Entanglement evolution after inhomogeneous quantum quenches, and the artic circle

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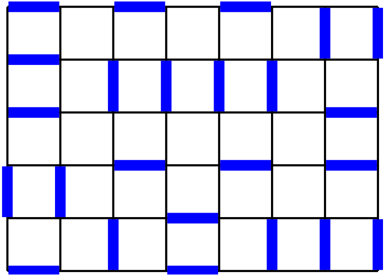
Workshop Benasque, 2016

JMS [J. Stat. Mech (2014) P05010]
J. Viti, JMS, J. Dubail and M. Haque [arXiv:1507.08132]
N. Allegra, J. Dubail, JMS and J. Viti [arXiv:1512.02872]
+ P. Calabrese.

Outline

1. Arctic circle in Statistical mechanics
   - Dimers
   - Six vertex
2. Filling fraction quantum quenches
   - Setup
   - Semiclassics
3. Arctic quench
   - Arctic circle
   - Field theory interpretation
   - Entanglement evolution

Fun with dimers



- dimers with hardcore constraint.
- Exactly solvable: free fermions.
- $Z = \mathrm{Pf}\,(\ldots)$ [Kasteleyn, Fisher (1963)]
- Critical $U(1)$ system

Long distance limit: Dirac field or free gaussian compact field

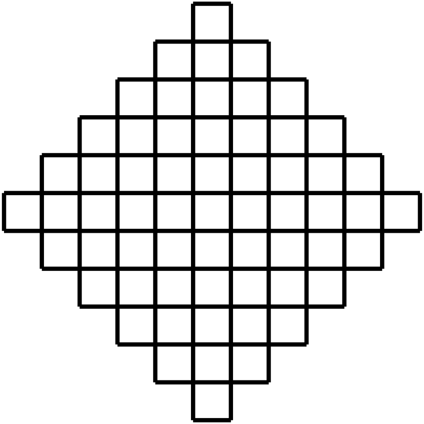
$$S = \frac{g}{4\pi}\int dxdy\,(\nabla\varphi)^2 \qquad , \qquad \varphi = \varphi + 2\pi$$

$$C_{\mathrm{dimer-dimer}}(\mathbf{r},\mathbf{r}') = \left|\mathbf{r}-\mathbf{r}'\right|^{-1/g}$$

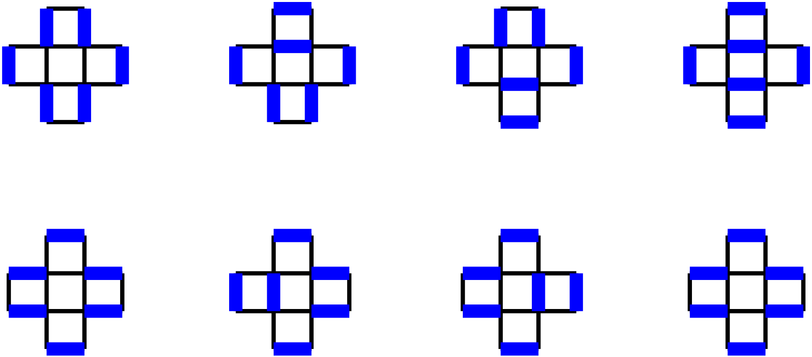
$\longrightarrow$ Well understood. Boundaries are usually not a problem.

Paving an Aztec diamond?

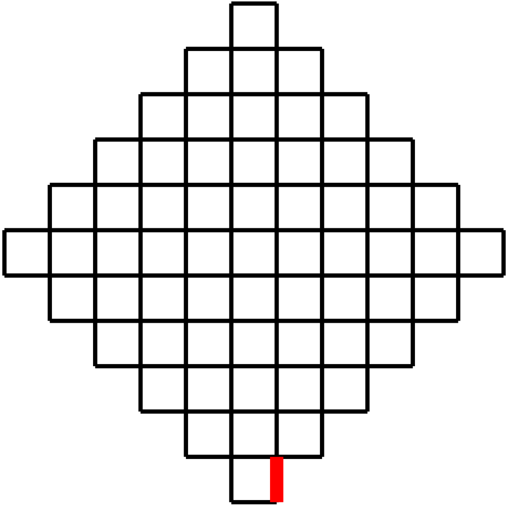
[Jokusch, Propp and Shor, 1995]



What happens? (1)



What happens? (2)



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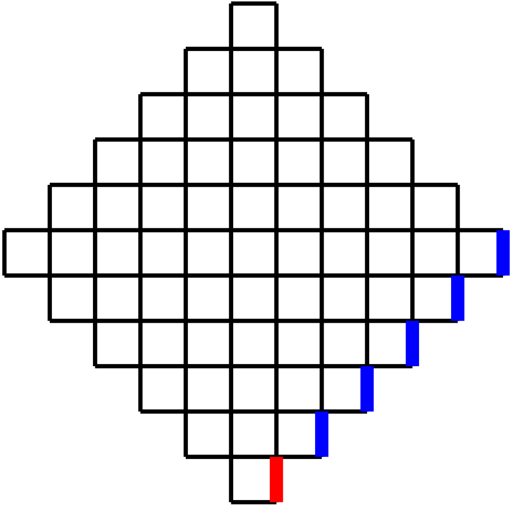

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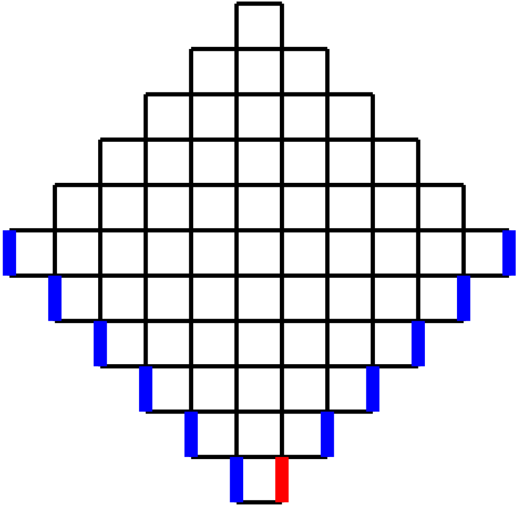
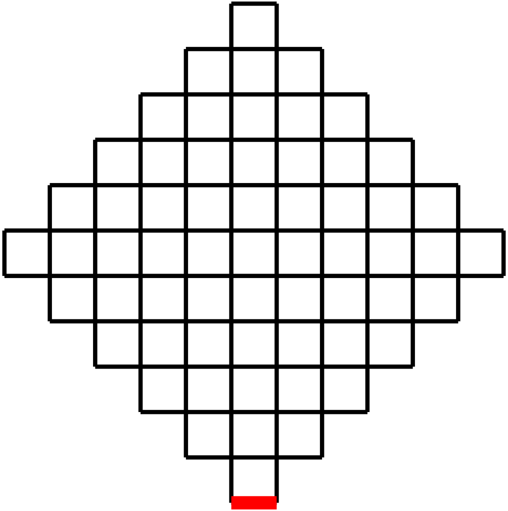


Very unlikely

What happens? (2)



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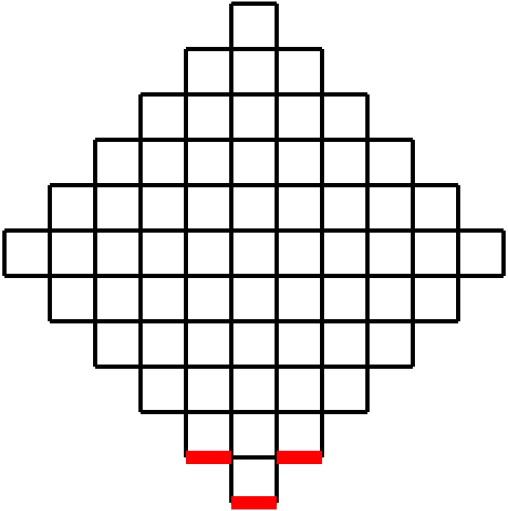
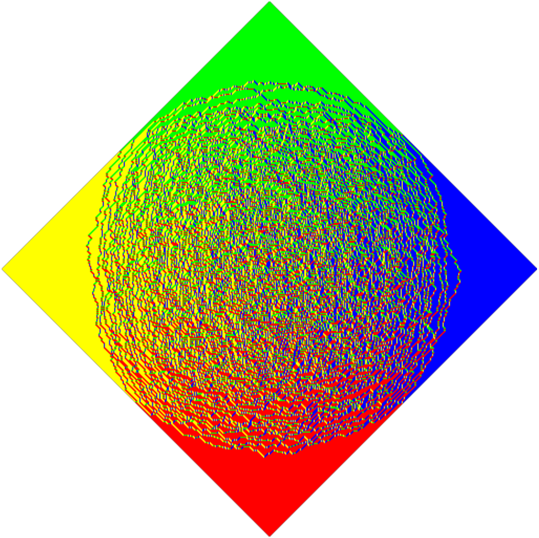
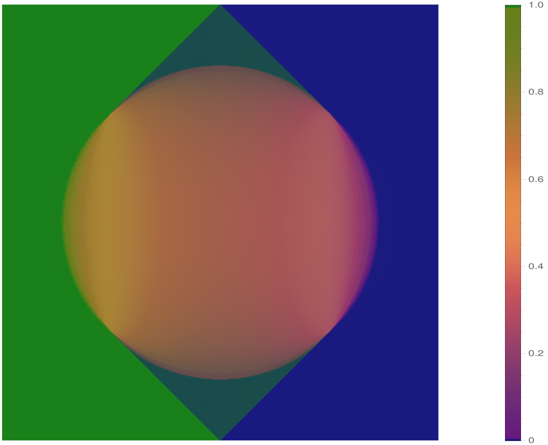


Very likely

The arctic circle



Density profile for (vertical) dimers

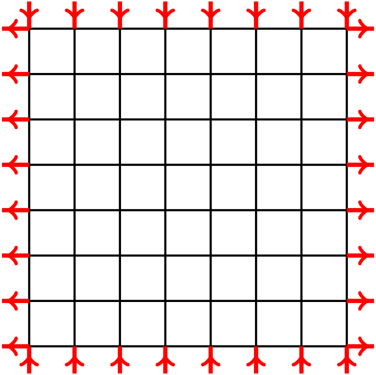


$$\rho(x,y) = \frac{1}{\pi} \arccos\left(\frac{R \pm x}{\sqrt{(R \pm x)^2 - y^2}}\right)$$

[Cohn, Elkies and Propp, Duke. Math. Journ 1996]

Six vertex with domain wall boundary conditions

[Korepin, Izergin, Zinn-Justin, Colomo, Pronko,. . .]

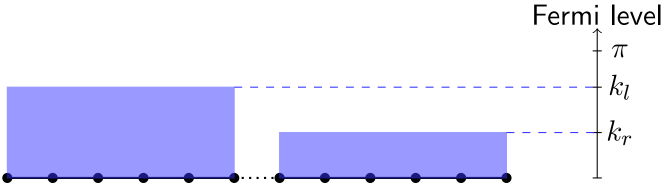


- Conjecture for the arctic curve [Colomo & Pronko, J. Stat 2010] [Colomo, Pronko & Zinn-Justin, J. Stat 2010]
- Density not known away from $\Delta = 0$ (!)

Field theory inside the circle?

Filling fraction quenches

$$H = -\sum_{j\in\mathbb{Z}} \left(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j \right) = -\int \frac{dk}{2\pi} \cos k \; c^\dagger(k) c(k)$$

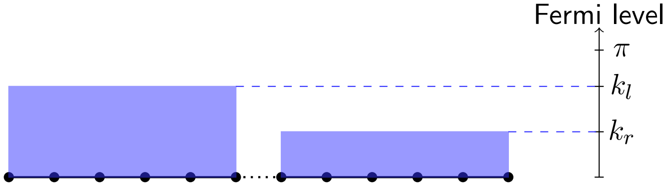


Fermions, with certain Fermi levels (k_l, k_r)

$$|\Psi_0\rangle = |k_l\rangle \otimes |k_r\rangle$$

and let evolve with $H(\frac{k_l+k_r}{2})$ at time $t > 0$

Filling fraction quenches



[Antal et al 1999/2008] [Gobert et al 2005] [Lancaster & Mitra 2010] [Sabetta & Misguich 2010] [Eisler et al 2008] [Calabrese & Cardy 2008] [Stéphan & Dubail 2011] ...

Limiting cases:

- $k_l = \pi$, $k_r = 0$ is the domain wall quench.
- $k_l = k_r$

Free case: stationary phase and semiclassics

$$\langle c_x^\dagger(t) c_{x'}(t)\rangle = \int \frac{dk dk'}{(2\pi)^2} e^{-ikx+it\varepsilon(k)} e^{ik'x'-it\varepsilon(k')} \langle \Psi_0 | c^\dagger(k) c(k') | \Psi_0 \rangle$$

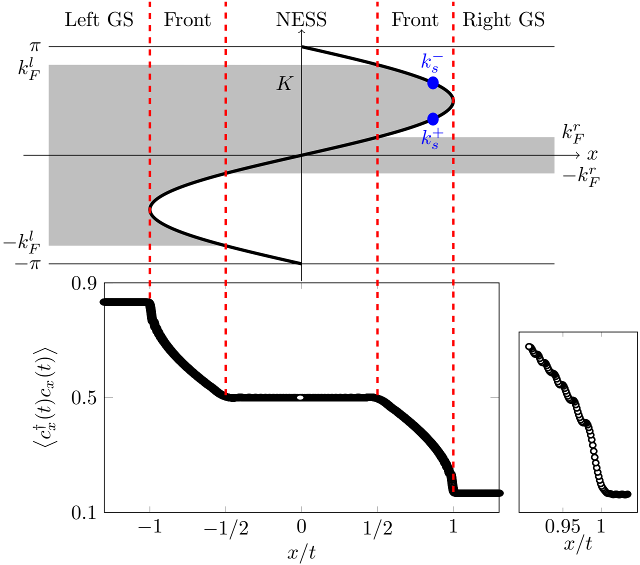
$$\langle \Psi_0 | c^\dagger(k) c(k') | \Psi_0 \rangle = \frac{\Theta(|k| - k_F^l) + \Theta(|k'| - k_F^r)}{2i \sin\left(\frac{k-k'}{2} + i0^+\right)} + \text{regular}(k, k')$$

Stationary phase treatment

$$x - t\frac{d\varepsilon(k)}{dk} = 0 \quad , \quad x' - t\frac{d\varepsilon(k')}{dk'} = 0$$

What matters is the second derivative of the phase.

Semiclassical picture



Quenches in boundary CFT

General approach: [Calabrese & Cardy, PRL 2006]

Type of quench studied

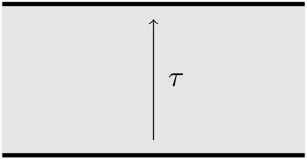
- Initial (translational invariant) state $|\Psi_0\rangle$ in a gapped phase.
- Let evolve with the critical Hamiltonian H.
- Look at large distances and late times.

Physical picture: entangled quasiparticles emitted everywhere.

Quenches in boundary CFT

General approach: [Calabrese & Cardy, PRL 2006]

$$\mathcal{L}(\tau) = \frac{\langle \Psi_0 | e^{-H\tau} | \Psi_0 \rangle}{\langle \Psi_0 | \Psi_0 \rangle}$$

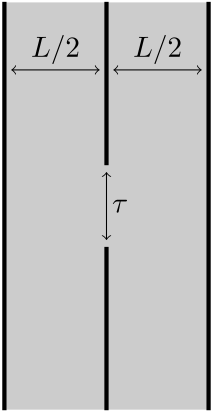


$|\Psi_0\rangle$ acts as a (conformal) boundary.

Another example: equal fillings

$$\mathcal{L}(\tau) = \left| \langle \Psi_0 | e^{-\tau H_{\text{tot}}} | \Psi_0 \rangle \right|^2$$

$\mathcal{L}(\tau)$ is a partition function [JMS & Dubail, J. Stat. Mech 2011]



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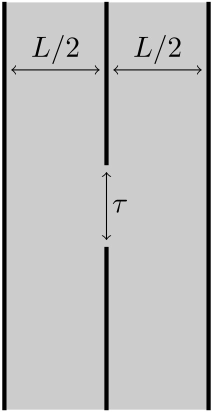

Loschmidt echo

$$\mathcal{L}(\tau) = \left|\frac{L}{\pi}\sinh\left(\frac{\pi\tau}{L}\right)\right|^{-c/4}$$

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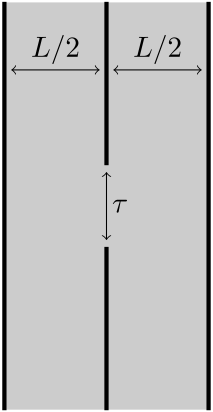

Loschmidt echo

$$\mathcal{L}(\tau) = \left| \frac{L}{\pi} \sinh\left(\frac{\pi\tau}{L} \right) \right|^{-c/4}$$

Back to real time

$$\mathcal{L}(t) = \left| \frac{L}{\pi} \sin\left(\frac{\pi v_F t}{L} \right) \right|^{-c/4}$$

Entropy

[Holzhey, Larsen & Wilczek 1994] [Calabrese & Cardy 2004,2007,2008]

- Global quench: $S(t) = \alpha t + \ldots$
- Local quench: $S(t) = \frac{c}{3} \log t + \ldots$

Here

$$S = \frac{c}{6} \log \left(d_{\text{conf}}(x, y) \right)$$

$$d_{\text{conf}}(x = 0, y) = \left| \frac{\pi \sinh\left[\frac{\pi \tau}{L}\right]}{2 \sinh\left[\frac{\pi}{2L}(\tau + 2y)\right] \sinh\left[\frac{\pi}{2L}(\tau - 2y)\right]} \right|$$

Analytic continuation:

$$S(t) = \frac{c}{6} \log \left(\frac{L}{\pi} \sin \frac{\pi v_F t}{L} \right)$$

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Imaginary time picture for the arctic quench?

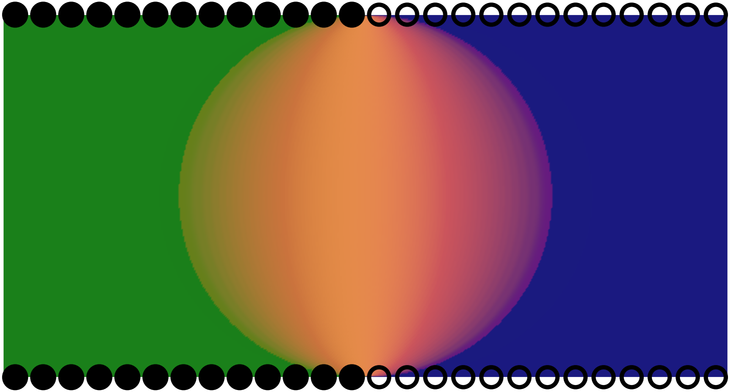
What happens in our case?

$$|\Psi_0\rangle = |\text{Domain Wall}\rangle = \prod_{x<0} c_x^\dagger |0\rangle$$

Exact calculation:
(separate left/right hoppings and use Baker-Campbell-Hausdorff)

$$\mathcal{L}(\tau) = e^{\tau^2/4} \qquad , \qquad \mathcal{L}(t) = e^{-t^2/4}$$

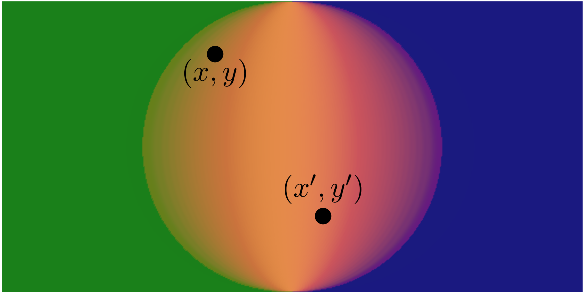
Imaginary time picture for the arctic quench?



Correlations inside the circle

$$\left\langle c_x^\dagger(y) c_{x'}(y') \right\rangle = \frac{\langle \Psi_0 | e^{-(R+y)H} c_x^\dagger e^{-(y-y')H} c_{x'} e^{-(R-y')H} | \Psi_0 \rangle}{\langle \Psi_0 | e^{-2RH} | \Psi_0 \rangle}$$

$$c^\dagger(k) = \sum_{x \in \mathbb{Z}} e^{-ikx} c_x^\dagger \qquad , \qquad [H, c^\dagger(k)] = \varepsilon(k) c^\dagger(k)$$



Bosonisation trick

Remark:

$e^{itH} c^\dagger(k) e^{-itH}$	Easy
$e^{-RH} c^\dagger(k) e^{-RH}$	Not so easy

Can still be done because $|\Psi_0\rangle = |\text{Domain Wall}\rangle$

$$a_n = \sum_{x\in\mathbb{Z}} c^\dagger_{x-n} c_x$$

$$\langle \Psi_0 \,|\, [a_n, a_m] \,|\, \Psi_0 \rangle = n\delta_{n+m,0}$$

Appearance of the Hilbert transform

$$\left\langle c^\dagger(k,y)c(k',y')\right\rangle = \frac{\left\langle \Psi_0 \,\middle|\, e^{-(R-y)H}c^\dagger(k)e^{-(y'-y)H}c(k')e^{-(R+y)H} \,\middle|\, \Psi_0\right\rangle}{\langle \Psi_0 \,|\, e^{-2RH} \,|\, \Psi_0\rangle}$$

One can show that

$$\boxed{\left\langle c^\dagger(k,y)c(k',y')\right\rangle = \frac{e^{-iR[\tilde{\varepsilon}(k)-\tilde{\varepsilon}(k')]}e^{y\varepsilon(k)-y'\varepsilon(k')}}{2i\sin\left(\frac{k-k'}{2}+i0^+\right)}}$$

$$\tilde{\varepsilon}(k) = \text{Hilbert transform of } \varepsilon(k) = \mathrm{pv}\int_{-\pi}^{\pi}\frac{dq}{2\pi}\varepsilon(q)\cot\frac{k-q}{2}.$$

Comments

- The result is exact.
- This completely solves the problem in principle.
- Real space correlations: inverse Fourier transform+stationary phase approximation.
- Stationary points

$$x + iy\frac{d\varepsilon(k)}{k} + R\frac{d\tilde{\varepsilon}(k)}{dk} = 0$$

Field theory inside the circle

$$c^{\dagger}_{x,y} = \frac{1}{\sqrt{2\pi}}\psi^{\dagger}(x,y) + \frac{1}{\sqrt{2\pi}}\overline{\psi}^{\dagger}(x,y)$$

where

$$\left\langle \psi^{\dagger}(x,y)\psi(x',y')\right\rangle = e^{-\frac{1}{2}[\sigma(x,y)+\sigma(x',y')]}\frac{e^{-i(\varphi(x,y)-\varphi(x',y'))}}{2i\sin\left(\frac{z(x,y)-z(x',y')}{2}\right)}$$

$$\begin{aligned} z(x,y) &= \arccos\frac{x}{\sqrt{R^2-y^2}} + i\,\mathrm{arcth}\frac{y}{R} \\ \varphi(x,y) &= -z(x,y)x - \sqrt{R^2-x^2-y^2} \\ e^{\sigma(x,y)} &= \sqrt{R^2-x^2-y^2} \end{aligned}$$

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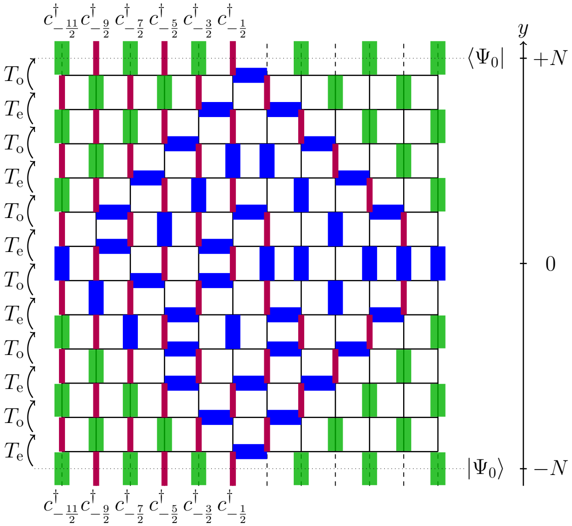
$\longrightarrow$ Dirac theory with a metric $ds^2 = e^{2\sigma}dzd\bar{z}$

Field theory inside the circle

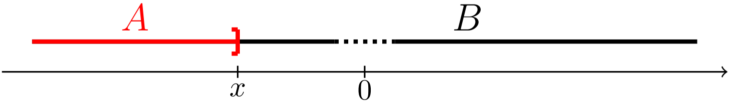
$$S = \frac{1}{2\pi}\int d^2\mathrm{x}\sqrt{g}\, e^{\mu}{}_{a}\left[\overline{\Psi}\gamma^{a}\left(\frac{1}{2}\overset{\leftrightarrow}{\partial}_{\mu} + iA_{\mu}^{(\mathrm{v})} + iA_{\mu}^{(\mathrm{a})}\gamma^{5}\right)\Psi\right].$$

Can identify all the parameters in the action.

Back to square dimers



Application: entanglement and fluctuations



Normal local quench [Calabrese & Cardy 2007]

$$S = \frac{1}{6} \log\left(t^2 - x^2\right)$$

Here this does not apply. Go still to imaginary time:

$$S(x,y) = \frac{1}{6} \log\left(d_{\rm conf}(x,y) \sin A_x^{(\rm a)}\right)$$

Application: entanglement and fluctuations

$$d_{\text{conf}}(x,y) = e^{\sigma(z,\bar{z})} \text{Im}\, g(z) \left| \frac{dg}{dz} \right|^{-1}$$

$$S(x,y) = \frac{1}{6} \log \left(\frac{\left[R^2 - x^2 - y^2\right]^{3/2}}{R^2 - y^2} \right)$$

Analytic continuation: $y = it$ and then $R \to 0$.

$$S(x,t) = \frac{1}{6} \log \left(t \left[1 - x^2/t^2\right]^{3/2} \right) \qquad , \qquad t > x$$

Same for the fluctuations $F = \langle (N_A - \langle N_A \rangle)^2 \rangle$

Interactions

- Similar curved space interpretation for all Δ.
- Exact computations, e. g. the metric are much more difficult.
- Can compute the metric numerically.
- Can predict the speed of the front in real time from the imaginary time arctic curve.

Conclusion

- Motivation from quantum quench problems.
- Arctic circle revisited: correlations.
- Inhomogeneous system, so inhomogeneous field theory.
- Consistent with free boson interpretation.
- Analytic continuation seems to work.
- Interactions.

Thank you!