On the status of classification of supergravity solutions: successes and open problems

George Papadopoulos

King's College London

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Why supergravity solutions

- Many of the developments in string theory and M-theory have been driven by solutions that preserve a fraction of spacetime supersymmetry
- Branes, Intersecting branes, Compactifications, solitons, instantons, localization
- black holes, near horizon geometries, uniqueness (or non-uniqueness) theorems
- AdS/CFT
- Applications to geometry: Geometries with skew-symmetric torsion, special geometric structures, generalized geometry

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Classification

- Why have supersymmetric solutions not been classified already?

A typical Killing spinor equation in type II theory is

$$\mathcal{D}\epsilon = \nabla\epsilon + Q(F)\epsilon = 0$$

where $\mathcal{D}$ is the supercovariant connection, F fluxes.

Holonomy of $\mathcal{D}$ for generic backgrounds in in a $\mathfrak{sl}(k, \mathbb{R})$ group; $k = 32$ for D=11, IIA and IIB. [Hull; Duff, Liu; Tsimpis, GP]

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What does it mean to solve the KSEs?

- Assume that a background is supersymmetric, ie there exist one or more $\epsilon \neq 0$ that solve the KSEs, $\mathcal{D}\epsilon = 0$. Solution of the KSEs means to find the restrictions on the fluxes and the geometry of spacetime such that such a spinor ϵ exists.
- For this typically some of the fluxes are expressed in terms of geometry, and the geometry of the spacetime must also be restricted.
- To find solutions of the theory, some of the field equations and Bianchi identities must also in addition be solved.

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Methods

- Investigate the integrability conditions to the KSEs

  Works well for maximally supersymmetric solutions

- The bilinears or G-structures method

  Works well for solving the KSEs for one Killing spinor. In special cases this can be extended to more spinors

- Spinorial geometry method

  It is very efficient for solving the KSEs for a small number of Killing spinors and for a very large number of Killing spinors (near maximal).

- There are other methods like use of twistors or generalized geometry that apply for a special class of theories or solutions.

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Example of spinorial geometry

Consider the KSE of a Euclidean 6-D gauge theory in spinorial geometry [Gillard, Gran, GP]

$$F_{CD}\Gamma^{CD}\epsilon = 0$$

Since $Spin(6) = SU(4)$ and ϵ in the **4** or $\bar{\mathbf{4}}$ (Weyl) representations.

- The **4** representation is identified with $\Lambda^{\text{ev}}(\mathbb{C}^3)$ and the gamma matrices are

$$\Gamma_\alpha = \sqrt{2}e_\alpha\wedge\,,\;\; \Gamma_{\bar\alpha} = \sqrt{2}e_\alpha\,\lrcorner\,,\;\;\; \Gamma_\alpha\Gamma_{\bar\beta} + \Gamma_{\bar\beta}\Gamma_\alpha = 2\delta_{\alpha\bar\beta}$$

- The covariance group is $SU(4)$ and has a single type of non-trivial orbit on **4** with isotropy group $SU(3)$, and so ϵ can be chosen as $\epsilon = 1$. This leads to a linear system

$$F_{CD}\Gamma^{CD}1 = F_{\bar\alpha\bar\beta}\Gamma^{\bar\alpha\bar\beta}1 + \delta^{\alpha\bar\beta}F_{\alpha\bar\beta}1 = 0 \Longrightarrow F_{\bar\alpha\bar\beta} = 0\,,\quad \delta^{\alpha\bar\beta}F_{\alpha\bar\beta} = 0$$

Maximally supersymmetric solutions

Investigating the integrability conditions of the KSEs one finds [Figueroa, GP]

- The maximal supersymmetric solutions of $D = 11$ supergravity are locally isometric to

  $\mathbb{R}^{10,1}$, $AdS_4 \times S^7$, $AdS_7 \times S^4$, plane wave

- Maximal supersymmetric solutions of IIB supergravity are locally isometric to

  $\mathbb{R}^{9,1}$, $AdS_5 \times S^5$, plane wave

- Maximal supersymmetric solutions of IIA supergravity are locally isometric to

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- Massive IIA supergravity does not have a maximally supersymmetric solution

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Nearly maximally supersymmetric solutions

Investigating the integrability conditions of the KSEs using spinorial geometry and the field equations and Bianchi identities, one finds

- $D = 11$ supergravity solutions preserving ≥ 30 supersymmetries are maximally supersymmetric [Gran, Gutowski, GP]
- IIB solutions preserving > 28 supersymmetries are maximally supersymmetric, and there is a unique solution, a plane wave, with strictly 28 supersymmetries. [Gran, Gutowski, Roest, GP]
- IIA solutions preserving ≥ 31 supersymmetries are locally maximally supersymmetric [Bandos, Azcarraga, Varela]

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Homogeneity

Conjecture: All solutions of a supergravity theory preserving more than half of the supersymmetry are homogenous. [Meessen]

Theorem: All solutions of $D = 11$, IIB and IIA supergravities that preserve strictly more than 16 supersymmetries are homogeneous [Figueroa, Hustler]

Proof: In the Euclidean case, the proof is simple. If the vector bilinears do not span the tangent space of the spacetime there is an X such that

$$X^M \langle \epsilon_1, \Gamma_M \epsilon_2 \rangle = \langle \epsilon_1, \not{X} \epsilon_2 \rangle = 0$$

Thus the spinors $\not{X}\epsilon$ for every Killing spinor ϵ are orthogonal to all Killing spinors, and so

$$\not{X} : \quad \mathcal{K} \to \mathcal{K}^\perp$$

But $\not{X}^2 = |X|^2 \mathbf{1}$ and as $X \neq 0$, the map is an injection. However this cannot be if $\dim \mathcal{K}^\perp < \dim \mathcal{K}$ which is the case for more than 16 supersymmetries. Thus $X = 0$ and the spacetime is homogenous.

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$N = 1$

The KSEs of D=11, IIB and IIA supergravities have been solved for one Killing spinor and the geometry of the spacetime has been identified.
There are several local geometries that can occur distinguished by the orbit of the Spin group in the space of spinors that the Killing spinor belongs.

- $D = 11$: There are two types of (local) solutions for which the isotropy group of the Killing spinor in $Spin(10,1)$ is either $SU(5)$ or $Spin(7) \ltimes \mathbb{R}^9$ [Pakis, Gauntlet, Gutowski; Gillard, Gran, GP]
- IIB: There are 3 types with Killing spinor isotropy groups in $Spin(9,1)$ either $Spin(7) \ltimes \mathbb{R}^8$, or $SU(4) \ltimes \mathbb{R}^8$, or G_2 [Gran, Gutowski, GP]

- IIA: There are 4 types of (local) solutions with Killing spinor isotropy groups in $Spin(9,1)$ either $Spin(7)$, or $Spin(7) \ltimes \mathbb{R}^8$, or $SU(4)$, or $G_2 \ltimes \mathbb{R}^8$. [Gran, von Schultz, GP]
- In IIA and IIB, there are special cases where the solution of the KSEs is especially simple.
- The requirements on the geometry in most cases are rather weak. For compact isotropy groups, a typical requirement is the existence of a time-like Killing vector field which leaves the fields invariant as well as the Killing spinor. A similar statement holds for the non-compact case with the difference that the Killing vector is null.

Heterotic supergravity

Heterotic supergravity: fields, a metric g, a closed 3-form field strength H, $dH = 0$, and dilaton Φ.
The Killing spinor equations of Heterotic supergravities are

$$\hat{\nabla}_\mu \epsilon = \nabla_\mu \epsilon - \frac{1}{8} H_{\mu\nu\rho} \Gamma^{\nu\rho} \epsilon = 0 \, ,$$
$$\Gamma^\mu \partial_\mu \Phi \epsilon - \frac{1}{24} H_{\mu\nu\rho} \Gamma^{\mu\nu\rho} \epsilon = 0 \, , \quad \epsilon \in \Delta^+_{\mathbf{16}}$$

- Holonomy of the supercovariant connection: $\mathrm{hol}(\hat{\nabla}) \subseteq Spin(9,1)$.

$$[\hat{\nabla}, \hat{\nabla}]\epsilon = \hat{R}\epsilon = 0$$

  So either parallel spinors have a non-trivial isotropy group in $Spin(9,1)$ or $\hat{R} = 0$ and the solutions are group manifolds.
- The KSEs of Heterotic supergravity have been solved in all cases [Gran, Lohrmann, GP; Gran, Roest, Sloane, GP].

Solution of KSE for $dH = 0$.

L	$\mathrm{Stab}(\epsilon_1, \dots, \epsilon_L)$	N
1	$Spin(7) \ltimes \mathbb{R}^8$	1
2	$SU(4) \ltimes \mathbb{R}^8$	$-,\ 2$
3	$Sp(2) \ltimes \mathbb{R}^8$	$-,\ -,\ 3$
4	$(\times^2 SU(2)) \ltimes \mathbb{R}^8$	$-,\ -,\ -,\ 4$
5	$SU(2) \ltimes \mathbb{R}^8$	$-,\ -,\ -,\ -,\ 5$
6	$U(1) \ltimes \mathbb{R}^8$	$-,\ -,\ -,\ -,\ -,\ 6$
8	$\mathbb{R}^8$	$-,\ -,\ -,\ -,\ -,\ -,\ -,\ 8$
2	G_2	$-,\ 2$
4	$SU(3)$	$-,\ 2,\ -,\ 4$
8	$SU(2)$	$-,\ 2,\ -,\ 4,\ -,\ 6,\ -,\ 8$
16	$\{1\}$	$8,\ 10,\ 12,\ 14,\ 16$

$SU(3)$

M admits 4 $\hat{\nabla}$-parallel 1-forms λ^a, and a 2-form ω and a (3,0)-form χ, fundamental forms of $SU(3)$.

$$i_a\omega = 0\,, \quad i_a\chi = 0\,, \quad \mathcal{L}_a\omega = 0\,, \quad \mathcal{L}_a\chi = k_a\chi$$

The Lie algebra $\mathfrak{g}$ of vector fields associate to λ^a is

$$\mathbb{R}^{3,1}\,, \quad \mathfrak{sl}(2,\mathbb{R})\oplus\mathbb{R}\,, \quad \mathfrak{su}(2)\oplus\mathbb{R}\,, \quad \mathfrak{cw}_4$$

The spacetime locally is $M = P(G, B^6; \pi)$, Lie $G = \mathfrak{g}$ equipped with connection λ^a and B^6 a Hermitian (KT) manifold with metric $d\tilde{s}^2_{(6)}$ and Hermitian form $\omega_{(6)} = \omega$. Then

$$ds^2 = \eta_{ab}\lambda^a\lambda^b + \pi^* d\tilde{s}^2_{(6)}\,, \quad H = CS(\lambda) + \pi^*\tilde{H}_{(6)}\,, \quad \tilde{H}_{(6)} = -i(\partial - \bar{\partial})\omega$$

<u>$\mathfrak{g}$ abelian</u>: B^6 is a Calabi-Yau with torsion, ie $\mathrm{hol}(\hat{\tilde{\nabla}}) \subseteq SU(3)$. Moreover

$$\tilde{\theta}_{\omega_{(6)}} = 2\tilde{d}\Phi\,, \quad \partial_a\Phi = 0\,, \quad \mathcal{F} \equiv d\lambda - \lambda^2 \in \mathfrak{su}(3)\,.$$

where $\theta = \star(\omega \wedge \star d\omega)$ is the Lee form.

$\mathfrak{g}$ non-abelian: B^6 is Hermitian (KT) and $\mathrm{hol}(\hat{\tilde{\nabla}}) \subseteq U(3)$. Moreover

$$\hat{\tilde{\rho}} = k_a \mathcal{F}^a \ , \quad \tilde{\theta}_{\omega_{(6)}} = 2\tilde{d}\Phi \ , \quad \partial_a \Phi = 0 \ , \quad \mathcal{F}^a \in \mathfrak{u}(3) \ .$$

The complex trace of $\mathcal{F}$ is related k which is dual to the structure constants of $\mathfrak{g}$.

- The geometry of the remaining cases is similarly known
- The half supersymmetric solutions associated with $\mathbb{R}^8$ and $SU(2)$ holonomies have been classified [GP]

Special backgrounds

Apart from the general classification problem, there are several others for special types of backgrounds for applications to black holes, AdS/CFT, compactifications and others.

- Classification of supersymmetric black hole solutions and their near horizon geometries
- Classification of warped AdS backgrounds (AdS/CFT, compactifications)
- Classification of warped, flux Minkowski compactifications (can arise as infinite AdS radius limits)
- Backgrounds used in localization techniques

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AdS backgrounds

The a priori number of sypersymmetries preserved by D=11, IIB and IIA AdS backgrounds are [Beck, Gutowski, GP]

AdS_n	N
$n=2$	$2k, k<16$
$n=3$	$2k, k<16$
$n=4$	$4k, k\leq 7, 32(D=11)$
$n=5$	$8, 16, 24, 32(IIB)$
$n=6$	16
$n=7$	$16, 32(D=11)$

Table: The proof for AdS_2 requires the maximum principle. For the rest, no such assumption is necessary. The bounds on k arise from the classification of solutions with near maximal supersymmetry.

AdS_5 , $N = 24$ and AdS_6

Theorem: There are no smooth AdS_5 solutions preserving $N = 24$ supersymmetries with compact without boundary internal space in all type II and $D = 11$ supergravities.

There are plenty of AdS_5 solutions apart from the IIB $AdS_5 \times S^5$ preserving less supersymmetry [Gauntlett, Martelli, Sparks, Waldram; Piltch, Warner; Maldacena Nunez; Itsios, Nunez, Sfetsos, Thomson] and a more systematic investigation was done by [Apruzzi, Fazzi, Passias, Tomasiello].

Theorem: There are no smooth AdS_6 solutions with compact without boundary internal space in (massive) type IIA and $D = 11$ supergravities.

However there are AdS_6 solutions in massive IIA [Brandhuber, Oz] and a systematic investigation has been done in [Apruzzi, Fassi, Passias, Rosa, Tomassiello]

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There are plenty of AdS_5 solutions apart from the IIB $AdS_5 \times S^5$ preserving less supersymmetry [Gauntlett, Martelli, Sparks, Waldram; Piltch, Warner; Maldacena Nunez; Itsios, Nunez, Sfetsos, Thomson] and a more systematic investigation was done by [Apruzzi, Fazzi, Passias, Tomasiello].

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Minkowski supersymmetries

The a priori number of supersymmetries preserved by warped, flux, Minkowski backgrounds in $D = 11$

$\mathbb{R}^{n-1,1} \times_w M^{11-n}$	N
$n = 2$	unrestricted
$n = 3$	$2k, k < 15$
$n = 4$	$4k, k \leq 7$
$n = 5$	$8, 16, 24$
$n = 6$	$8, 16, 24$
$n = 7$	16

Table: There is no a priori restriction on the number of supersymmetries preserved by $\mathbb{R}^{1,1} \times_w M^9$ backgrounds as the global techniques do not apply. The bounds on k arise from the classification of supersymmetric solutions with near maximal supersymmetry.

Heterotic

Theorem: In heterotic theory with $dH = 0$

- There are no AdS_n, $n > 3$, supersymmetric backgrounds
- There are no smooth AdS_2 backgrounds for which the internal space is compact without boundary
- AdS_3 backgrounds preserve 2,4,6 and 8 supersymmetries
- Smooth AdS_3 backgrounds preserving 8 supersymmetries with compact without boundary internal space are locally isometric to either $AdS_3 \times S^3 \times T^4$ or $AdS_3 \times S^3 \times K_3$
- Although there is no classification of all possible backgrounds, there is a clear overview of all possibilities and what equations should be solved to achieve the task.

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Geometry

The geometry of AdS_3 backgrounds is as follows:

N	M^7	B^k	*fibre*
2	G_2	$-$	$-$
4	$SU(3)$	$U(3)$	S^1
6	$SU(2)$	$self-dual-Weyl$	S^3
8	$SU(2)$	$hyper-Kahler$	S^3

Table: The G-structure of M^7 is compatible with a connection with skew-symmetric torsion. For $N = 4, 6, 8$, M^7 is a local (twisted) fibration over a base space B^k with fibre either S^1 or S^3 . The base spaces B are conformally balanced with respect to the associated fundamental forms.

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Regularity

Regularity of certain classes of solutions, like compactifications, AdS solutions and others is desirable as otherwise singularities require some interpretation.

Regularity together with some topological assumptions on the spaces involved provide powerful tools to solve field equations and KSEs

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Regularity of compactifications

Theorem: There are no flat or de-Sitter smooth warp flux compactifications of 10- and 11-dimensional supergravities for which the internal space is compact without boundary [Gibbons; Maldacena, Nunez]

Proof: It easily follows from the Einstein equations and in particular from the field equation of the warp factor A,

$$ds^2 = A^2 ds^2(M_{st}) + ds^2(M_{in})$$

which can be written as

$$\nabla^2 A + X^i A^{-1} \partial_i A - A^2 R_{st} = Q(F, A) \geq 0$$

Applying the maximum principle leads to an inconsistency for $R_{st} > 0$ and for $R_{st} = 0$ the fluxes must vanish $F = 0$.

- Higher order correction can alter this as well as allowing singularities
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New Lichnerowicz type of theorems for AdS

These arise in the investigation of AdS solutions and are instrumental in understanding the symmetries of near horizon geometries

The warp, flux, AdS_n, $n > 2$, backgrounds can be written as

$$ds^2 = 2du(dr + rh) + A^2(dz^2 + e^{2z/\ell}\sum_{a=1}^{n-3}(dx^a)^2) + ds^2(M^{11-n}) ,$$

with

$$e^+ = du ,\ \ e^- = dr + rh ,\quad h = -\frac{2}{\ell}dz - 2A^{-1}dA ,$$

In these coordinates the Killing spinors of $AdS_n \times_w M^{11-n}$ backgrounds can be written as

$$\epsilon = \sigma_+ - \ell^{-1}\sum_{a=1}^{n-3}x^a\Gamma_{az}\tau_+ + e^{-\frac{z}{\ell}}\tau_+ + \sigma_- + e^{\frac{z}{\ell}}(\tau_- - \ell^{-1}\sum_{a=1}^{n-3}x^a\Gamma_{az}\sigma_-) - \ell^{-1}uA^{-1}\Gamma_{+z}\sigma_- - \ell^{-1}rA^{-1}e^{-\frac{z}{\ell}}\Gamma_{-z}\tau_+ ,$$

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The remaining independent KSEs on M^{11-n} are

$$D_i^{(\pm)}\sigma_\pm = 0\,,\quad D_i^{(\pm)}\tau_\pm = 0\,,$$

and

$$\mathcal{A}^{(\pm)}\sigma_\pm = 0\,,\quad \mathcal{B}^{(\pm)}\tau_\pm = 0\,,$$

Example AdS_6

$$\begin{aligned} D_i^{(\pm)} &= D_i \pm \frac{1}{2}\partial_i \log A - \frac{1}{288}\Gamma_i{}^{j_1j_2j_3j_4}X_{j_1j_2j_3j_4} \\ \mathcal{A}^{(\pm)} &= -\frac{1}{2}\Gamma_z\Gamma^i\partial_i A \pm \frac{1}{2\ell} + \frac{1}{288}\Gamma_z A\Gamma^{j_1j_2j_3j_4}X_{j_1j_2j_3j_4} \\ \mathcal{B}^{(\pm)} &= \mathcal{A}^{(\pm)} \pm \frac{1}{\ell} \end{aligned}$$

One can establish new Lichnerowicz type theorems as

$$\mathscr{D}^{(\pm)}\sigma_\pm = 0 \Longleftrightarrow D_i^{(\pm)}\sigma_\pm = 0\,, \quad \mathcal{A}^{(\pm)}\sigma_\pm = 0\,,$$

These are based on maximum principle formulae

$$D^2 \parallel A^{-1}\sigma_- \parallel^2 + nA^{-1}\partial^i A \partial_i \parallel A^{-1}\sigma_- \parallel^2 = 2A^{-2}\langle \mathbb{D}_i^{(-)}\sigma_-, \mathbb{D}^{(-)i}\sigma_-\rangle + 2\frac{9n-18}{11-n}A^{-2} \parallel \mathcal{A}^{(-)}\psi \parallel^2\,,$$

where $\mathbb{D}_i^{(-)} = D_i^{(-)} + \frac{2-n}{11-n}\Gamma_i\mathcal{A}^{(-)}$ and $\mathscr{D}^{(\pm)} = \Gamma^i\mathbb{D}_i^{(\pm)}$.

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Conclusion

- Apart from a few cases, there is no handle over the geometry of the vast majority of supersymmetric backgrounds in type II theories
- But there is a guiding paradigm that of the classification in heterotic theory
- There are stronger results for special backgrounds, like AdS, but again the results are not complete specially in type II theories
- Progress for the classification of black holes and black hole horizons is less advanced. For the latter part of it is the classification of AdS_2 backgrounds. Stronger results are known in lower dimensions.
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