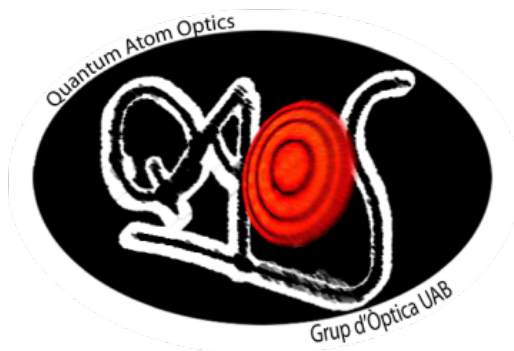


Ultracold atoms carrying orbital angular momentum in sided coupled cylindrically symmetric potentials

Verònica Ahufinger

J. Polo, J. Mompart and V. A., Phys. Rev. A **93**, 033613 (2016)

G. Pelegrí, J. Polo, A. Turpin, M. Lewenstein, J. Mompart, V. A., Phys. Rev. A **95**, 013614 (2017)



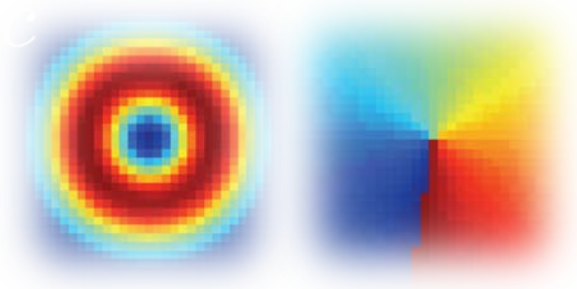
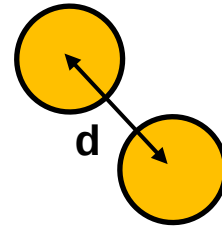
UAB
Universitat Autònoma
de Barcelona

Introduction

Tunneling dynamics

+

Orbital angular momentum



Outline

▪ Geometrically induced complex tunneling

- Introduction
- Two in-line ring potentials
- Three ring potentials in a triangular configuration
- Geometrically engineered spatial dark states

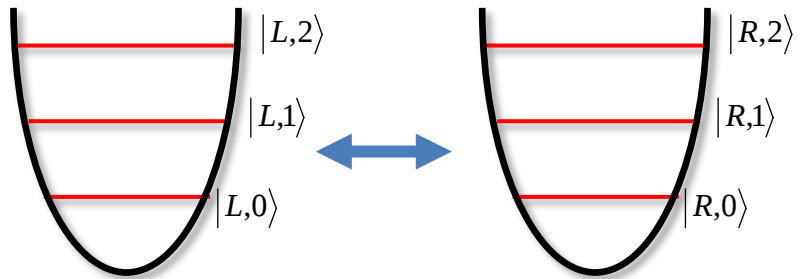
▪ Edge-like states in an optical ribbon

- Optical ribbon
- Ground state manifold
- OAM manifold
- Robustness

▪ Conclusions

Introduction

Tunneling amplitudes between two traps in 1D $\Rightarrow$ Real



The diagram shows two 1D harmonic traps, each represented by a black parabola. The left trap has three red horizontal lines representing energy levels labeled $|L,0\rangle$, $|L,1\rangle$, and $|L,2\rangle$ from bottom to top. The right trap has three red horizontal lines labeled $|R,0\rangle$, $|R,1\rangle$, and $|R,2\rangle$. A blue double-headed arrow connects the two traps, indicating tunneling.

$$J_{j,n}^{k,p} \equiv \langle k, p | H | j, n \rangle = \int_{-\infty}^{+\infty} \psi_{k,p}^*(x) H \psi_{j,n}(x) dx$$

$\uparrow$
 $j, k = L, R$

Tunneling amplitudes between two traps in 2D for OAM states?

Cylindrical symmetry:

$\downarrow$ Real



J. Polo *et al.*, New Journal of Physics **18**, 015010 (2016)

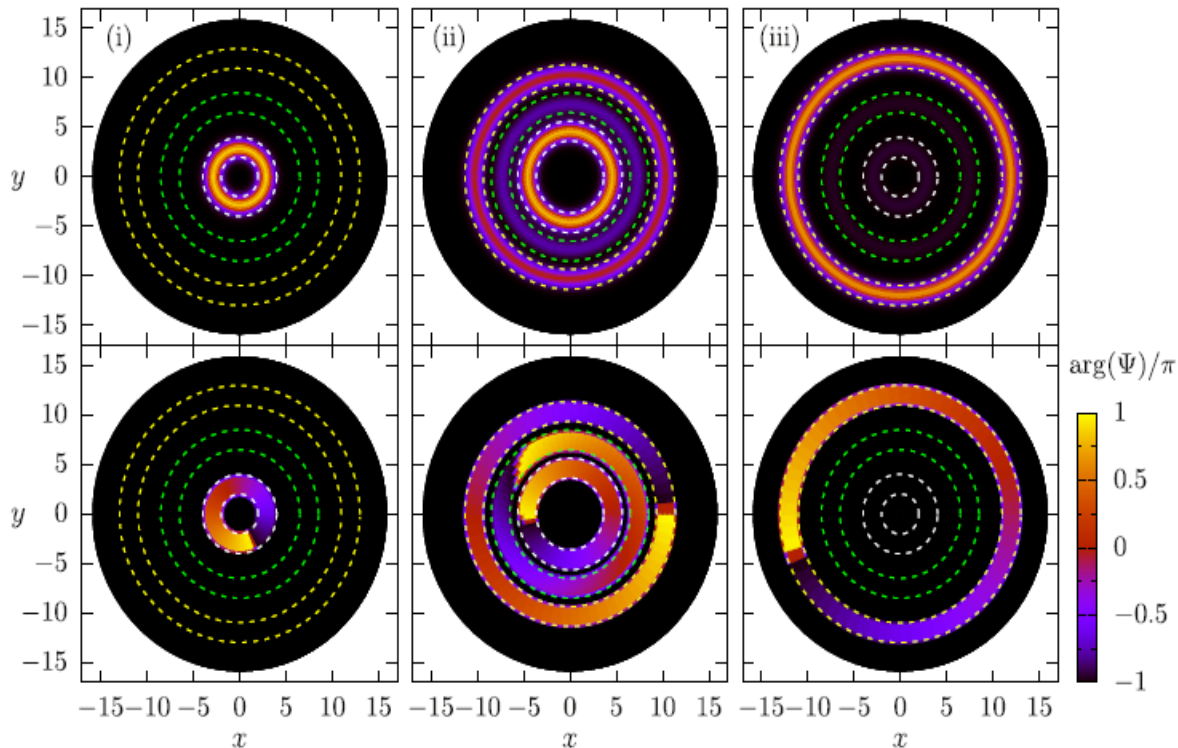
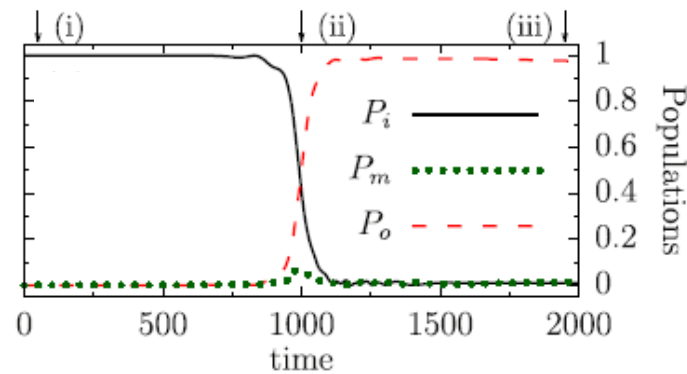
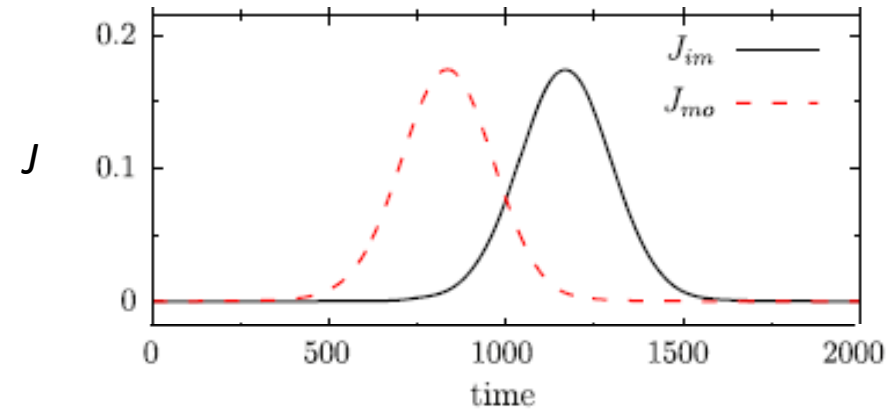
J. Brand *et al.*, Phys. Rev. A **80**, 011602 (2009)

Introduction

J. Polo, A. Benseny, T. Busch, V. Ahufinger, J. Mompart, New Journal of Physics **18**, 015010 (2016)

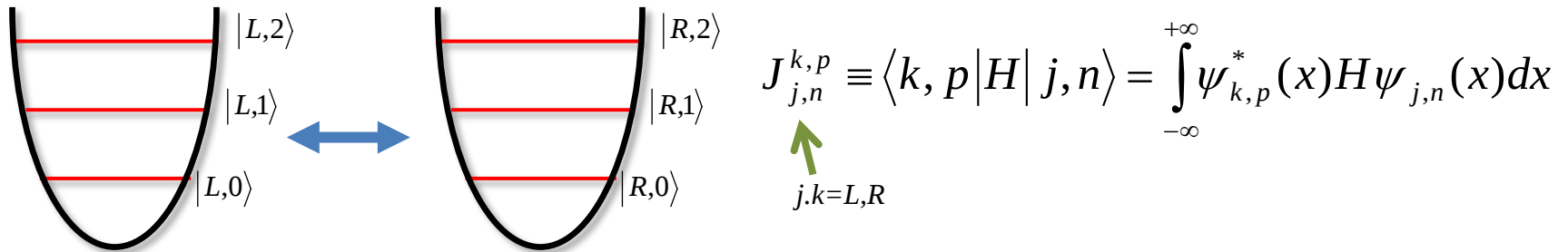
Review on Spatial Adiabatic passage:

R. Menchon-Enrich A. Benseny, V. Ahufinger, A. D. Greentree, T. Busch, J. Mompart, Reports on Progress in Physics **79**, 074401 (2016)



Introduction

Tunneling amplitudes between two traps in 1D $\Rightarrow$ Real



Tunneling amplitudes between two traps in 2D for OAM states?

Cylindrical symmetry:

$\downarrow$ Real



J. Polo *et al.*, New Journal of Physics **18**, 015010 (2016)
J. Brand *et al.*, Phys. Rev. A **80**, 011602 (2009)



I. Lesanovsky and W. von Klitzing, Phys. Rev. Lett. **98**, 050401 (2007)
J. Brand *et al.*, Phys. Rev. A **81**, 025602 (2010)
L. Amico *et al.*, Scientific Reports **4**, 4298 (2013)

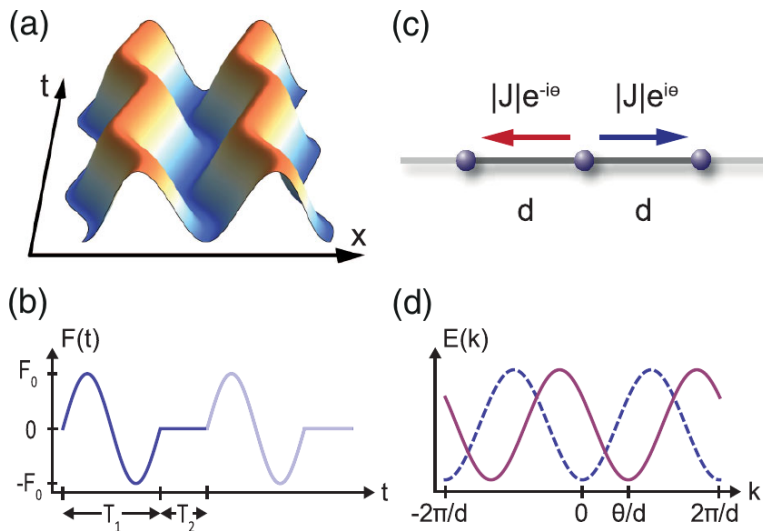
Breaking the cylindrical symmetry with more than two traps $\Rightarrow$ Complex

Introduction

How to create complex tunnelings?

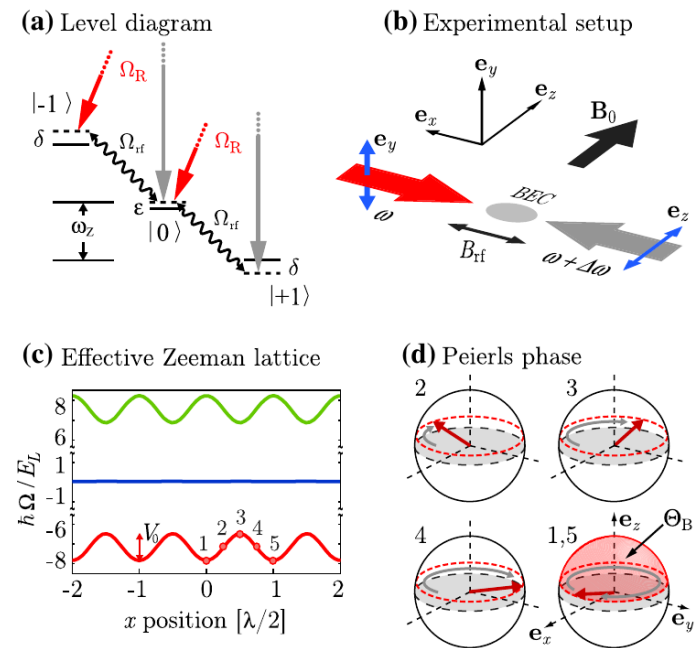
Suitable forcing of the optical lattice

P. Windpassinger *et al.*, Phys. Rev. Lett. **108**,225304 (2012)



Combination of radio frequency and Raman fields that couple to the internal states of the atom

I.B.Spielman *et al.*, Phys. Rev. Lett. **108**,225303(2012)



Geometrically induced complex tunneling through orbital angular momentum states. J.Polo,J.Mompart,V.Ahufinger, Phys. Rev. A **93**, 033613 (2016)

Introduction

Interest of complex tunnelings?

- **Generation of artificial vector gauge potentials**

J. Dalibard *et al.*, Rev. Mod. Phys. **83**, 1523 (2011)

- **Generation of staggered fluxes**

I. Bloch *et al.*, Phys. Rev. Lett. **107**, 255301 (2011)

L. Mathey *et al.*, Nature Physics **9**, 738 (2013)

- **Implementation of the Hofstadter, Harper and Weyl Hamiltonians**

I. Bloch *et al.*, Phys. Rev. Lett. **111**, 185301 (2013)

W. Ketterle *et al.*, Phys. Rev. Lett. **111**, 185302 (2013)

- **Realization of the topological Haldane model**

T. Esslinger *et al.*, Nature **515**, 237 (2014)

- **Geometrical engineering of spatial dark states**

J. Polo, J. Mompert, V. Ahufinger, Phys. Rev. A **93**, 033613 (2016)

Outline

▪ Geometrically induced complex tunneling

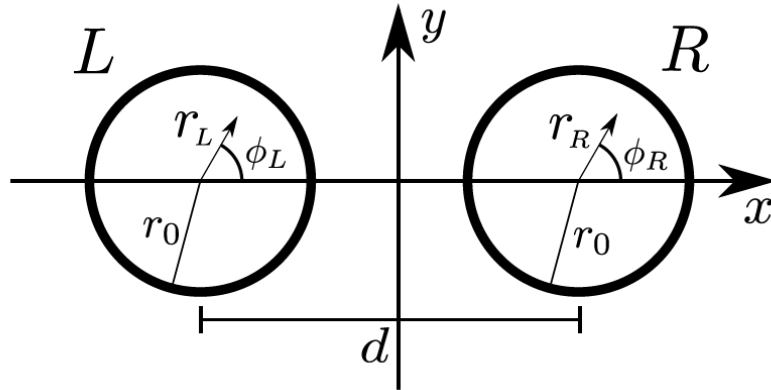
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▪ Conclusions

Two in-line ring potentials



$$\begin{array}{c}
 \text{Energy} \uparrow \\
 \begin{array}{cccc}
 (m, l) & \overline{|L, m, n\rangle} & \overline{|L, m, -n\rangle} & \overline{|R, m, n\rangle} & \overline{|R, m, -n\rangle} & \hat{H}^{m,l} \\
 \vdots & \vdots & \vdots & \vdots & \vdots & \\
 (0, 1) & \overline{|L, 0, 1\rangle} & \overline{|L, 0, -1\rangle} & \overline{|R, 0, 1\rangle} & \overline{|R, 0, -1\rangle} & \hat{H}^{0,1} \\
 (0, 0) & & \overline{|L, 0, 0\rangle} & \overline{|R, 0, 0\rangle} & & \hat{H}^{0,0}
 \end{array}
 \end{array}$$

$$l = |n|$$

winding number, $n = \pm l$, l being the orbital angular momentum quantum number

$$\Psi_{j,m}^n(r_j, \phi_j) = \langle \vec{r} | j, m, n \rangle = \frac{1}{\sqrt{N}} \psi_m(r_j) e^{in(\phi_j - \phi_0)}$$

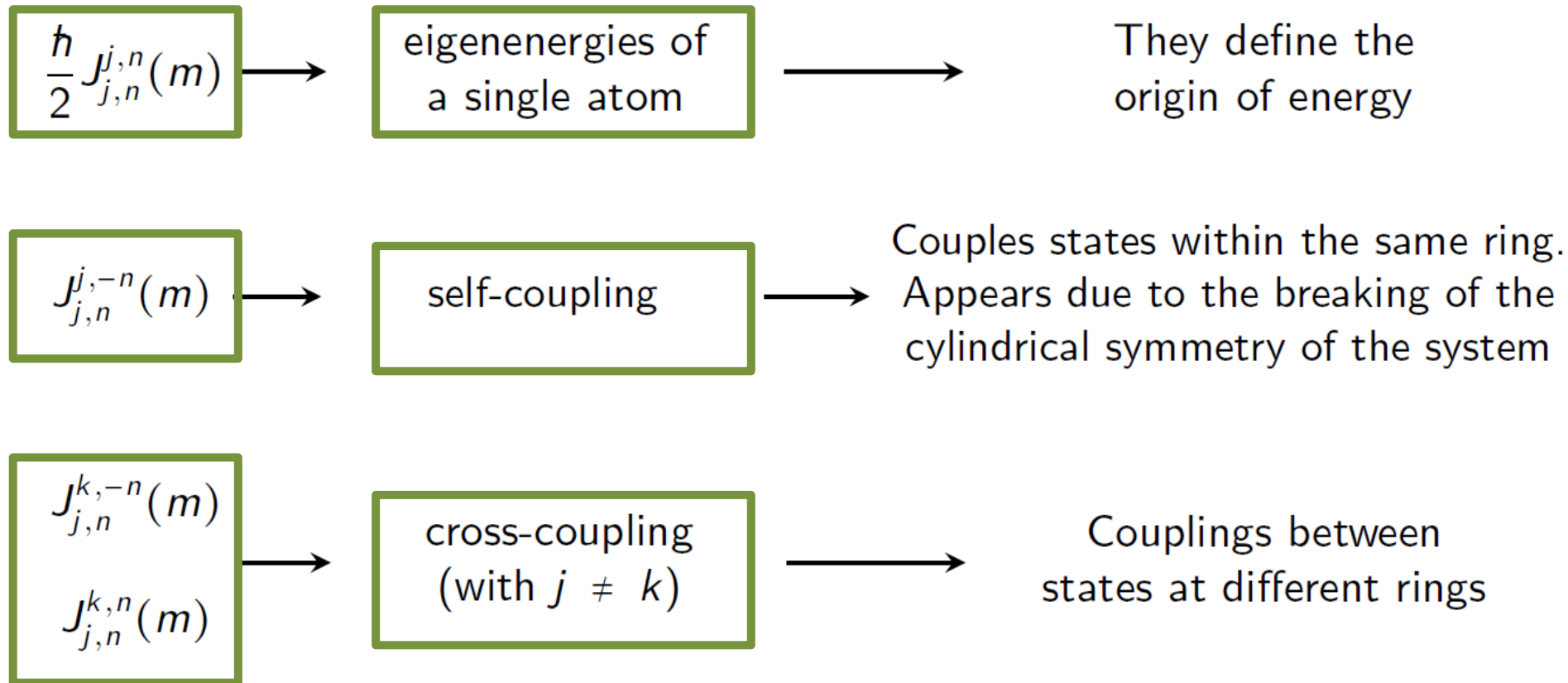
$j=R, L$ transverse vibrational state
 radial part
 azimuthal phase origin

Assuming $\sigma_m \ll r_0 \ll d$

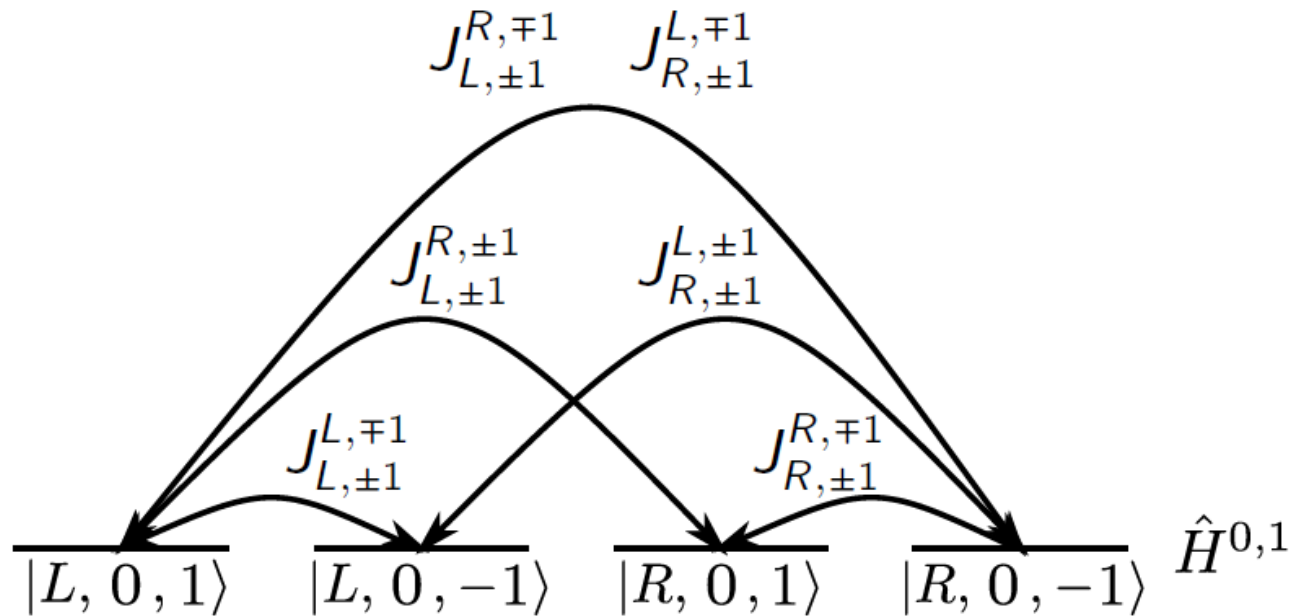
$$\hat{H}_T = \sum_{m \geq 0} \hat{H}^{m,0} + \sum_{m \geq 0} \sum_{l > 0} \hat{H}^{m,l}$$

Two in-line ring potentials

$$\hat{H}^{m,l} = \frac{\hbar}{2} \sum_{j,k=L,R} \sum_{n=\pm l} \left(J_{j,n}^{k,n}(m) |j, m, n\rangle \langle k, m, n| + J_{j,n}^{k,-n}(m) |j, m, n\rangle \langle k, m, -n| \right)$$



Two in-line ring potentials



$$\hat{H}^{m,l} = \frac{\hbar}{2} \begin{pmatrix} 0 & J_{L,n}^{L,-n} & J_{L,n}^{R,n} & J_{L,n}^{R,-n} \\ J_{L,-n}^{L,n} & 0 & J_{L,-n}^{R,n} & J_{L,-n}^{R,-n} \\ J_{R,n}^{L,n} & J_{R,n}^{L,-n} & 0 & J_{R,n}^{R,-n} \\ J_{R,-n}^{L,n} & J_{R,-n}^{L,-n} & J_{R,-n}^{R,n} & 0 \end{pmatrix} \quad |L, 0, +1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Two in-line ring potentials

Hermiticity of
the Hamiltonian

$$J_{j,n}^{k,p} = (J_{k,p}^{j,n})^*$$



Six independent couplings

Symmetries $[\hat{H}, \hat{M}_x] = [\hat{H}, \hat{M}_y] = 0$

$$J_{j,n}^{k,p} = \langle k, p | \hat{H} | j, n \rangle = \langle k, p | \hat{M}_y^{-1} \hat{H} \hat{M}_y | j, n \rangle \\ = \langle k, p | \hat{M}_x^{-1} \hat{H} \hat{M}_x | j, n \rangle$$

$$(x, y) \xrightarrow{\hat{M}_x} (x, -y) \longrightarrow \hat{M}_x |j, m, n\rangle = e^{-2in\phi_0} |j, m, -n\rangle$$

$$(x, y) \xrightarrow{\hat{M}_y} (-x, y) \longrightarrow \hat{M}_y |j, m, n\rangle = e^{-2in\phi_0} e^{in\pi} |k, m, -n\rangle \text{ for } j \neq k$$

$$J_{L,n}^{L,-n} = |J_{L,n}^{L,-n}| e^{2in\phi_0}$$

$$J_{L,n}^{R,n}$$

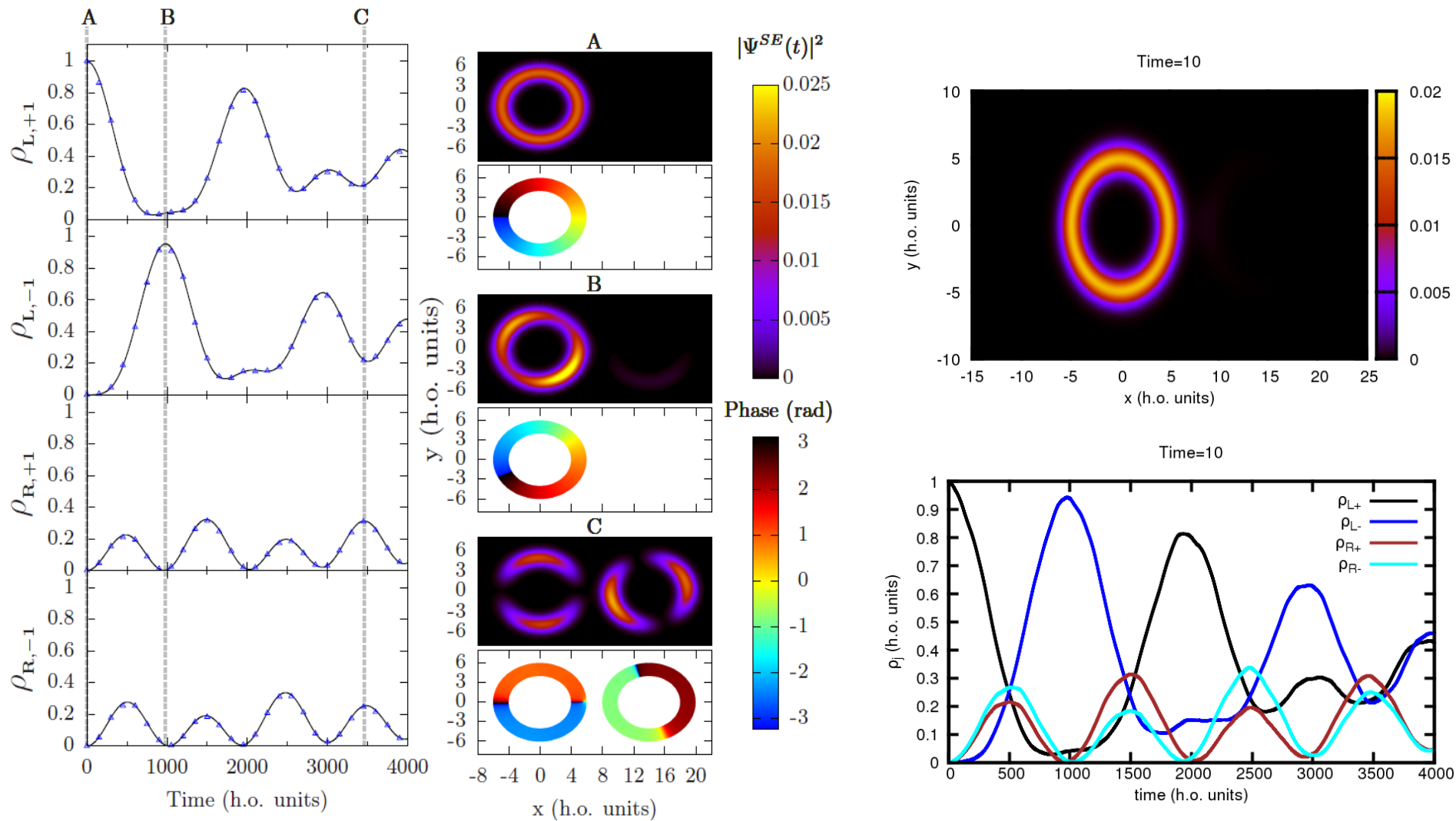
$$J_{L,n}^{R,-n} = |J_{L,n}^{R,-n}| e^{2in\phi_0}$$



$$\hat{H}^{m,l} = \frac{\hbar}{2} \begin{pmatrix} 0 & J_{L,n}^{L,-n} & J_{L,n}^{R,n} & J_{L,n}^{R,-n} \\ J_{L,-n}^{L,n} & 0 & J_{L,-n}^{R,n} & J_{L,-n}^{R,-n} \\ J_{R,n}^{L,n} & J_{R,n}^{L,-n} & 0 & J_{R,n}^{R,-n} \\ J_{R,-n}^{L,n} & J_{R,-n}^{L,-n} & J_{R,-n}^{R,n} & 0 \end{pmatrix}$$

$$\phi_0 = 0$$

Two in-line ring potentials



$$r_0=5 \quad \text{and} \quad J_{L,1}^{R,1} = 3.06 \times 10^{-3} \quad J_{L,1}^{R,-1} = 3.28 \times 10^{-3}$$

$$d=14 \quad J_{L,1}^{L,-1} = -4.06 \times 10^{-4} \text{ in h.o. units}$$

Outline

▪ **Geometrically induced complex tunneling**

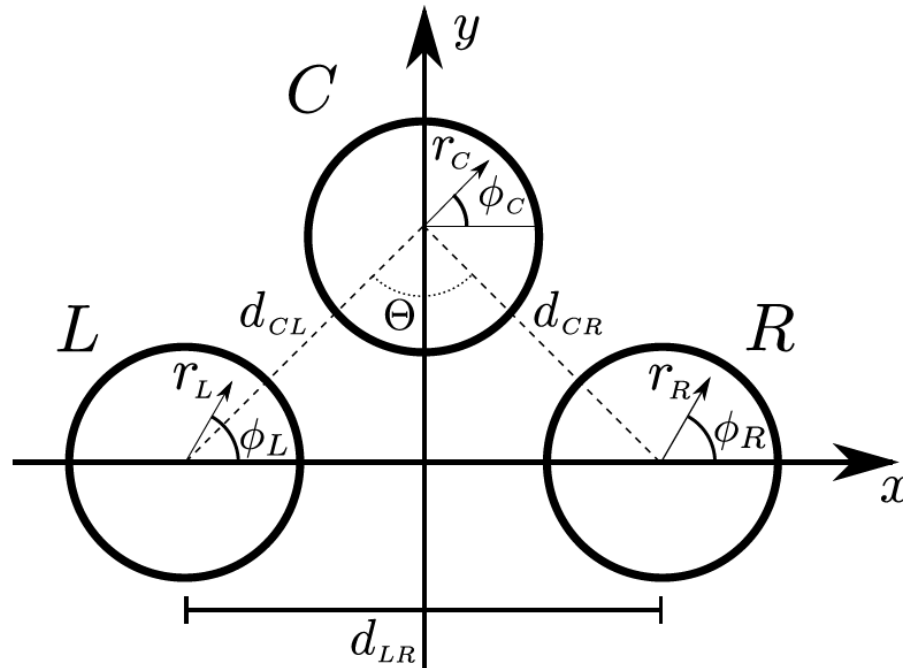
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- OAM manifold
- Robustness

▪ **Conclusions**

Triangular configuration

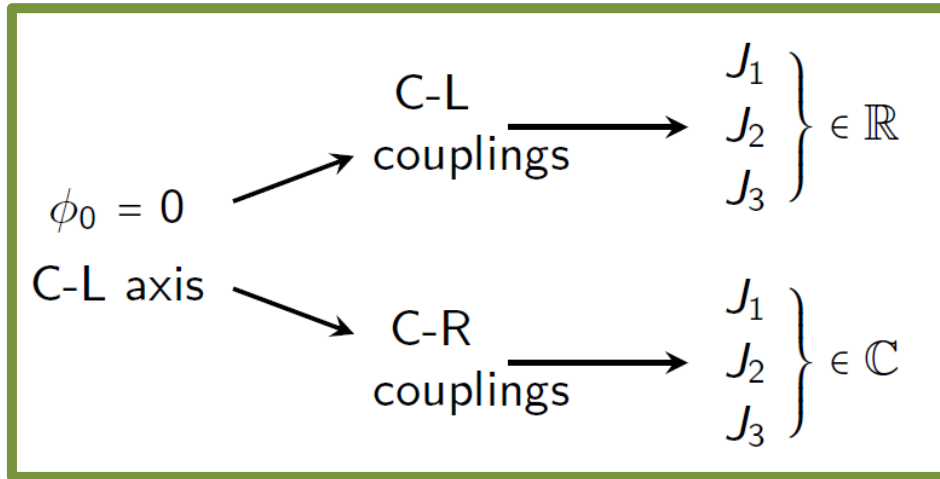
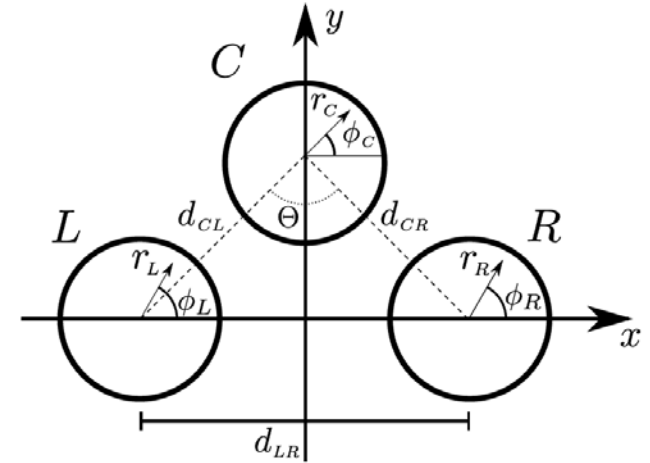


Energy	(m, l)	$\overline{ L, m, n\rangle}$	$\overline{ L, m, -n\rangle}$	$\overline{ C, m, n\rangle}$	$\overline{ C, m, -n\rangle}$	$\overline{ R, m, n\rangle}$	$\overline{ R, m, -n\rangle}$	$\hat{H}^{m,l}$
	$(0, 1)$	$\overline{ L, 0, 1\rangle}$	$\overline{ L, 0, -1\rangle}$	$\overline{ C, 0, 1\rangle}$	$\overline{ C, 0, -1\rangle}$	$\overline{ R, 0, 1\rangle}$	$\overline{ R, 0, -1\rangle}$	$\hat{H}^{0,1}$
	$(0, 0)$		$\overline{ L, 0, 0\rangle}$	$\overline{ C, 0, 0\rangle}$	$\overline{ R, 0, 0\rangle}$			$\hat{H}^{0,0}$

Triangular configuration

$$d_{CL} = d_{CR} \equiv d \text{ and } d_{LR} = 2d \sin(\Theta/2)$$

Assumptions: left and right rings are decoupled, i.e., $d_{LR} \gg d$



$$J_{C,n}^{C,-n} = J_{L,+n}^{L,-n} (1 + e^{2in\Theta})$$

$$\text{For } n\Theta = \pi/2 \longrightarrow J_{C,n}^{C,-n} = 0$$

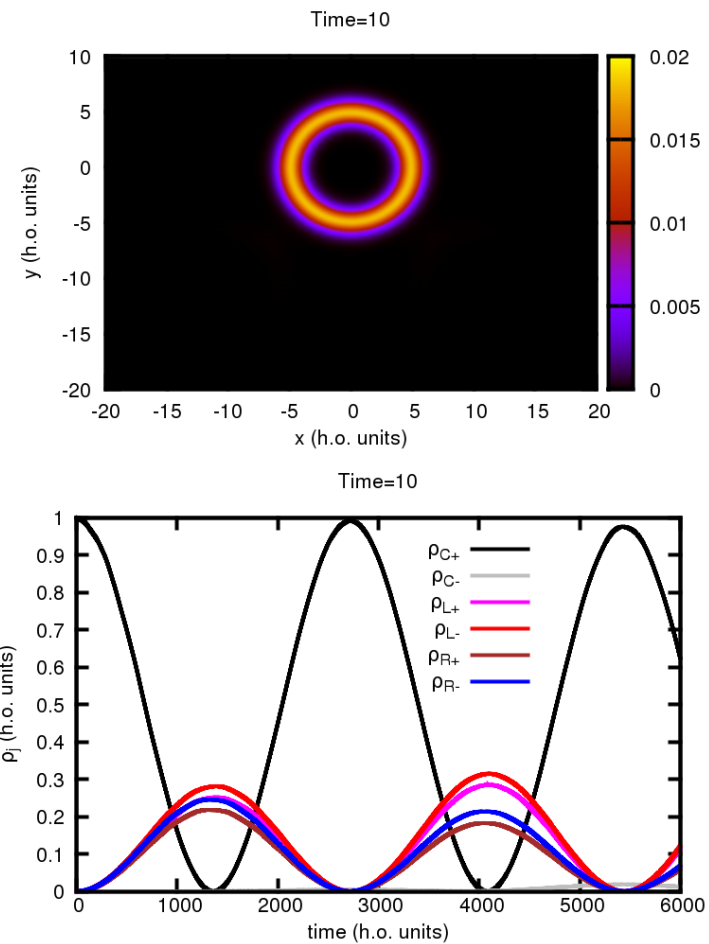
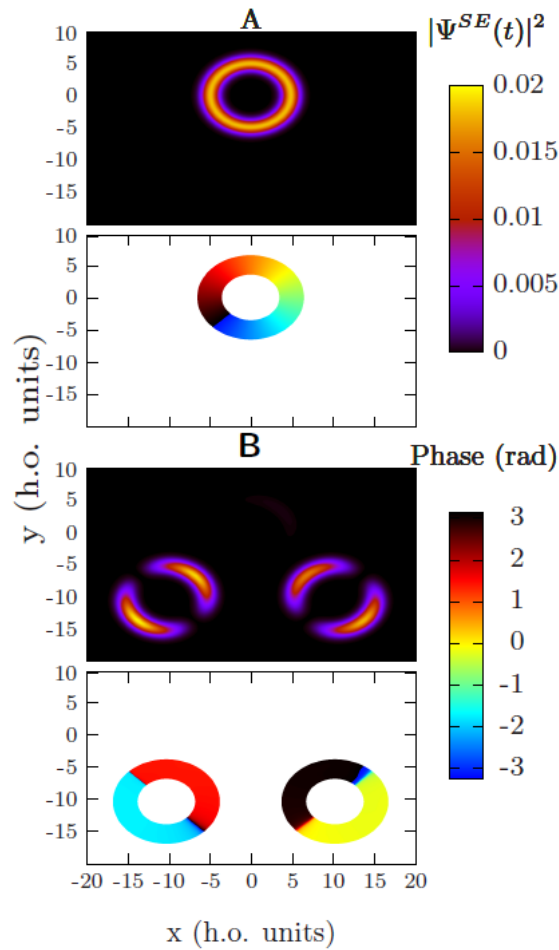
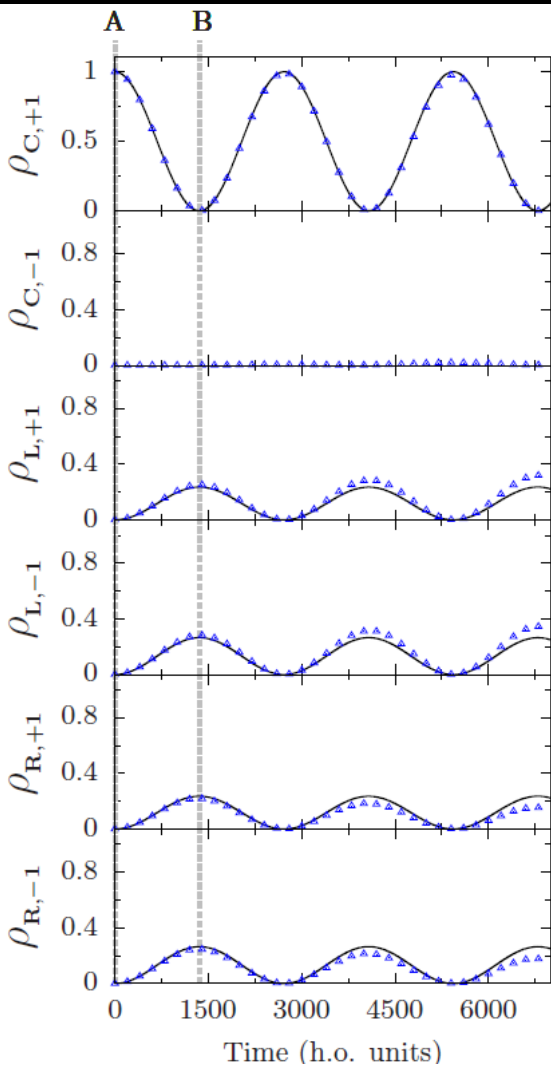
where $J_1 = J_{L,n}^{L,-n}$

$$J_2 = J_{L,n}^{C,n}$$

$$J_3 = J_{L,n}^{C,-n}$$

$$\hat{H}^{m,l} = \frac{\hbar}{2} \begin{pmatrix} 0 & J_1 & J_2 & J_3 & 0 & 0 \\ J_1 & 0 & J_3 & J_2 & 0 & 0 \\ J_2 & J_3 & 0 & 0 & J_2 & |J_3|e^{-in\pi} \\ J_3 & J_2 & 0 & 0 & |J_3|e^{in\pi} & J_2 \\ 0 & 0 & J_2 & J_3e^{-in\pi} & 0 & J_1e^{-in\pi} \\ 0 & 0 & J_3e^{in\pi} & J_2 & J_1e^{in\pi} & 0 \end{pmatrix}$$

Triangular configuration



$$r_0=5 \quad \text{and} \quad |J_{j,1}^{C,1}| = 3.06 \times 10^{-3} \quad |J_{j,1}^{C,-1}| = 3.28 \times 10^{-3} \\ |J_{j,1}^{j,-1}| = 4.06 \times 10^{-4} \text{ in h.o. units}$$

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▪ **Edge-like states in an optical ribbon**

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- Ground state manifold
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- Robustness

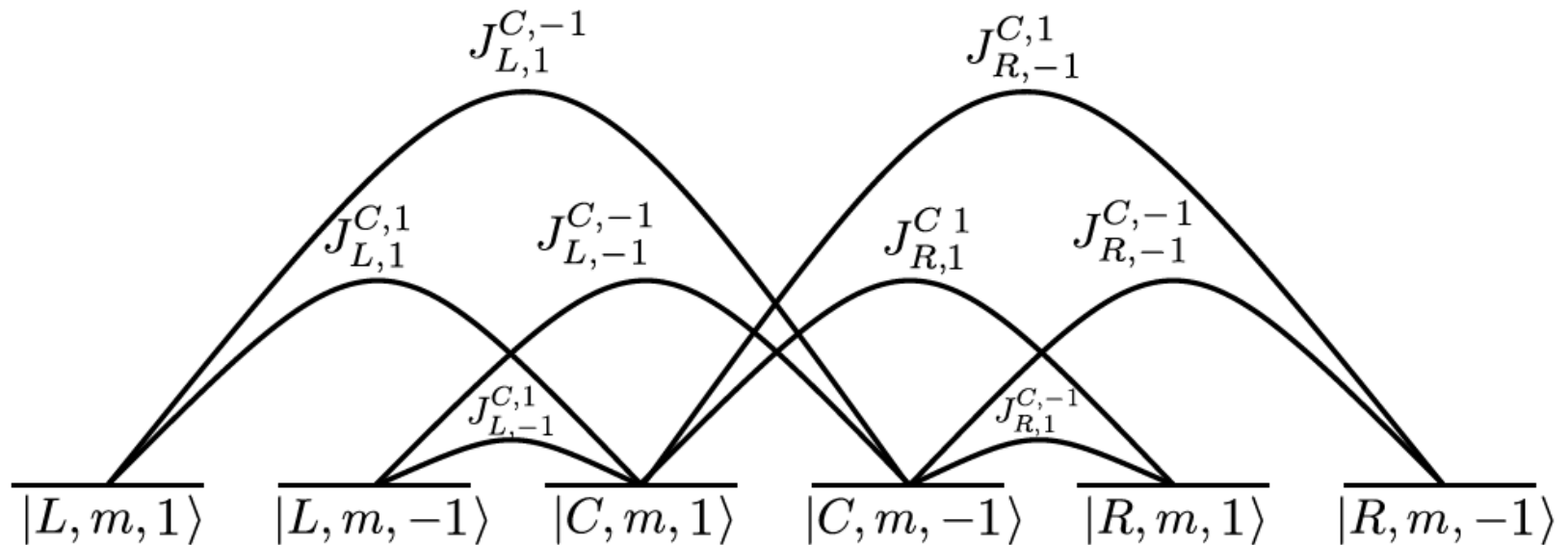
▪ **Conclusions**

Triangular configuration

Assumptions $\rightarrow$ L-R decoupled
 $\rightarrow J_{L+}^{L-} = J_{R+}^{R-} = 0$

$$\Theta = \pi/2 \quad \longrightarrow \quad l = 1$$

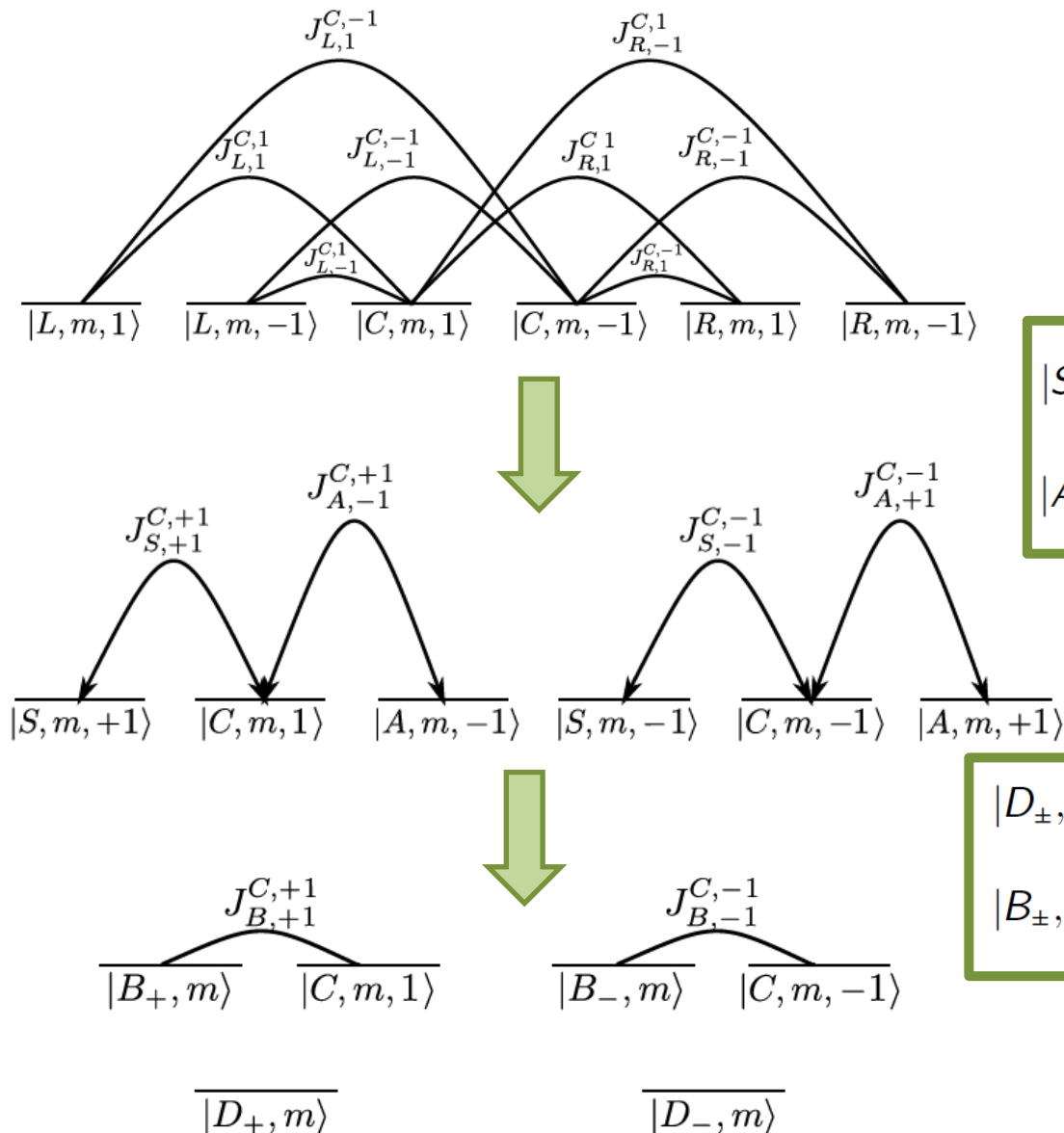
$$\begin{aligned} J_{C,+1}^{C,-1} &= 0 \\ J_{L,+1}^{C,1} &\in \mathbb{R} \\ J_{C,+1}^{R,-1} &= |J_{C,+1}^{L,-1}| e^{i\pi} \end{aligned}$$



Triangular configuration

$$\Theta = \pi/2$$

$$l = 1$$



$$|S, m, \pm 1\rangle = \frac{1}{\sqrt{2}} (|L, m, \pm 1\rangle + |R, m, \pm 1\rangle)$$

$$|A, m, \pm 1\rangle = \frac{1}{\sqrt{2}} (|L, m, \pm 1\rangle - |R, m, \pm 1\rangle)$$

$$|D_{\pm}, m\rangle \equiv \frac{1}{J} \left(J_{L,n}^{C,-n} |S, m, \pm 1\rangle - J_{L,n}^{C,n} |A, m, \mp 1\rangle \right)$$

$$|B_{\pm}, m\rangle \equiv \frac{1}{J} \left(J_{L,n}^{C,n} |S, m, \pm 1\rangle + J_{L,n}^{C,-n} |A, m, \mp 1\rangle \right)$$

$$\text{where } J = \sqrt{(J_{L,n}^{C,-n})^2 + (J_{L,n}^{C,n})^2}$$

Triangular configuration

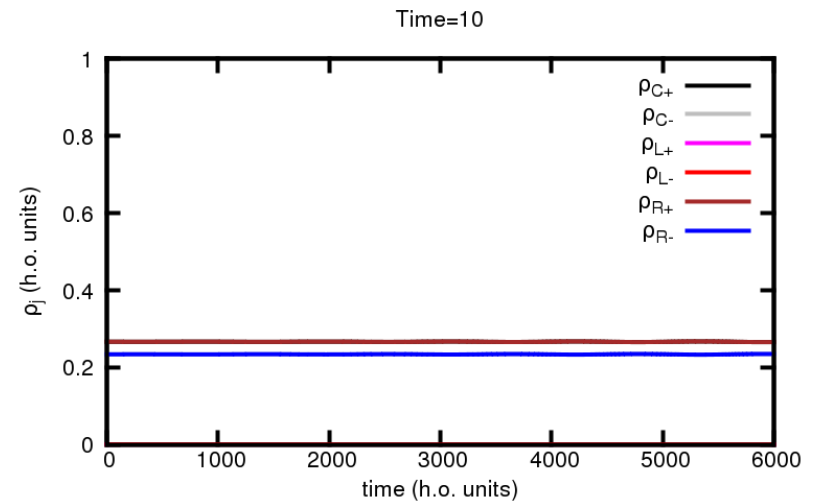
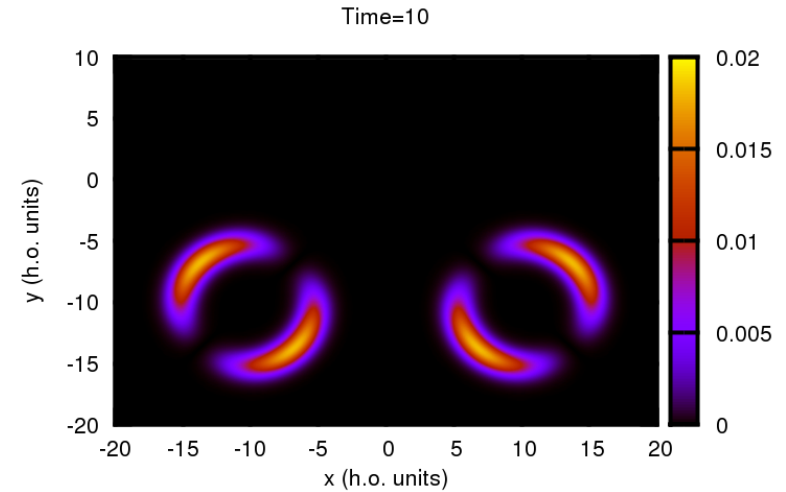
$$\begin{array}{ccc}
 \begin{array}{c} J_{B,+1}^{C,+1} \\ \text{---} \quad \text{---} \\ |B_+, m\rangle \quad |C, m, 1\rangle \\ \text{---} \end{array} & & \begin{array}{c} J_{B,-1}^{C,-1} \\ \text{---} \quad \text{---} \\ |B_-, m\rangle \quad |C, m, -1\rangle \\ \text{---} \end{array} \\
 \overline{|D_+, m\rangle} & & \overline{|D_-, m\rangle}
 \end{array}$$

$$|D_+, m\rangle \equiv \frac{1}{J} \left(J_{L,n}^{C,-n} |S, m, +1\rangle - J_{L,n}^{C,n} |A, m, -1\rangle \right)$$

where

$$|S, m, +1\rangle = \frac{1}{\sqrt{2}} (|L, m, +1\rangle + |R, m, +1\rangle)$$

$$|A, m, -1\rangle = \frac{1}{\sqrt{2}} (|L, m, -1\rangle - |R, m, -1\rangle)$$



Triangular configuration

What happens for $\Theta \neq \pi/2$ or for $|n| \neq 1$?

For an arbitrary angle we see that:

$$J_{D,\pm n}^{C,\pm n} = \frac{J_2 J_3}{\sqrt{2} \sqrt{J_2^2 + J_3^2}} \left(1 + e^{\pm 2in\Theta} \right)$$

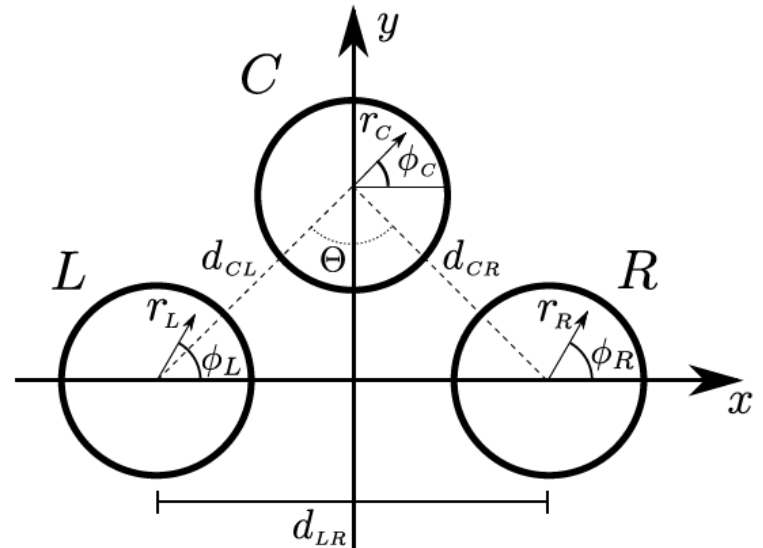
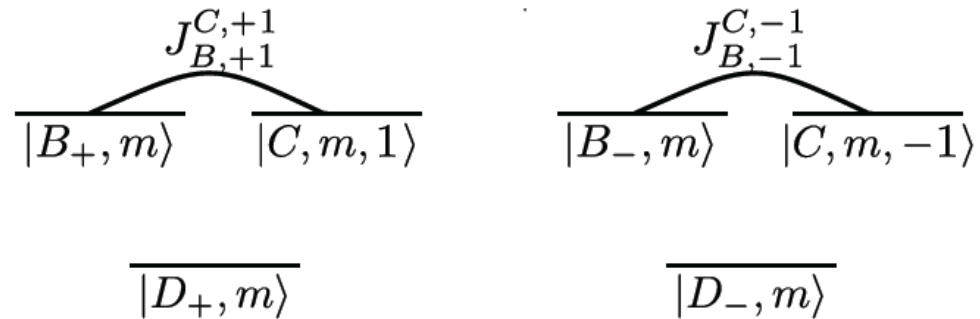
$$J_{D,\pm n}^{C,\mp n} = \frac{J_3^2}{\sqrt{2} \sqrt{J_2^2 + J_3^2}} \left(1 + e^{\pm 2in\Theta} \right)$$

$$\Theta = \frac{\pi}{2} \left(\frac{2k+1}{n} \right) \text{ with } k \in \mathbb{N}$$

where $J_1 = J_{L,n}^{L,-n}$

$$J_2 = J_{L,n}^{C,n}$$

$$J_3 = J_{L,n}^{C,-n}$$



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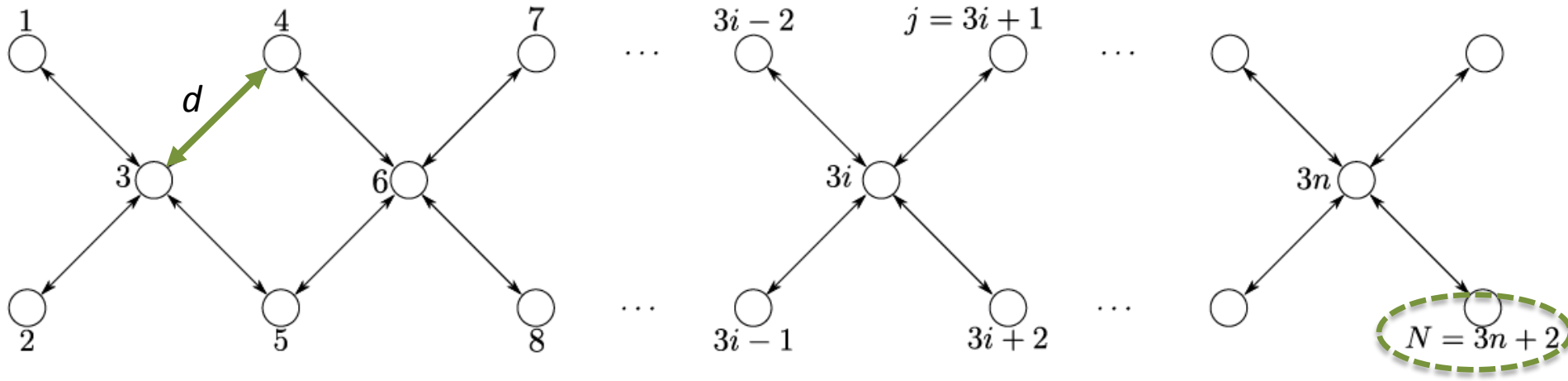
Edge-like states in an optical ribbon

Edge-like states



robust due to quantum interference effects

Ribbon of n cells



Manifolds of $N(l+1)$ degenerated states



$$\hat{H}_{\text{ribbon}} = \sum_{l=0}^{\infty} \hat{H}_l$$

$$l=0$$

$$l=1$$

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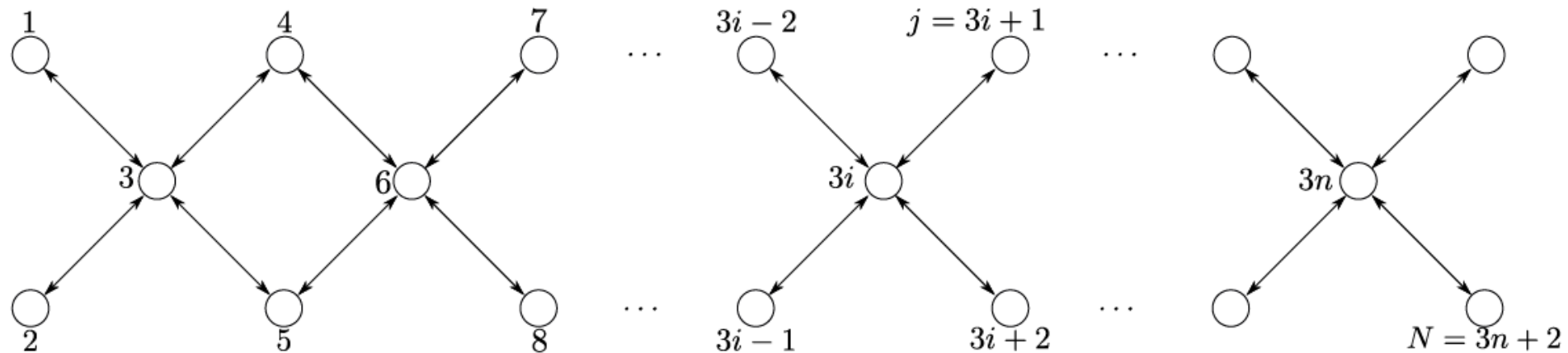
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Edge-like states in an optical ribbon

Manifold of ground states, $l = 0$ $\{|j\rangle\}$ $j = 1, \dots, N$

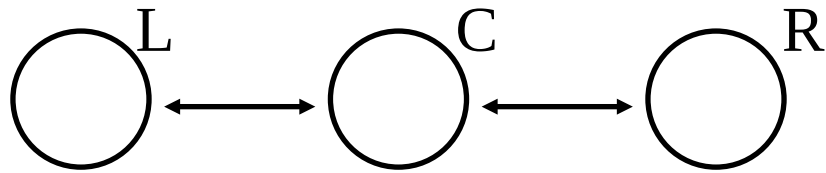
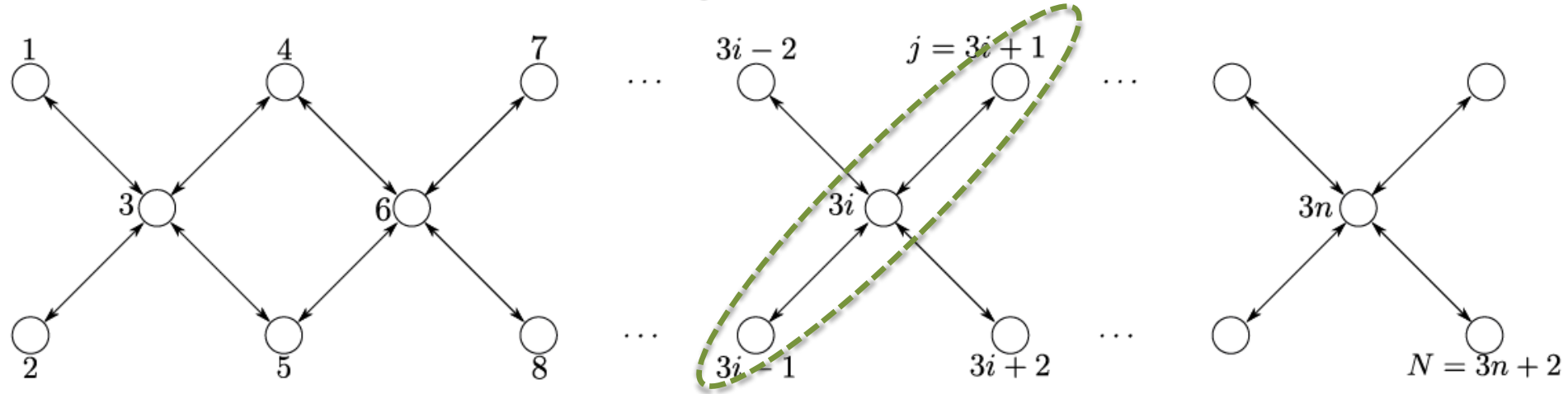


$$\hat{H}_0 = -\hbar J \sum_{i=1}^n \left[(\hat{a}_{3i-2}^\dagger \hat{a}_{3i} + \hat{a}_{3i-1}^\dagger \hat{a}_{3i} + \hat{a}_{3i+1}^\dagger \hat{a}_{3i} + \hat{a}_{3i+2}^\dagger \hat{a}_{3i}) + h.c. \right]$$

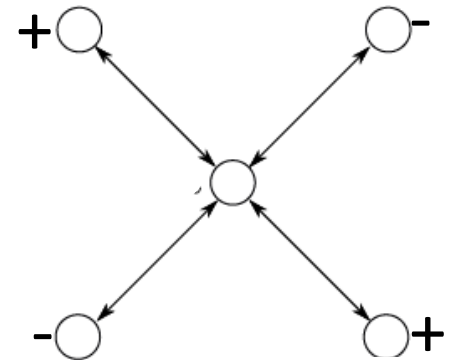
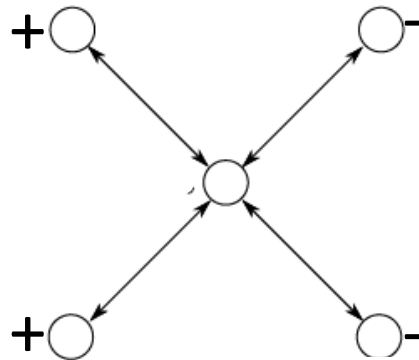
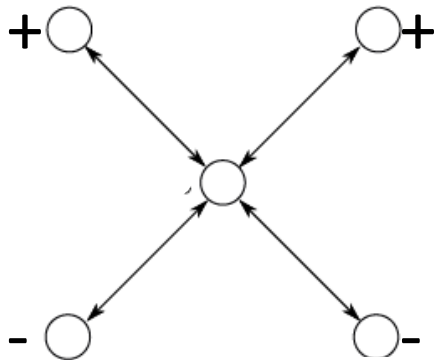
$$\hat{a}_k^\dagger \hat{a}_j |h\rangle = |k\rangle \delta_{jh}$$

Edge-like states in an optical ribbon

Manifold of ground states, $l = 0$ $\{|j\rangle\}$ $j = 1, \dots, N$



$$|D\rangle = \frac{1}{\sqrt{2}}(|L\rangle + e^{i\pi} |R\rangle)$$



Edge-like states in an optical ribbon

Manifold of ground states, $l = 0$

For a ribbon of n cells



$2^n + 1$ possible edge-like states



eigenstates of H_0

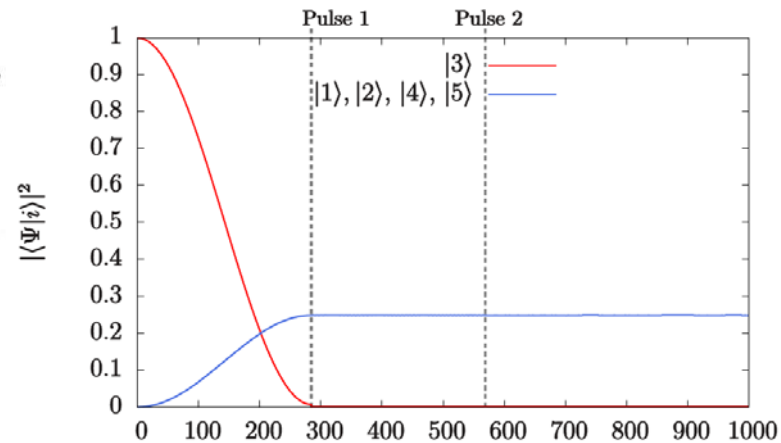
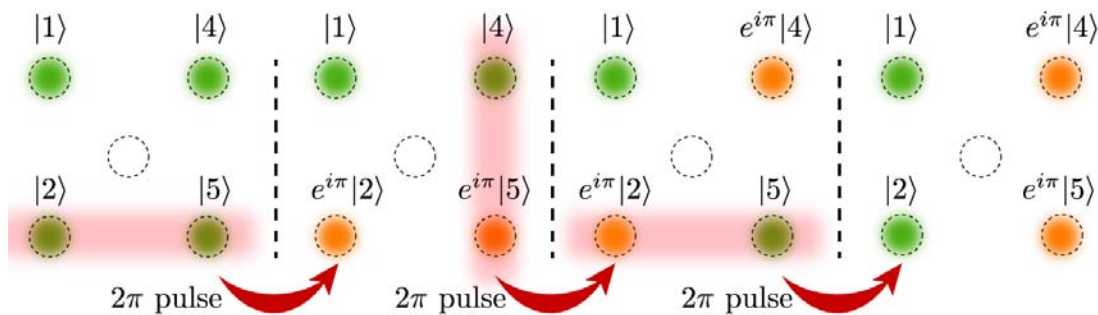
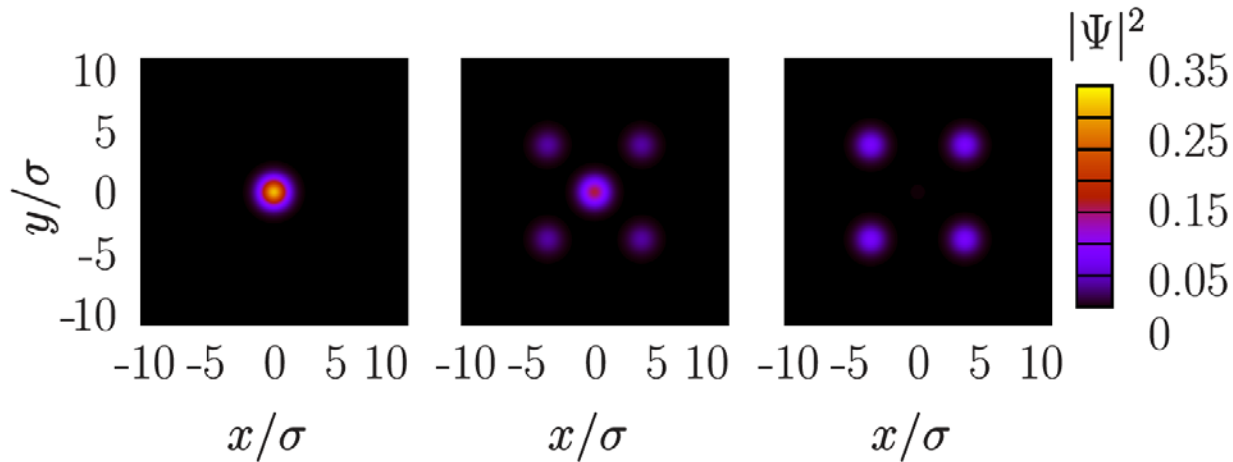
$$|D_k\rangle_{l=0} = \frac{1}{\sqrt{2(n+1)}} (|1\rangle + e^{i\pi} |2\rangle + \sum_{j=1}^n (-1)^{B_k^j(n)} (|3j+1\rangle + e^{i\pi} |3j+2\rangle))$$

$$|D_{2^n}\rangle_{l=0} = \frac{1}{\sqrt{2(n+1)}} \sum_{j=0}^n e^{j \cdot i\pi} (|3j+1\rangle + |3j+2\rangle)$$

where $k = 0, \dots, 2^n - 1$ and $B_k^j(n)$ is the j th digit (starting from the left) of the binary representation of k using a total of n digits, i.e.,
 $B_7(n=4) = 0111$, $B_7^1(n=4) = 0$ and $B_7^2(n=4) = 1$.

Edge-like states in an optical ribbon

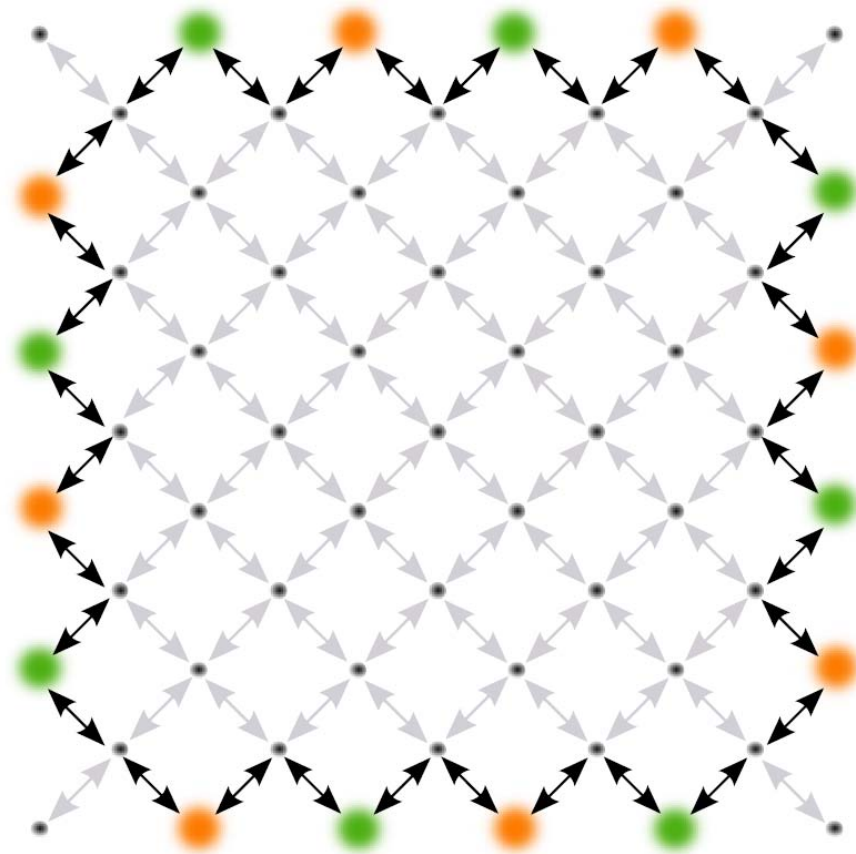
Manifold of ground states, $l = 0$



Edge-like states in an optical ribbon

Manifold of ground states, $l = 0$

Generalization to other geometries



Outline

▪ Geometrically induced complex tunneling

- Introduction
- Two in-line ring potentials
- Three ring potentials in a triangular configuration
- Geometrically engineered spatial dark states

▪ Edge-like states in an optical ribbon

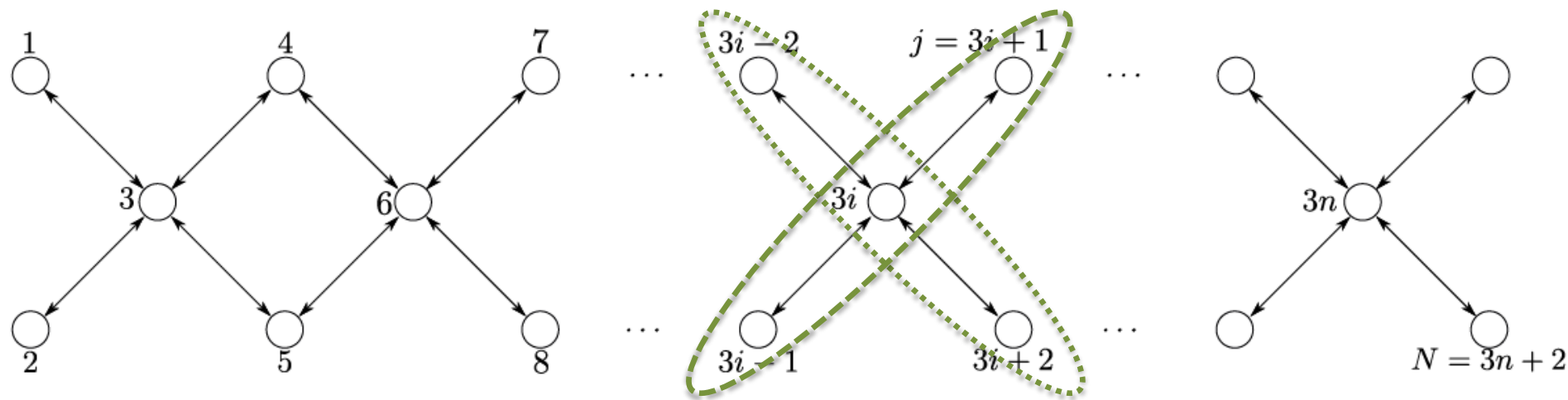
- Optical ribbon
- Ground state manifold
- OAM manifold
- Robustness

▪ Conclusions

Edge-like states in an optical ribbon

Manifold of OAM states $l = 1$ $\{|j, +\rangle, |j, -\rangle\} \quad j = 1, \dots, N$

$$\langle \vec{r} | j, \pm \rangle \sim \Psi(r_j) e^{\pm i(\phi_j - \phi_0)}$$



Self-coupling

$$J_1 \sim |\langle j, \pm | \hat{H} | j, \mp \rangle|$$

Cross-couplings

$$J_2 \sim |\langle j, \pm | \hat{H} | k, \pm \rangle|$$

$$J_3 \sim |\langle j, \pm | \hat{H} | k, \mp \rangle|$$

$$\phi_0 = -\pi/4$$

$$3i - 2 \leftrightarrow 3i$$

$$3i \leftrightarrow 3i + 2$$

Real

$$3i - 1 \leftrightarrow 3i$$

$$3i \leftrightarrow 3i + 1$$

π phase

Edge-like states in an optical ribbon

Manifold of OAM states $l = 1$

$$\hat{a}_{j,\alpha''}^\dagger \hat{a}_{k,\alpha'} |h,\alpha\rangle = |j,\alpha''\rangle \delta_{kh} \delta_{\alpha'\alpha}$$

$$\begin{aligned} \hat{H}_1 = & -\hbar \sum_{i=1}^n \sum_{\alpha,\alpha'=\pm 1} (U_1)_{\alpha\alpha'} (\hat{a}_{3i,\alpha}^\dagger \hat{a}_{3i-1,\alpha'} + \hat{a}_{3i,\alpha}^\dagger \hat{a}_{3i+1,\alpha'}) \\ & -\hbar \sum_{i=1}^n \sum_{\alpha,\alpha'=\pm 1} (U_2)_{\alpha\alpha'} (\hat{a}_{3i,\alpha}^\dagger \hat{a}_{3i-2,\alpha'} + \hat{a}_{3i,\alpha}^\dagger \hat{a}_{3i+2,\alpha'}) \\ & -\hbar \sum_{\alpha,\alpha'} (S_1)_{\alpha\alpha'} (\hat{a}_{1,\alpha}^\dagger \hat{a}_{1,\alpha'} + \hat{a}_{N-1,\alpha}^\dagger \hat{a}_{N-1,\alpha'}) \\ & -\hbar \sum_{\alpha,\alpha'} (S_2)_{\alpha\alpha'} (\hat{a}_{2,\alpha}^\dagger \hat{a}_{2,\alpha'} + \hat{a}_{N,\alpha}^\dagger \hat{a}_{N,\alpha'}) + \text{H.c.}, \end{aligned}$$

$$U_1 = \begin{pmatrix} J_2 & J_3 e^{-i\pi} \\ J_3 e^{i\pi} & J_2 \end{pmatrix}$$

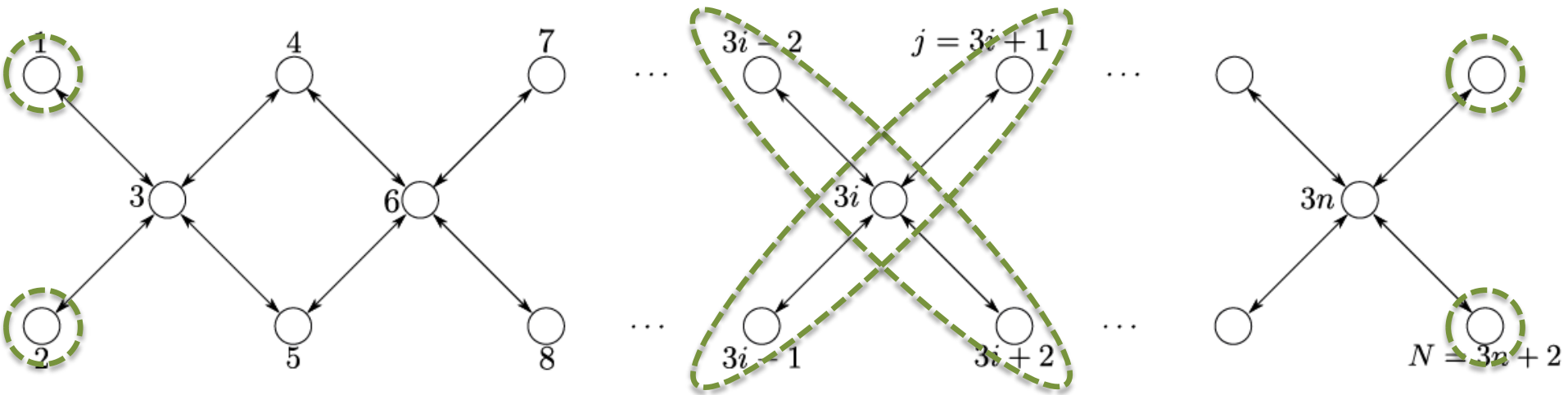
$$S_1 = \begin{pmatrix} 0 & J_1 \\ J_1 & 0 \end{pmatrix}$$

$$U_2 = \begin{pmatrix} J_2 & J_3 \\ J_3 & J_2 \end{pmatrix}$$

$$S_2 = \begin{pmatrix} 0 & J_1 e^{-i\pi} \\ J_1 e^{i\pi} & 0 \end{pmatrix}$$

Edge-like states in an optical ribbon

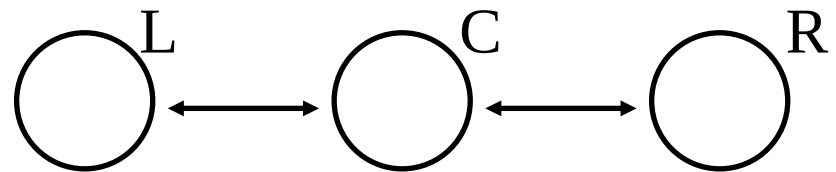
Manifold of OAM states $l = 1$ $\phi_0 = -\pi/4$



J_1 only at the corners

$$|J_1| \ll |J_2|, |J_3|$$

$$|J_2| \approx |J_3|$$



$$|D+\rangle = \frac{1}{2}(|L, +\rangle + |L, -\rangle - |R, +\rangle - |R, -\rangle)$$

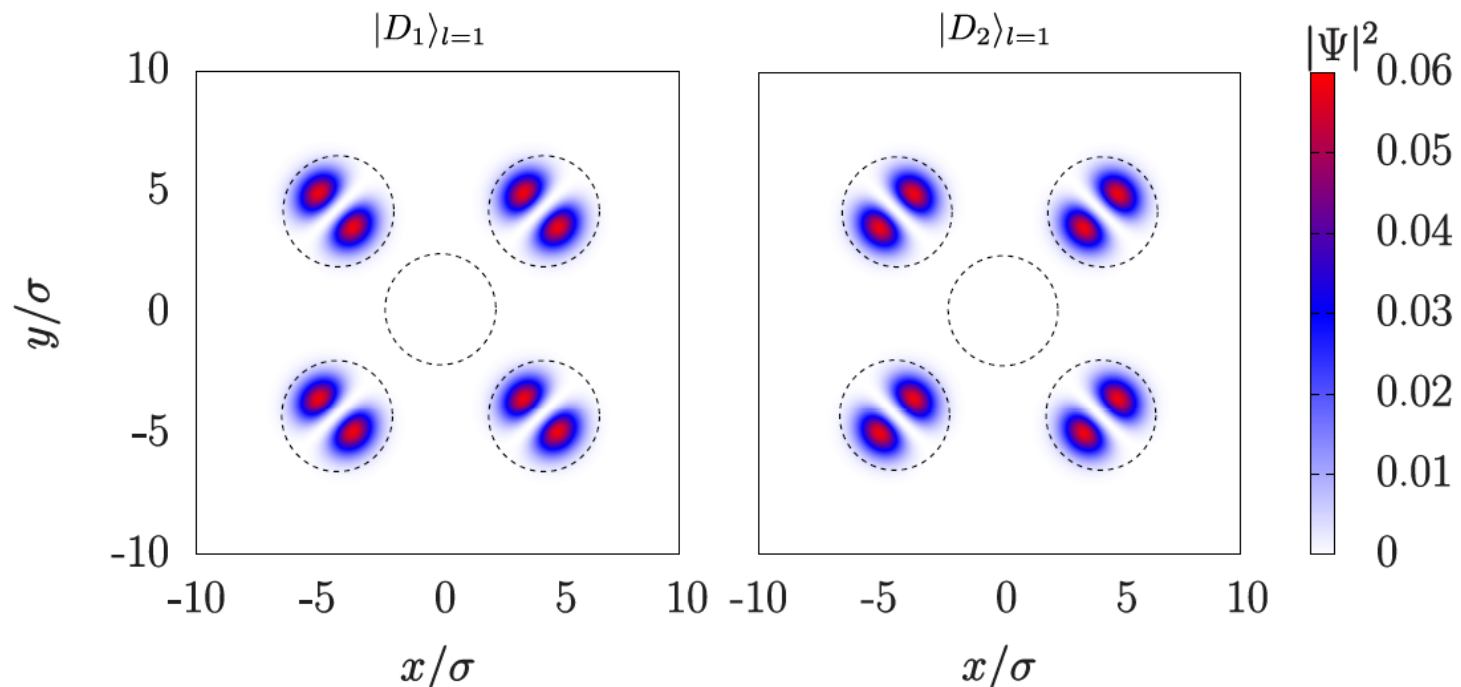
$$|D-\rangle = \frac{1}{2}(|L, +\rangle - |L, -\rangle - |R, +\rangle + |R, -\rangle)$$

Edge-like states in an optical ribbon

Manifold of OAM states $l = 1$ $\phi_0 = -\pi/4$

$$|D_1\rangle_{l=1} = \frac{1}{\sqrt{4(n+1)}} \sum_{j=0}^n ((|3j+1, +\rangle + |3j+1, -\rangle) + e^{i\pi} (|3j+2, +\rangle + |3j+2, -\rangle))$$

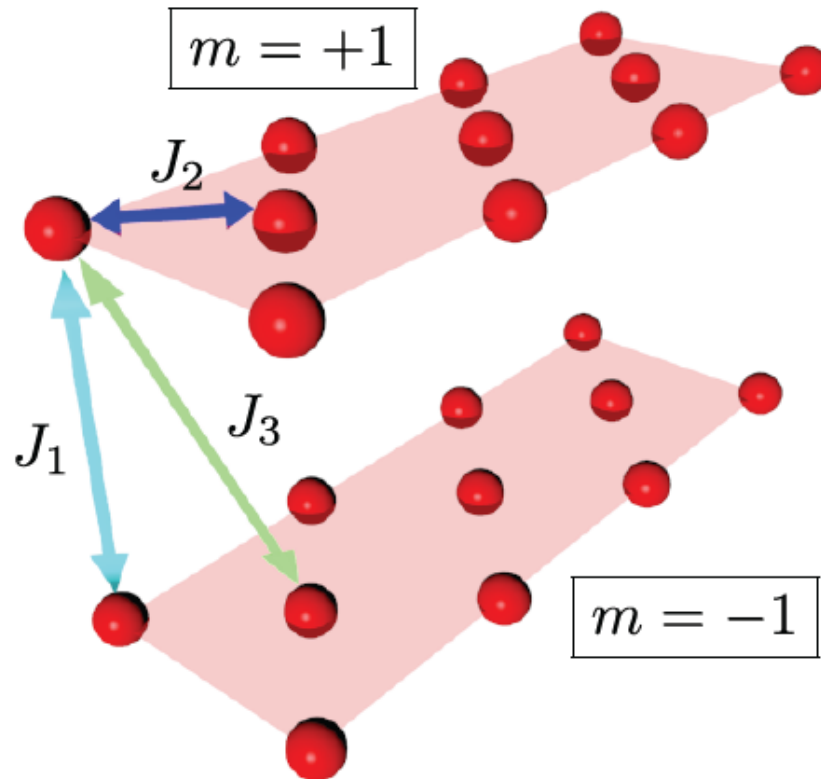
$$|D_2\rangle_{l=1} = \frac{1}{\sqrt{4(n+1)}} \sum_{j=0}^n ((|3j+1, +\rangle - |3j+1, -\rangle) + e^{i\pi} (|3j+2, +\rangle - |3j+2, -\rangle))$$



Edge-like states in an optical ribbon

Manifold of OAM states $l = 1$

Winding number associated to the orbital angular momentum as synthetic dimension



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- OAM manifold
- Robustness

▪ Conclusions

Edge-like states in an optical ribbon

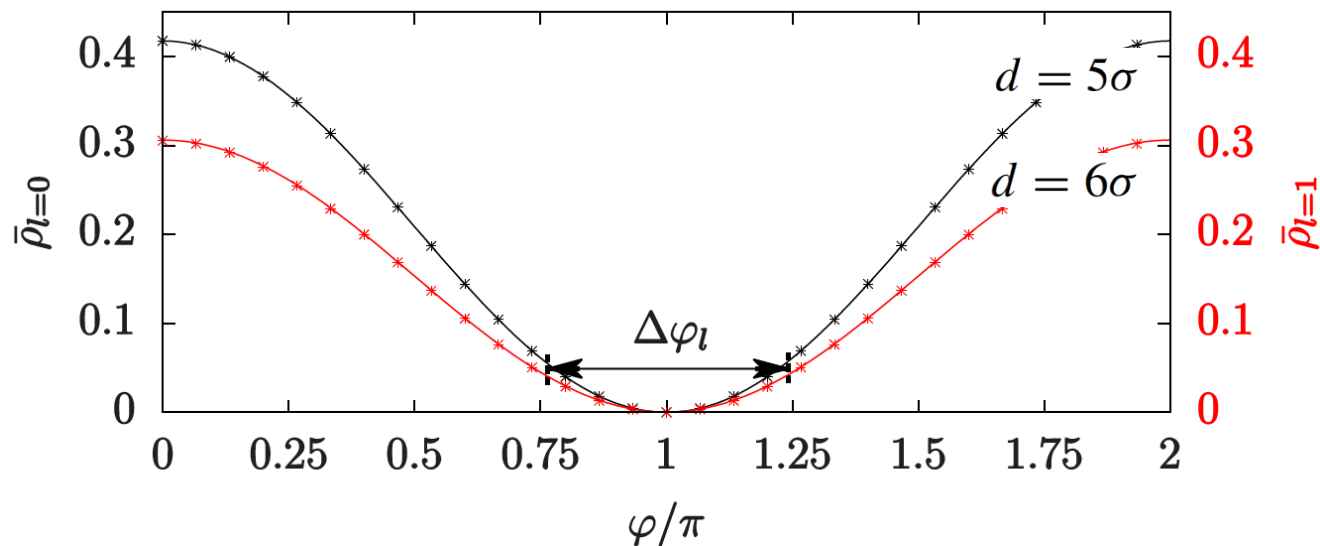
Robustness

➡ against phase differences

Trial states in a ribbon of 2 cells:

$$l = 0 \quad |\Psi(\varphi)\rangle_{l=0} = \frac{1}{\sqrt{6}} \sum_{j=0}^2 (|3j+1\rangle + e^{i\varphi} |3j+2\rangle)$$

$$l = 1 \quad |\Psi(\varphi)\rangle_{l=1} = \frac{1}{\sqrt{12}} \sum_{j=0}^2 ((|3j+1, +\rangle + |3j+1, -\rangle) + e^{i\varphi} (|3j+2, +\rangle + |3j+2, -\rangle))$$



$$\bar{\rho}_l(\varphi) = \sum_{i=1,2} \frac{1}{T} \int_0^T dt \left| \langle 3i | e^{-i\hat{H}t/\hbar} | \Psi(\varphi) \rangle_l \right|^2$$

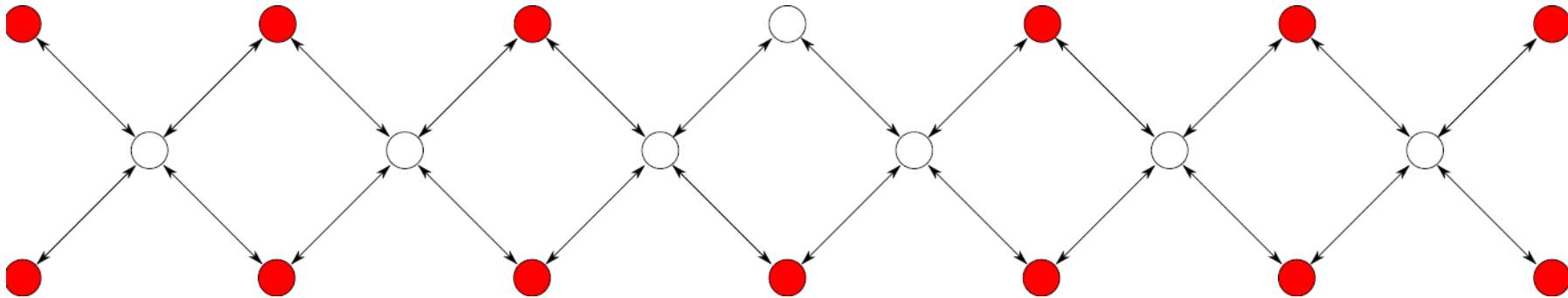
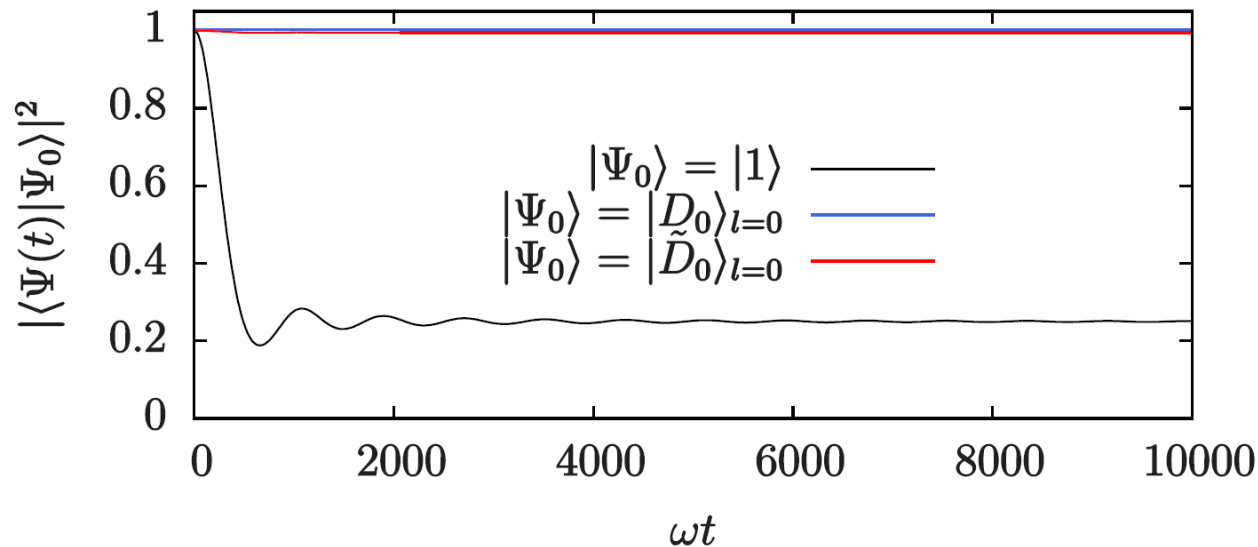
average value of the population of the central sites

Edge-like states in an optical ribbon

Robustness

➡ against local perturbations

ribbon of 100 cells

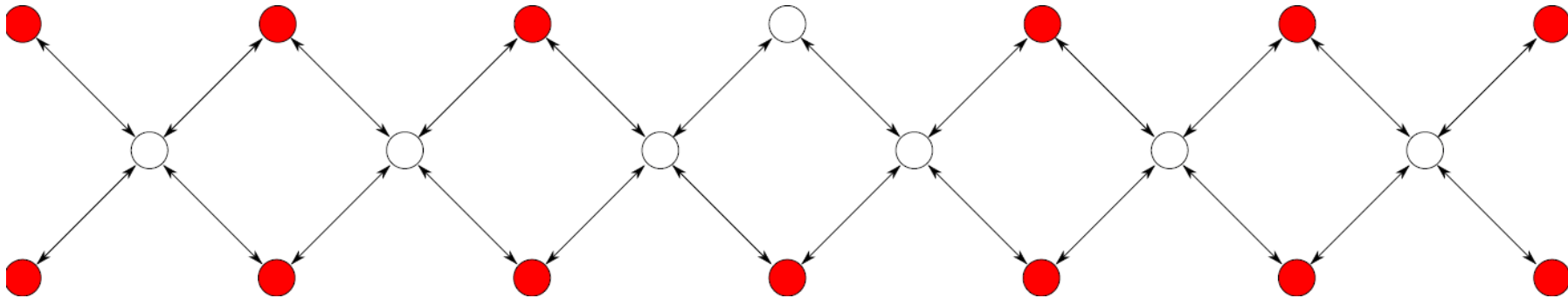
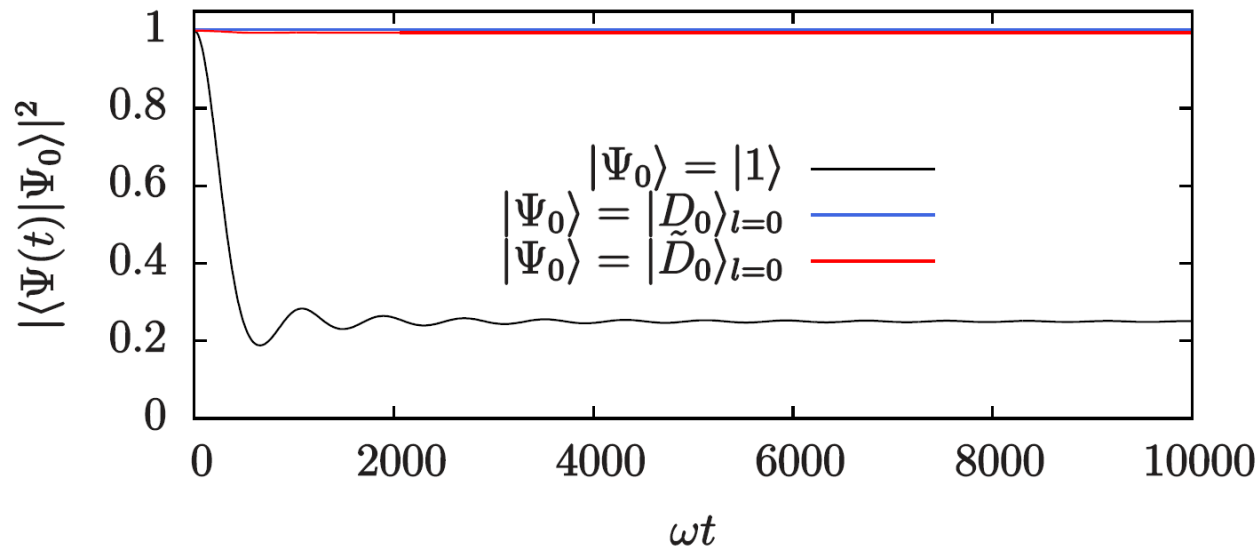


Edge-like states in an optical ribbon

Robustness

➡ against local perturbations

ribbon of 100 cells

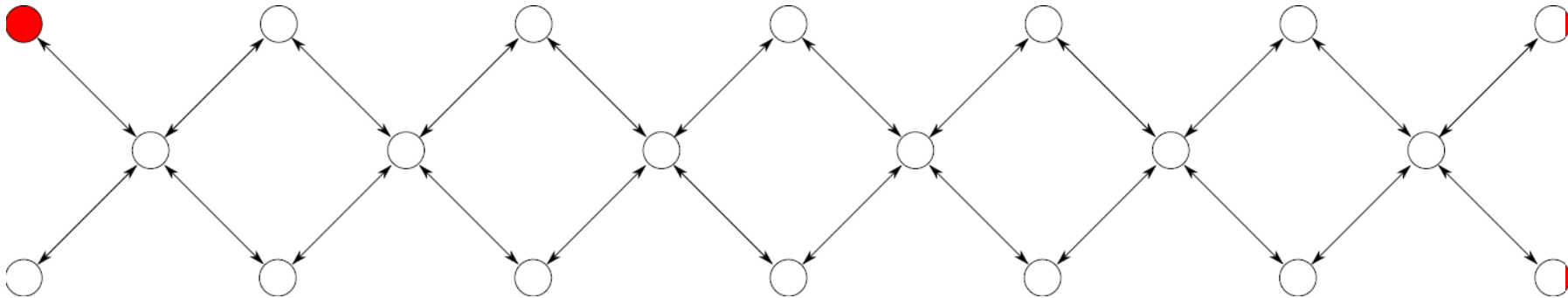
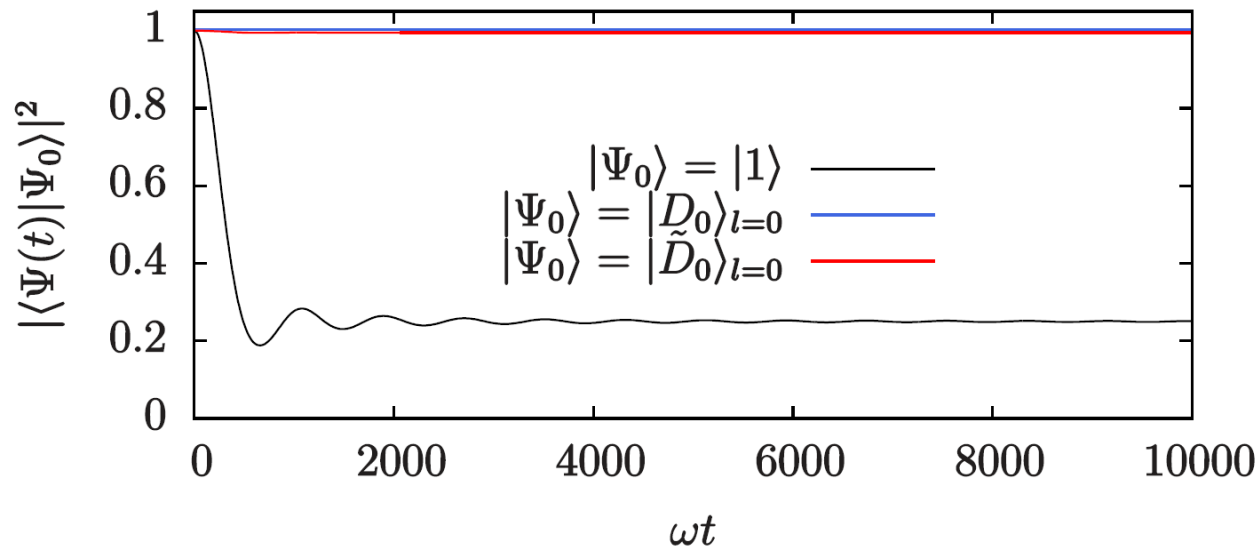


Edge-like states in an optical ribbon

Robustness

➡ against local perturbations

ribbon of 100 cells



Outline

▪ **Geometrically induced complex tunneling**

- Introduction
- Two in-line ring potentials
- Three ring potentials in a triangular configuration
- Geometrically engineered spatial dark states

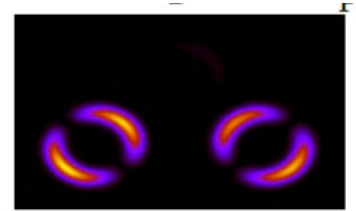
▪ **Edge-like states in an optical ribbon**

- Optical ribbon
- Ground state manifold
- OAM manifold
- Robustness

▪ **Conclusions**

Conclusions

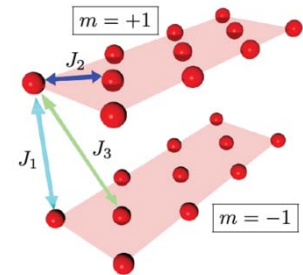
Geometrically induced complex tunneling



- By means of geometrical symmetries it is shown that angular momentum states in sided-coupled cylindrically symmetric potentials exhibit complex tunneling amplitudes.
- In a system of three rings the complex nature of the tunneling plays crucial role in the dynamics.
- In a triangular ring configuration, complex tunneling amplitudes allow engineering spatial dark states via quantum interference.

Conclusions

Edge-like states in an optical ribbon



- ELS using states with $l = 0$ and $l = 1$ in a ribbon based on spatial dark states are predicted.
- The experimental implementation of these ELS for $l = 0$ and the switch from one ELS to another using laser pulses is discussed.
- The robustness of these states under relative phase variations and when including a defect on the lattice is demonstrated.
- The winding number can be regarded as a synthetic dimension.

QUANTUM ATOM OPTICS GROUP (UAB)

<http://grupsderecerca.uab.cat/qaos/>



Juan Luis Rubio

Josep Cabedo

Gerard Pelegrí

Verònica Ahufinger

Jordi Mompart

Ramón Corbalán

Gerard Queraltó

Todor Kirilov



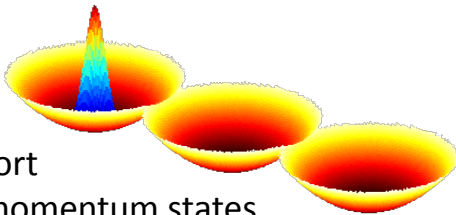
J. Polo
(now in Grenoble)



A. Turpin
(now in Bonn)

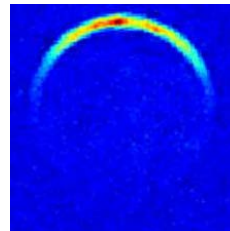
Ultracold atoms

Quantum transport
Orbital angular momentum states
Complex tunneling



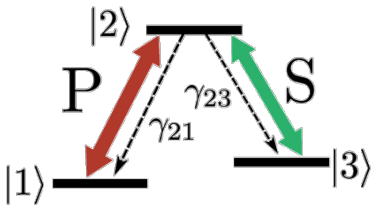
Conical Refraction

Fundamentals: theory and experiment
Applications: trapping microparticles and BECs



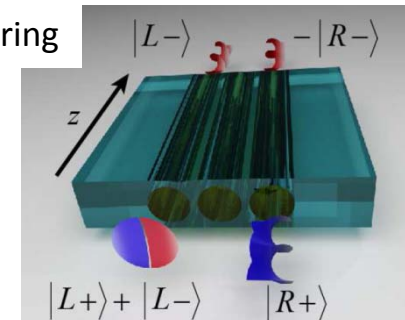
Laser-matter interaction

Nanoscopy
Atomic frequency combs
Spin-orbit coupling



Light propagation in coupled optical waveguides

Dark and bright OAM modes
SUSY techniques for mode filtering





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