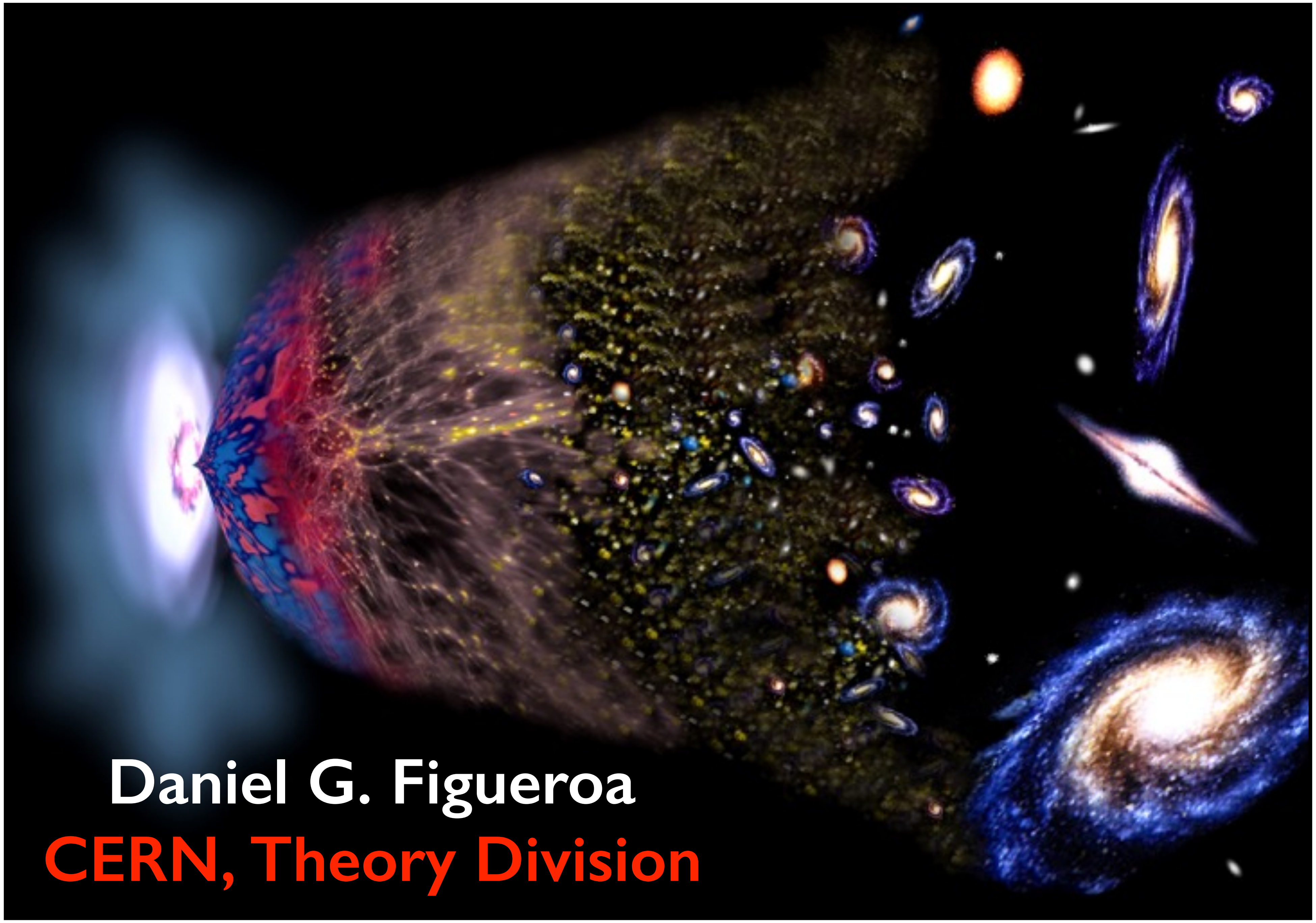
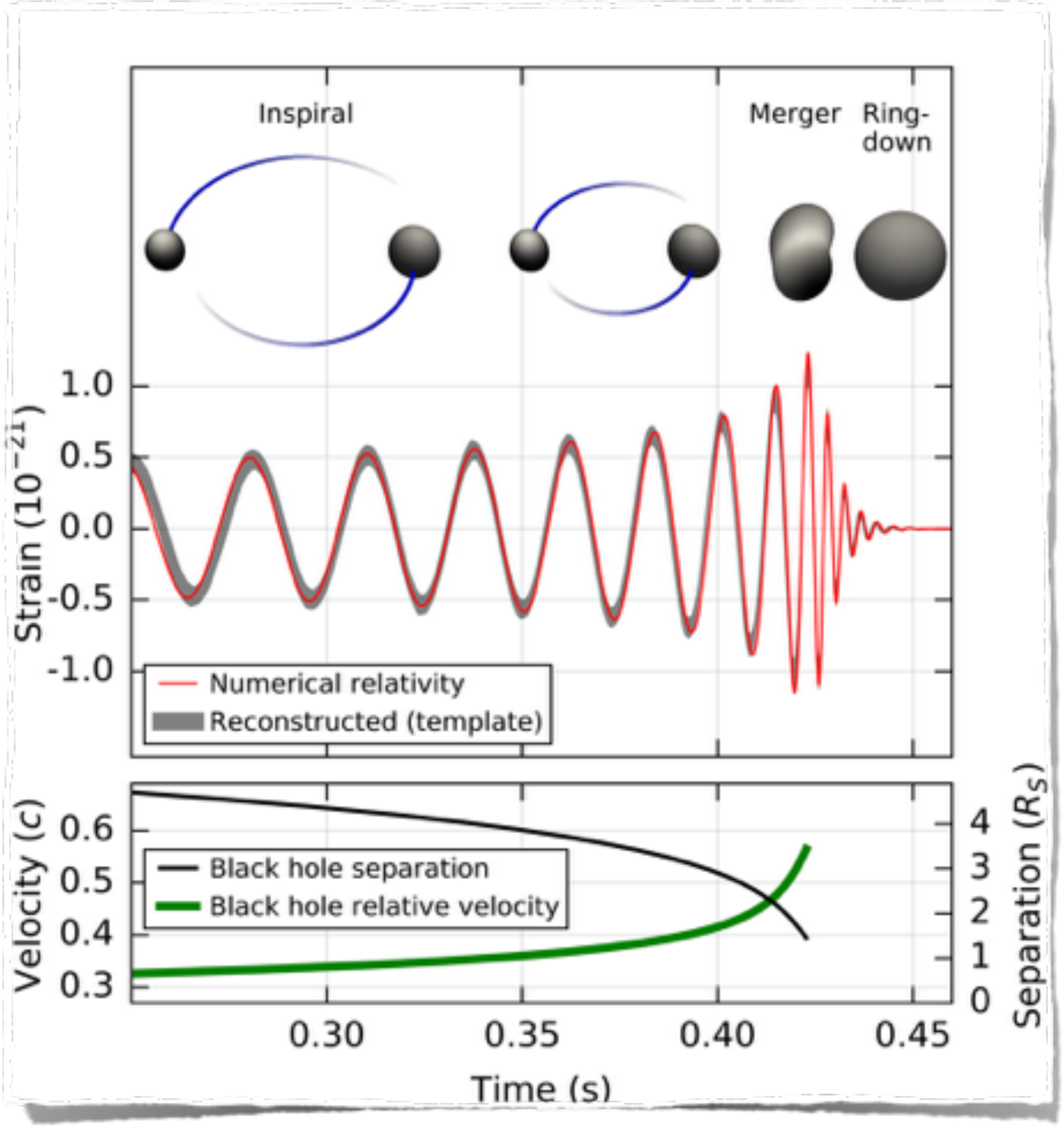
GRAVITATIONAL WAVES
PROBE OF THE EARLY UNIVERSE



Daniel G. Figueroa
CERN, Theory Division

School on Gravitational Waves for Cosmology and Astrophysics, Benasque, May 28 - June 10, 2017

Straight to the point …

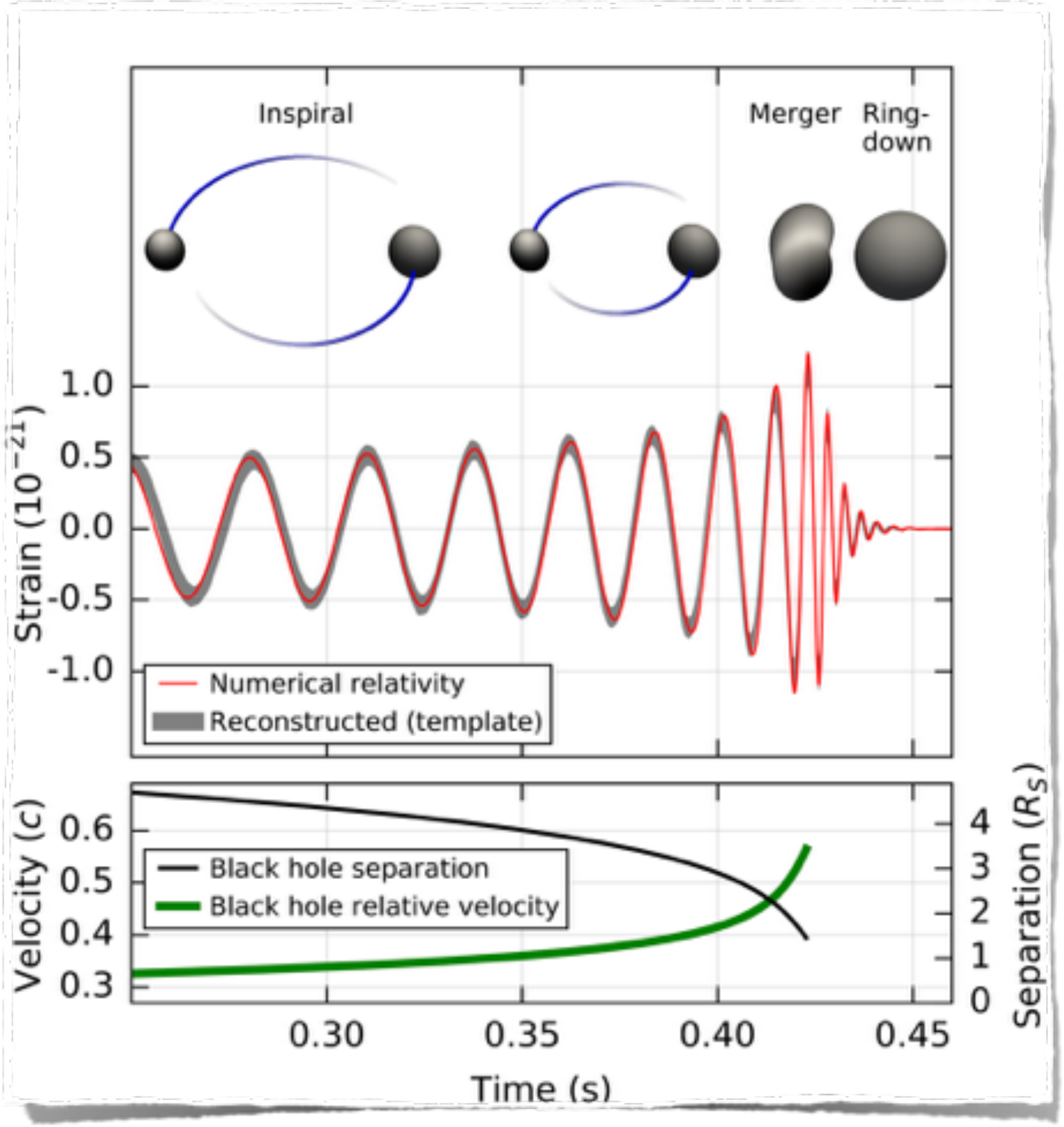


Gravitational Waves (GWs) detected !

[by aLIGO]

[LIGO & Virgo Scientific Collaborations (**arXiv:1602.03841**)]

Straight to the point …



Gravit al

)

aLIGO]

milestone
in physics

[LIGO & Virgo Scientific Collaborations
(**arXiv:1602.03841**)]

Einstein 1916 … aLIGO 2015/16

Straight to the point …

* O(10) Solar mass Black Holes (BH) exist

* We can test the BH's paradigm (or Neutron Star physics)

* We can further test General Relativity (GR)
[so far no deviation]

* We can observe the Universe through GWs

* …

milestone in physics

Straight to the point …

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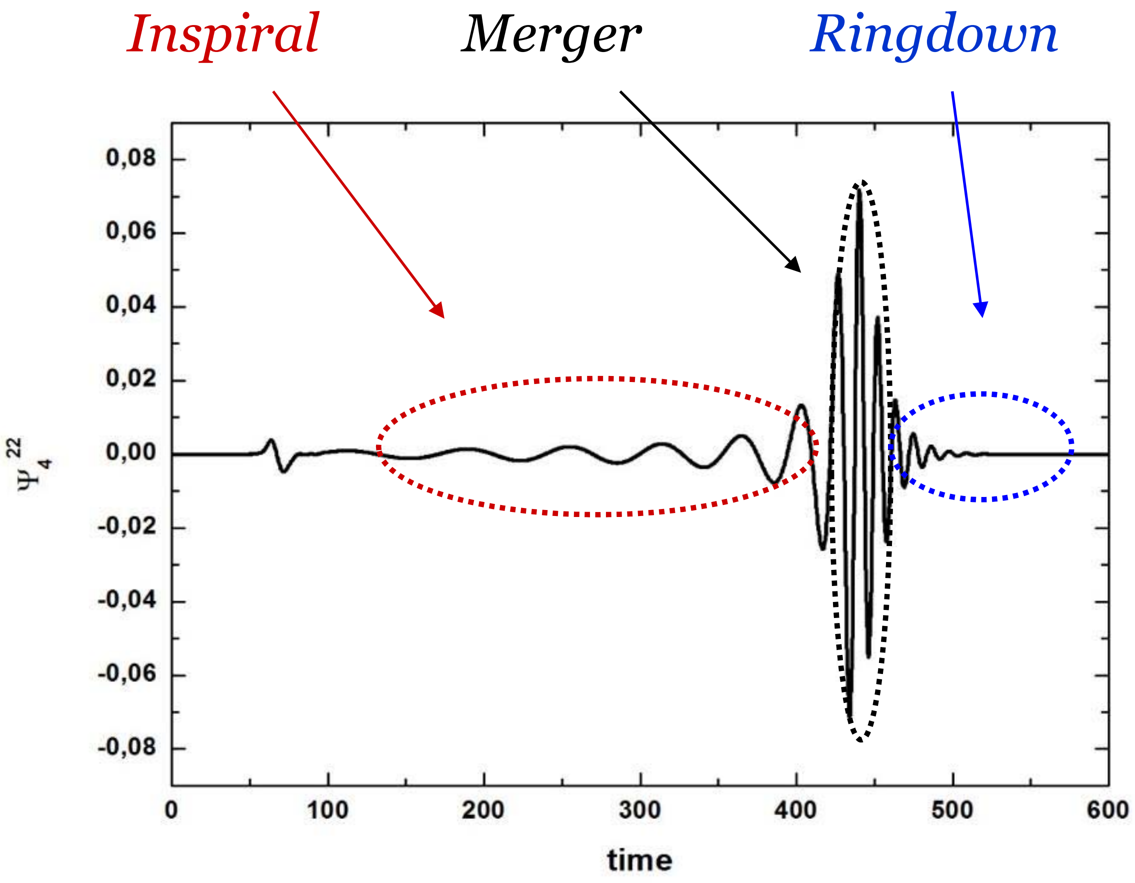
Binaries



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(binaries)

Extremely interesting !
but …

* We can observe the Universe through GWs

* …

Straight to the point …

- **O(10) Solar mass Black Holes (BH) exist**
- **We can test the BH's paradigm (or Neutron Star physics)**
- **We can further test General Relativity (GR)** [so far no deviation]
- **We can observe the Universe through GWs**
- **…**

(binaries)

Extremely interesting !

but …

… We will focus on something else !

Straight to the point …

- * O(10) Solar mass Black Holes (BH) exist
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→ (binaries)

Extremely interesting !

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- * We can observe the Universe through GWs
- * …

Straight to the point …

* O(10) So s
Blac st

N (binaries)

* W
Gen ity (GR)
[so no deviation]

See Lectures by

WITEK
SESANA
BLANCHET
…

* We can observe the Universe through GWs

* …

Extremely interesting !
but …

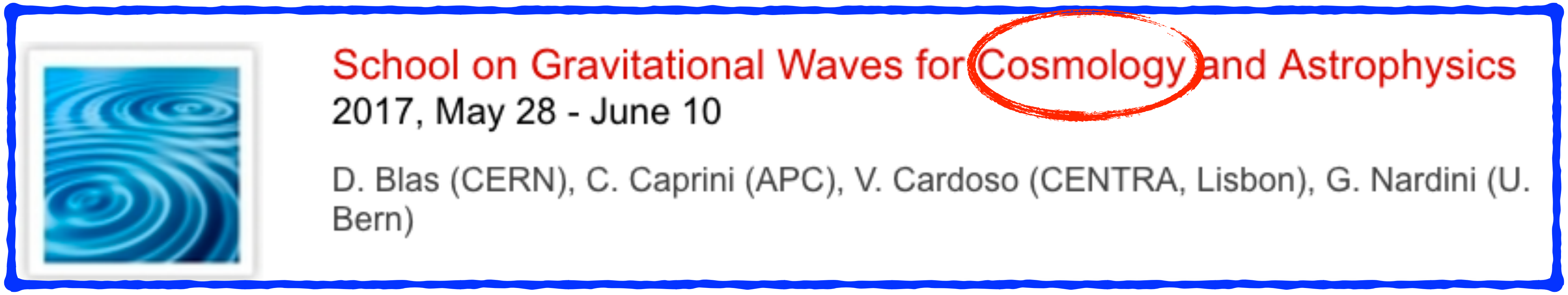
… We will focus on something else !

* We can observe the Universe through GWs

* **Cosmology** with **GWs**

* We can observe the Universe through GWs

* **Cosmology with GWs**



School on Gravitational Waves for Cosmology and Astrophysics
2017, May 28 - June 10

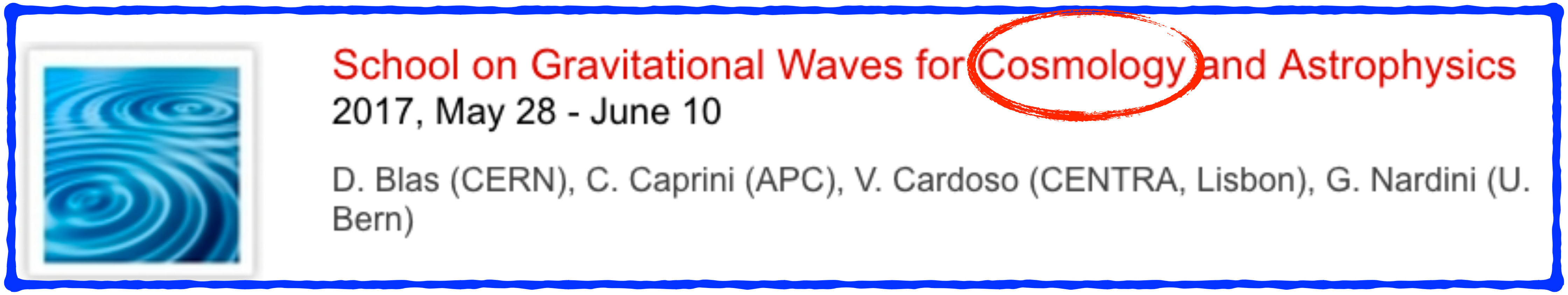
D. Blas (CERN), C. Caprini (APC), V. Cardoso (CENTRA, Lisbon), G. Nardini (U. Bern)

* We can observe the Universe through GWs

* Cosmology with GWs

* Late Universe: Hubble diagram from Binaries

* Early Universe: High Energy Particle Physics



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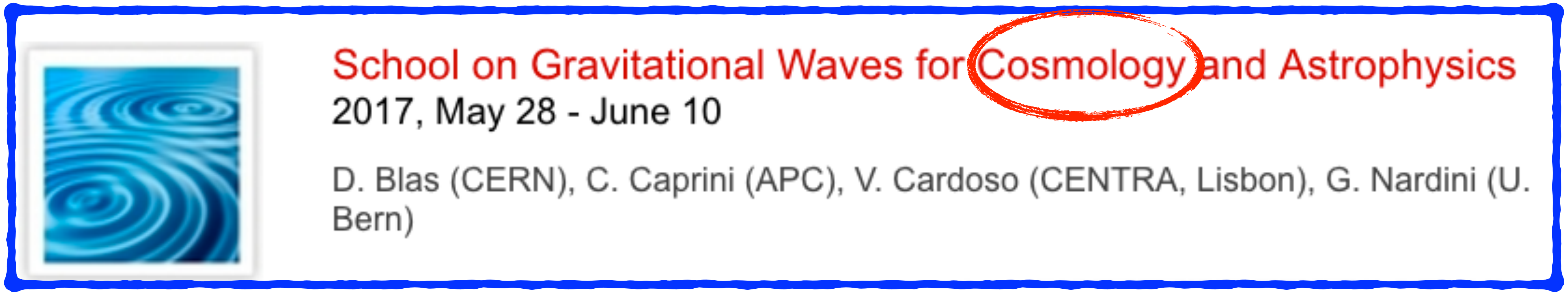
Lectures by BONVIN

* Early Universe: High Energy Particle Physics



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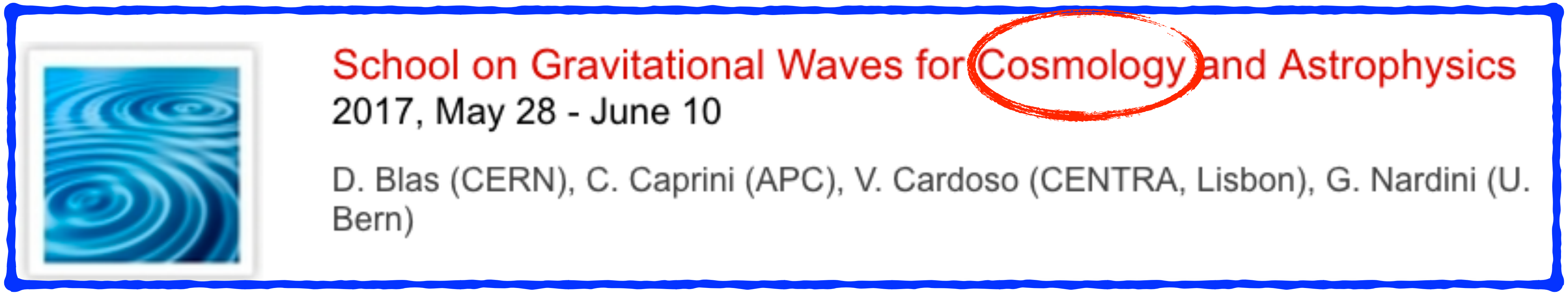

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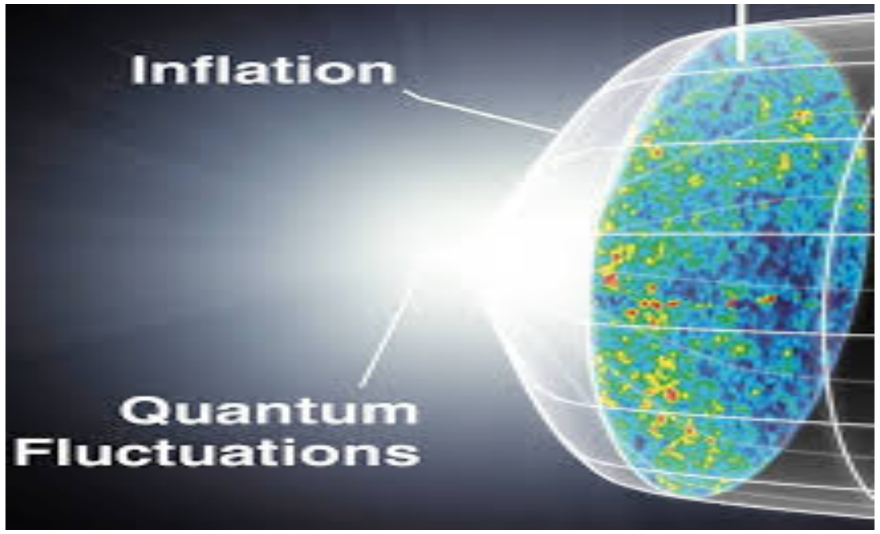
Gravitational Waves as a probe of the early Universe

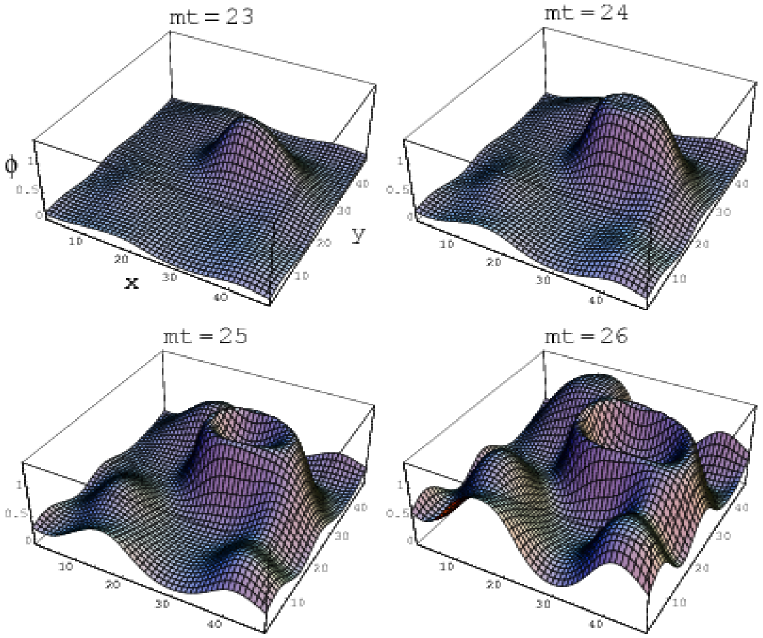
Gravitational Waves as a probe of the early Universe

Inflationary Period



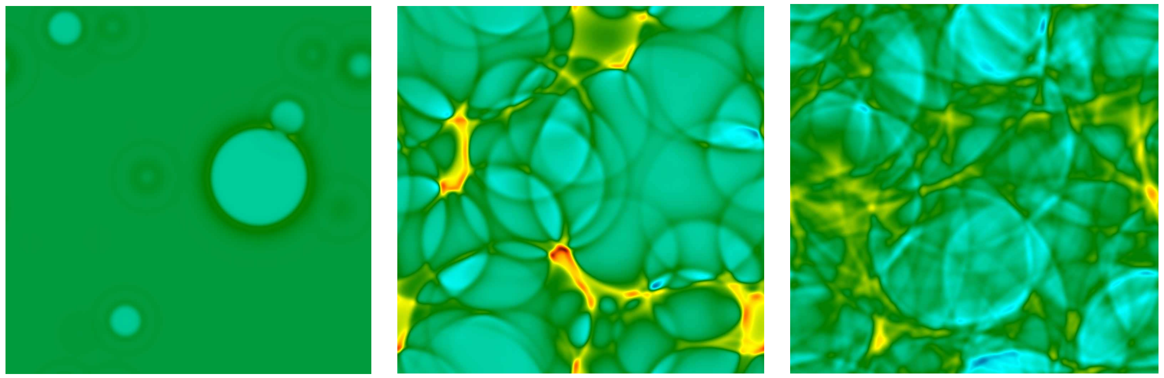
(Image: Google Search)

Preheating



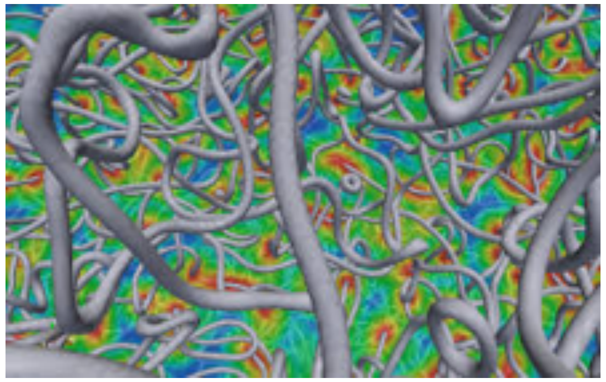
(Fig. credit: Phys.Rev. D67 103501)

Phase Transitions



(Image: PRL 112 (2014) 041301)

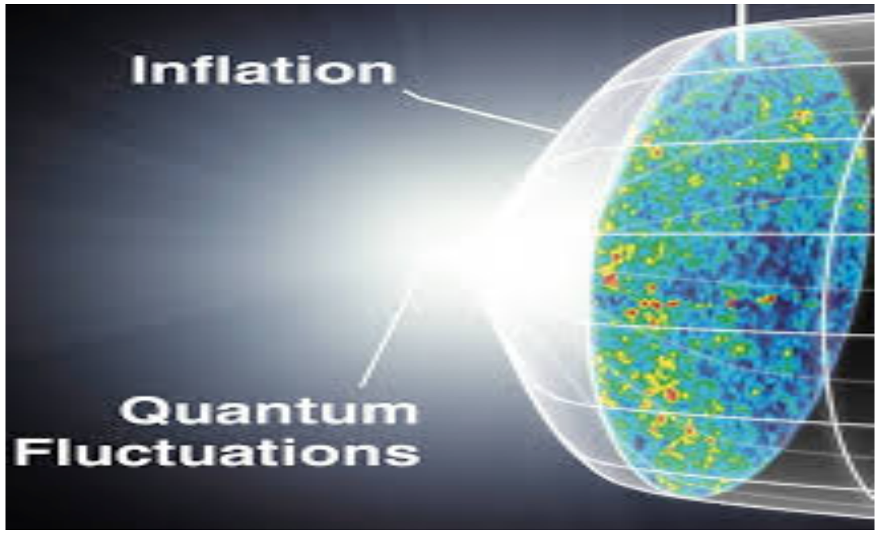
Cosmic Defects



(Image: Daverio et al, 2013)

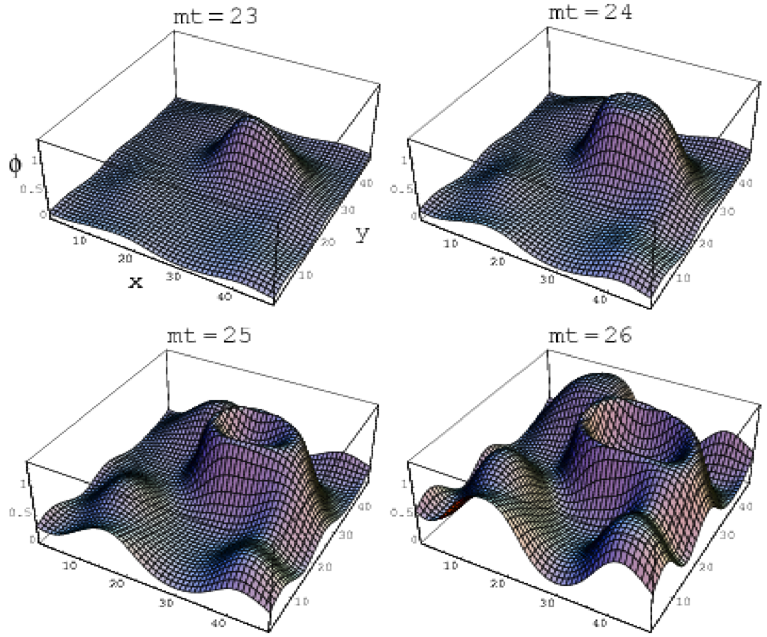
Gravitational Waves as a probe of the early Universe

Inflationary Period



(Image: Google Search)

Preheating



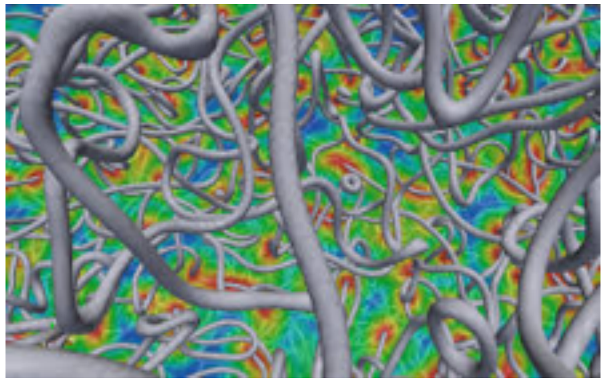
(Fig. credit: Phys.Rev. D67 103501)

Phase Tra ns

See lectures by HINDMARSH

(I ge: PRL 112 (2014) 041301)

Cosmic Defects



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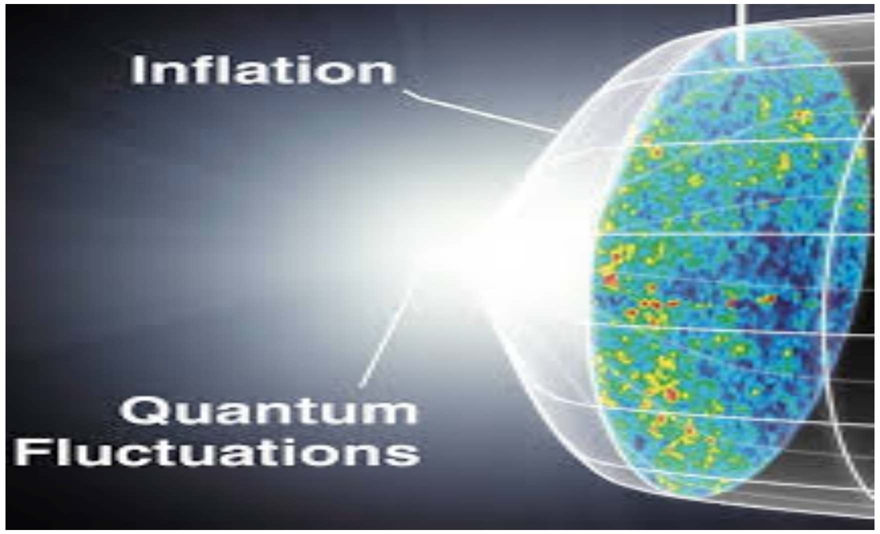
Gravitational Waves as a probe of the early Universe

Inflationary Period



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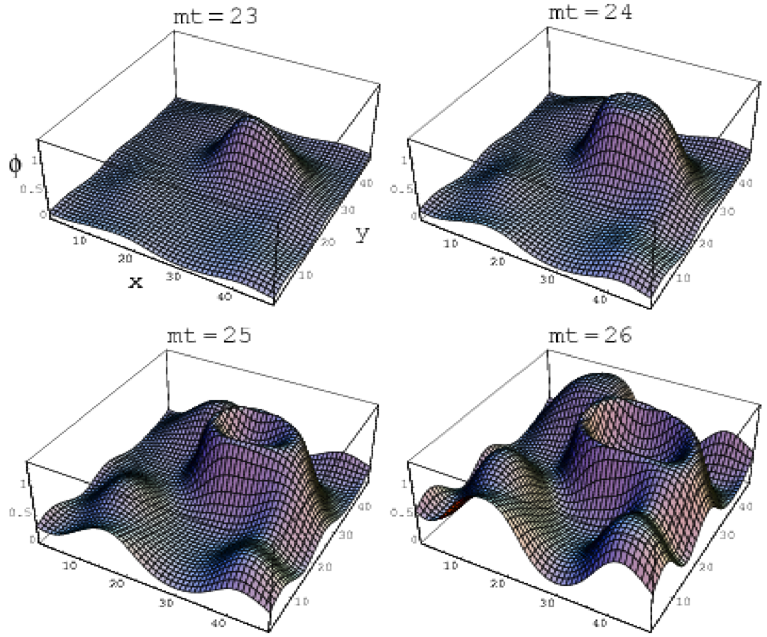
Preheating



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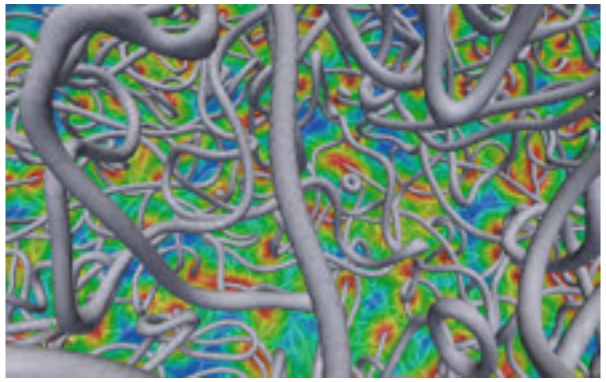
See lectures by HINDMARSH

Cosmic Defects



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Gravitational Waves as a probe of the early Universe

OUTLINE

Early Universe

1) GWs from Inflation
2) GWs from Preheating
3) GWs from Phase Transitions
4) GWs from Cosmic Defects

Gravitational Waves as a probe of the early Universe

OUTLINE

0) Cosmology

Early Universe

1) GWs from Inflation

2) GWs from Preheating

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Gravitational Waves as a probe of the early Universe

OUTLINE

0) Cosmology

1st lecture (1.5 h)

Early Universe

1) GWs from Inflation

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Gravitational Waves as a probe of the early Universe

OUTLINE

0) Cosmology

1st lecture (1.5 h)

Early Universe

1) GWs from Inflation

2nd lecture (1.5 h)
+ 3rd lecture (1 h)

2) GWs from Preheating

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Gravitational Waves as a probe of the early Universe

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1st lecture (1.5 h)

Early Universe

1) GWs from Inflation

2nd lecture (1.5 h)
+ 3rd lecture (1 h)

2) GWs from Preheating

3rd lecture (0.5 h)
+ 4th lecture (1 h)

3) GWs from Phase Transitions

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Gravitational Waves as a probe of the early Universe

OUTLINE

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1st lecture (1.5 h)

Early Universe

1) GWs from Inflation

2nd lecture (1.5 h)
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3rd lecture (0.5 h)
+ 4th lecture (1 h)

3) GWs from Phase Transitions

HINDMARSH
(3 lectures)

4) GWs from Cosmic Defects

Gravitational Waves as a probe of the early Universe

OUTLINE

0) Cosmology — 1st lecture (1.5 h)

Early Universe

1) GWs from Inflation — 2nd lecture (1.5 h) + 3rd lecture (1 h)

2) GWs from Preheating — 3rd lecture (0.5 h) + 4th lecture (1 h)

3) GWs from Phase Transitions — HINDMARSH (3 lectures)

4) GWs from Cosmic Defects — 4th lecture (0.5 h) + 5th lecture (1 h)

Gravitational Waves as a probe of the early Universe

OUTLINE

0) Cosmology — 1st lecture (1.5 h)

Early Universe

1) GWs from Inflation — 2nd lecture (1.5 h) + 3rd lecture (1 h)

2) GWs from Preheating — 3rd lecture (0.5 h) + 4th lecture (1 h)

3) GWs from Phase Transitions — HINDMARSH (3 lectures)

4) GWs from Cosmic Defects — 4th lecture (0.5 h) + 5th lecture (1 h)

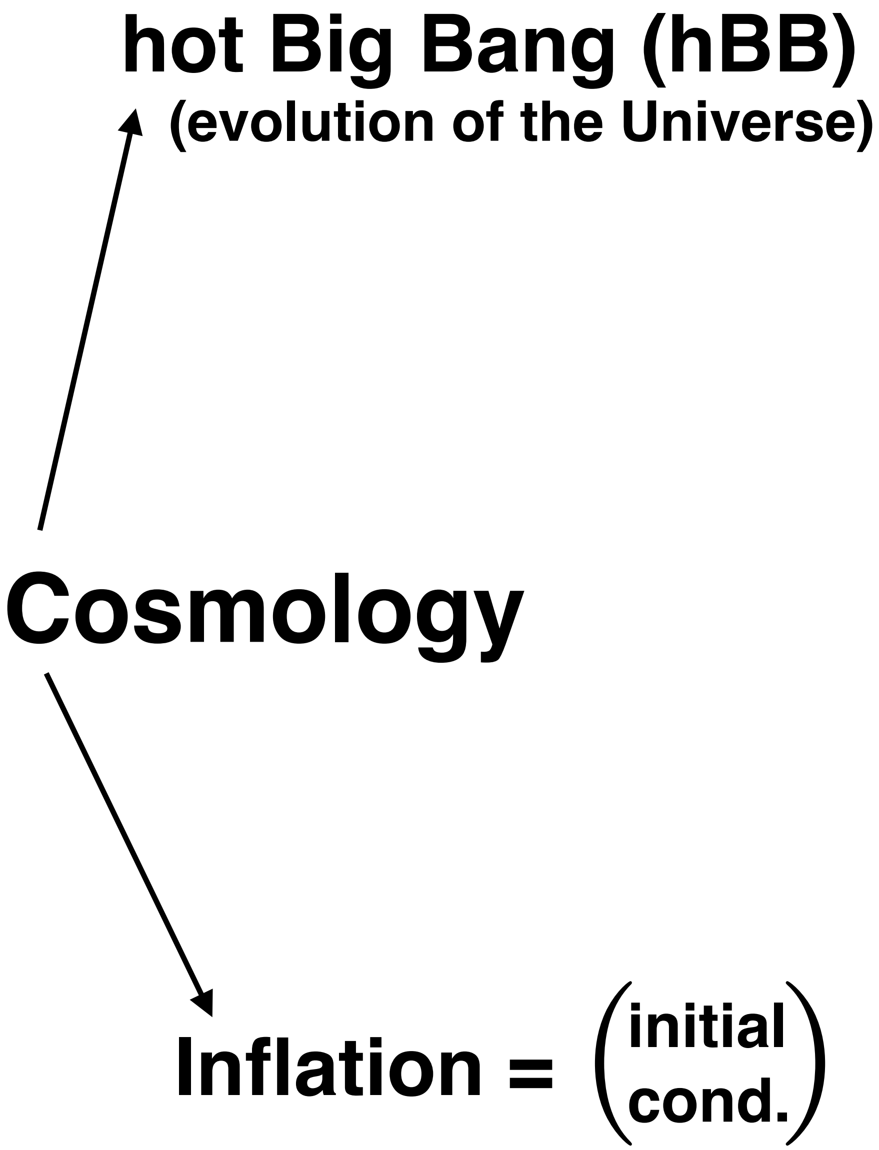
Discussion

5) Summary & Perspectives — 5th lecture (last 0.5 h)

A

COSMOLOGY PRIMER

BASICS of COSMOLOGY



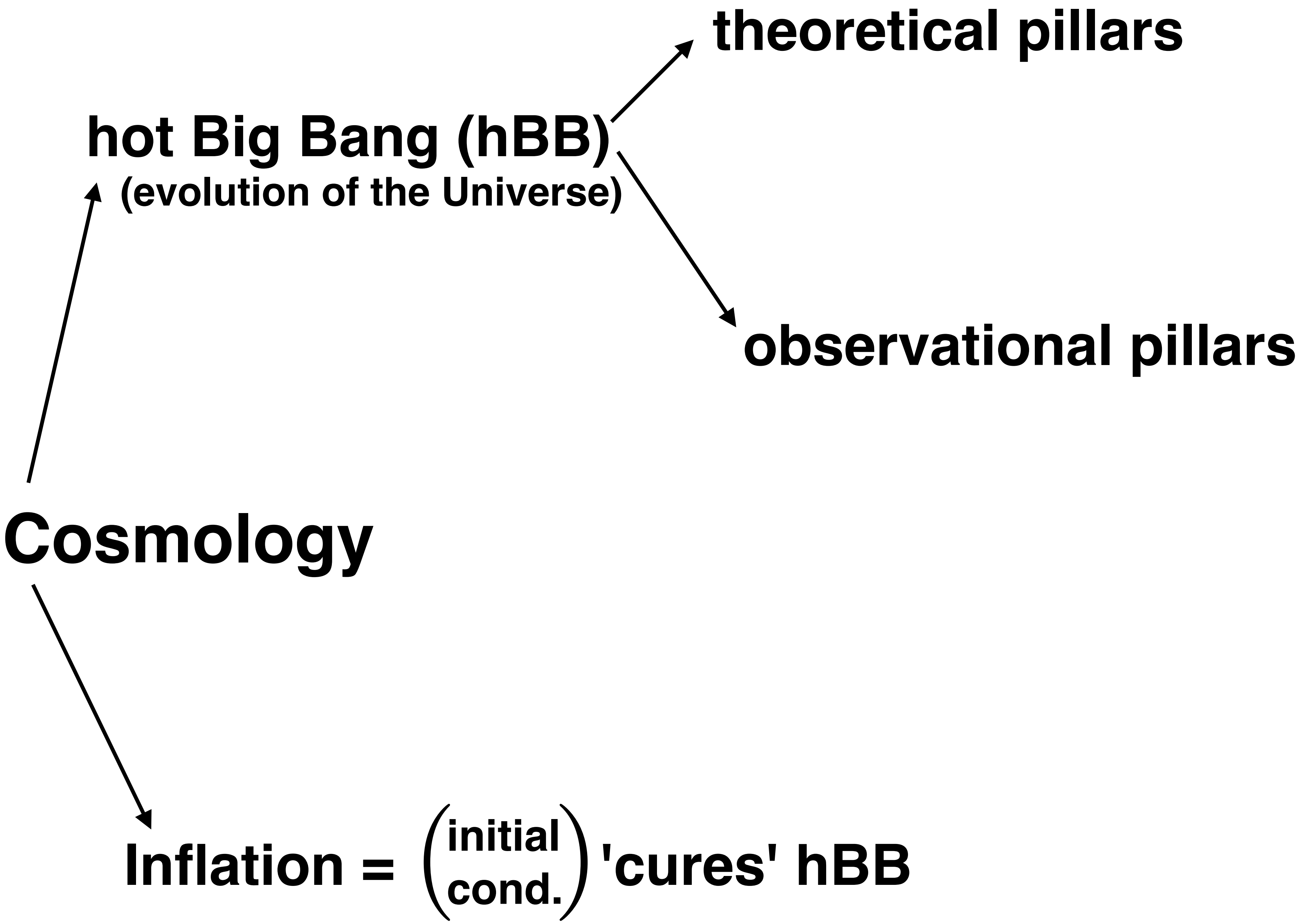
hot Big Bang (hBB)

(evolution of the Universe)

Cosmology

Inflation = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$

BASICS of COSMOLOGY



theoretical pillars

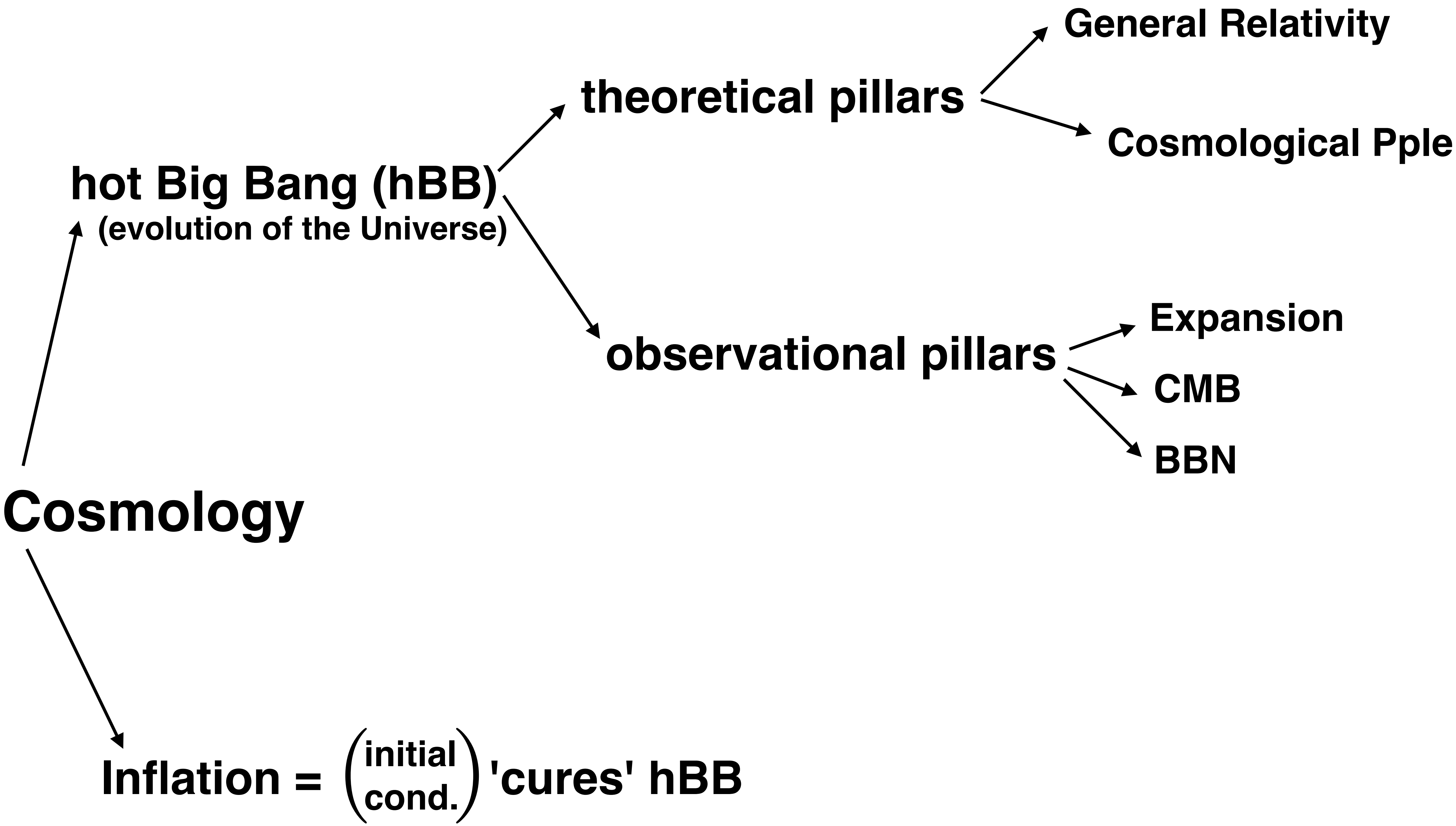
hot Big Bang (hBB)
(evolution of the Universe)

observational pillars

Cosmology

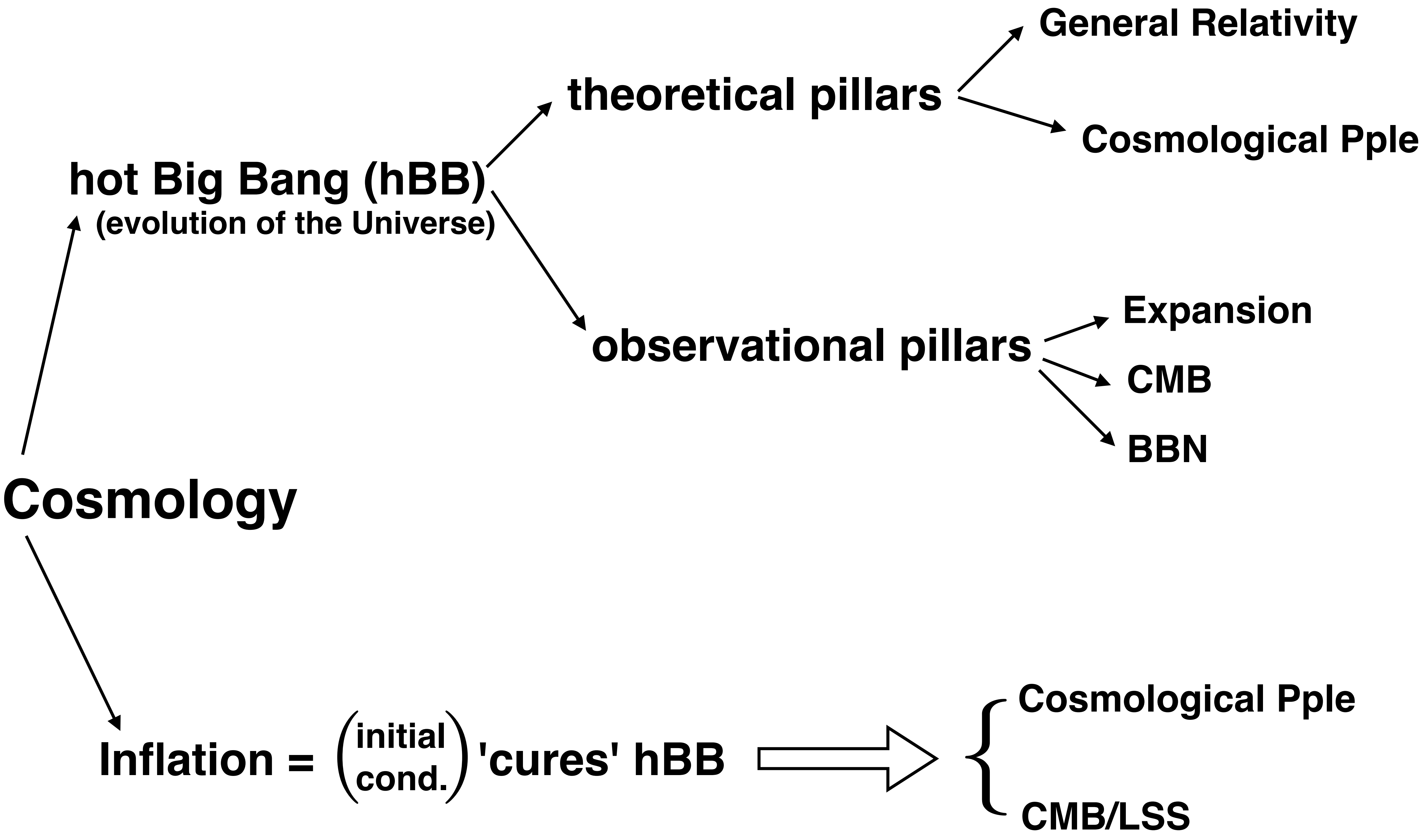
Inflation = $\begin{pmatrix}\text{initial} \\ \text{cond.}\end{pmatrix}$ 'cures' hBB

BASICS of COSMOLOGY



Cosmology
- hot Big Bang (hBB) (evolution of the Universe)
  - theoretical pillars
    - General Relativity
    - Cosmological Pple
  - observational pillars
    - Expansion
    - CMB
    - BBN
- Inflation = $\begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ 'cures' hBB

BASICS of COSMOLOGY



Cosmology

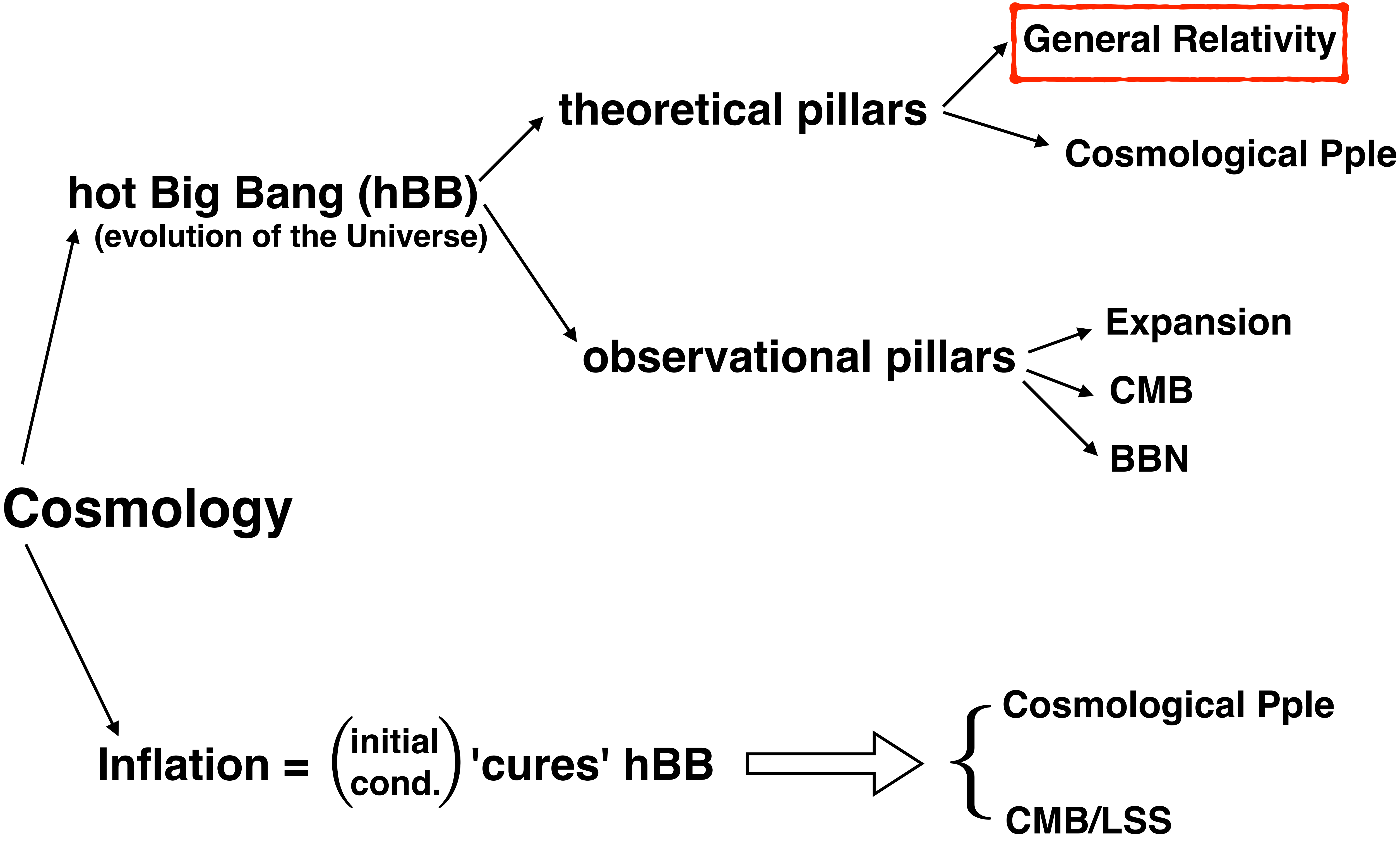
hot Big Bang (hBB)
(evolution of the Universe)

theoretical pillars
- General Relativity
- Cosmological Pple

observational pillars
- Expansion
- CMB
- BBN

Inflation = $\left(\begin{array}{c}\text{initial}\\ \text{cond.}\end{array}\right)$ 'cures' hBB ⇒
- Cosmological Pple
- CMB/LSS

BASICS of COSMOLOGY



Cosmology

hot Big Bang (hBB)
(evolution of the Universe)

theoretical pillars

General Relativity

Cosmological Pple

observational pillars

Expansion

CMB

BBN

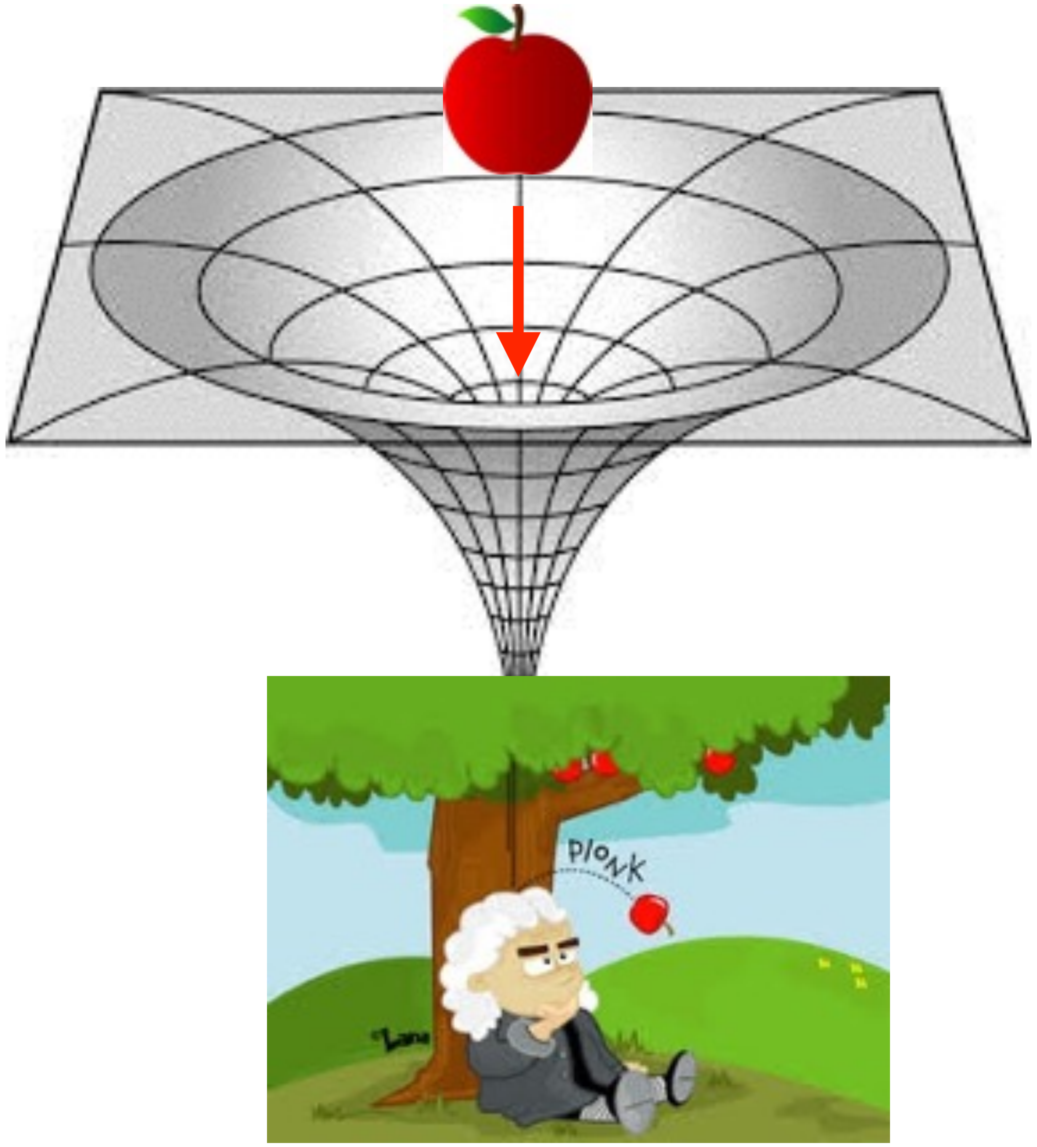
Inflation = (initial cond.) 'cures' hBB

Cosmological Pple

CMB/LSS

Gravitational Framework

General Relativity (GR)



$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry matter

$$\left[m_p = (8\pi G)^{-1/2} = 2.44 \cdot 10^{18}\,\mathrm{GeV} \right]$$

Reduced Planck mass

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

$$\mathrm{DIFF}: \quad x^\mu \to x'^\mu(x)$$

symmetry

Gravitational Framework

General Relativity (GR)

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry matter

metric

$$G_{\mu\nu} \equiv \mathcal{D}[\overset{\uparrow}{g}_{\alpha\beta}] \quad = \quad m_p^{-2} T_{\mu\nu}(\mathrm{Matt}, \mathrm{Rad}, \mathrm{Top.Defects}, \mathrm{DarkEnergy}, ...)$$

$\downarrow$

2nd order, non-Linear

source

Gravitational Framework

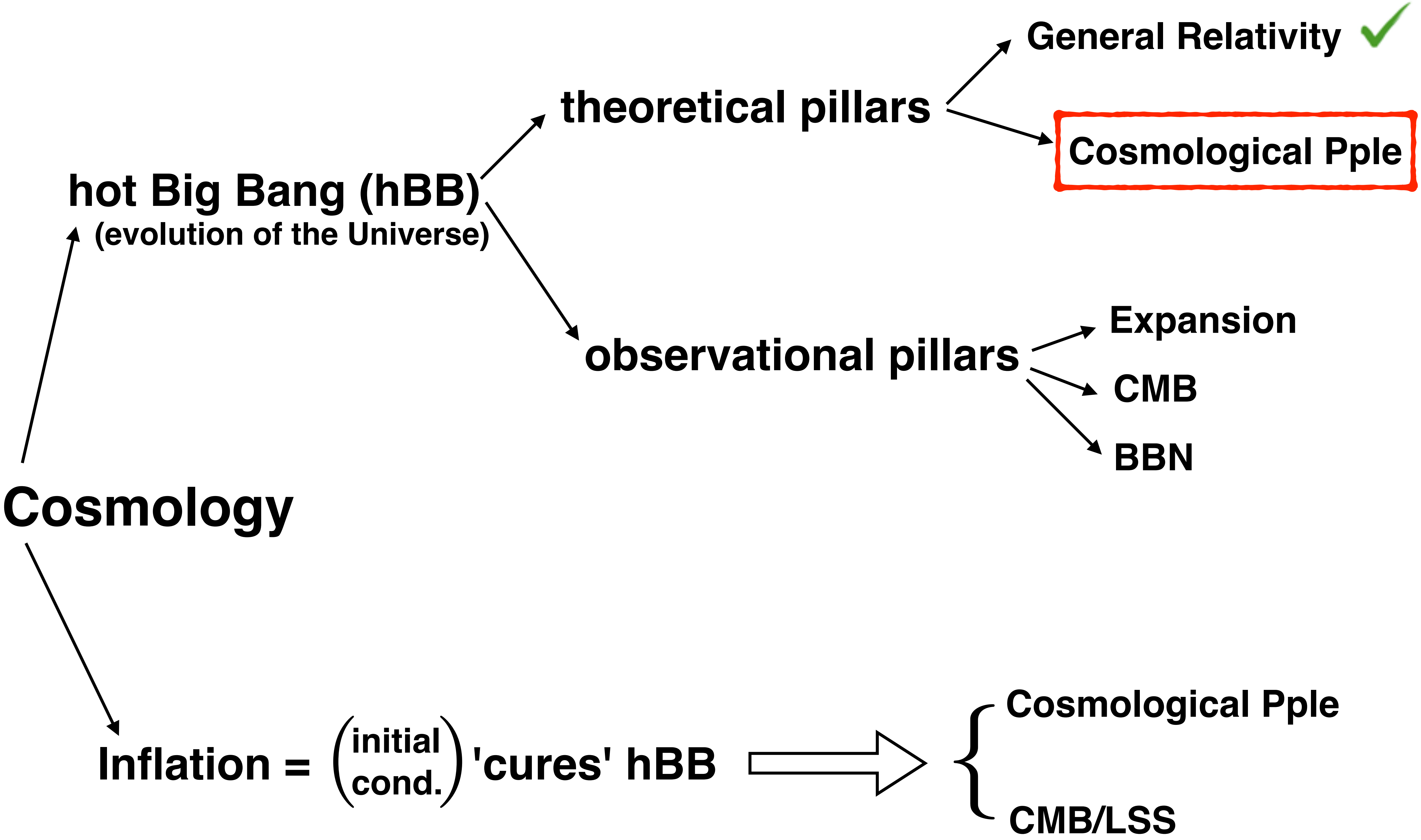
General Relativity (GR)

$$\underset{\text{geometry}}{G_{\mu\nu}} = \frac{1}{m_p^2} \underset{\text{matter}}{T_{\mu\nu}}$$

$$G_{\mu\nu} \equiv \underset{\substack{\downarrow \\ \text{2nd order, non-Linear}}}{\mathcal{D}}[\overset{\substack{\text{metric} \\ \uparrow}}{g_{\alpha\beta}}] \quad = \quad \underset{\text{source}}{m_p^{-2} T_{\mu\nu}(\mathrm{Matt}, \mathrm{Rad}, \mathrm{Top.Defects}, \mathrm{DarkEnergy}, \ldots)}$$

$$\begin{tikzcd}
 & T^{[U]}_{\mu\nu} \arrow[dd, bend left] \\
\textbf{Universe ?} \arrow[ur] \arrow[dr] & \\
 & g^{[U]}_{\mu\nu}
\end{tikzcd}$$

BASICS of COSMOLOGY



Cosmology

hot Big Bang (hBB)
(evolution of the Universe)

theoretical pillars
- General Relativity ✓
- Cosmological Pple

observational pillars
- Expansion
- CMB
- BBN

Inflation = (initial cond.) 'cures' hBB ⟹ { Cosmological Pple, CMB/LSS }

Cosmological Principle

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry of the Universe

matter within the Universe

Cosmological Principle

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu} \longrightarrow$$

geometry of the Universe

matter within the Universe

Principle of Symmetry:

The Universe is Homogeneous & Isotropic

Cosmological Principle

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geometry of the Universe

matter within the Universe

Principle of Symmetry:

The Universe is Homogeneous & Isotropic

$$g_{\mu\nu}^{[U]} \equiv \mathrm{diag}\left(-1, \frac{a^2(t)}{1-kr^2}, a^2(t)r^2, a^2(t)r^2\sin^2\theta\right)$$

FLRW

Friedmann-Lemaître -Robertson-Walker

Cosmological Principle

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu}$$

geometry of the Universe

matter within the Universe

→

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FLRW

Friedmann-Lemaître
-Robertson-Walker

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left\{\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right\}$$

***2 dof* :**

Scale Factor
(dynamical)

Curvature
(const.)

$$\left\{\begin{array}{l} k<0, \text{Open} \\ k=0, \text{Flat} \\ k>0, \text{Close} \end{array}\right.$$

Cosmological Principle

$$G_{\mu\nu} = \frac{1}{m_p^2} T_{\mu\nu} \longrightarrow$$

geometry of the Universe

matter within the Universe

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The Universe is Homogeneous & Isotropic

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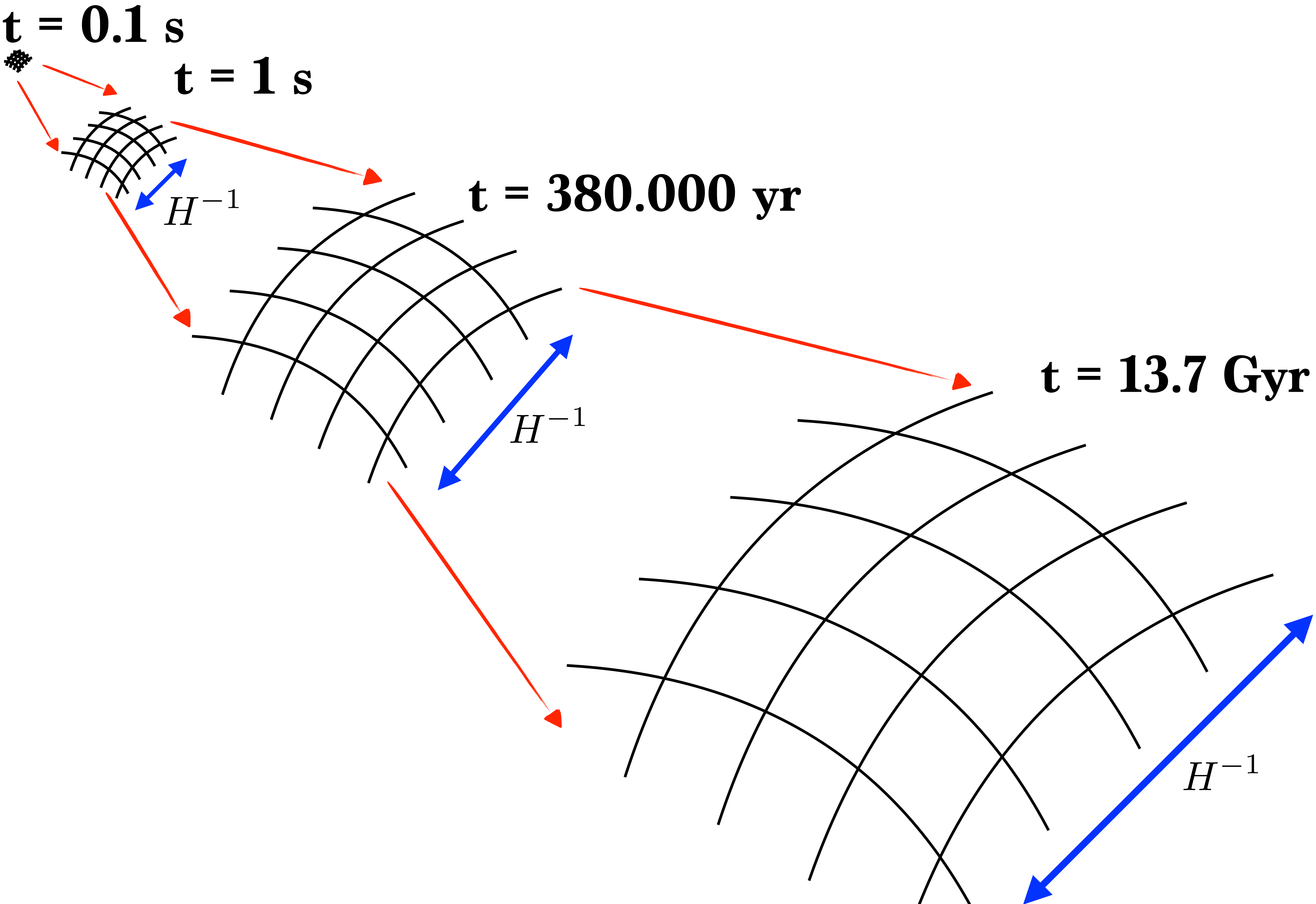
invariant: $(c = \mathrm{const.})$

$$\left\{\begin{array}{l} k \to k/c^2 \\ r \to c\cdot r \\ a \to a/c \end{array}\right. \Longrightarrow \left\{\begin{array}{l} a, r, k \quad \text{unphysical} \\ \frac{k}{a^2},\; a\cdot r,\; kr^2 \quad \text{physical} \end{array}\right.$$

Expanding Universe



Cosmological Principle

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Principle of Symmetry:

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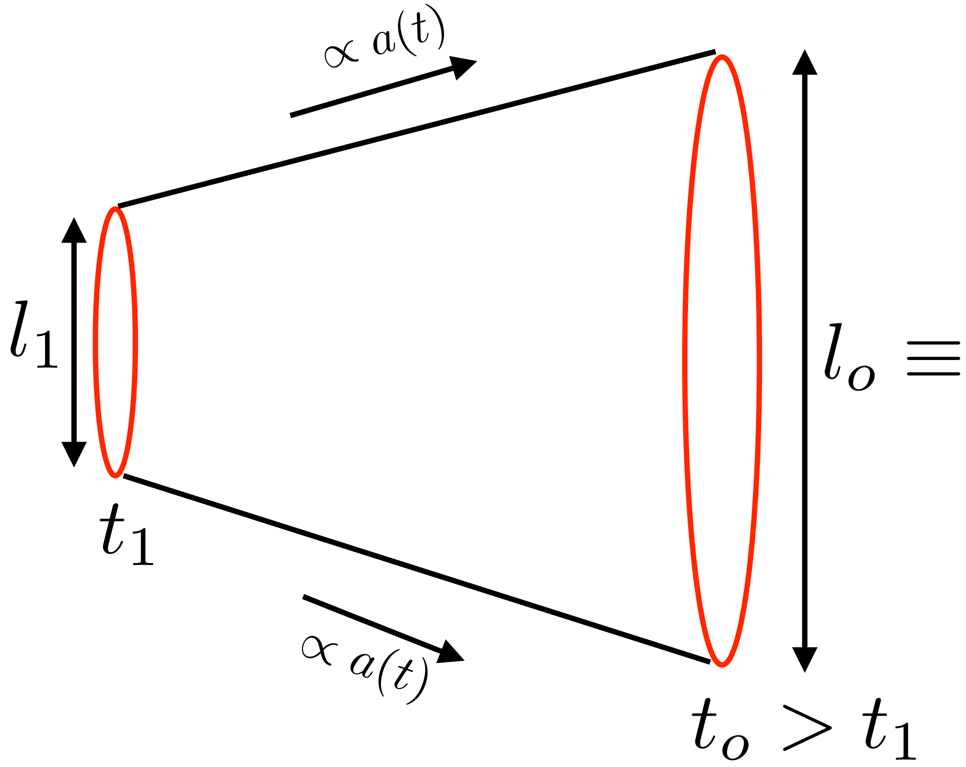


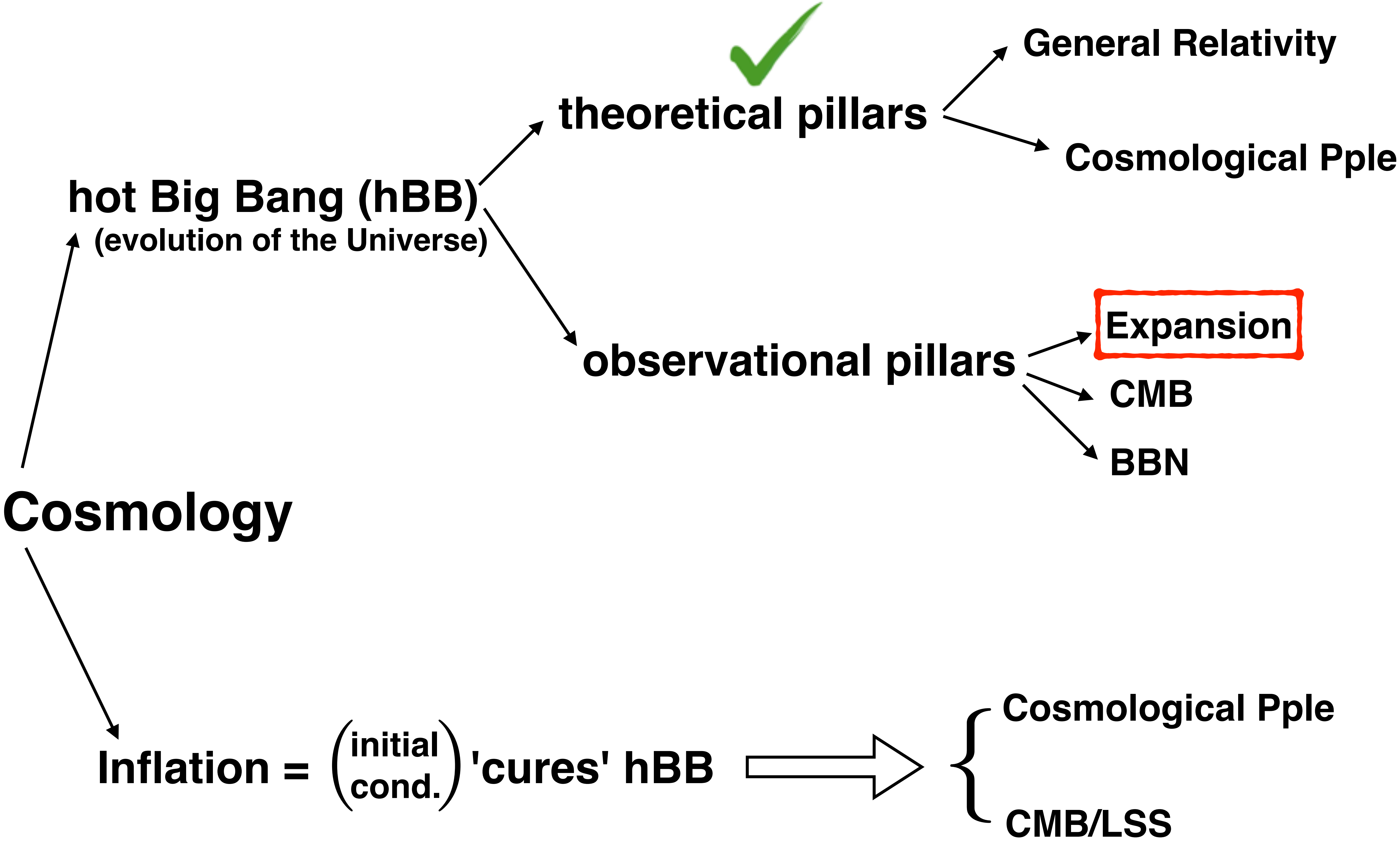
$$l_o \equiv \left(\frac{a(t_o)}{a(t_1)} \right) l_1$$

Redshift

$$z_1 \equiv \frac{a_o - a_1}{a_1}$$

$$\boxed{1 + z \equiv \frac{a(t_o)}{a(t)}}$$

BASICS of COSMOLOGY



Cosmology

hot Big Bang (hBB)
(evolution of the Universe)

✓ theoretical pillars
- General Relativity
- Cosmological Pple

observational pillars
- Expansion
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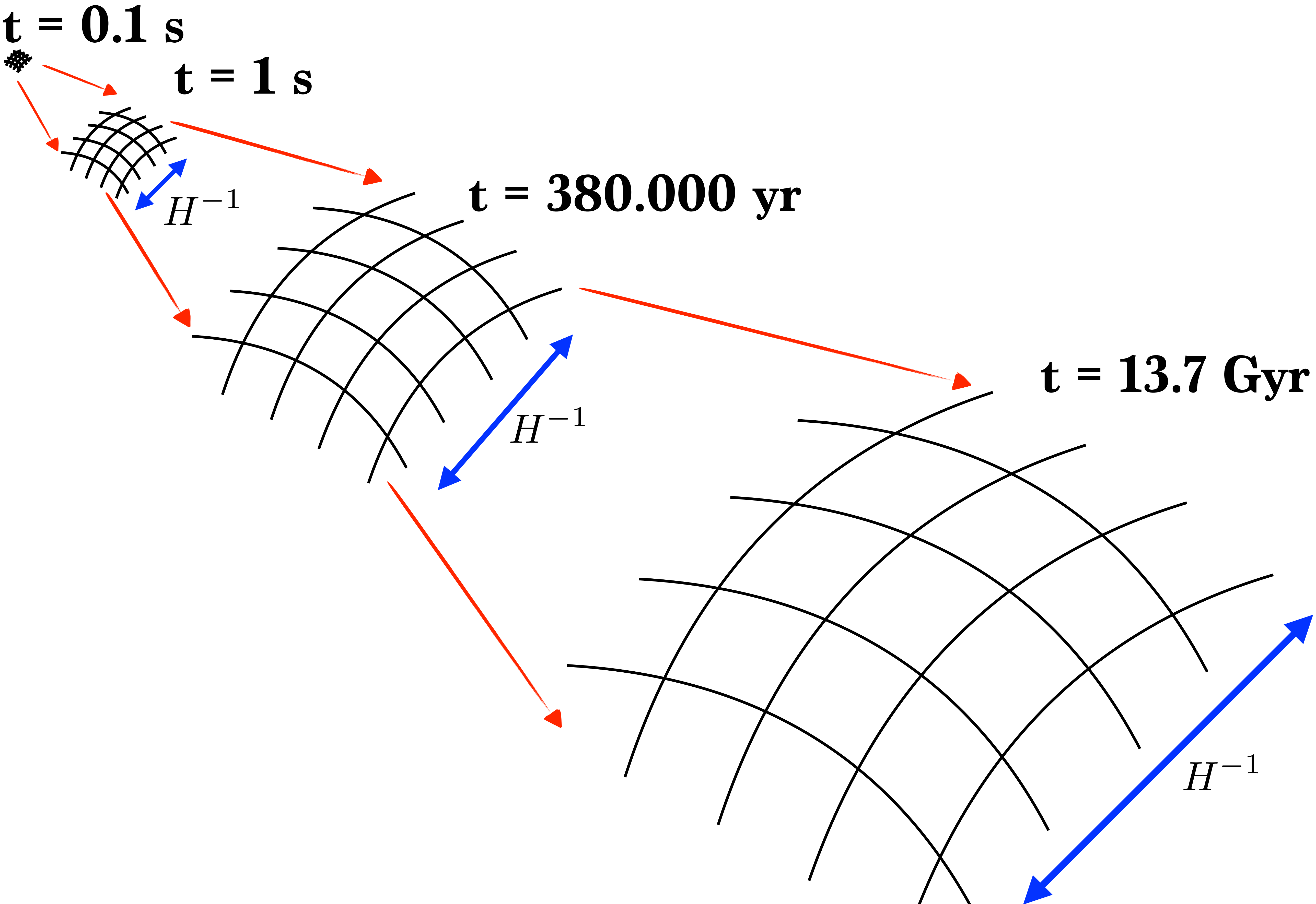
Inflation = $\begin{pmatrix}\text{initial}\\\text{cond.}\end{pmatrix}$ 'cures' hBB $\Longrightarrow \begin{cases}\text{Cosmological Pple}\\\text{CMB/LSS}\end{cases}$

Expanding Universe



Expanding Universe

H & I

$$T^\mu_\nu \equiv \mathrm{diag}(-\rho, p, p, p)$$

⇩

$$m_p^2 G^\mu_\nu \left[g_{**}^{(FRW)} \right] = T^\mu_\nu$$

Expanding Universe

H & I

$$T^{\mu}_{\nu} \equiv \mathrm{diag}(-\rho, p, p, p)$$

$$\Downarrow$$

$$m_p^2 G^{\mu}_{\nu}\left[g_{**}^{(FRW)}\right] = T^{\mu}_{\nu}$$

$$\Rightarrow$$

Friedmann Equations

$$\boxed{\begin{aligned} \frac{1}{a}\frac{d^2a}{dt^2} &= -\frac{\rho}{6m_p^2}(1+3w) \\ H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 &= \frac{\rho}{3m_p^2} - \frac{K}{a^2} \end{aligned}} \quad \begin{aligned} &(\mathrm{I}) \\ &(\mathrm{II}) \end{aligned}$$

$\left(w \equiv \frac{p}{\rho}\right)$ Equation of State (EoS)

Expanding Universe

H & I

$$T^{\mu}_{\nu} \equiv \mathrm{diag}(-\rho, p, p, p)$$

$$\Downarrow$$

$$m_p^2 G^{\mu}_{\nu}\left[g_{**}^{(FRW)}\right] = T^{\mu}_{\nu} \quad \Rightarrow$$

Friedmann Equations

$$\boxed{\begin{aligned} \frac{1}{a}\frac{d^2a}{dt^2} &= -\frac{\rho}{6m_p^2}(1+3w) & \text{(I)} \\ H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 &= \frac{\rho}{3m_p^2} - \frac{K}{a^2} & \text{(II)} \end{aligned}}$$

$$\left(w \equiv \frac{p}{\rho}\right)$$ Equation of State (EoS)

$$\text{(I)} + \text{(II)} \Rightarrow \boxed{\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)} \quad \text{(III)}$$

Expanding Universe

UNIVERSE:

1) GR

+

2) H & I

Friedmann Equations

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w) \tag{I}$$

$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \tag{II}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w) \tag{III}$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

Equation of State (EoS)

Expanding Universe

(II) $$H^2 \equiv \left(\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$$

Critical density $(\rho = \rho_c \Leftrightarrow K = 0)$

Expanding Universe

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$$\rho = \sum_i \rho_i \;\; ; \;\; \Omega_i \equiv \frac{\rho_i}{\rho_c} \Rightarrow \Omega \equiv \frac{\rho}{\rho_c} = \sum_i \Omega_i \Rightarrow \boxed{\Omega - 1 \equiv \frac{k}{a^2 H^2}}$$

Cosmic Sum

Expanding Universe

$$\text{(II)}\quad H^2 \equiv \left(\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$$

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Cosmic Sum

$$\begin{cases} \Omega > 1 \Rightarrow \mathrm{Close}(k > 0) \\ \Omega = 1 \Rightarrow \mathrm{Flat}(k = 0) \\ \Omega < 1 \Rightarrow \mathrm{Open}(k < 0) \end{cases}$$

one-to-one
correlation

Expanding Universe

(II) $$H^2 \equiv \left(\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$$

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Cosmic Sum

(III)

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w) \;\Rightarrow\; \rho \propto e^{-3\int \frac{da}{a}(1+w)} = \begin{cases} 1/a^3 & \text{, Mat.}(w=0) \\ 1/a^4 & \text{, Rad.}(w=1/3) \\ \text{const.} & \text{, C.C.}(w=-1) \end{cases}$$

Expanding Universe

(II) $H^2 \equiv \left(\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2} \longrightarrow \boxed{\rho_c \equiv 3m_p^2 H^2}$

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Cosmic Sum

(III) + (II) :

$$H^2(a) = H_o^2 \left\{ \Omega_{\rm R}^{(o)} \left(\frac{a_o}{a}\right)^4 + \Omega_{\rm M}^{(o)} \left(\frac{a_o}{a}\right)^3 + \Omega_{\rm k}^{(o)} \left(\frac{a_o}{a}\right)^2 + \Omega_{\rm DE}^{(o)} e^{-3\int \frac{da}{a}(1+w)} \right\}$$

$$\equiv H_o^2 E^2(a)$$

$$E(a) \equiv \sqrt{\Omega_{\rm R}^{(o)} \left(\frac{a_o}{a}\right)^4 + \Omega_{\rm M}^{(o)} \left(\frac{a_o}{a}\right)^3 + \Omega_{\rm k}^{(o)} \left(\frac{a_o}{a}\right)^2 + \Omega_{\rm DE}^{(o)} e^{-3\int \frac{da}{a}(1+w)}} \qquad \Omega_{\rm k}^{(o)} \equiv -\frac{k}{a_o^2 H_o^2}$$

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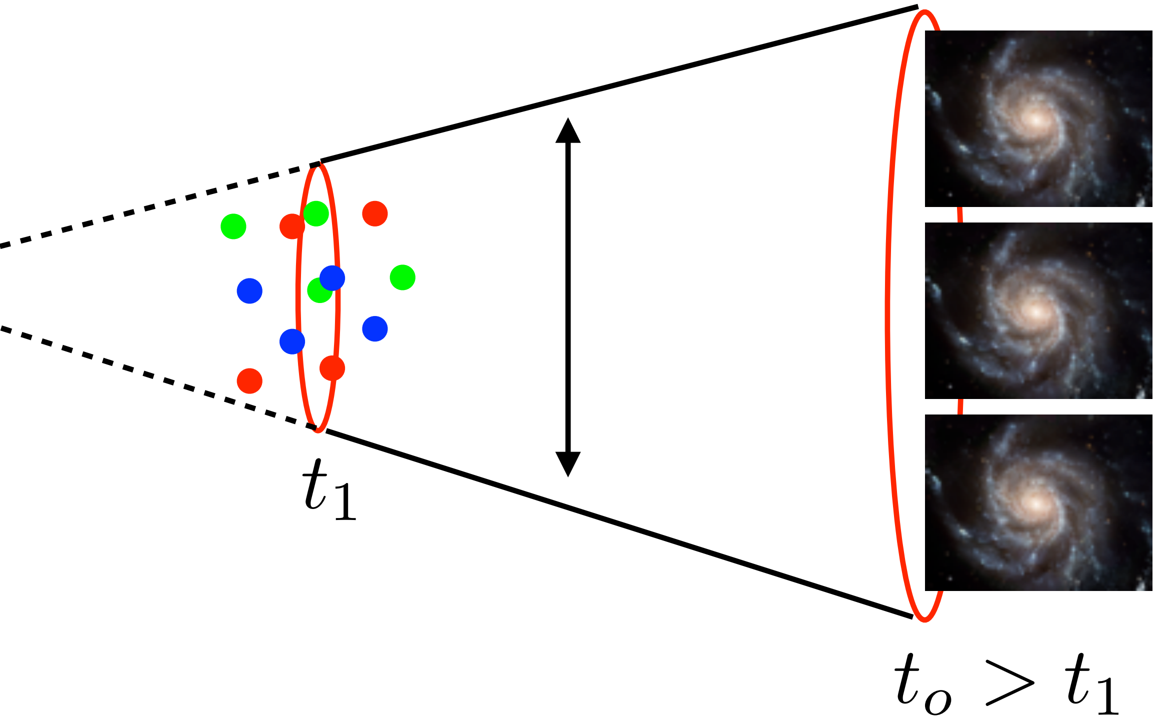
$$\equiv H_o^2 E^2(a)$$

Lecture(s) by BONVIN

$$E(a) \equiv \sqrt{\Omega_{\rm R}^{(o)} \left(\frac{a_o}{a}\right)^4 + \Omega_{\rm M}^{(o)} \left(\frac{a_o}{a}\right)^3 + \Omega_{\rm k}^{(o)} \left(\frac{a_o}{a}\right)^2 + \Omega_{\rm DE}^{(o)} e^{-3\int \frac{da}{a}(1+w)}} \qquad \Omega_{\rm k}^{(o)} \equiv -\frac{k}{a_o^2 H_o^2}$$

Expansion History

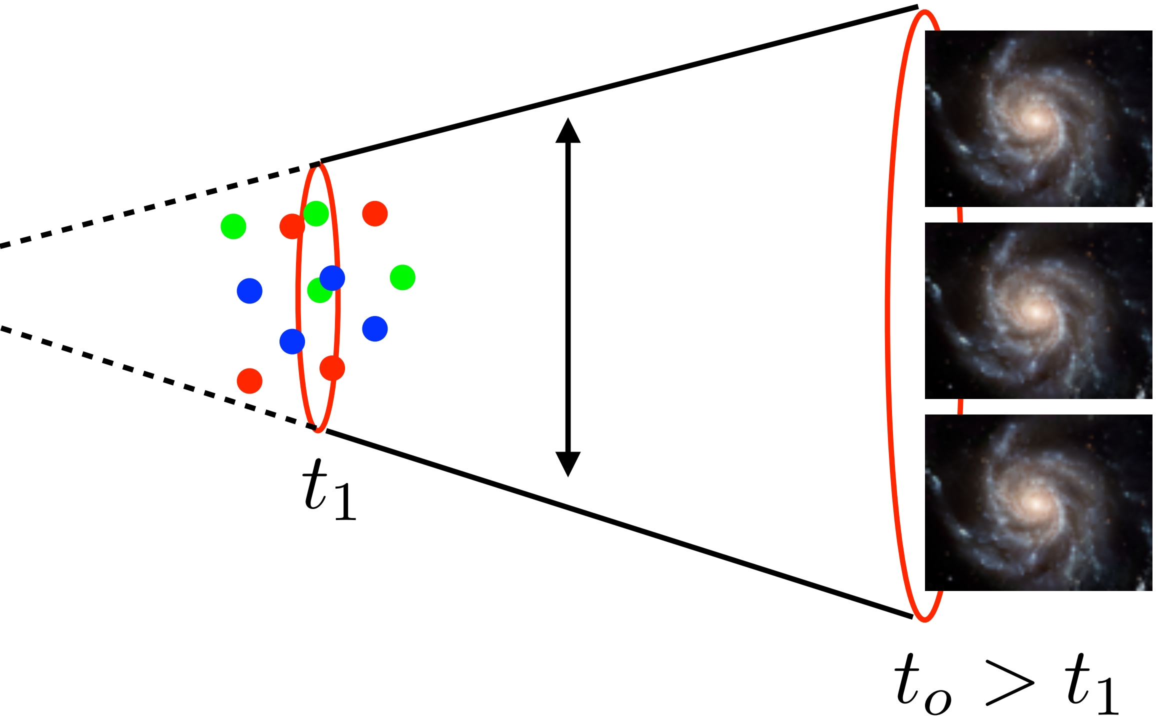


Past: particle ensemble

Statistical Mechanics

$$\text{(III)}\quad \frac{d\rho}{dt} + 3H(\rho + p) = 0 \longrightarrow \quad \frac{dU}{dt} + p\frac{dV}{dt} = 0\,, \qquad \begin{cases} U = a^3\rho\,, \\ V = a^3 \end{cases}$$

Expansion History



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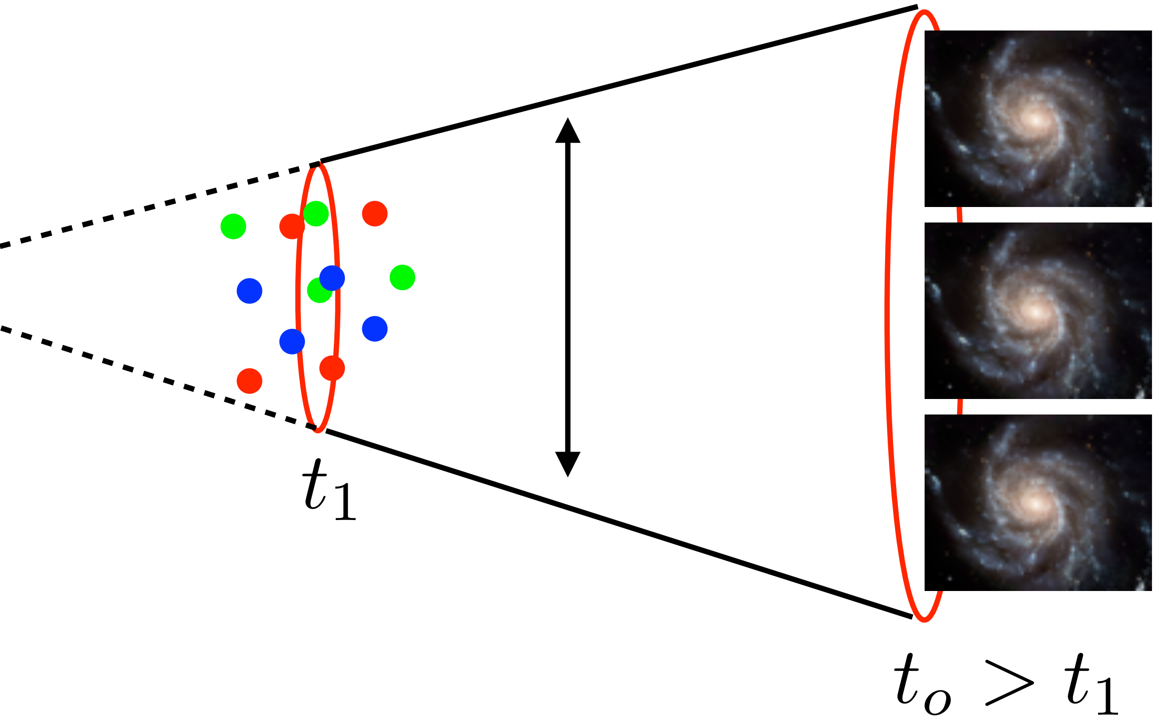
$$\Longrightarrow \begin{cases} \frac{dU}{dt} + p\frac{dV}{dt} = T\frac{dS}{dt}\,, \longrightarrow \textbf{Thermal Eq.} \\ \frac{dS}{dt} = 0\,, \longrightarrow \textbf{Adiabatic Exp.} \end{cases}$$

Expansion History



Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, \\ \rho = g_* \int d\vec{p}\, E(\vec{p})\, f(\vec{p})\,, \\ p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})}\, f(\vec{p})\,, \end{cases}$$

number

energy

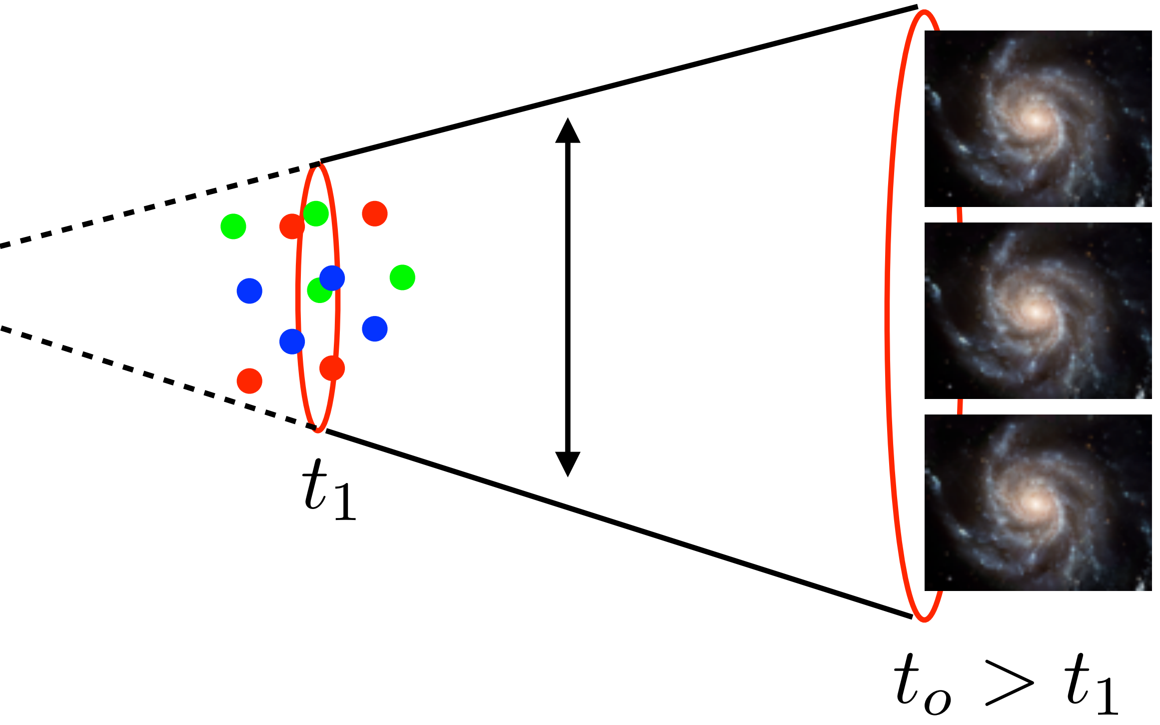
pressure

dof

Dispersion relation

Statistical Distribution

Expansion History



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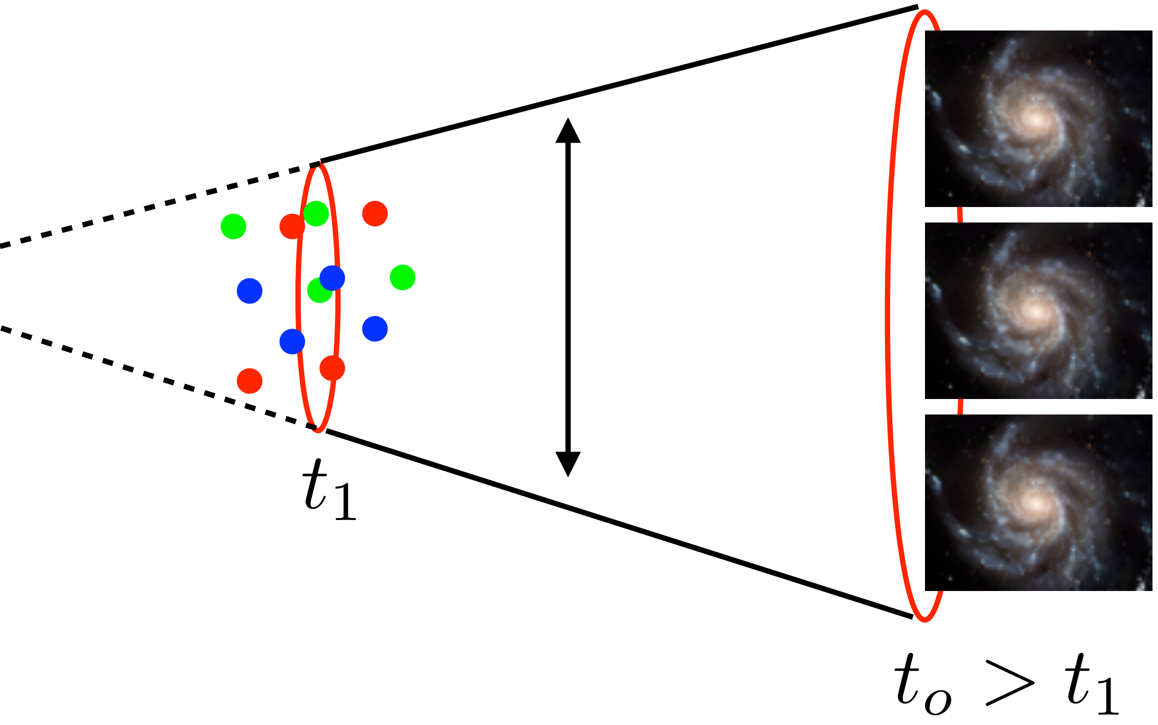
dof — *Dispersion relation* — *Statistical Distribution*

Bose-Einstein / Fermi-Dirac: $f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$

Expansion History



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dof *Dispersion relation* *Statistical Distribution*

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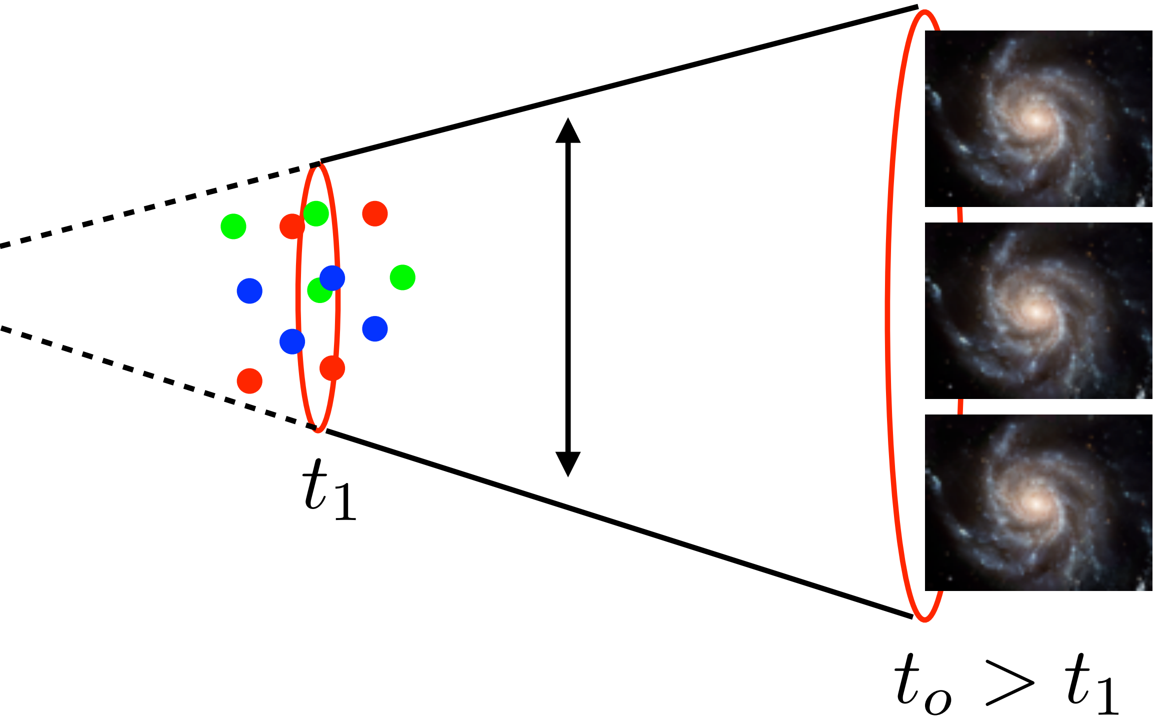
$$\frac{\rho_R \propto 1/a^4}{\rho_M \propto 1/a^3} \propto 1/a\,, \quad \Rightarrow \quad z \geq z_{\rm EQ}\ (t \leq t_{\rm EQ})\,, \quad \rho_R > \rho_M$$

Expansion History



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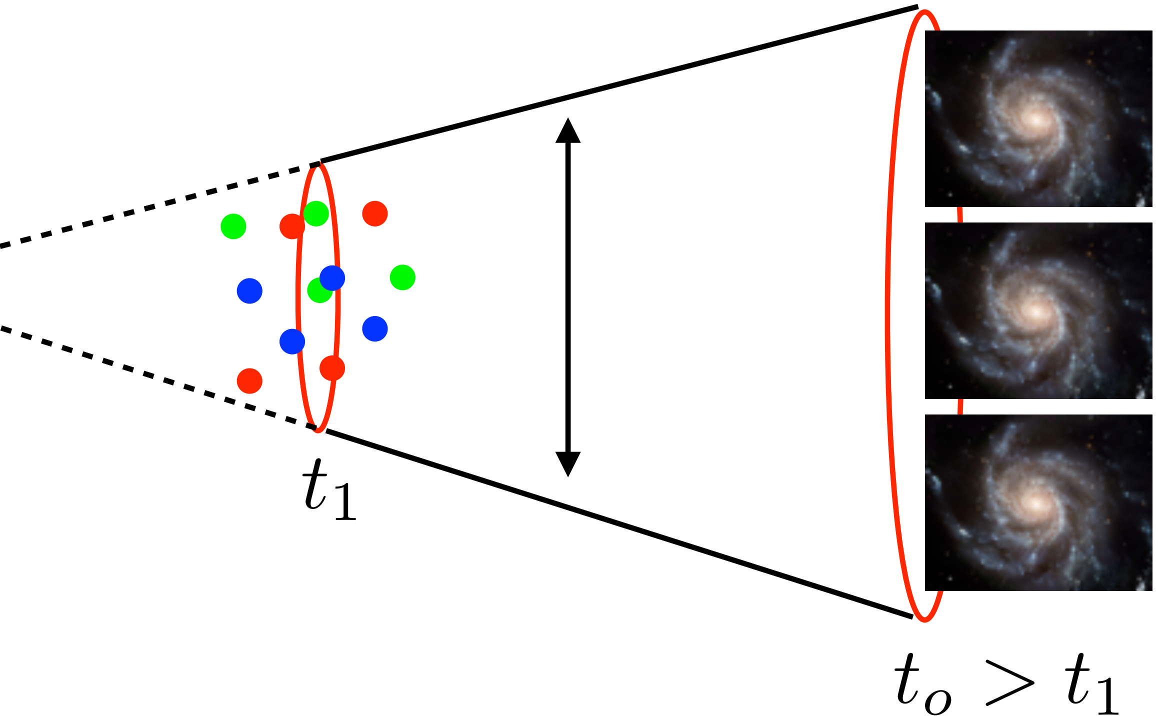
Past: Radiation Domination ! $\boxed{1 + z_{\rm EQ} = \Omega^{(o)}_{\rm M}/\Omega^{(o)}_{\rm Rad} \sim 3400}$

Expansion History



Thermal Eq. **(densities)**

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = g_* \int d\vec{p}\, \frac{|\vec{p}|^2}{3E(\vec{p})} f(\vec{p})\,, & \textbf{pressure} \end{cases}$$

dof *Dispersion relation* *Statistical Distribution*

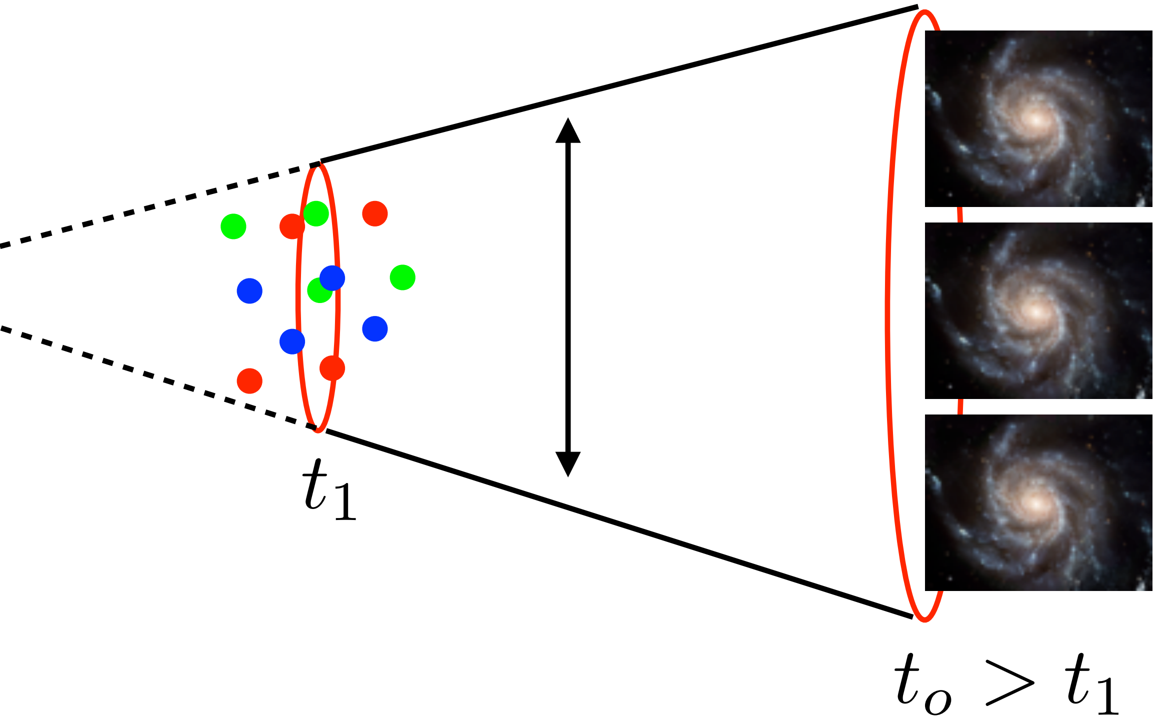
Bose-Einstein / Fermi-Dirac:

$$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

Past: Radiation Domination (RD)

$$\rho_R^{(i)} = f_i g_*^{(i)} \frac{\pi^2}{30} T_i^4\,, \quad f_i = \begin{cases} 1, & \mathrm{B} \\ \frac{7}{8}, & \mathrm{F} \end{cases}$$

Expansion History



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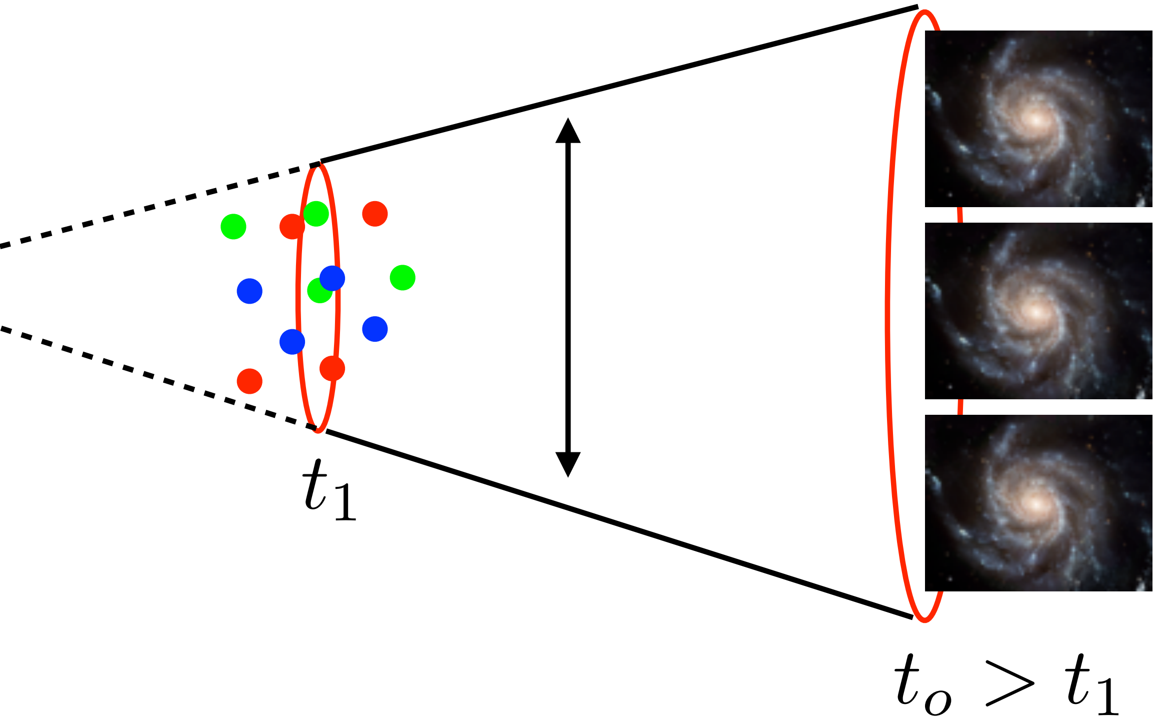
Expansion History



Thermal Eq.

(densities)

$$\begin{cases} n = g_* \int d\vec{p}\, f(\vec{p})\,, & \textbf{number} \\ \rho = g_* \int d\vec{p}\, E(\vec{p}) f(\vec{p})\,, & \textbf{energy} \\ p = \underbrace{g_*}_{\text{dof}} \int d\vec{p}\, \underbrace{\frac{|\vec{p}|^2}{3E(\vec{p})}}_{\text{Dispersion relation}} \underbrace{f(\vec{p})}_{\text{Statistical Distribution}}\,, & \textbf{pressure} \end{cases}$$

Bose-Einstein / Fermi-Dirac: $$f(\vec{p}) = \left(e^{E(\vec{p})/T} \pm 1\right)^{-1}\,, \begin{cases} F(+) & \textbf{[fermions]} \\ B(-) & \textbf{[bosons]} \end{cases}$$

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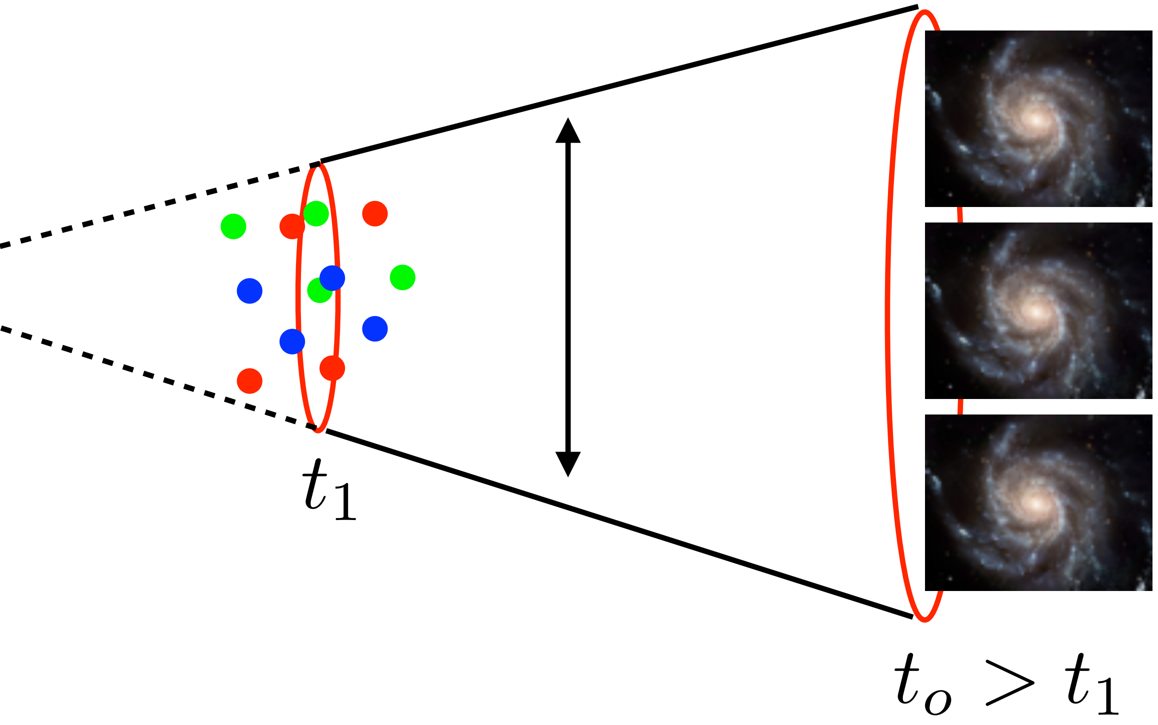
$$\boxed{\rho_R = \sum_i \rho_R^{(i)} \equiv g_*(T) \frac{\pi^2}{30} T^4}$$

$$g_*(T) \equiv \sum_i g_{*,i}^{(B)} \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_i g_{*,i}^{(F)} \left(\frac{T_i}{T}\right)^4$$

Expansion History



Adiabatic Exp:

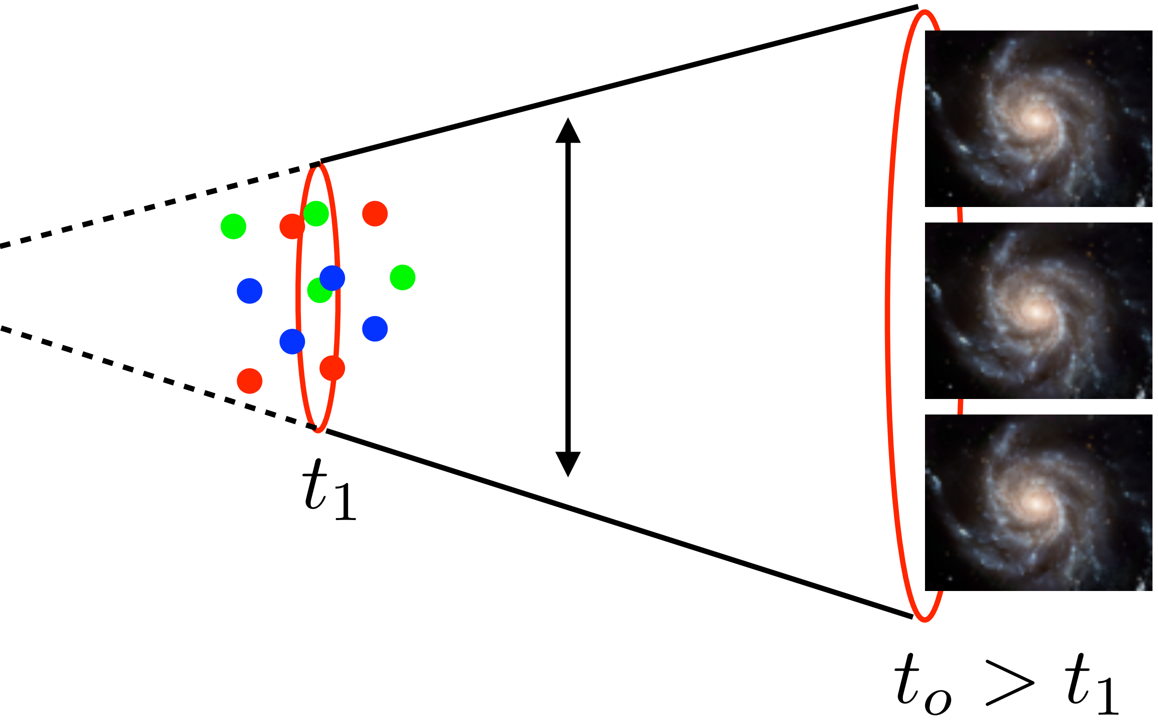
$$S = \frac{a^3(\rho + p)}{T} = const.$$

$$\Downarrow$$

$$\boxed{a^3 T^3 g_*^{(s)}(T) = const.}$$

$$g_*^{(s)}(T) \equiv \sum_i g_{*,i}^{(B)} \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_i g_{*,i}^{(F)} \left(\frac{T_i}{T}\right)^3$$

Expansion History



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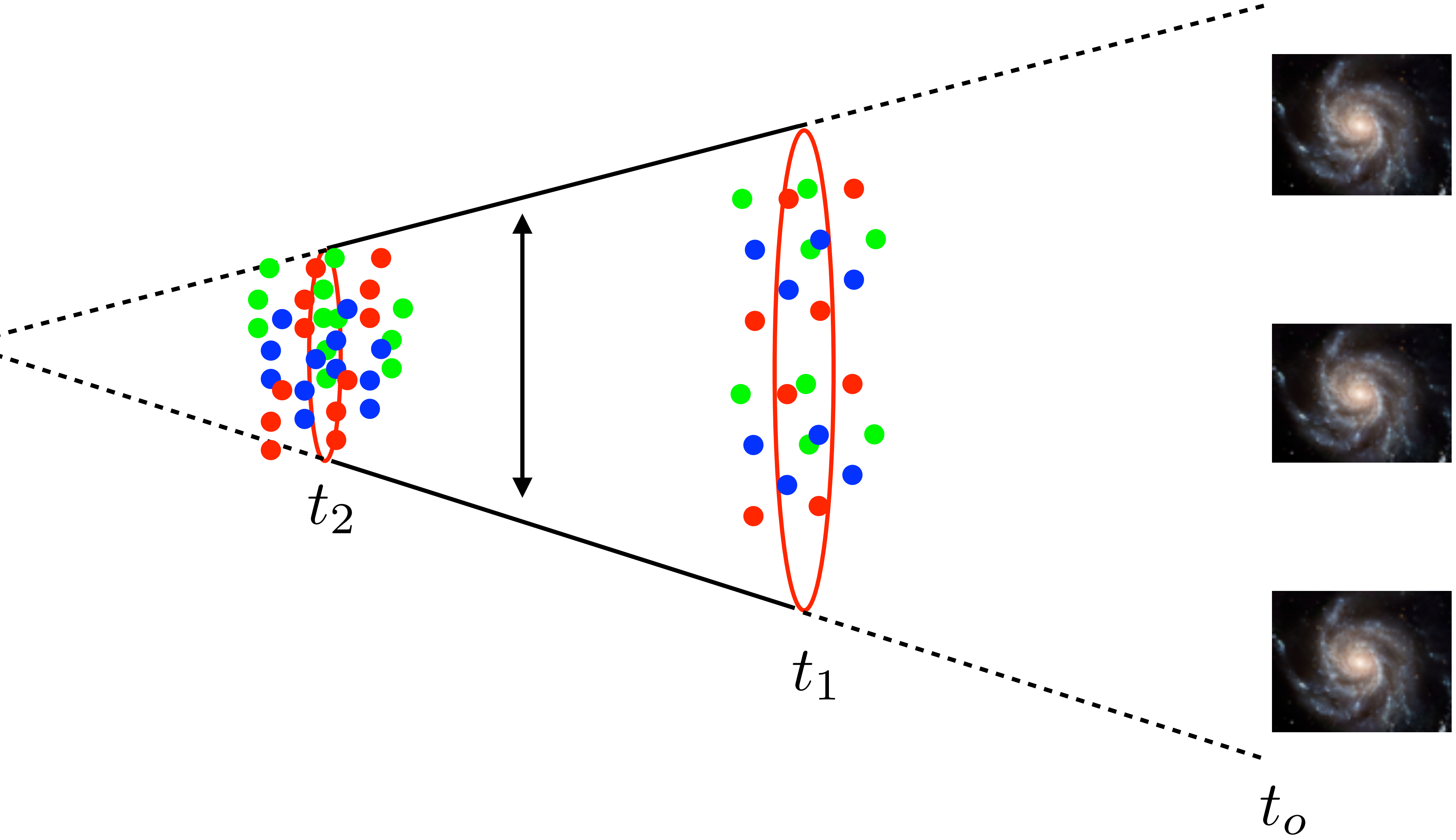
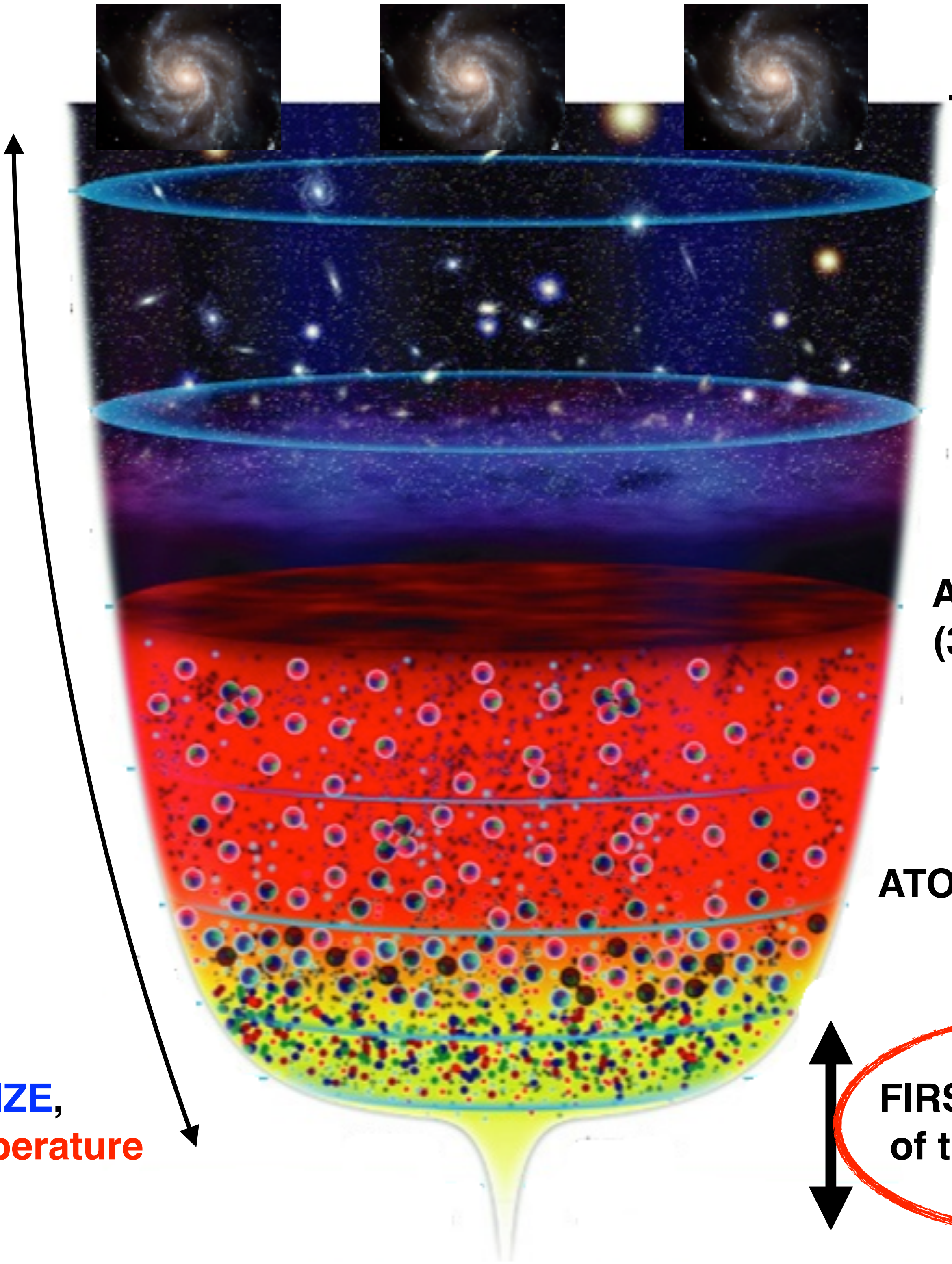
When do $g_*(T), g_*^{(s)}(T)$ change ?

- 1) Species Decoupling , $T \to T_i$,
- 2) Mass threshold , $T < 2m_i$,

Expansion History



Expansion History



BiGGER size,
SMALLER Temp

TODAY [Galaxies, Clusters, …]
(13.700 Million years)

FIRST GALAXIES
(500 Millions years)

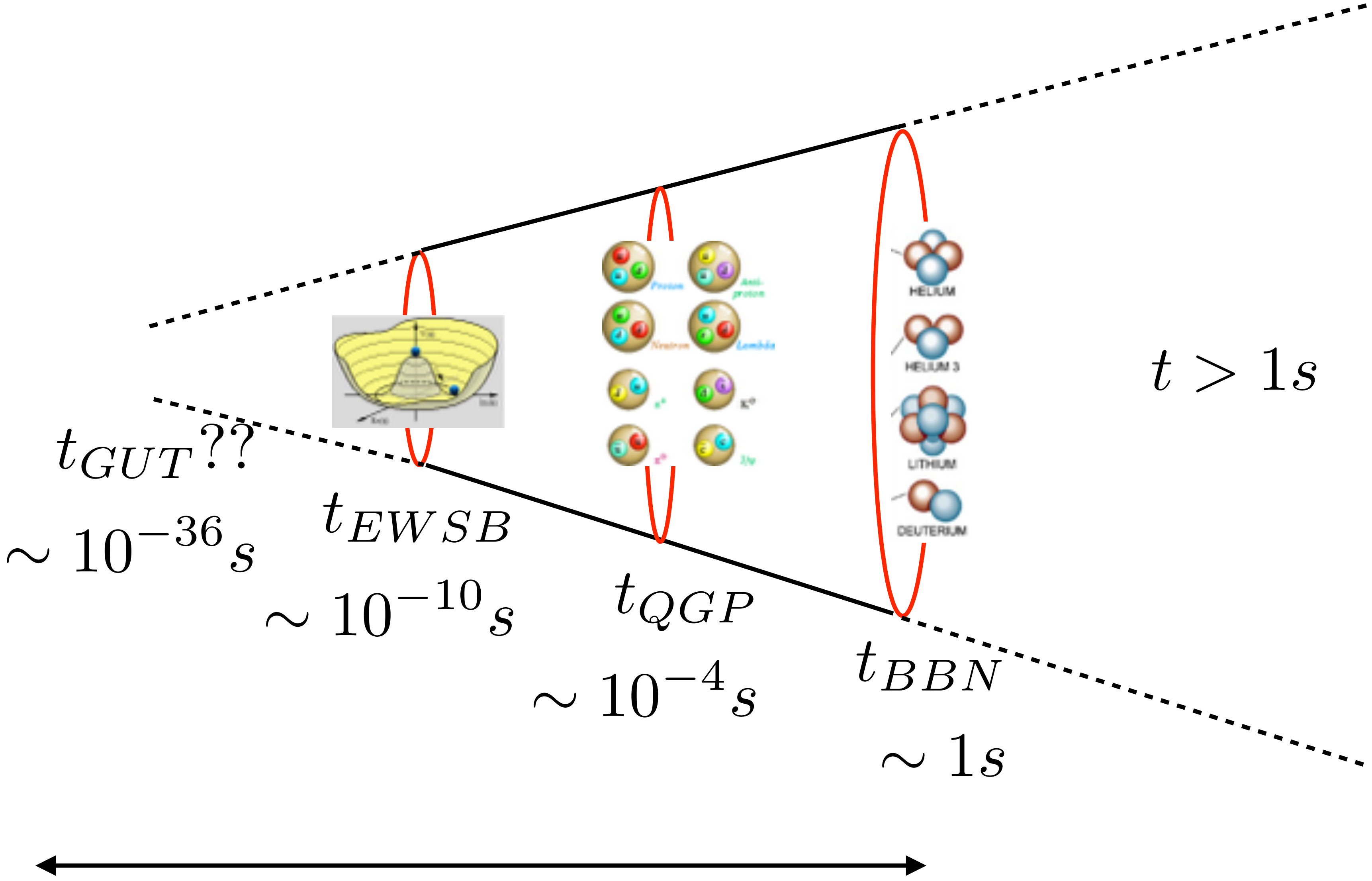
ATOMS CREATION
(300.000-400.000 years)

ATOMIC NUCLEI CREATION
(3 minutes !)

FIRST SECOND
of the UNIVERSE !

SMALLER SIZE,
LARGER Temperature

Expansion History



$t_{GUT}??$

$\sim 10^{-36}s$

t_{EWSB}

$\sim 10^{-10}s$

t_{QGP}

$\sim 10^{-4}s$

t_{BBN}

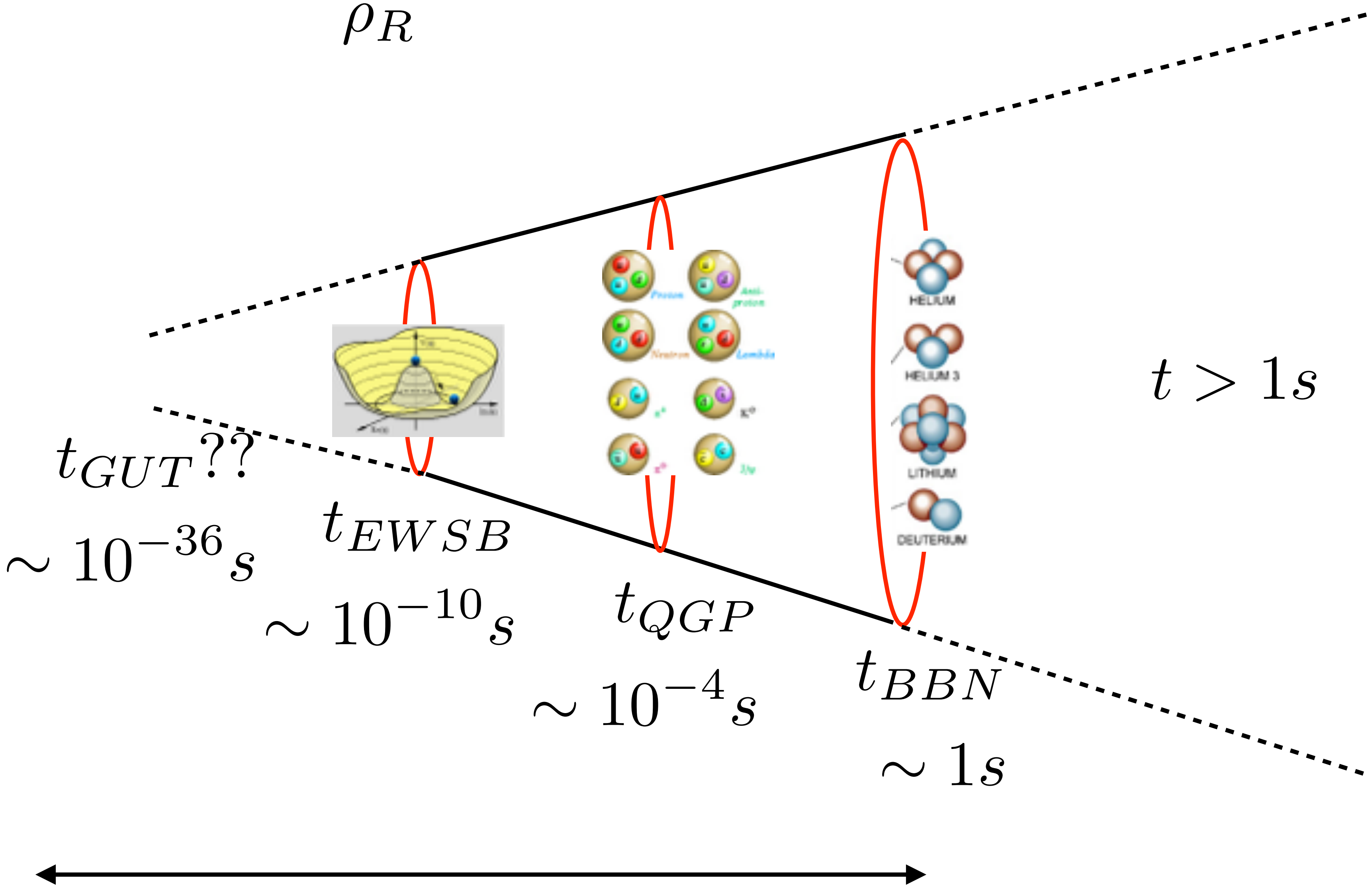
$\sim 1s$

$t > 1s$

hot early Universe $(\sim 1s)$

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}g_*(T)T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$



t_{GUT}?? $\sim 10^{-36}s$

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T	$1\ \mathrm{MeV}$	$200\ \mathrm{MeV}$	$\mathcal{O}(10^2)\ \mathrm{GeV}$	$\mathcal{O}(10^{16})\ \mathrm{GeV}$	$\mathcal{O}(10^{19})\ \mathrm{GeV}$
t	$1\ \mathrm{s}$	$10^{-5}\ \mathrm{s}$	$10^{-10}\ \mathrm{s}$	$10^{-37}\ \mathrm{s}$	$10^{-43}\ \mathrm{s}$
	BBN	**QGP**	**EWSB**	**GUT**	**QG**
	↓	↓	↓	↓	↓
	observational evidence !	theoretical input	theoretical input	theoretical speculation	theoretical speculation

Expansion History

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When do $g_*(T), g_*^{(s)}(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,,\ \ T \to T_i\ , \\ 2)\ \text{Mass threshold}\,, T < 2m_i\ , \end{cases}$$

Expansion History

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Decoupling:

$$\Gamma_{\rm int} = \sigma \times \langle nv \rangle \quad \longrightarrow \quad N_{\rm int} = \int_t^{t+\Delta t} dt' \Gamma_{\rm int}(t') \sim \frac{\Gamma_{\rm int}(t)}{H(t)} \begin{cases} \ll 1 \Rightarrow \text{decoupling} \\ \gg 1 \Rightarrow \text{Thermal Eq.} \end{cases}$$

cross section

number density

velocity (c = 1)

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}g_*(T)T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \ \text{s}$$

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cross section $\sim \begin{cases} \frac{(g^2/4\pi)^2}{T^2}, & \text{massless gauge} \\ T^2/M_*^4, & \text{massive gauge} \end{cases}$

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30}g_*(T)T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\text{MeV}}\right)^{-2} \ \text{s}$$

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Neutrino Decoupling:

$$\Gamma_\nu = \underset{\sim G_F^2 T^2}{\sigma_{\rm EW}} \times \underset{\sim T^3}{\langle n \rangle} \sim T^5/M_W^4 \lesssim H(t) \quad \Leftrightarrow \quad T \lesssim T_{\nu-\rm dec} = 0.8\text{MeV}$$

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2} \underbrace{\frac{\pi^2}{30} g_*(T) T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}} \right)^{-2} \ \mathrm{s}$$

When do $g_*(T), g_*^{(s)}(T)$ change ?

$$\begin{cases} 1)\ \text{Species Decoupling}\,, \ T \to T_i\,, \\ 2)\ \text{Mass threshold}\,, T < 2m_i\,, \end{cases}$$

Annihilation (mass threshold):

$T < 2m_i \ \Rightarrow$ Can’t produce it anymore

[Boltzman Supression $\sim e^{-m/T}$]

Expansion History

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- -

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[Boltzman Supression $\sim e^{-m/T}$]

Example e+/e- Annihilation

$$e^+ + e^- \leftrightarrow 2\gamma \, , \quad T > 511 \ \mathrm{keV}$$

$$\cancel{e^+ + e^-} \rightarrow 2\gamma \, , \quad T < 511 \ \mathrm{keV}$$

$$T_\gamma(t > t_{e^\pm}) = \left(\frac{g_s^<}{g_s^>}\right)^{1/3} T_\nu = \left(\frac{11}{4}\right)^{1/3} T_\nu$$

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\underbrace{\frac{\pi^2}{30} g_*(T)T^4}_{\rho_R} \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$

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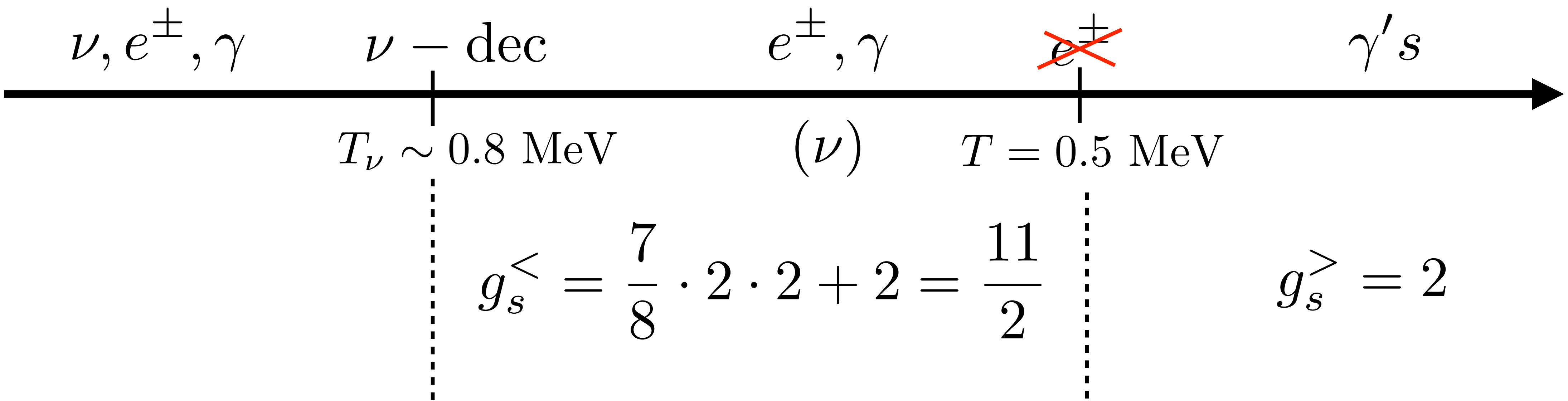
$$= (11/4)^{1/3} T_\nu$$



$\nu, e^\pm, \gamma$ $\quad$ $\nu - \mathrm{dec}$ $\quad$ $e^\pm, \gamma$ $\quad$ $\cancel{e^\pm}$ $\quad$ $\gamma' s$

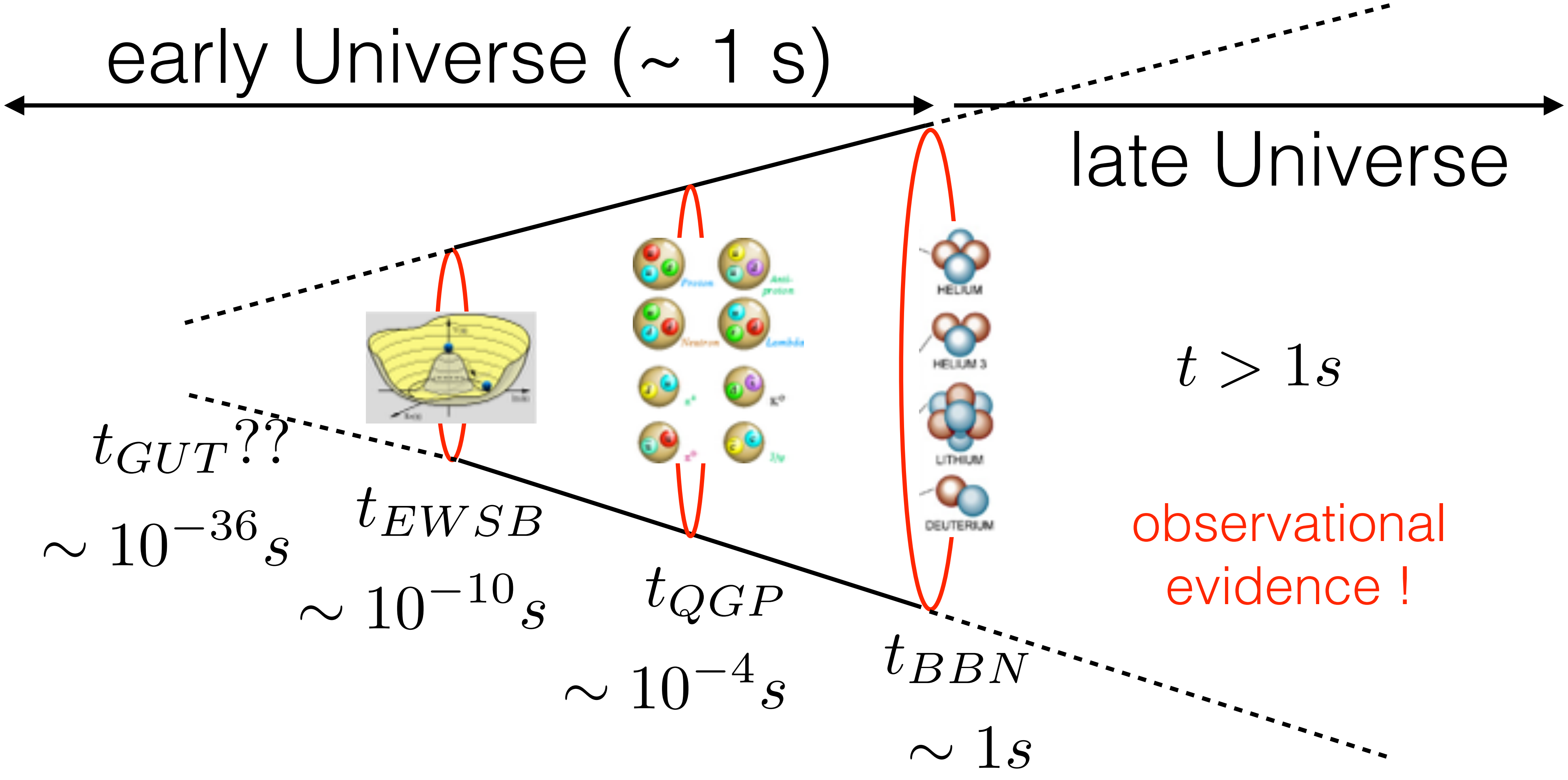
$T_\nu \sim 0.8$ MeV $\quad$ (ν) $\quad$ $T = 0.5$ MeV

$$g_s^< = \frac{7}{8} \cdot 2 \cdot 2 + 2 = \frac{11}{2}$$

$$g_s^> = 2$$

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$



early Universe (~ 1 s)

late Universe

t_{GUT}?? $\sim 10^{-36} s$

t_{EWSB} $\sim 10^{-10} s$

t_{QGP} $\sim 10^{-4} s$

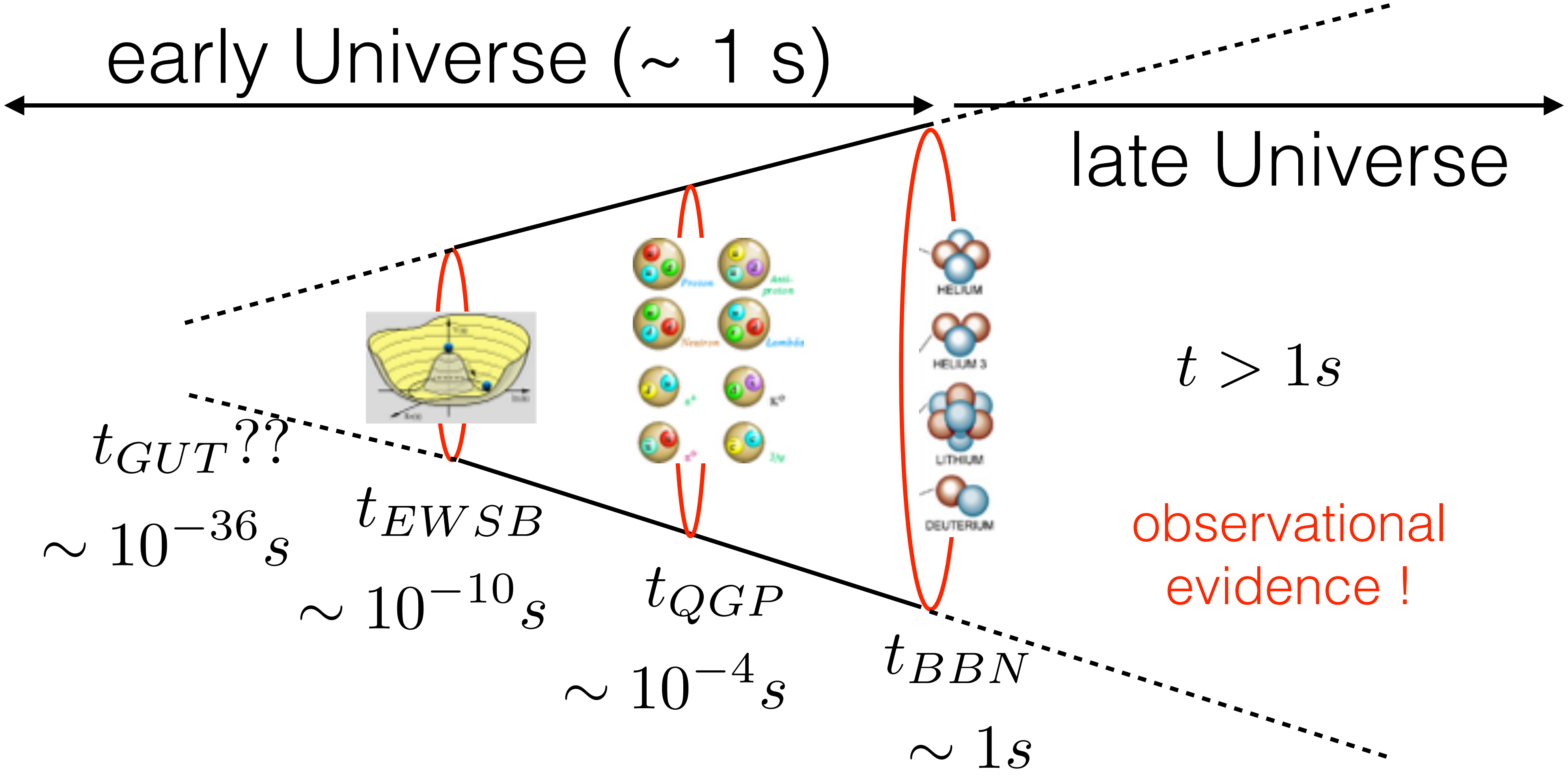
t_{BBN} $\sim 1 s$

$t > 1 s$

observational evidence !

Expansion History

$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$



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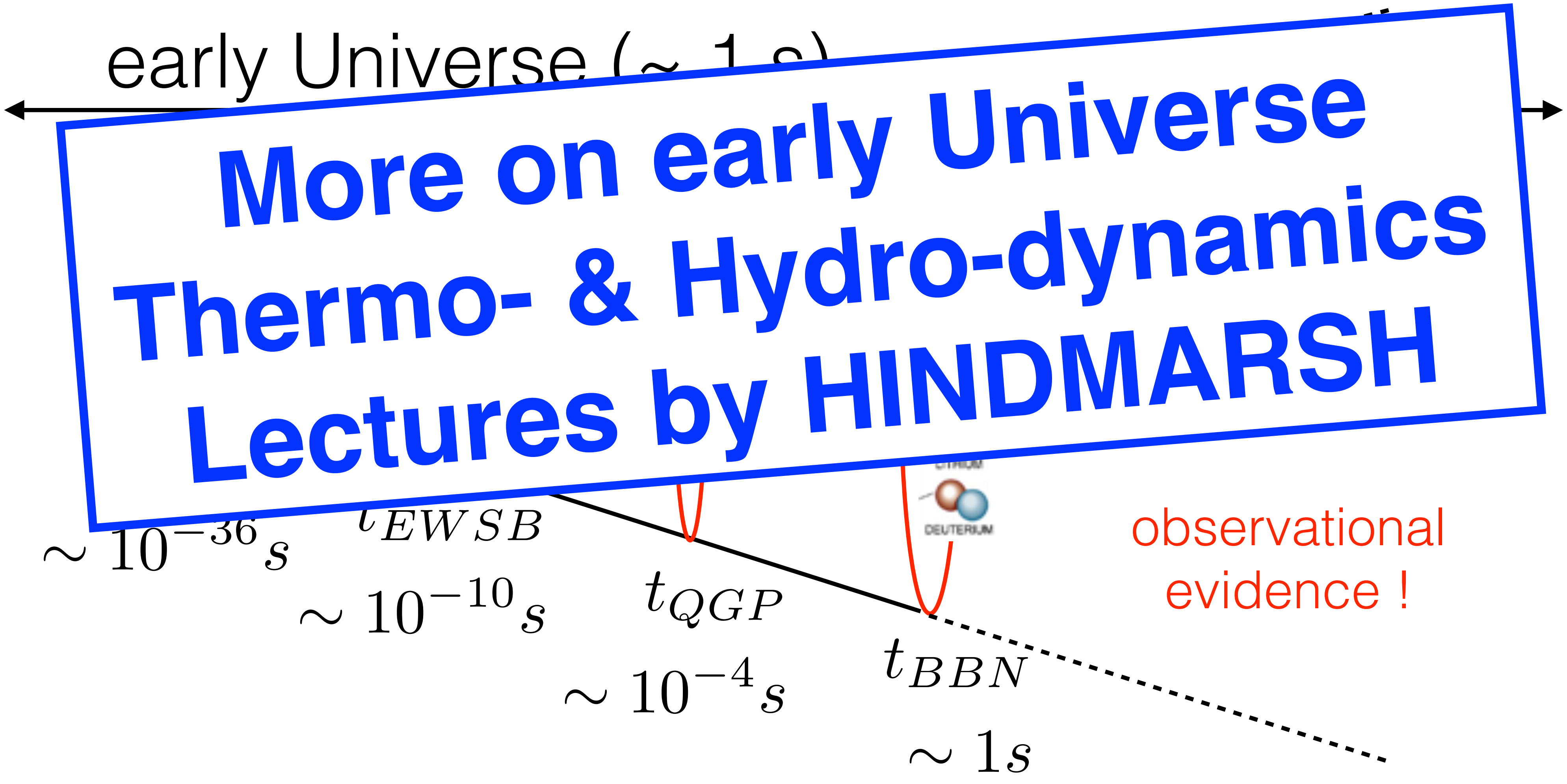
observational evidence !

How can we probe the early Universe ? GWs !

Expansion History

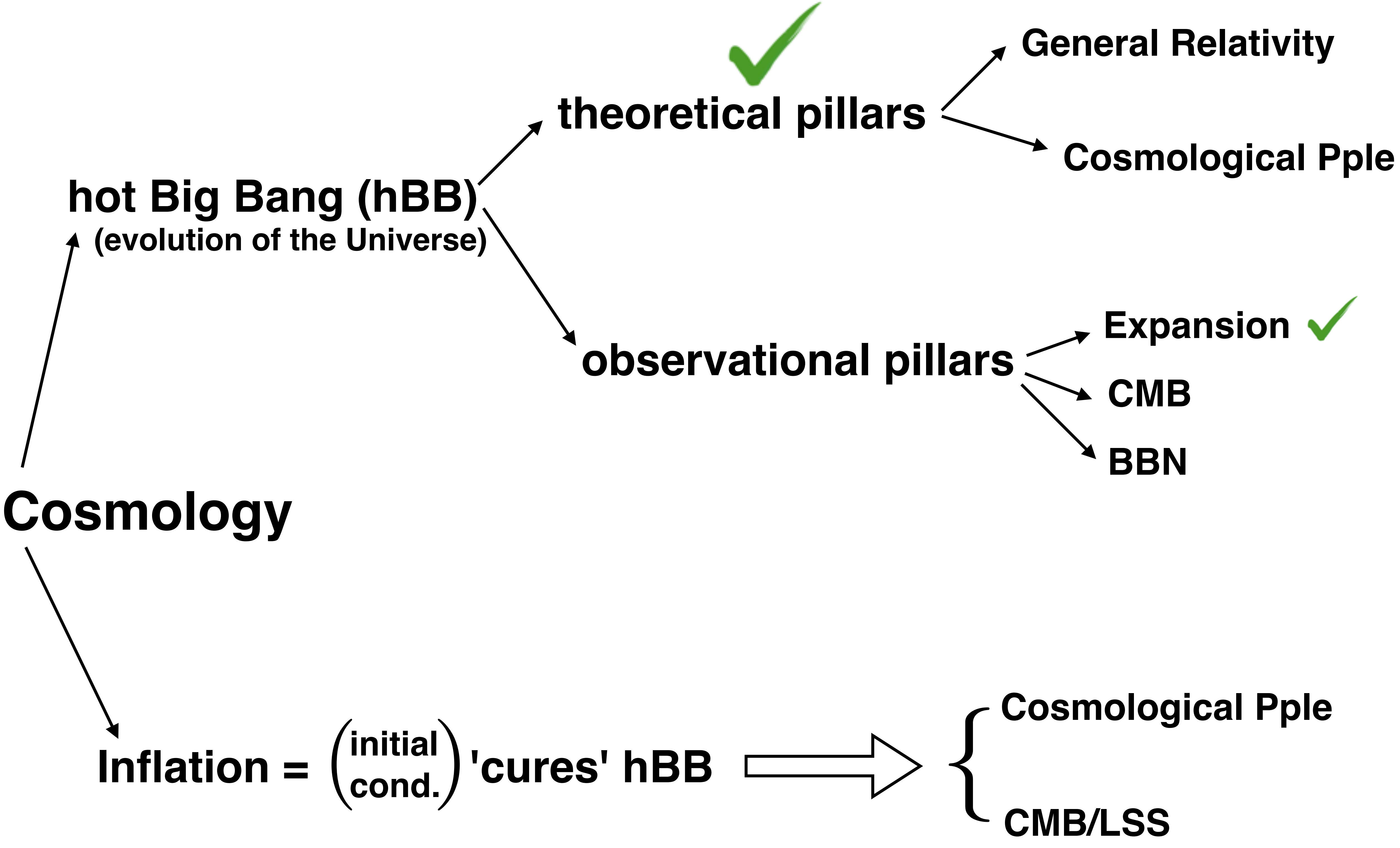
$$H^2 = \frac{1}{(2t)^2} = \frac{1}{3m_p^2}\frac{\pi^2}{30}g_*(T)T^4 \quad \Rightarrow \quad t \sim \left(\frac{T}{\mathrm{MeV}}\right)^{-2} \ \mathrm{s}$$



More on early Universe Thermo- & Hydro-dynamics Lectures by HINDMARSH

How can we probe the early Universe ? GWs !

BASICS of COSMOLOGY



General Relativity

✓ theoretical pillars

Cosmological Pple

hot Big Bang (hBB)
(evolution of the Universe)

Expansion ✓

observational pillars

CMB

BBN

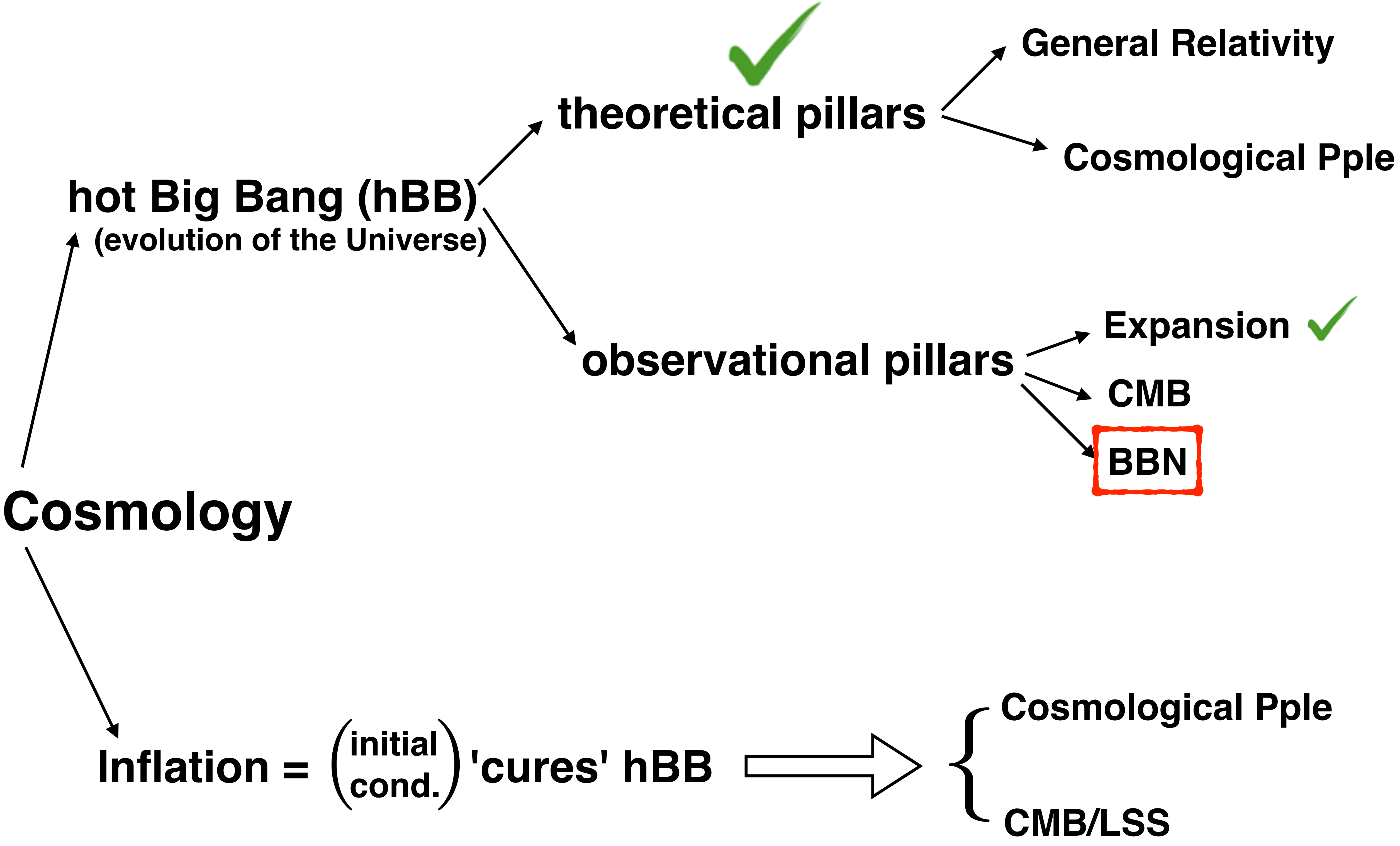
Cosmology

Cosmological Pple

Inflation = $\left(\begin{matrix}\text{initial}\\ \text{cond.}\end{matrix}\right)$ 'cures' hBB $\Longrightarrow$ $\left\{\begin{matrix}\text{Cosmological Pple}\\ \text{CMB/LSS}\end{matrix}\right.$

CMB/LSS

BASICS of COSMOLOGY



Cosmology

hot Big Bang (hBB)
(evolution of the Universe)

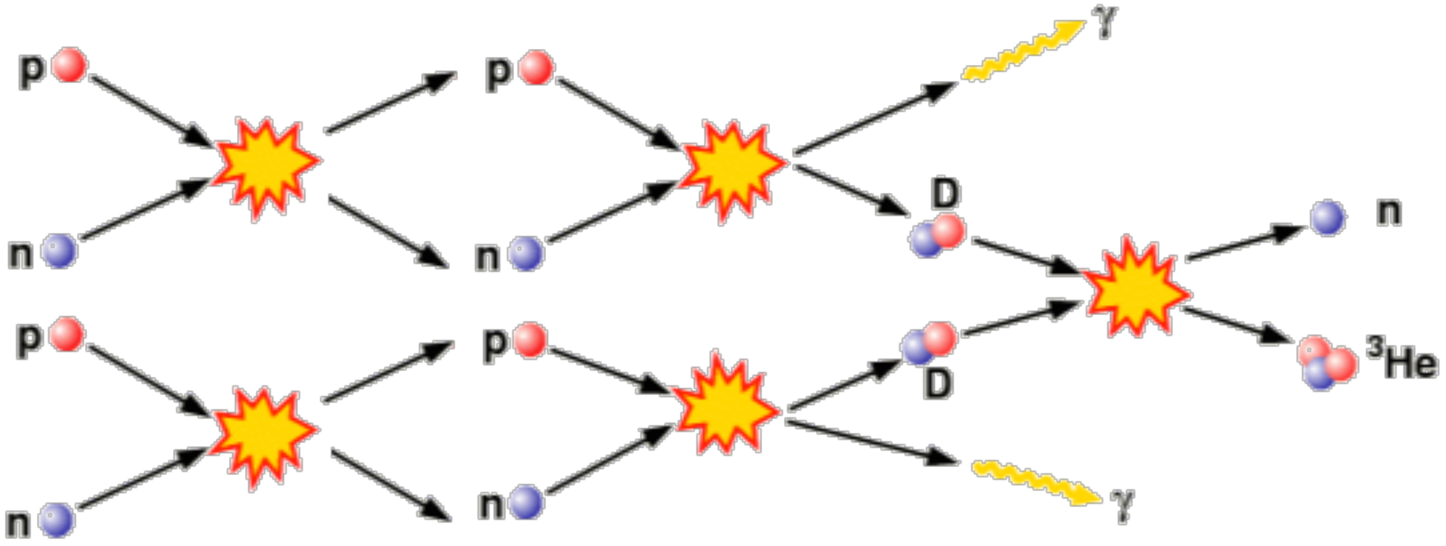
theoretical pillars ✓
- General Relativity
- Cosmological Pple

observational pillars
- Expansion ✓
- CMB
- BBN

Inflation = (initial cond.) 'cures' hBB ⇒
- Cosmological Pple
- CMB/LSS

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



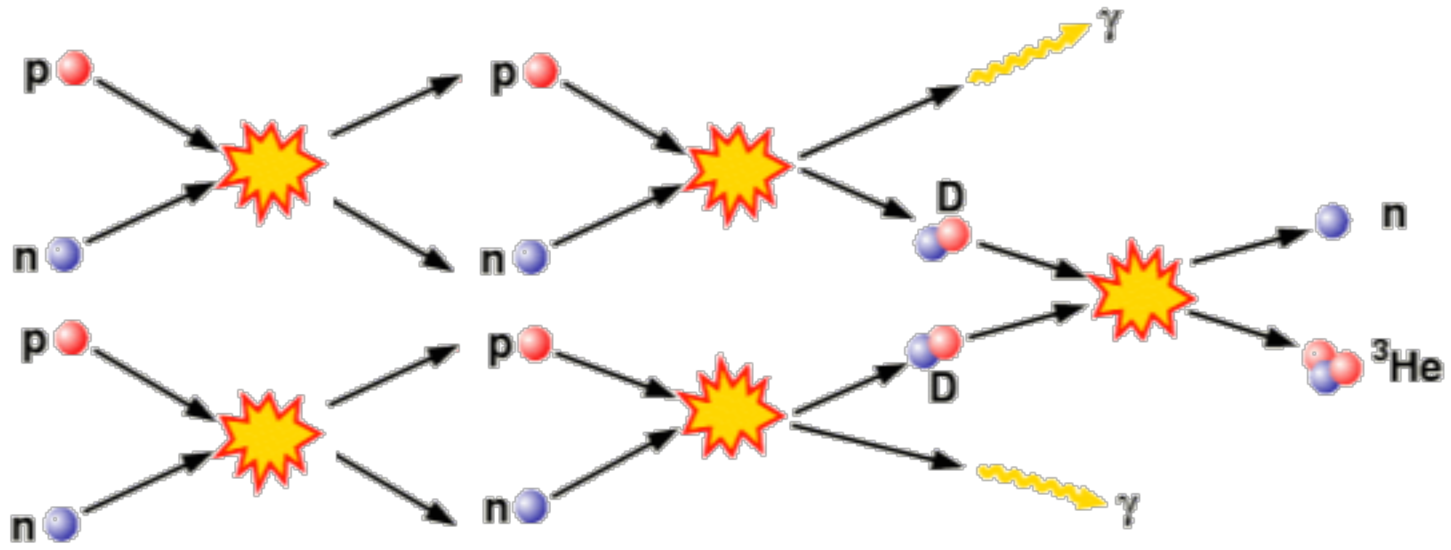
Past,
hotter
(1 second)

Protons,
Neutrons

Interact
strongly

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



Past,
hotter
(1 second)

Protons, Neutrons

Interact strongly

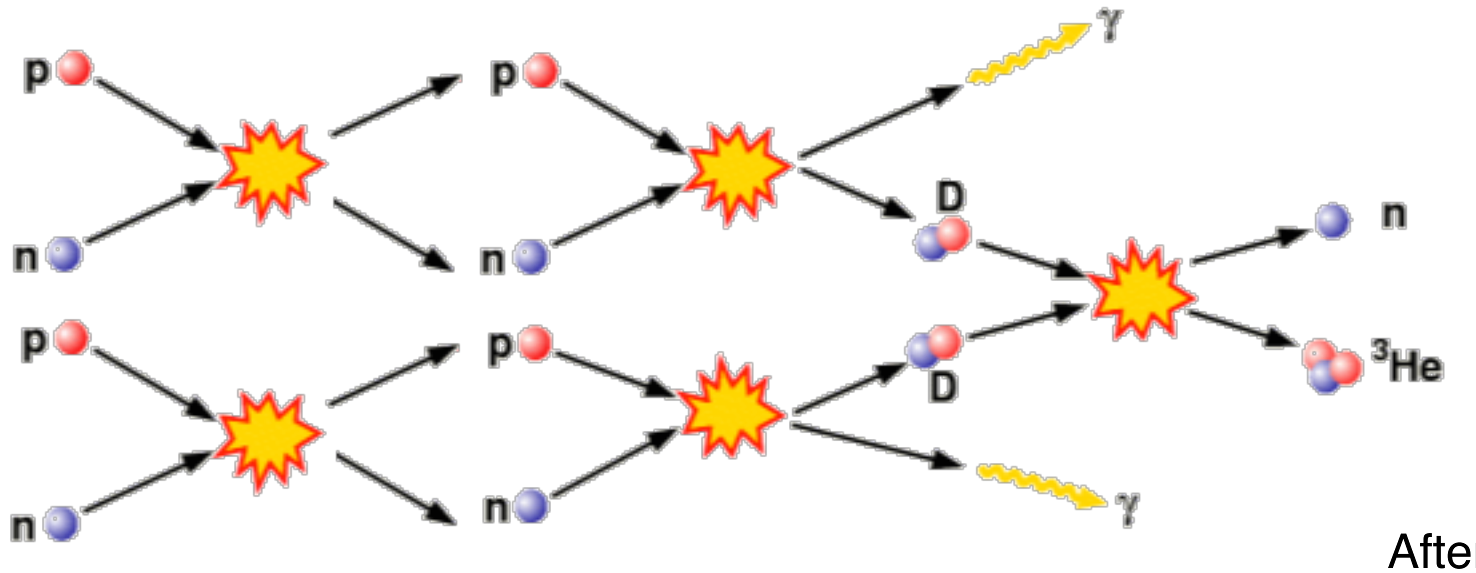
Universe cools down…

… protons and neutrons **don't have sufficient energy anymore**

Then **they join together forming atomic nuclei**: Nuclear Physics!

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



Past,
hotter
(1 second)

Afterwards, colder
(3 minutes)

Protons, Neutrons

Interact strongly

Universe cools down…

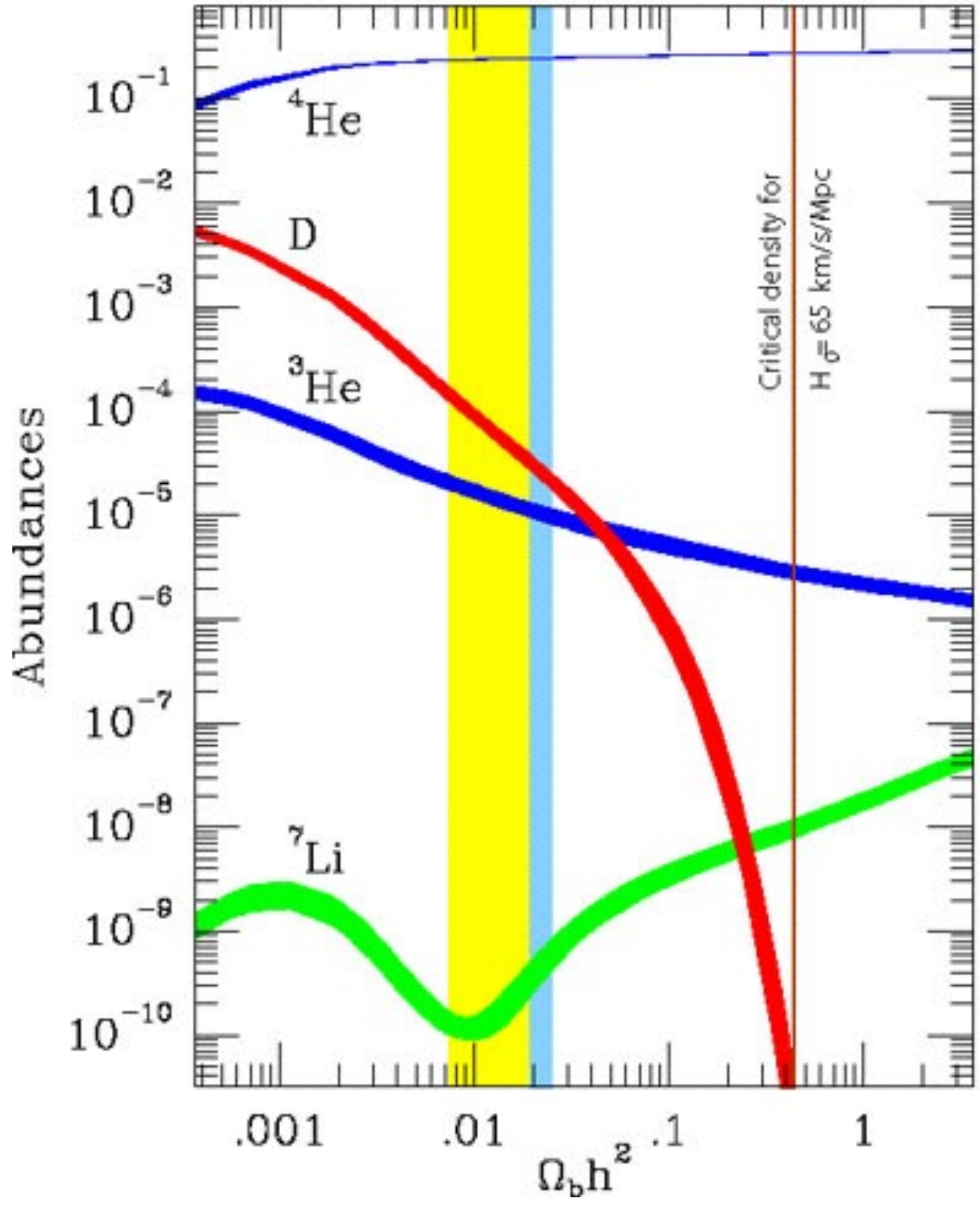
… protons and neutrons **don't have sufficient energy anymore**

Then **they join together forming atomic nuclei**: Nuclear Physics!

Atomic Nuclei created !

Big Bang Nucleosynthesis

Formation of atomic nuclei (1s - 3 mins)



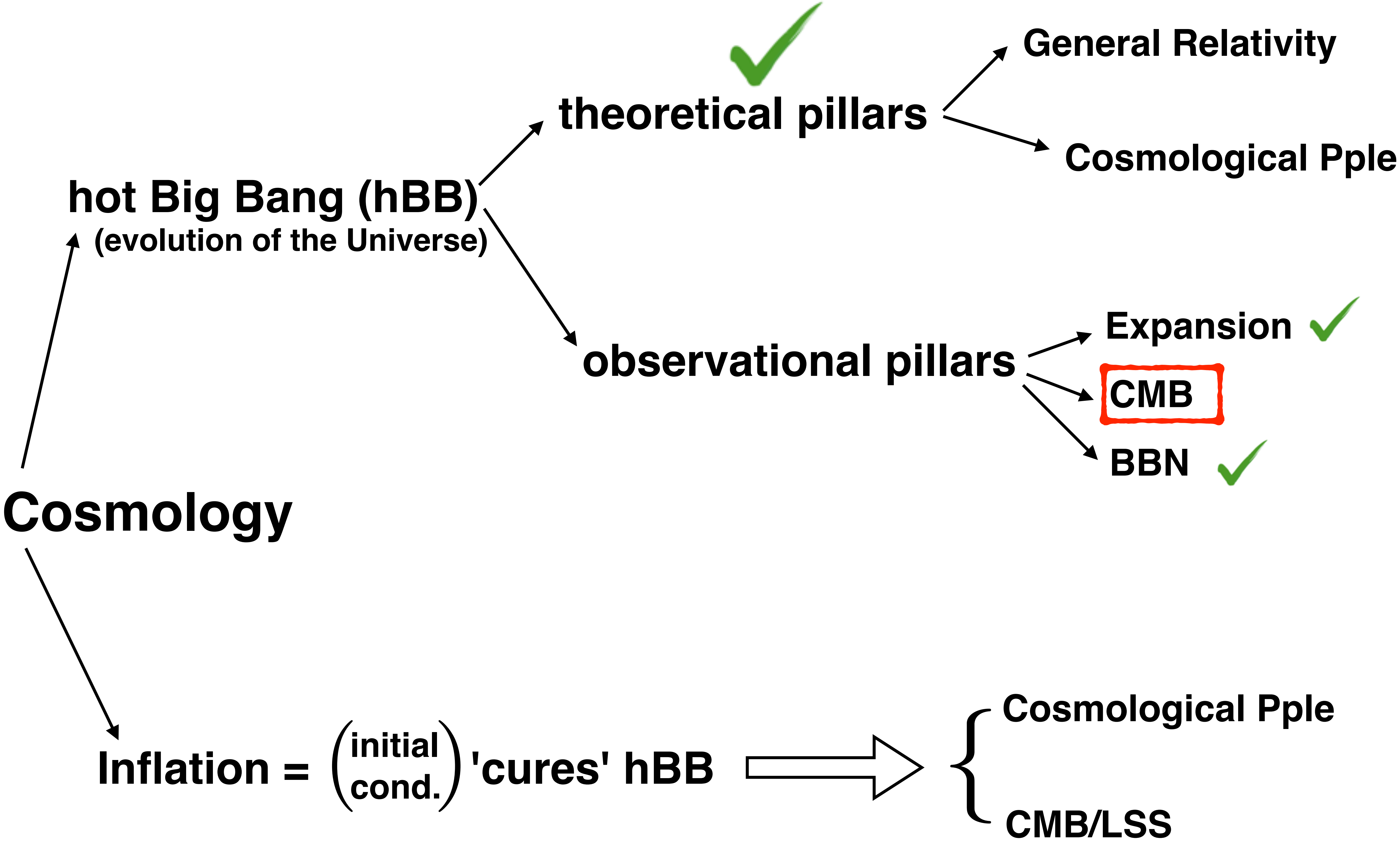
NUCLEAR PHYSICS
(measured in the lab)

Leads to predict abundances of

$$H,\ {}^4He,\ D,\ {}^3He,\ {}^7Li, \ldots$$

Another definitive proof of hot Big Bang framework !

BASICS of COSMOLOGY



✓ theoretical pillars
- General Relativity
- Cosmological Pple

hot Big Bang (hBB)
(evolution of the Universe)

observational pillars
- Expansion ✓
- CMB
- BBN ✓

Cosmology

Inflation = (initial cond.) 'cures' hBB ⇒
- Cosmological Pple
- CMB/LSS

Recombination & release of Cosmic Microwave Background (CMB)

Protons,
Electrons,
Photons

$$e^-,\ p + \ \longleftrightarrow\ \gamma$$

Interact electro-magnetically

Thomson Scattering

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{(4\pi)^2 m^2}$$

Recombination & release of Cosmic Microwave Background (CMB)

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$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{(4\pi)^2 m^2}$$

$$m_e \sim 10^{-3} m_p \ \Rightarrow \sigma_T^{(e)} \sim 10^6 \sigma_T^{(p)}$$

Only electro-photon scattering matters !

Recombination & release of Cosmic Microwave Background (CMB)

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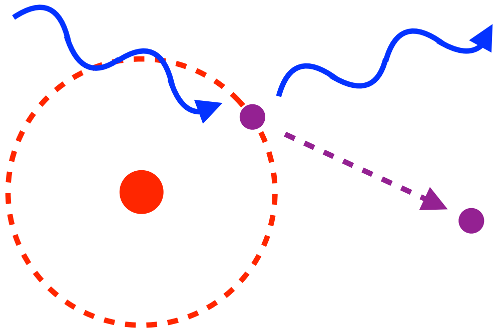

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$$X_e \equiv \frac{n_e}{n_H}$$

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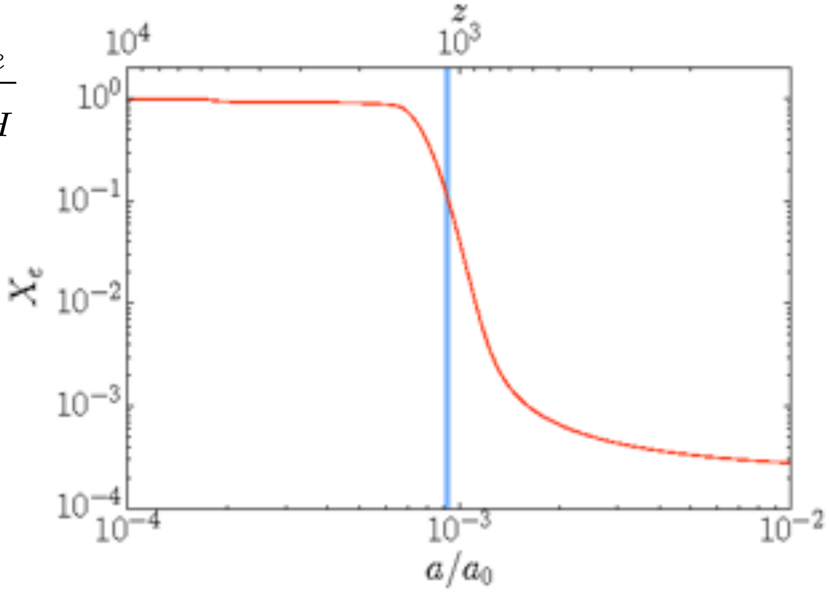

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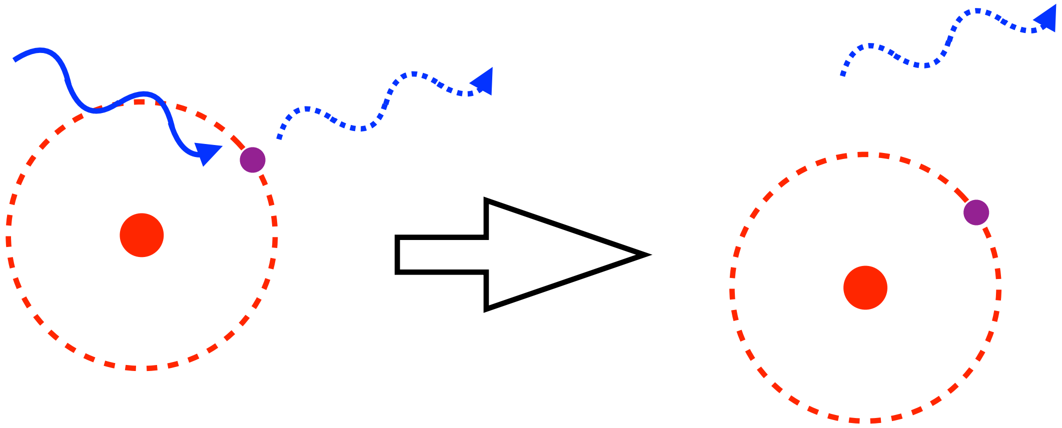
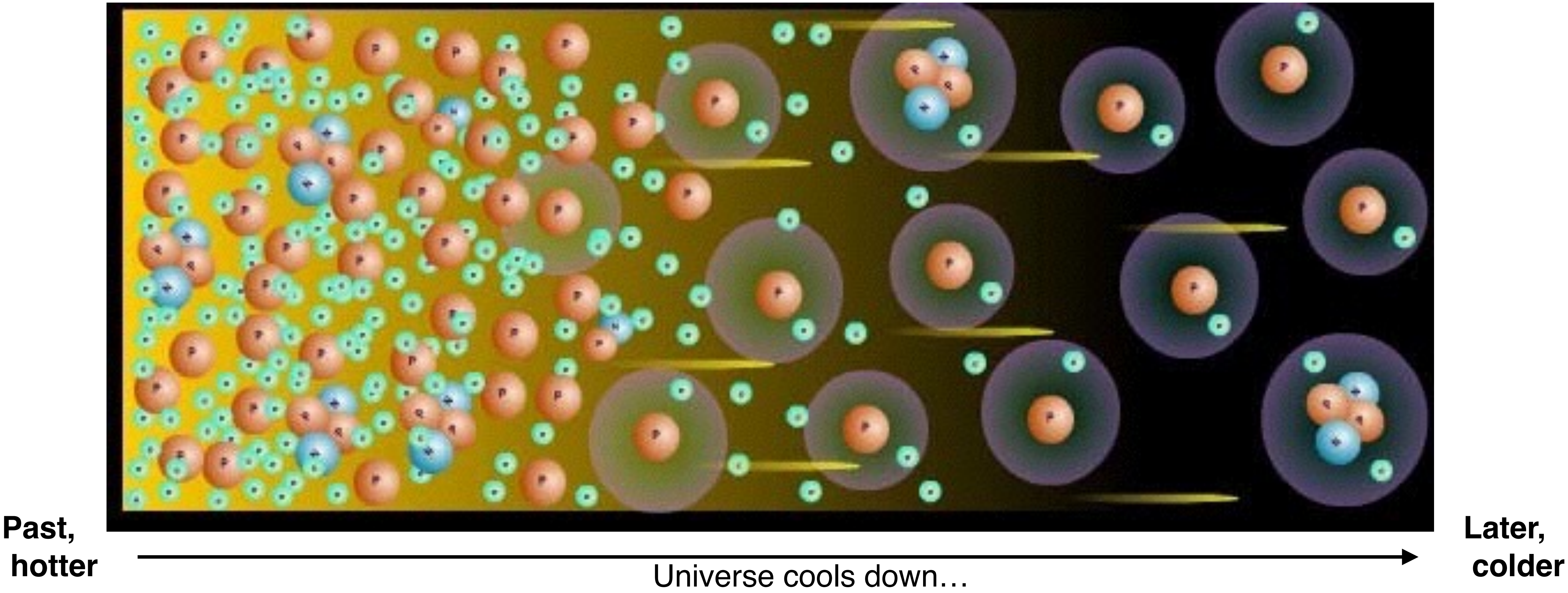


Recombination & release of Cosmic Microwave Background (CMB)



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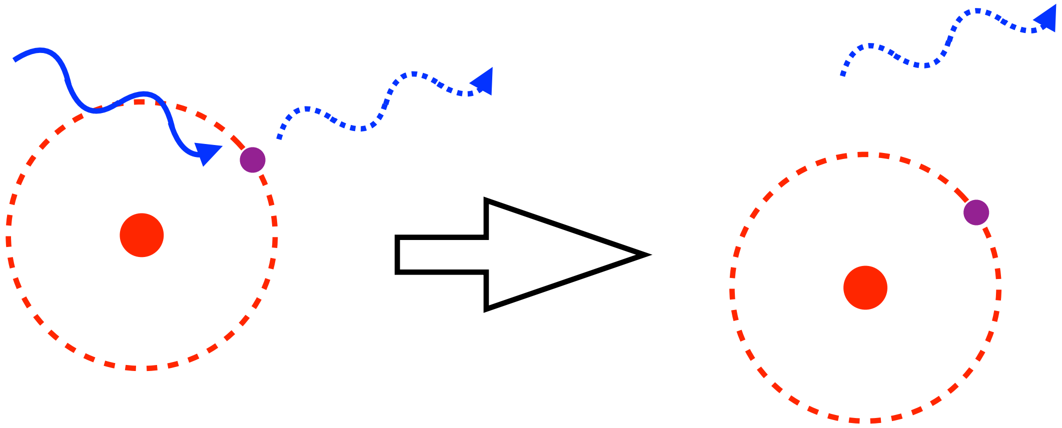

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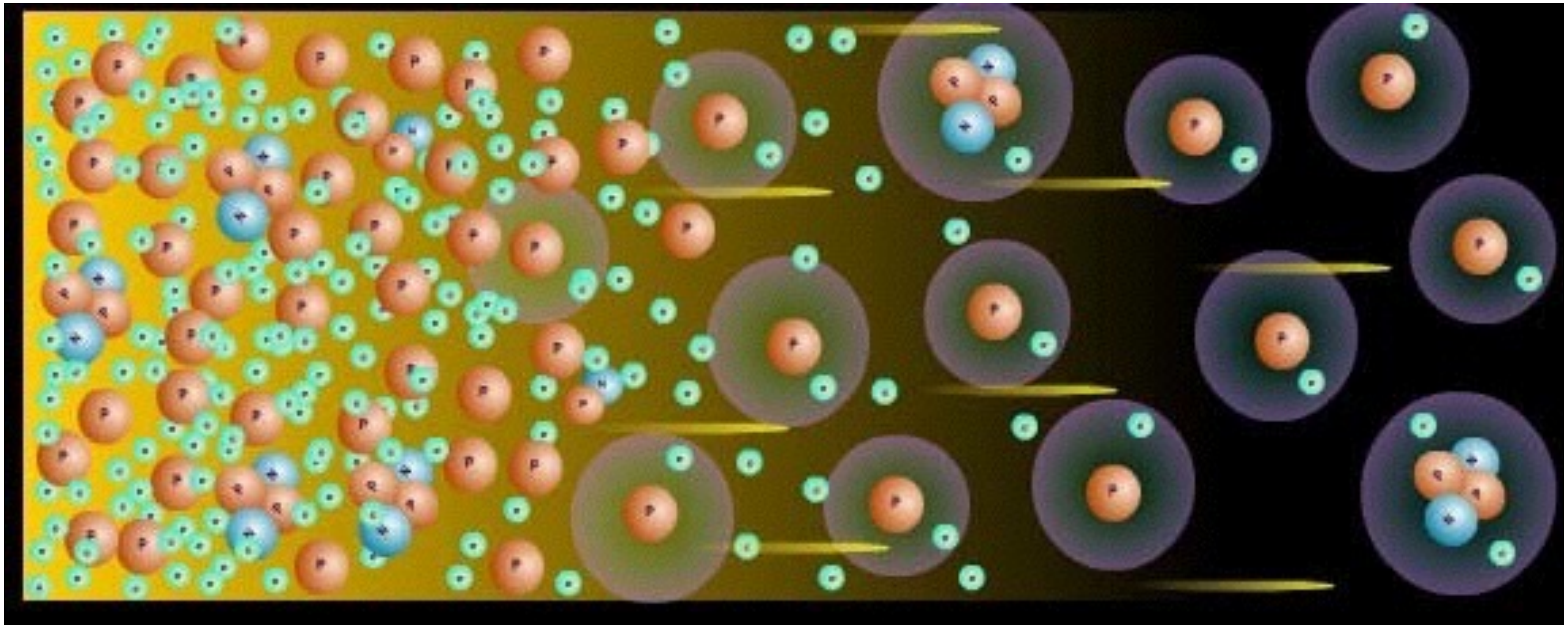


Past, hotter

Universe cools down…

Later, colder

Protons,
Electrons,
Photons

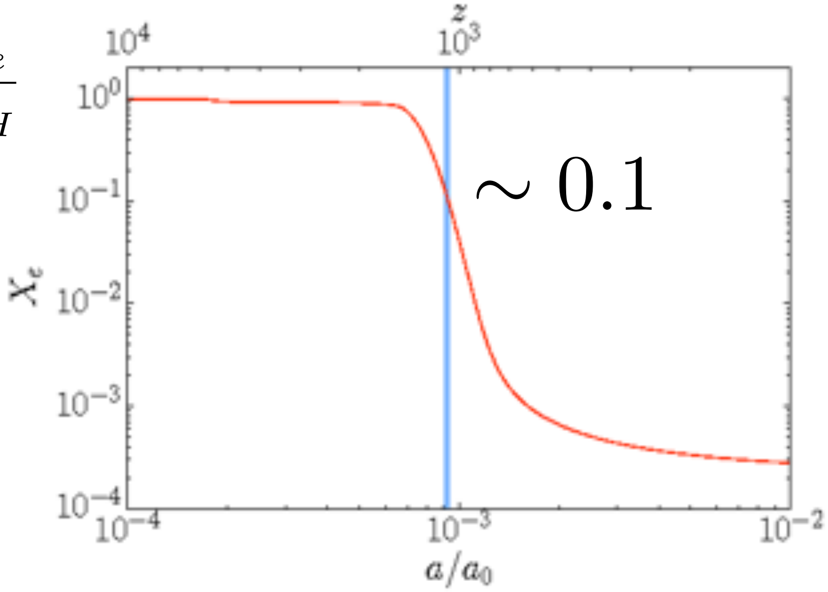
$$e^-, \; p+ \quad \longleftrightarrow \quad \gamma$$

Interact
electro-
magnetically

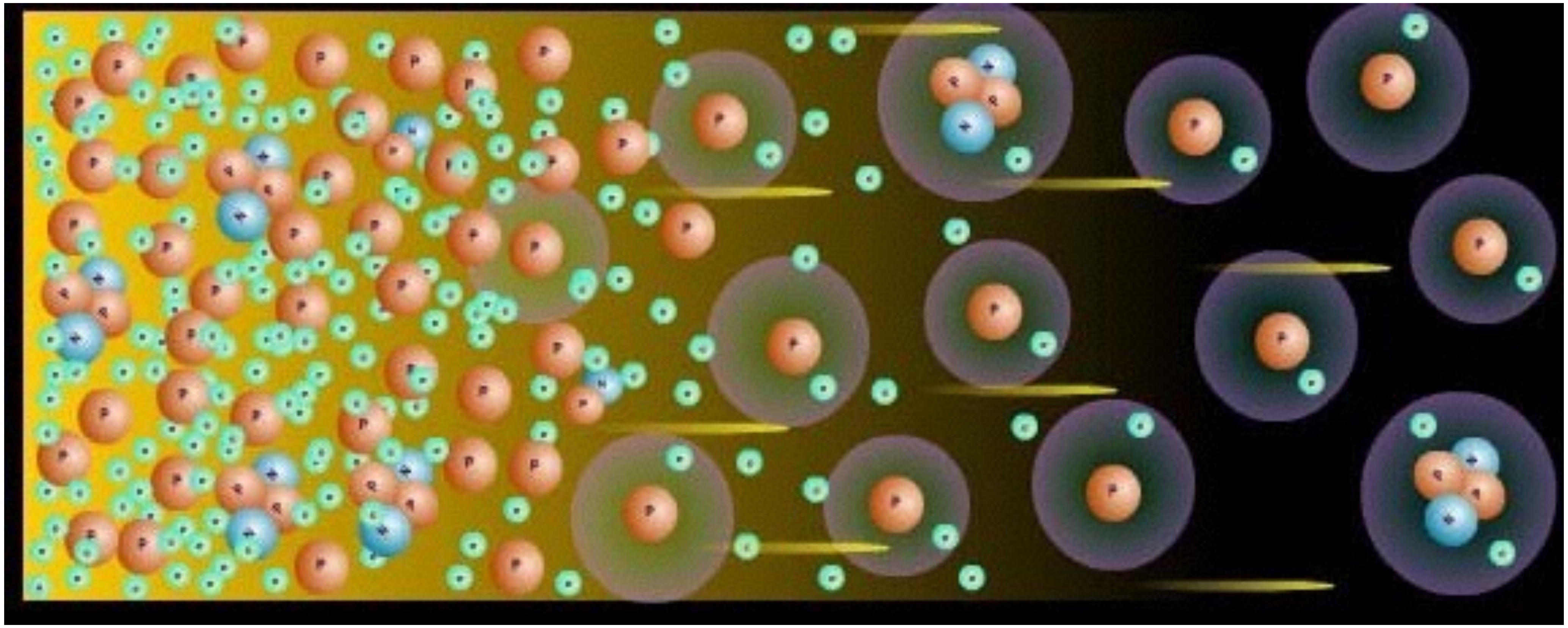
Thomson Scattering

$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{(4\pi)^2 m^2}$$

$$X_e \equiv \frac{n_e}{n_H}$$



Recombination & release of Cosmic Microwave Background (CMB)



Past, hotter

Universe cools down…

Later, colder

Protons, Electrons, Photons

$$e^-, \; p+ \quad \longleftrightarrow \quad \gamma$$

$$X_e \equiv \frac{n_e}{n_H} \sim 0.1$$

Moment of Recombination

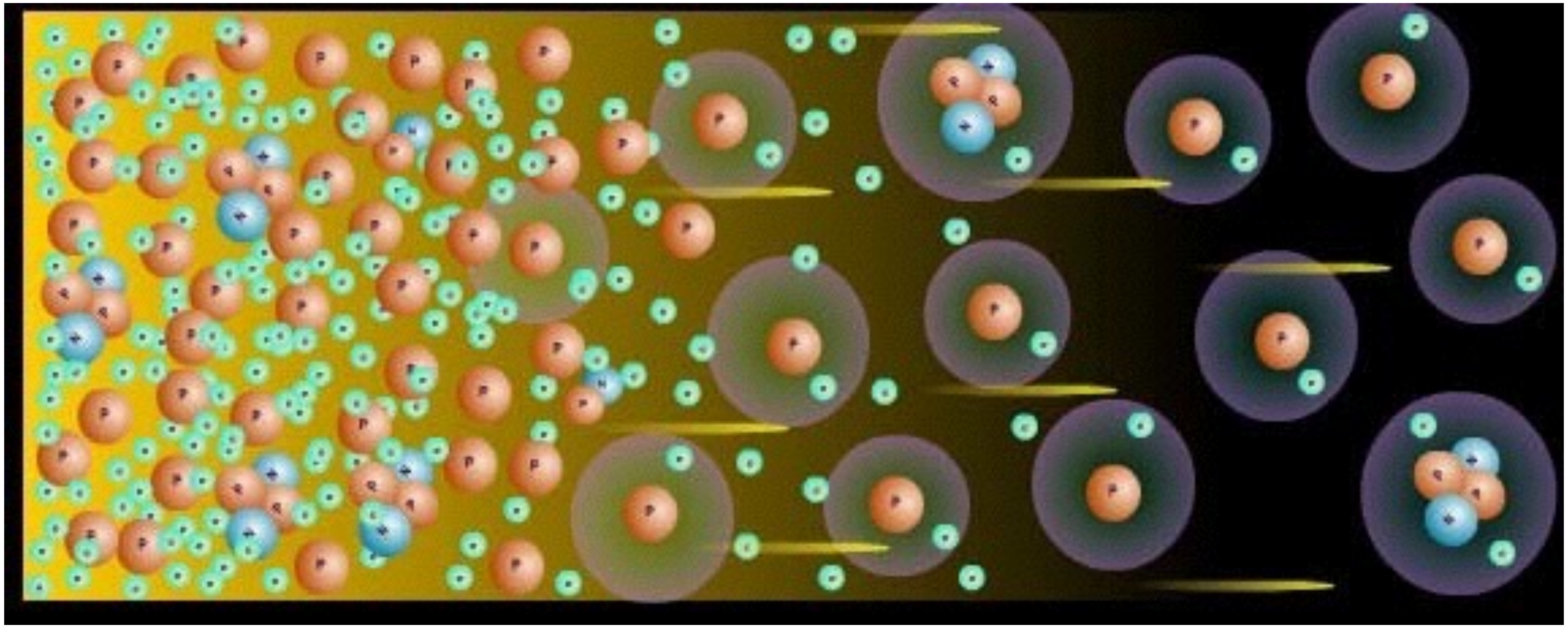
Interact electro-magnetically

Thomson Scattering

$$\sigma_T = \frac{8\pi}{3} \frac{e^4}{(4\pi)^2 m^2}$$

$$z_{\rm rec} \simeq 1330 \quad (T_{\rm rec} \sim 0.31 eV)$$

Recombination & release of Cosmic Microwave Background (CMB)



Past, hotter

Universe cools down…

Later, colder

Protons,
Electrons,
Photons

$$e^{-},\ p+\ \longleftrightarrow\ \gamma$$

Interact electro-magnetically

Thomson Scattering

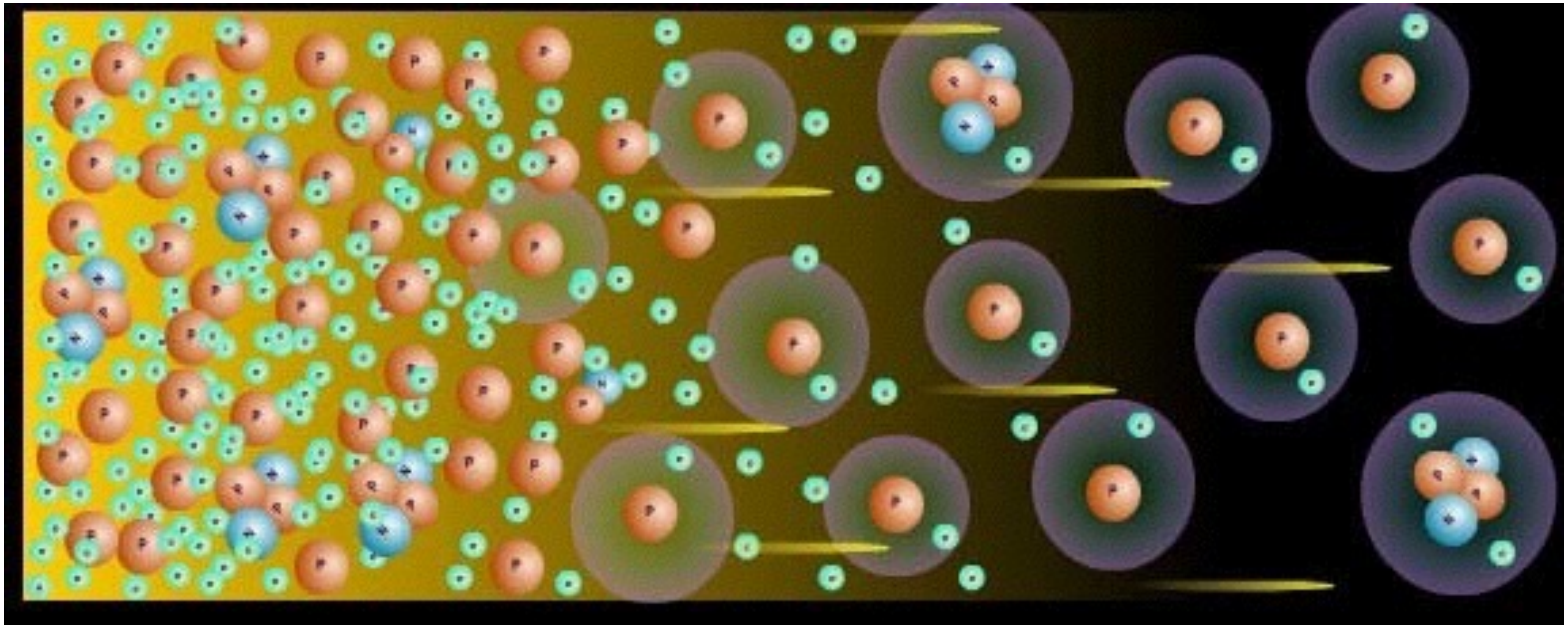
$$\sigma_T = \frac{8\pi}{3}\frac{e^4}{(4\pi)^2 m^2}$$

Consequence of all this ?

Atoms form (first time) !

Photon background freed !

Recombination & release of Cosmic Microwave Background (CMB)



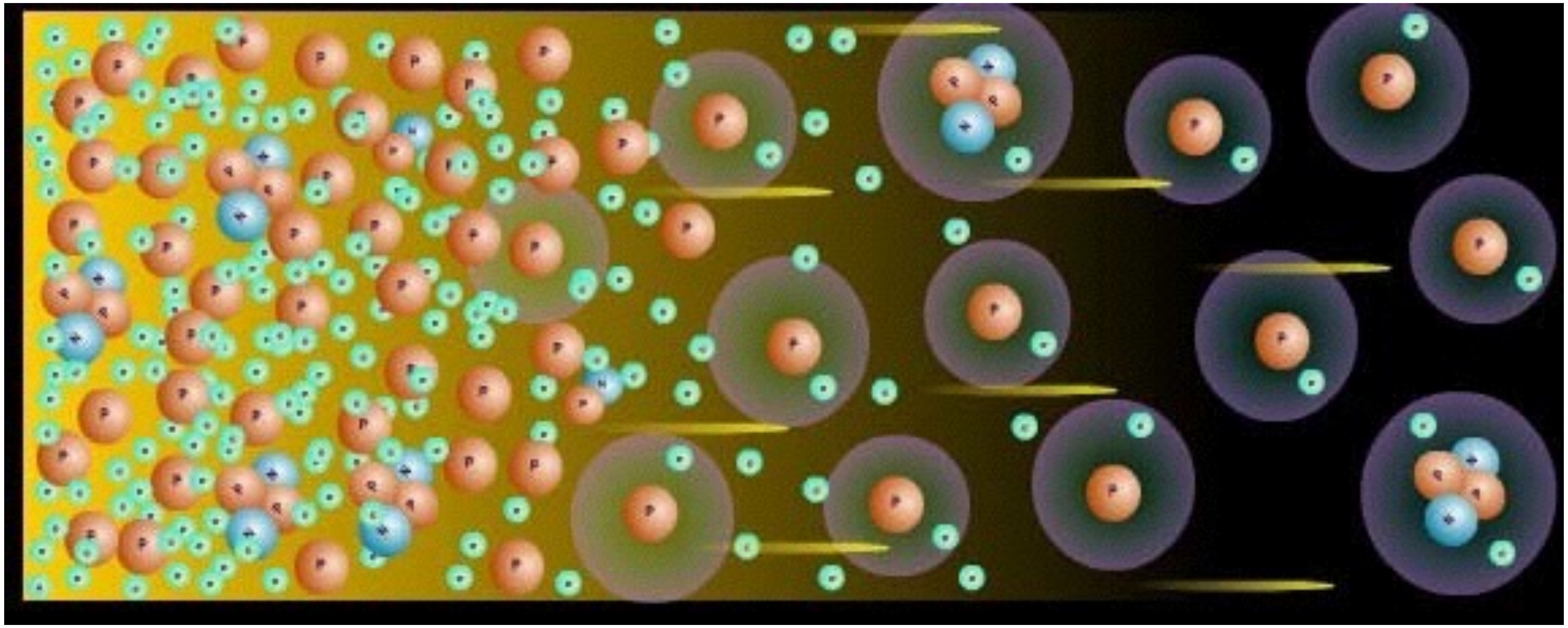
Past, hotter

Universe cools down…

Later, colder

Photon background set free ! ⟶ Origin of CMB !

Recombination & release of Cosmic Microwave Background (CMB)



Past, hotter

Universe cools down…

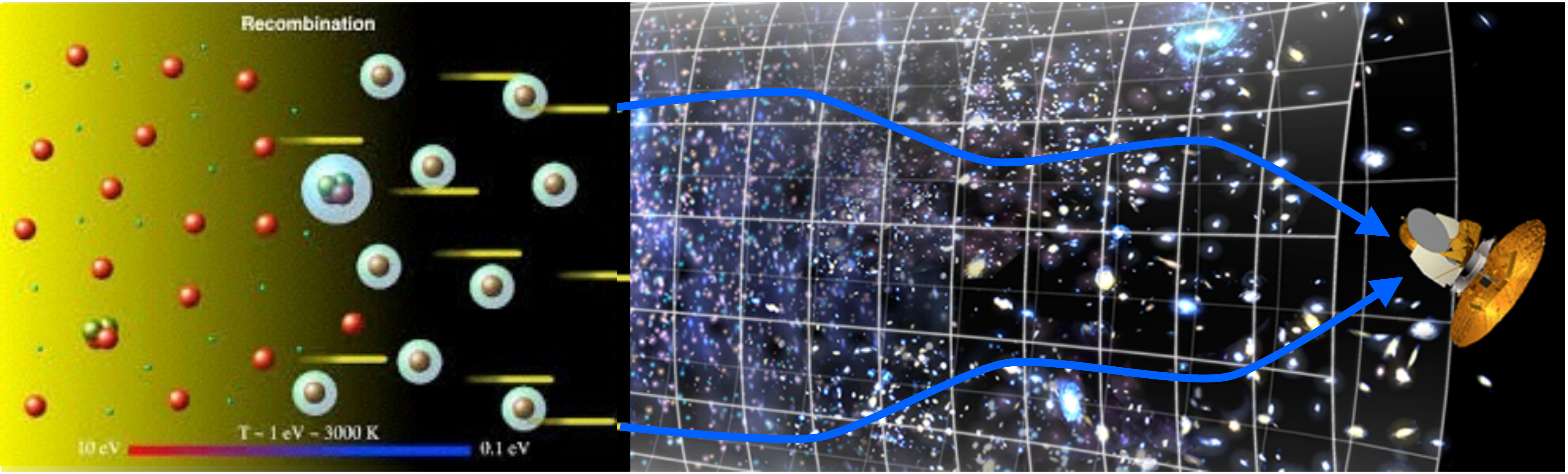
Later, colder

Photon background set free ! ⟶ Origin of CMB !

$$\Gamma_\gamma = \sigma_T^{(e)} \langle n_e v \rangle = \sigma_T^{(e)} X_e \eta_b n_\gamma \leq H(T) \quad \Leftrightarrow \quad T \leq T_{\rm dec}^{(\gamma)} = 0.26 \text{ eV}$$

$$[\text{at } z_{\rm dec} = 1100]$$

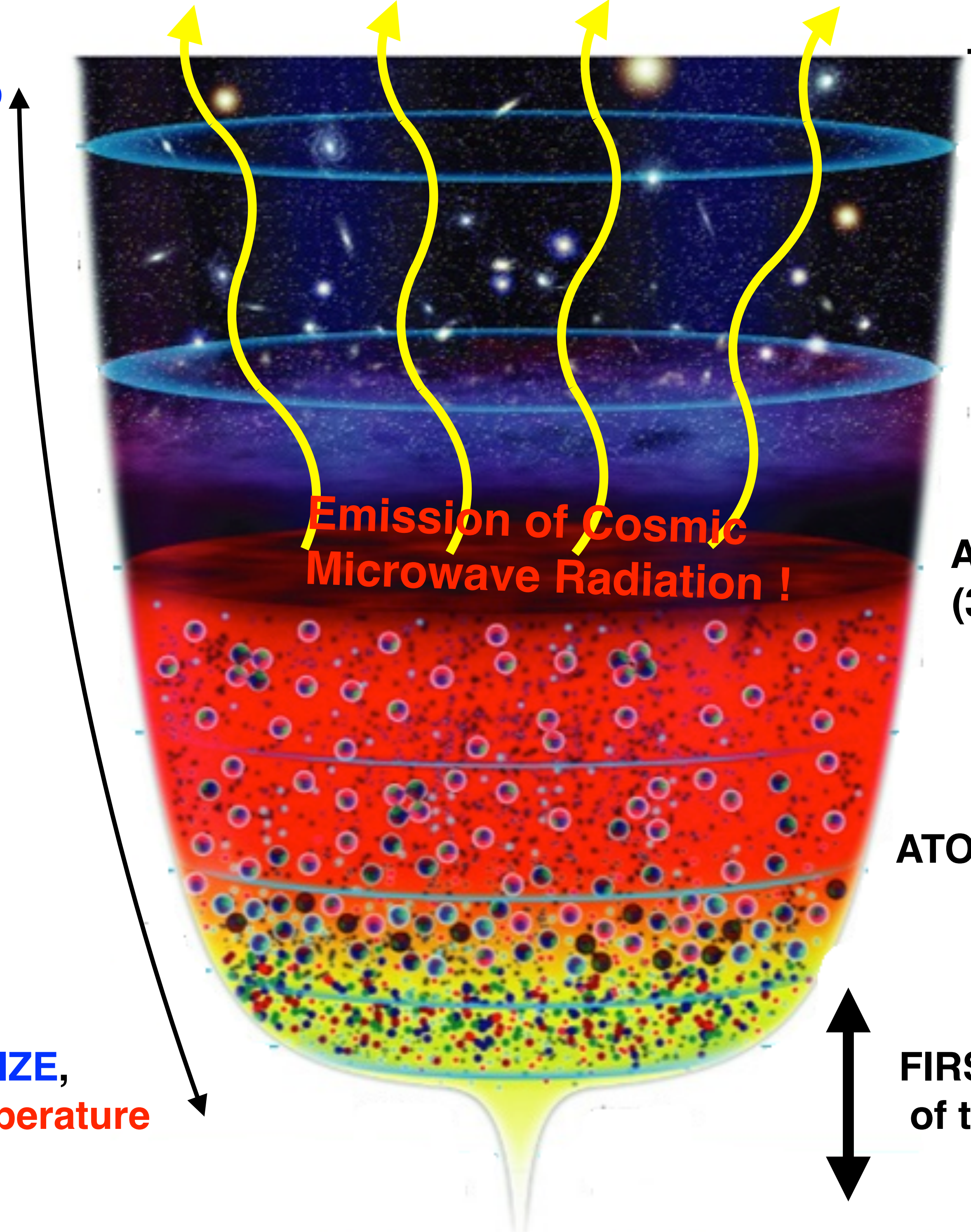
Recombination & release of Cosmic Microwave Background (CMB)



Atom Formation: Free propagation of light !
(Recombination)

Cosmic Microwave Background (CMB)

BiGGER size,
SMALLER Temp



TODAY [Galaxies, Clusters, …]
(13.700 Million years)

FIRST GALAXIES
(500 Millions years)

ATOMS FORMATIO
(300.000-400.000 ye

Definitive proof of the
Big Bang Framework!

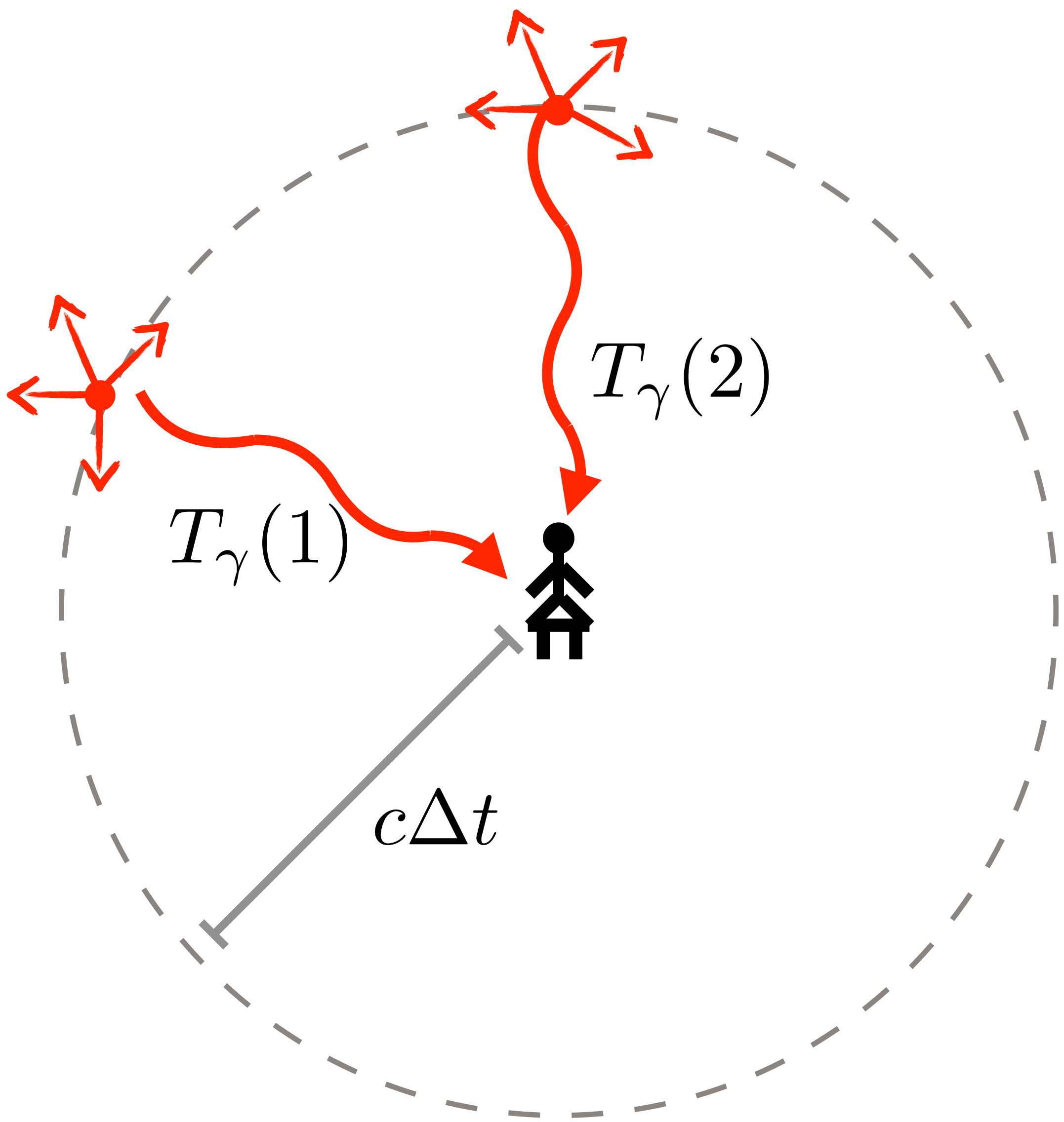
ATOMIC NUCLEI FORMATION
(3 minutes !!!)

FIRST SECOND
of the UNIVERSE !!

SMALLER SIZE,
LARGER Temperature

Cosmic Microwave Background (CMB)

¿ Where is that light? Everywhere!



Cosmic Microwave Background (CMB)

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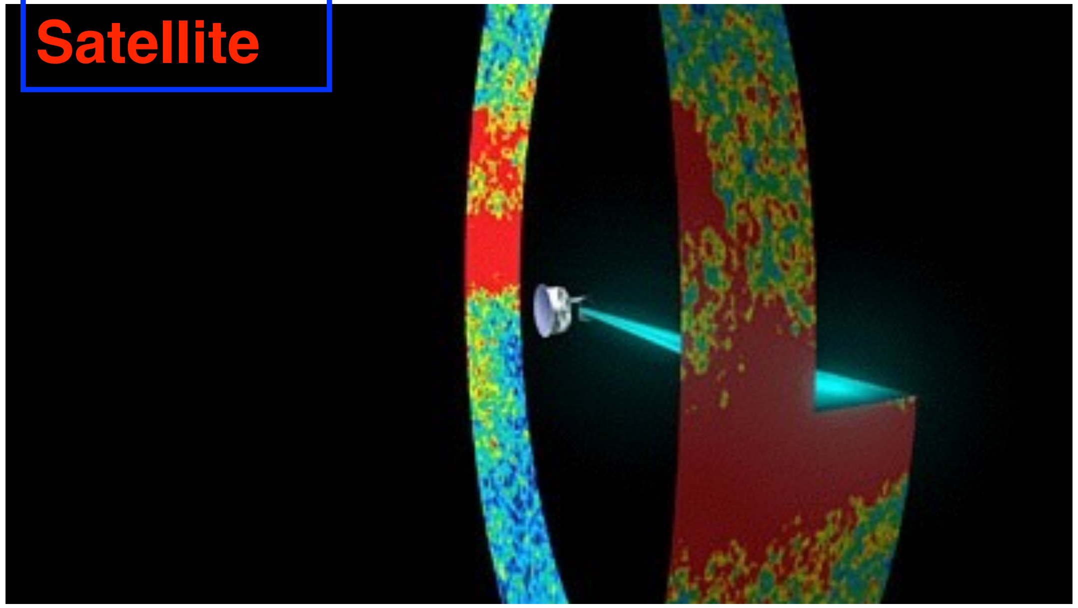
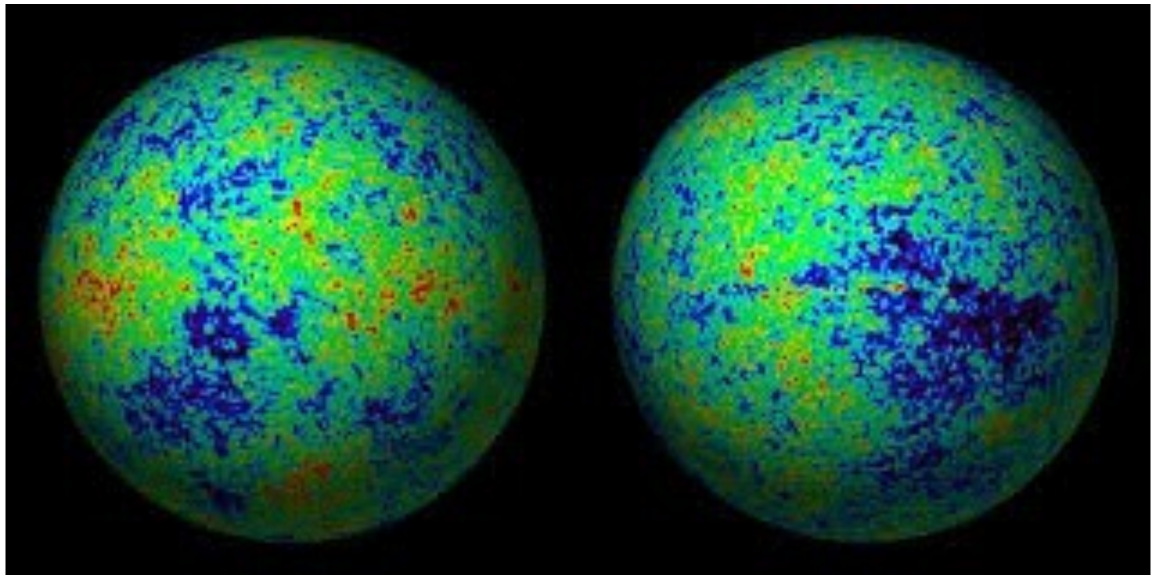


PLANCK Satellite

Cosmic Microwave Background (CMB)

Definitive proof of the Big Bang Framework!

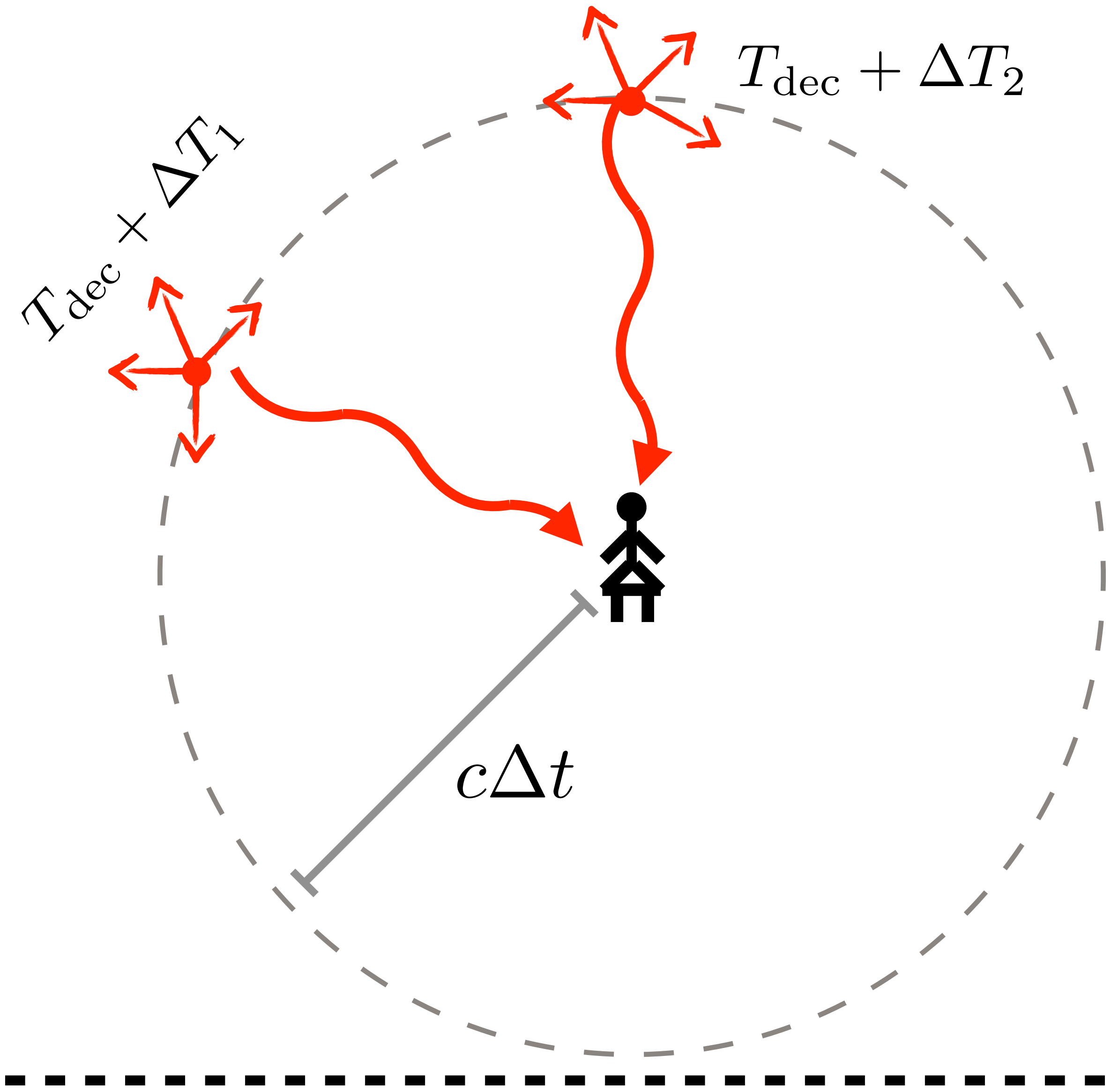


(almost-)ISOTROPIC

But ….

There are small ‘Anisotropies’ (variations of 1/100.000 only !)

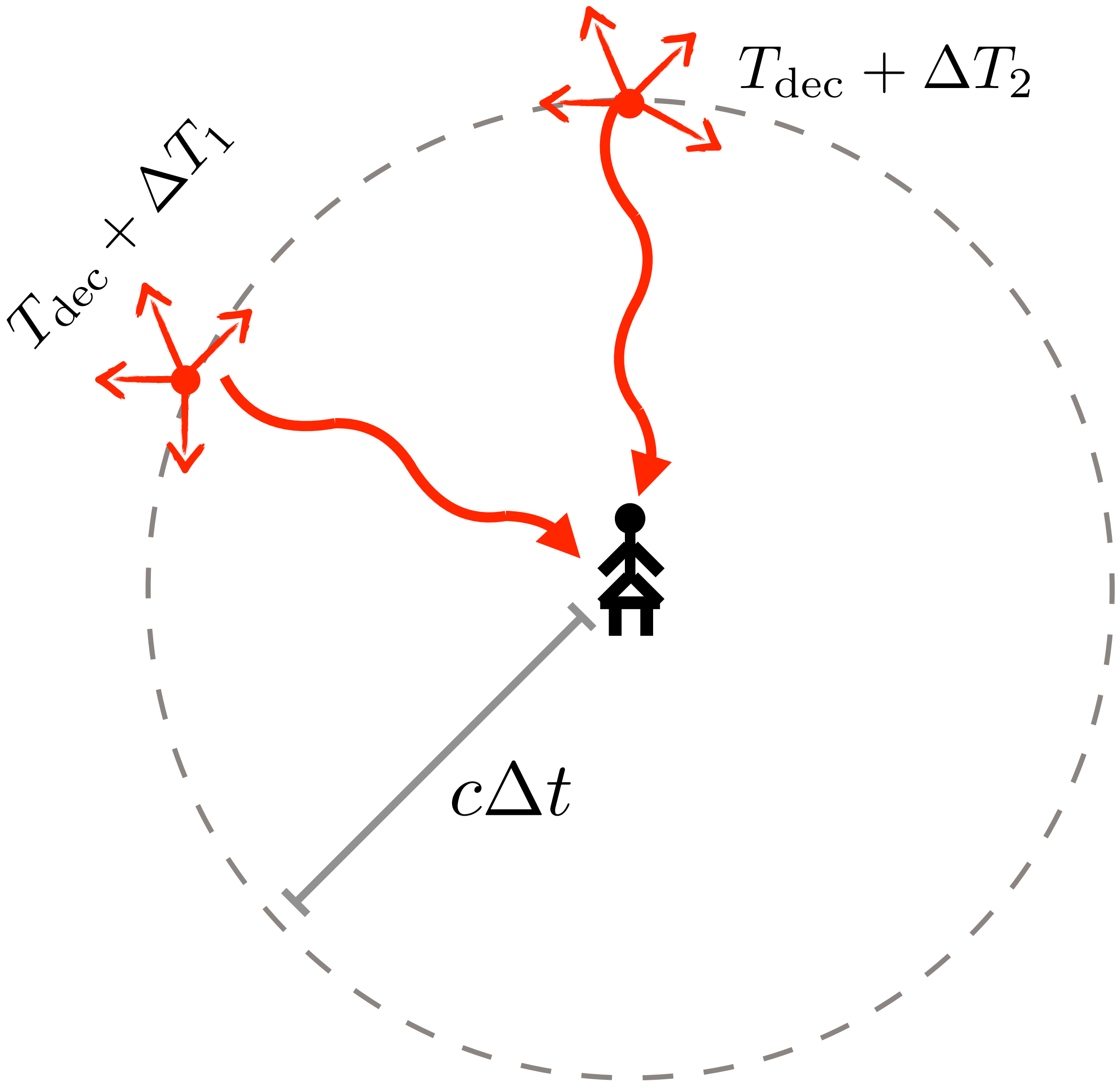
Cosmic Microwave Background (CMB)



$$T_{\text{dec}} + \Delta T \quad \Leftrightarrow \quad \rho_\gamma (1 + \delta\rho_\gamma)$$

$$\Delta T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}_{\text{CMB}}}{\bar{T}_{\text{CMB}}}$$

Cosmic Microwave Background (CMB)



$$T_{\rm dec} + \Delta T \quad \Leftrightarrow \quad \rho_\gamma(1 + \delta\rho_\gamma)$$

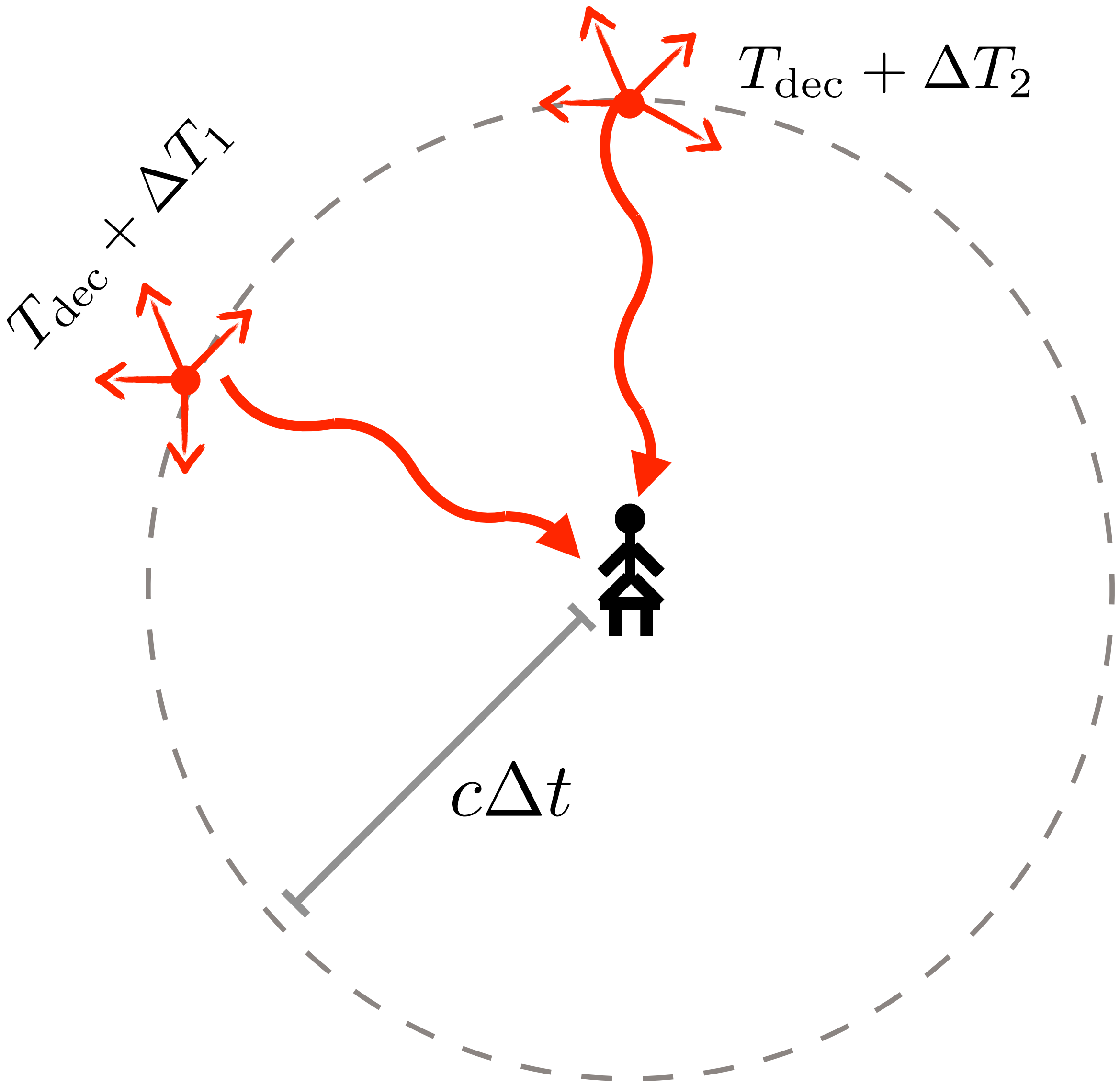
$$\Delta T(\hat{n}) \equiv \frac{T(\hat{n}) - \bar{T}_{\rm CMB}}{\bar{T}_{\rm CMB}}$$

$$\Delta T(\hat{n}) = \sum_{l=1}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\hat{n})$$

$$a_{lm} \equiv \int d\hat{\Omega} Y^*_{lm}(\hat{n}) \Delta T(\hat{n})$$

$$\langle \Delta T(\hat{n}) \rangle_{4\pi} = 0 \ \Rightarrow \ \langle a_{lm} \rangle_{4\pi} = 0$$

Cosmic Microwave Background (CMB)



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$$\langle \Delta T(\hat{n}) \rangle_{4\pi} = 0 \;\; \Rightarrow \;\; \langle a_{lm} \rangle_{4\pi} = 0$$

However…

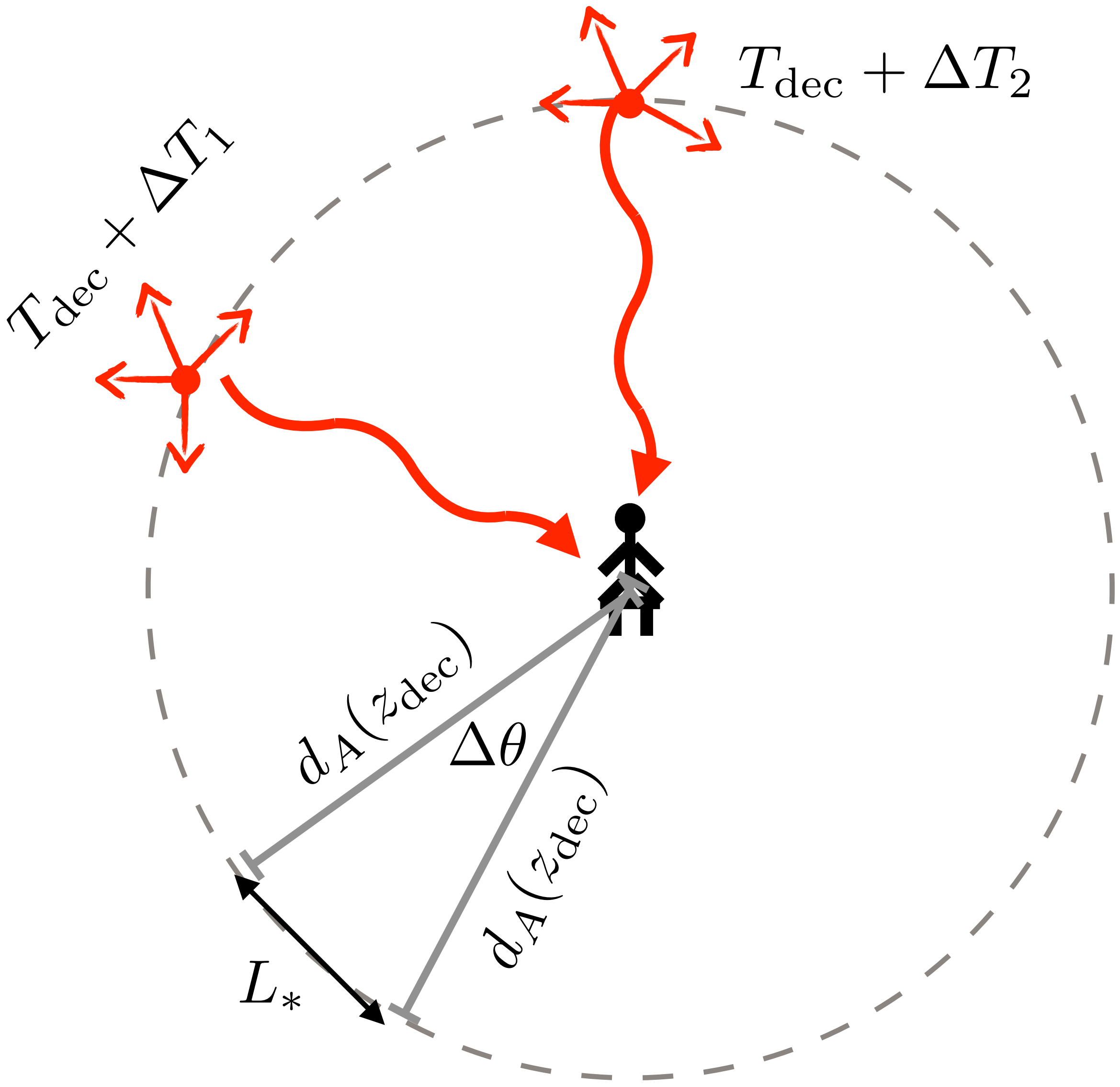
$$\langle \Delta T(\hat{n}) T(\hat{m}) \rangle_{4\pi} = F(\hat{n} \cdot \hat{m}) \neq 0$$

$$\langle a_{lm} a^*_{l'm,} \rangle = C_l \delta_{ll'} \delta_{mm'}$$

$$F(\hat{n} \cdot \hat{m}) = \sum_l \frac{(2l+1)}{4\pi} C_l P_l(\hat{n} \cdot \hat{m})$$

$$C_l \equiv \int d\hat{\Omega} F(\hat{n} \cdot \hat{m}) P_l(\hat{n} \cdot \hat{m})$$

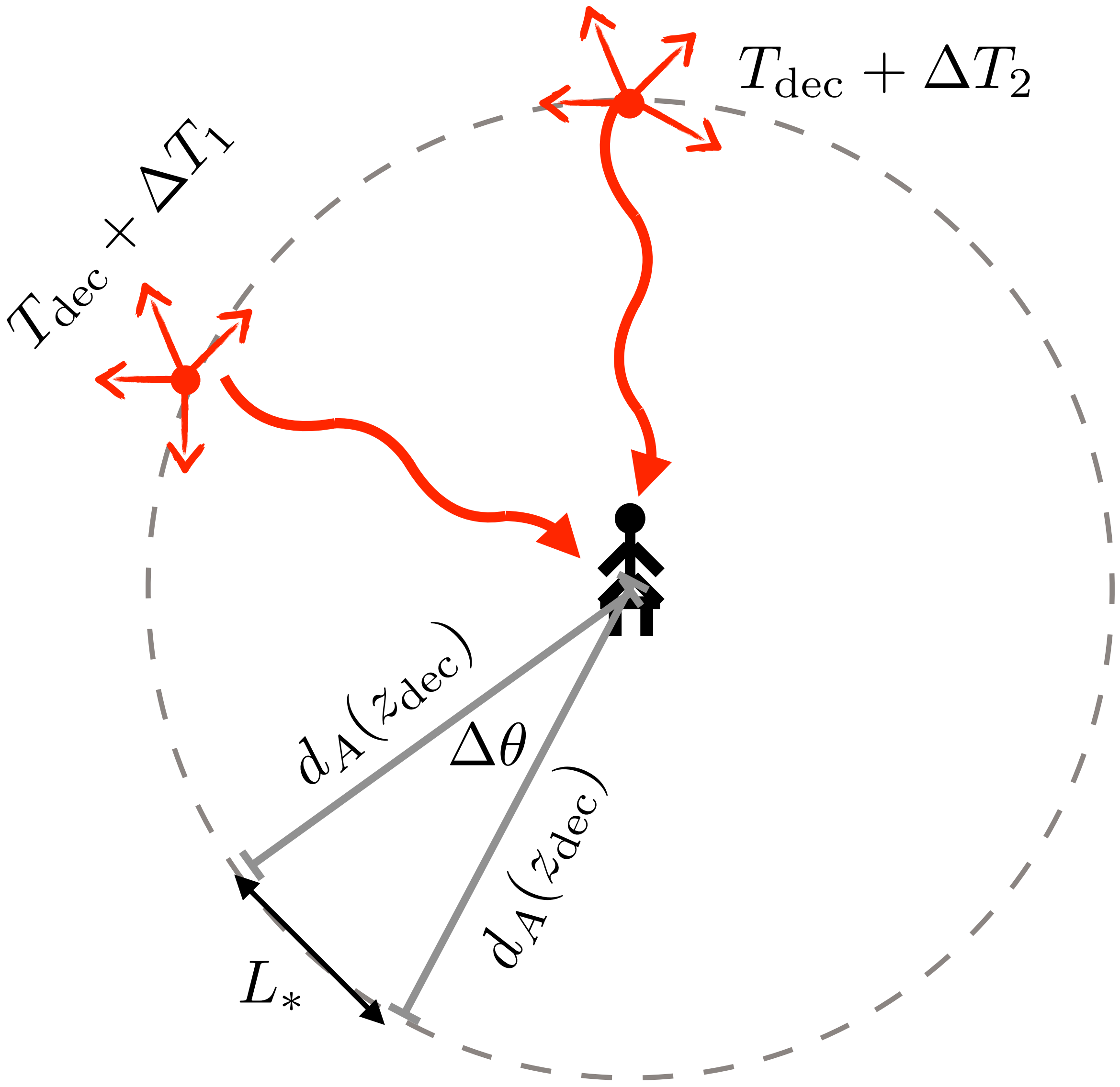
Cosmic Microwave Background (CMB)



$$\langle |\Delta T(\hat{n})|^2 \rangle = \sum_l \frac{(2l+1)}{4\pi} C_l$$

$$Y_{lm}(\hat{n}) \quad \longrightarrow \quad 2l\ 'zeros' : \Delta\theta \sim \frac{\pi}{l}$$

Cosmic Microwave Background (CMB)



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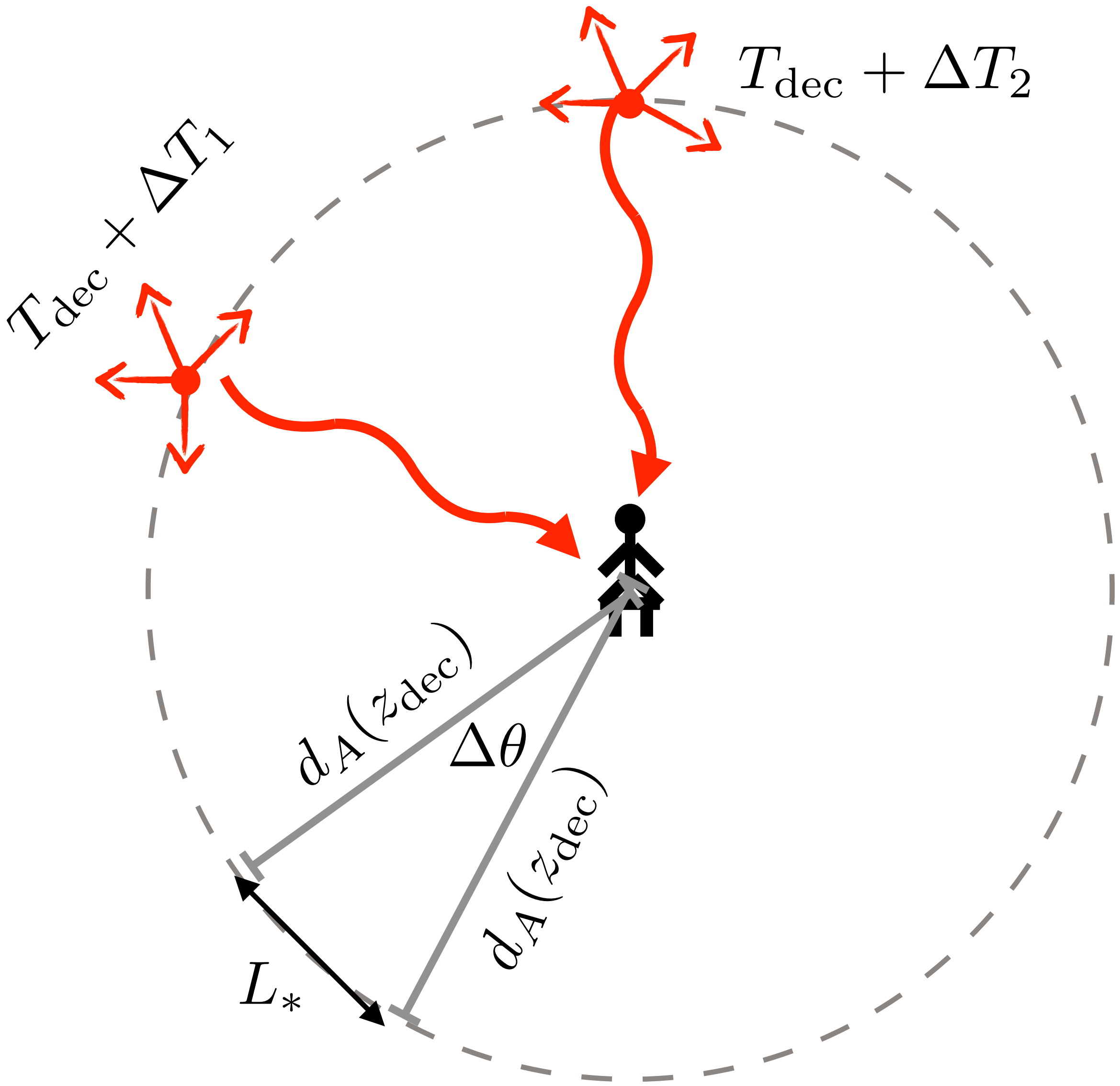
$$Y_{lm}(\hat{n}) \quad \longrightarrow \quad 2l\ 'zeros' : \Delta\theta \sim \frac{\pi}{l}$$

$$\frac{L_*(\Omega_M, \Omega_R, \Omega_k, ...)}{d_A(z_{\rm dec})} \equiv \Delta\theta_* = \frac{\pi}{l_*}$$

Photon-Decoupling: Sound horizon (and multiples)

$$l_1 \quad \leftrightarrow \quad c_s H^{-1}_{\rm dec}(\Omega_M, \Omega_R, ...)$$

Cosmic Microwave Background (CMB)



$$\langle|\Delta T(\hat{n})|^2\rangle = \sum_l \frac{(2l+1)}{4\pi} C_l$$

$$Y_{lm}(\hat{n}) \quad \longrightarrow \quad 2l\ 'zeros' : \Delta\theta \sim \frac{\pi}{l}$$

$$\frac{L_*(\Omega_M, \Omega_R, \Omega_k, ...)}{d_A(z_{\rm dec})} \equiv \Delta\theta_* = \frac{\pi}{l_*}$$

Photon-Decoupling: Sound horizon (and multiples)

$$l_1 \quad \leftrightarrow \quad c_s H^{-1}_{\rm dec}(\Omega_M, \Omega_R, ...)$$

$$l_* = \frac{\pi d_A(z_{\rm dec})}{L_*(\Omega_M, \Omega_R, \Omega_k, ...)}$$

measurement

prediction

$$d_A(z_{\rm dec}) = \frac{H_o^{-1}}{(1+z_{\rm dec})|\Omega_k|^{1/2}} \mathrm{sinn}\left\{|\Omega_k|^{1/2} \int_0^{z_{\rm dec}} \frac{dz'}{E(z')}\right\}$$

$$E(a) \equiv \sqrt{\Omega_{\rm R}^{(o)}\left(\frac{a_o}{a}\right)^4 + \Omega_{\rm M}^{(o)}\left(\frac{a_o}{a}\right)^3 + \Omega_{\rm k}^{(o)}\left(\frac{a_o}{a}\right)^2 + \Omega_{\rm DE}^{(o)} e^{-3\int \frac{da}{a}(1+w)}}$$

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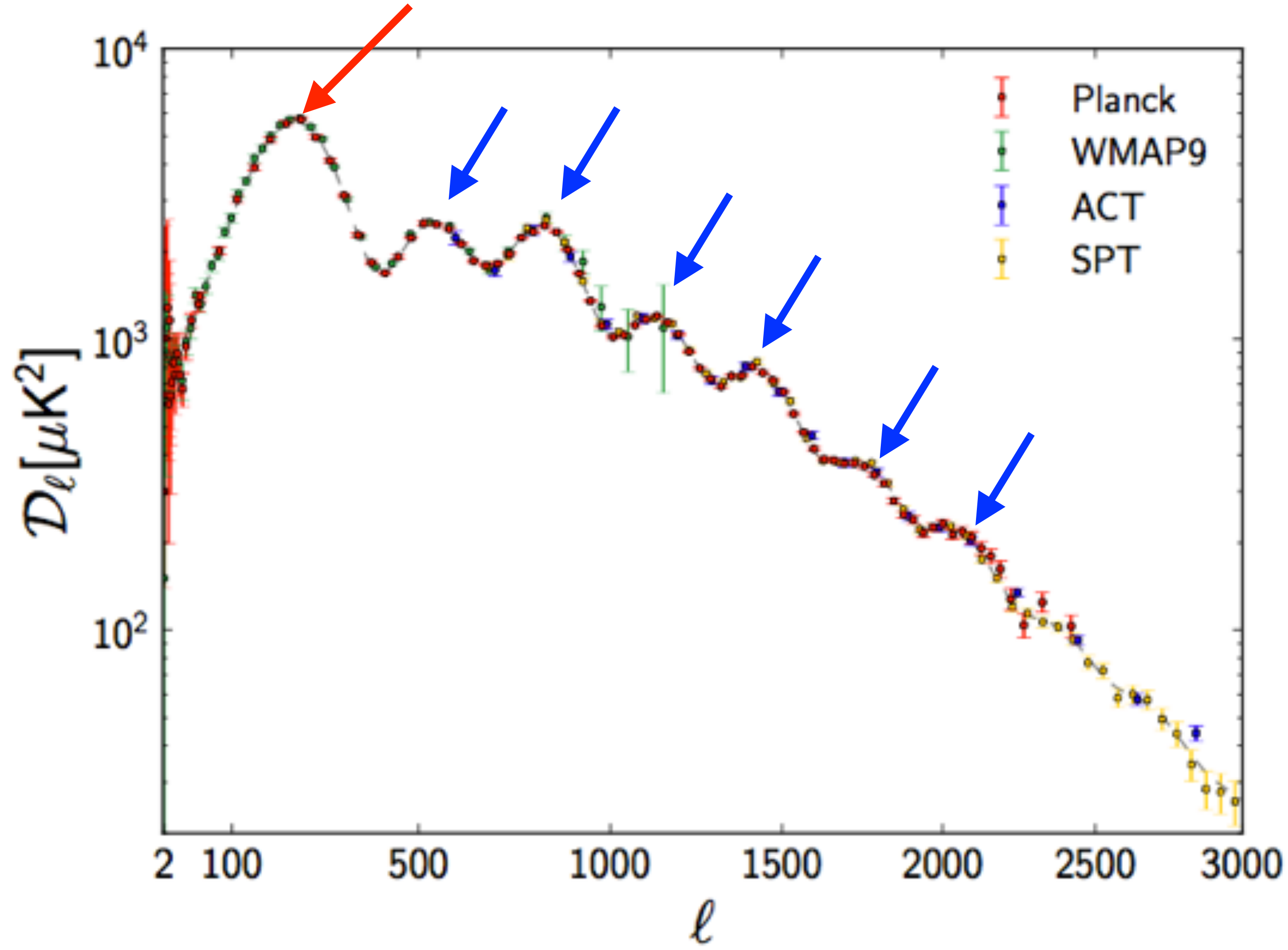

Acoustic Peaks $\begin{cases} C_l^{(1)} \ @ \ l = l_1 \\ C_l^{(2)} \ @ \ l = l_2 \\ C_l^{(3)} \ @ \ l = l_3 \\ \ldots \end{cases}$

$$\Omega_M, \Omega_R, \Omega_k, z_{\rm dec}, \ldots$$

$$l_* = \frac{\pi d_A(z_{\rm dec})}{L_*(\Omega_M, \Omega_R, \Omega_k, \ldots)}$$

measurement

prediction

$$d_A(z_{\rm dec}) = \frac{H_o^{-1}}{(1+z_{\rm dec})|\Omega_k|^{1/2}} \mathrm{sinn} \left\{ |\Omega_k|^{1/2} \int_0^{z_{\rm dec}} \frac{dz'}{E(z')} \right\}$$

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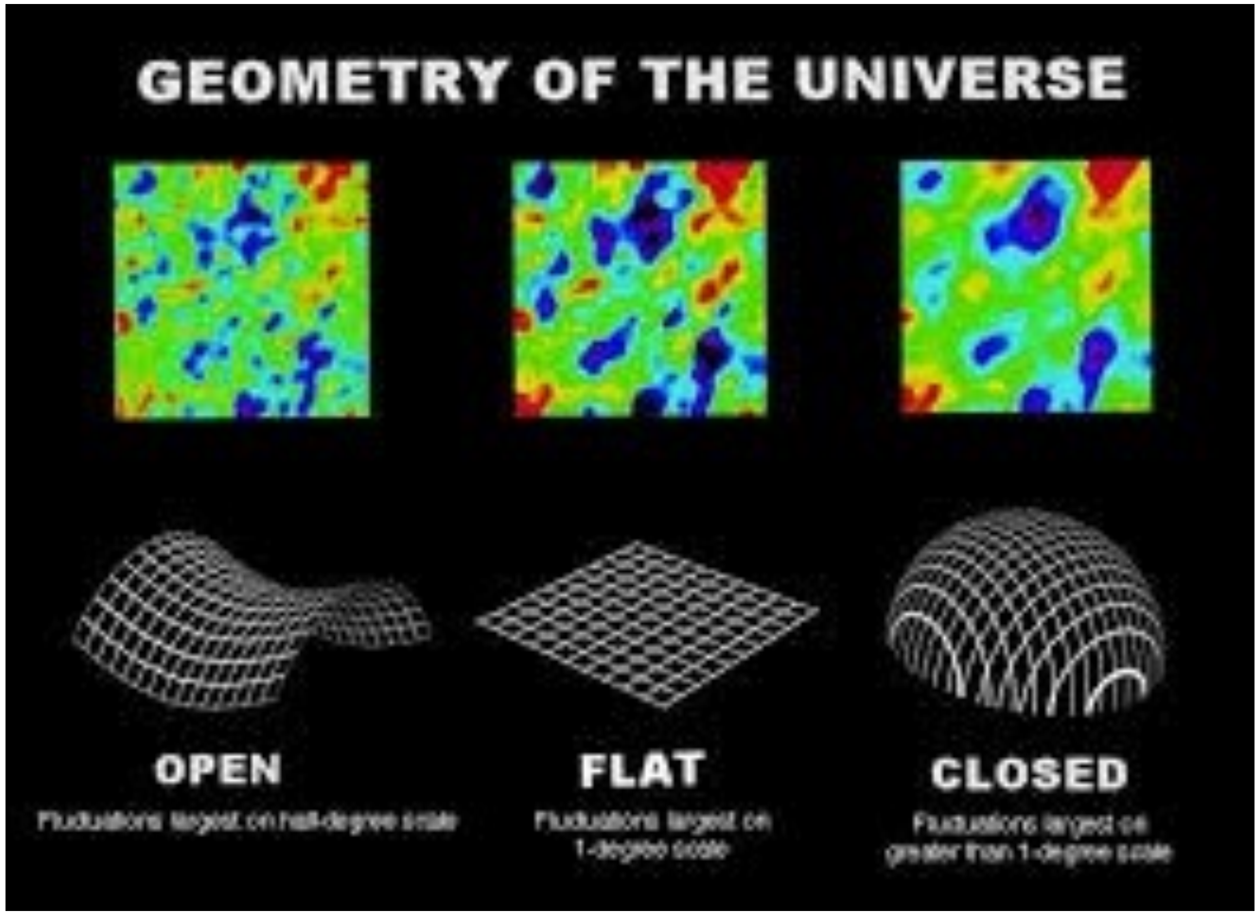
Cosmic Microwave Background (CMB)



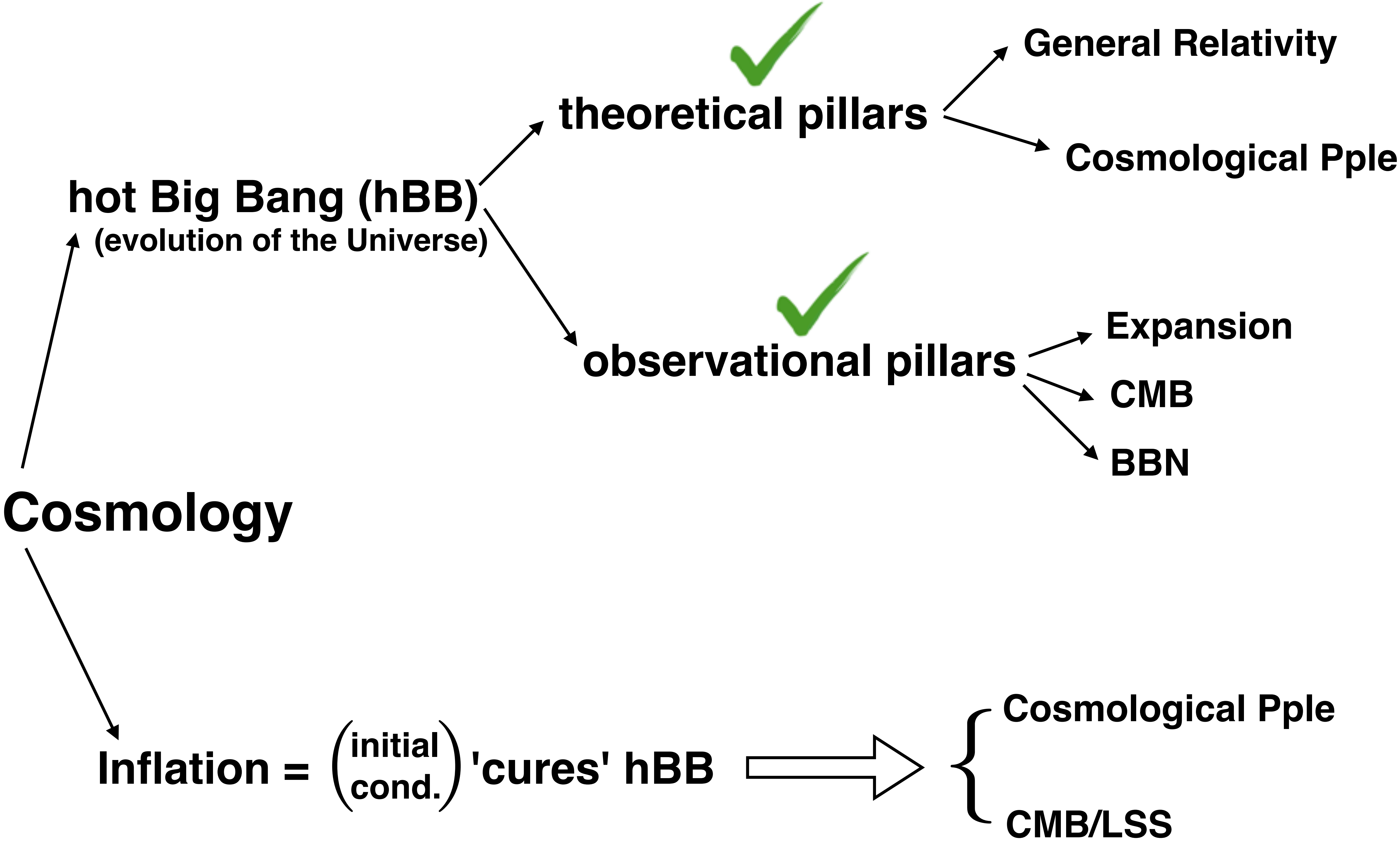
Properties of the Anisotropies
(position of 1st acoustic peak)

then ….

Geometry of the Universe !

The UNIVERSE has FLAT GEOMETRY (k = 0)!

BASICS of COSMOLOGY



Cosmology

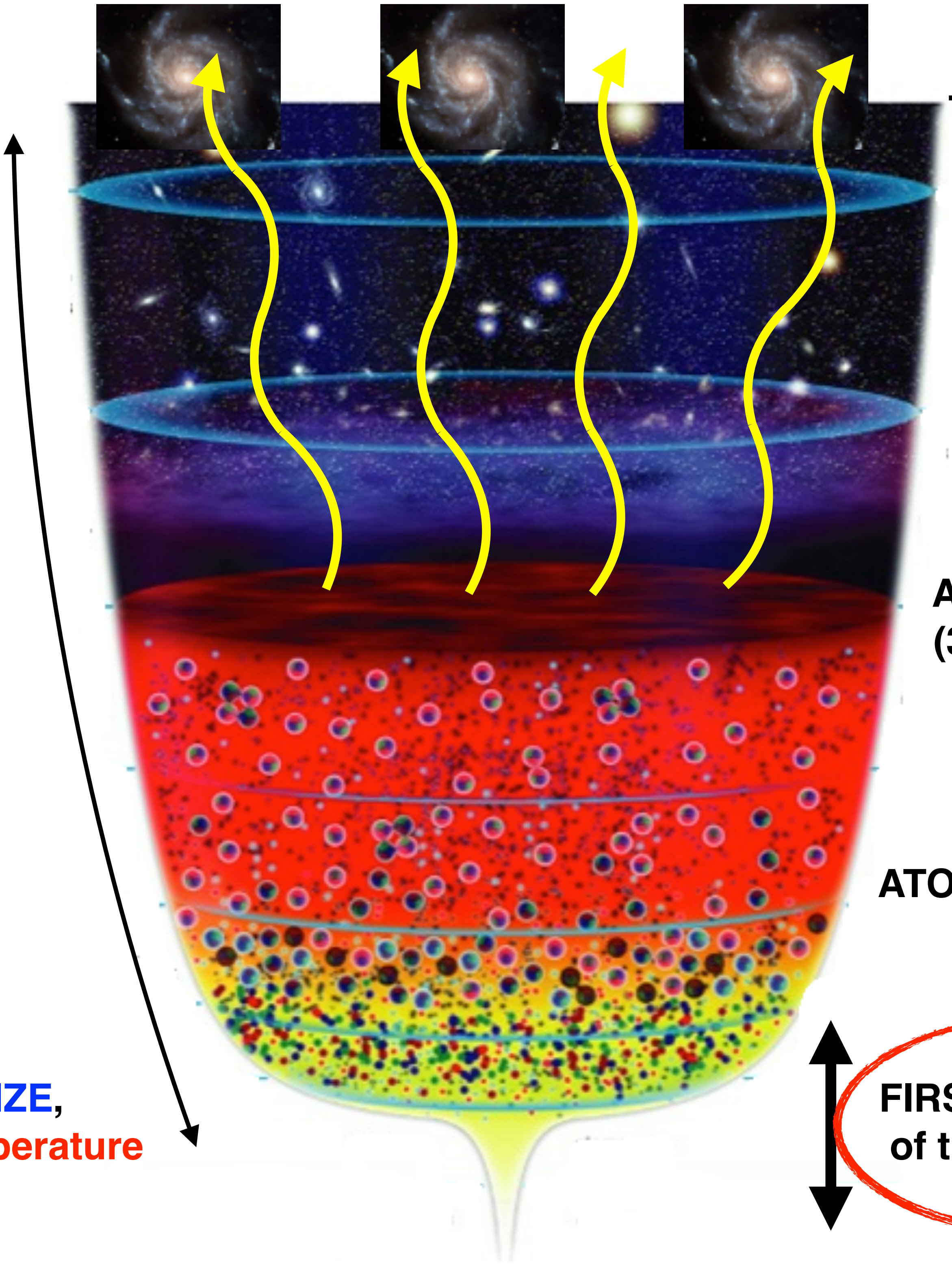
hot Big Bang (hBB)
(evolution of the Universe)

theoretical pillars ✓
- General Relativity
- Cosmological Pple

observational pillars ✓
- Expansion
- CMB
- BBN

Inflation = (initial cond.) 'cures' hBB ⟹ { Cosmological Pple, CMB/LSS }

hot Big Bang (hBB)



BiGGER size,
SMALLER Temp

TODAY [Galaxies, Clusters, …]
(13.700 Million years)

FIRST GALAXIES
(500 Millions years)

ATOMS CREATION
(300.000-400.000 years)

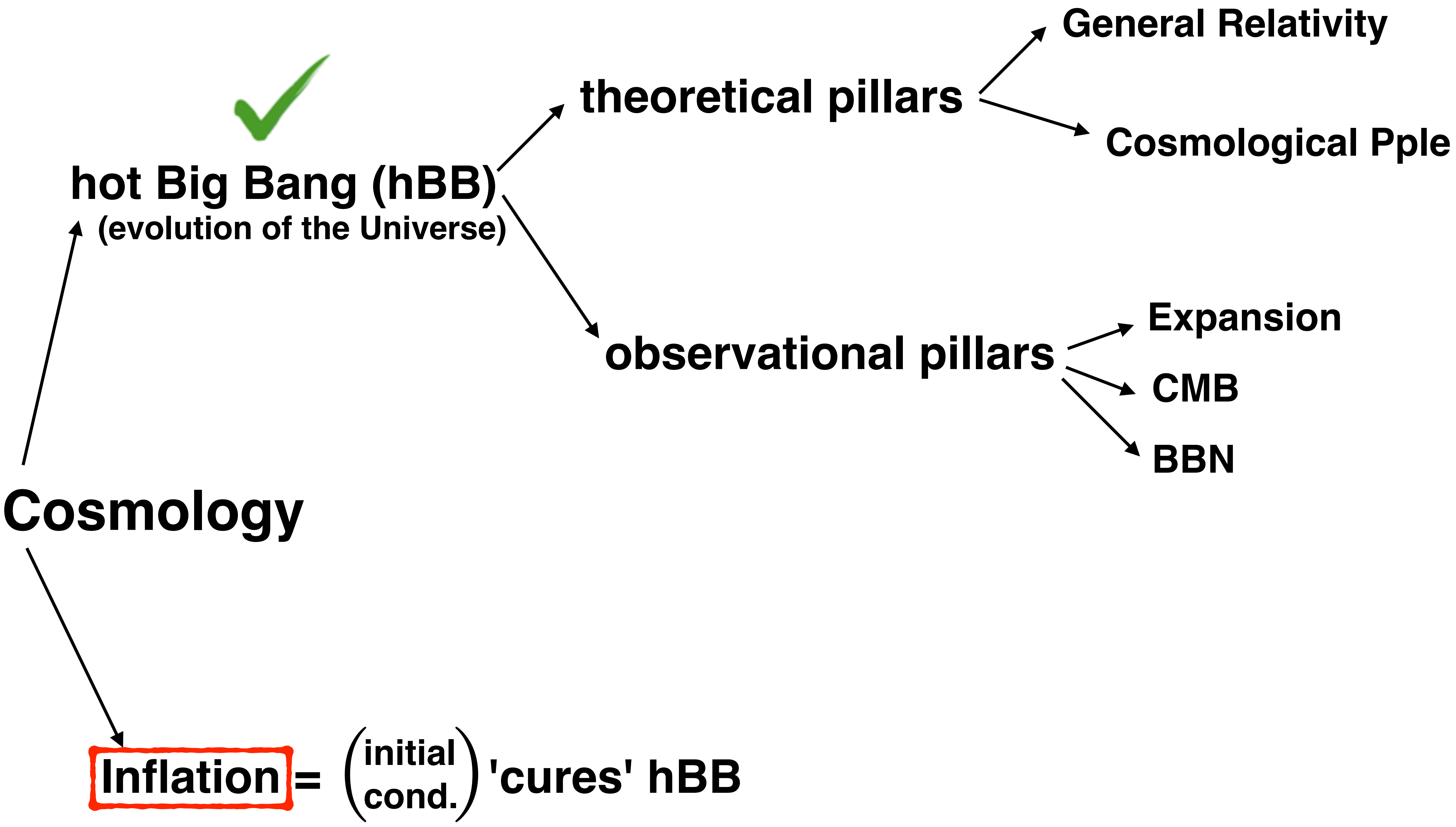
CMB emission

ATOMIC NUCLEI CREATION
(3 minutes !)

FIRST SECOND
of the UNIVERSE !

SMALLER SIZE,
LARGER Temperature

BASICS of COSMOLOGY



✓

hot Big Bang (hBB)
(evolution of the Universe)

theoretical pillars
- General Relativity
- Cosmological Pple

observational pillars
- Expansion
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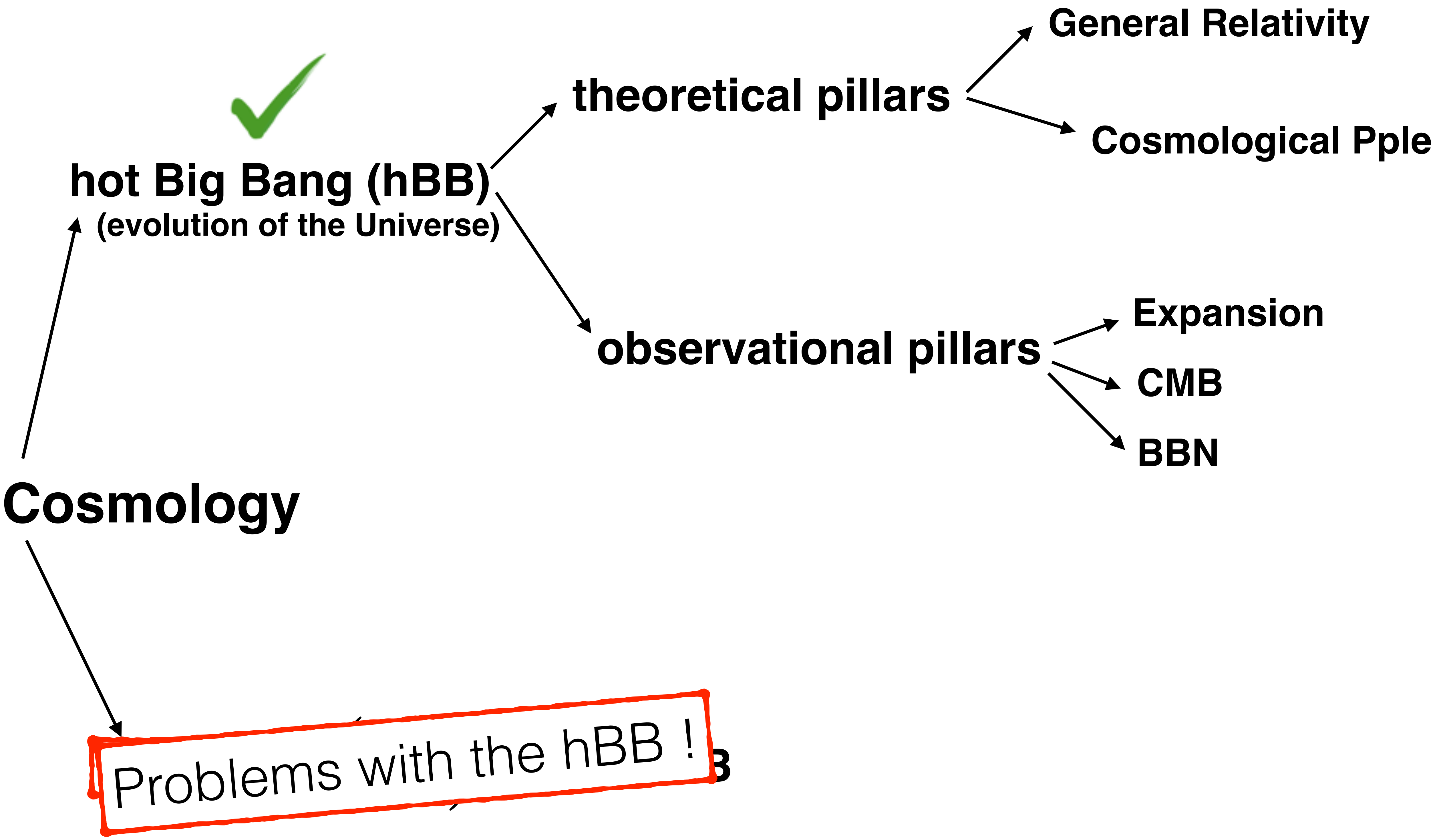
Cosmology

$\boxed{\text{Inflation}} = \begin{pmatrix}\text{initial}\\ \text{cond.}\end{pmatrix}$ 'cures' hBB

BASICS of COSMOLOGY



Cosmology

- **hot Big Bang (hBB)** ✓
  (evolution of the Universe)
  - **theoretical pillars**
    - **General Relativity**
    - **Cosmological Pple**
  - **observational pillars**
    - **Expansion**
    - **CMB**
    - **BBN**

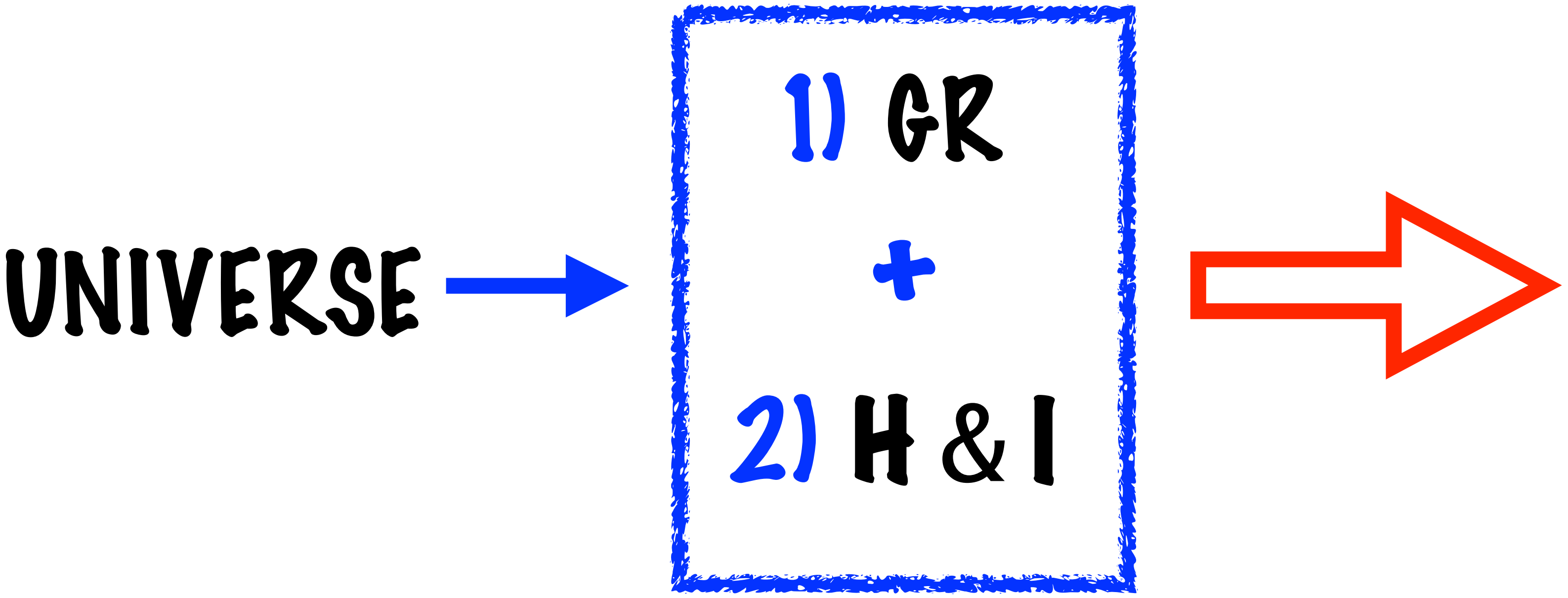
Problems with the hBB !

B

Shortcomings of the hBB framework

hBB shortcomings

(motivation for inflation)



Friedmann Equations

$$\frac{1}{a}\frac{d^2a}{dt^2} = -\frac{\rho}{6m_p^2}(1+3w)$$

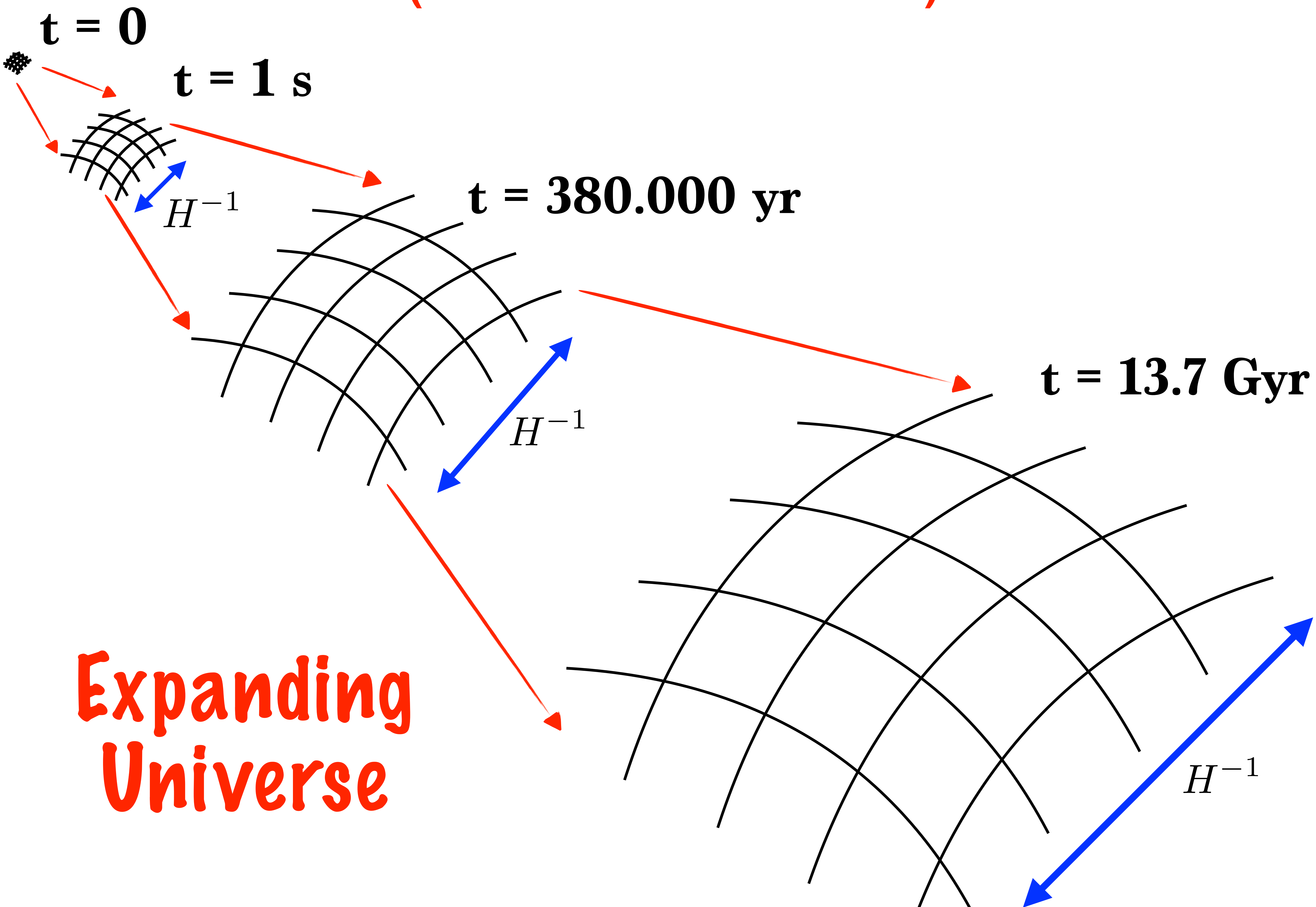
$$H^2 \equiv \left(\frac{1}{a}\frac{da}{dt}\right)^2 = \frac{\rho}{3m_p^2} - \frac{K}{a^2}$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = -\frac{3}{a}\frac{da}{dt}(1+w)$$

$$\left(w \equiv \frac{p}{\rho}\right)$$

hBB shortcomings

(motivation for inflation)



hBB shortcomings

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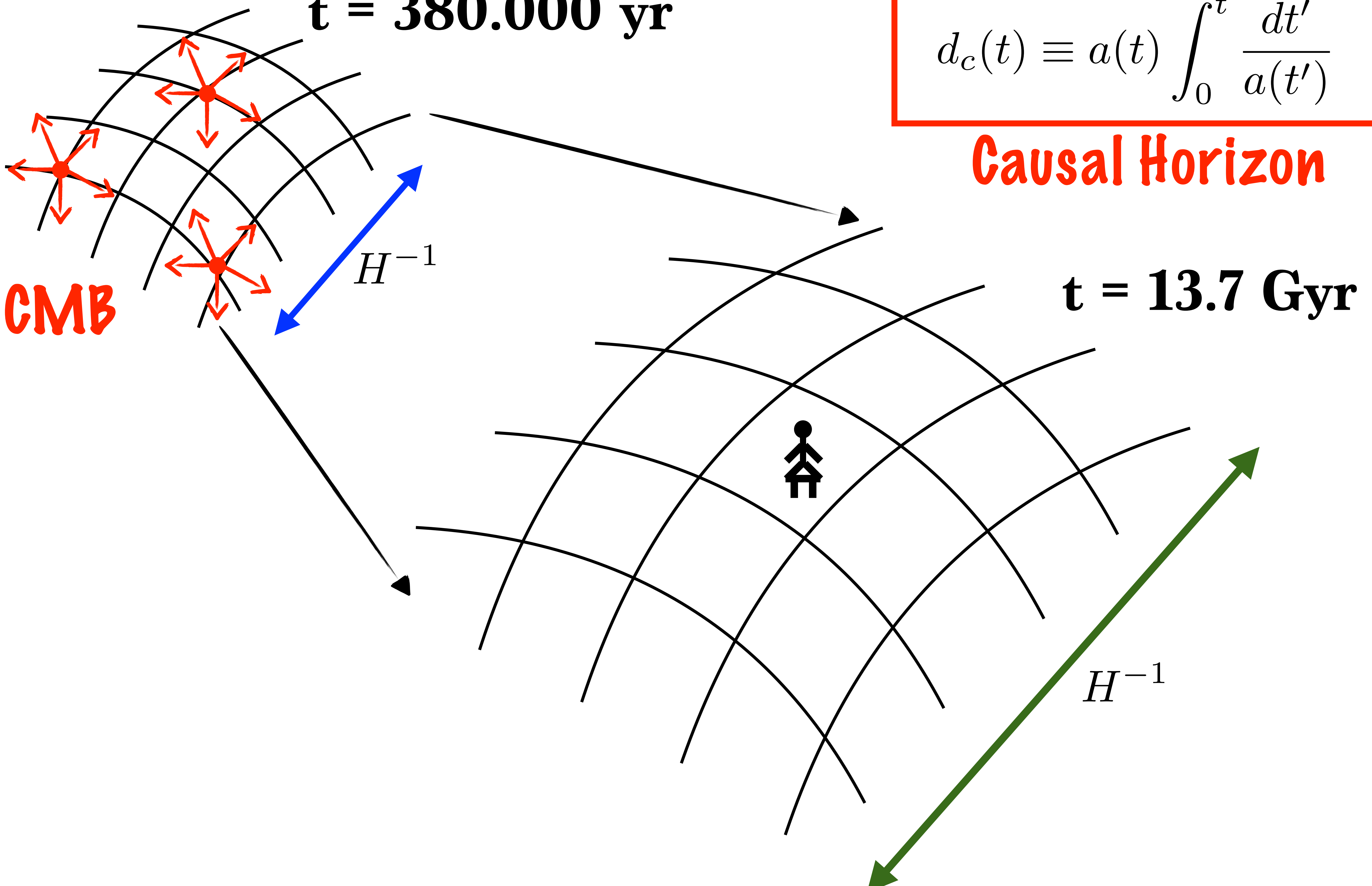


$$d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}$$

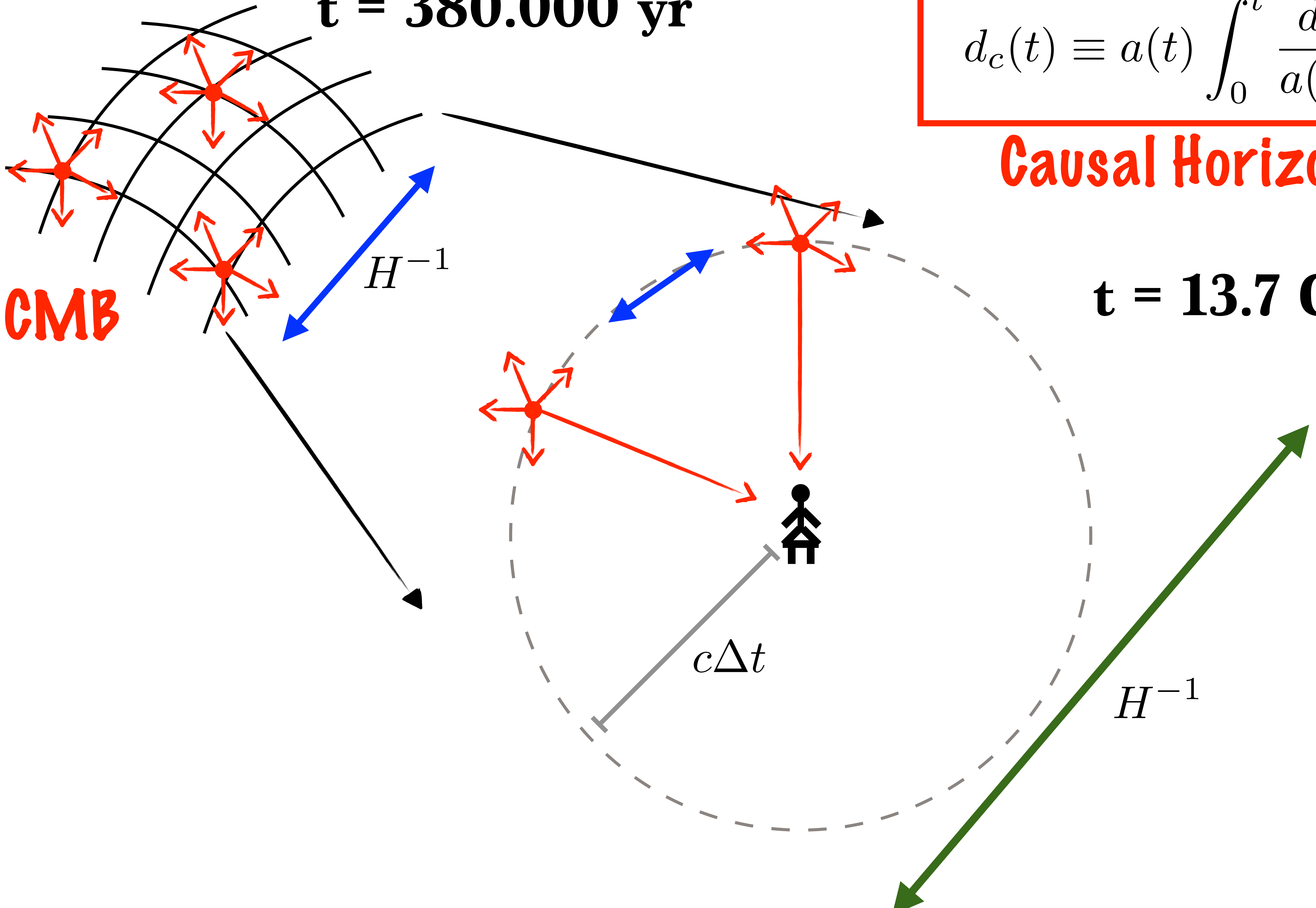
Causal Horizon

hBB shortcomings

(motivation for inflation)

$$\boxed{d_c(t) \equiv a(t) \int_0^t \frac{dt'}{a(t')}}$$

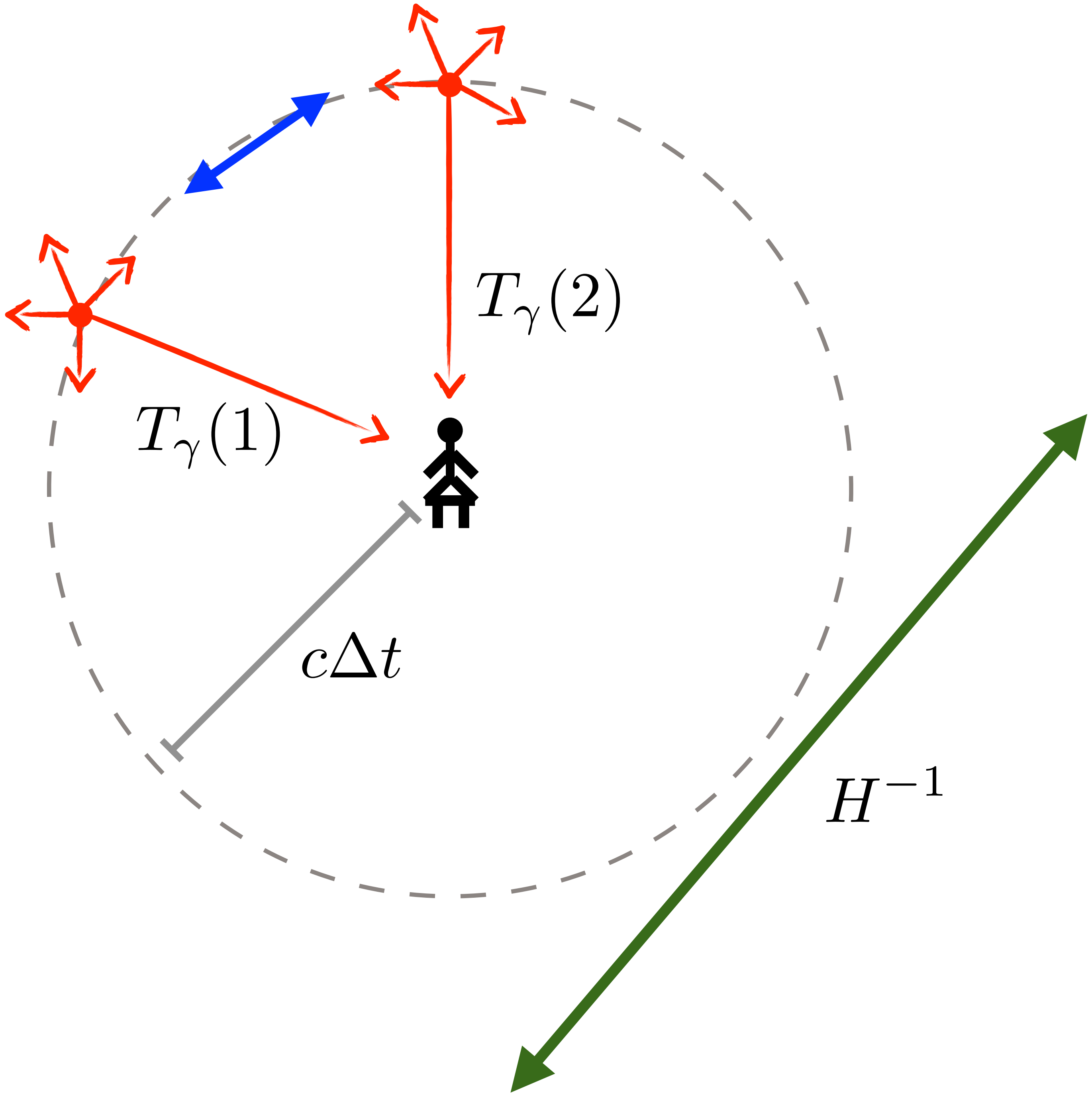
Causal Horizon



hBB shortcomings

(motivation for inflation)

t = 13.7 Gyr



IF $\boxed{T_\gamma(1) = T_\gamma(2)}$

CAUSALITY VIOLATION !

hBB:

H&I @ Scales >> 1/H

iLL-defined!

hBB shortcomings

(motivation for inflation)

1) Horizon Problem → Causality Violation !

hBB

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation!

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings

(motivation for inflation)

hBB

1) **Horizon Problem** → **Causality Violation !**

2) **Curvature Problem**

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

hBB shortcomings

(motivation for inflation)

hBB

1) **Horizon Problem** → **Causality Violation !**

2) **Curvature Problem**

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1$$

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem $\longrightarrow$ **Causality Violation !**

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

$$\text{Today}: \;\; |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \quad |\Omega - 1|_{\rm BBN} = \left(\frac{a_{\rm eq}}{a_o}\right)\left(\frac{a_{\rm BBN}}{a_{\rm eq}}\right)^2 |\Omega - 1|_o$$

$$\simeq \frac{1}{(1+z_{\rm eq})}\left(\frac{T_{\rm eq}}{T_{\rm BBN}}\right)^2 |\Omega - 1|_o$$

$$\simeq 10^{-3}10^{-12}|\Omega - 1|_o \lesssim 10^{-18}$$

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem ⟶ **Causality Violation !**

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \text{RD} \\ \propto a, & \text{MD} \end{cases}$$

$$\text{Today}: \;\; |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \boxed{|\Omega - 1|_{\text{BBN}}} = \left(\frac{a_{\text{eq}}}{a_o}\right)\left(\frac{a_{\text{BBN}}}{a_{\text{eq}}}\right)^2 |\Omega - 1|_o$$

$$\simeq \frac{1}{(1 + z_{\text{eq}})} \left(\frac{T_{\text{eq}}}{T_{\text{BBN}}}\right)^2 |\Omega - 1|_o$$

$$\simeq 10^{-3} 10^{-12} |\Omega - 1|_o \boxed{\lesssim 10^{-18}}$$

hBB shortcomings

(motivation for inflation)

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hBB shortcomings

(motivation for inflation)

hBB

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(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \begin{cases} \propto a^2, & \mathrm{RD} \\ \propto a, & \mathrm{MD} \end{cases}$$

$$\mathrm{Today}: \; |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \quad |\Omega - 1|_{\rm BBN} \lesssim 10^{-18}$$

$$\boxed{|\Omega - 1|_{\rm GUT}} = \left(\frac{a_{\rm GUT}}{a_{\rm BBN}}\right)^2 |\Omega - 1|_{\rm BBN} \sim (T_{\rm BBN}/T_{\rm GUT})^2 |\Omega - 1|_{\rm BBN}$$

$$\simeq 10^{-38} |\Omega - 1|_{\rm BBN} \boxed{\lesssim 10^{-56}}$$

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem

(If curvature $\neq 0$, it grows unstable!)

$$|\Omega - 1| = \frac{|k|}{a^2H^2} \begin{cases} \propto a^2, \ \text{RD} \\ \propto a, \ \text{MD} \end{cases}$$

$$\text{Today}: \ |\Omega - 1|_o \lesssim 0.1 \quad \Rightarrow \boxed{|\Omega - 1|_{\text{BBN}} \lesssim 10^{-18}} \ \boxed{|\Omega - 1|_{\text{GUT}} \lesssim 10^{-56}}$$

It might well be that k = 0 ...

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

hBB shortcomings

(motivation for inflation)

hBB

1) Horizon Problem → Causality Violation !

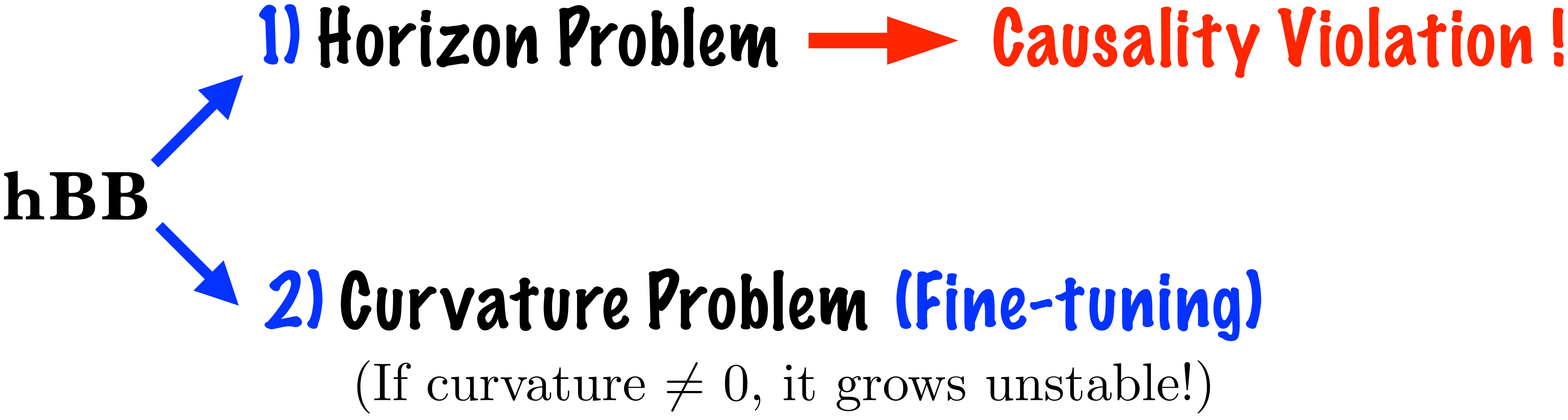
2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

Need extra 'Ingredient' ! → INFLATION !

hBB shortcomings

(motivation for inflation)



hBB

1) Horizon Problem → Causality Violation !

2) Curvature Problem (Fine-tuning)

(If curvature $\neq 0$, it grows unstable!)

N … ATION !

To Be Continued