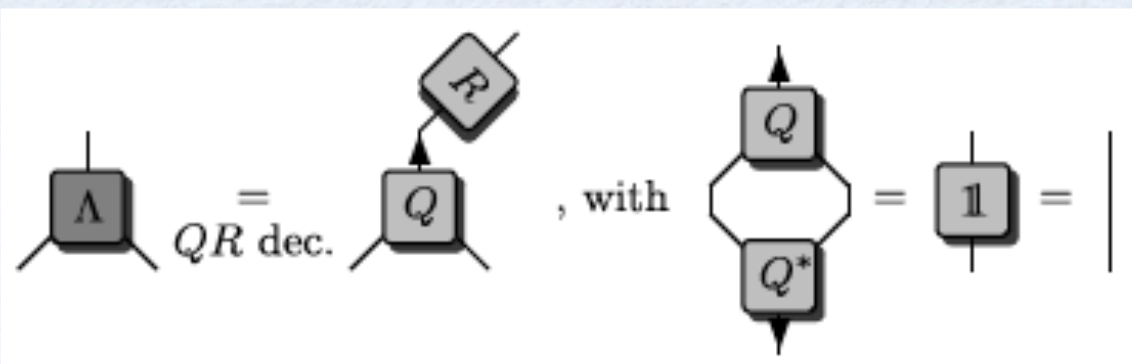
(feasible on all legs …)



M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

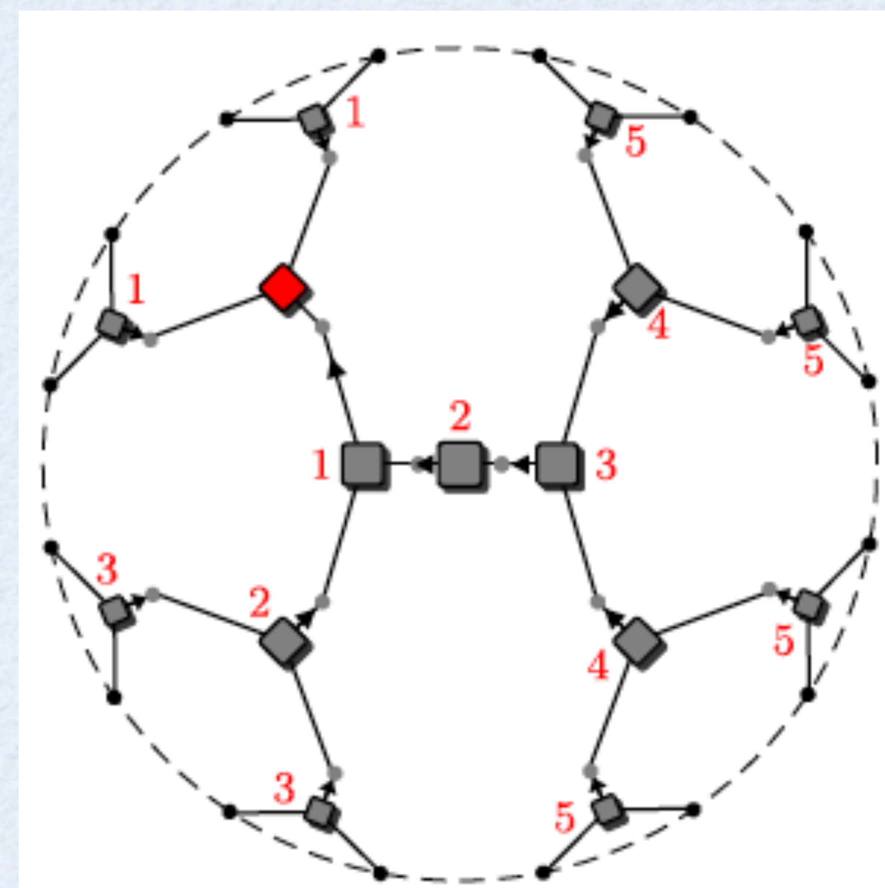
Adaptive gauging of a TTN

- **loop-free structure** allows for more ! ***Central gauge*** towards <u>any</u> tensor !
- INGREDIENT: Isometrization of a single tensor via QR decomposition



(feasible on all legs …)

- RECIPE:
  a) for each tensor determine distance from “nucleus”
  b) perform QR in order & direction of decreasing distance
  c) adsorb the R remainder in the not yet isometrized tensors



M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

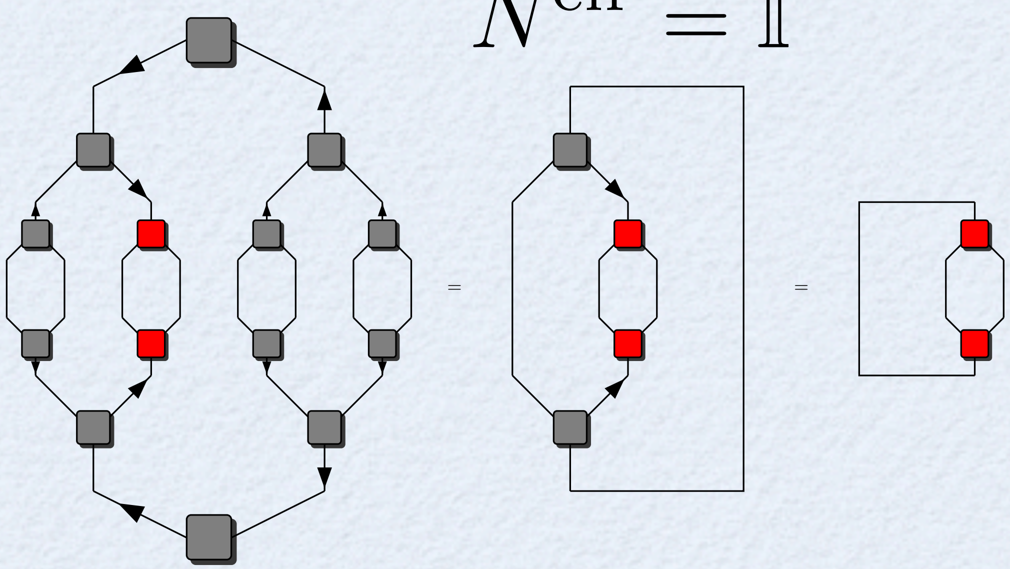
Ground state variational algorithm

- **loop-free structure** allows for more ! ***Central gauge*** towards <u>any</u> tensor !
- ADVANTAGE: effective norm operator disappears in the secular problem i.e., no need for less stable generalized eigenvalue problem solver !

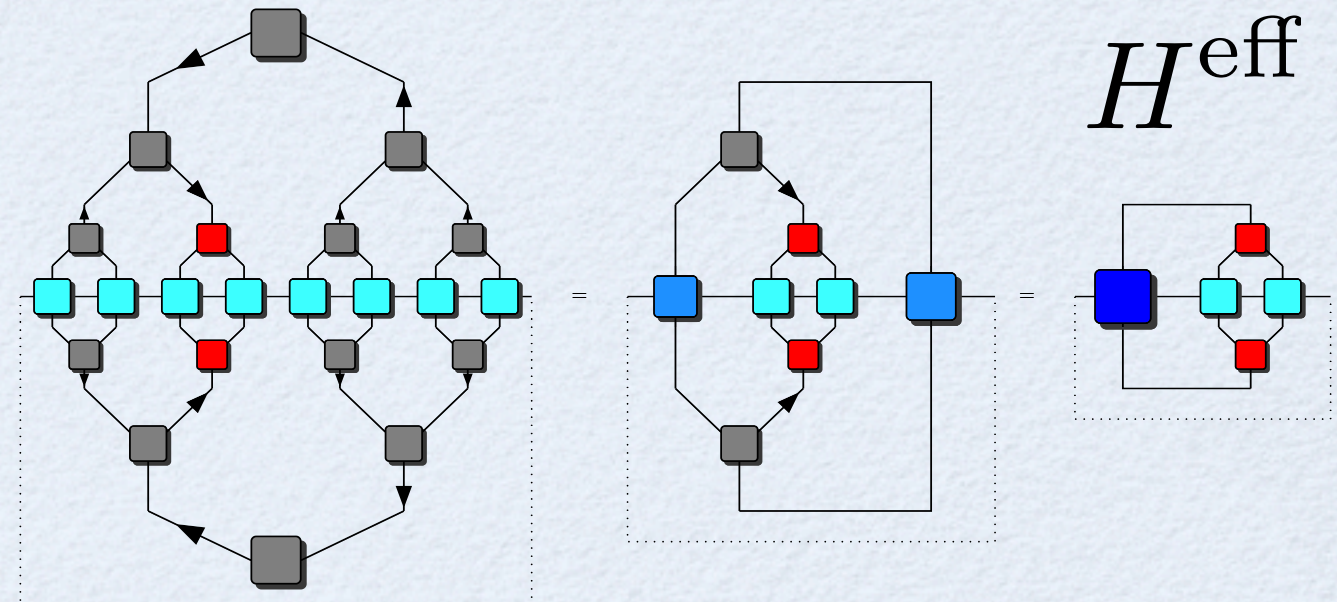
$$\min_{|\Psi\rangle \in \mathrm{TN}} \langle \Psi | H | \Psi \rangle \,/\, \langle \Psi | \Psi \rangle \quad \Rightarrow \quad H_j^{\mathrm{eff}} A_{[j]} = \varepsilon \, N_j^{\mathrm{eff}} A_{[j]}$$

(cycle on j)

$$N^{\mathrm{eff}} = \mathbb{I}$$

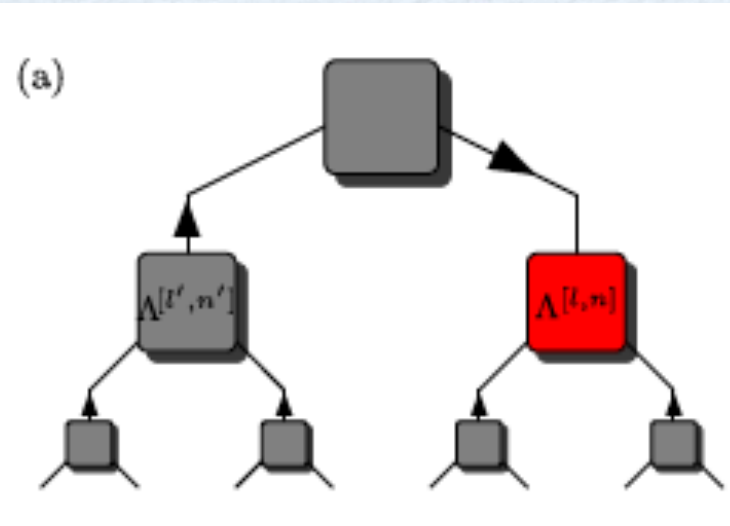


$$H^{\mathrm{eff}}$$



M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

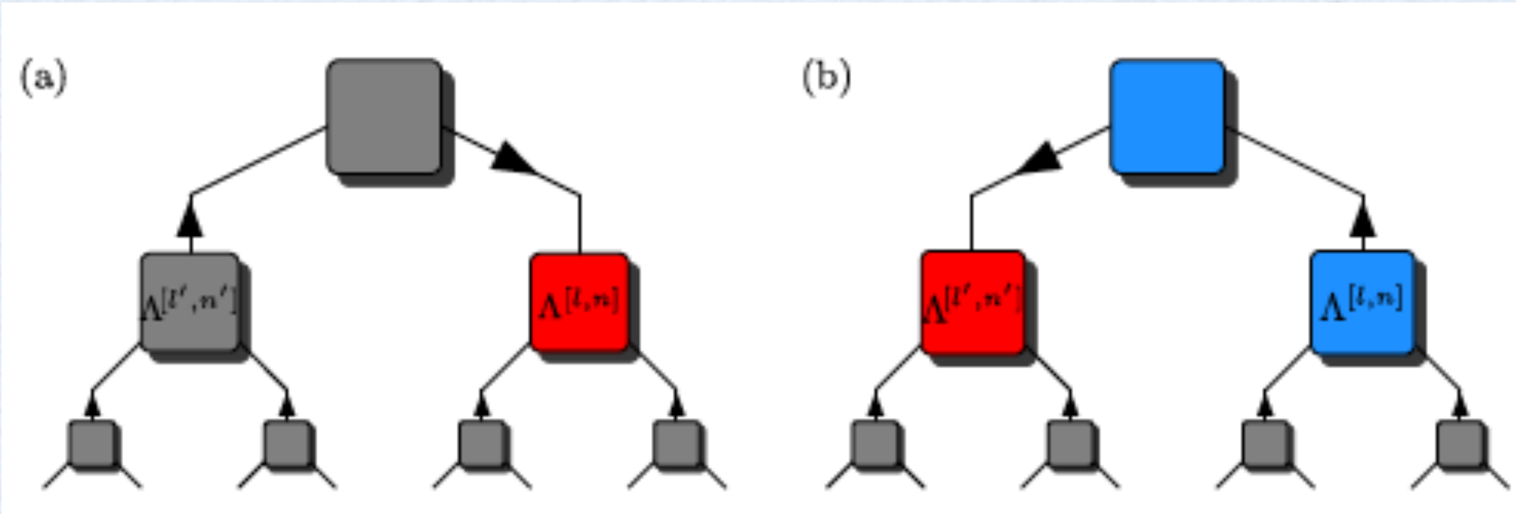
Ground state variational algorithm



(a) optimize $\Lambda^{[l,n]}$, i.e., solve the secular problem $H^{\text{eff}}_{[l,n]}\Lambda_{[l,n]} = \varepsilon\,\Lambda_{[l,n]}$

M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Ground state variational algorithm

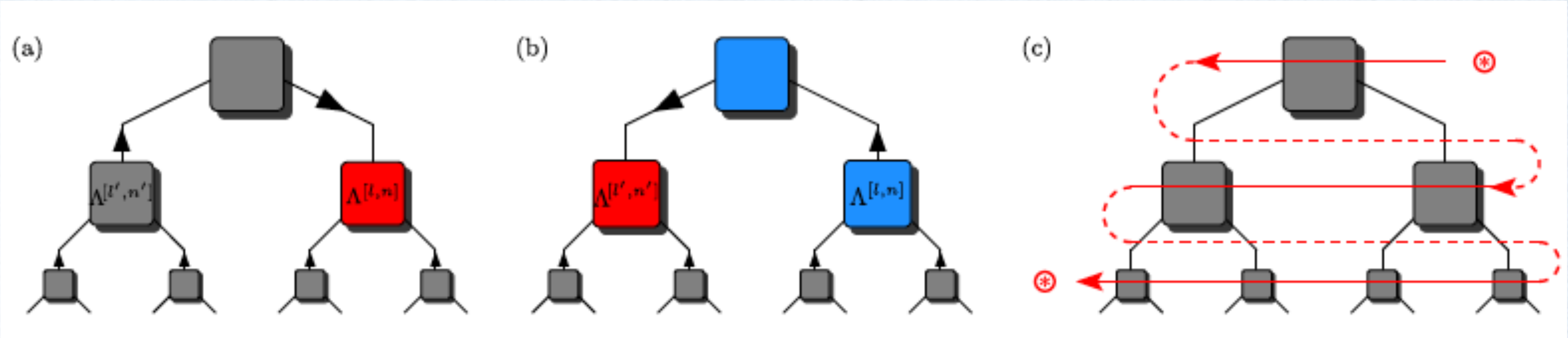


(a) optimize $\Lambda^{[l,n]}$, i.e., solve the secular problem $H^{\text{eff}}_{[l,n]}\Lambda_{[l,n]} = \varepsilon\,\Lambda_{[l,n]}$

(b) target the next tensor $\Lambda^{[l',n']}$ by adjusting isometrization and updating effective Hamiltonian terms

M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Ground state variational algorithm



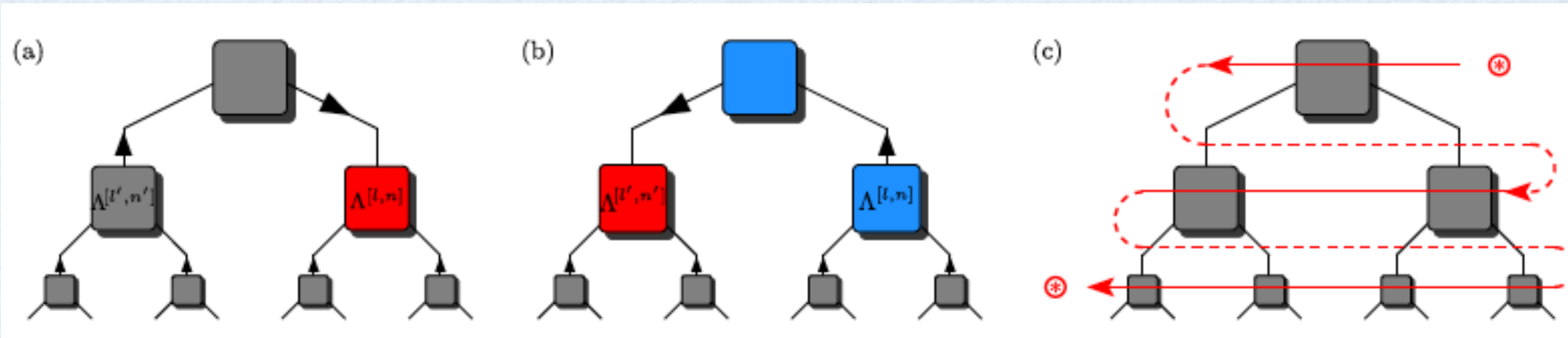
(a) optimize $\Lambda^{[l,n]}$, i.e., solve the secular problem $H^{\text{eff}}_{[l,n]}\Lambda_{[l,n]} = \varepsilon\,\Lambda_{[l,n]}$

(b) target the next tensor $\Lambda^{[l',n']}$ by adjusting isometrization and updating effective Hamiltonian terms

(c) repeat for each tensor (= "sweep") and stop when converged

M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Ground state variational algorithm



(a) optimize $\Lambda^{[l,n]}$, i.e., solve the secular problem $H^{\text{eff}}_{[l,n]}\Lambda_{[l,n]} = \varepsilon\,\Lambda_{[l,n]}$

(b) target the next tensor $\Lambda^{[l',n']}$ by adjusting isometrization and updating effective Hamiltonian terms

(c) repeat for each tensor (= "sweep") and stop when converged

- **Numerically stable!** Only ingredients needed:
  QR + eigensolver (Arnoldi/Lanczos) + tensor contraction routines
  (+ SVD in case of two-tensor optimization)

- ternary tensors + loop-free ==> **easily implemented symmetries !**

M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

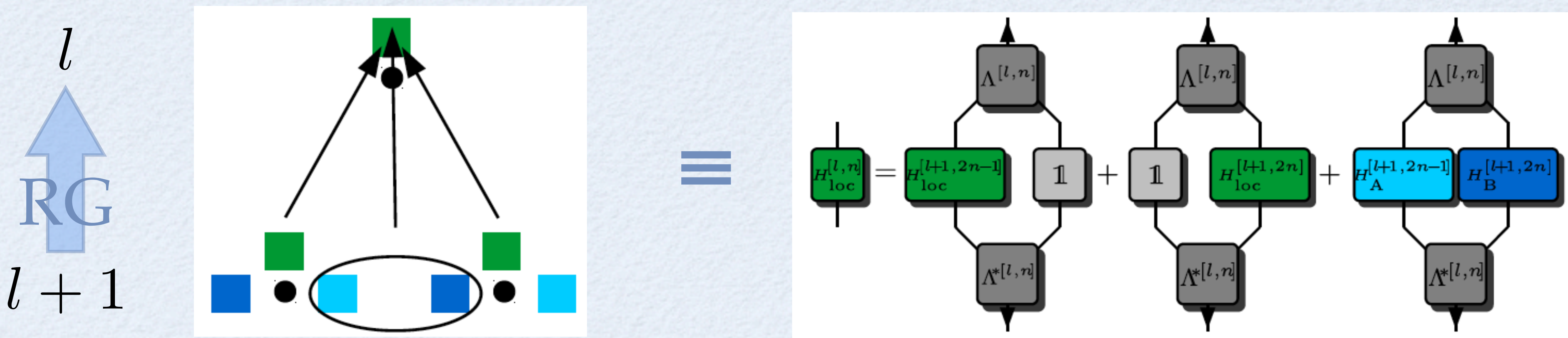
Y.-Y. Shi, et al., PRA 74, 022320 (2006)

Effective Hamiltonian Construction

- Particularly simple with nearest-neighbor terms only (and easily generalizable also to MPOs)

$$H = \sum_i H^i_{\mathrm{loc}} + \sum_i H^i_{\mathrm{A}} H^{i+1}_{\mathrm{B}}$$

- Mapping from one level to the next:



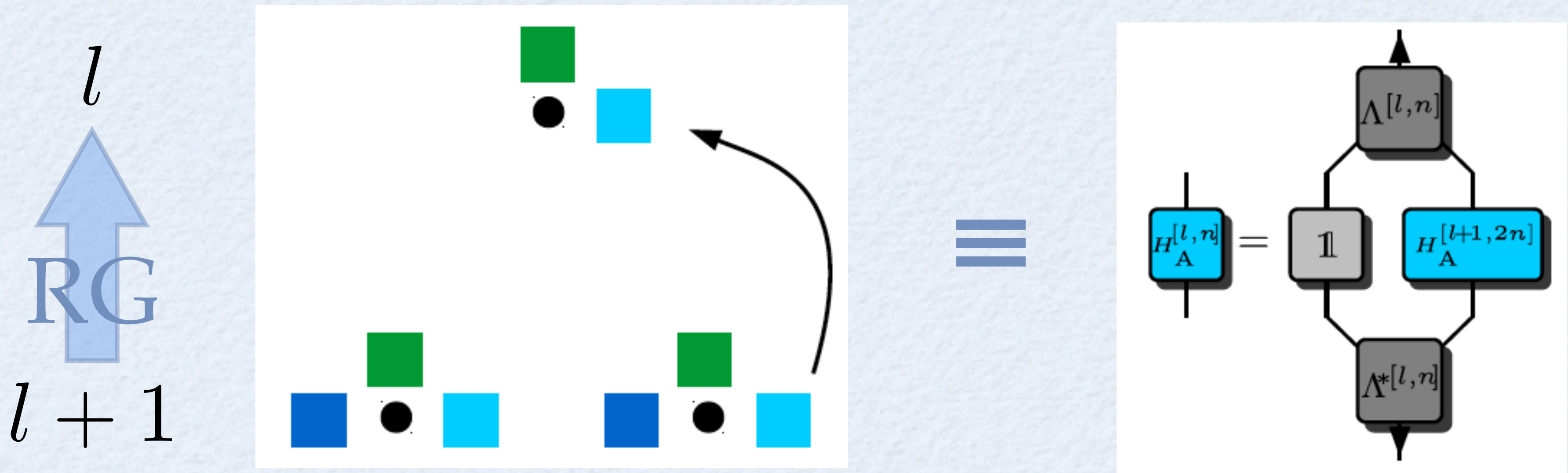
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Effective Hamiltonian Construction

- Particularly simple with nearest-neighbor terms only (and easily generalizable also to MPOs)

$$H = \sum_i H^i_{\text{loc}} + \sum_i H^i_{\text{A}} H^{i+1}_{\text{B}}$$

- Mapping from one level to the next:



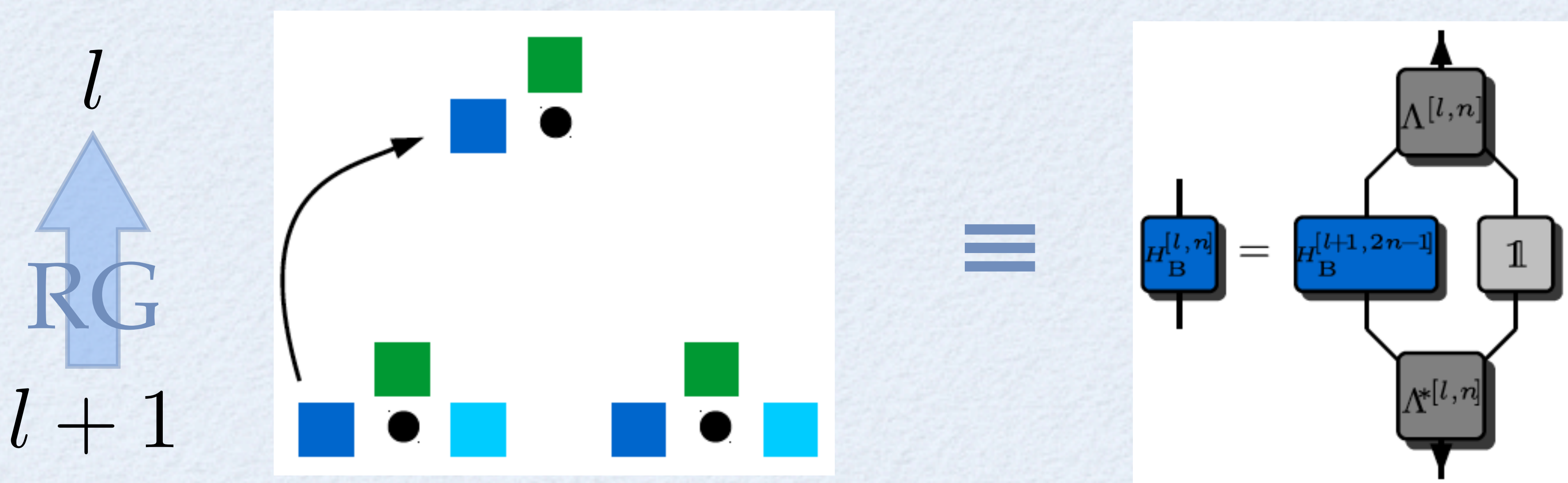
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Effective Hamiltonian Construction

- Particularly simple with nearest-neighbor terms only (and easily generalizable also to MPOs)

$$H = \sum_i H^i_{\mathrm{loc}} + \sum_i H^i_{\mathrm{A}} H^{i+1}_{\mathrm{B}}$$

- Mapping from one level to the next:



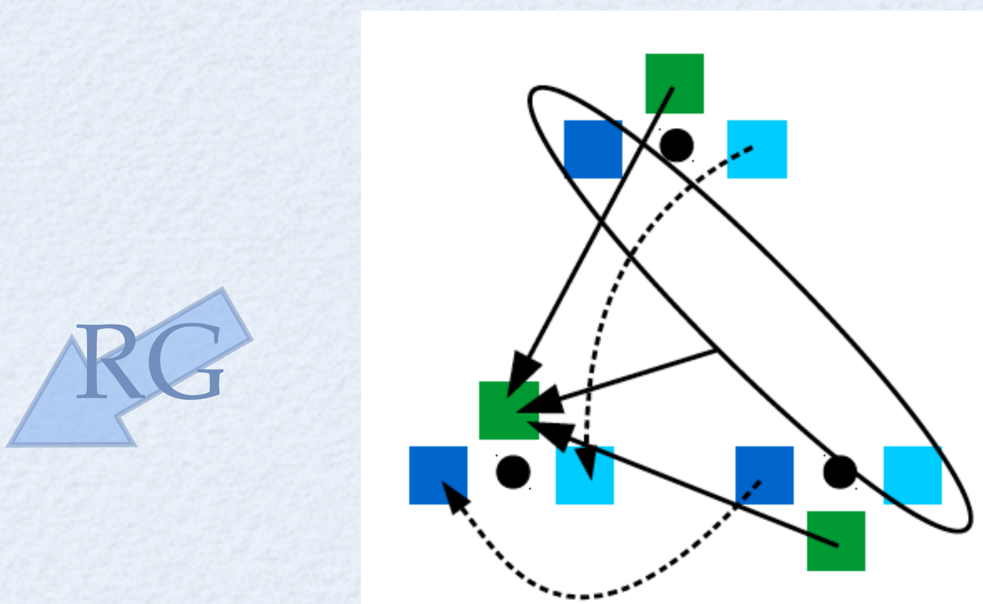
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

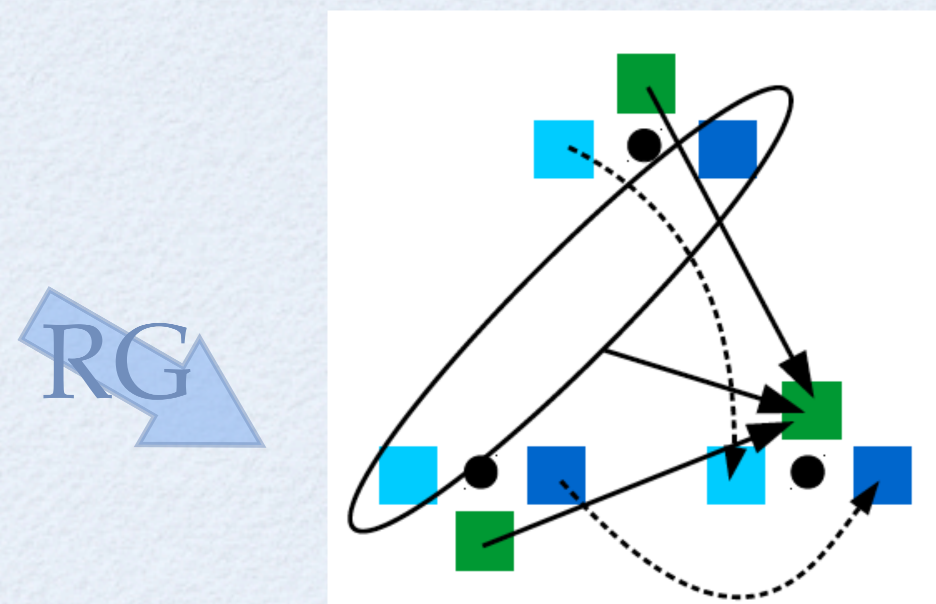
Effective Hamiltonian Construction

- Particularly simple with nearest-neighbor terms only (and easily generalizable also to MPOs)

$$H = \sum_i H^i_{\rm loc} + \sum_i H^i_{\rm A} H^{i+1}_{\rm B}$$

- Mapping works identically in all directions !



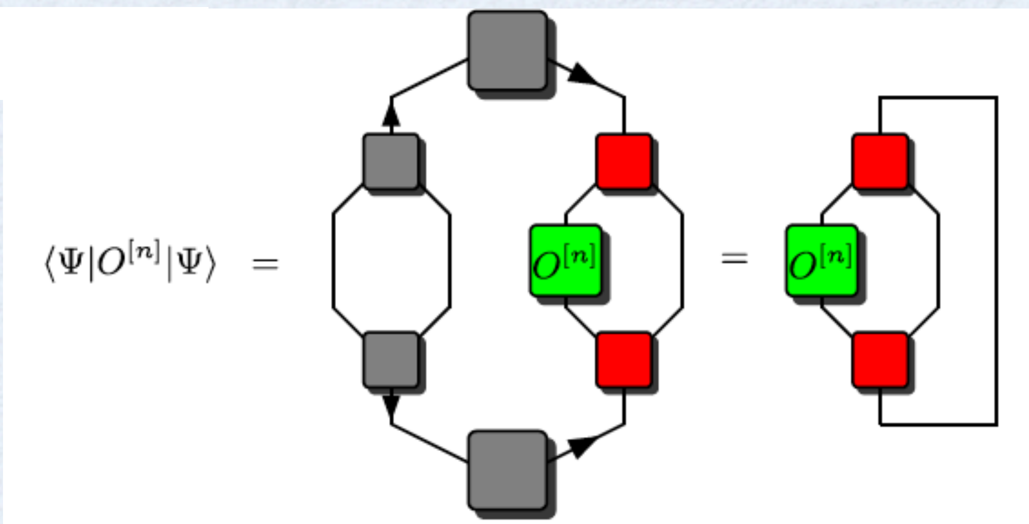


- Mapping should be performed in the same order as isometrization (i.e., from furthest to closest tensors to the “nucleus”)

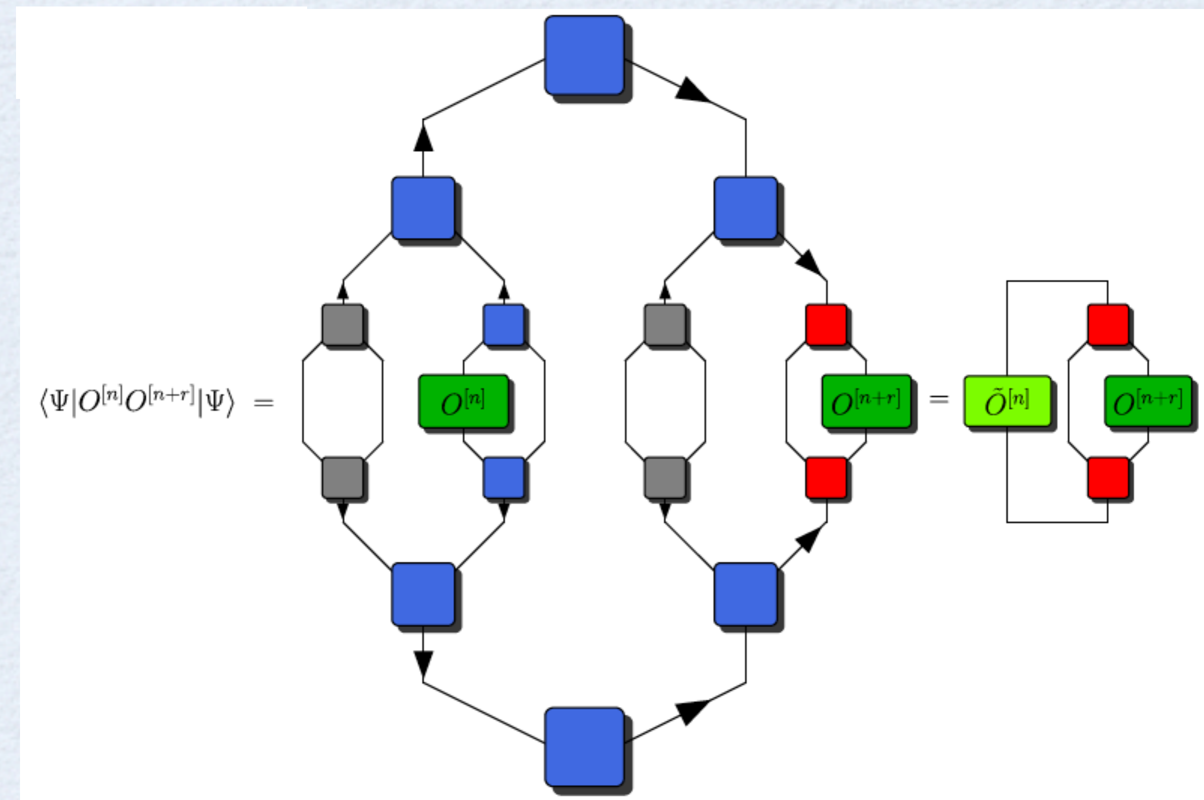
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Measurement of observables

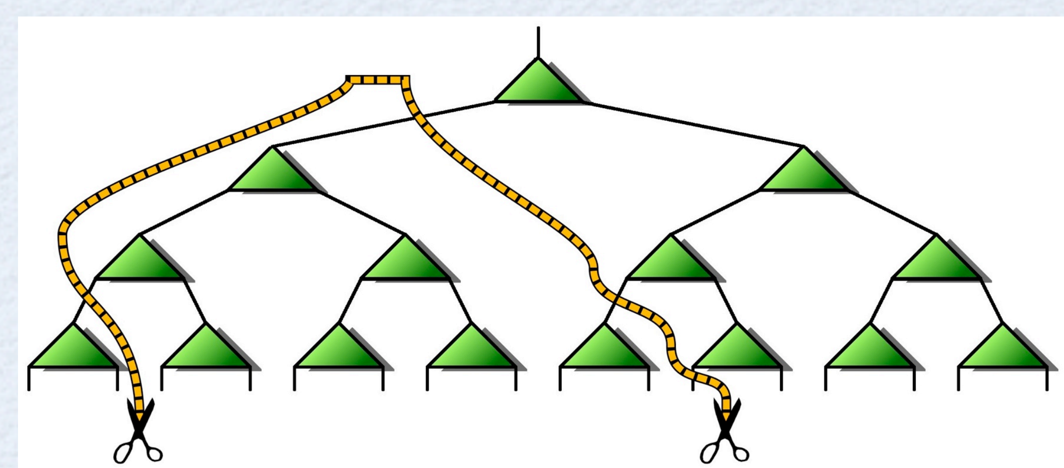
- local observables:
  only 3 contractions



- correlation functions:
  only O(log N) contractions



- hierarchical structure could encompass critical properties
- log divergence of entang. entropy captured "on average"



P. Silvi, MR, et al. Phys. Rev. A 81, 062335 (2010)

OUTLINE

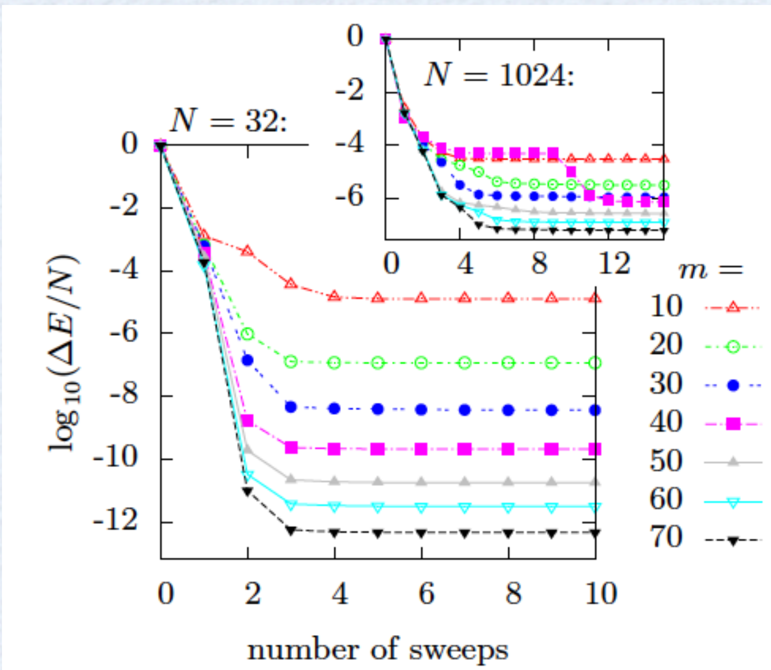
- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

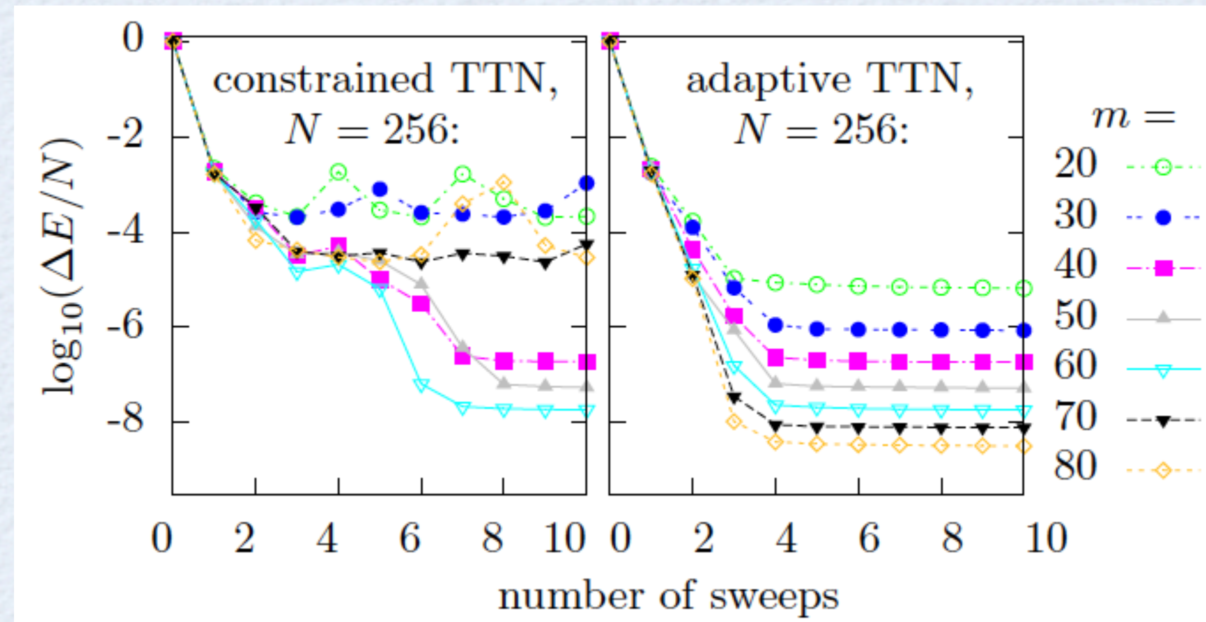
OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

Ising in transverse field: energy

- Convergence in few sweeps -- more stable than *standard* isometric TTN



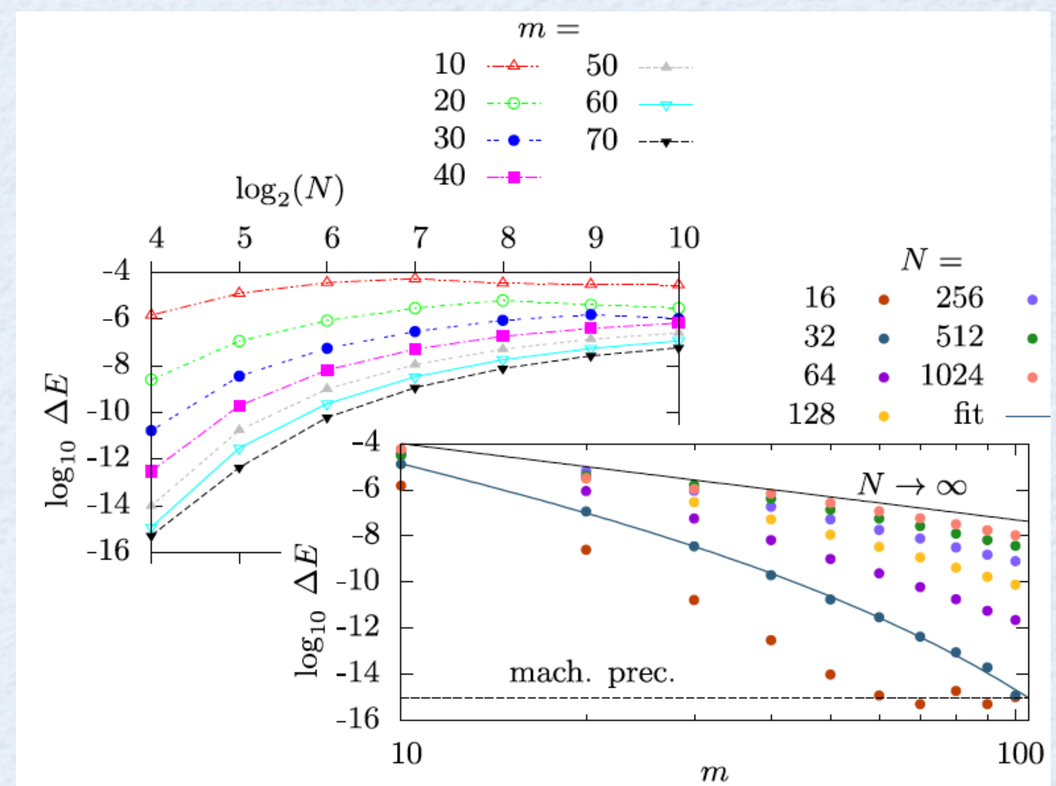


- Power-law convergence in the bond-dimension

$$\Delta E(m) \propto m^{-a}\, e^{-b\, m}$$

b vanishing at TL

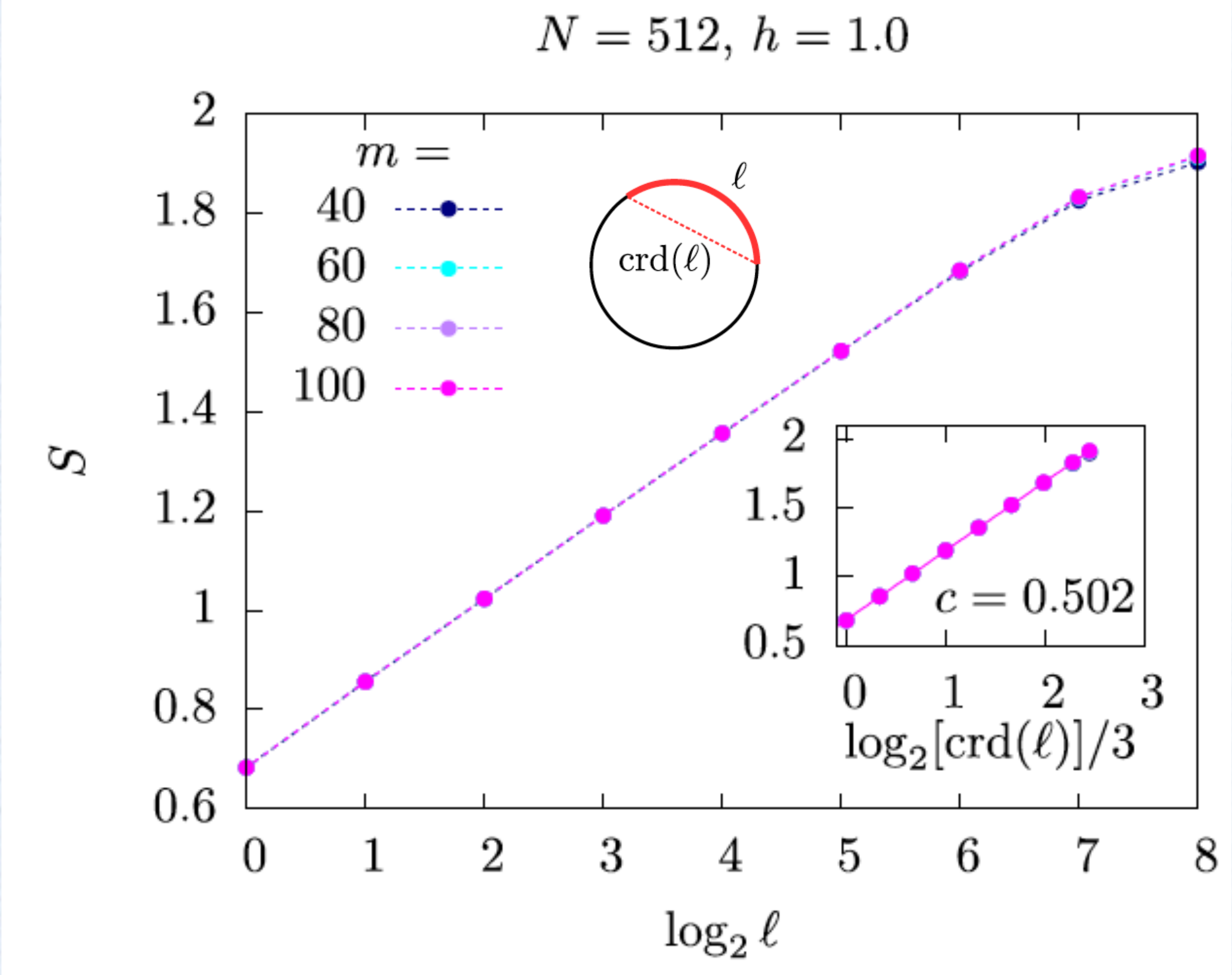
a ~ 3 at TL



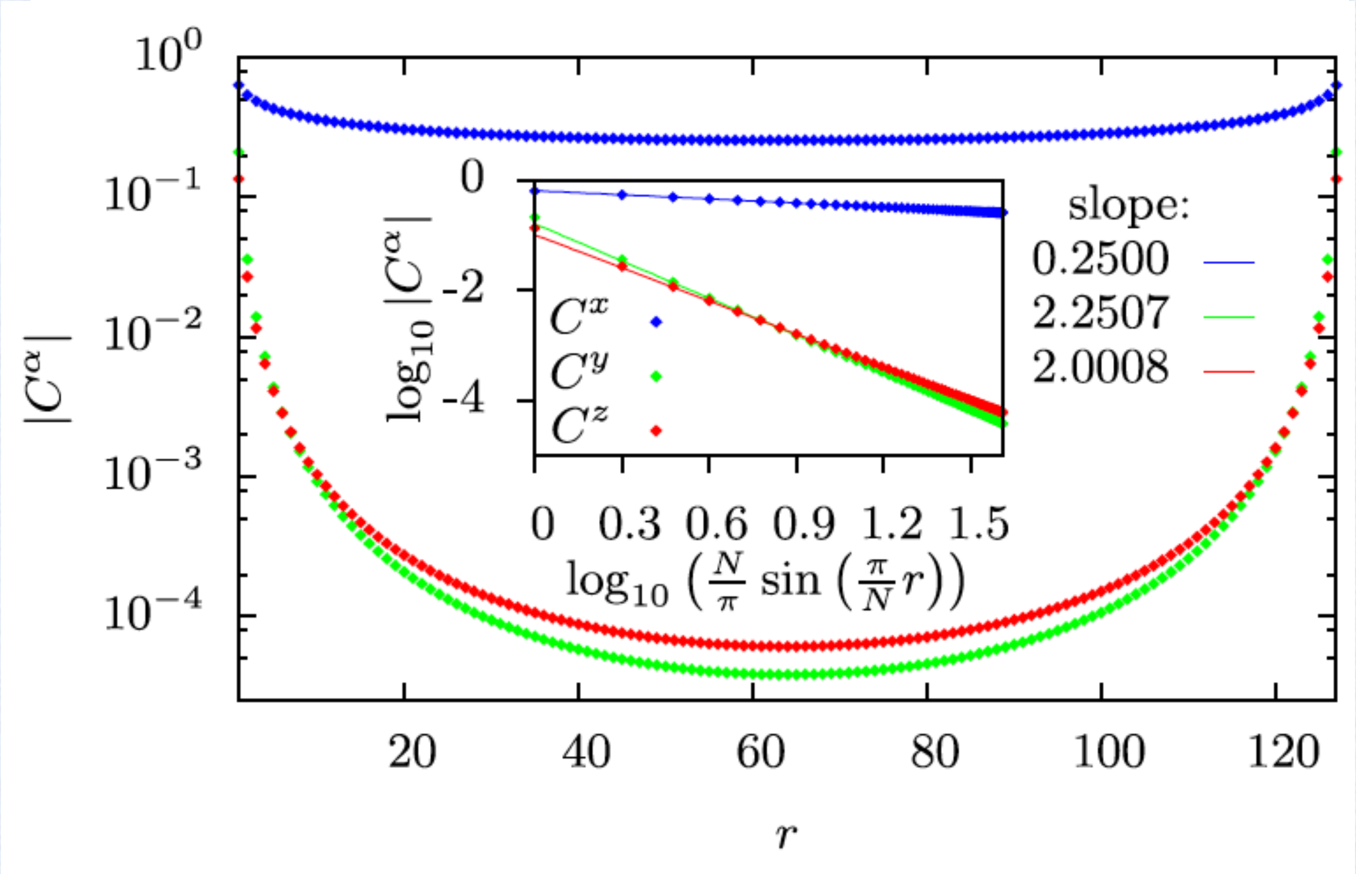
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Ising in transverse field: correlations

- Critical behaviour captured with high accuracy



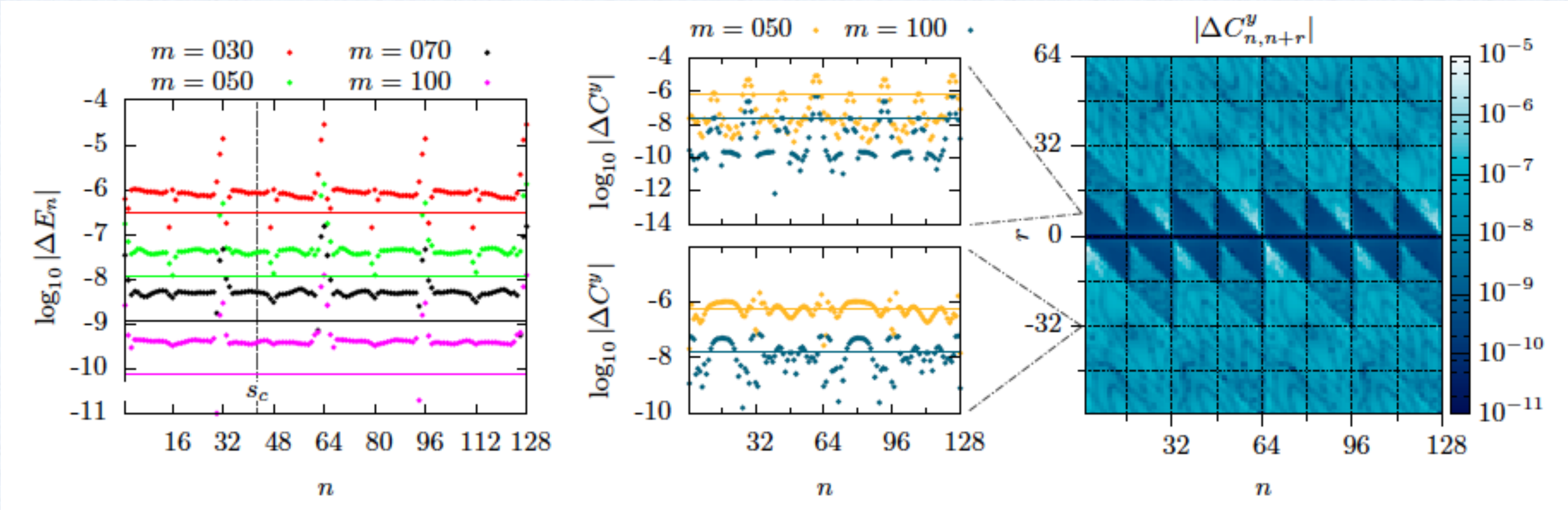
$$C^{\alpha}(r) = \sigma_i^{\alpha}\sigma_{i+r}^{\alpha} \propto r^{-\eta_\alpha}\ , \quad \alpha \in \{x, y, z\}$$



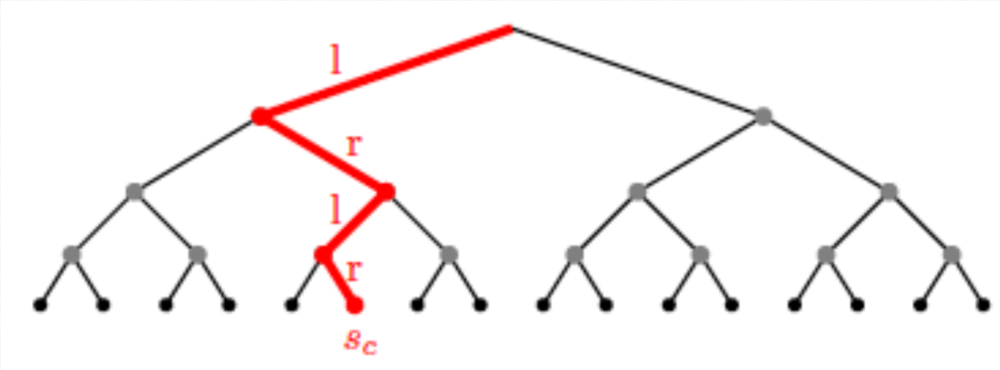
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

Ising in transverse field: translations

- breaking of translational invariance compensated by large bond dimens.



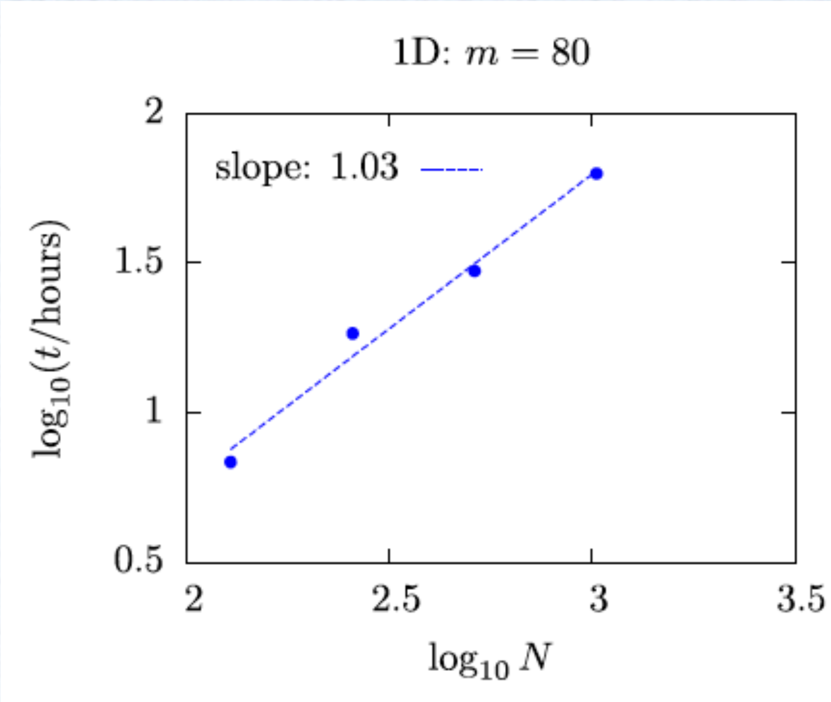
- no strict need to introduce a "central" site...



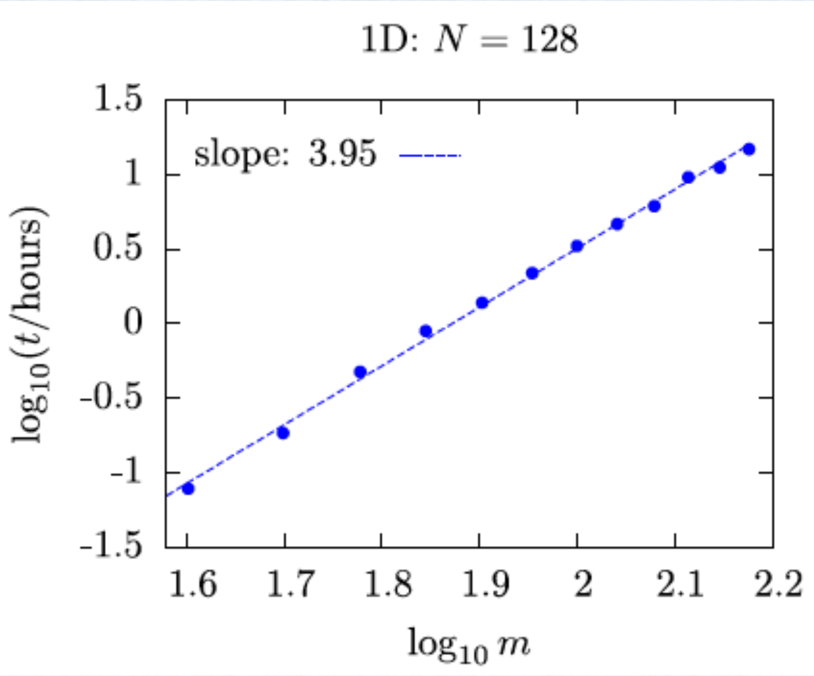
M. Gerster, MR, et al. Phys. Rev. B 90, 125154 (2014)

CPU running time

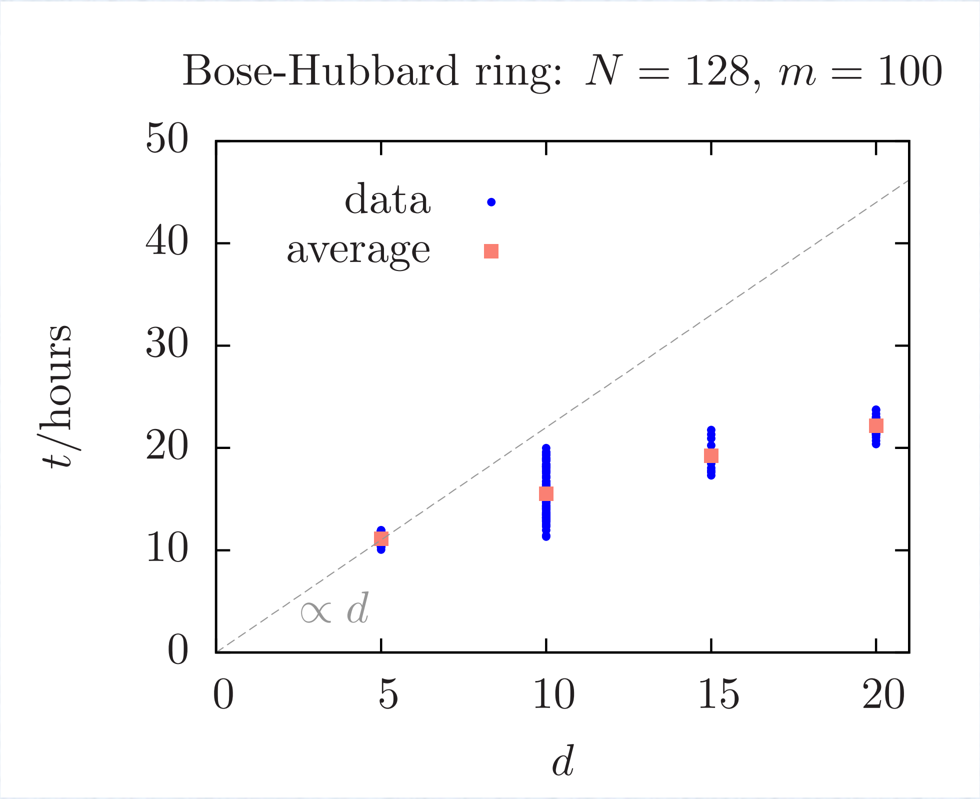
- linear scaling with system size



- $O(m^4)$ scaling with bond dim.



- sub-linear scaling with local dimension ! [OBC is O (d^n m^3)]



OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

The Hamiltonian

$$\mathcal{H} = -t\sum_j \left(b_j^\dagger b_{j+1} + \text{h.c.}\right) + \frac{U}{2}\sum_j n_j(n_j - 1) - \sum_j V_j n_j$$

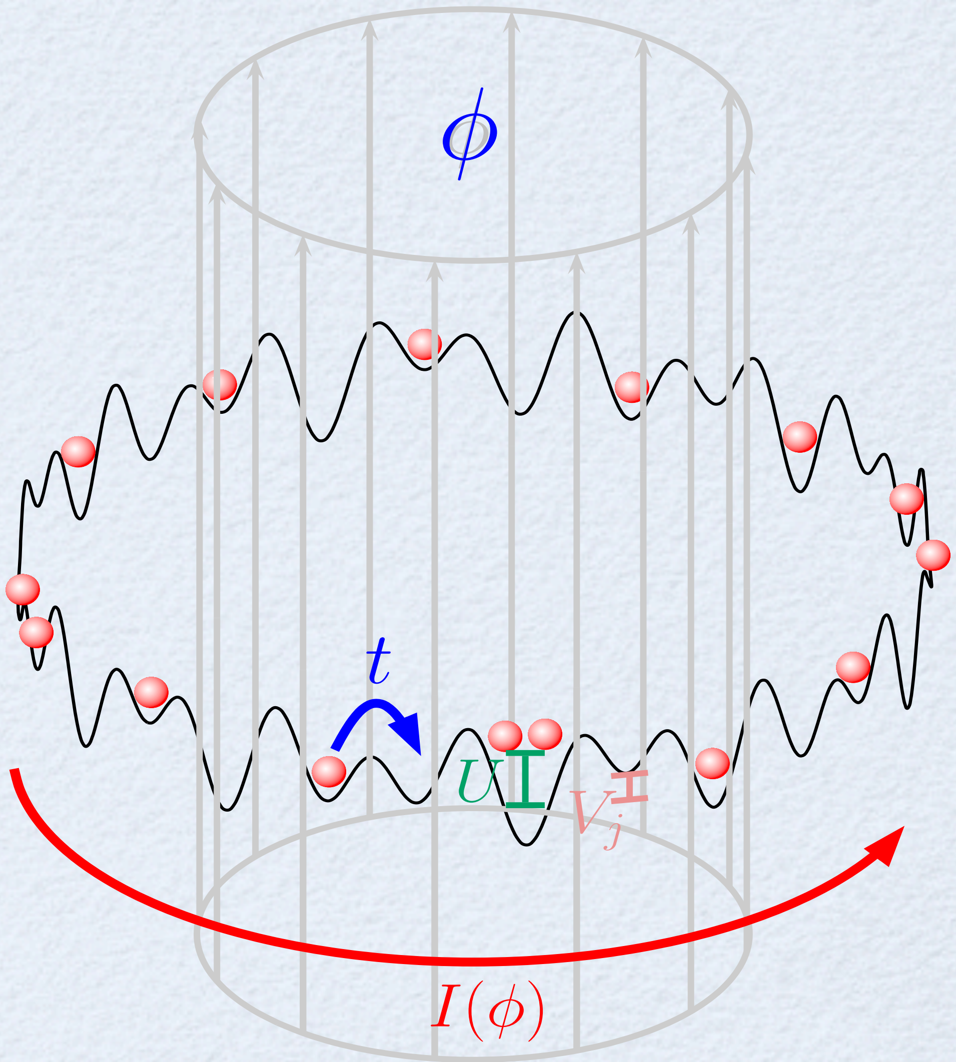
Commensurate particle density $n = N/L = 1$

Uniform disorder distribution $V_j \in [-\Delta, +\Delta]$

On-site interactions

Peierls substitution $t \longrightarrow t\, \mathrm{e}^{-2\pi i\phi/N_s}$

Superfluid stiffness $\rho_s = \frac{N_s}{8\pi^2}\frac{\partial^2 E_0}{\partial\phi^2}\Big|_{\phi=0}$



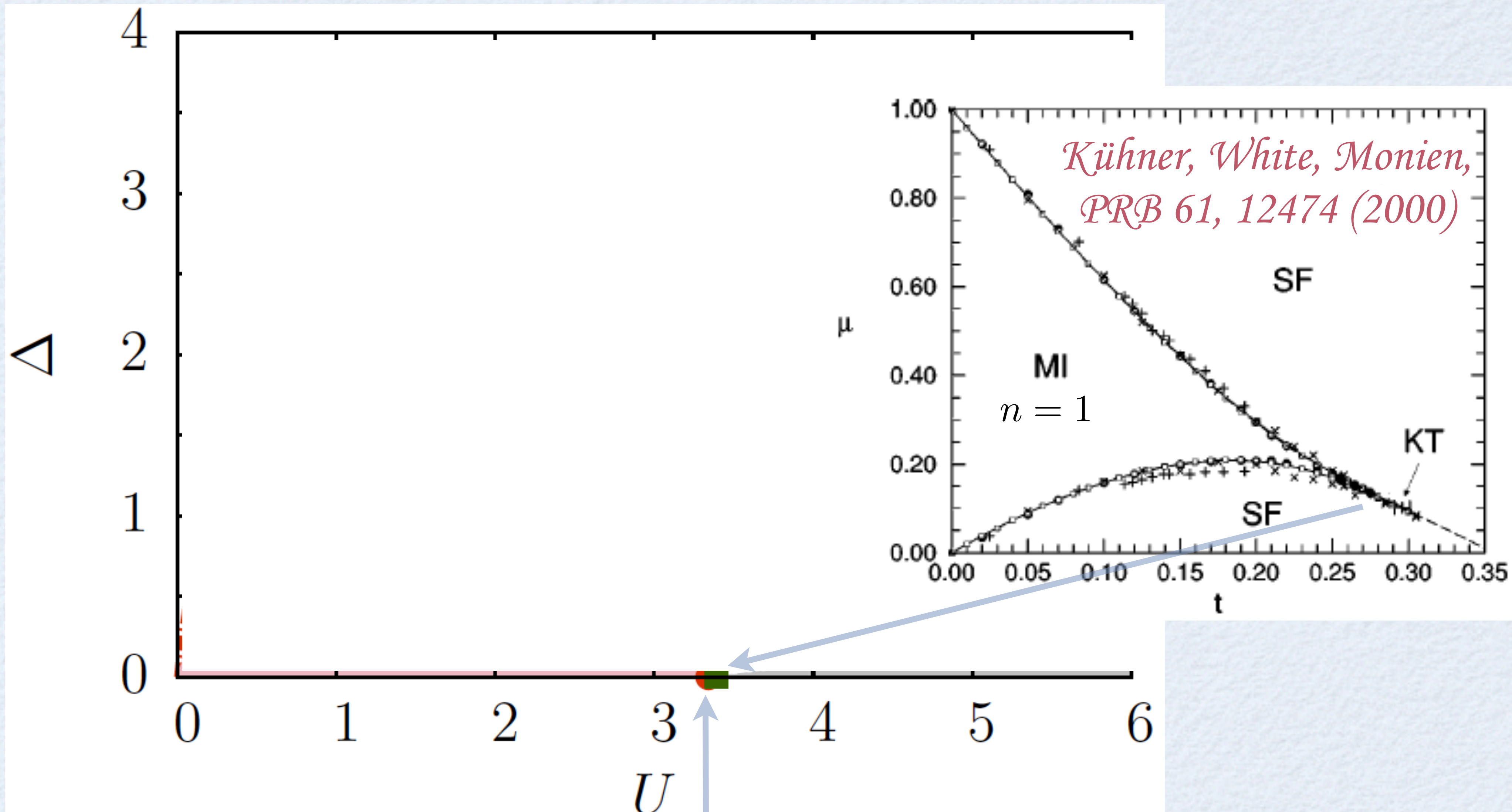
Fisher, Barber, Jasnow, PRA **8,** 1111 (1973)

The phase diagram

$U = 0$

Anderson Localization

$\Delta \neq 0$



$\Delta = 0$

Superfluid

Mott Insulator

BKT transition

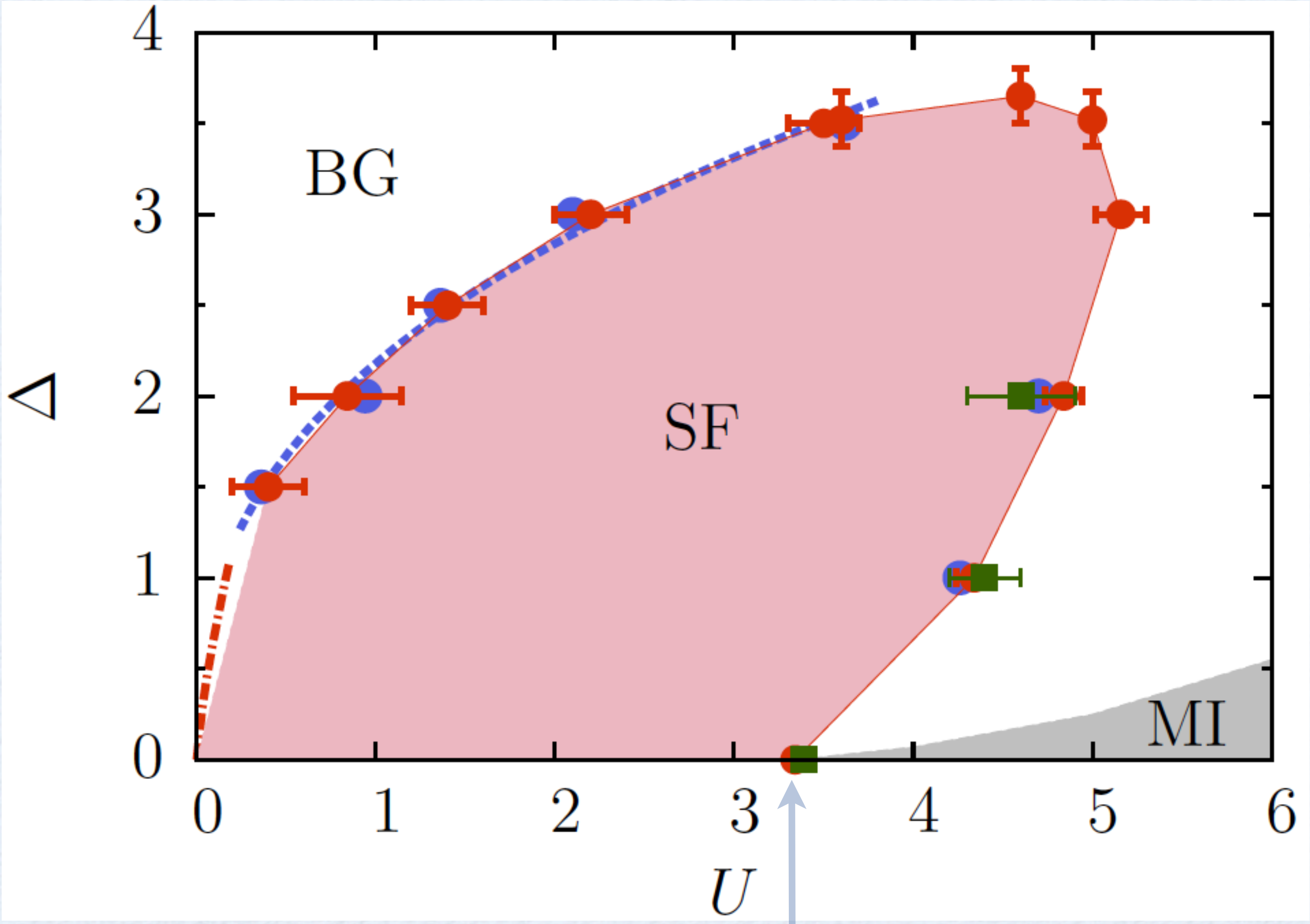
$U_c \simeq 3.3\,t$

The phase diagram

$U = 0$

Anderson Localization

$\Delta \neq 0$



Bose Glass

- compressible
- not coherent !

Fisher, et al. ('89)
Giamarchi, Schulz,
Altman, Zwerger,
Prokof'ev, Svistunov,
Pollet, & many others

$\Delta = 0$ → Superfluid | Mott Insulator

BKT transition

$U_c \simeq 3.3\, t$

M. Gerster, MR, et al. NJP 18 015015 (2016)

N.V. Prokof'ev, B.V. Svistunov, PRL 80, 4355 (1998)

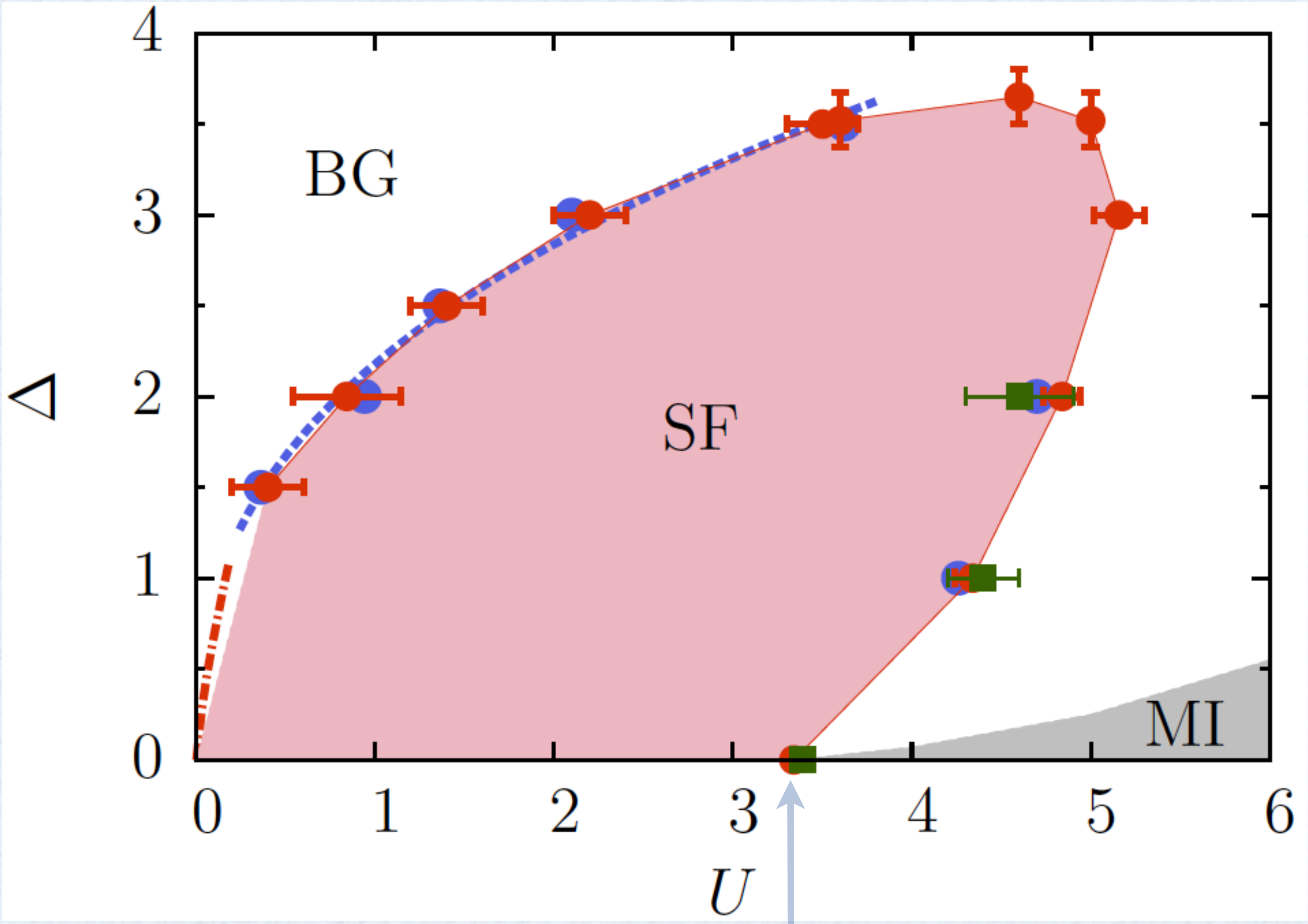
The phase diagram
cold atoms experiments !?

$U = 0$

Anderson Localization

$\Delta \neq 0$

J. Billy, et al. , Nature 453, 891 (2008); G. Roati, et al. , Nature 453, 895 (2008).



Bose Glass

- compressible
- not coherent !

Fisher, et al. ('89) Giamarchi, Schulz, Altman, Zwerger, Prokof'ev, Svistunov, Pollet, & many others

C. D'Errico, et al., PRL 113, 095301 (2014)

$\Delta = 0$ ⇒ Superfluid   Mott Insulator

M. Greiner, et al. , Nature 415 , 39 (2002).

BKT transition

$U_c \simeq 3.3\, t$

M. Gerster, MR, et al. NJP 18 015015 (2016)

N.V. Prokof'ev, B.V. Svistunov, PRL 80, 4355 (1998)

Indicator 1: superfluid stiffness

$$\rho_s = \frac{N_s}{8\pi^2}\frac{\partial^2 E_0}{\partial \phi^2}\bigg|_{\phi=0}$$

universal jump
(~ classical 2D XY)

$$\rho_s^c = \left(2/\pi^2\right) U_c$$

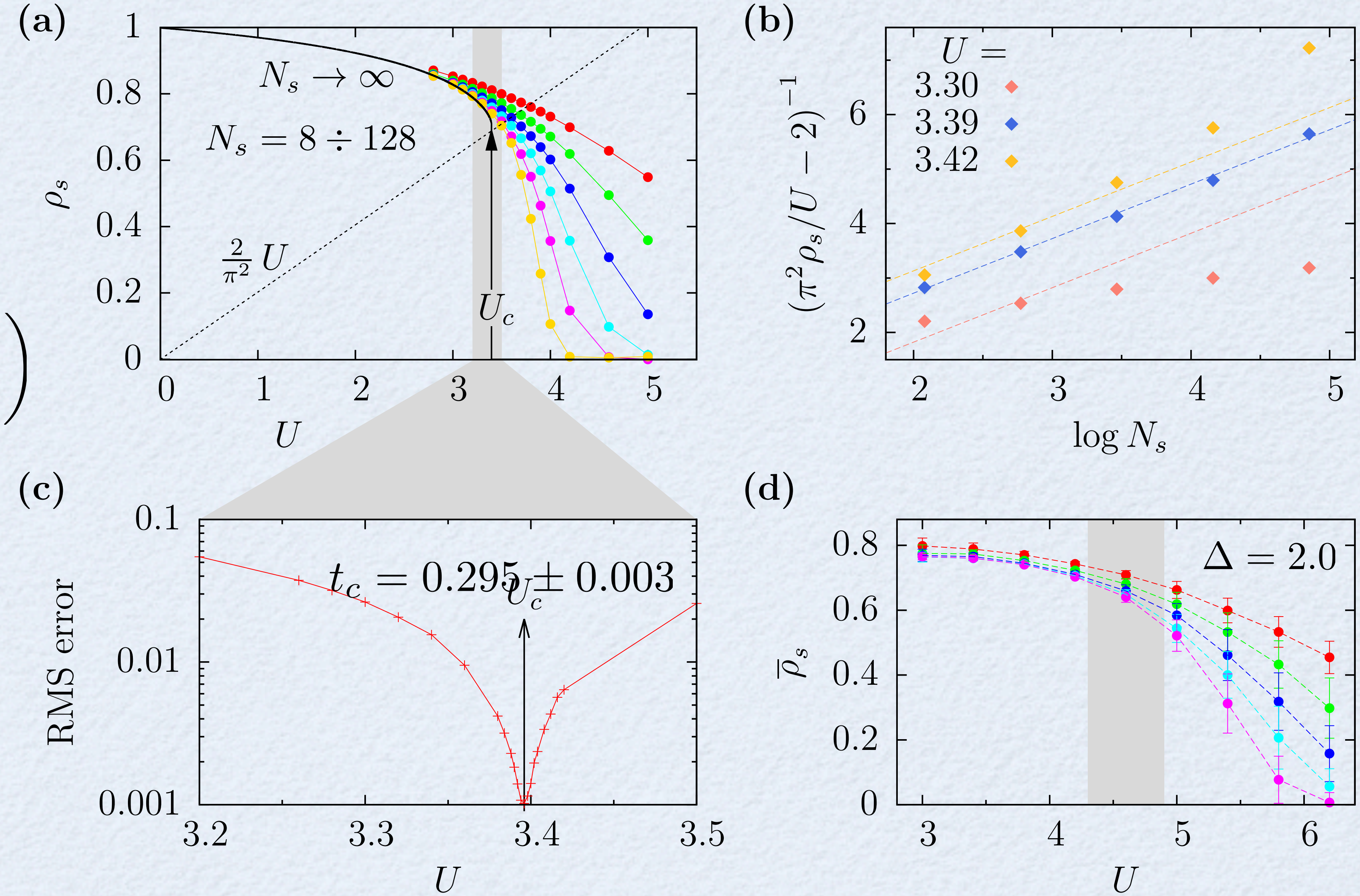
Nelson & Kosterlitz, PRL.39, 1201 (1977)

M. Fisher, et al., PRB 40, 546 (1989)

Finite size scaling

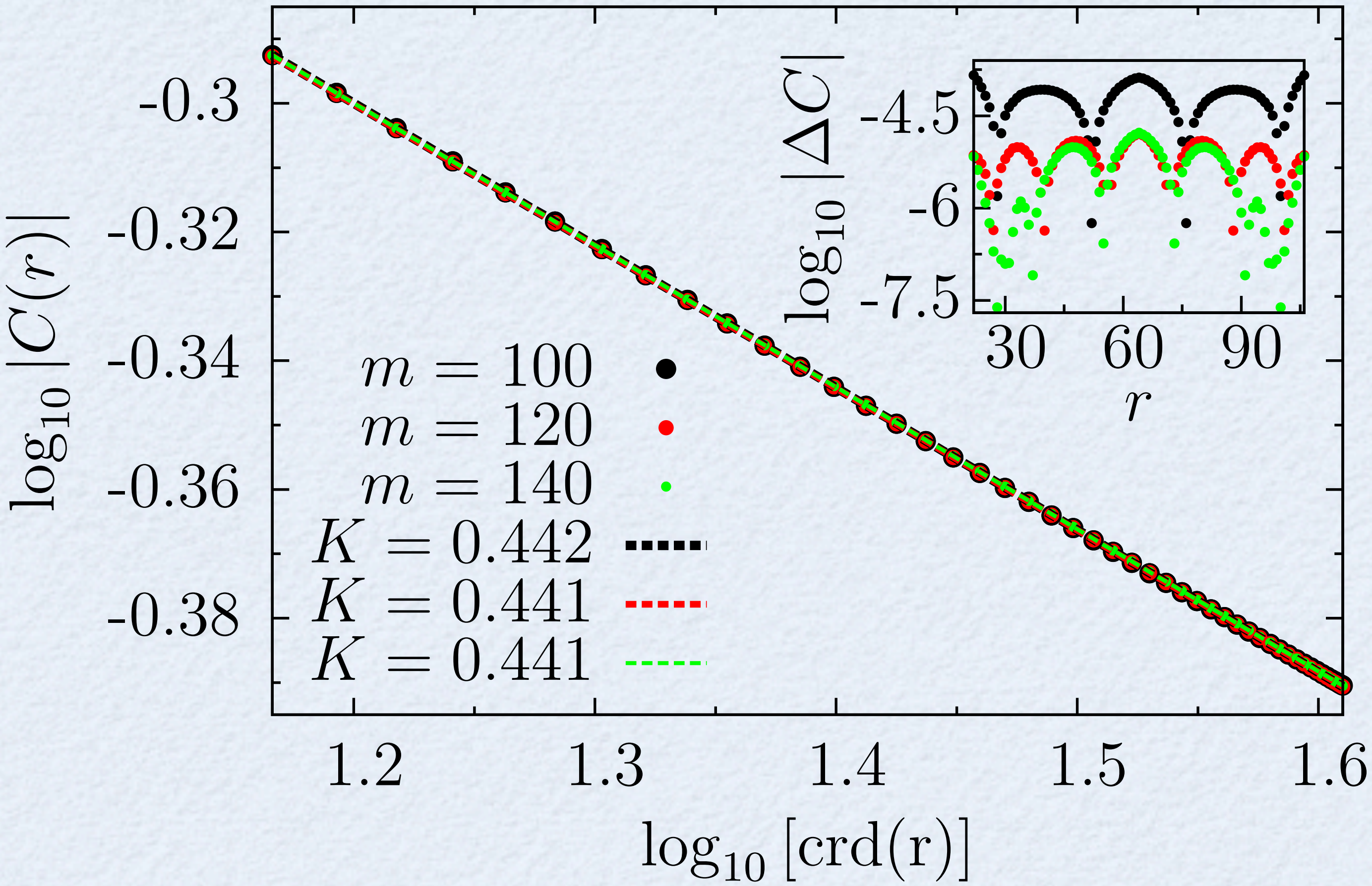
$$\rho_s^c(N) = \frac{2U}{\pi^2}\left(1 + \frac{1}{2}\frac{1}{\log N + b}\right)$$

H. Weber, P. Minnhagen, PRB 37, 5986 (1988).



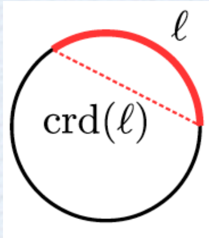
M. Gerster, MR, et al. NJP 18 015015 (2016)

Indicator 2: hopping correlation decay (K)



Luttinger liquid theory

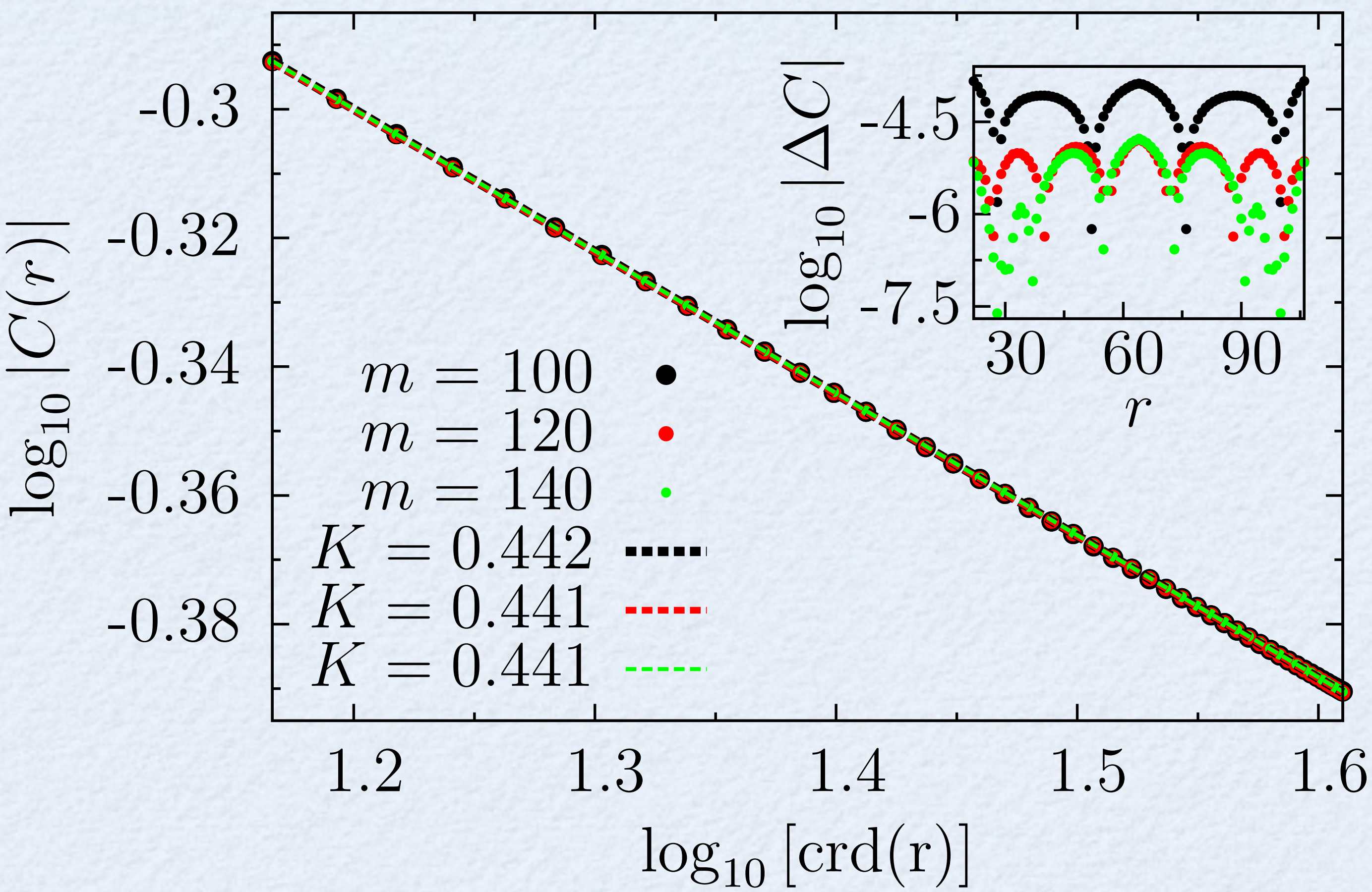
$$C(r) = \langle b_j^\dagger b_{j+r} \rangle \propto \mathrm{crd}(r)^{-K/2}$$



PBC → chord distance

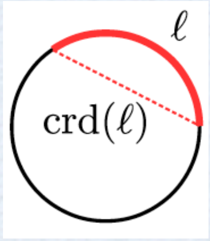
$$\mathrm{crd}(r) = N_s/\pi \, \sin(\pi r / N_s)$$

Indicator 2: hopping correlation decay (K)



Luttinger liquid theory

$$C(r) = \langle b_j^\dagger b_{j+r} \rangle \propto \mathrm{crd}(r)^{-K/2}$$



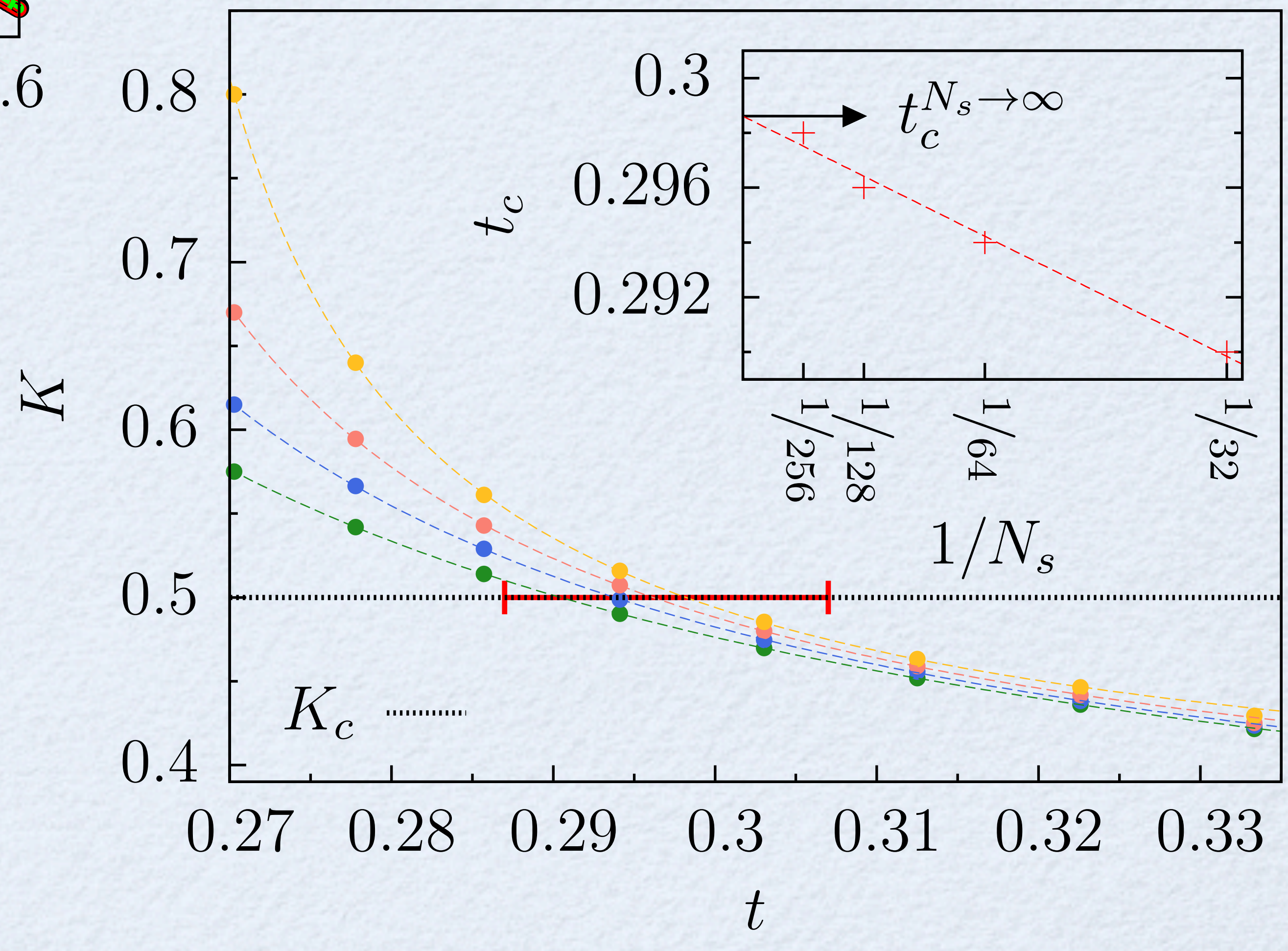
PBC → chord distance

$$\mathrm{crd}(r) = N_s/\pi \, \sin(\pi r/N_s)$$

"Clean" Mott - Superfluid trans.

$$K_c = 1/2$$

$$t_c = 0.299 \pm 0.002$$



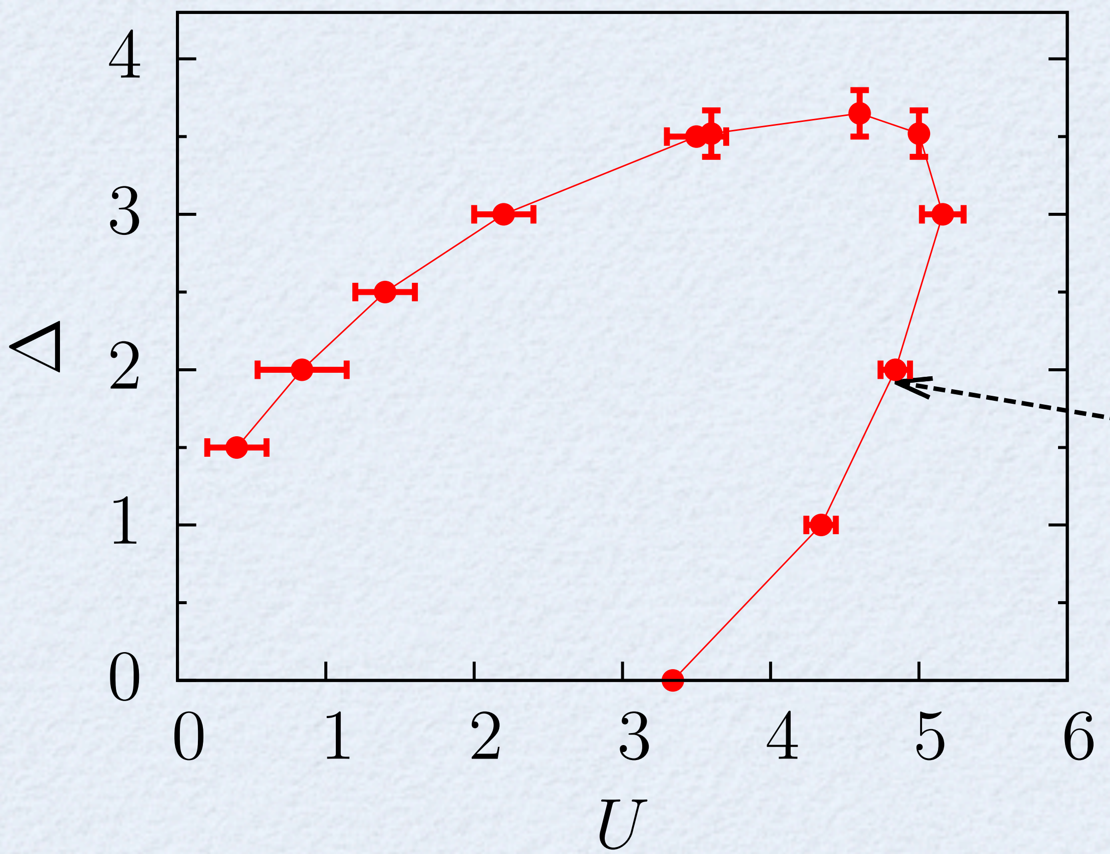
M. Gerster, MR, et al. NJP 18 015015 (2016)

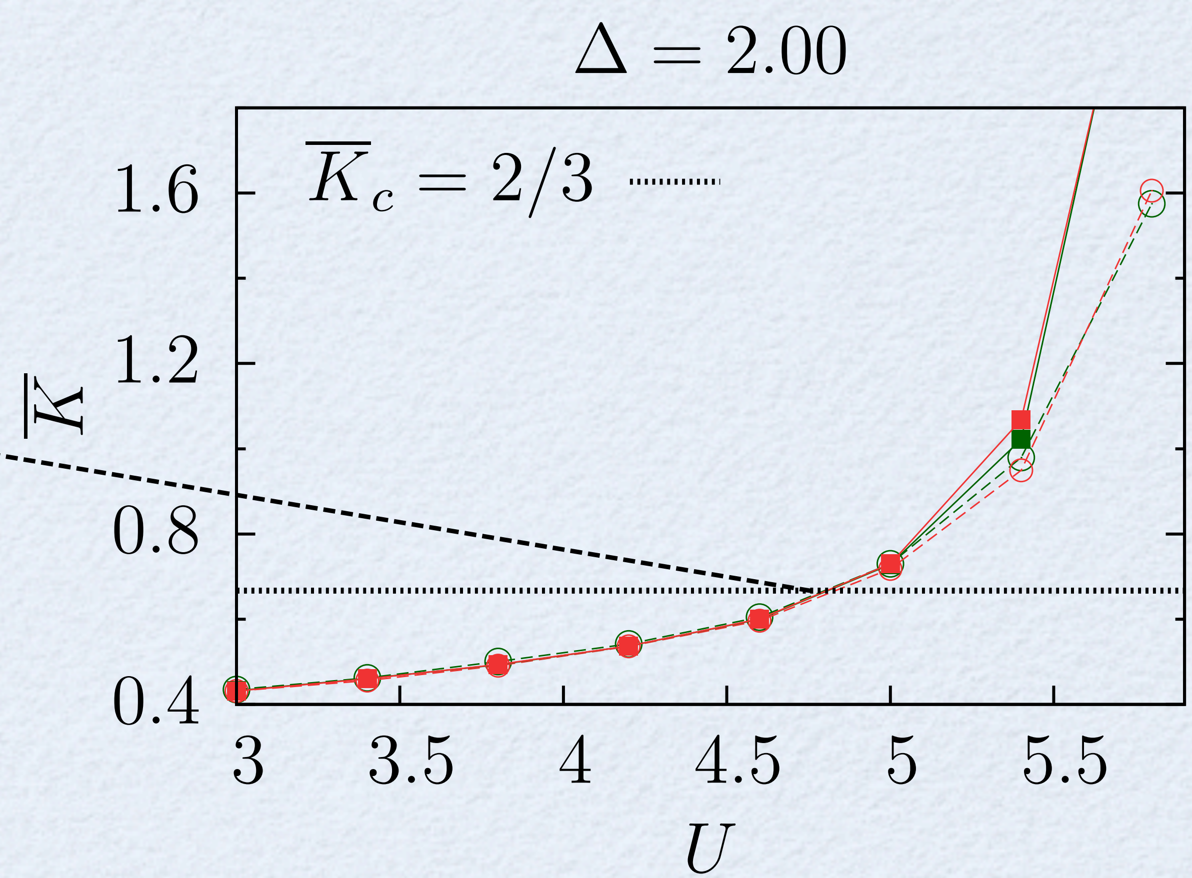
Indicator 2: hopping correlation decay (K)

Disorder-averages $\overline{A}_s \equiv \frac{1}{n_r}\sum_{\gamma=1}^{n_r}\langle A\rangle_\gamma$ over "reasonable" $n_r = 10^2 \div 10^3$

Giamarchi-Schulz SF criterion $\overline{K}_c = 2/3$

T. Giamarchi, H.J. Schulz, PRB 37, 325 (1988).





M. Gerster, MR, et al. NJP 18 015015 (2016)

Indicator 2: hopping correlation decay (K)

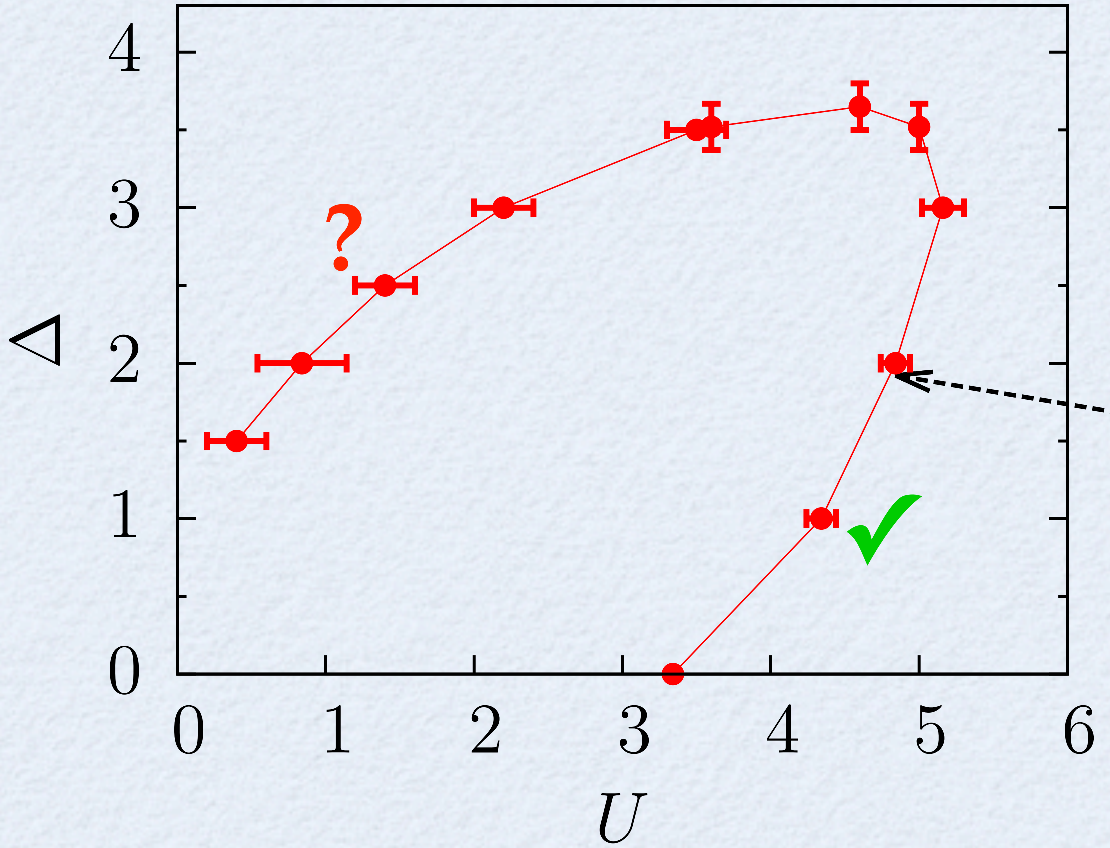
Disorder-averages $\overline{A}_s \equiv \frac{1}{n_r} \sum_{\gamma=1}^{n_r} \langle A \rangle_\gamma$ over "reasonable" $n_r = 10^2 \div 10^3$

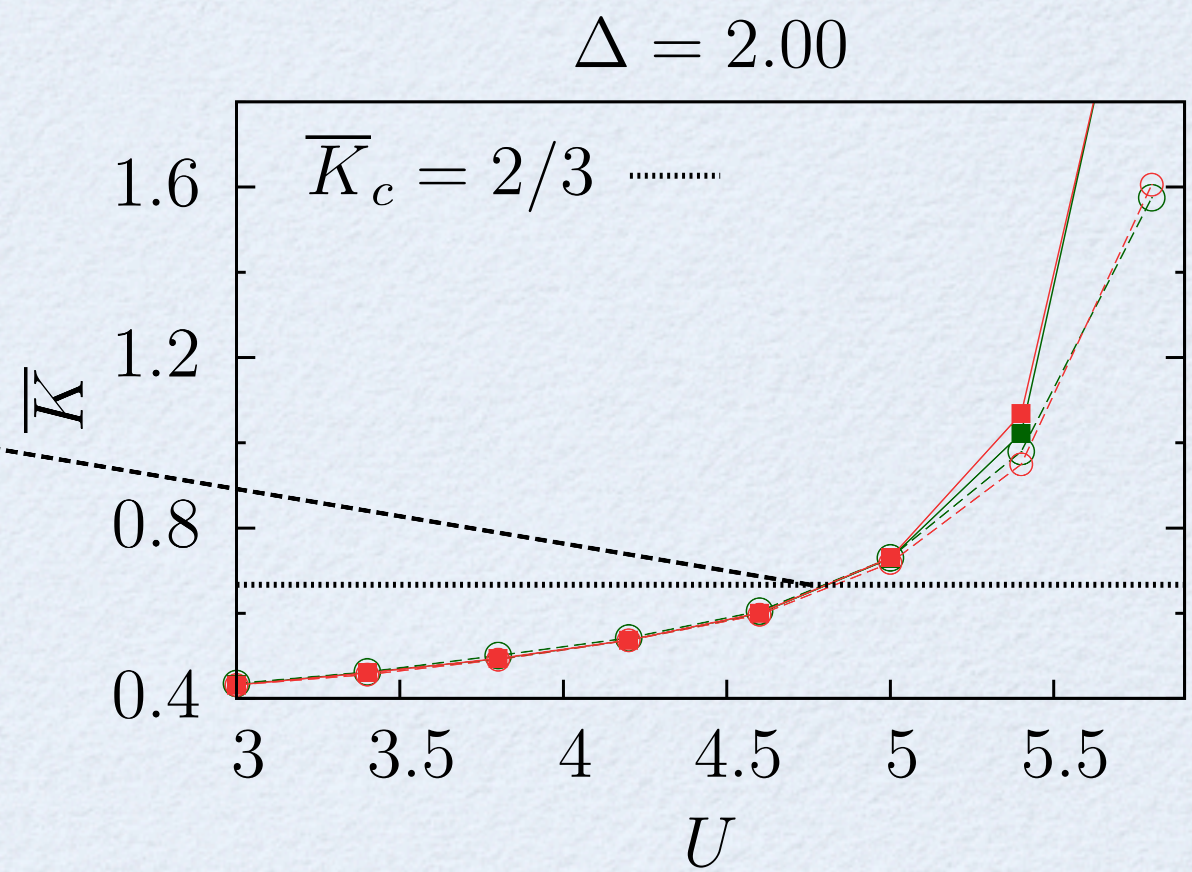
Giamarchi-Schulz SF criterion $\overline{K}_c = 2/3$

T. Giamarchi, H.J. Schulz, PRB 37, 325 (1988).

- ✓ appropriate $\Delta/U \ll 1$
- ? (highly) debated $\Delta/U \gg 1$

L. Pollet, Comptes Rendus Physique 14, 712 (2013)

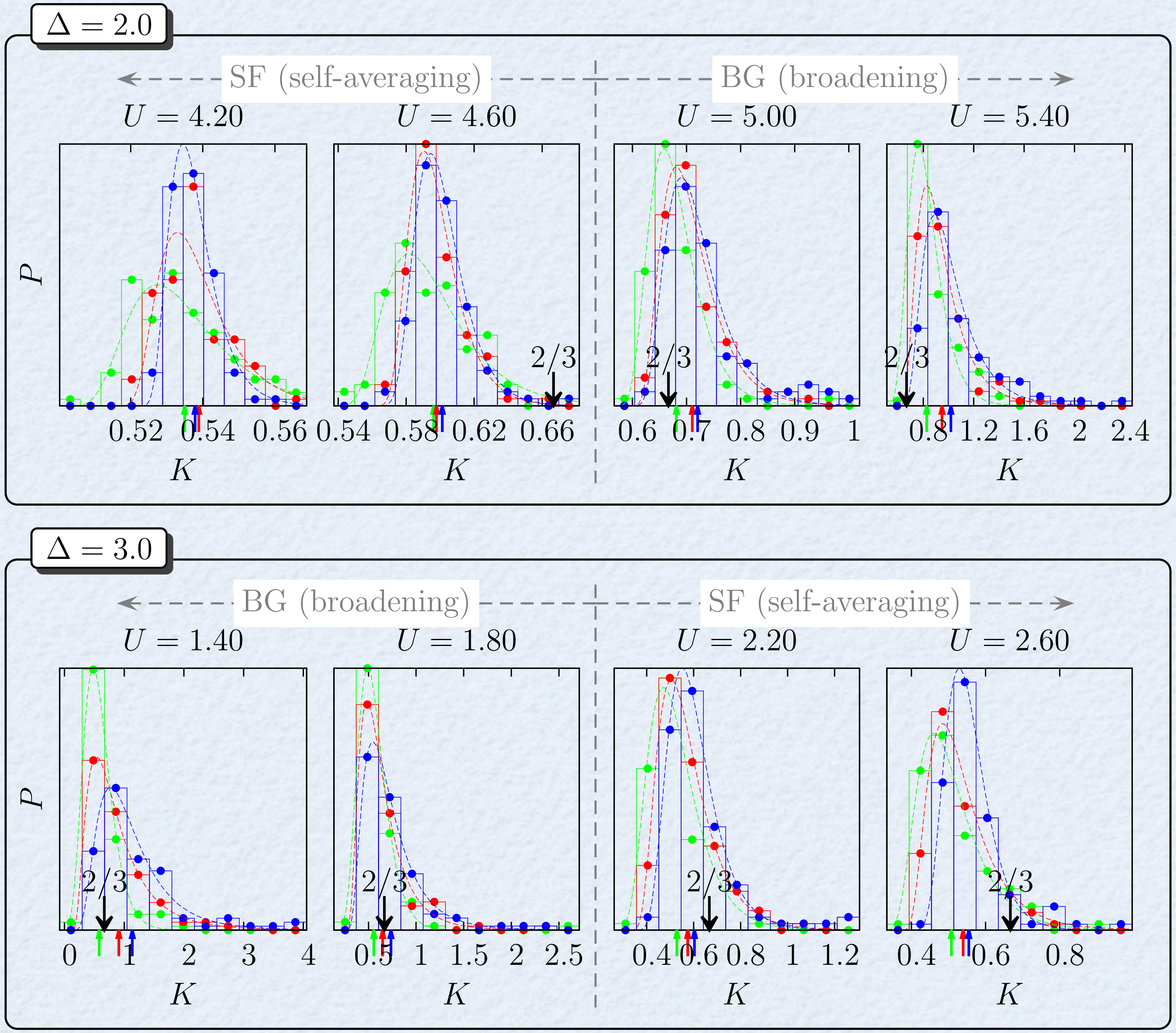




M. Gerster, MR, et al. NJP 18 015015 (2016)

Indicator 3: K-distribution properties

New universality class: XY-scratched !? → Lobe shrinking due to rare events → Broadening vs. averaging with system size



Pollet, Prokof'ev, Svistunov, PRB 87, 144203 (2013)

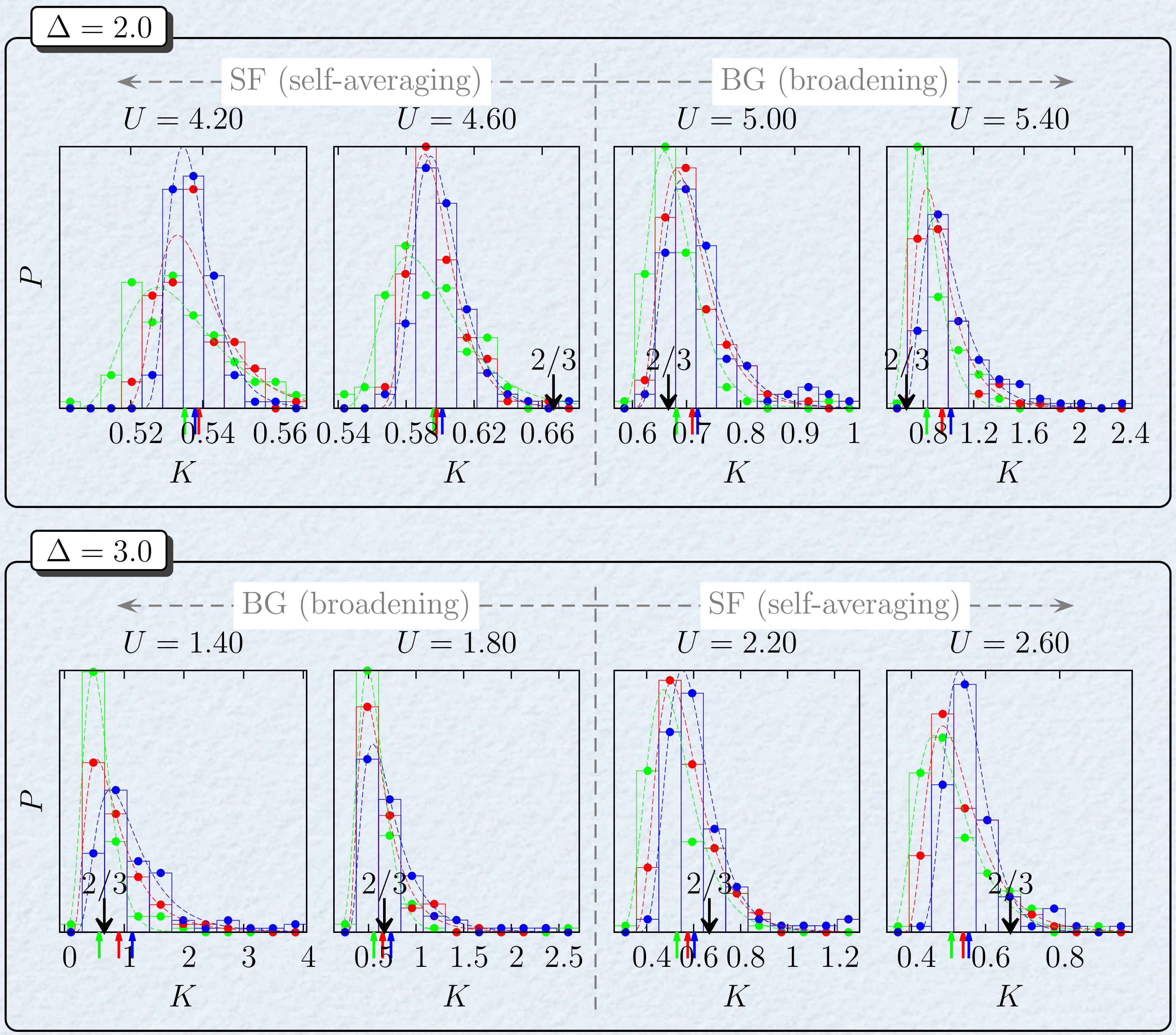
$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} \frac{1}{x} \exp\left(-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right)$$

$$x = K - K_{\min}$$

M. Gerster, MR, et al. NJP 18 015015 (2016)

Indicator 3: K-distribution properties

New universality class: XY-scratched !? → Lobe shrinking due to rare events → Broadening vs. averaging with system size



M. Gerster, MR, et al. NJP 18 015015 (2016)

Pollet, Prokof'ev, Svistunov, PRB 87, 144203 (2013)

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}}\frac{1}{x}\exp\left(-\frac{1}{2}\left(\frac{\log x - \mu}{\sigma}\right)^2\right)$$

$$x = K - K_{\min}$$

For "reasonable" sampling

$\sigma(N_s) \simeq \sigma(N_s/2)$ $\qquad$ $\overline{K} \simeq \overline{K}_c = 2/3$

↓

$n_r \gg N_s$

$n_r \simeq 10^5$

to see deviations !

↓

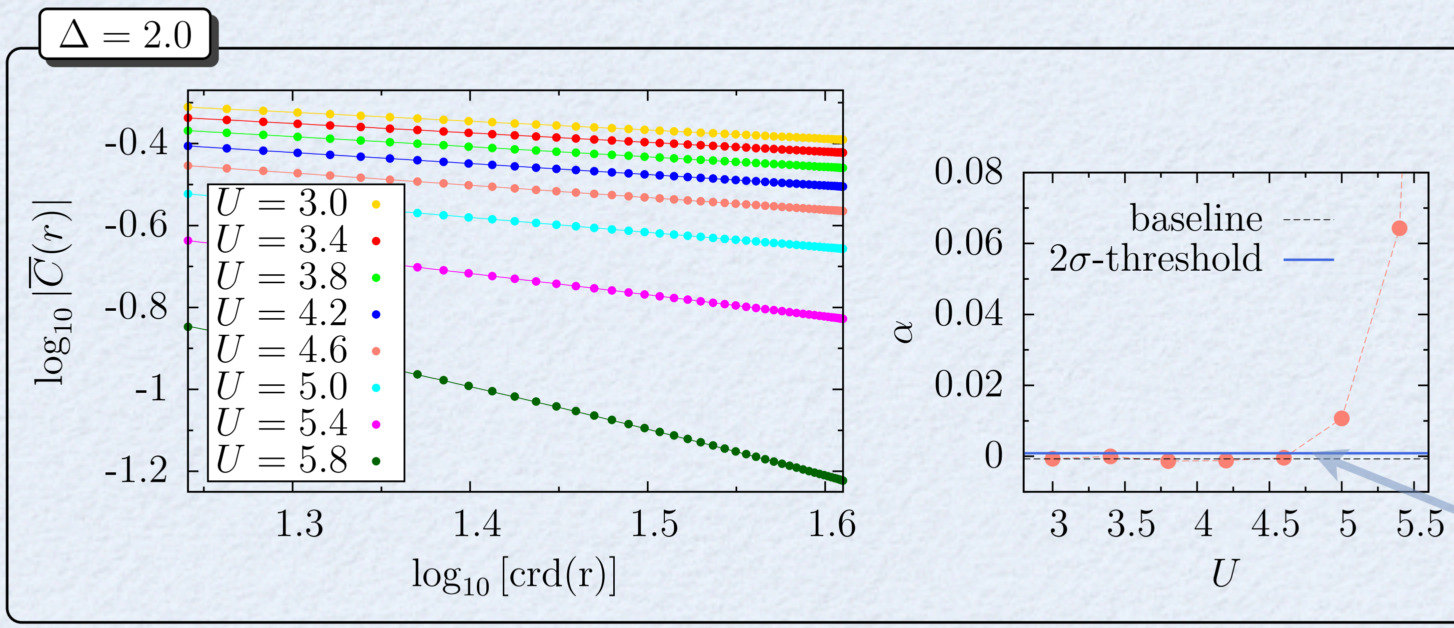
Out of experimental reach !?

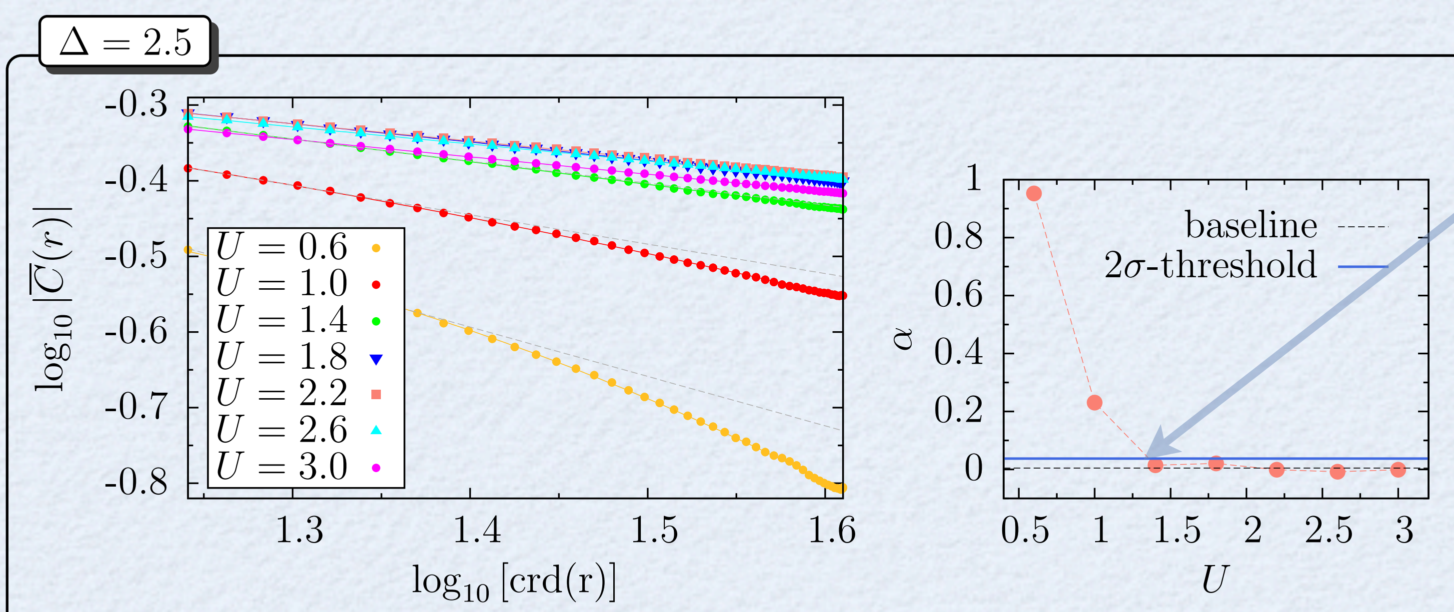
Indicator 4: averaged hopping correlations

No assumption on transition class $\log\left[\,\overline{C}(r)\right] = -\alpha\,\tilde{r}^2 - \frac{K}{2}\,\tilde{r} + \text{const.}$

$\tilde{r} = \text{crd}(r)$

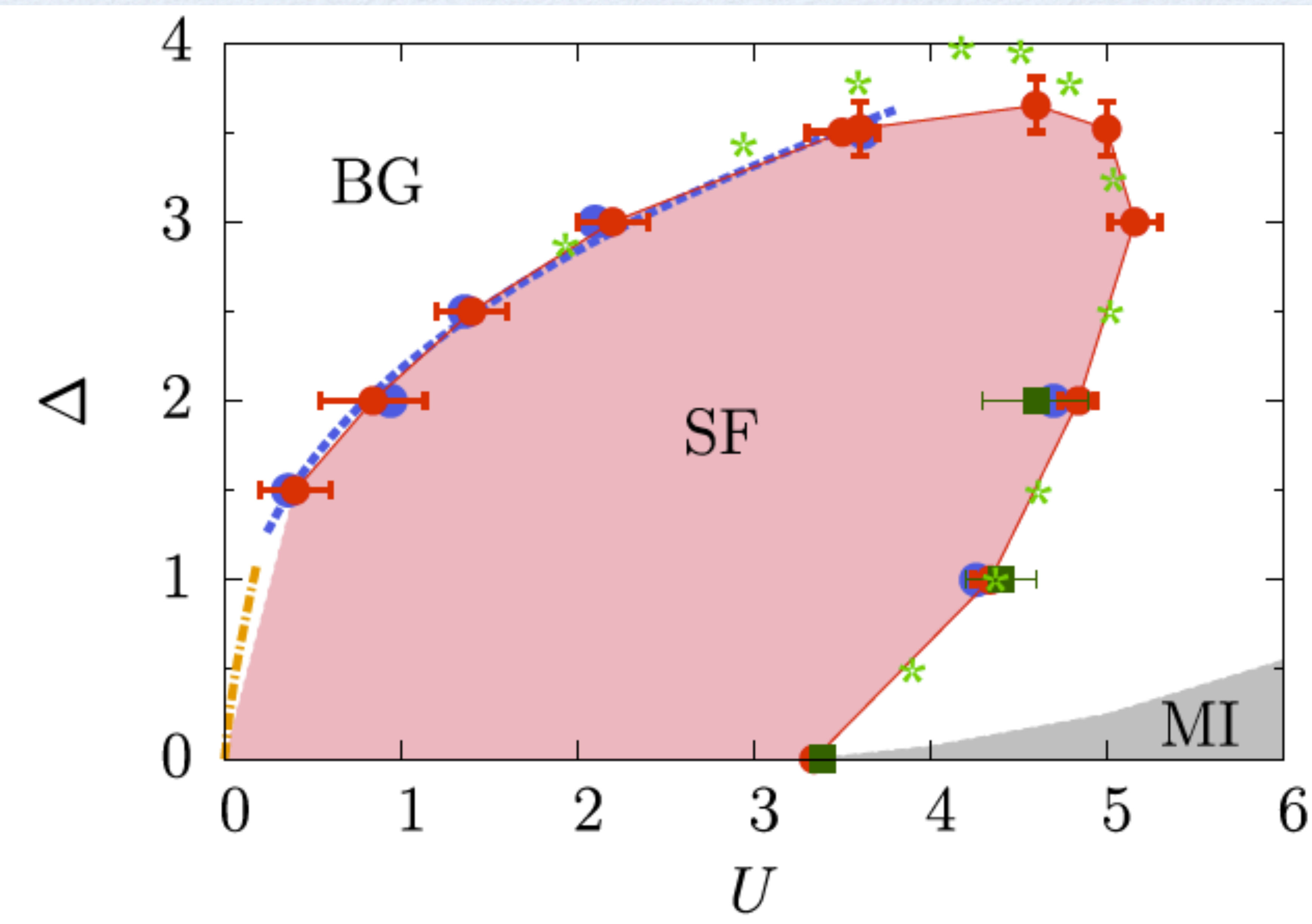
$\alpha = 0$ Superfluid $\alpha \neq 0$ Insulator (BG)





guessed U_c

The phase diagram



jump in $\overline{\rho}_s$

$\overline{K} = 2/3$

$\overline{C}(r)$ crossover

fit

$\Delta_c \sim U^{3/4}$

Quantum MonteCarlo

N.V. Prokof'ev, B.V. Svistunov, PRL 80, 4355 (1998)

"scratched XY" conjecture

$\Delta_c \propto U^{3/4}$

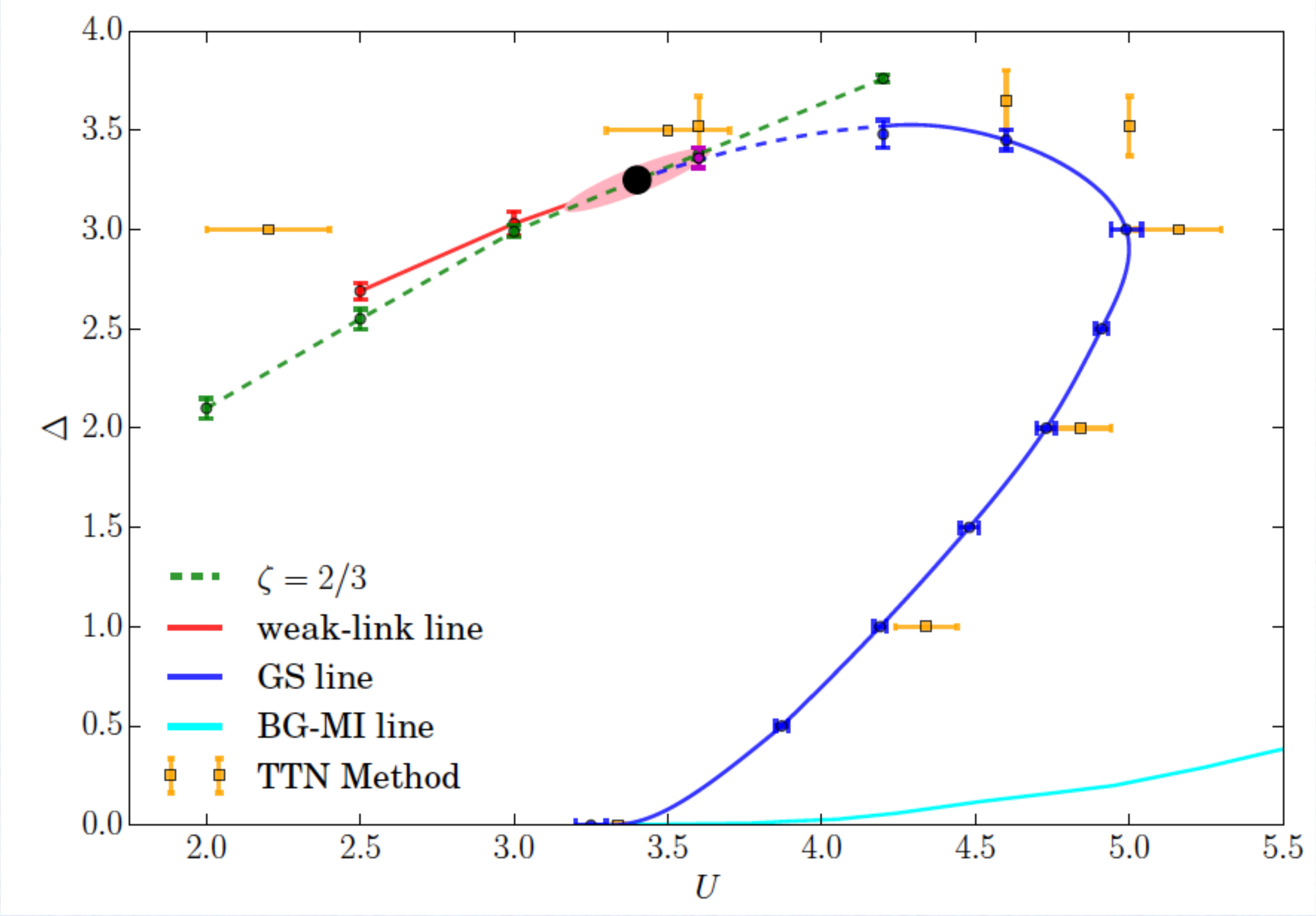
L. Pollet, Comptes Rendus Physique 14, 712 (2013)

M. Gerster, MR, et al. NJP 18 015015 (2016)

The phase diagram

Most recent QMC results - averages over 50.000 realizations

Z. Yao, et al., arXiv:1601.06185



M. Gerster, MR, et al. NJP 18 015015 (2016)

OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

OUTLINE

- Motivation: why periodic boundaries?
- Tree Tensor Networks & adaptive gauge picture
- Some benchmark data
- Disordered Bose-Hubbard model
- Summary & Future perspectives

Persistent currents in Dirac rings

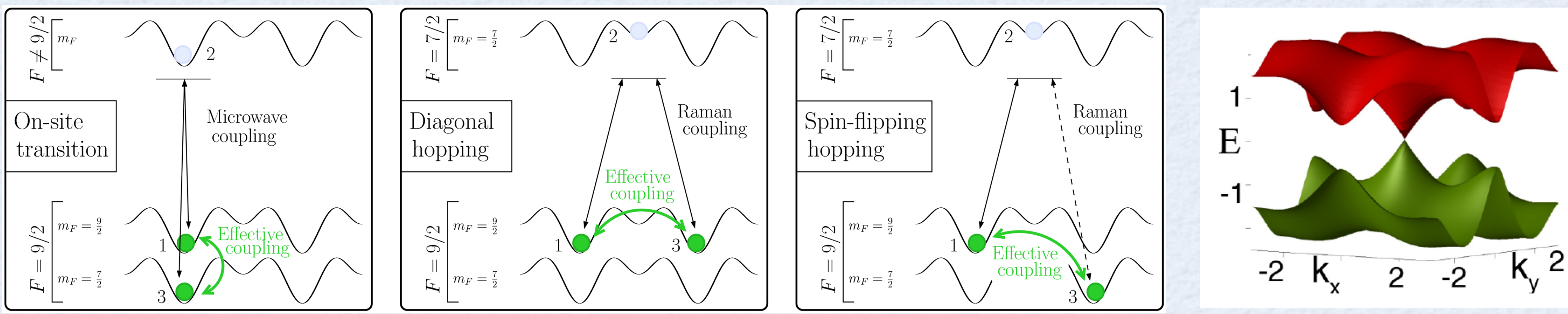
orbital magnetic susceptibility of 1D cold **Dirac fermions**

⇨ orbital paramagnets of pure many-body origin !?

J. Jünemann

M. Bischoff

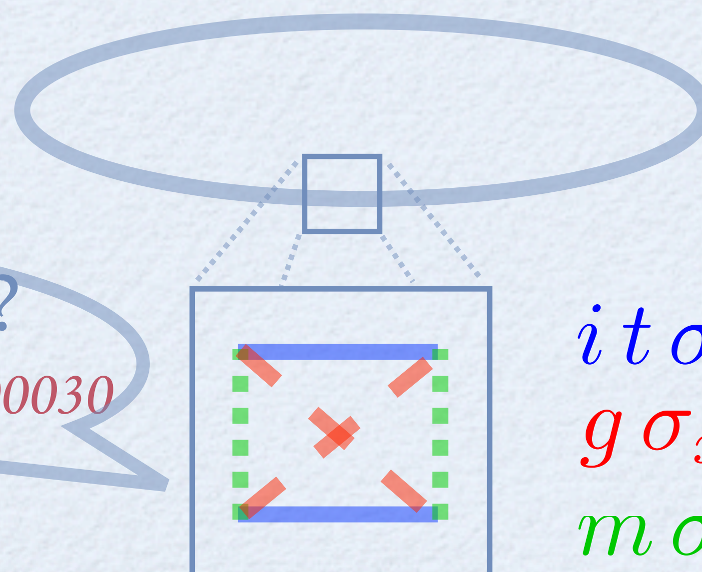
tunable gauge potentials with cold atoms



M. Rizzi, POS (SISSA) 193 (QCD-TNT-III), 036 (2013)

all that on rings!?

M. Łącki, et al. arXiv:1507.00030



$i\,t\,\sigma_z$

$g\,\sigma_x$

$m\,\sigma_x$

Creutz ladder at m=g=t

⇩

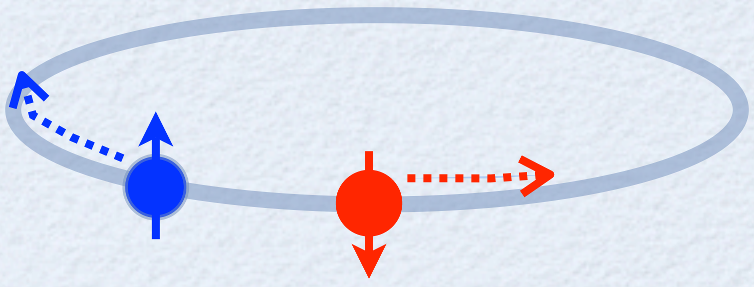
isolated cone at k=0

⇩

consider interactions

Future: time-dependent problems

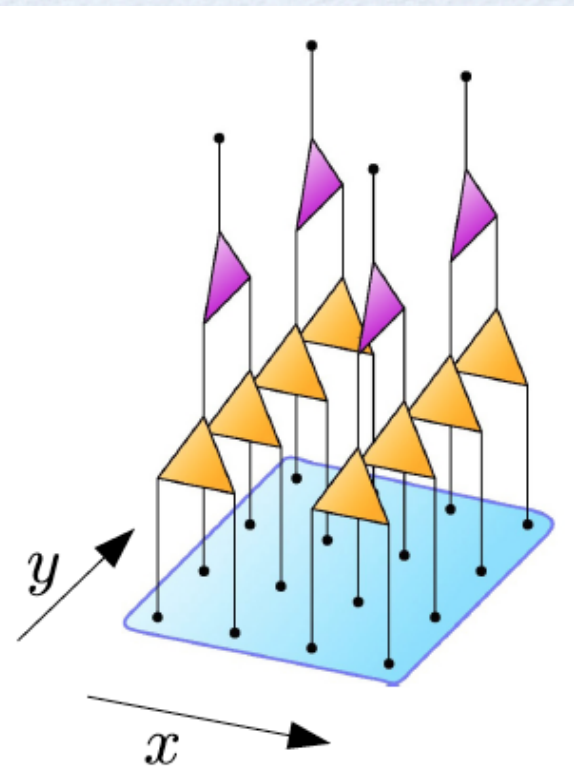
- implement time-dependence in TTN's (e.g., TEBD or TDVP like)
- study generation and decay of orbital and spin currents: spin-drag, dynamical instabilities, pairing effects, ...

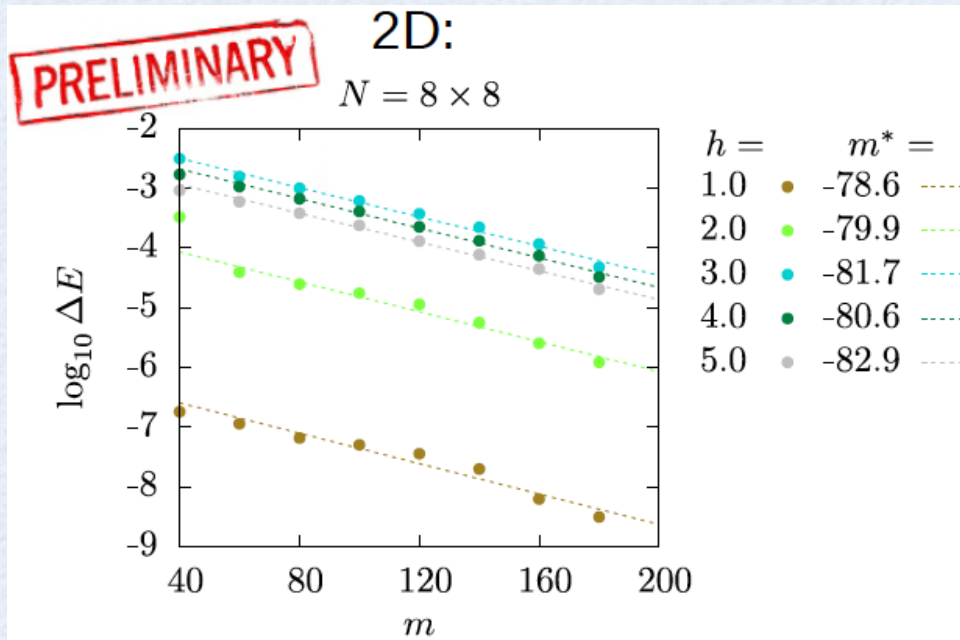


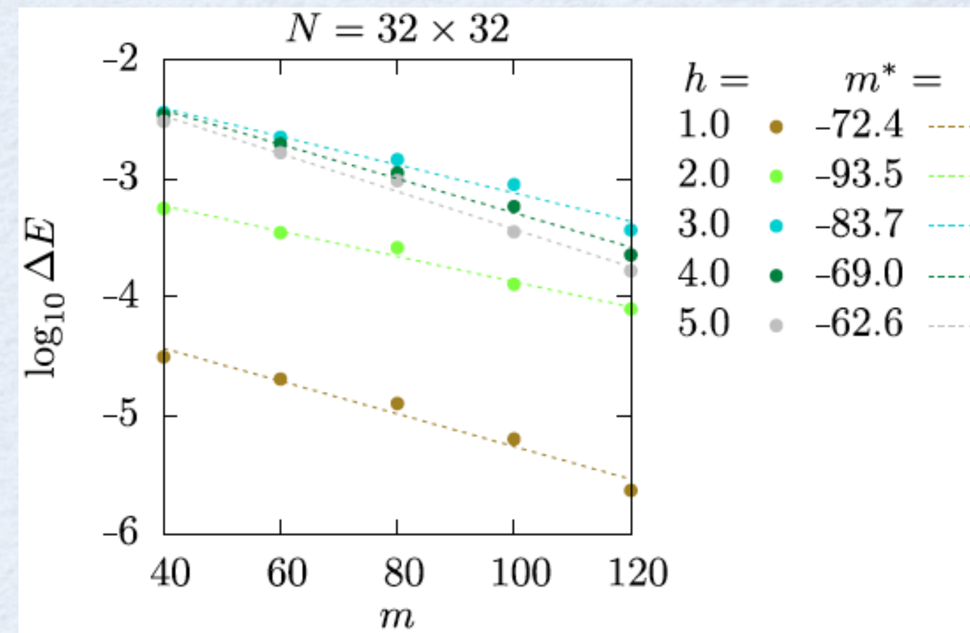
- study quenches and relaxation with disorder (MBL!?)

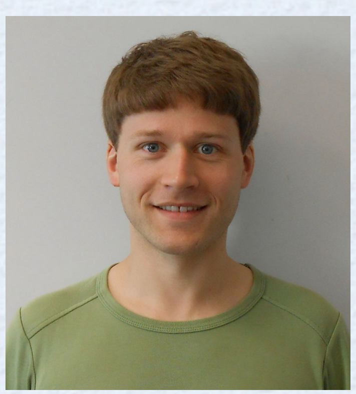
Future: what about 2D ?

- it should **not** work, but what if brute force helps ?
  on-going tests on Ising in transverse field ($h_c \sim 3.04$)

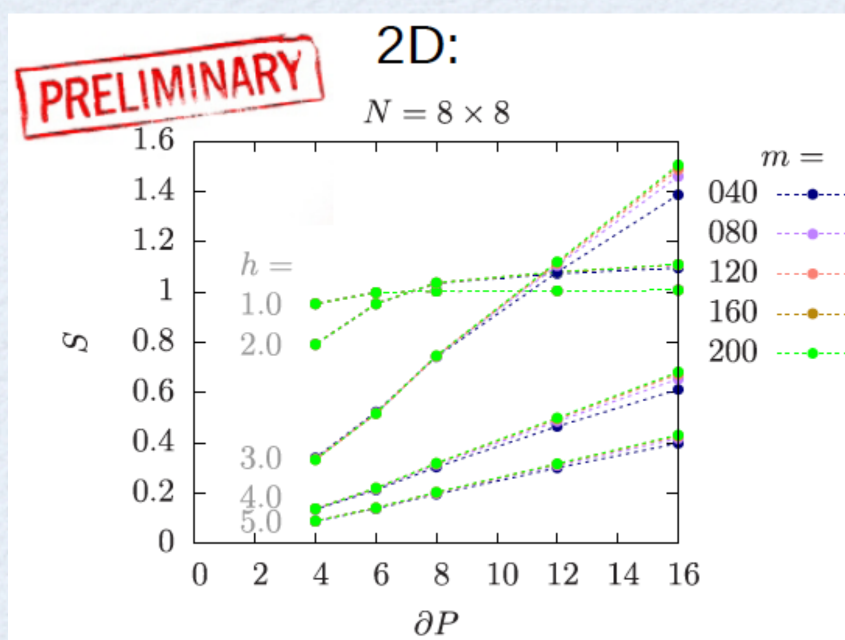


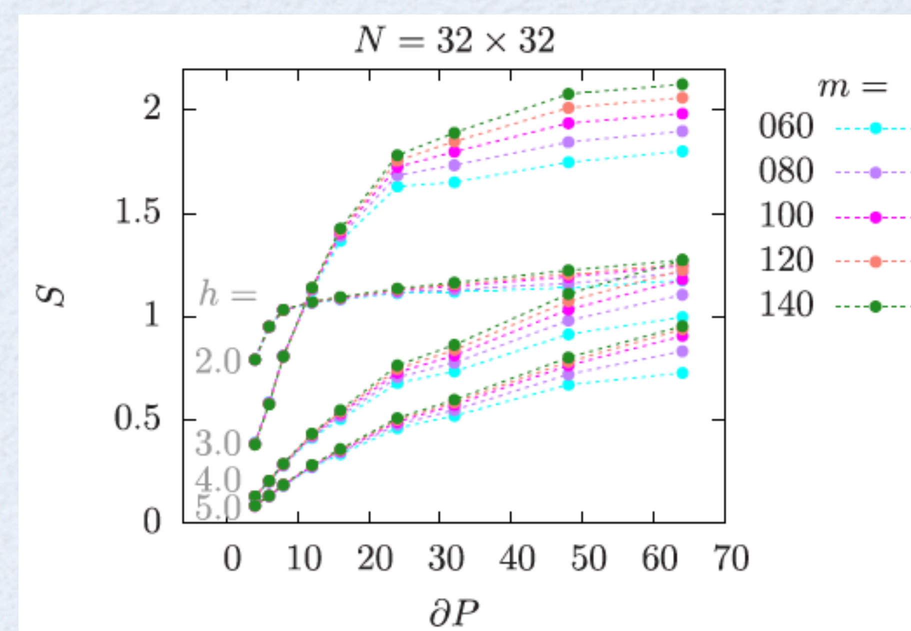






M. Gerster (Ulm)

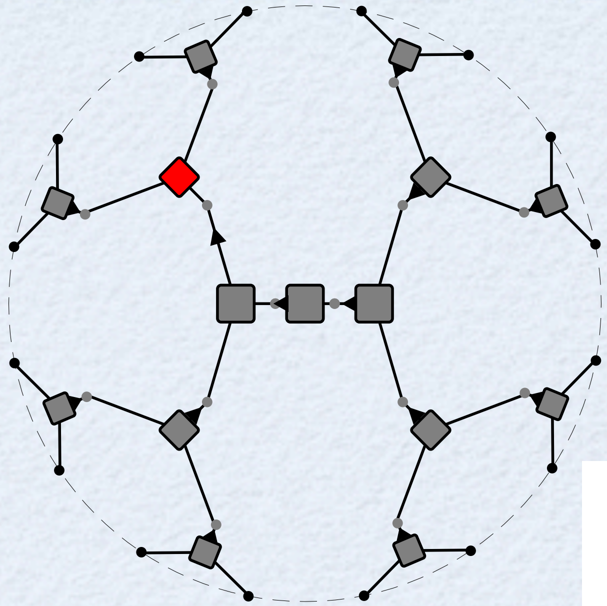




Summary

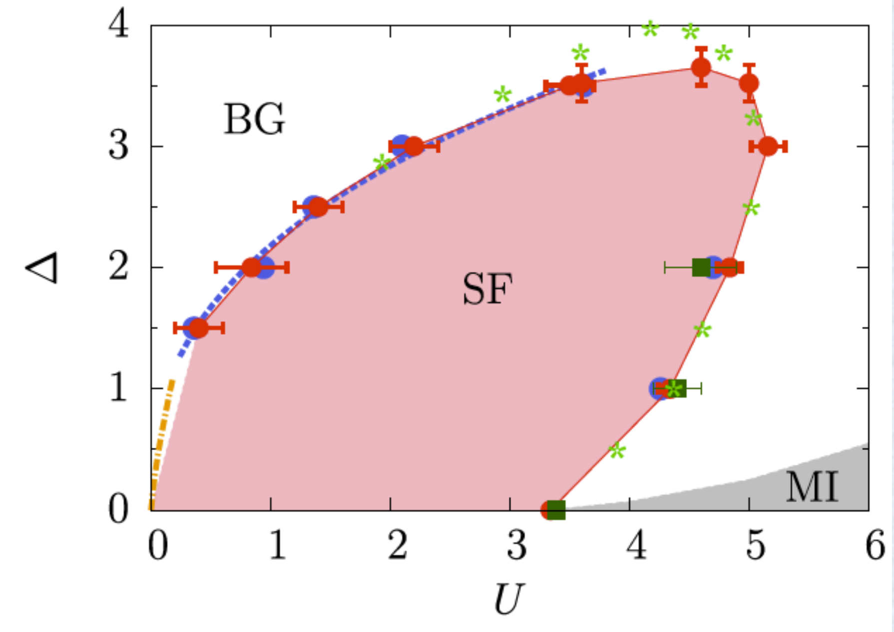
- Adaptive gauge strategy for Tree Tensor Networks: ***pbc*** enhanced performance !

  PRB 90, 125154 (2015)



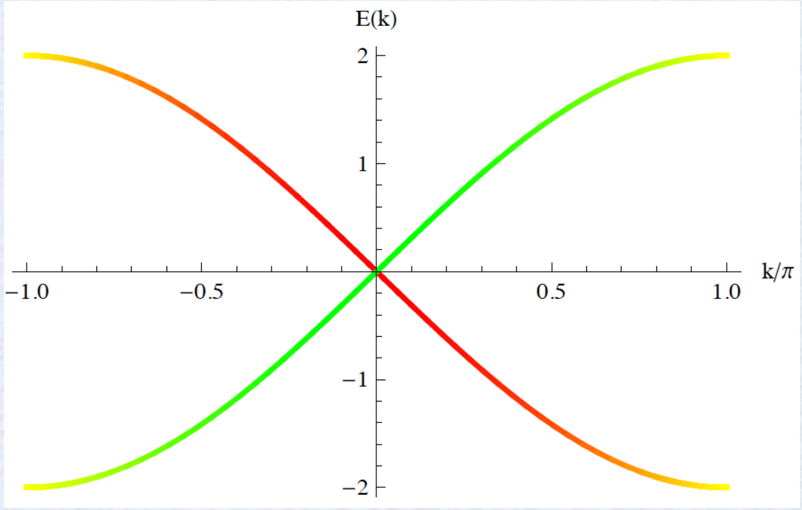
- 1D disordered Bose-Hubbard model: a TTN study on Bose-Glass transition

  NJP 18 015015 (2016)



- Future studies on persistent currents in Dirac rings & other systems

  POSTER by J. Jünemann (arriving tomorrow)



Thanks to ...

F. Hekking (Grenoble)
A. Minguzzi (Grenoble)
D. Rossini (SNS-Pisa)
L. Amico (Catania)
L.C. Kwek (Singapore)

M. Gerster (Ulm)
P. Silvi (Ulm)
S. Montangero (Ulm)
T. Calarco (Ulm)
R. Fazio (ICTP)

A. Sartori (Trento)
A. Recati (TUM & Trento)
M. Polini (IIT-Genova)
D. Draxler (Wien)

P. van Dongen, R. Orus, and all **KOMET337**

My group: J. Jünemann, A. Haller, M. Bishoff

$$$:

... and all of you for your attention !