Atomtronic Rotational Sensing *with* Bose-Einstein Condensates

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  - David Carty
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  - Ifan Hughes
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  - Viv Kendon
  - Robert Potvliege
  - Kevin Weatherill
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  - David Carty
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  - Eckart Wrede
- **Newcastle Mathematics and Statistics**
  - Andrew Baggaley
  - Carlo Barenghi
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  - Nick Parker
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  - Paolo Comaron
  - Sarah Jowett
  - Fabrizio Larcher
  - Klejdja Xhani

1: BEC & INTERFEROMETRY BASICS

Gross-Pitaevskii Equation (GPE)

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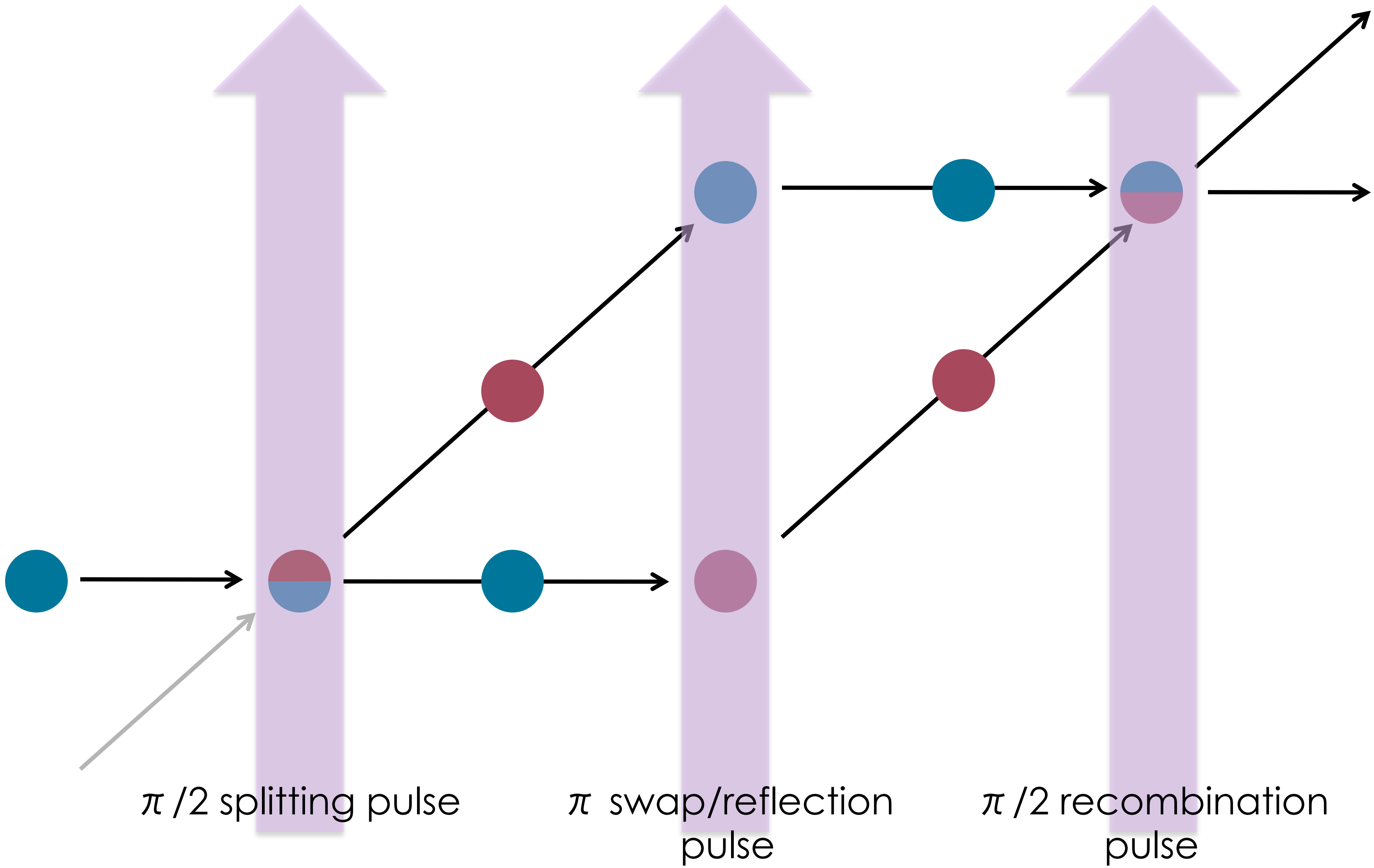
- In the case of **Bose-Einstein condensation** in a dilute atomic gas:
  - "almost everything" in the **same spatial mode**
  - transition to a "macroscopic, classical" regime

- Quantum field well-described by a **classical field**

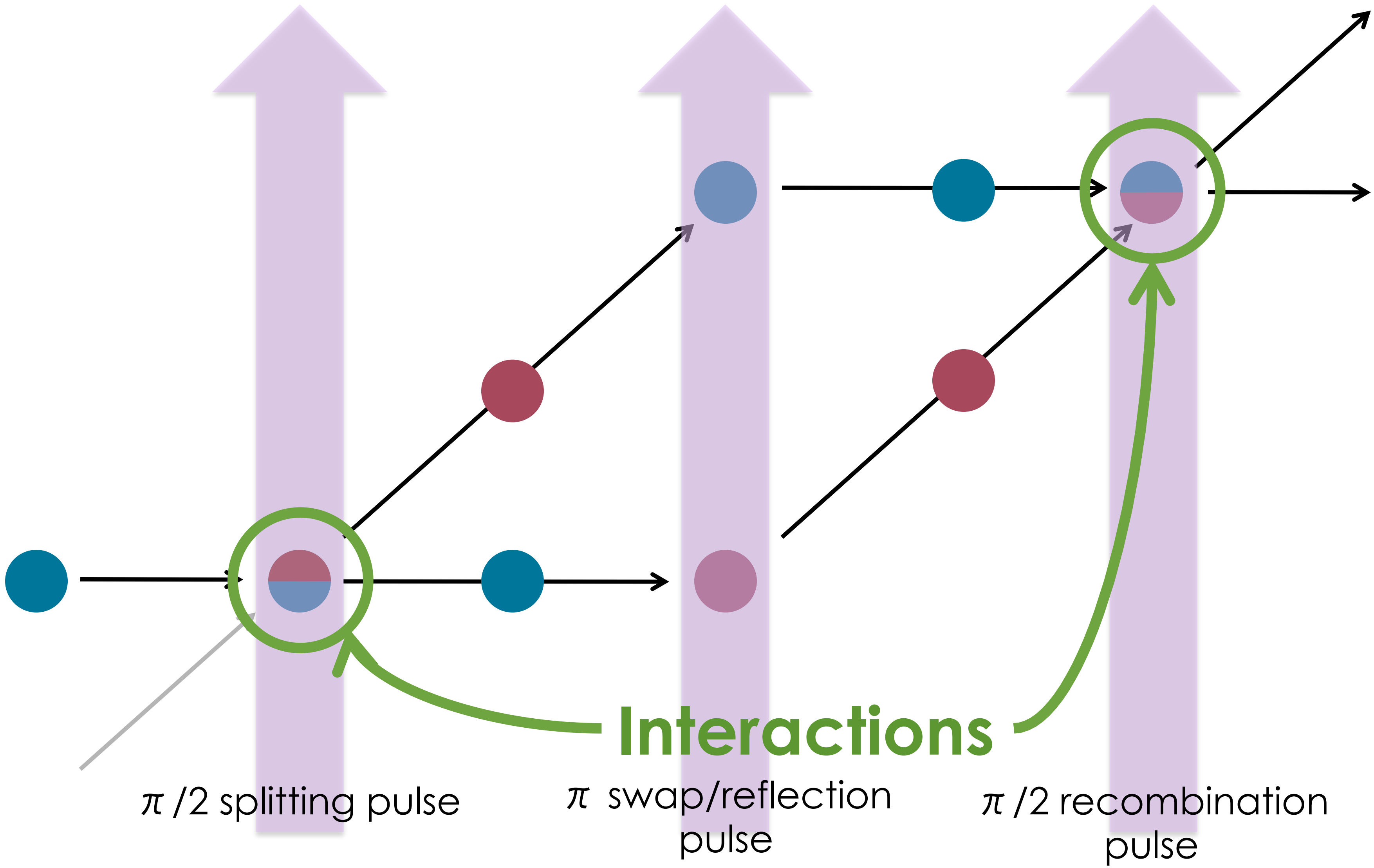
$$i\hbar\frac{\partial\Psi(\mathbf{r})}{\partial t} = \left[-\frac{\hbar^2\nabla^2}{2M} + V(\mathbf{r}) + \frac{4\pi\hbar^2 aN}{M}|\Psi(\mathbf{r})|^2\right]\Psi(\mathbf{r}),$$

(have simplified interactions to a contact term quantified by the s-wave scattering length a – **low-energy** scattering approximation)

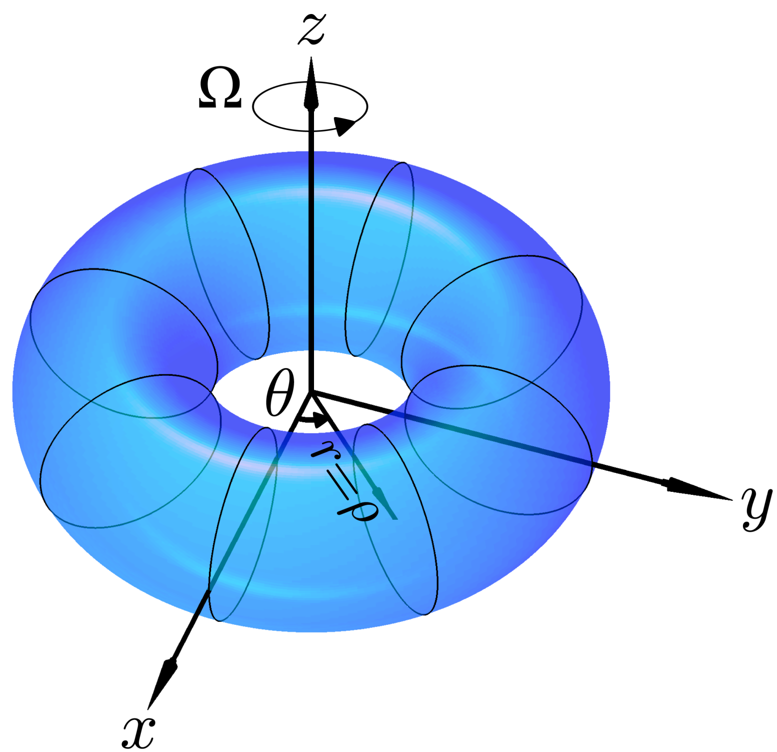
Atom Interferometry



Atom Interferometry: Interactions Bad!



Angular Motion in a Toroidal Trap



- Consider trap to be in a **rotating frame** (frequency Ω)
- Invites possibility of **Sagnac interferometry** (optically, light split & propagates around in opposing directions)

See also Burke, Sackett: Phys. Rev. A **80**, 061603 (2009)
Stevenson *et al.*, Phys. Rev. Lett. **115**, 163001 (2015)
Nolan *et al.*, Phys. Rev. A **93**, 023616 (2016)
Bell *et al.*, New J. Phys. **18**, 035003 (2016)

2: ATTRACTIVE INTERACTIONS (BRIGHT SOLITONS)

Properties of Bright Solitons

Joint Quantum Centre
www.jqc.org.uk

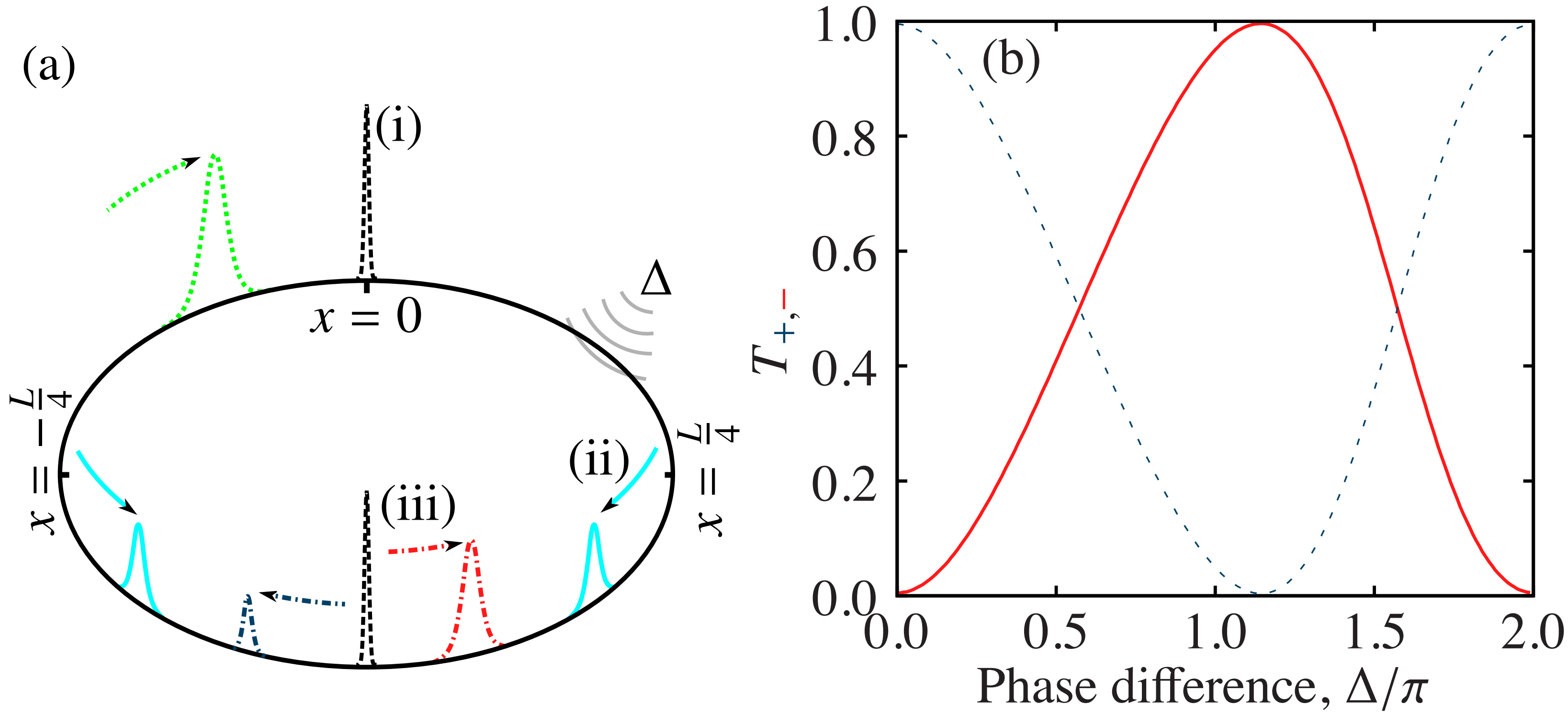
Zakharov, Shabat, Sov. Phys. JETP **34**, 62 (1972)
Gordon, Opt. Lett. **8**, 596 (1983)

- Solitary waves (including solitons) **propagate without dispersion**
- The 1D nonlinear Schrödinger equation (NLSE) is **integrable** (as many constants of the motion as there are degrees of freedom)
- **Protection** that conservation laws offer means solitons (here meaning solitary wave solutions of the 1D NLSE) are **robust to collisions**
- **Particle-like** behaviour and name, "soliton"

Martin, Adams, Gardiner
Phys. Rev. Lett. **98**, 020402 (2007); Phys. Rev. A **77**, 013620 (2008)

Splitting (& Recombining) on Narrow Barriers

Helm, Billam, Gardiner, Phys. Rev. A **85**, 053621 (2012)



See also Martin, Ruostekoski, New J. Phys **14**, 043040 (2012)
Polo, Ahufinger, Phys. Rev. A **88**, 053628 (2013)

Scaling to "Soliton Units"

- Units of
  - Position: $\hbar^2 / mgN$
  - Time: $\hbar^3 / mg^2 N^2$
  - Energy: $mg^2 N^2 / \hbar^2$

- Dimensionless 1D Gross-Pitaevskii equation (GPE)

$$i\frac{\partial \psi(x)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{q}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2} + \frac{\omega_x^2 x^2}{2} - |\psi(x)|^2 \right]\psi(x)$$

Splitting on an Asymptotically Narrow Barrier

$$i\frac{\partial\psi(x)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \frac{q}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2} - |\psi(x)|^2\right]\psi(x)$$

- **Delta-function barrier** tended to as $\sigma \to 0$
- Transmission coefficient for a soliton with **incoming velocity** v approximately (exact for $v \to \infty$)

$$T_q(v) = \frac{v^2}{v^2+q^2} = \frac{1}{1+\alpha^2} \qquad (\alpha = q/v)$$

- Outgoing waves are a decaying radiation term and 1 or 2 **solitons**, with amplitudes

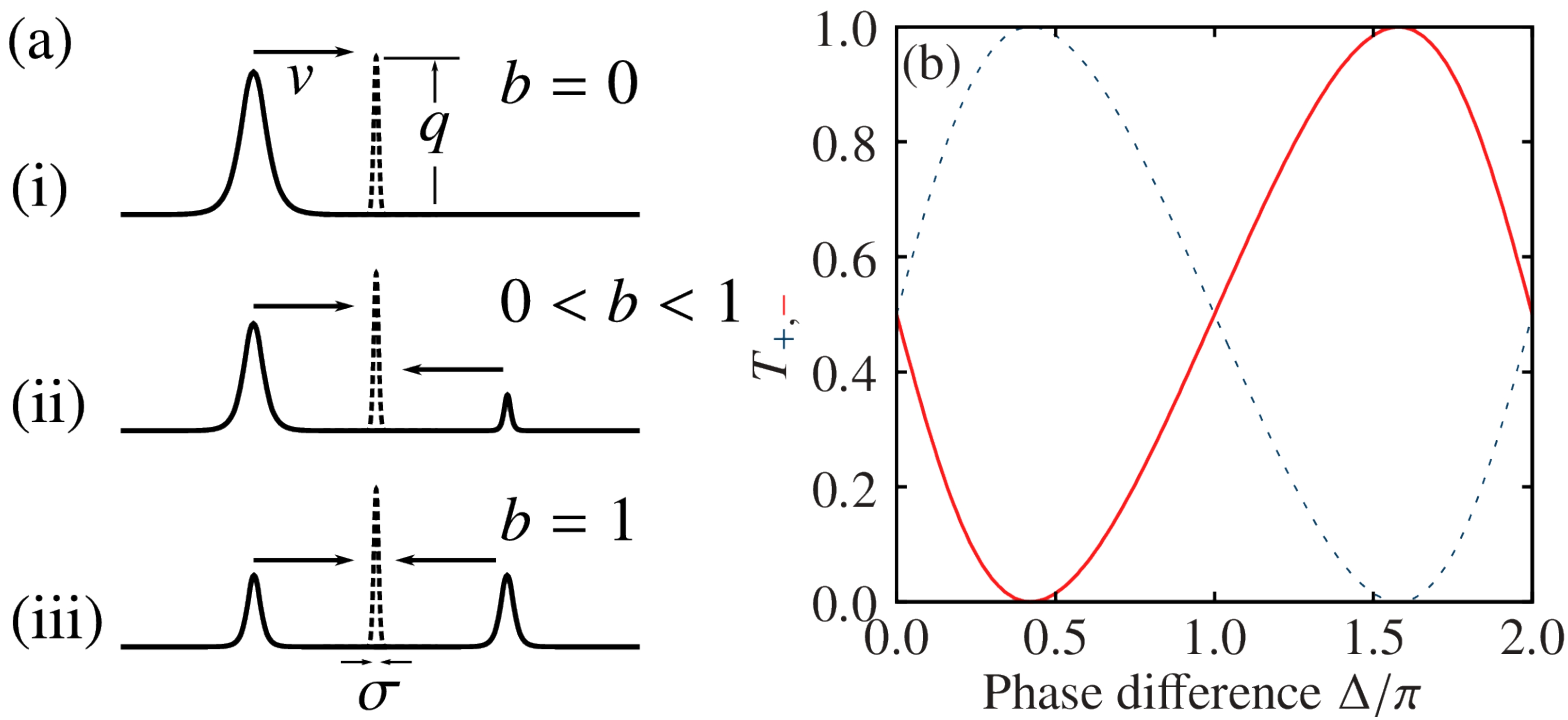
$$A_T = \max\left(0, 2\sqrt{T_q(v)} - 1\right), \qquad A_R = \max\left(0, 2\sqrt{1-T_q(v)} - 1\right)$$

Holmer, Marzuola, Zworski, Comm. Math. Phys. **274**, 187 (2007)
Holmer, Marzuola, Zworski, J. Nonlin. Sci. **17**, 349 (2007)

Collisions at a Narrow Barrier

$$\psi(x,0) = \psi_+(x) + \psi_-(x),$$

$$\psi_-(x) = \frac{1}{2+2b}\operatorname{sech}\left(\frac{x+x_0}{2+2b}\right)e^{ivx}, \qquad \psi_+(x) = \frac{b}{2+2b}\operatorname{sech}\left(\frac{b(x-x_0)}{2+2b}\right)e^{-i[vx+\Delta]}$$



$b = 1, \quad v = 2, \quad \sigma = 0.28$

Dynamics in a Rotating Frame (One *or* Two Barriers)

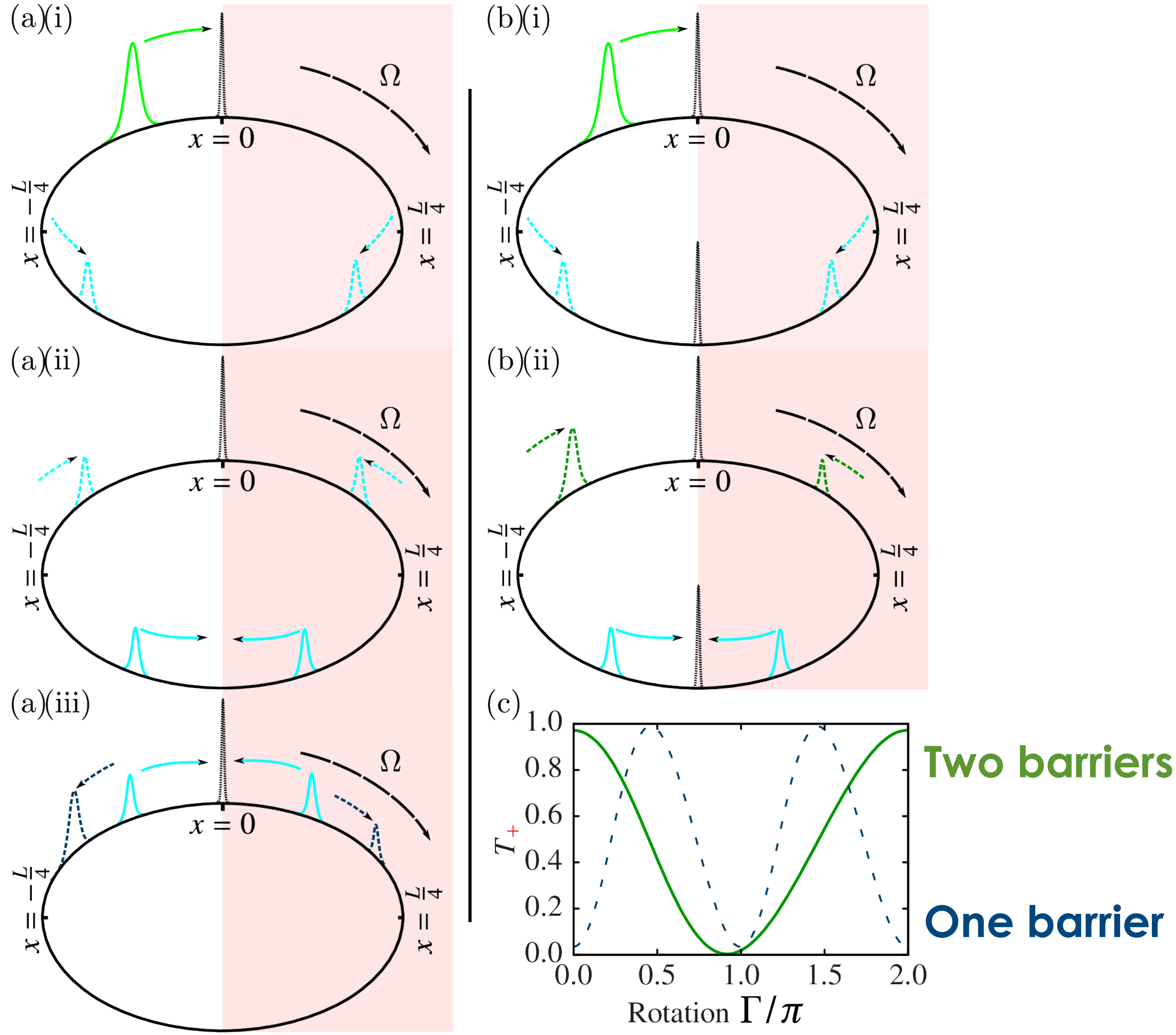
- Consider GPE in a **ring geometry** (periodic boundary conditions with period L_D)

$$i\frac{\partial\psi(x)}{\partial t}=\Big[-\frac{1}{2}\frac{\partial^2}{\partial x^2}+i\Gamma\frac{\partial}{\partial x}+\frac{q}{\sigma\sqrt{2\pi}}e^{-x^2/2\sigma^2} + (n_\mathrm{b}-1)\frac{q}{\sigma\sqrt{2\pi}}e^{-(x\pm L/2)^2/2\sigma^2}-|\psi(x)|^2\Big]\psi(x)$$

- Magnitude of **rotational term** defined through

$$2\pi\Gamma=\Omega L=\frac{\hbar}{|g_{1\mathrm{D}}|N}\Omega_\mathrm{D}L_\mathrm{D}$$

Rotational Sensing and Sagnac Interferometry

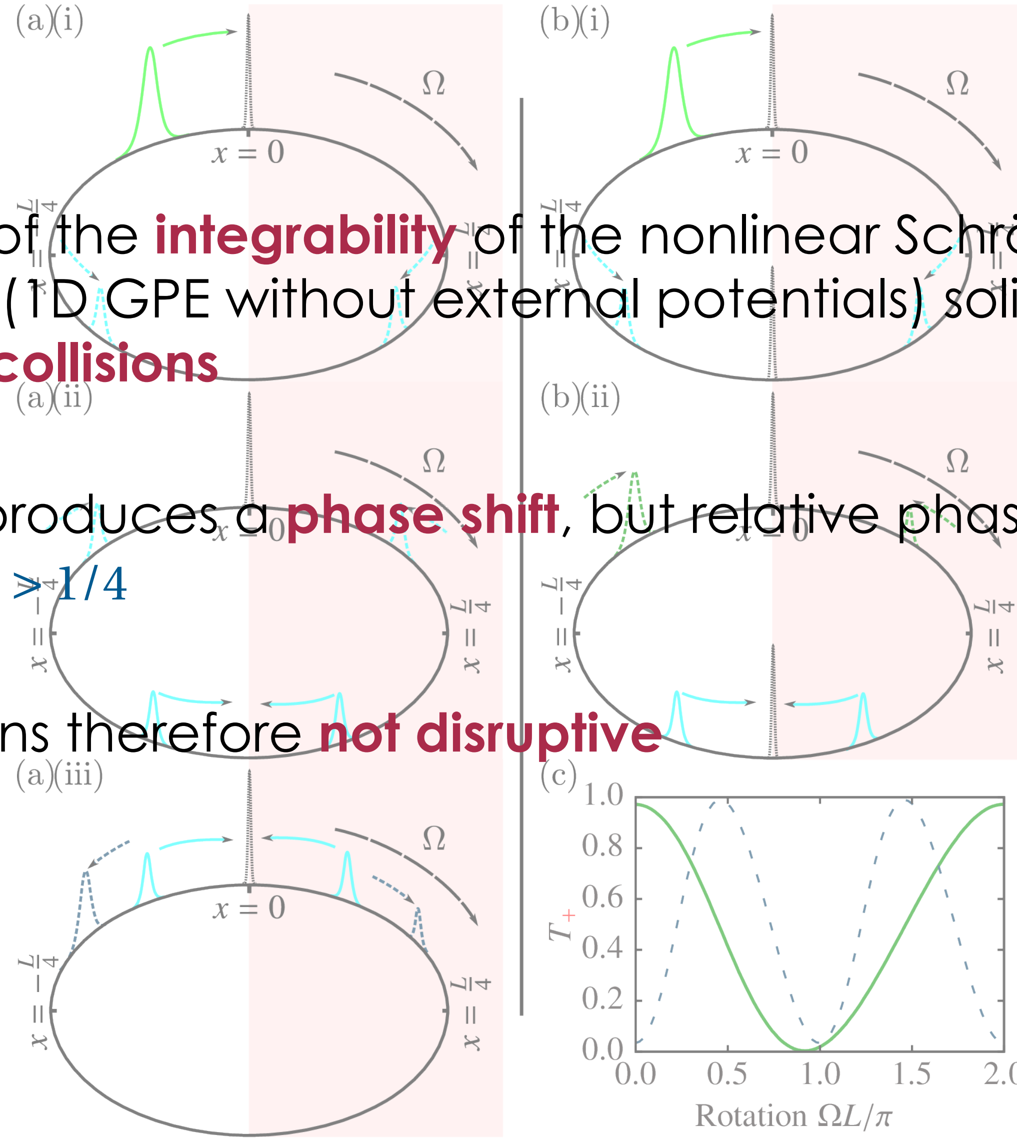


Collision without a Barrier

- Because of the **integrability** of the nonlinear Schrödinger equation (1D GPE without external potentials) solitons are **robust to collisions**

- Collision produces a **phase shift**, but relative phase difference is **zero** if $v > 1/4$

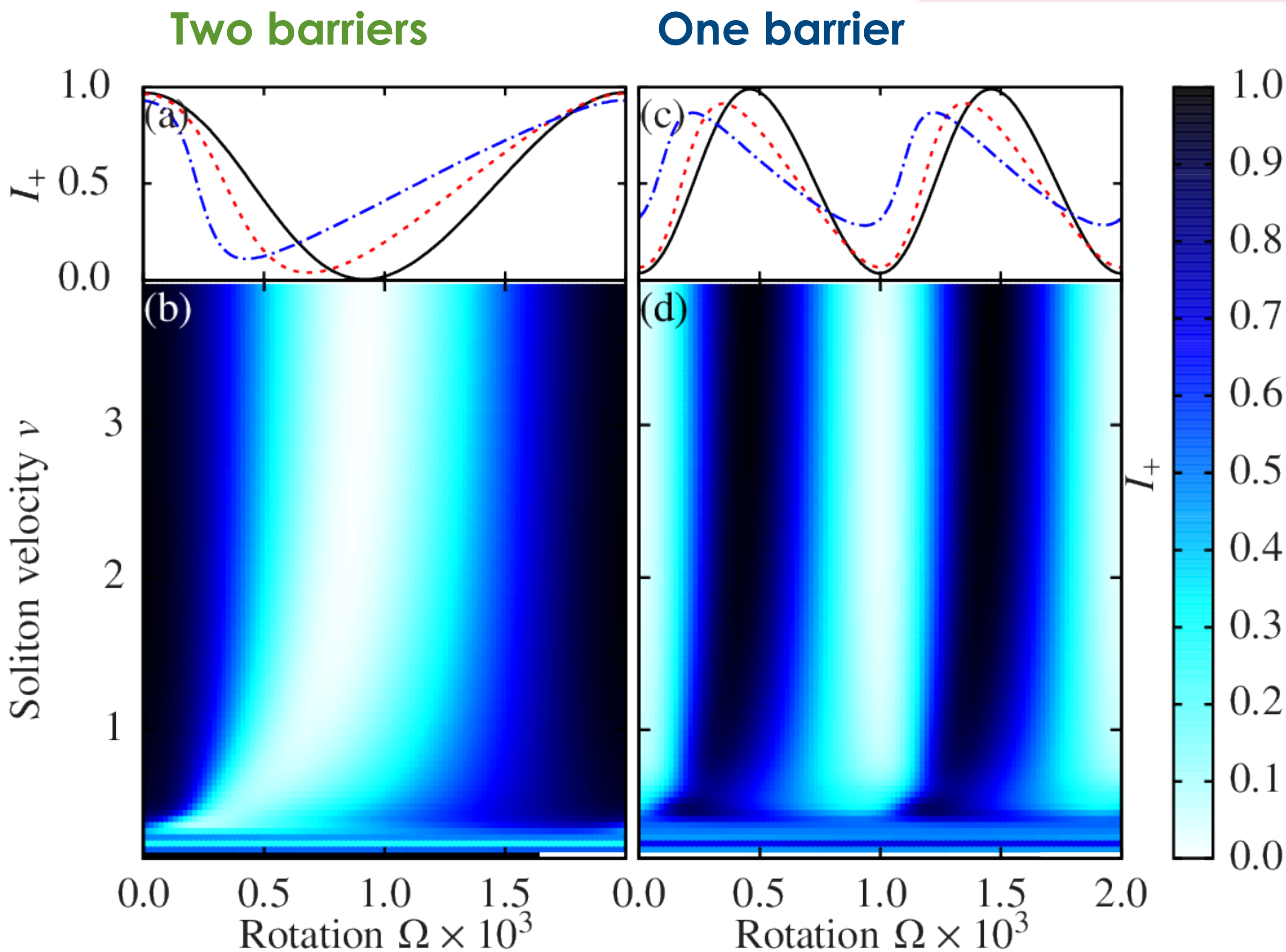
- Interactions therefore **not disruptive**



Interferometric Response

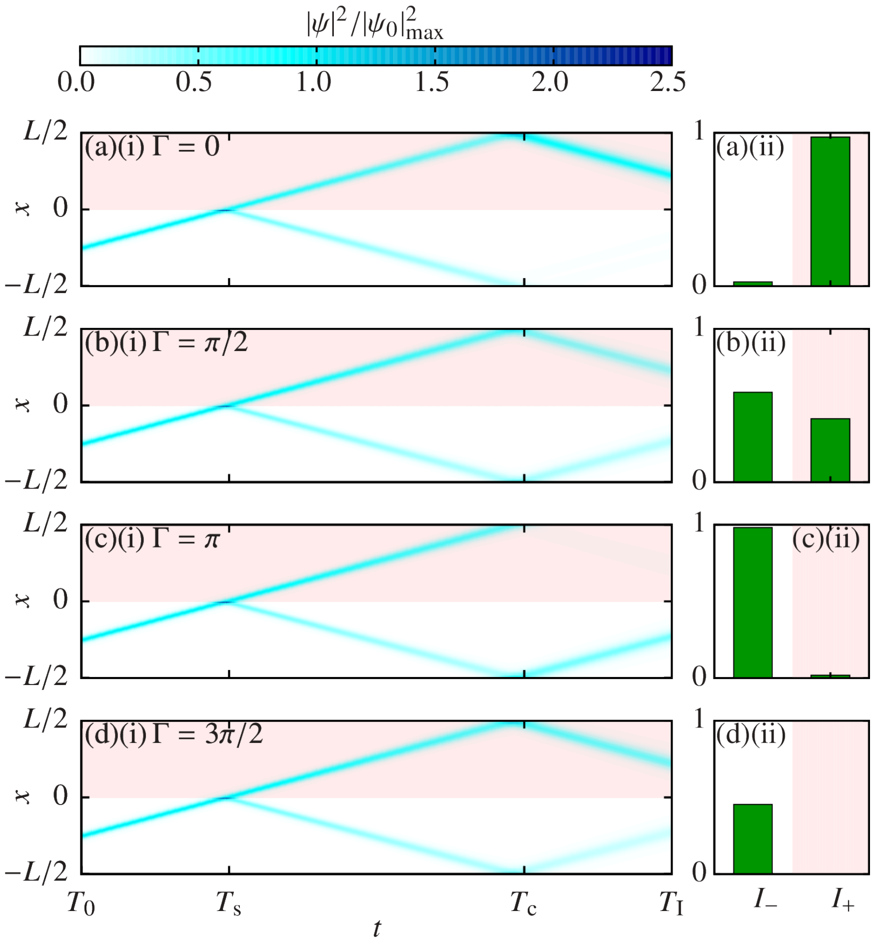
v = 0.52, 1, 4

Helm, Cornish, Gardiner,
Phys. Rev. Lett. **114**, 134101 (2015)

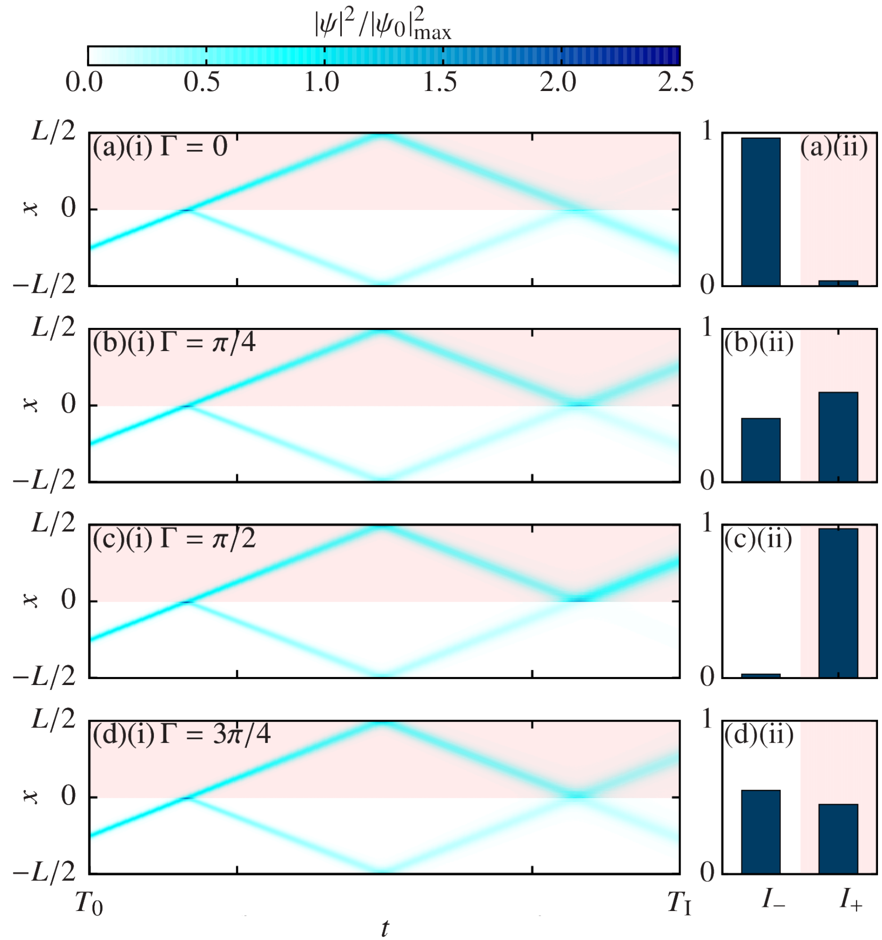


Sample Outputs

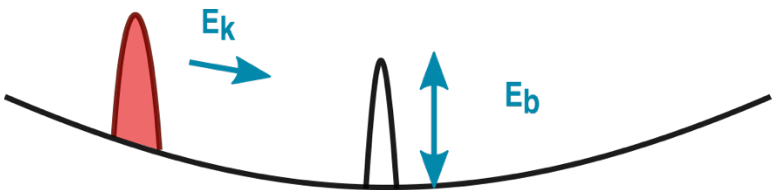
Two barriers

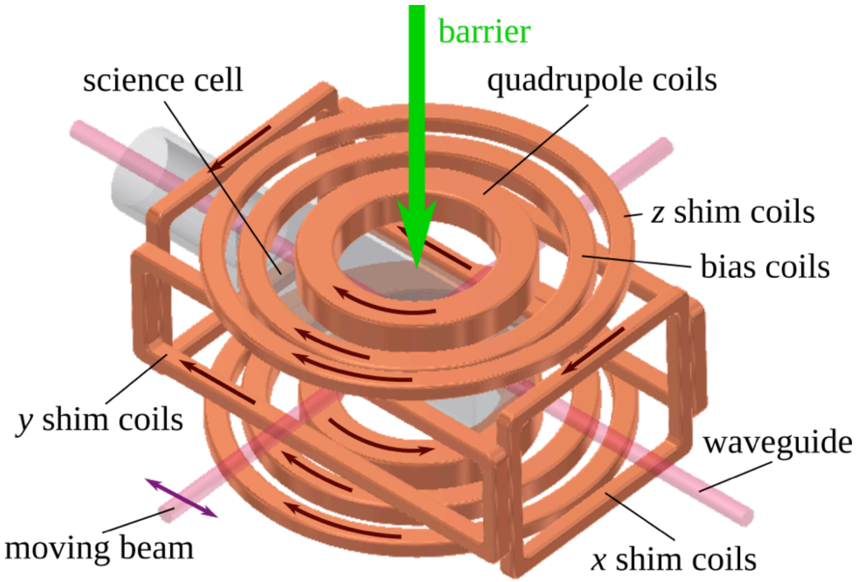


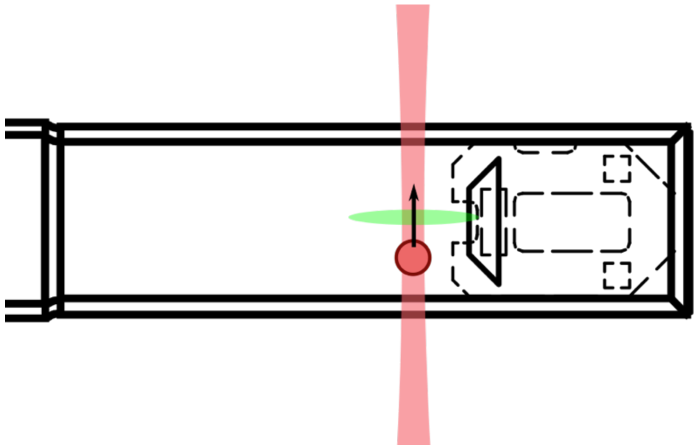
One barrier



Durham ^{85}Rb Soliton-with-Barrier Setup

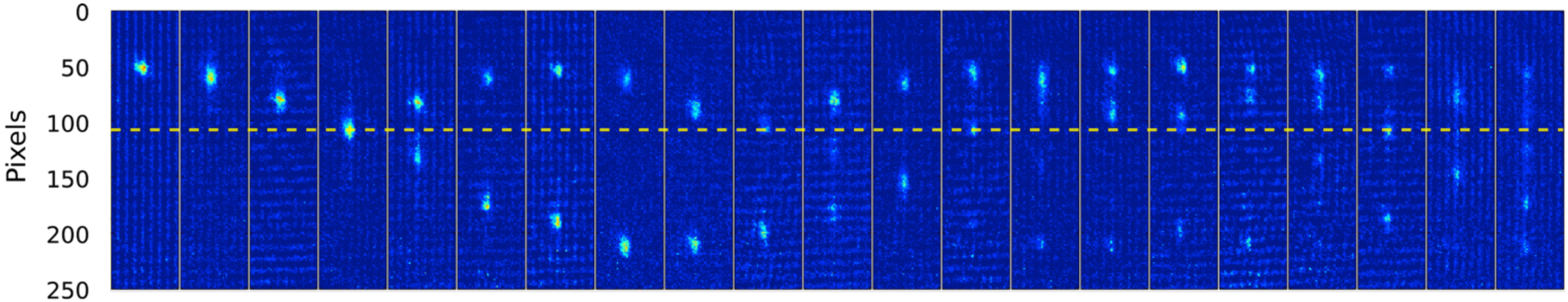


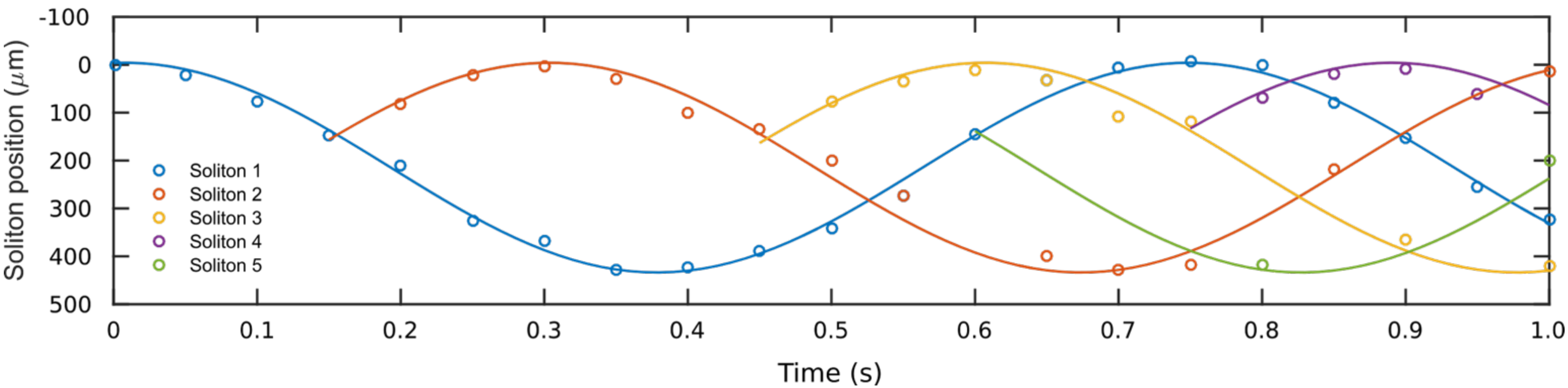




- Make BEC in **crossed-dipole** trap
- Ramp scattering length to $a=0$
- Simultaneously jump to negative scattering length and turn waveguide off ➔ soliton!

First Experiment

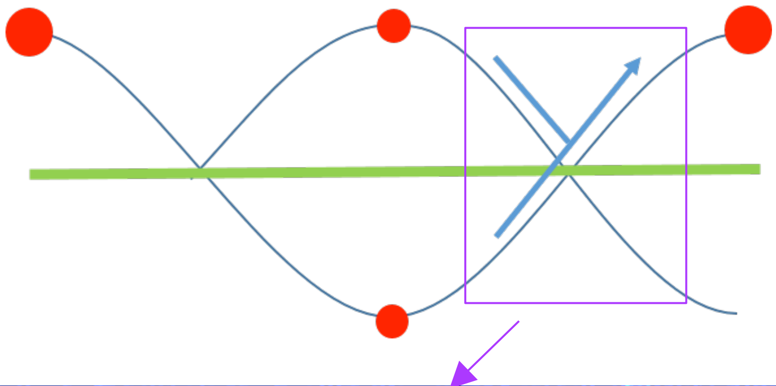


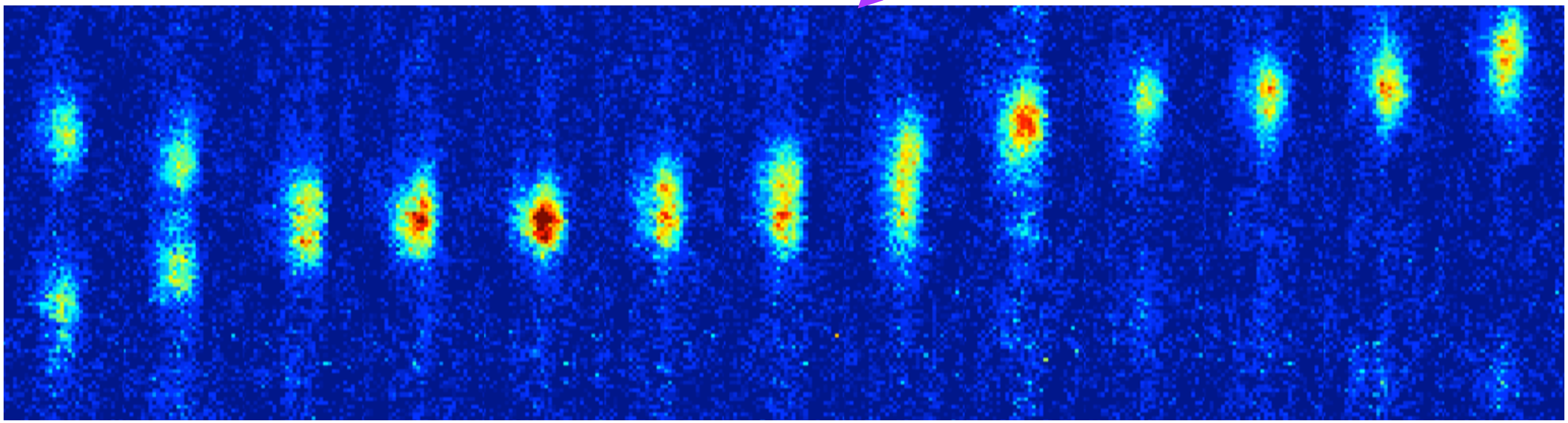


- Release soliton into **axially weak** harmonic trap with the **barrier on** – and see what happens!

Recombination (Preliminary Data)

- Split 50/50 at the barrier, then recombine at the barrier

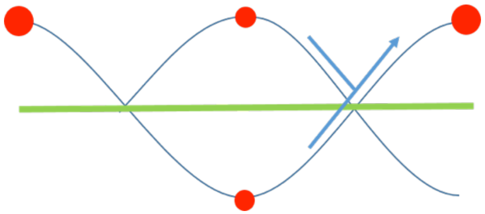


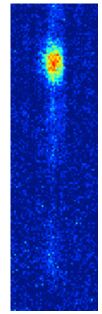


Recombination (Preliminary Data)

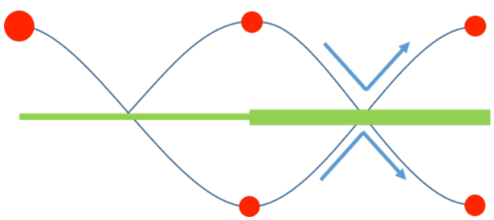
50/50 initial splitting:

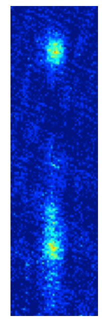
Recombination



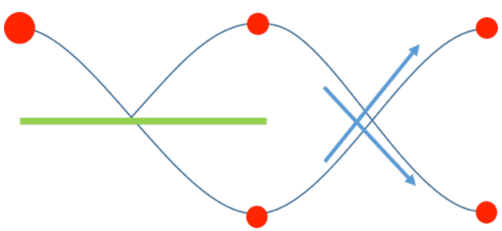


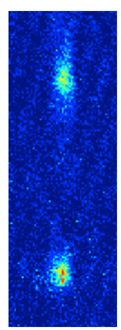
High barrier ➔ both reflected





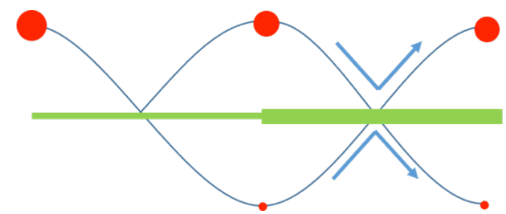
No barrier ➔ both transmitted

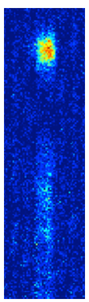




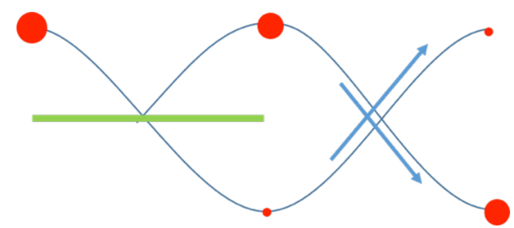
~ 80/20 initial splitting:

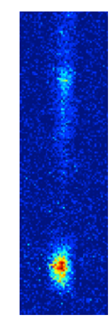
High barrier ➔ both reflected





No barrier ➔ both transmitted

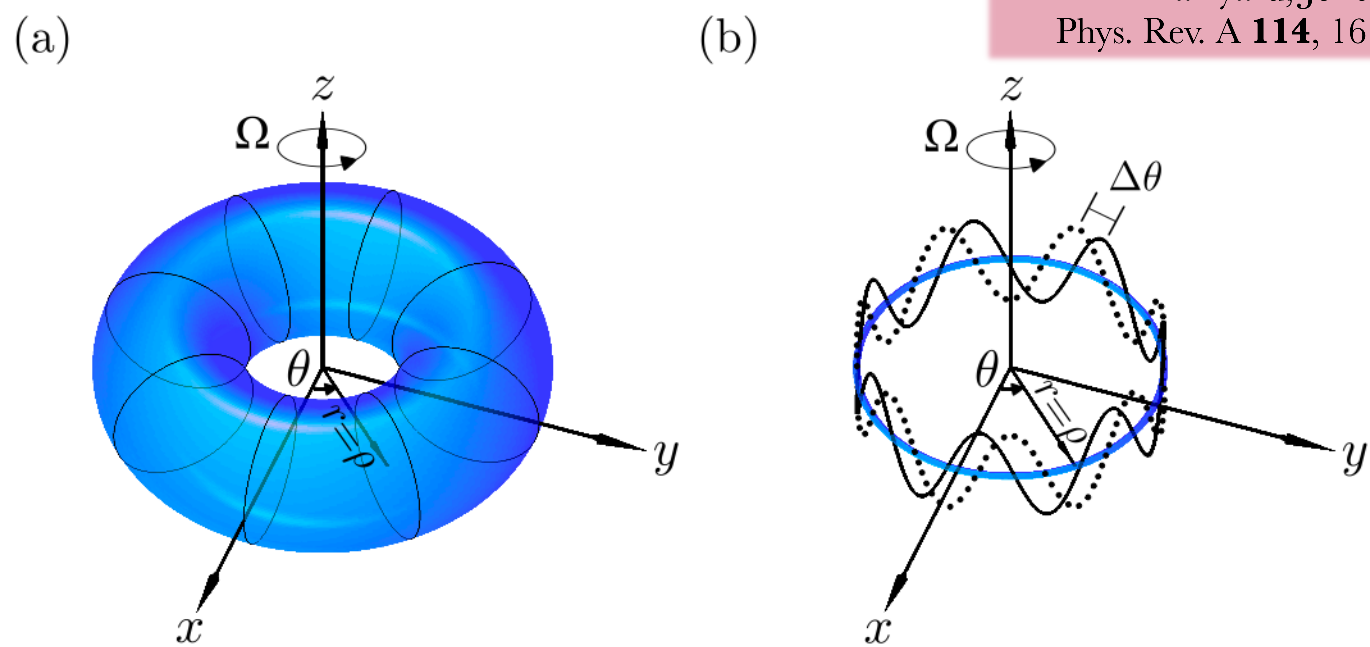




3: REPULSIVE INTERACTIONS

Two-State Condensate on a Ring

Halkyard, Jones, Gardiner,
Phys. Rev. A **114**, 161602 (2010)



- Consider (internal) **two-state** condensate

$$i\hbar\frac{\partial\Psi_j(\mathbf{r})}{\partial t}=\left[-\frac{\hbar^2\nabla^2}{2M}+V(\mathbf{r})+(-1)^j\frac{\hbar\omega}{2}+\frac{4\pi\hbar^2 N}{M}\sum_{k=1}^{2}a_{jk}|\Psi_k(\mathbf{r})|^2\right]\Psi_j(\mathbf{r}),$$

$$V(\mathbf{r})=M[\omega_r^2(r-\rho)^2+\omega_z^2z^2]/2$$

Rescaling

- Assume quasi-1D regime (r, z **frozen out**)
- Time in units of $\tau = M\rho^2/\hbar$; hence

$$i\frac{\partial\psi_j(\theta)}{\partial t} = \left[-\frac{1}{2}\frac{\partial^2}{\partial\theta^2} - i\Omega\frac{\partial}{\partial\theta} + (-1)^j\frac{\omega}{2} + \sum_{k=1}^{2} g_{jk}|\psi_k(\theta)|^2\right]\psi_j(\theta)$$

where

$$g_{jk} = 2MN\rho a_{jk}\sqrt{\omega_r\omega_z}/\hbar \qquad \sum_{j=1}^{2}\int_0^{2\pi} d\theta|\psi_j(\theta)|^2 = 1$$

Interferometric Protocol

1. Apply **splitting** $\pi/2$ pulse $U_{\pi/2}$ to initial state $\psi_1^I = 1/\sqrt{2\pi},\ \psi_2^I = 0$

$$\vec{\psi}^{\pi/2}(\theta) = \frac{1}{2\sqrt{\pi}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad U_{\pi/2} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

2. Imprint **angular momenta** (e.g. transfer of light OAM)

$$\vec{\psi}^{\ell m}(\theta) = \frac{e^{im\theta}}{2\sqrt{\pi}}\begin{pmatrix} e^{i\ell\theta} \\ e^{-i\ell\theta} \end{pmatrix}, \quad U_{\ell m} = \begin{pmatrix} e^{i(m+\ell)\theta} & 0 \\ 0 & e^{i(m-\ell)\theta} \end{pmatrix}$$

3. Allow **free evolution** $f(T/2)$

Anderson *et al.*, Phys. Rev. Lett. **97** 170406 (2007)

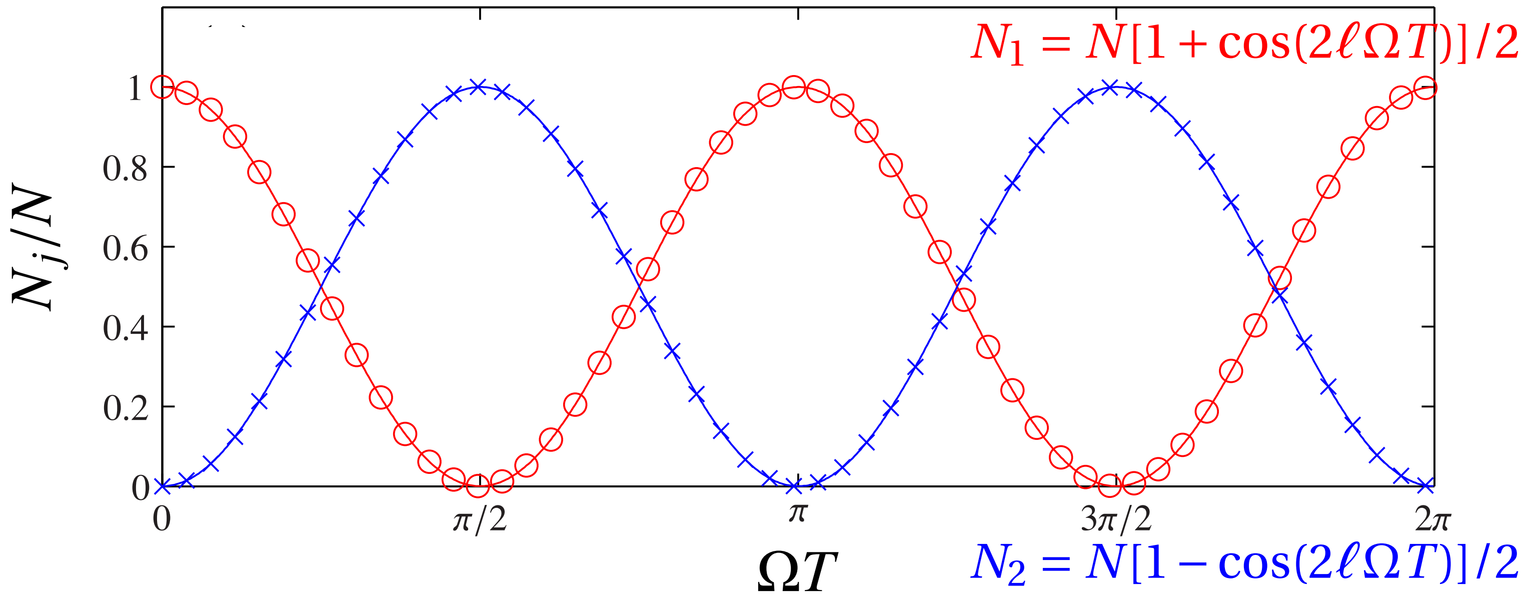
$$\vec{\psi}^{T/2}(\theta) = \frac{e^{-i\varphi_1 T/2} e^{im(\theta+\Omega T/2)}}{2\sqrt{\pi}}\begin{pmatrix} e^{i\varphi_2 T/2} e^{i\ell(\theta+\Omega T/2)} \\ e^{-i\varphi_2 T/2} e^{-i\ell(\theta+\Omega T/2)} \end{pmatrix}$$

4. Apply **swap** π pulse, followed by a second **free evolution**

$$\vec{\psi}^{T}(\theta) = \frac{e^{-i\varphi_1 T} e^{im(\theta+\Omega T)}}{2\sqrt{\pi}}\begin{pmatrix} e^{-i\ell(\theta+\Omega T)} \\ e^{i\ell(\theta+\Omega T)} \end{pmatrix}$$

5. **Repeat** AM imprinting and apply **recombination** $\pi/2$ pulse

$$\vec{\psi}^{R}(\theta) = \frac{e^{-i\varphi_1 T} e^{im(2\theta+\Omega T)}}{\sqrt{2\pi}}\begin{pmatrix} \cos(\ell\Omega T) \\ -i\sin(\ell\Omega T) \end{pmatrix}$$



- Following $S_N(T)$ sequence, populations **oscillate**

$$S_N(T) \equiv U_{\pi/2} U_{\ell m} f(T/2) U_\pi f(T/2) U_{\ell m} U_{\pi/2}$$

- **Any experimentally significant** change of N_2 from zero is a positive response
- **Same response** obtained when AM imprinted on ψ_1 only (*m*=*l*)

Sensitivity Considerations

- Optically, fringe shift relative to fringe width a common measure of sensitivity $\delta_L = 4A\Omega/\lambda_L c$ (A the **enclosed area**)
- Present equivalent is $\delta \equiv \Delta\theta / w = 2\ell\Omega T/\pi$ (counts instances population **alternates between** 0 and *N* over [0,*T*])
- **No apparent area dependence** – due to fact that interferometer may be interrogated at **any time**, not just when discrete, split wavepackets recombine
- Atomic **shot noise** also places a fundamental limit

4: SPIN-ORBIT COUPLED INTERFEROMETRY (SOCI)

System: Spinor Condensate

- **Vector** order parameter $\mathbf{\Psi} = \sum_{j=-1}^{1} \Psi_j |j\rangle$

Isoshima *et al.*, Phys. Rev. A **61**, 063610 (2000)
Ray *et al.*, Nature **505**, 657 (2014)

- Gross–Pitaevskii equation

$$i\hbar \frac{\partial}{\partial t}\Psi_j = \left[-\frac{\hbar^2}{2m}\nabla^2 + V - i\hbar(\mathbf{r}\times\mathbf{\Omega})\cdot\nabla + g_\mathrm{n}\mathbf{\Psi}^*\cdot\mathbf{\Psi} \right]\Psi_j + \left[\left(g_\mathrm{s}\bar{\mathbf{F}}\cdot\mathbf{F} - \mu_\mathrm{B} g_F \mathbf{B}\cdot\mathbf{F}\right)\mathbf{\Psi}\right]_j$$

$$\bar{F}_\alpha = \sum_{j,k} \Psi_k^* \Psi_j \langle k|F_\alpha|j\rangle$$

- **Toroidal** trapping configuration $V = m\omega_\perp^2[(\rho - R_0)^2 + z^2]/2$

- **Normal** and **spin-flipping** scattering

$$g_\mathrm{n} = 4\pi\hbar^2(a_0 + 2a_2)/3m, \;\; g_\mathrm{s} = 4\pi\hbar^2(a_2 - a_0)/3m$$

- Consider **^{87}Rb** in $F = 1$

Magnetic Field: Ioffe-Pritchard Configuration

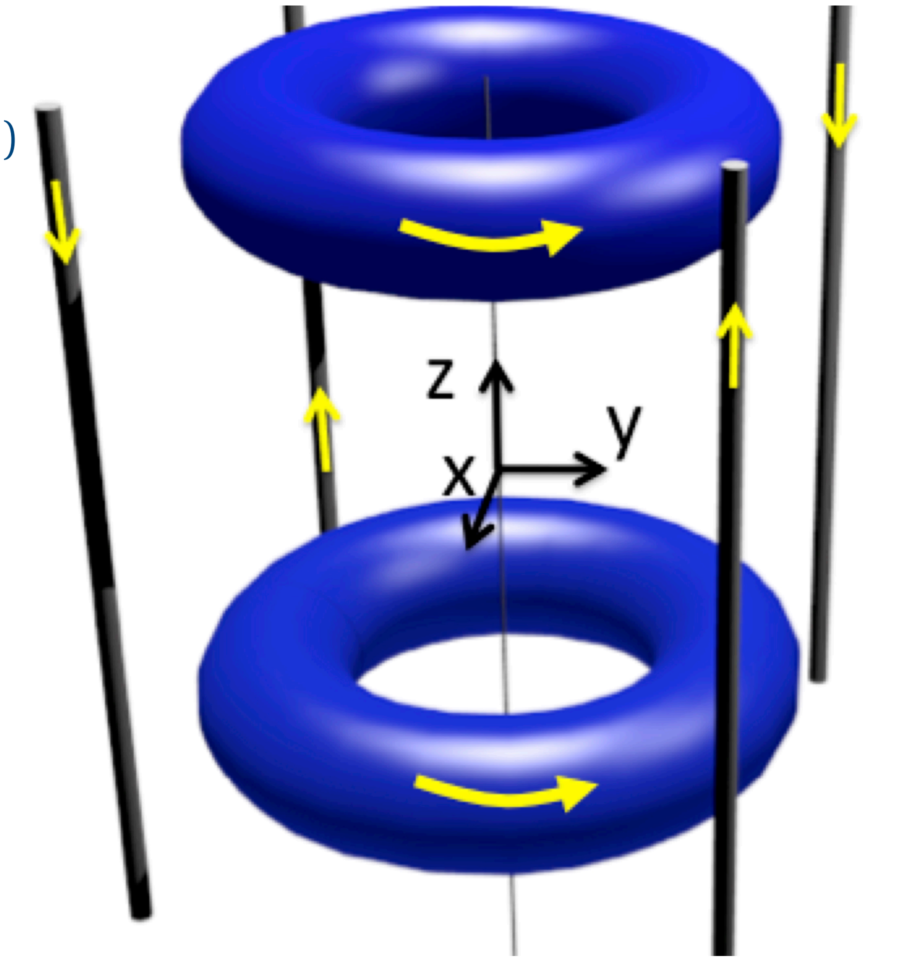
Perez-Rios, Sanz
Am. J. Phys. **81**, 836 (2013)

- Ioffe–Pritchard **field texture**

  $$\mathbf{B} = (B_\mathrm{q}(\rho)\cos(\phi), -B_\mathrm{q}(\rho)\sin(\phi), B_z)$$

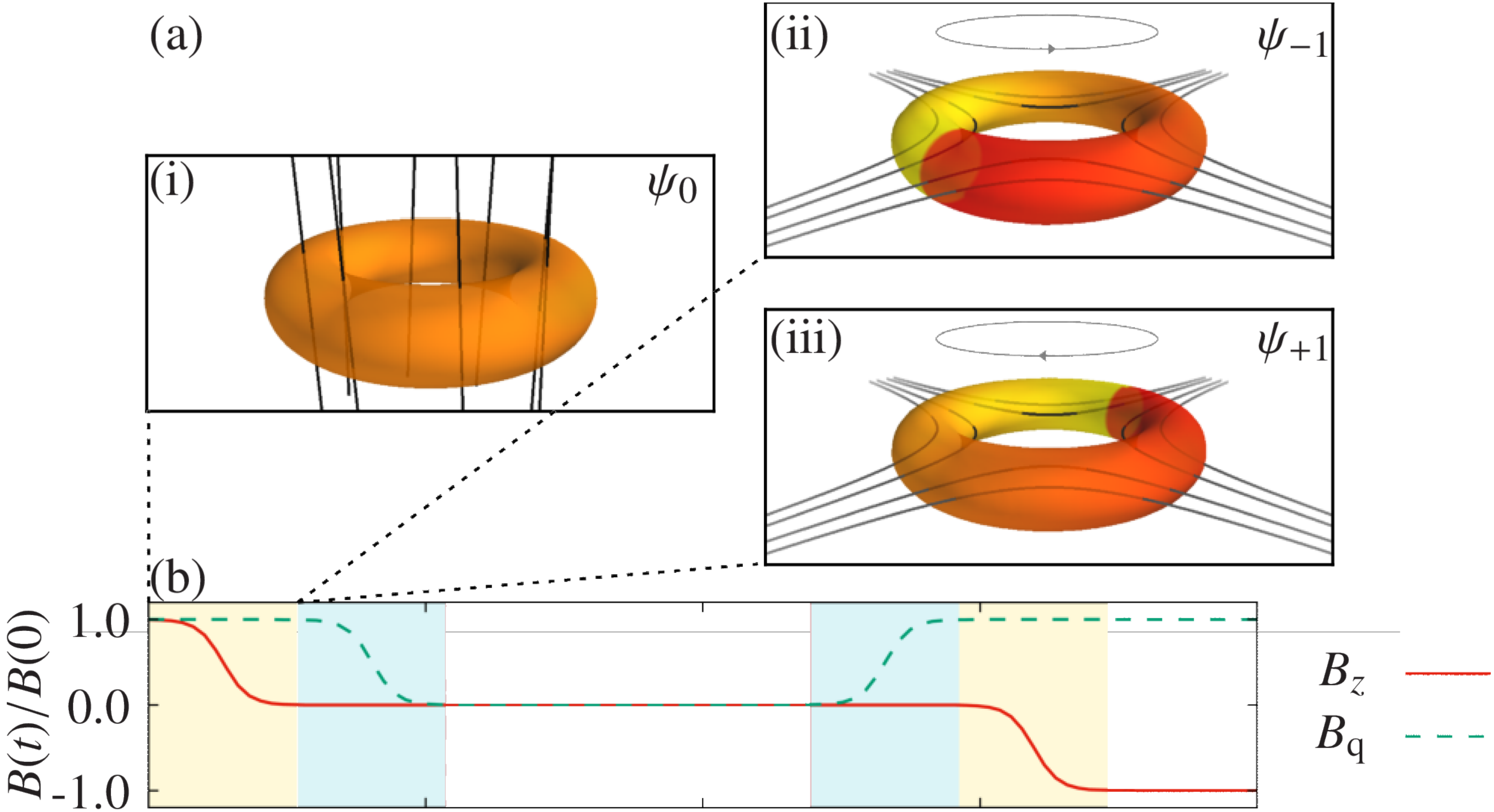
- **Quadrupole** field $B_\mathrm{q}(\rho) = b'\rho$, **bias** field B_z

- Vary b', B_z with time



Magnetic Imprinting in a Spin 1 Condensate

Helm, Billam, Rakonjac, Cornish, Gardiner
arXiv:1701.02154



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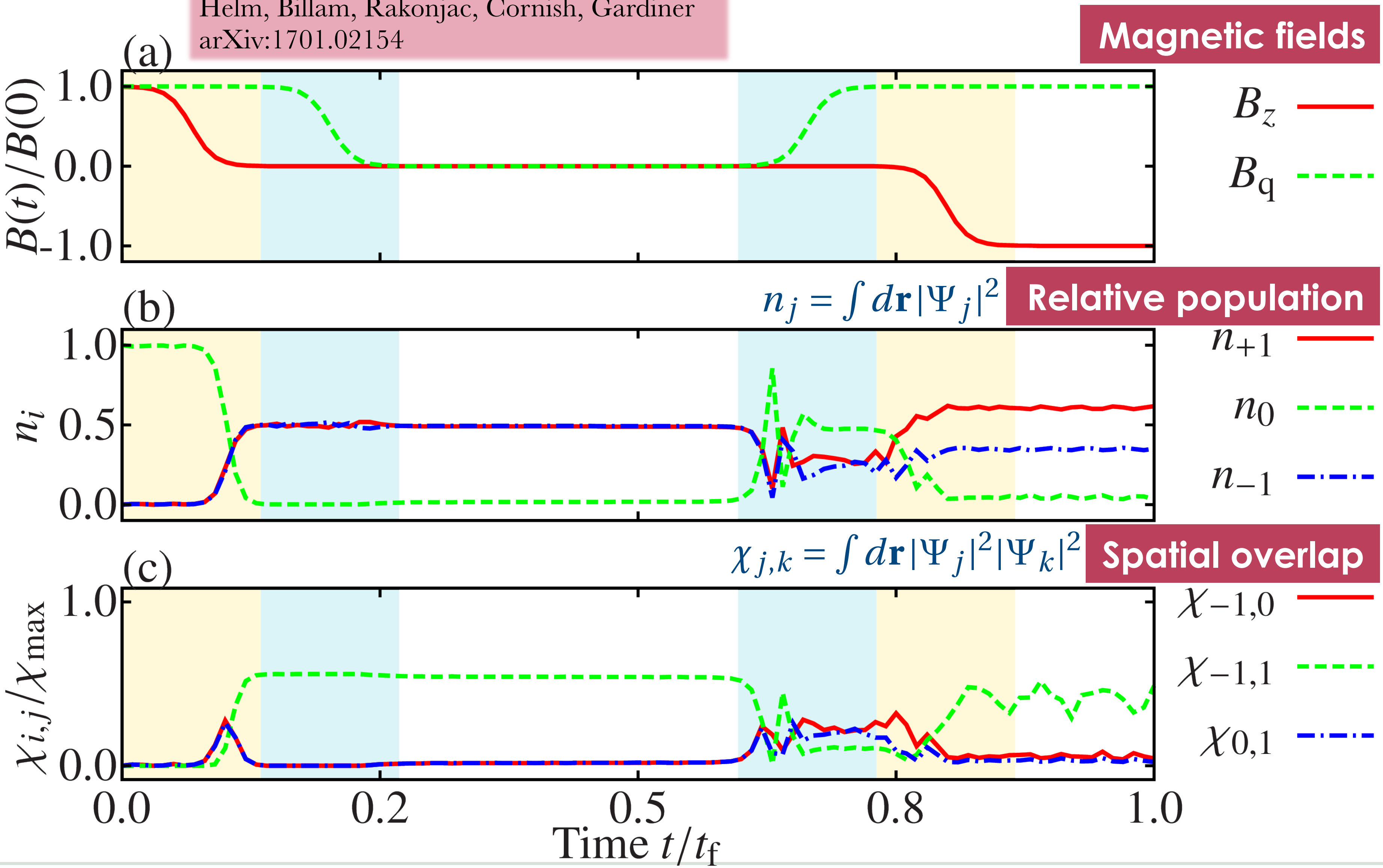


Adiabatic Transfer

- GPE dynamics **dominated by**

$$\mathbf{B}\cdot\mathbf{F} = \begin{pmatrix} B_z & B_\mathrm{q} e^{i\phi}/\sqrt{2} & 0 \\ B_\mathrm{q} e^{-i\phi}/\sqrt{2} & 0 & B_\mathrm{q} e^{i\phi}/\sqrt{2} \\ 0 & B_\mathrm{q} e^{-i\phi}/\sqrt{2} & -B_z \end{pmatrix}$$

- Spatially dependent **eigenstates**

$$|\pm B\rangle = ([B \pm B_z]e^{i\phi}, \pm\sqrt{2}B_\mathrm{q}, [B \mp B_z]e^{-i\phi})/2B$$

$$|Z\rangle = (-B_\mathrm{q} e^{i\phi}, \sqrt{2}B_z, B_\mathrm{q} e^{-i\phi})/\sqrt{2}B$$

$$B = \sqrt{B_\mathrm{q}^2 + B_z^2}$$

- **Beam split** achieved by preparing condensate in $|0\rangle$ state with dominating B_z , then ramping $|B_z| \to 0$

Interferometry

- Imprint relative **phase difference** δ

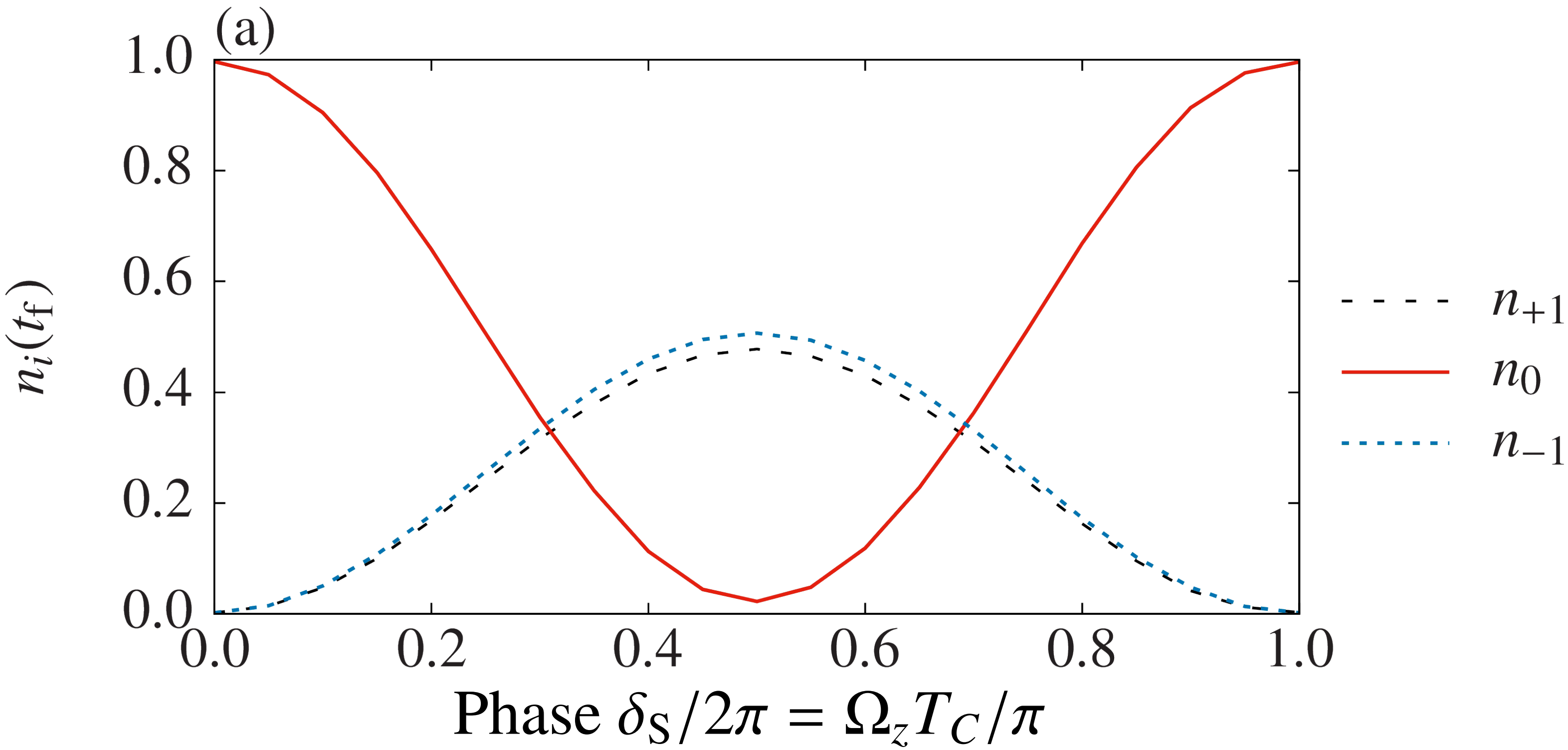
$$|\Psi\rangle = \left(-e^{i(\phi+\delta/2)}, 0, e^{-i(\phi+\delta/2)}\right)$$
$$= \sqrt{\frac{1+\cos(\delta)}{2}}|Z\rangle - i\sqrt{\frac{1-\cos(\delta)}{2}}\left(\frac{|+B\rangle + |-B\rangle}{\sqrt{2}}\right)$$

- Reverse beam-splitting (**recombination**)

$$\int d\mathbf{r}|\Psi_0(t_f)|^2 \approx \frac{[1+\cos(\delta)]}{2}$$

- To permit accumulation of **Sagnac phase** $\delta_S = \Omega_z T_{\mathrm{I}}$, must also diabatically ramp $B_{\mathrm{q}} \to 0$

Interferometric Response



- **Vary** Ω_z particular interrogation time $T_I = T_C$
- Note $T_C = 2\pi R_0^2 m/\hbar$ timescale for a particle to **circumnavigate** the ring

5: FISHER INFORMATION ANALYSES

Quantum & Classical Fisher Information

Haine, Phys. Rev. Lett. **116** 230404 (2016)

- In a Sagnac interferometer, **smallest resolvable difference** in Ω_z given by (Quantum Cramer–Rao bound)

$$\delta\Omega_z = \frac{1}{\sqrt{\mathscr{F}_Q}}$$

- The **Quantum Fisher Information** $\mathscr{F}_Q$ for GPE-described atomic BEC reasonably approximated by $\mathscr{F}_Q = NF_Q$, where

$$F_Q(t) = 4\left[\int d\mathbf{r}\left|\frac{\partial\Psi(\mathbf{r},t)}{\partial\Omega_z}\right|^2 - \left|\int d\mathbf{r}\,\Psi^*(\mathbf{r},t)\frac{\partial\Psi(\mathbf{r},t)}{\partial\Omega_z}\right|^2\right]$$

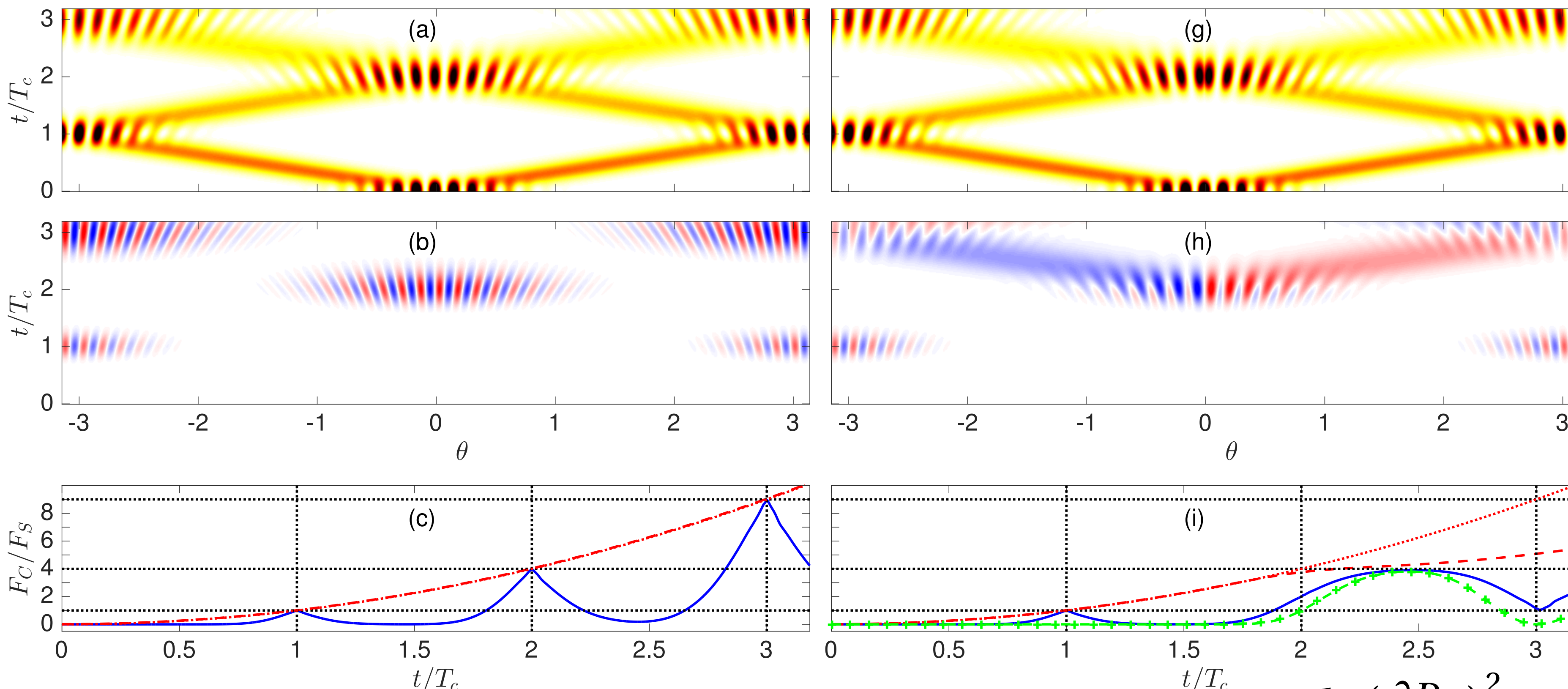
- If limited to measuring e.g. 2D spatial density $P(x,y)$, sensitivity limited to $\Delta\Omega_z = 1/\sqrt{\mathscr{F}_C}$, where the **Classical Fisher Information** $\mathscr{F}_C = NF_C$ and

$$F_C(t) = \int dx\,dy\,\frac{1}{P(x,y,t)}\left[\frac{\partial P(x,y,t)}{\partial\Omega_z}\right]^2$$

Split Gaussian Wavepackets on a Ring

Superposition & Fringes

Barrier & Populations



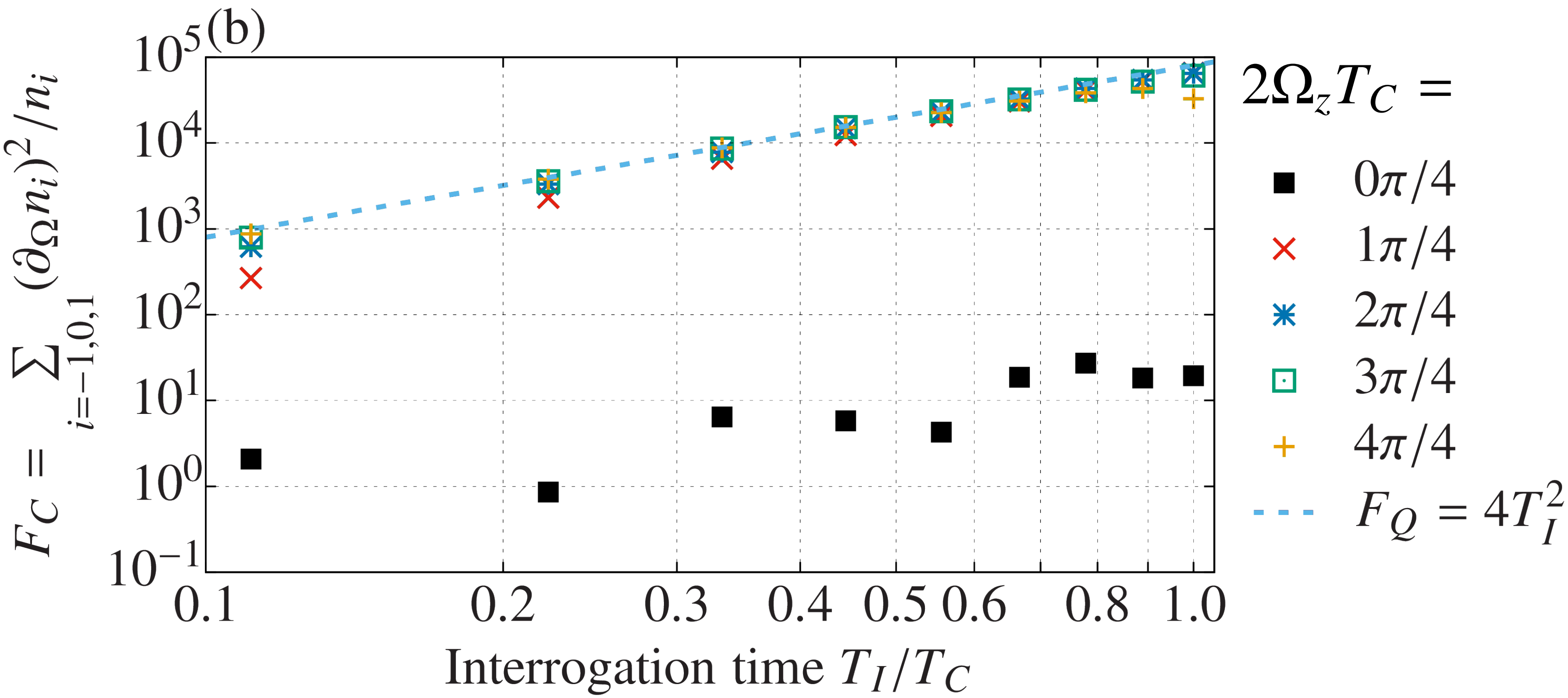
- Blue: F_C
- Red dashed: F_Q
- Dotted: "ideal" $F_Q = 4F_S(T_I/T_C)^2$

- Green: $F_{LR} = \sum_{j=L,R} \frac{1}{P_j}\left(\frac{\partial P_j}{\partial \Omega_z}\right)^2$

$$P_L = \int_{-\pi}^{0} d\theta\, P(\theta), \quad P_R = \int_{0}^{\pi} d\theta\, P(\theta)$$

Haine, Phys. Rev. Lett. **116** 230404 (2016)

SOCI Fisher Information



- For **idealized** 1D Halkyard-Jones-Gardiner protocol

$$F_C = F_Q = 4\ell^2 T_I^2 = 4F_S(T_I/T_C)^2$$

Conclusions

- Can avoid issues of **interactions** in BEC interferometry
- Possibilities for **rotational sensing** with **toroidally trapped** atomic BECs
- Time-dependent magnetic fields can imprint **counterflow states**
- **Coherent soliton experiments!**

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Collision without a Barrier

- Because of the **integrability** of the nonlinear Schrödinger equation (1D GPE without external potentials) solitons are **robust to collisions**

- Collision produces a **phase shift**, but relative phase difference is **zero** if $v > 1/4$

- Interactions therefore **not disruptive**

Equivalency between Sagnac and Zeeman Phase

- Appropriately **scale** magnetic field and angular velocity

$$\tilde{\mathbf{B}} = \mu_{\mathrm{B}} g_F \mathbf{B}/\hbar\omega_\perp, \quad \tilde{\mathbf{\Omega}} = \mathbf{\Omega}/\omega_\perp$$

- If angular velocity present, can effectively **substitute**

$$\tilde{B}_z \to \tilde{B}_z + \tilde{\Omega}_z$$

- Hence, equivalency between interrogating z-components of angular velocity (**Sagnac phase**) and magnetic field (**Zeeman phase**)