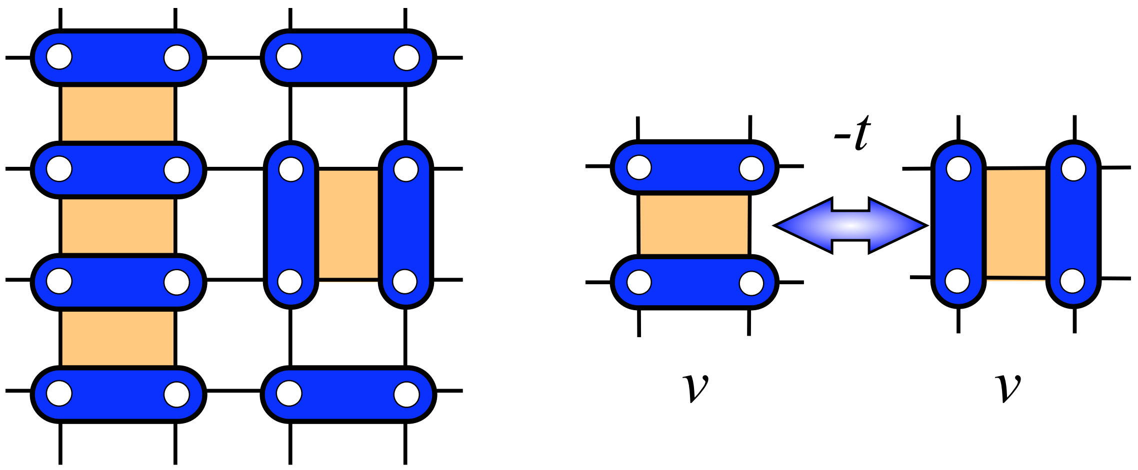
Quantum dimer models and their links to SU(2) spin models

Sylvain Capponi
Toulouse university



UNIVERSITÉ TOULOUSE III PAUL SABATIER
Université de Toulouse

Benasque, Feb. 22nd 2016

Outline of the talk

- Frustrated quantum magnetism in a nutshell
- Some examples of constrained models and their phase diagrams
- Revisiting the simplest case: quantum dimer model (QDM) on a square lattice
- Engineering SU(2) spin-1/2 microscopic models to realize *any* phase of its QDM counterpart
- Conclusion and outlook

Collaborators

- Quantum Dimer Models
  - D. Schwandt (former PhD student), Toulouse university
  - S. Isakov, ETH Zurich
  - A. Läuchli, Innsbruck
  - R. Moessner, MPI PKS Dresden

unpublished

- Engineering Spin models
  - F. Alet, Toulouse univ.
  - M. Mambrini, Toulouse univ.

Mambrini, Capponi, Alet, Phys. Rev. B ‘15

Part 1 : Quantum Magnetism

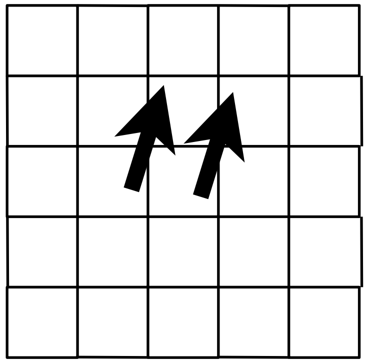
see also talks by Misguich and Becca

Quantum magnetism

- Mott insulator : magnetism well described by spin lattice models

Simplest example: Heisenberg model

$$H = \sum_{i,j} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$



- Antiferromagnetism $J > 0$
- In most compounds, few coupling constant are important.

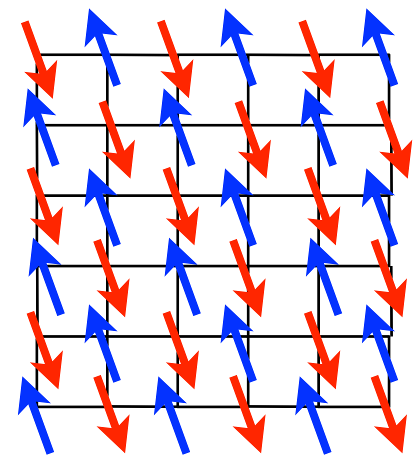
$$J_{ij} \neq 0 \quad \text{for} \quad \langle i,j \rangle, \langle\langle i,j \rangle\rangle ...$$

- Quantum effects are important for low spin values and low dimension

mostly $S = 1/2,\ d = 2$ in this talk

S=1/2 Heisenberg models

- Bipartite lattices, nearest-neighbor J
  - Under control (e.g. with spin waves, Quantum Monte Carlo)
  - $d > 1$ and $S = 1/2$ per unit cell : Antiferromagnetic long range order
  - Simplest example : square lattice

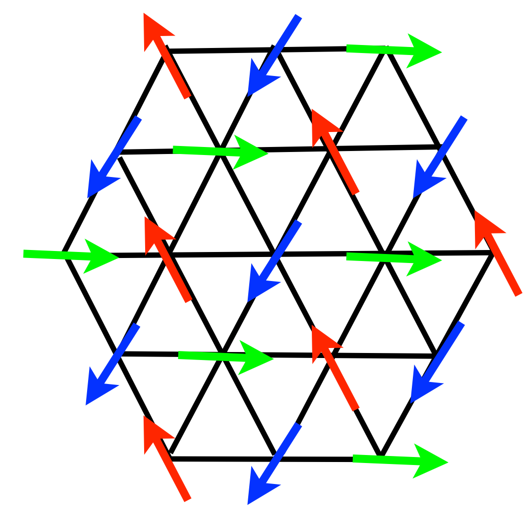


$m_s = 0.30793(3)$

S=1/2 Heisenberg models

- Bipartite lattices, nearest-neighbor J
  - Under control (e.g. with spin waves, Quantum Monte Carlo)
  - $d > 1$ and $S = 1/2$ per unit cell : Antiferromagnetic long range order
- Non-bipartite lattices: frustration
  - No longer under control
  - Antiferromagnetic order may survive

    e.g. triangular lattice

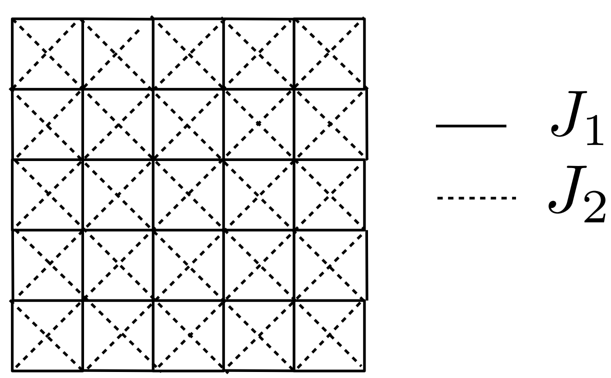


$m_s \simeq 0.205$

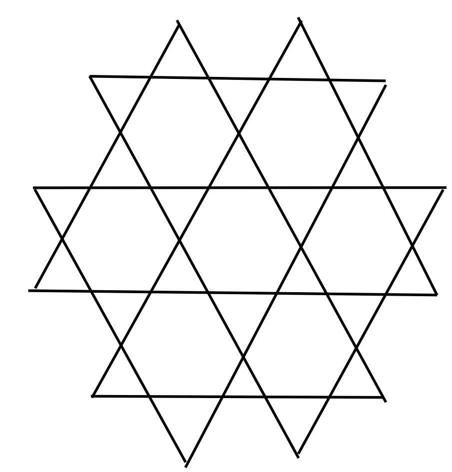
- Most of the time, antiferromagnetism dies but ground-state not understood

J1-J2 model for

$J_2 \sim 0.5 J_1$



kagome



- Guess : spin is no longer the good degree of freedom at low energy

Exotic Quantum Phases

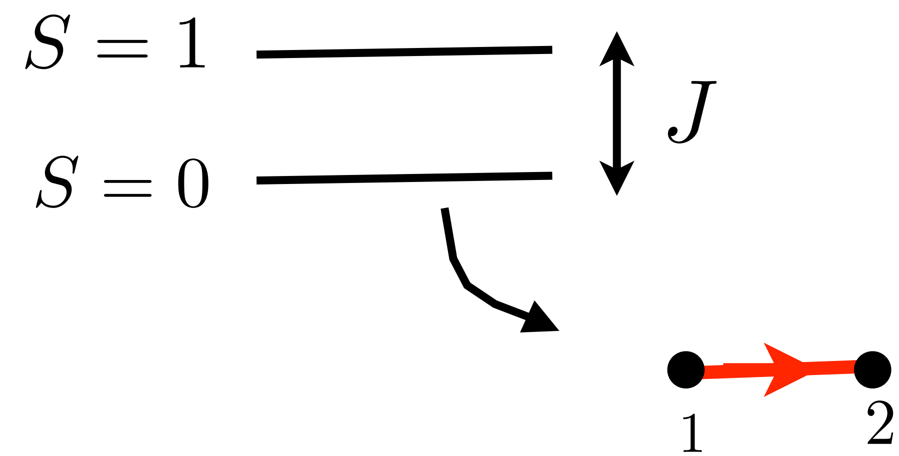
Recent list (inventory à la Prévert)

- J1-J2 square lattice:
  - ✦ Z2 spin liquid — H. Jiang, H. Yao and L. Balents, PRB (2012).
  - ✦ Valence Bond Crystal (VBC) with J3 — M. Mambrini, A. Läuchli, D. Poilblanc, F. Mila, PRB (2006).
  - ✦ Both phases — Gong et al., PRL (2014)
- J1-J2 honeycomb lattice:
  - ✦ Valence Bond Crystal (VBC) with additional J3 — F. Albuquerque et al., PRB (2011).
  - ✦ VBC — Zhu, Huse, White, PRL (2013) Gong, Sheng, Motrunich, Fisher, PRB (2013)
- kagome lattice:
  - ✦ Z2 spin liquid — Yan, White and Huse, Science (2011); Jiang, Wang, Balents, Nat. Phys. (2012) Depenbrock, McCulloch, Schollwöck PRL (2012)
  - ✦ U(1) spin liquid — Iqbal, Becca, Sorella, Poilblanc, PRB (2013)
  - ✦ VBC — Evenbly and Vidal, PRL (2010)
  - ✦ p6 chiral RVB — Capponi, Chandra, Auerbach, Weinstein, PRB (2013)

Singlet physics

- 2 sites example

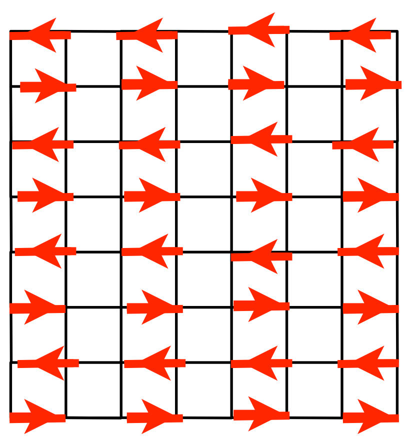
$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2$$



  - form a singlet $\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

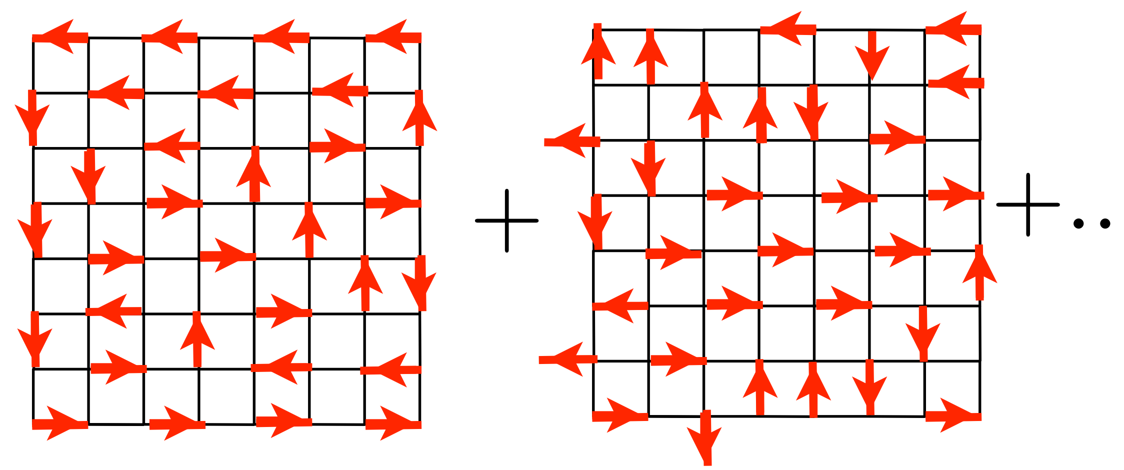
  a.k.a Valence bond (VB), SU(2) dimer

- Good ansatz for non magnetic states (S=0)



Valence Bond crystal

(rotation, translation broken)



...

« Spin liquid »

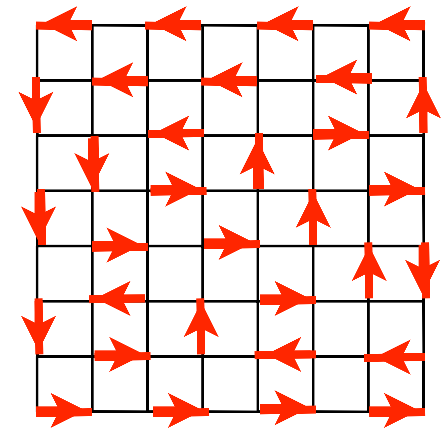
(no broken symmetry)

- Caution : Caricatures (not always n.n. bonds), other states are possible ...

- In some cases, n. n. VBs are the «good» degrees of freedom

Formalize this!

- Assume nearest neighbors valence bonds are important
  - Method 1 : Diagonalize in this variational subspace
    - Problem : Valence bonds are non-orthogonal



$$\mathcal{O}_{12} = \langle \mathrm{VB}_1 | \mathrm{VB}_2 \rangle \neq 0 \quad \forall \ \mathrm{VB}_1, \mathrm{VB}_2$$

$$\mathcal{H}|\Psi\rangle = E\mathcal{O}|\Psi\rangle$$

  - Method II : Overlap expansion

Rokhsar, Kivelson, '88

- Exploit the hierarchy of overlap matrix elements

$$\mathcal{O}_{ij} \in \{1, \frac{1}{2}, \frac{1}{4} \dots \frac{1}{2^{N/2-1}}\}$$

- Expand overlap matrix and Heisenberg Hamiltonian in powers of $x = 1/\sqrt{2}$

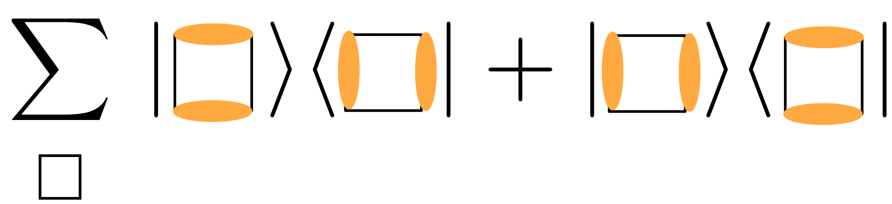
- Leads to effective (orthogonal) Quantum Dimer Models
- Original scheme recently revisited

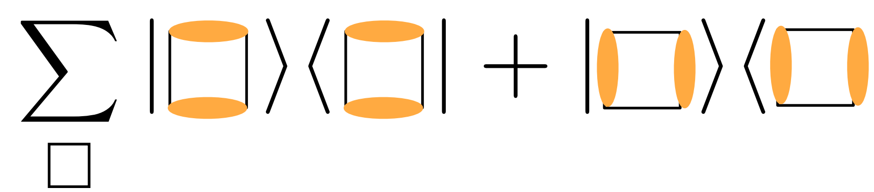
Schwandt, Mambrini, Poilblanc '10

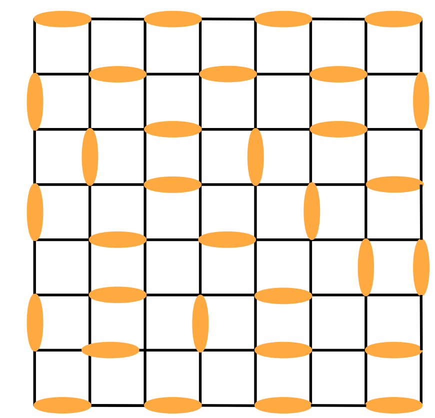
Quantum dimer models

- Hamiltonian for orthogonal hardcore dimer coverings

$$H_{\mathrm{QDM}} = -t\,\square + v\,\square$$

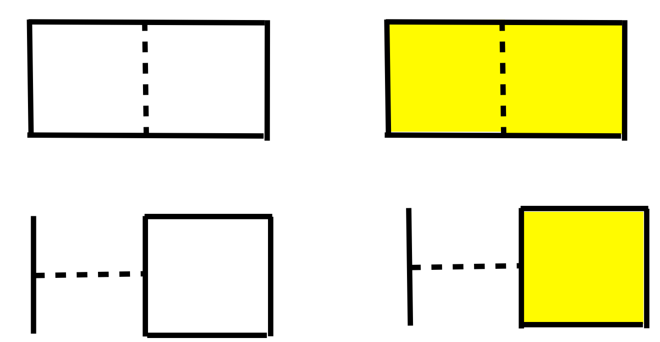






- Overlap expansion of Heisenberg model : $t = x^2 J \quad v = x^4 J$

- New terms appear at higher order :



$\cdots$

Generalized QDM

Quantum dimer models

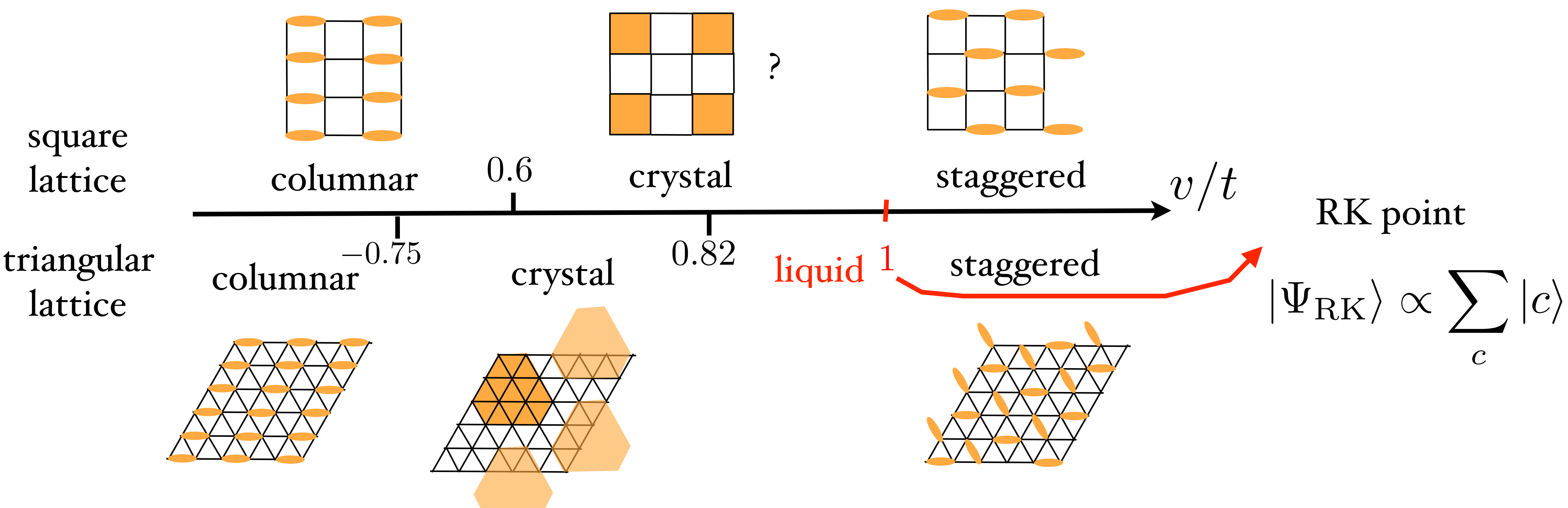
- Hamiltonian for orthogonal hardcore dimer coverings

$$H_{\mathrm{QDM}} = -t \, \boxed{} + v \, \boxed{}$$

- QDM interesting on their own

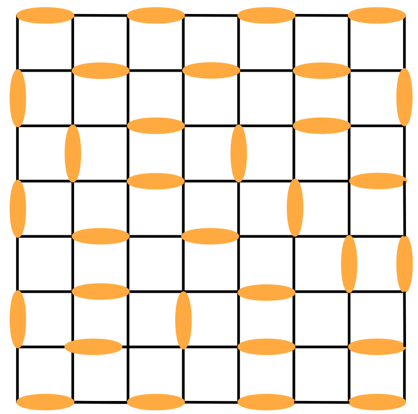
Rokshar, Kivelson, Moessner, Sondhi ...

  - Phase diagrams



- Effectively captures a great deal of physics (new phases)
- Easy to play with : field theories, numerics-friendly

But wait ...



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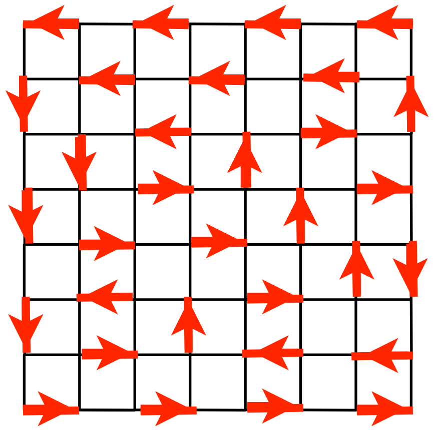


- Q1 : How well are the phase diagram established ?
- Q2 : Spin models with quantum dimer phases ?

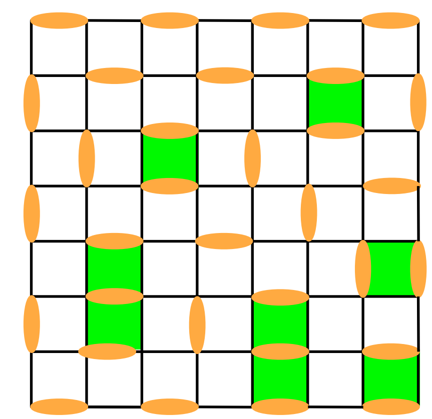
Part 2 : Snapshots of phase diagrams for some constrained models

Classical dimers in 2d

- Simplest classical model on 2d (square) lattice

$$Z = \sum_c \exp(-E_c/T)$$

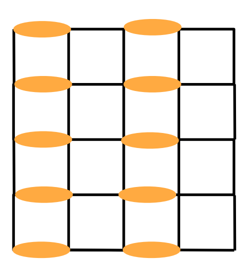
$$E_c = v\left[N^c(\square) + N^c(\square)\right]$$

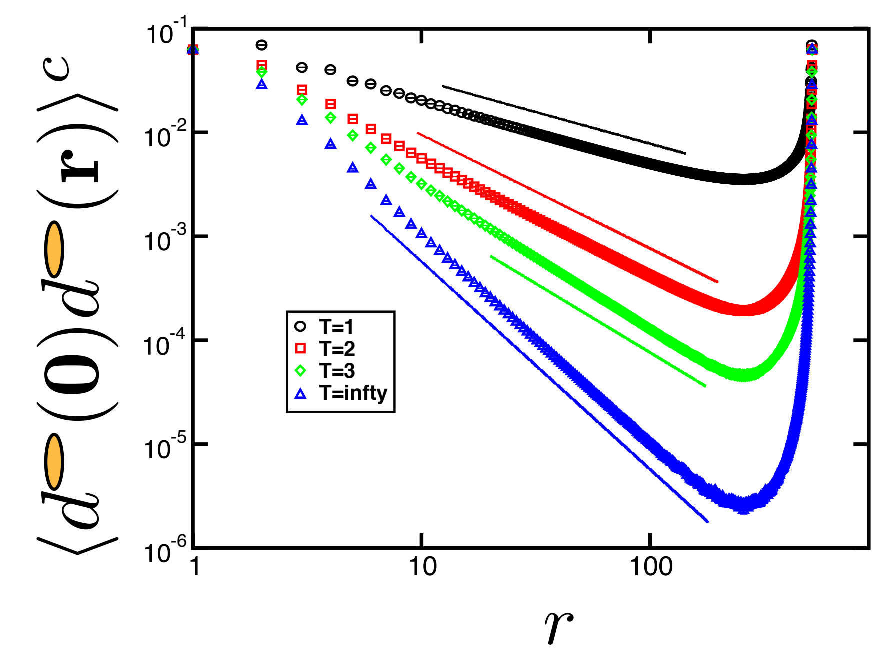


- Phase diagram for attractive $v = -1$

columnar crystal

critical phase





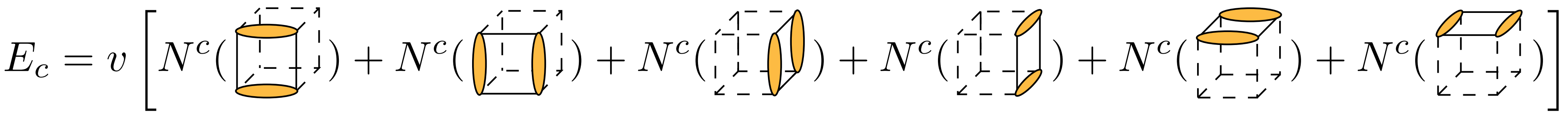
- In the critical phase, exponents vary continuously
- Kosterlitz-Thouless transition, sine-Gordon theory

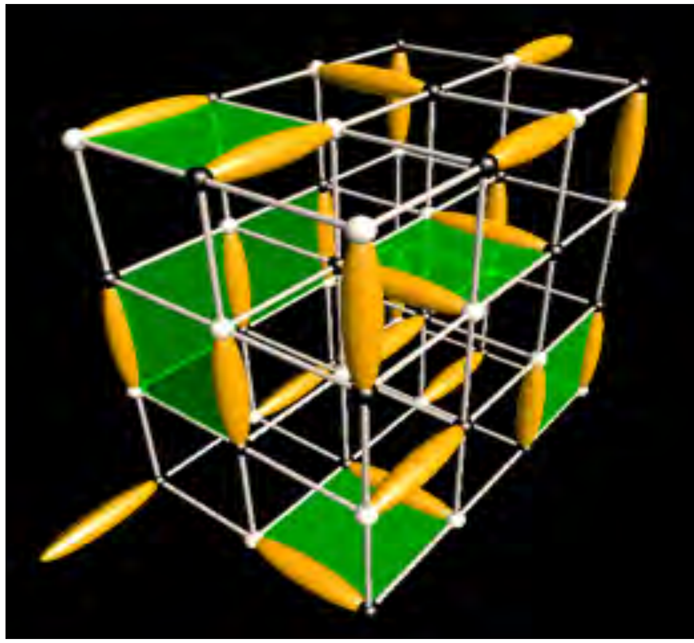
Alet et al. ‘05

Classical dimers in 3d

- Same model …

$$Z = \sum_c \exp(-E_c/T)$$





- … but new physics !

$T = 0$ — columnar crystal — $T_c = 1.675$ — $T = \infty$

dimer “Coulomb” liquid

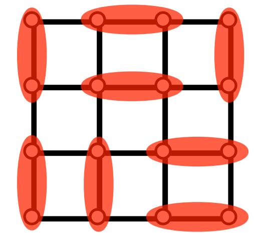
- degeneracy = 6
- Long-range order

$$\mathbf{m}(\mathbf{r}) = \begin{pmatrix} (-)^{r_x}[d^x(\mathbf{r}) - d^{-x}(\mathbf{r})] \\ (-)^{r_y}[d^y(\mathbf{r}) - d^{-y}(\mathbf{r})] \\ (-)^{r_z}[d^z(\mathbf{r}) - d^{-z}(\mathbf{r})] \end{pmatrix}$$

unconventional phase transition

Alet, Misguich, Pasquier, Moessner, & Jacobsen ‘06

2d nearest-neighbor RVB **quantum** wave-function

$$|\Psi_{\mathrm{RVB}}\rangle = \sum_c |$$  $$\rangle_c$$

 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

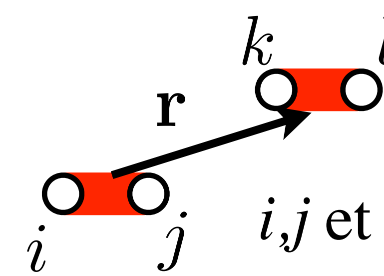
paradigm for spin liquid

➡ Short-range spin correlations

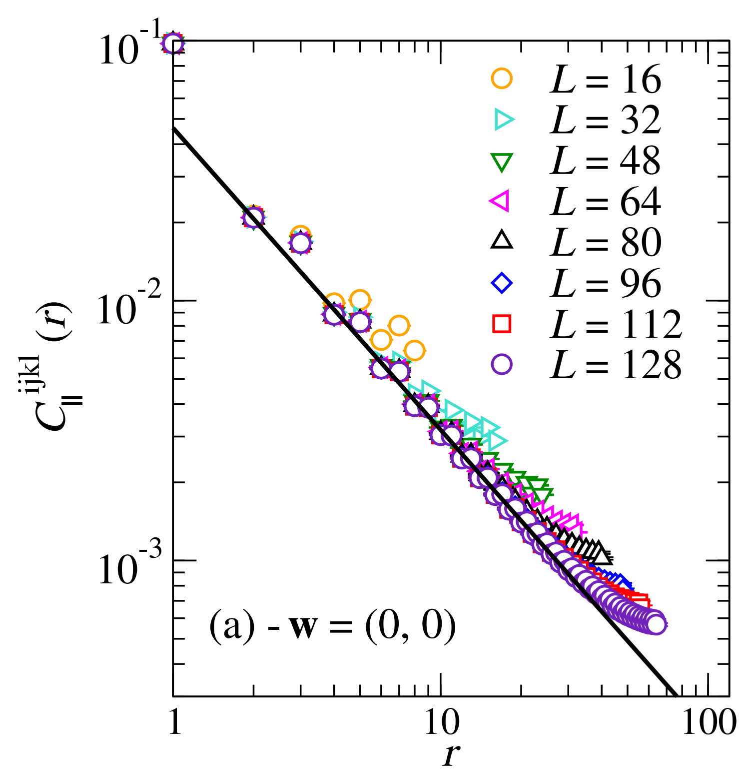
$$\langle \mathbf{S_0} \cdot \mathbf{S_r} \rangle \propto (-)^r e^{-r/\xi} \qquad \xi \sim 1.35$$

➡ But critical dimer-dimer correlations !

$$C^{ijkl} = \langle (\mathbf{S}_i \cdot \mathbf{S}_j)(\mathbf{S}_k \cdot \mathbf{S}_l) \rangle - \langle \mathbf{S}_i \cdot \mathbf{S}_j \rangle \langle \mathbf{S}_k \cdot \mathbf{S}_l \rangle$$



i,j et k,l
nearest-neighbor



$$C^{ijkl}(\mathbf{r}) \simeq 1/|\mathbf{r}|^\alpha$$

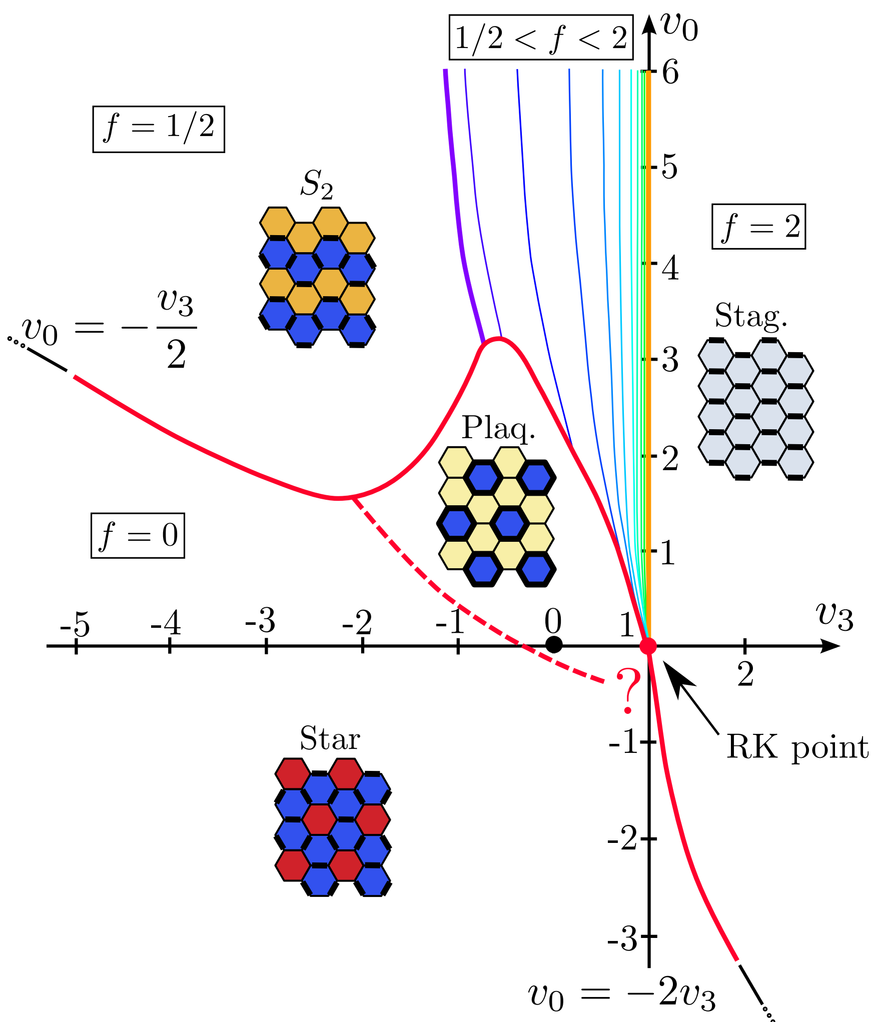
$$\alpha \simeq 1.16$$

Albuquerque and Alet '10
Tang, Sandvik, Henley '11

Unusual spin liquid :
finite spin gap but gapless singlet excitations

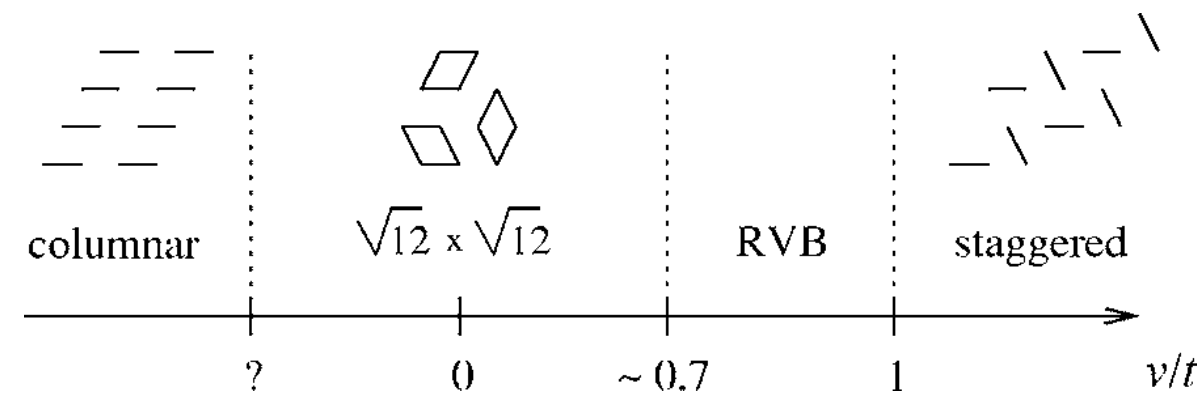
2d QDM

Phase diagram on a honeycomb lattice (bipartite case)



Moessner, Sondhi, Chandra '01
Schlittler, Barthel, Misguich, Vidal, Mosseri '15

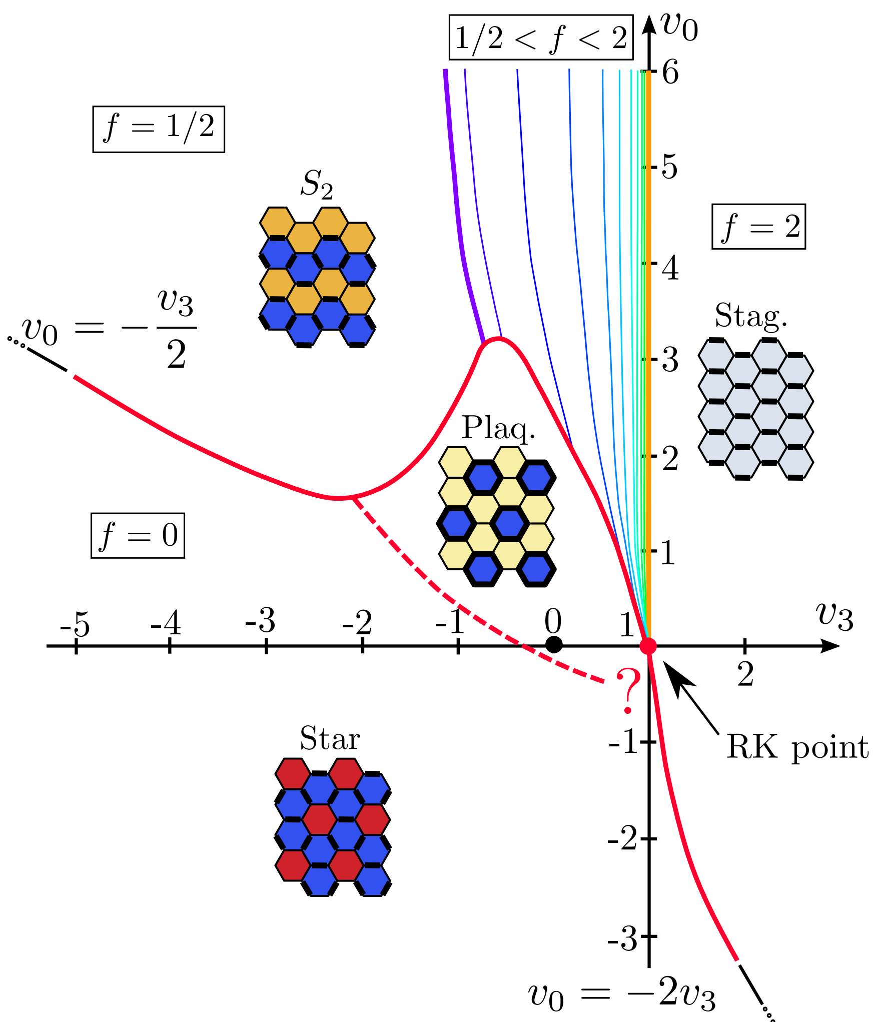
Phase diagram on a triangular lattice (non-bipartite case)



Moessner and Sondhi '01
Ralko et al. '05

2d QDM

Phase diagram on a honeycomb lattice (bipartite case)



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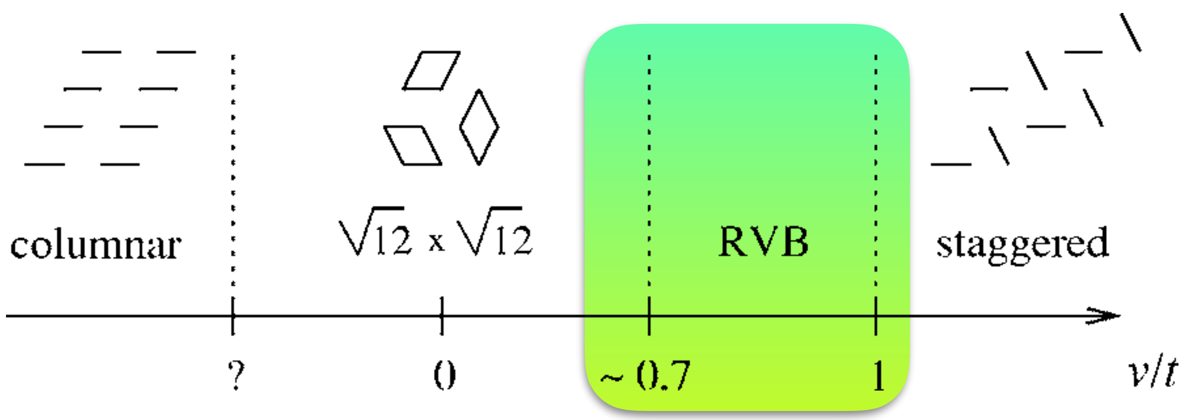
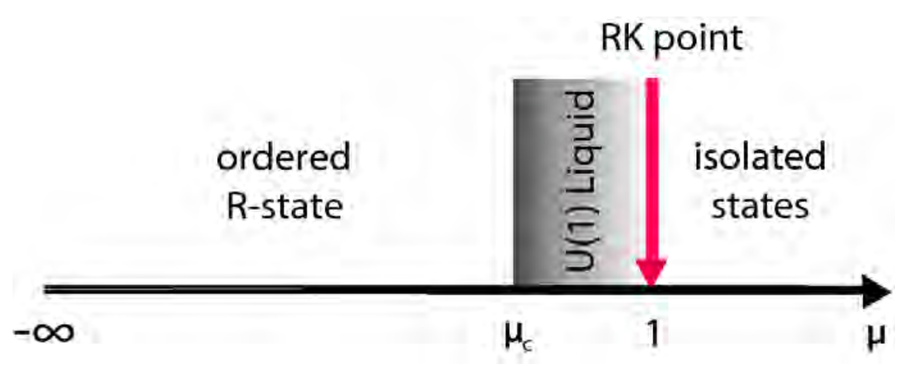


topological Z2 liquid !

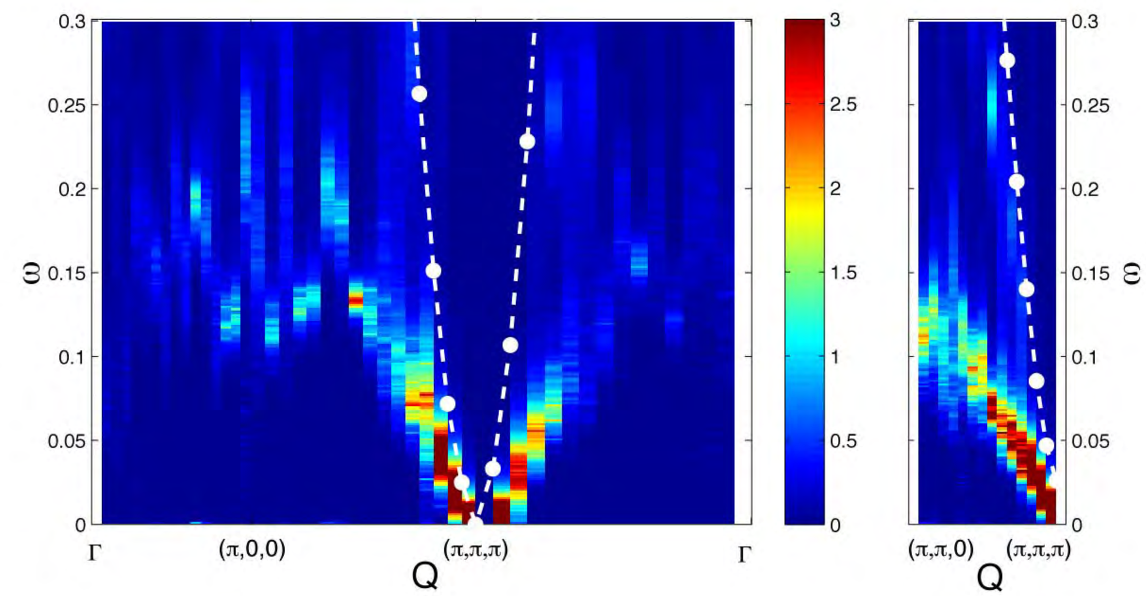
3d QDM

Phase diagram
on a diamond lattice
(bipartite case)



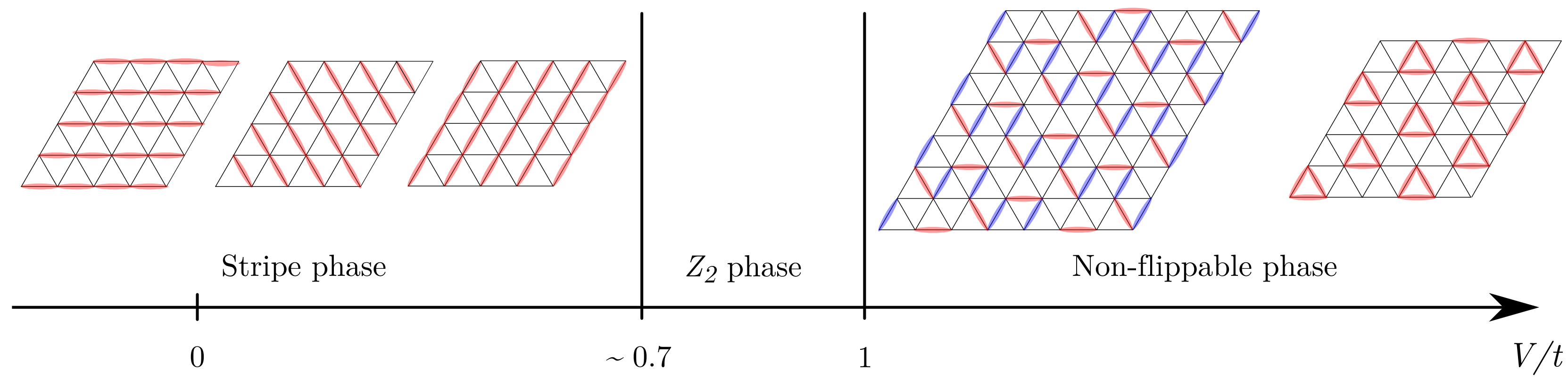
Sikora, Shannon, Pollmann, Penc, Fulde '11

Dynamics at the RK point:
emergence of a photon !

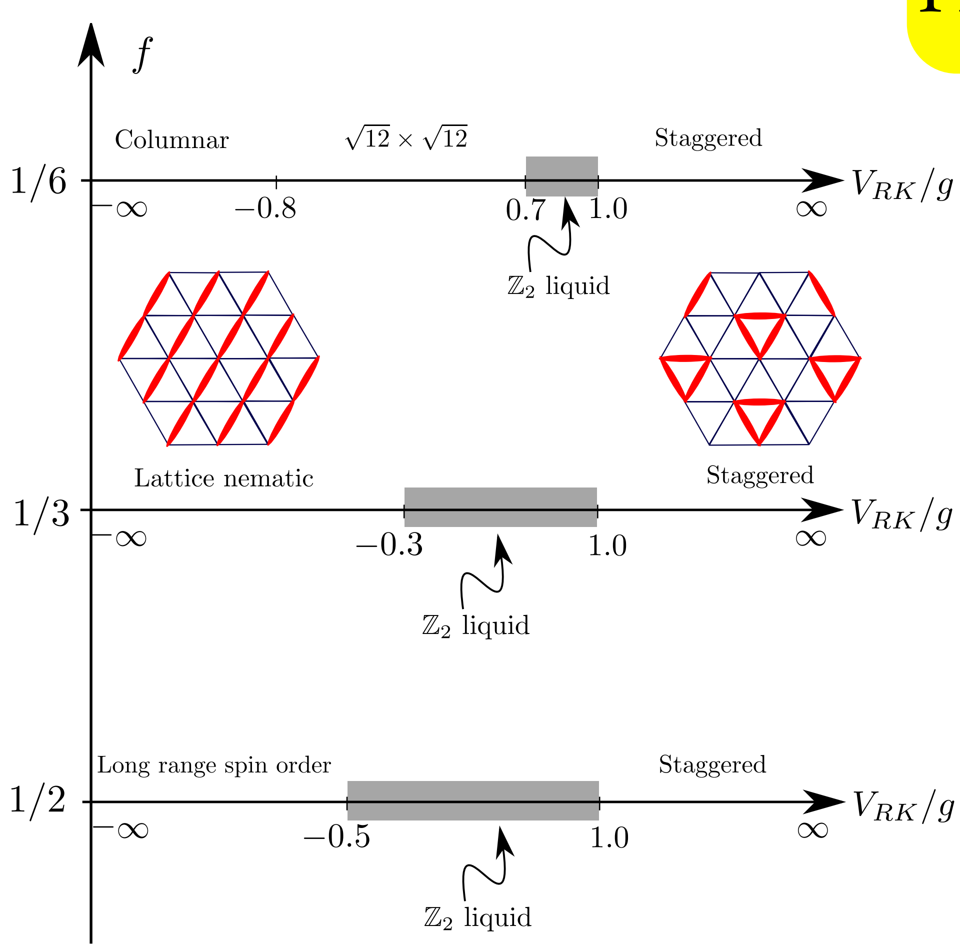


Läuchli, Capponi, Assaad '08

Phase diagram of the quantum loop model on the triangular lattice



Plat, Alet, Capponi, Totsuka '15, see Totsuka's talk !



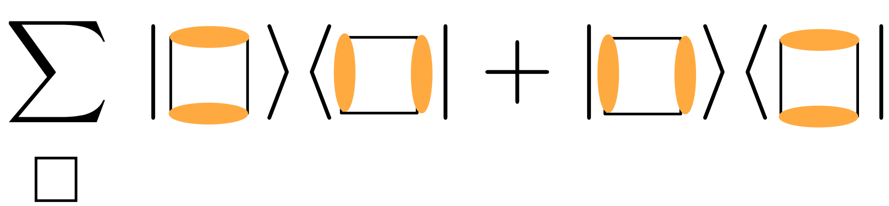
Various QDM's on the triangular lattice

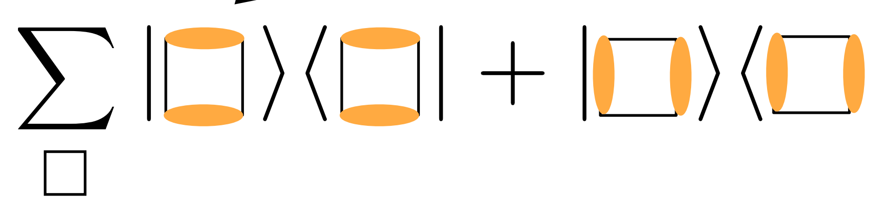
Roychowdhury, Bhattacharjee, Pollmann '15

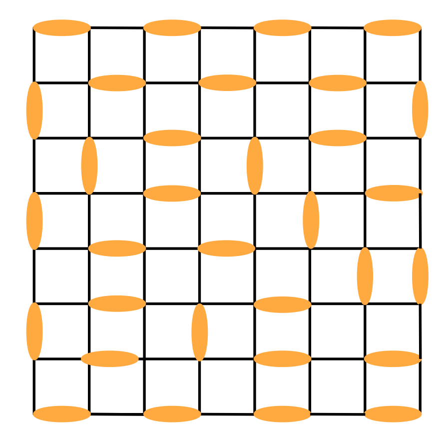
Part 3 : QDM revisited

D. Schwandt, S.V. Isakov, S. Capponi, R. Moessner, A.M. Läuchli

$$H_{\mathrm{QDM}} = -t\ \square + v\ \blacksquare$$

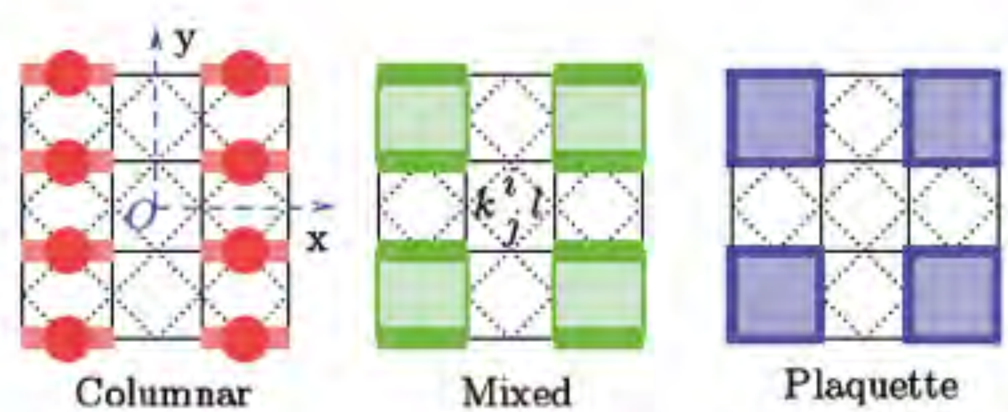


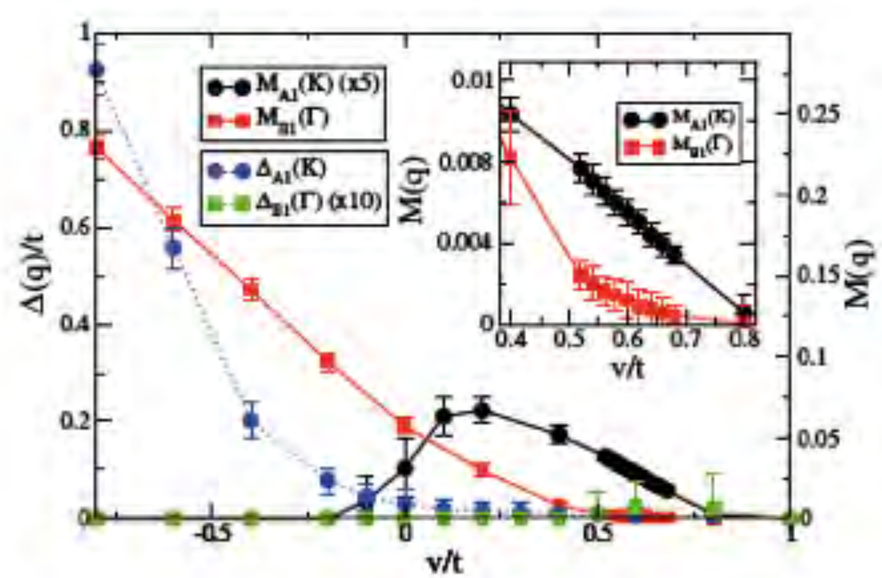




Rokhsar-Kivelson QDM on the square lattice

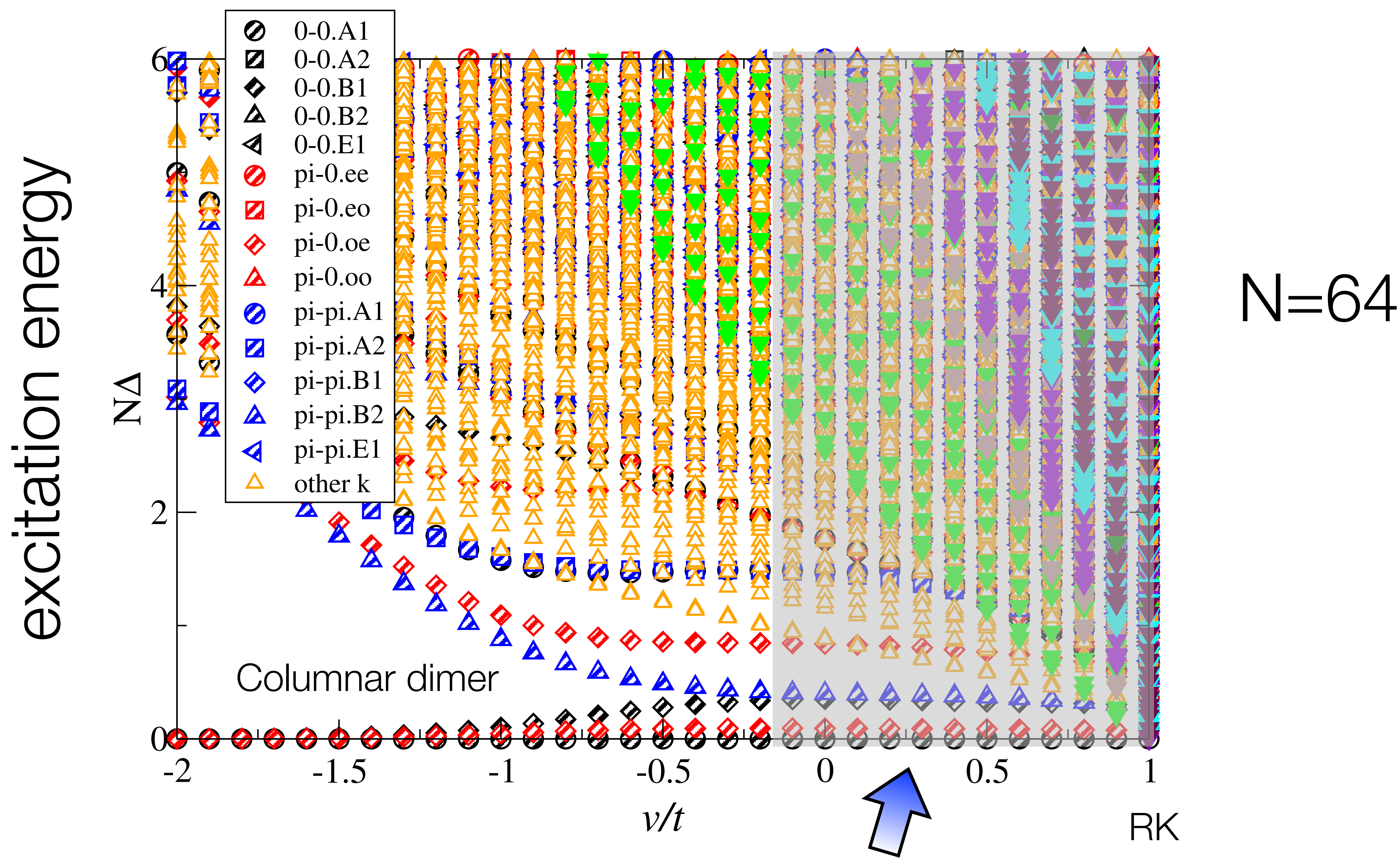
- Phase diagram: long history of numerical simulations
  - S Sachdev, PRB **40** 5204 (1989)
  - PW Leung, KC Chiu, KJ Runge, PRB **54** 12938 (1996)
  - OJ Syljuasen, PRB **71** 020401 (2005), PRB **73** 245105 (2006)
  - A Ralko, D Poilblanc, R Moessner PRL **100** 037201 (2008)





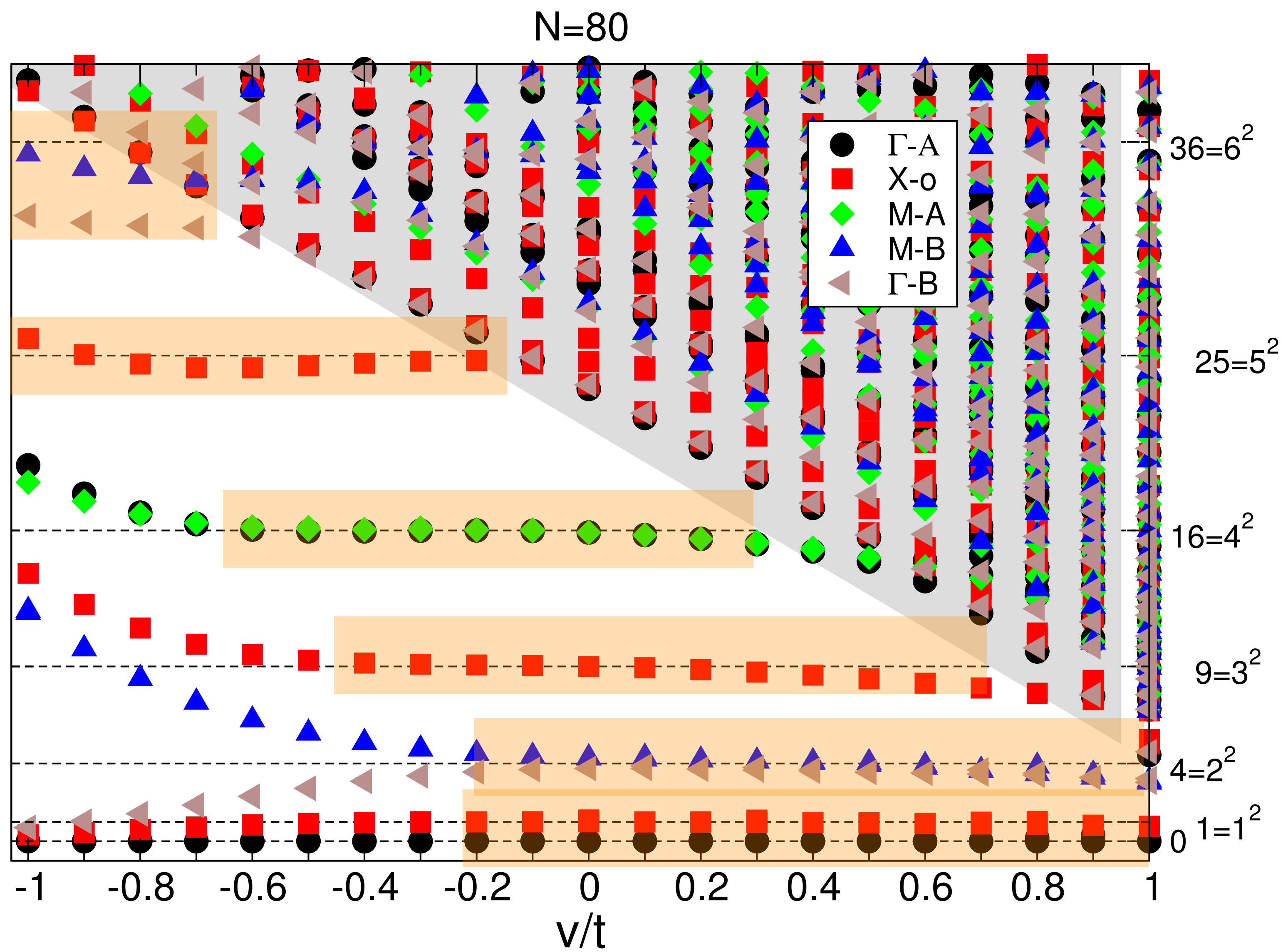
- RK point (v=t): GS are equal-weight superpositions of dimer coverings, with **critical dimer correlations**

Low energy exact diagonalization spectrum



- apparent low energy spectrum of a d=2+1 “Coulomb” phase !

Zero winding number sector



- Characteristic modes exhibiting a (“quantum number”)^2 dependence (and a 1/N dependence). Is this a U(1) tower of states of some kind ?

Field theory

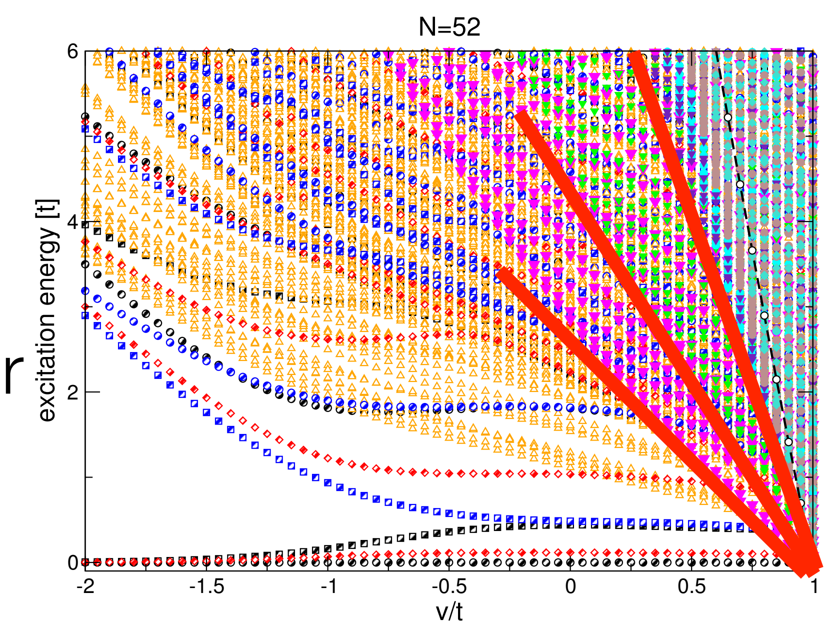
- in the vicinity of the RK point, field theory in terms of the height variable

$$\mathcal{L} = \frac{1}{2}(\partial_\tau h)^2 + \frac{1}{2}\rho_2(\nabla h)^2 + \frac{1}{2}\rho_4(\nabla^2 h)^2 + \lambda \cos(2\pi h)$$

E. Fradkin et al.
PRB 69 224415 (2004)

- Neglect the last two terms, Hamiltonian formulation:

$$\mathcal{H} = K_B(\nabla \times \vec{a})^2 + K_E e^2$$



  - Unit electric flux ⇒ $e_E = K_E$

    where e_E is the energy in the first winding number sector

  - Unit magnetic flux ⇒

  - speed of light ⇒

Field theory

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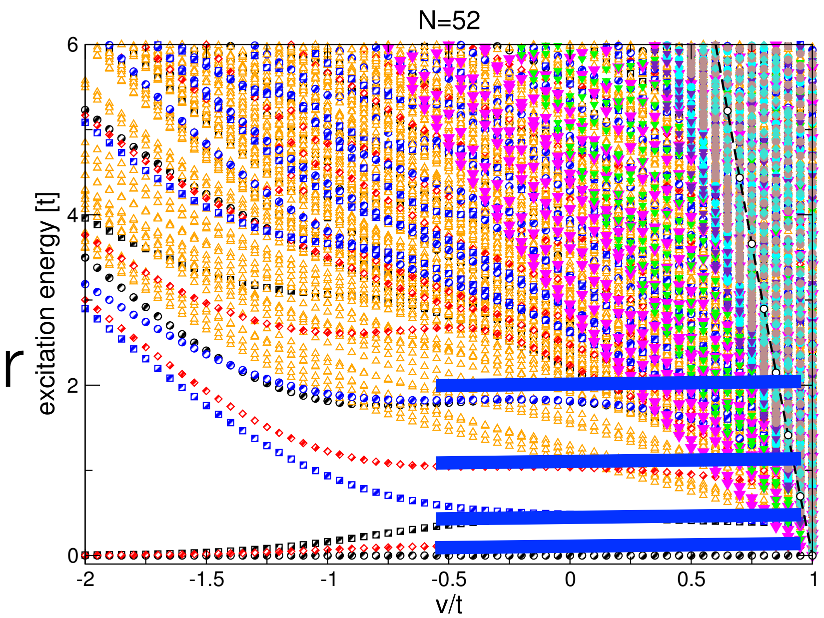

  - Unit electric flux ⇒ $e_E = K_E$

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  - Unit magnetic flux ⇒ $e_B/L^2 = \left(\frac{2\pi}{L}\right)^2 K_B$

  - speed of light ⇒

Field theory

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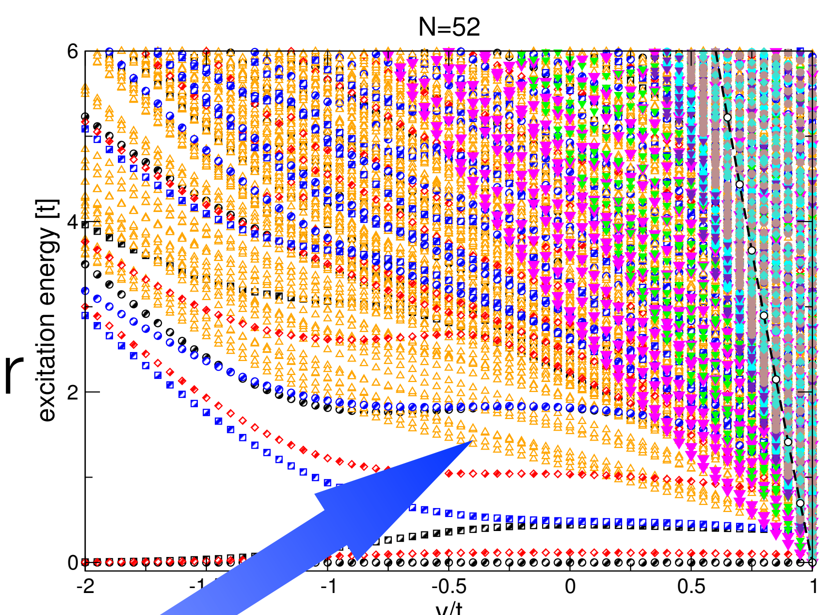
  - Unit electric flux $\Rightarrow e_E = K_E$

    where e_E is the energy in the first winding number sector

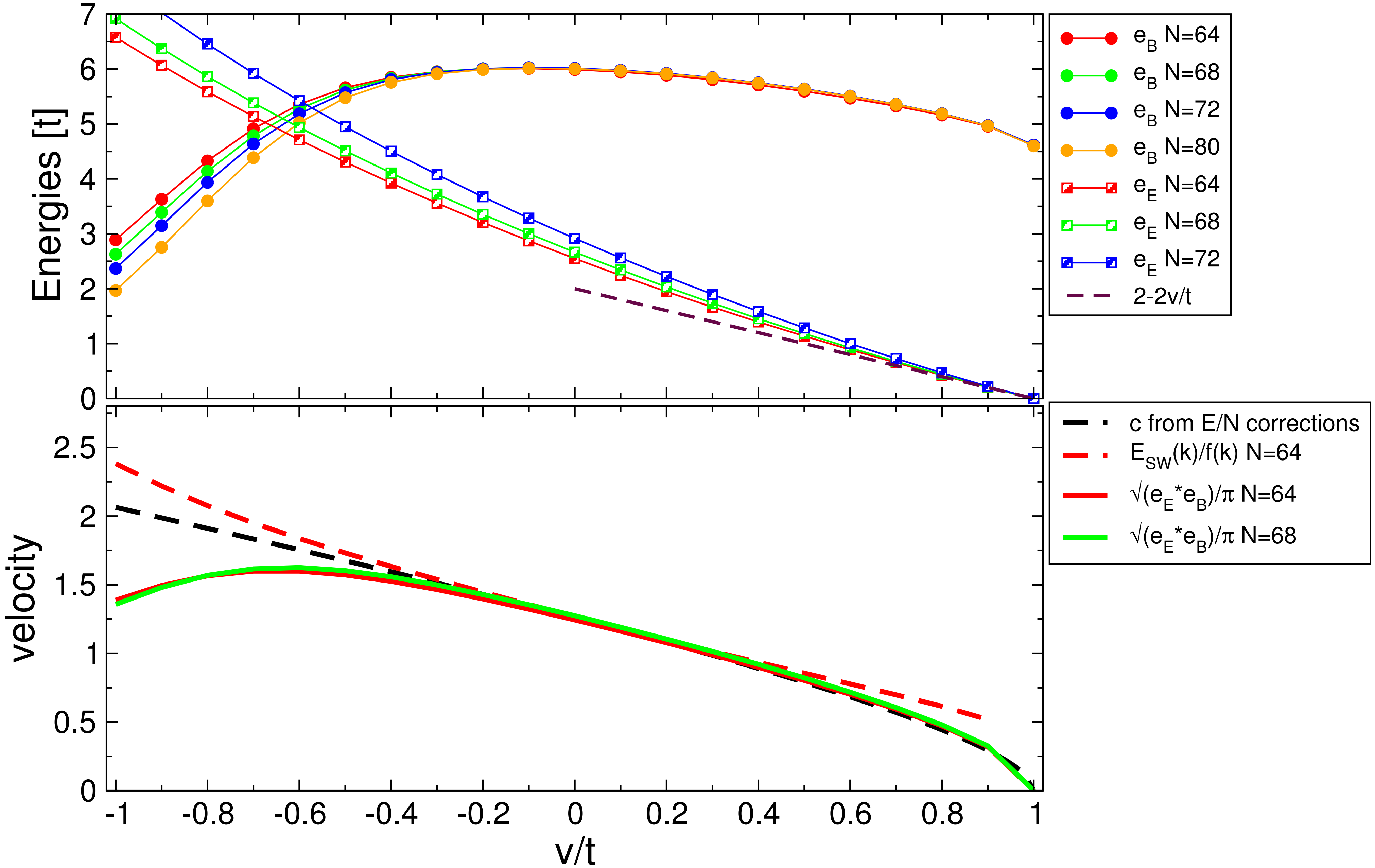
  - Unit magnetic flux $\Rightarrow e_B/L^2 = \left(\frac{2\pi}{L}\right)^2 K_B$

  - speed of light $\Rightarrow \quad c = \frac{2}{2\pi}\sqrt{e_E e_B}$



These are the emergent photons in the QDM

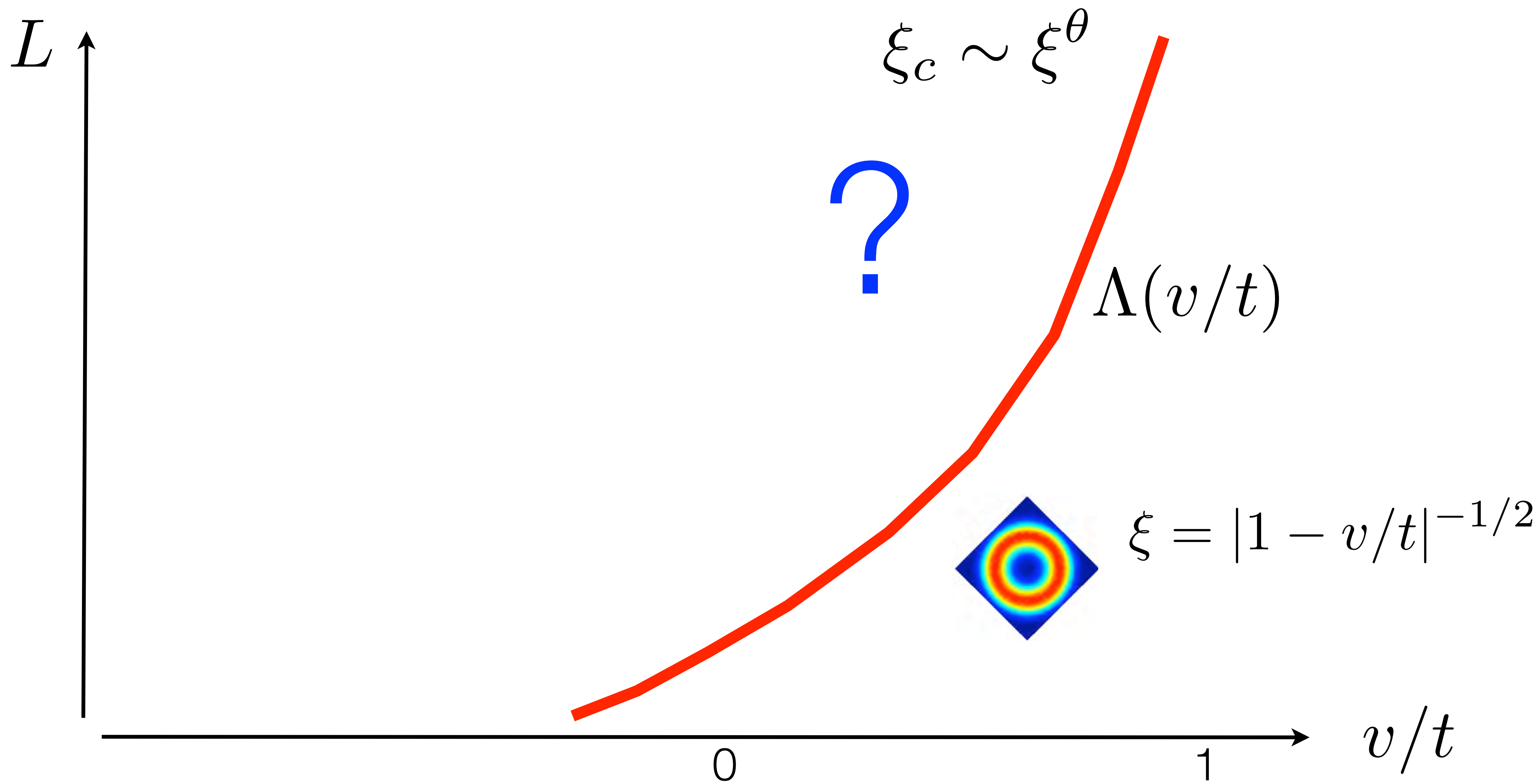
Parameters of the field theory



- Very nice agreement between 3 different ways to measure the speed of light!

What happens beyond the crossover scale ?

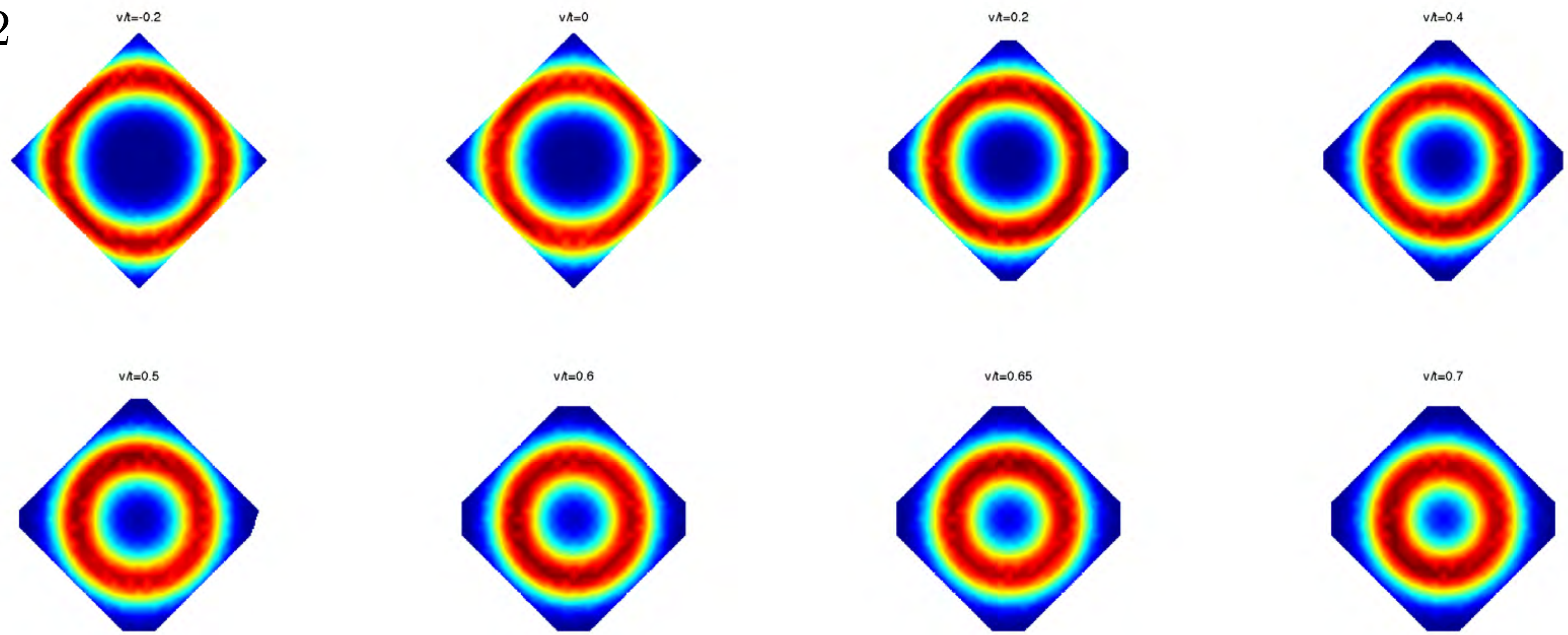
E. Fradkin et al.
PRB 69 224415 (2004)



- Quantum Monte Carlo simulations of a **fully frustrated transverse field Ising** model which reduces to the quantum dimer model in the limit of small transverse field.

L=8

$v/t = -0.2$ $v/t = 0.4$

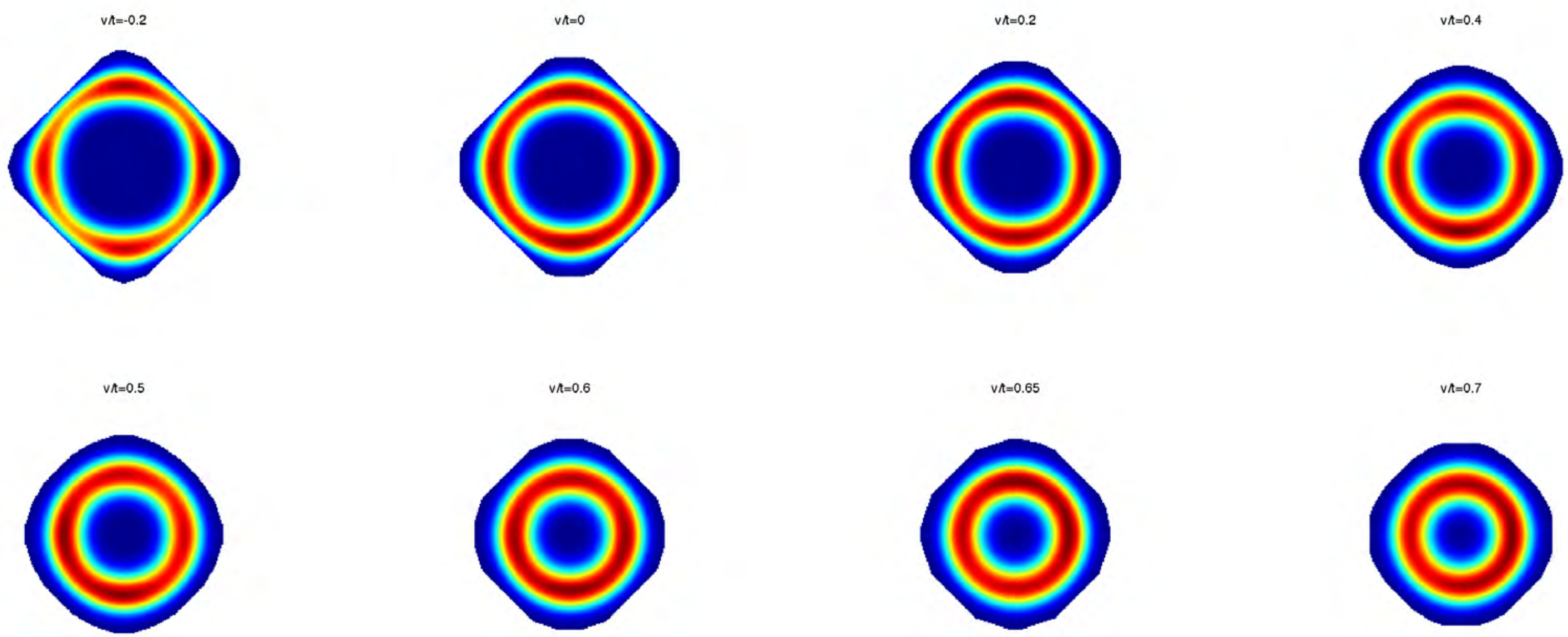


$v/t = 0.5$ $v/t = 0.7$

L=12

$v/t = -0.2$

$v/t = 0.4$



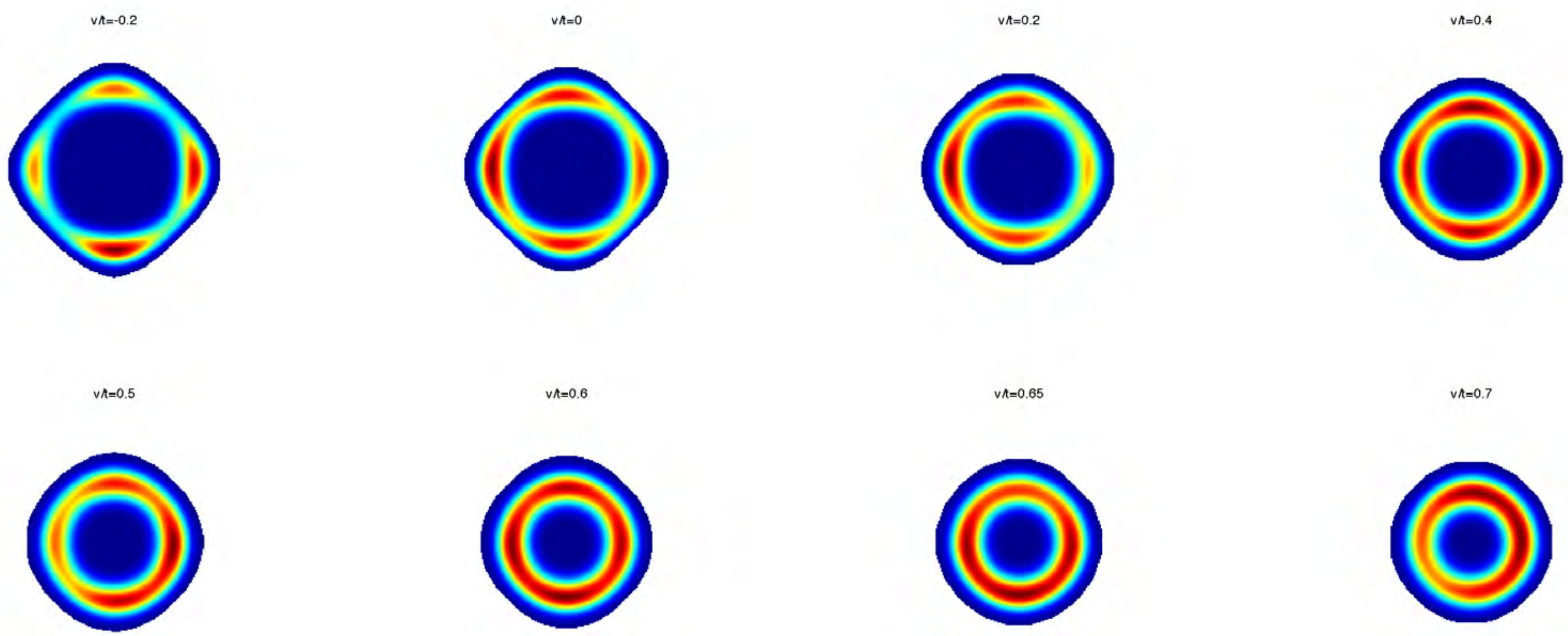
$v/t = 0.5$

$v/t = 0.7$

L=16

$v/t = -0.2$

$v/t = 0.4$



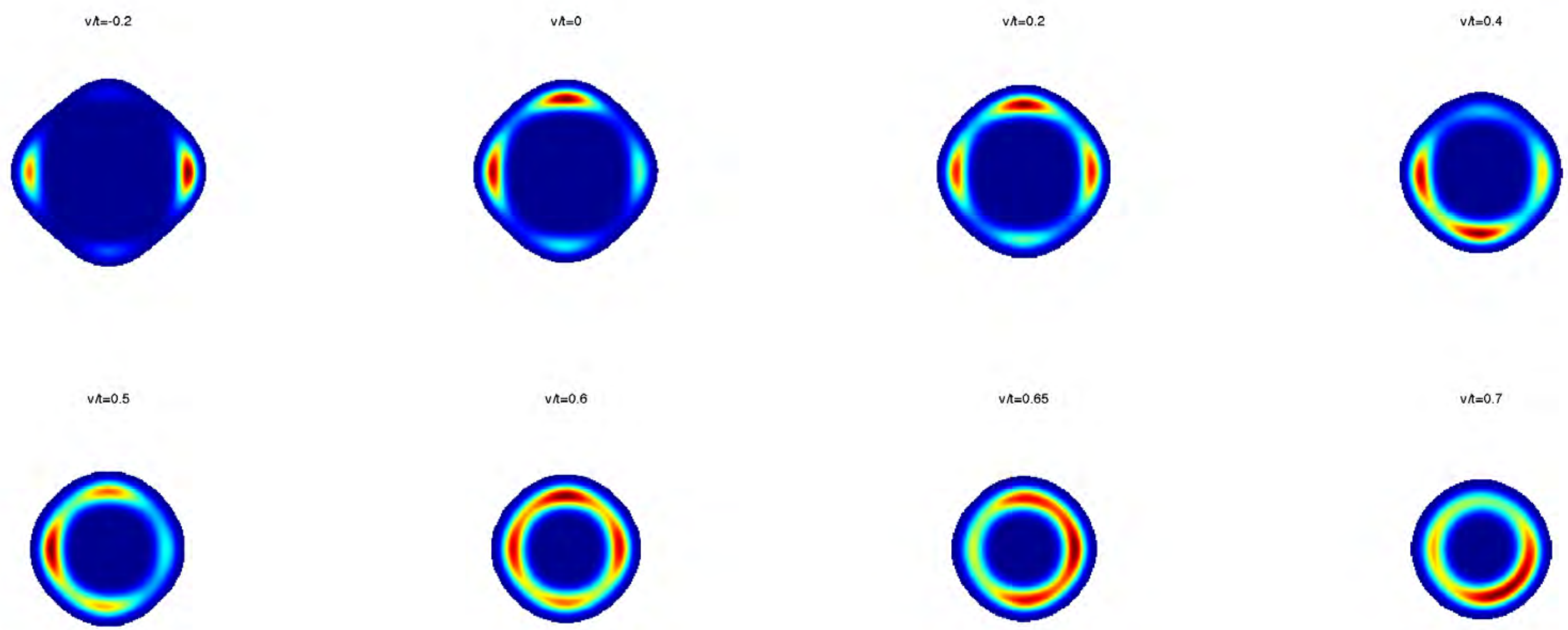
$v/t = 0.5$

$v/t = 0.7$

L=24

$v/t = -0.2$

$v/t = 0.4$



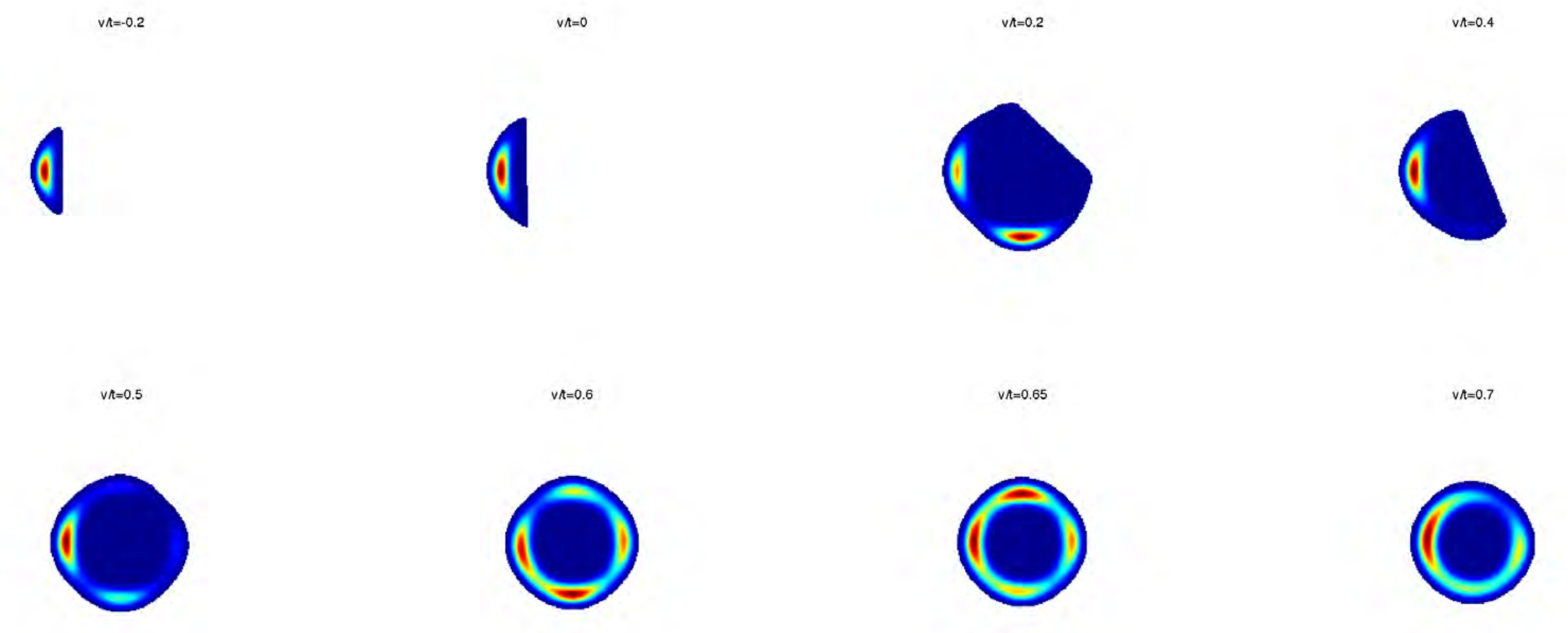
$v/t = 0.5$

$v/t = 0.7$

L=32

$v/t = -0.2$

$v/t = 0.4$

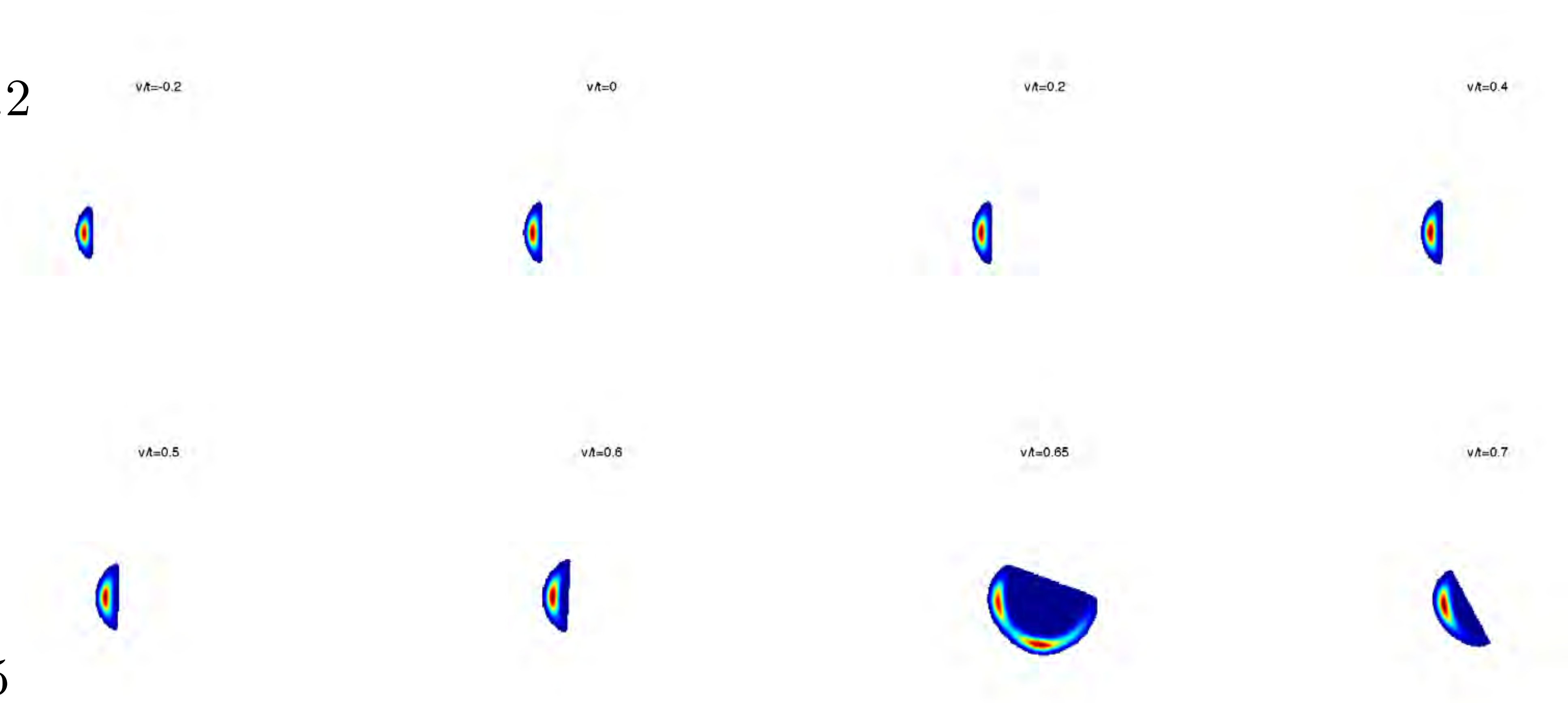


$v/t = 0.5$

$v/t = 0.7$

L=48

$v/t = -0.2$ $v/t = 0.4$

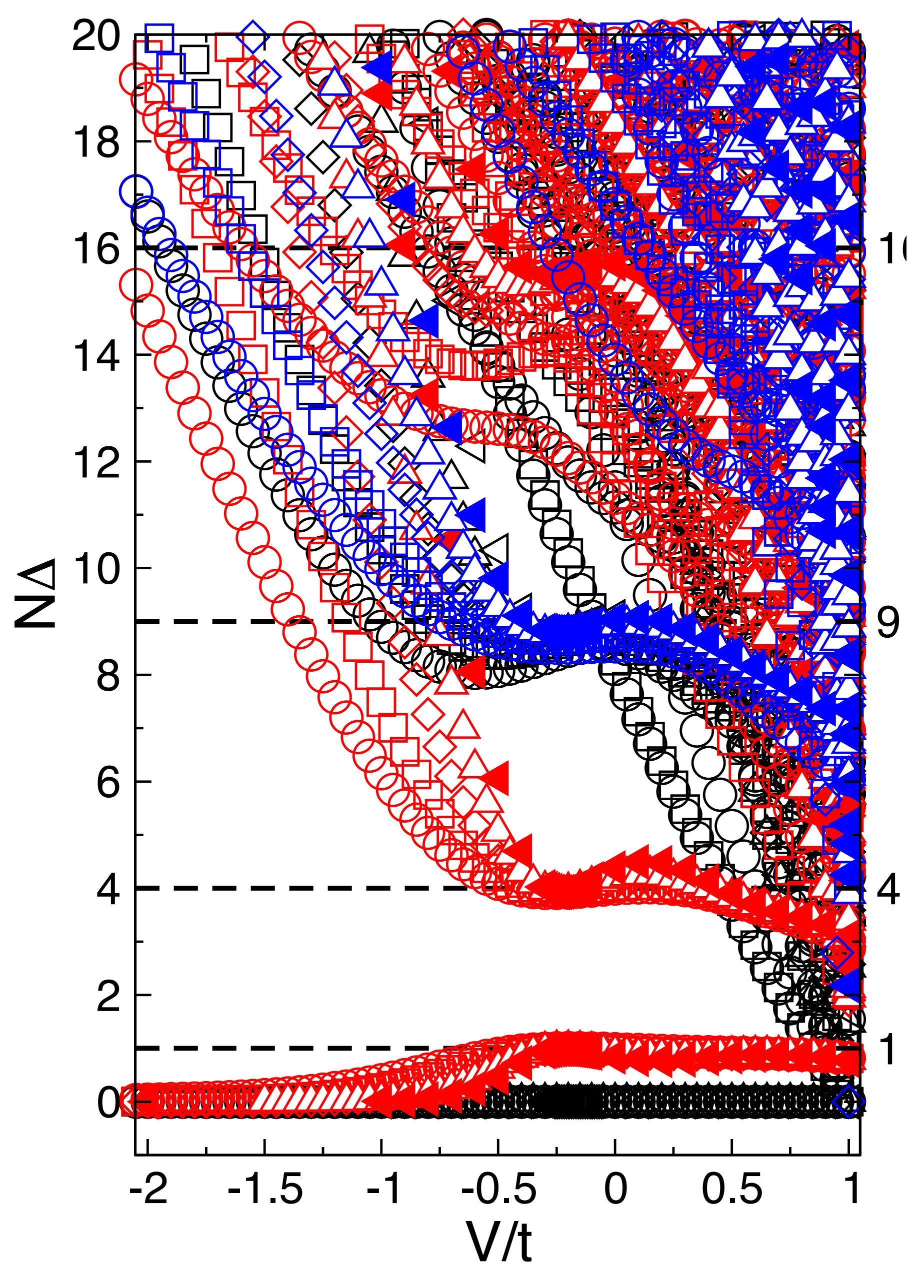


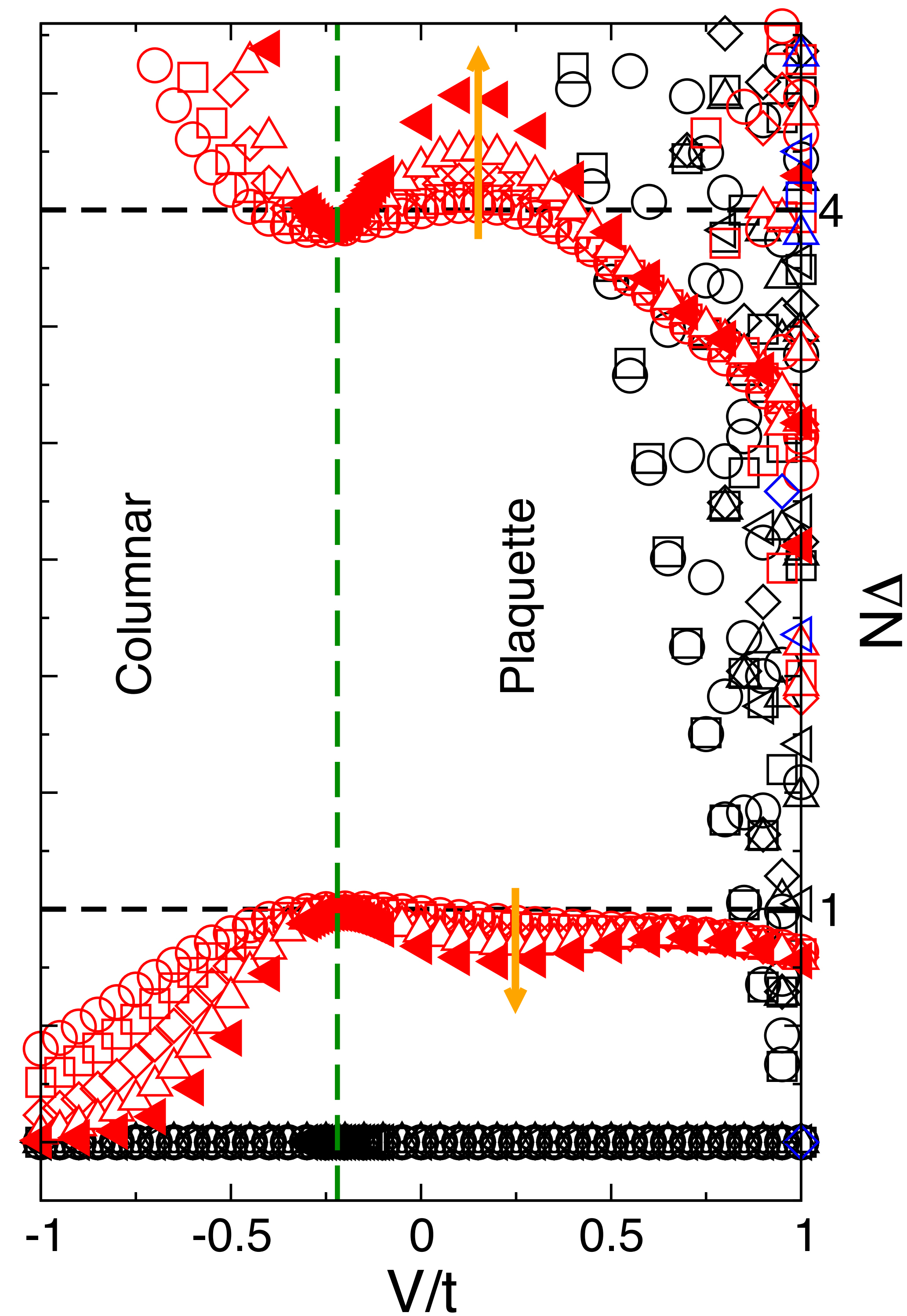
$v/t = 0.5$ $v/t = 0.7$

Locking into a columnar state in a large part of the phase diagram; ergodic issues

RK QDM on the honeycomb lattice

- Let us briefly compare to what happens on the honeycomb lattice (ED, N=126):
  - ED Energy spectrum (up to





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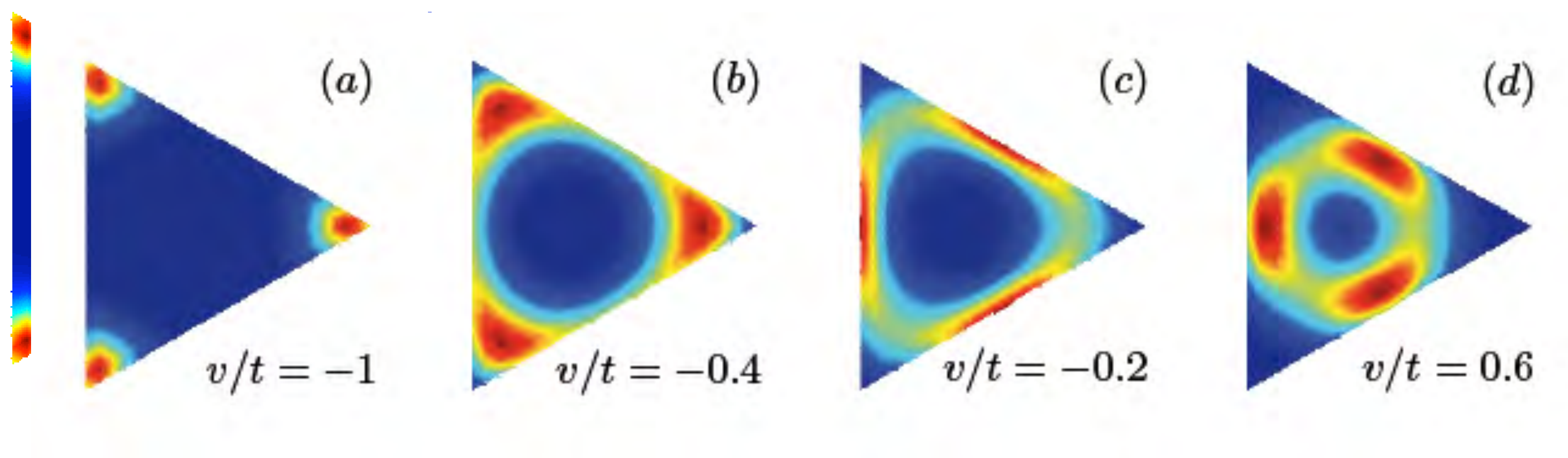


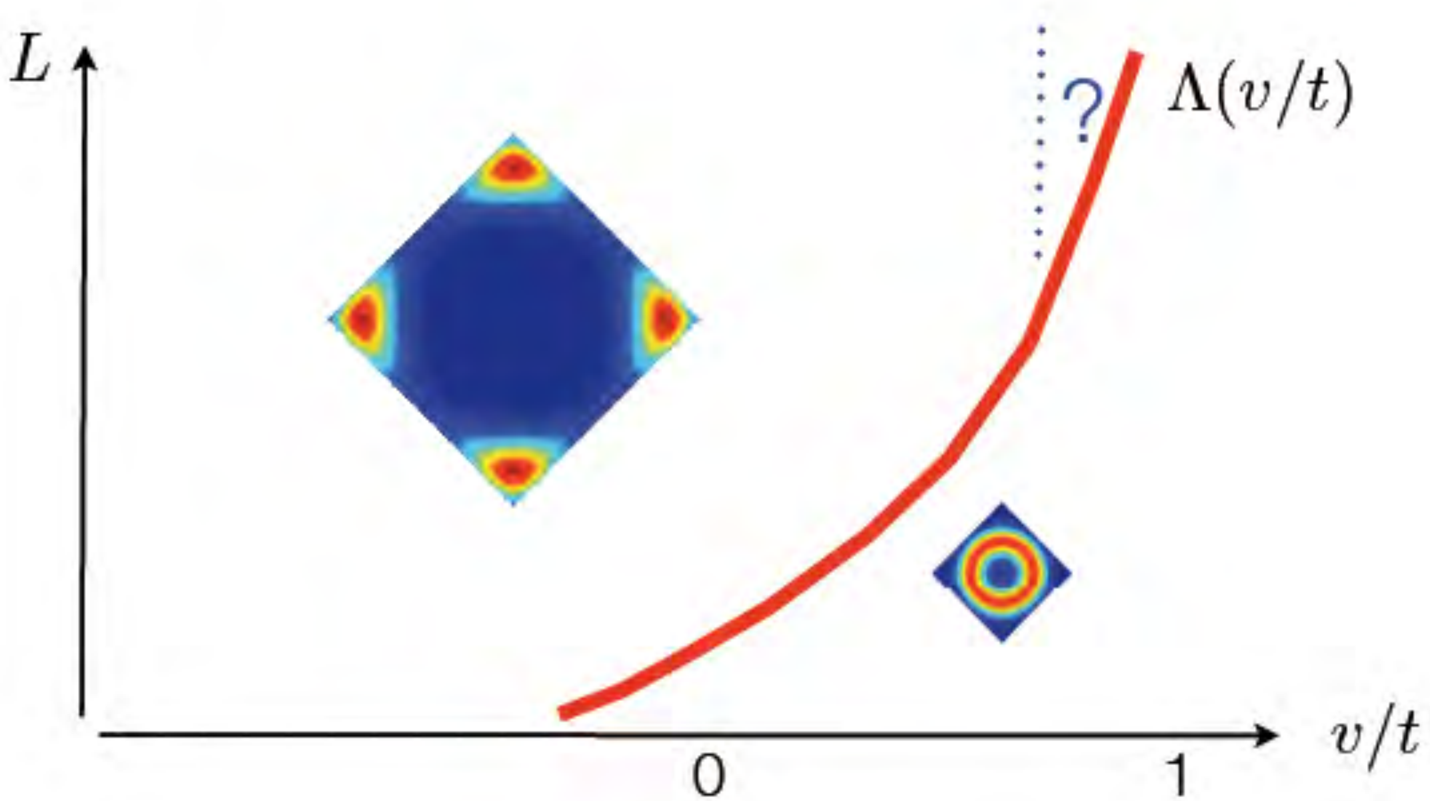
- Columnar phase, plaquette phase, first order transition between them, as expected....

cf R. Moessner, S. L. Sondhi, and P. Chandra, PRB 64, 144416 (2001).

- ED can detect both phases. Indeed $\xi_c \sim \xi^\theta$ with $\theta = 5/2$ (honeycomb)

  compared to $\theta = 6$ (square)

Conclusion 2D bipartite RK QDMs



No evidence for mixed or plaquette phases...

see also Banerjee et al. '15

- U(1) symmetric behavior seen in bipartite lattice QDM
- identification of parameters of effective U(1) description (useful for d=3+1)
- what happens in frustrated square lattice spin models ?
- Similarity to U(1)-like histograms seen in some VBS-Néel transitions ?

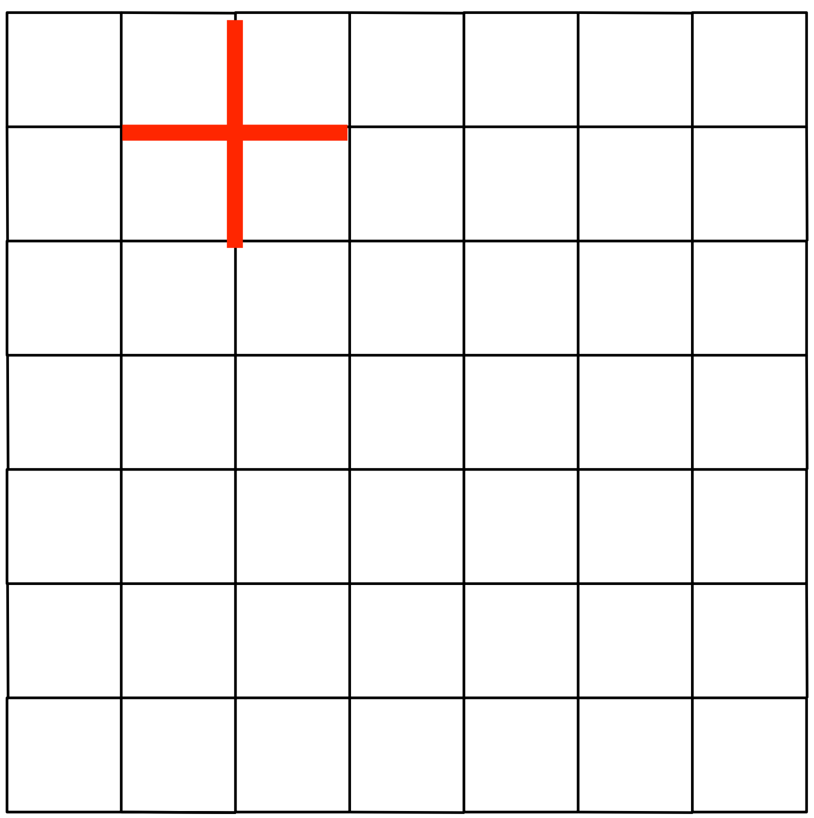
Part 4 : Back to spins

Cano-Fendley models

- Local S=1/2 spin models on the square lattice

Cano & Fendley, '10

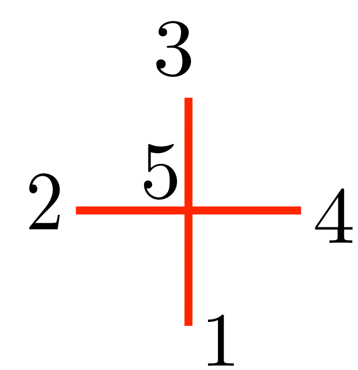
$$H_{\mathrm{CF0/1}} = \sum_{+} H_{+} + \sum_{\square} H_{\square} + H_{\square}$$



$$H_{+} = P^{S_{+}=5/2}$$

$$\propto (\mathbf{S}_{+}\cdot\mathbf{S}_{+} - 3/4)(\mathbf{S}_{+}\cdot\mathbf{S}_{+} - 15/4)$$

$$\mathbf{S}_{+} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4 + \mathbf{S}_5$$



Annihilates all nearest-neighbors valence bond states

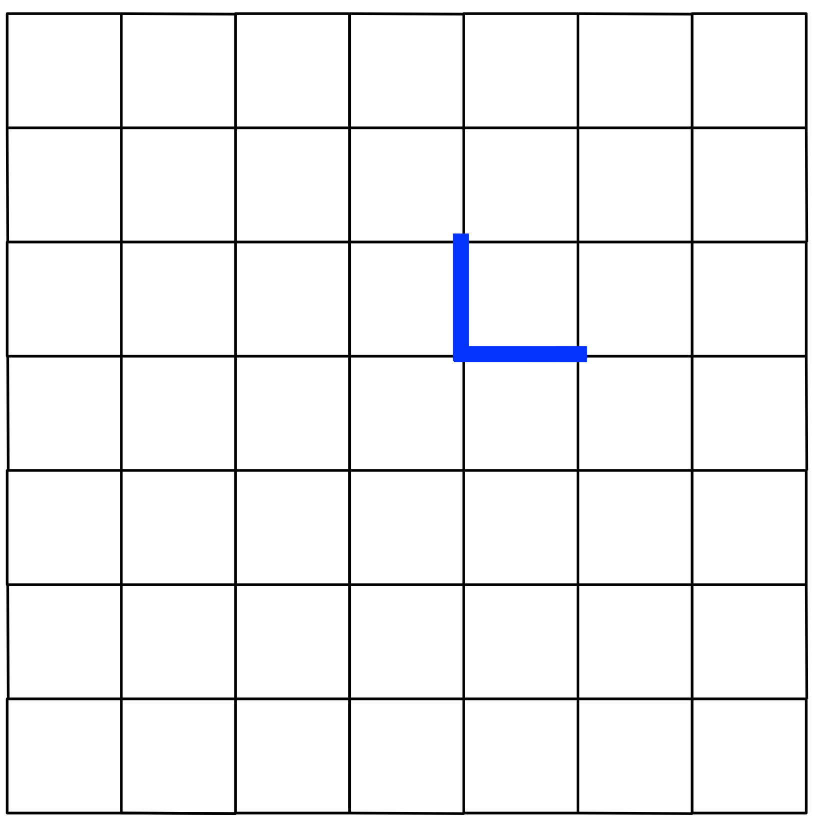
("Klein term")

and others ...

Cano-Fendley models

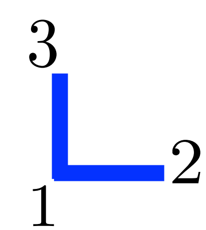
- Local S=1/2 spin models on the square lattice

$$H_{\mathrm{CF0/1}} = \sum_{+} H_{+} + \sum_{\square} H_{\square} + H_{\square}$$



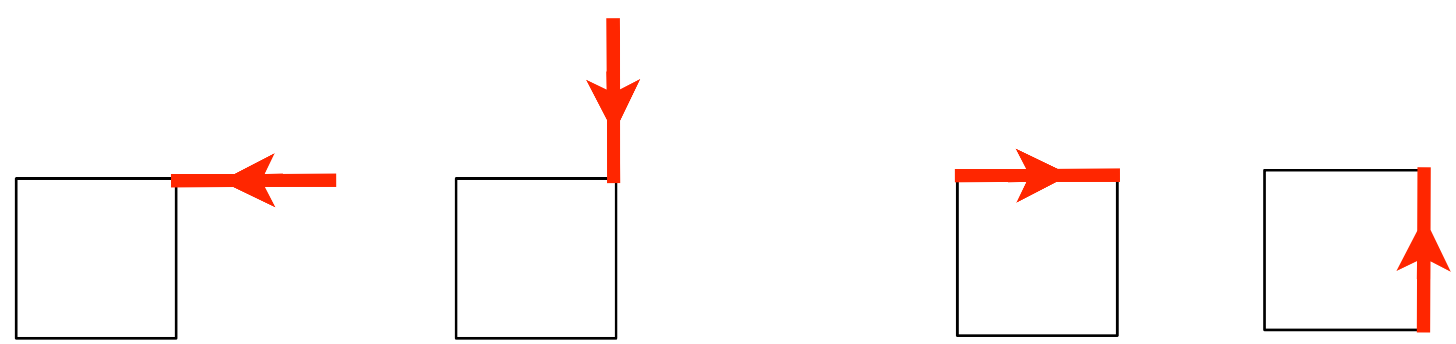
$$H_{\square\llcorner} = P^{S_{\llcorner}=3/2} \propto (\mathbf{S}_{\llcorner}\cdot\mathbf{S}_{\llcorner} - 3/4)$$

$$\mathbf{S}_{\llcorner} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$



In the nearest-neighbors VB subspace:

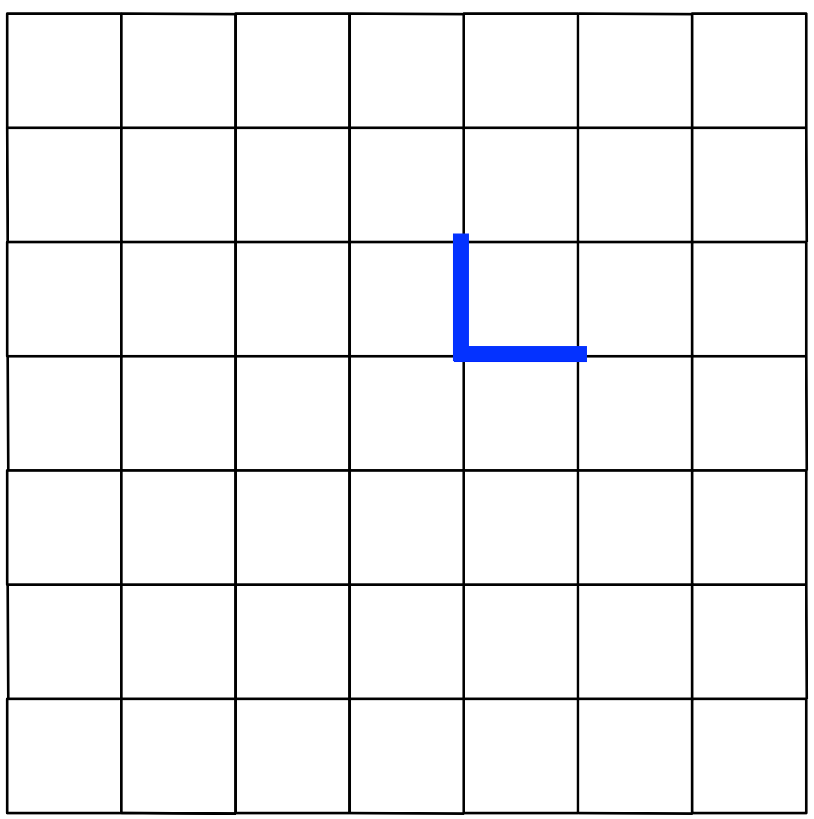
Will force a dimer on the plaquette



Cano-Fendley models

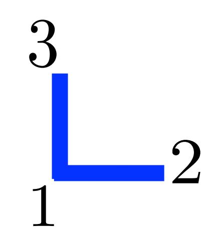
- Local S=1/2 spin models on the square lattice

$$H_{\mathrm{CF}0/1} = \sum_{+} H_{+} + \sum_{\square} H_{\llcorner\,\square\,\urcorner} + H_{\lrcorner\,\square\,\ulcorner}$$



$$H_{\square\llcorner} = P^{S_{\llcorner}=3/2} \propto (\mathbf{S}_{\llcorner} \cdot \mathbf{S}_{\llcorner} - 3/4)$$

$$\mathbf{S}_{\llcorner} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3$$



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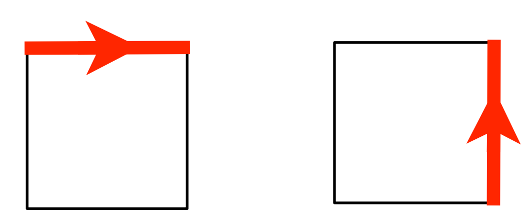
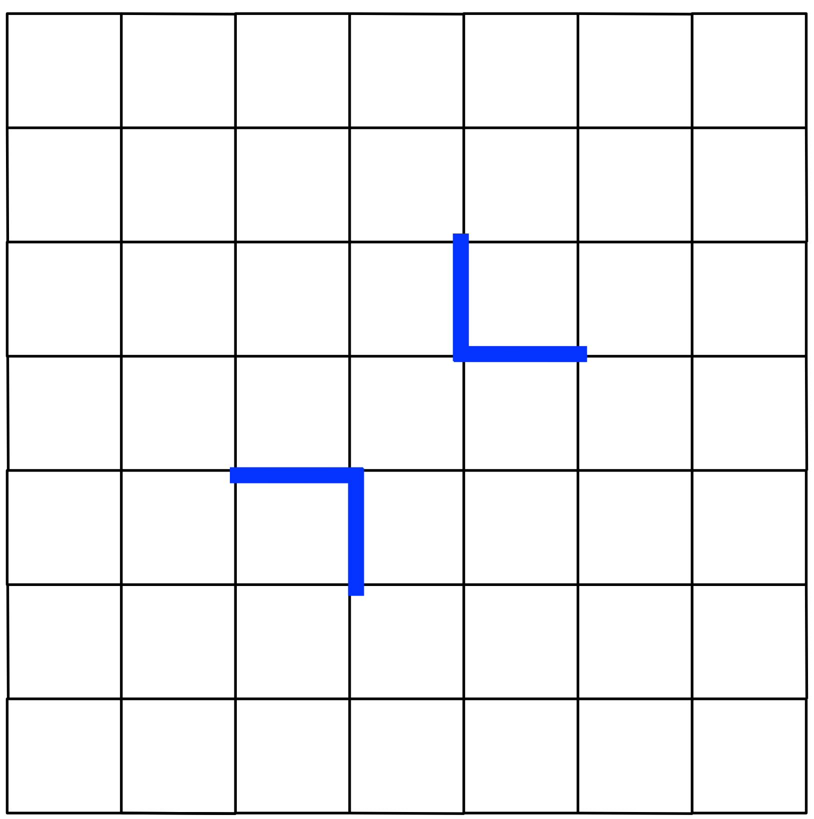


Cano-Fendley models

- Local S=1/2 spin models on the square lattice

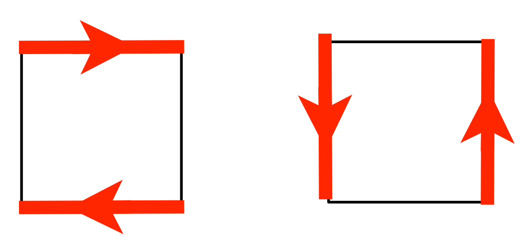
$$H_{\mathrm{CF0/1}} = \sum_{+} H_{+} + \sum_{\square} H_{\square} + H_{\square}$$



$$H_{\square} = P^{S_{\llcorner}=3/2} P^{S_{\urcorner}=3/2}$$

In the nearest-neighbors VB subspace:

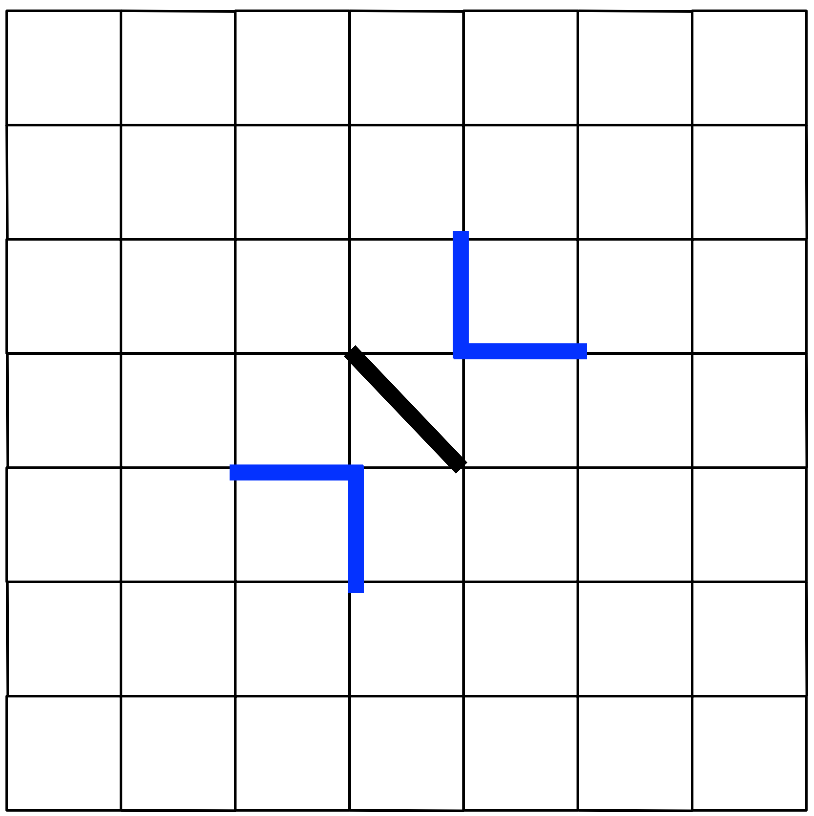
Will force flippable plaquettes



Cano-Fendley models

- Local S=1/2 spin models on the square lattice

$$H_{\mathrm{CF0/1}} = \sum_{+} H_{+} + \sum_{\square} H_{\boxbslash} + H_{\boxslash}$$



$$H_{\boxbslash} = P^{S_{\llcorner}=3/2}\, P^{S_{\urcorner}=3/2}\, P^{S_{\boxbslash}=0/1}$$

$$P^{S_{\boxbslash}=0} \propto (\mathbf{S}_{\boxbslash}\cdot\mathbf{S}_{\boxbslash} - 2)$$

$$P^{S_{\boxbslash}=1} \propto \mathbf{S}_{\boxbslash}\cdot\mathbf{S}_{\boxbslash}$$

In the nearest-neighbors VB subspace:

flip plaquettes

$$P^{S_{\boxbslash}=0}\,|\,\square\,\rangle = -\,|\,\square\,\rangle + |\,\square\,\rangle$$

$$P^{S_{\boxbslash}=1}\,|\,\square\,\rangle = +\,|\,\square\,\rangle + |\,\square\,\rangle$$

Cano-Fendley models

- In the nearest-neighbors VB subspace:

Look similar to Quantum Dimer Model (at RK point)

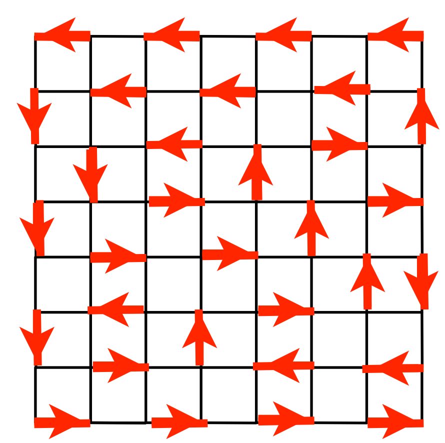
$$H_{\rm RK} = \sum_{\Box} |\Box\rangle\langle\Box| - |\Box\rangle\langle\Box|$$

Let's see later ...

- Therefore expect Sutherland ground-states !

$$|\Psi\rangle = \sum_c |c\rangle$$

$|c\rangle$ covering of the square lattice with nearest-neighbour valence bonds



- What about outside this subspace ?
  - Study with exact diagonalization : full and n.n. VB restricted
  - Compare with Quantum Dimer Model

Ground-state degeneracies

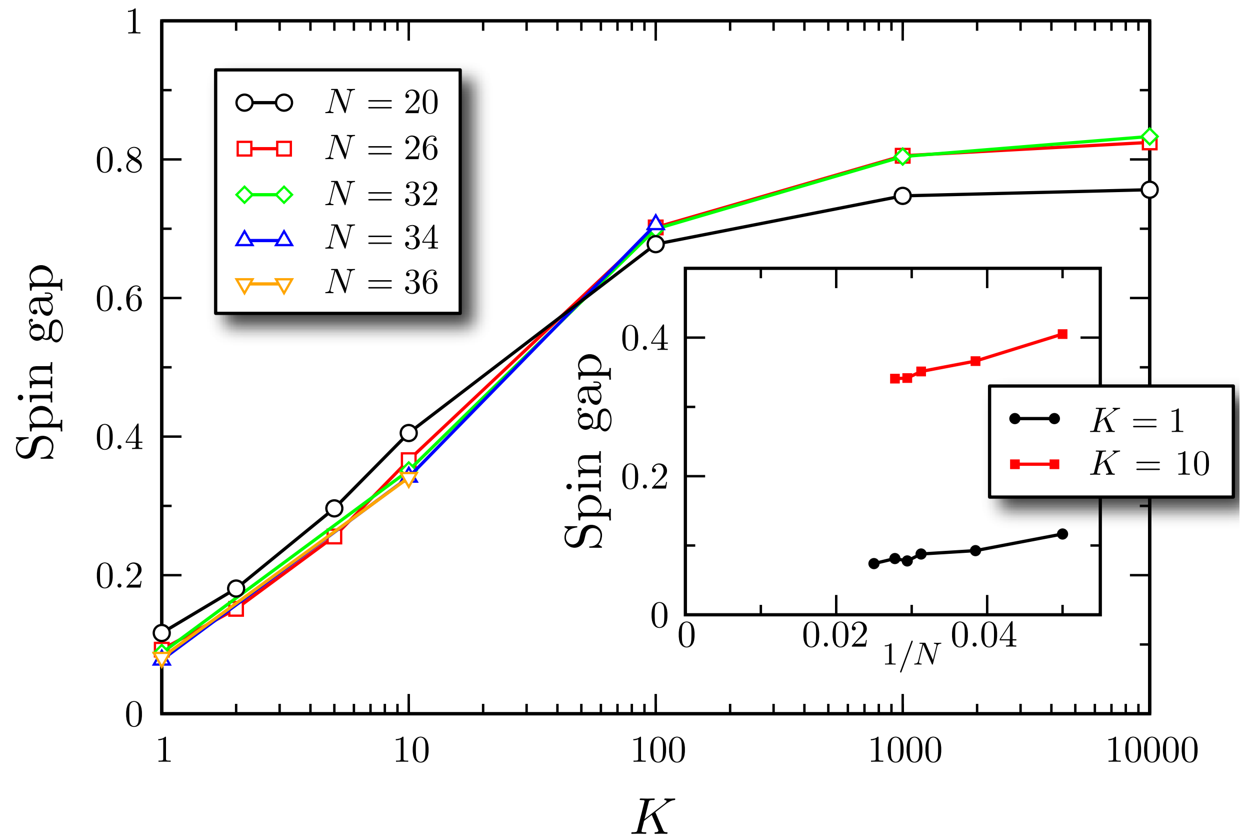
- Zero energy states degeneracies on a torus (N sites)

	Cano-Fendley (full)	n.n. VB restricted	QDM
N=16	23	17	17
N=20	13	13	13
N=26	16	16	16
N=32	69	69	69
N=36	-	41	41
N=40	-	29	29
N=50	-	47	47

- Same ground-states for VB variational subspace and QDM
- Extensive torus degeneracy easily understood from QDM
- No spurious ground-states for the full model (except N=16)

Spin gap

- Role of the Klein term amplitude K for one of the models

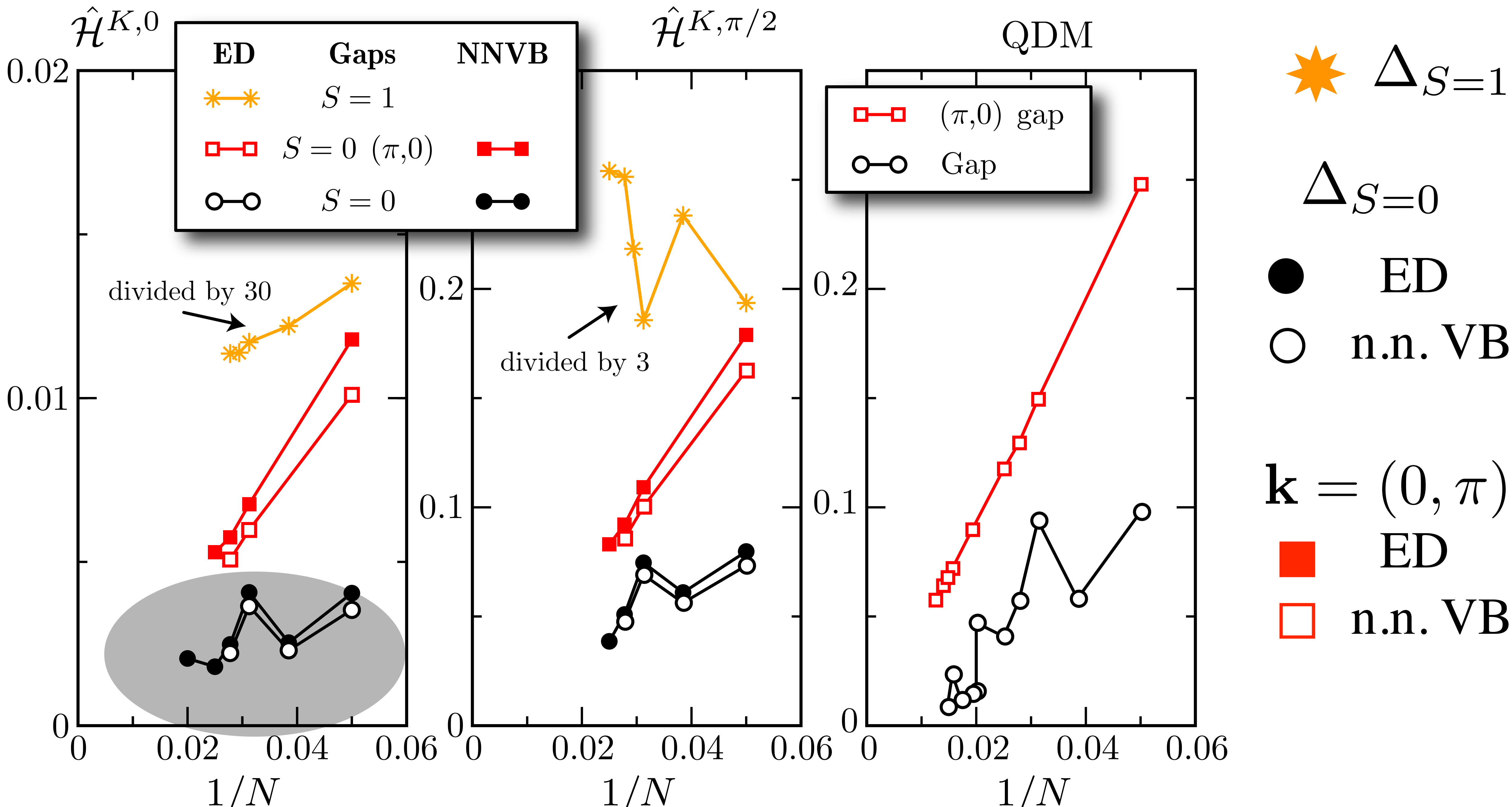


K=10 in the following

- Generalization of the two Cano-Fendley models

$$\hat{\mathcal{H}}^{K,\theta} = K\hat{\mathcal{H}}_1 + \frac{1}{n(\theta)}\left(\cos\theta\,\hat{\mathcal{H}}_2^0 + \sin\theta\,\hat{\mathcal{H}}_2^1\right)$$

Gaps



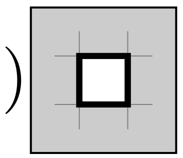
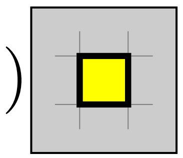
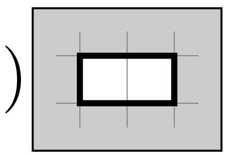
- Gapless S=0 excitations (very likely)
- Finite spin gap

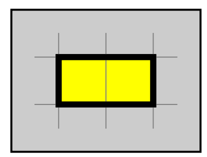
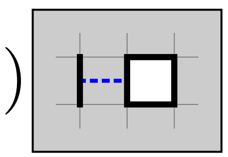
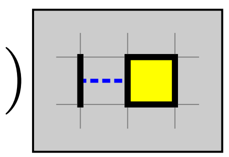
- QDM (gapless) and CF have similar low-lying singlet structure
- Can push variational NNVBs up to N=50

Generalized QDM mapping

- Systematic derivation of the effective QDM model

Mambrini et al.

$\mathcal{H}^{\mathrm{eff},\theta} = -t_4(\theta)$  $+ v_4(\theta)$  $- t_6(\theta)$ 

$+ v_6(\theta)$  $- t'_4(\theta)$  $+ v'_4(\theta)$ 

$+$ larger loops

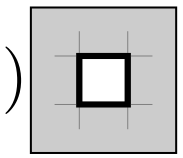
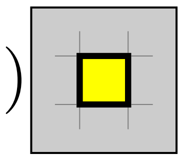
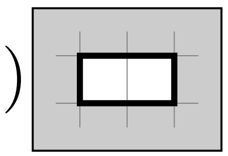
	$\hat{\mathcal{H}}^0$	$\hat{\mathcal{H}}^1$	**A**	**B**	**C**
	$\theta = 0$	$\theta = \pi/2$	$\theta_A = -\arccos\left(-\frac{3}{\sqrt{10}}\right)$	$\theta_B = -\arccos\left(\frac{3}{\sqrt{10}}\right)$	$\theta_C = \arccos\left(\frac{1}{\sqrt{10}}\right)$
t_4	0.25	-0.25	-0.125	0.25	-0.125
v_4	0.25	0.25	-0.25	0.125	0.25
t_6	-0.0885651	0.202914	0.015695	-0.117152	0.130044
v_6	-0.031203	-0.0308274	0.0311091	-0.0156954	-0.0309213
t'_4	0.063808	0.0742716	-0.0664239	0.0292881	0.0716557
v'_4	0.0364349	0.0779137	-0.0468046	0.00784772	0.067544
$v_4/\lvert t_4\rvert$	1	1	$-2.$	0.5	2.

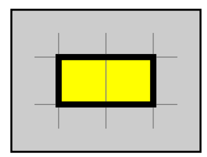
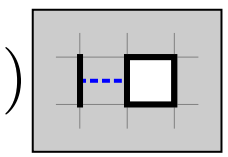
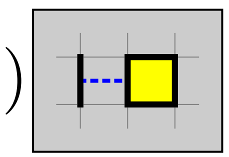
Original Cano-Fendley models maps approx. to RK point of t-V QDM

Generalized QDM mapping

- Systematic derivation of the effective QDM model

Mambrini et al.

$\mathcal{H}^{\text{eff},\theta} = -t_4(\theta)$  $+\, v_4(\theta)$  $-\, t_6(\theta)$ 

$+\, v_6(\theta)$  $-\, t'_4(\theta)$  $+\, v'_4(\theta)$ 

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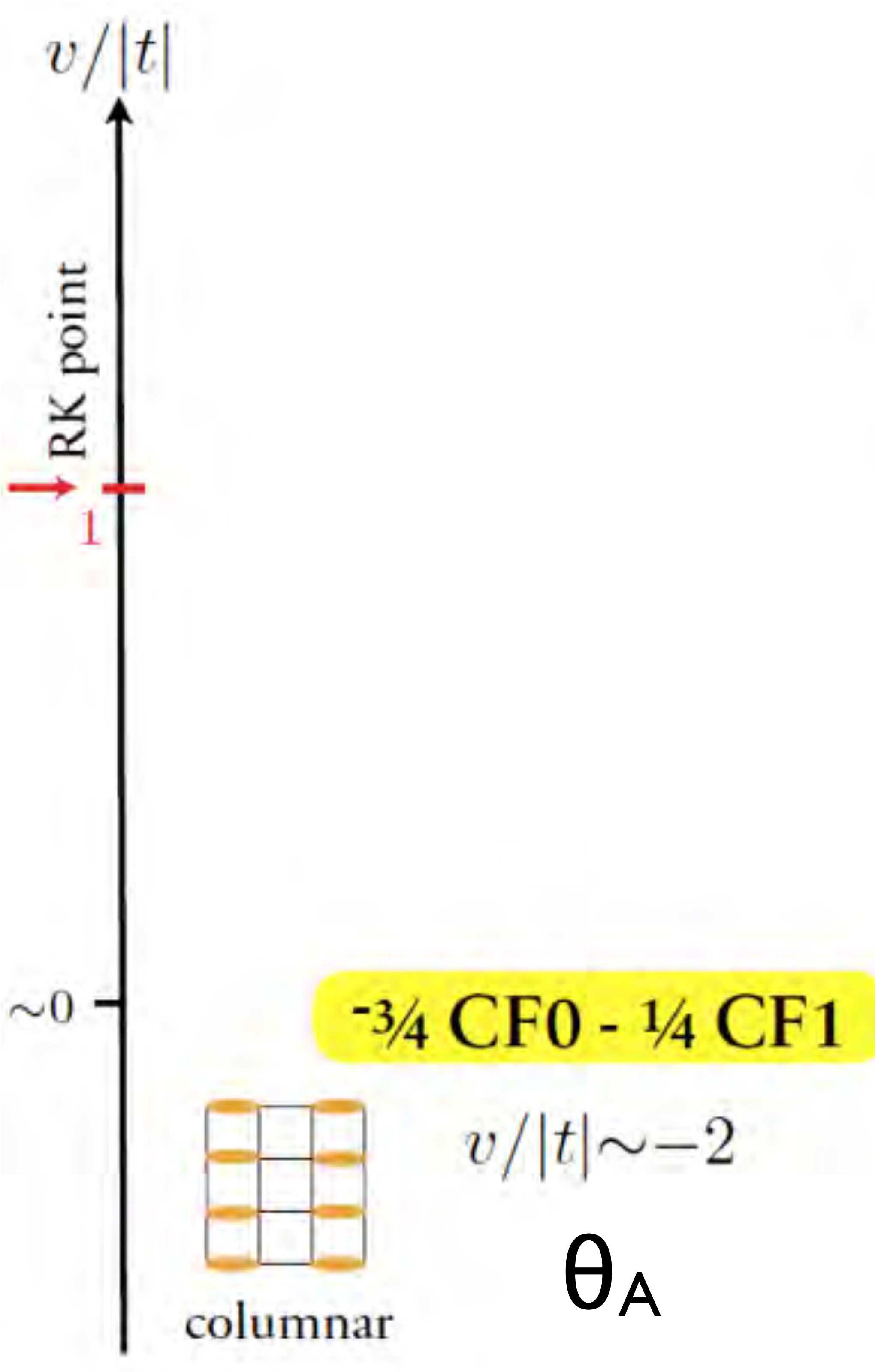
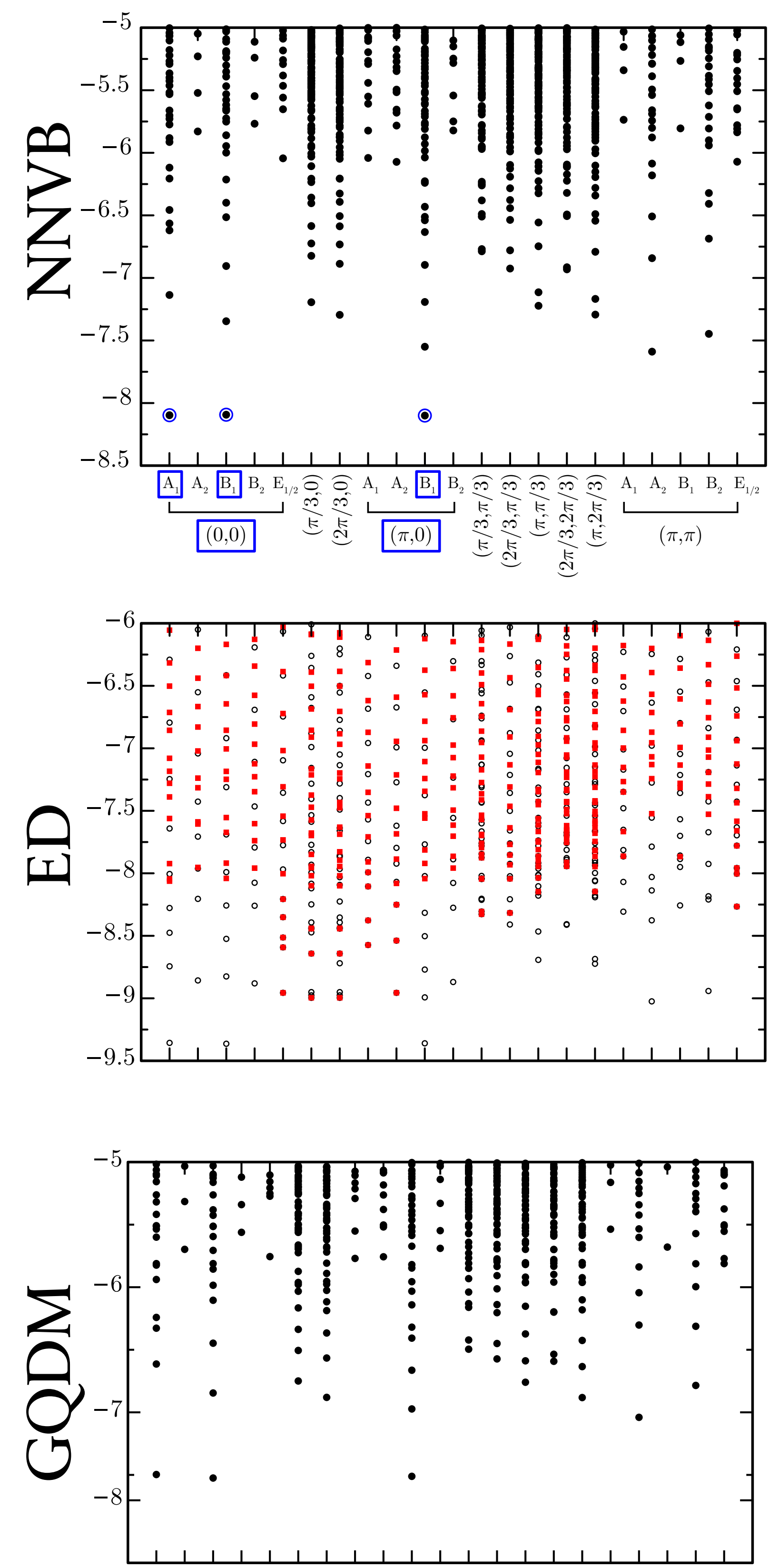
v'_4	0.0364349	0.0779137	-0.0468046	0.00784772	0.067544
$v_4/\lvert t_4\rvert$	1	1	$-2.$	0.5	2.

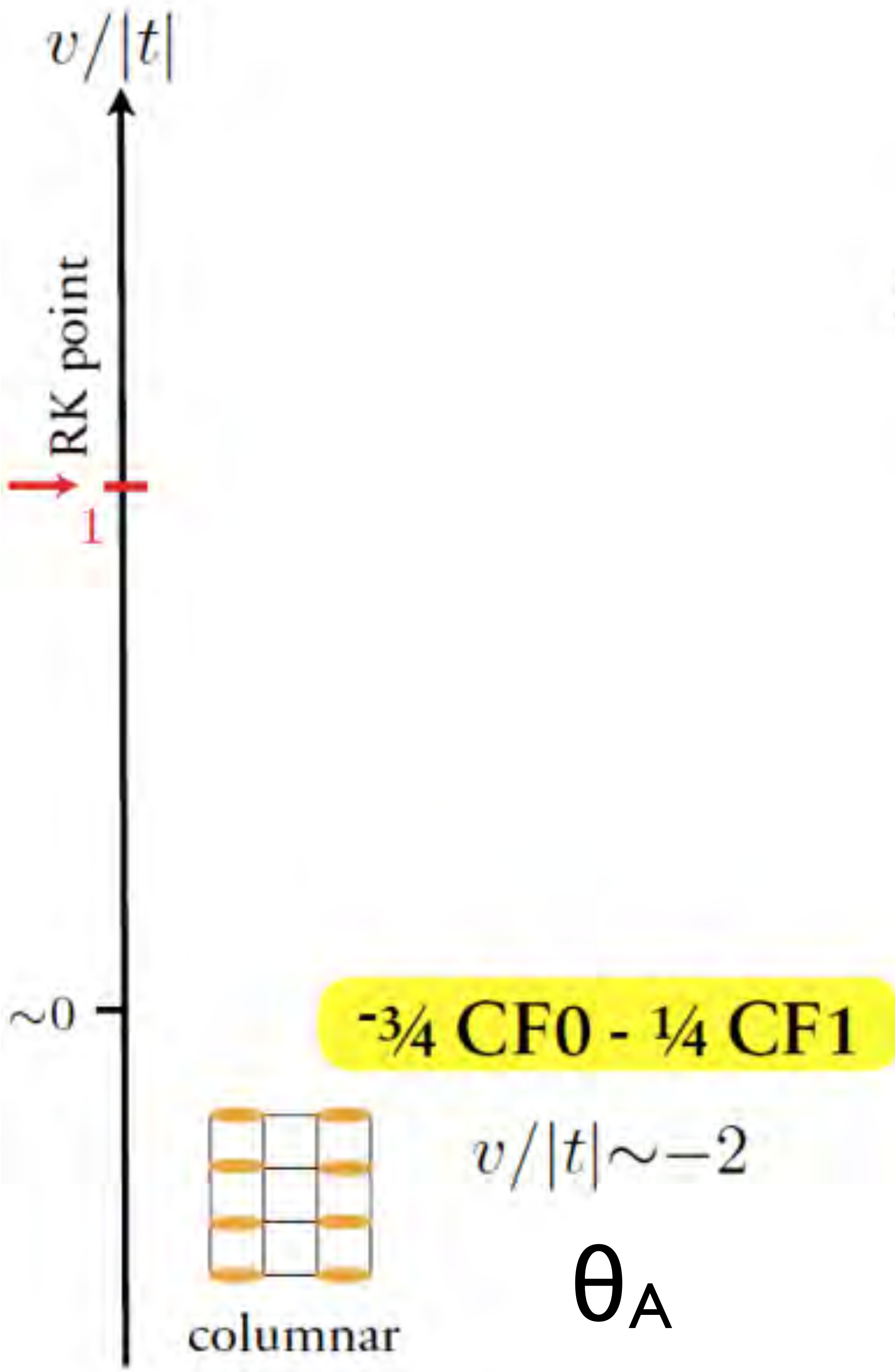
Generalizations map approx
onto *any* point in the phase diagram

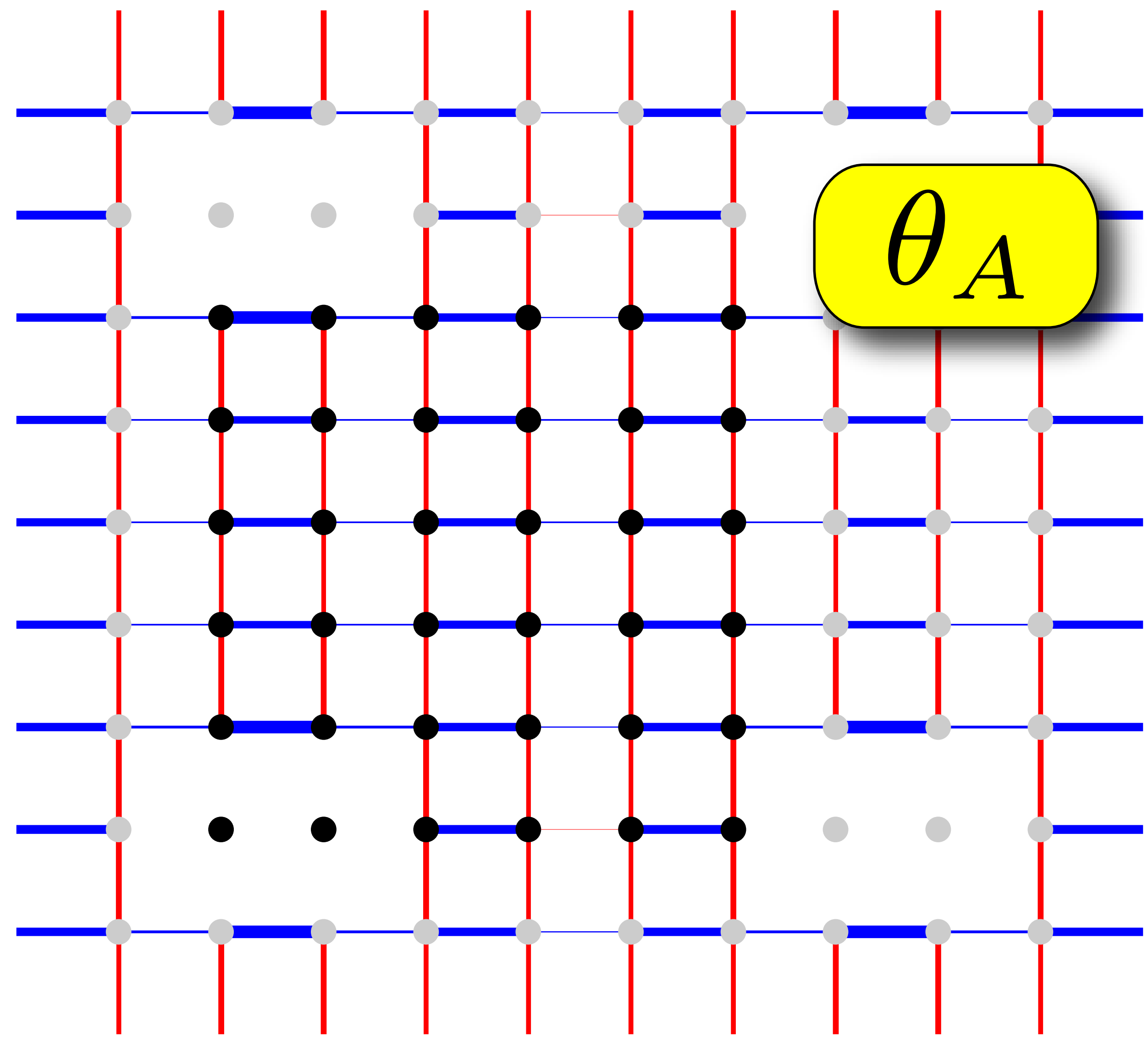
Mixing CF models



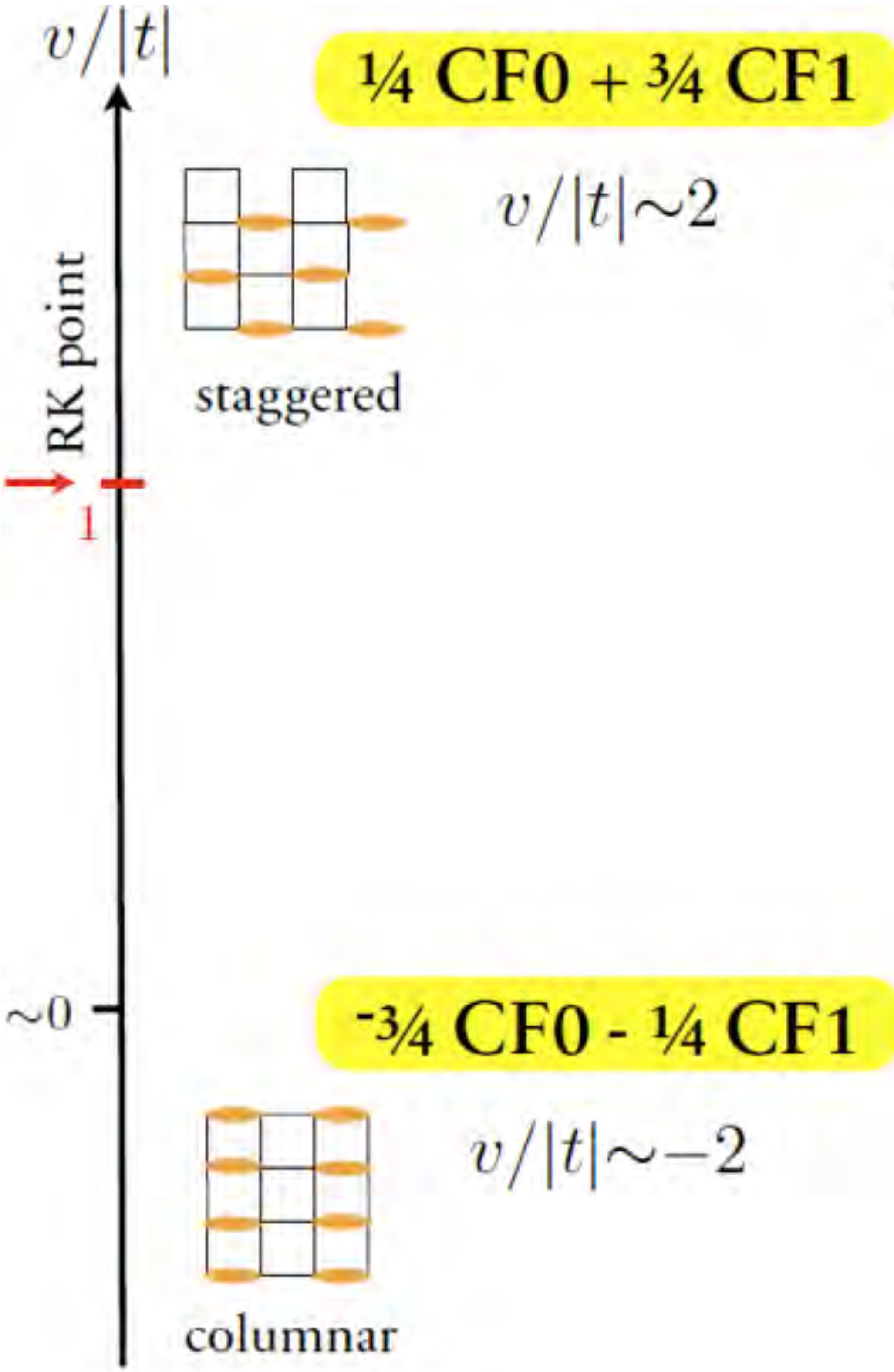


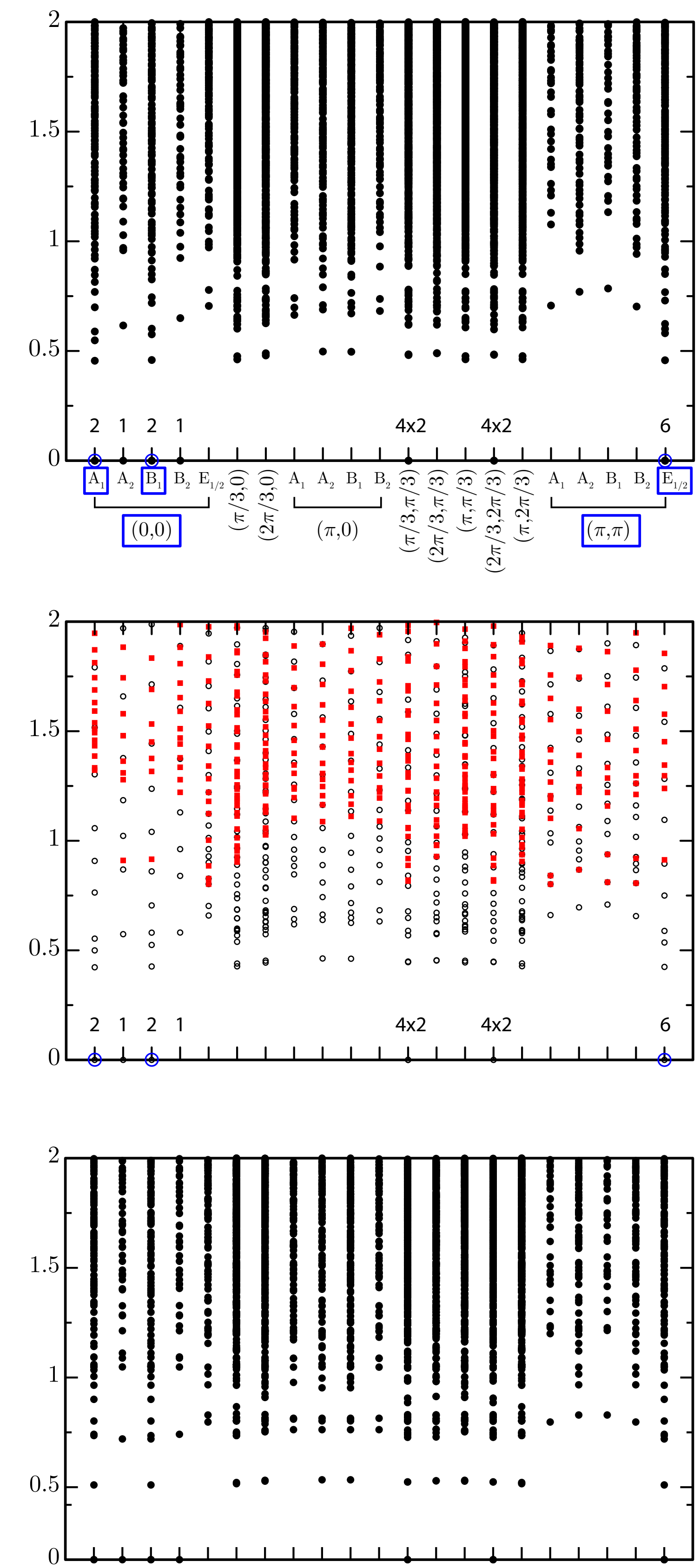
Mixing CF models





Mixing CF models





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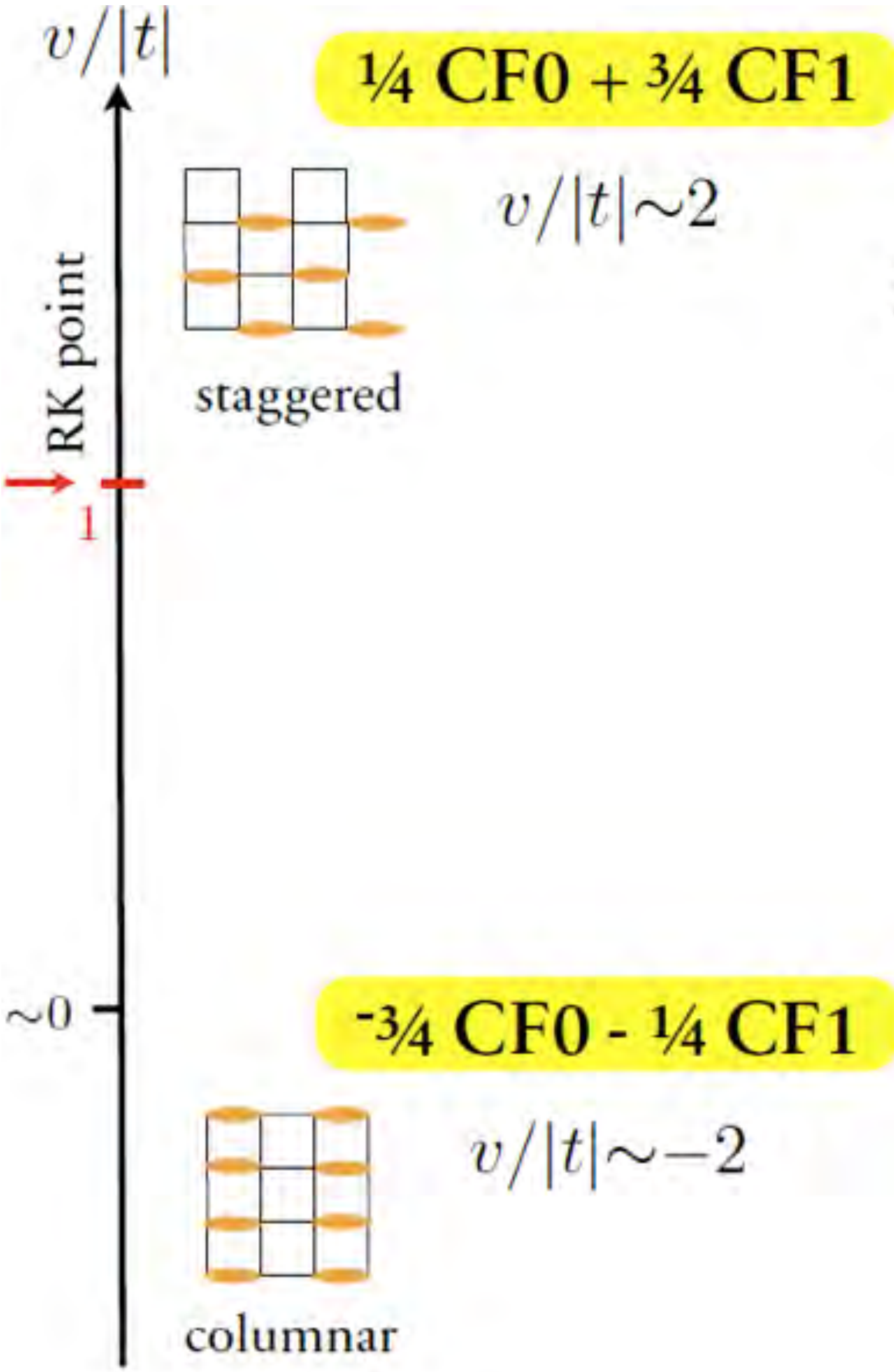
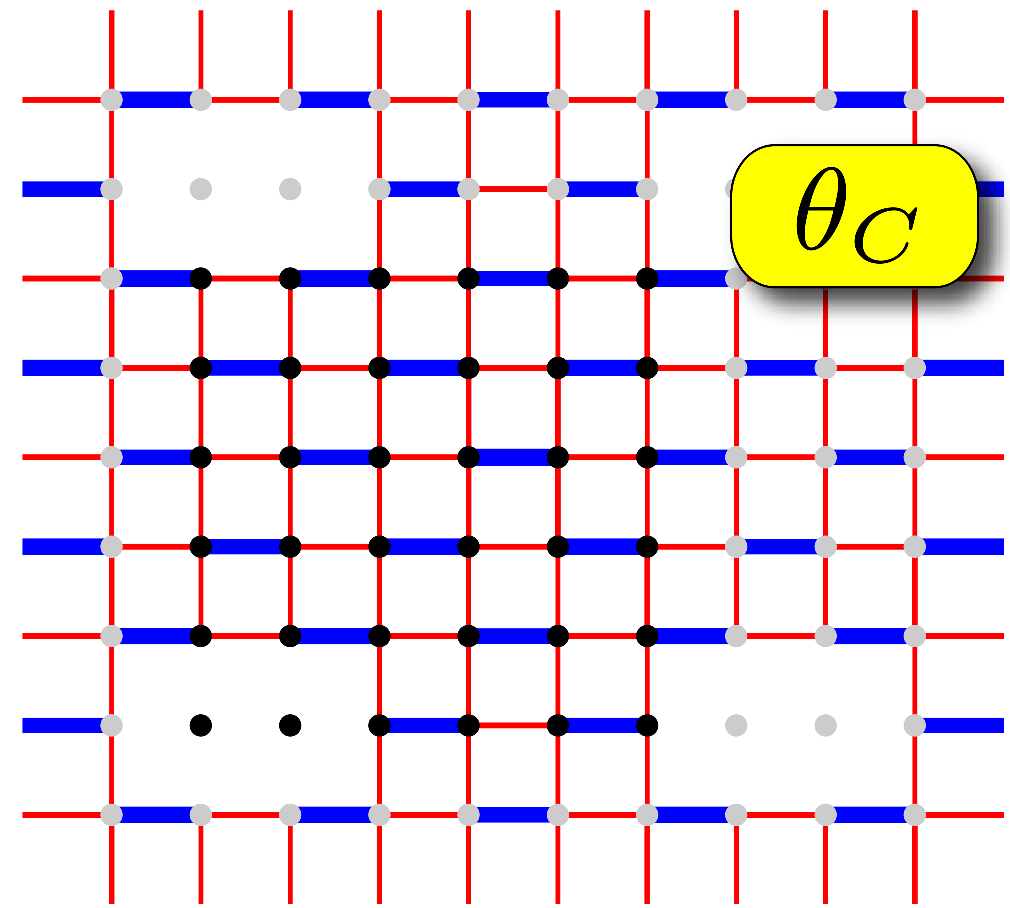
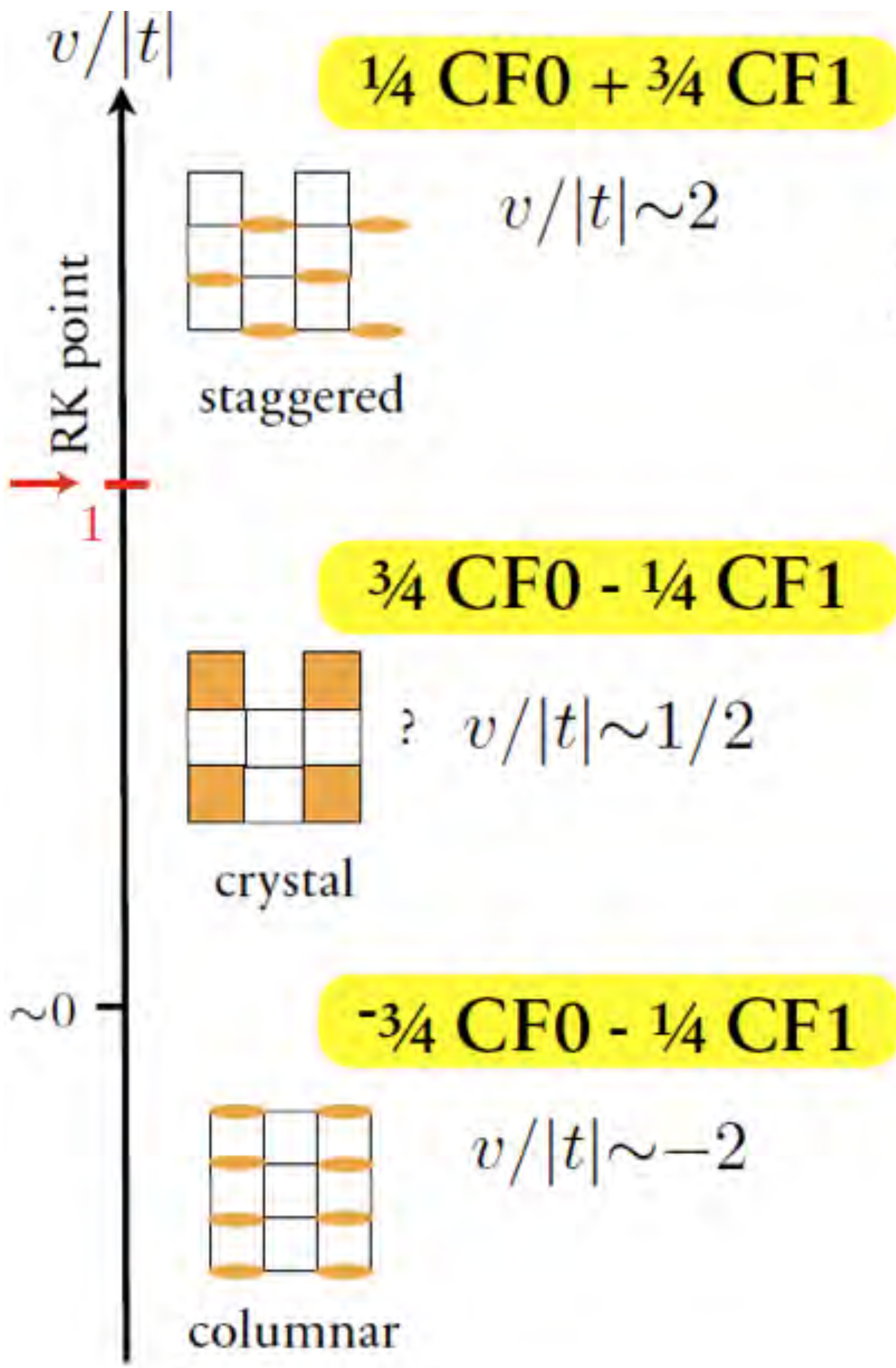
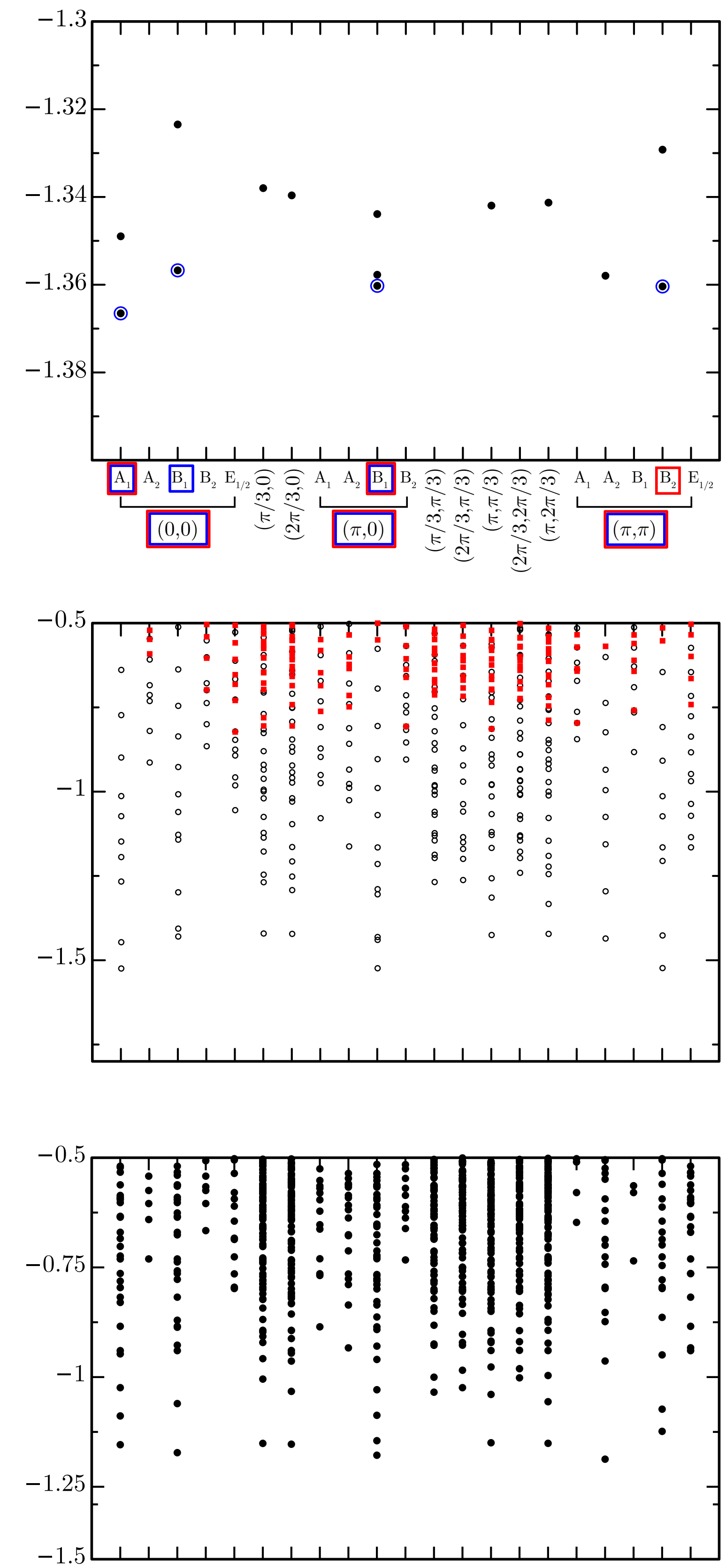




Mixing CF models





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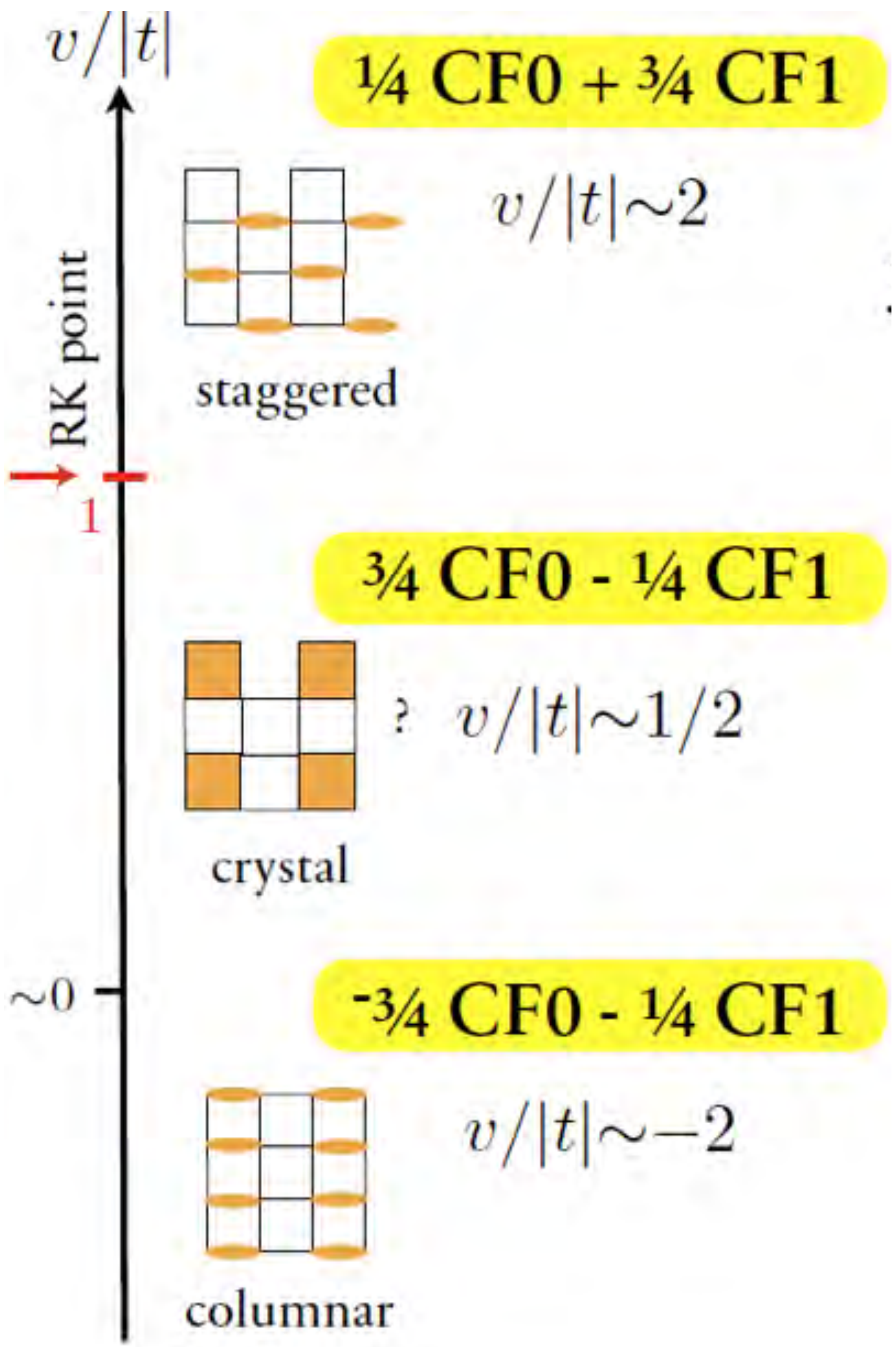
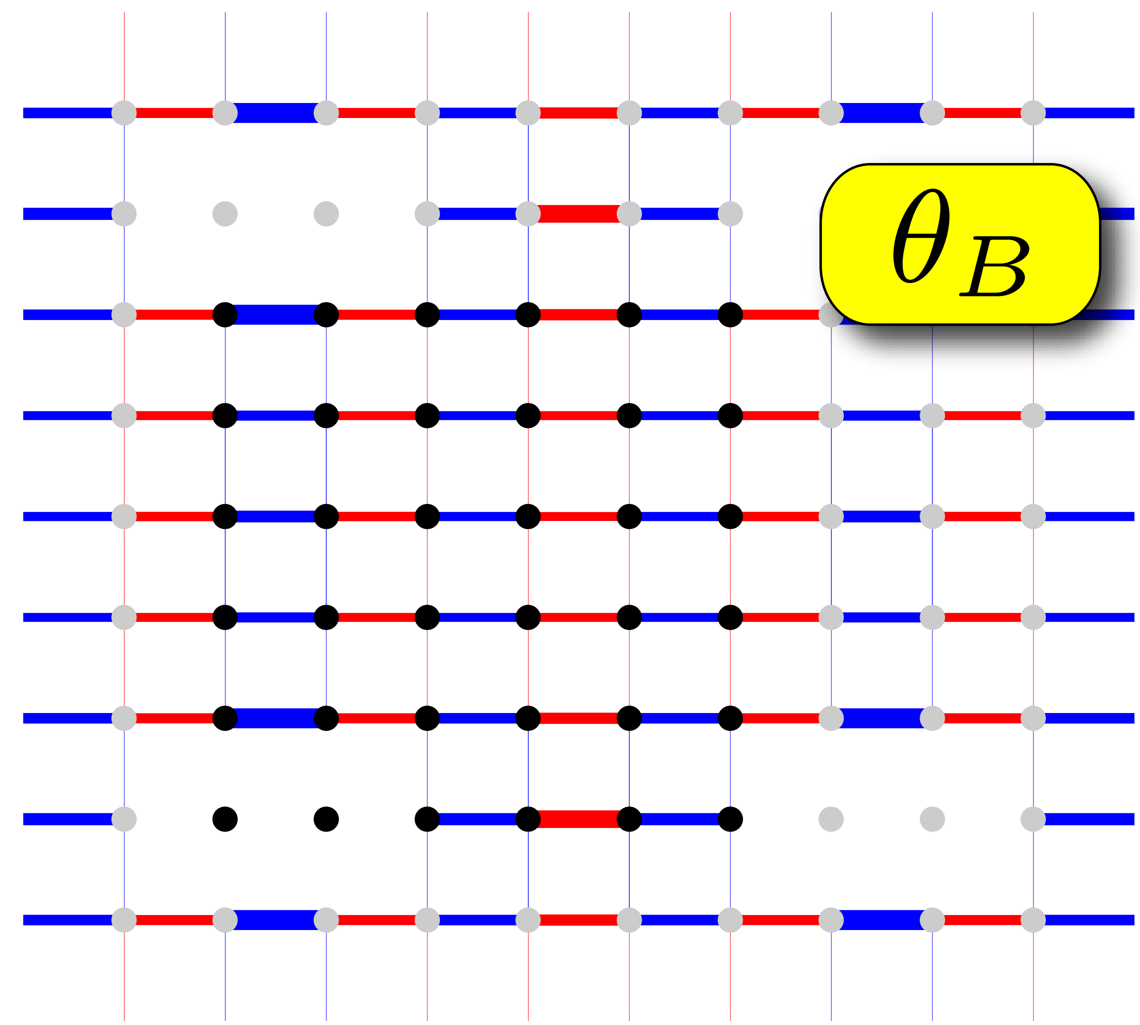
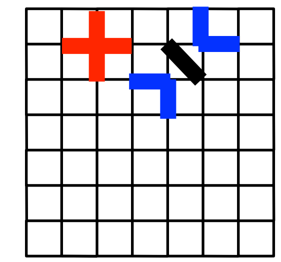




ambiguous signal

Conclusions

- Ground-states of the local S=1/2 SU(2) CF spin models
  - Unusual type of spin liquid
    - spin gap, but no S=0 gap
    - QDM and CF have similar low-lying singlets
    - Mixing angle θ: all phases of QDM can be found !



QDM ⟶ QDM-like Spin Model ⟶ GQDM

(Almost a round trip)

- Extensions
  - Stability w.r.t. Heisenberg interactions: spin liquid to VBC then Néel
  - Extend CF scheme to build a robust Spin Liquid on other geometries
  - nn RVB states can be written as PEPS

Schuch, Poilblanc, Cirac, Perez-Garcia ‘12