Flavour Changing Neutral Currents in the Higgs Sector

M. N. Rebelo
CFTP/IST, U. Lisboa

22 May 2017

Flavour Physics at LHC run II
Centro de Ciencias de Benasque
Pedro Pascual

Work done in collaborations with
F. J. Botella, G. C. Branco, A. Carmona, M. Nebot and L. Pedro

arXiv:1508.05101, arXiv:1401.6147, arXiv:1210.8163, arXiv:1102.0520,
arXiv:0911.1753

Work Partially supported by:

FCT Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

europa.eu
European Union

QREN
QUADRO DE REFERÊNCIA ESTRATÉGICO NACIONAL
PORTUGAL 2007.2013

COMPETE
PROGRAMA OPERACIONAL FACTORES DE COMPETITIVIDADE

FCT
Fundação para a Ciência e a Tecnologia
MINISTÉRIO DA EDUCAÇÃO E CIÊNCIA

Two Higgs Doublet Models

Several motivations

- New sources of CP violation

SM cannot account for BAU

- Possibility of having spontaneous CP violation

EW symmetry breaking and CP violation same footing

T. D. Lee 1973, Kobayashi and Maskawa 1973

- Strong CP Problem, Peccei-Quinn

- Supersymmetry

LHC important role

In general two Higgs doublet models have FCNC

Neutral currents have played an important rôle in the construction and experimental tests of unified gauge theories

EPS Prize in 2009 to Gargamelle, CERN

In the Standard Model Flavour changing neutral currents (FCNC) are forbidden at tree level

- in the gauge sector, no ZFCNC

- in the Higgs sector, no HFCNC

Models with two or more Higgs doublets have potentially large HFCNC

Strict limits on FCNC processes!

Two Higgs Doublet Models

Despite several good motivations,
there is the need to suppress potentially dangerous FCNC:

Without HFCNC

- discrete symmetry leading to NFC

Weinberg, Glashow (1977); Paschos (1977)

- aligned two Higgs doublet model Pich, Tuzon (2009)

With HFCNC

- assume existence of suppression factors

Cheng and Sher (1987)

Antaramian, Hall, Rasin (1992); Hall, Weinberg (1993); Joshipura, Rindani (1991)

- first models of this type obtaining in a natural way suppression by small elements of VCKM

Branco, Grimus, Lavoura (1996)

Minimal Flavour Violation

Notation

Yukawa Interactions

$$\mathcal{L}_Y = -\overline{Q_L^0}\ \Gamma_1 \Phi_1 d_R^0 - \overline{Q_L^0}\ \Gamma_2 \Phi_2 d_R^0 - \overline{Q_L^0}\ \Delta_1 \tilde{\Phi}_1 u_R^0 - \overline{Q_L^0}\ \Delta_2 \tilde{\Phi}_2 u_R^0 \text{ + h.c.}$$

$$\tilde{\Phi}_i = -i\tau_2 \Phi_i^*$$

Quark mass matrices

$$M_d = \frac{1}{\sqrt{2}}(v_1 \Gamma_1 + v_2 e^{i\theta} \Gamma_2), \quad M_u = \frac{1}{\sqrt{2}}(v_1 \Delta_1 + v_2 e^{-i\theta} \Delta_2),$$

Diagonalised by:

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag}\ (m_d, m_s, m_b)\ ,$$
$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag}\ (m_u, m_c, m_t)\ .$$

Leptonic Sector

$$-\overline{L_L^0}\ \Pi_1\Phi_1\ell_R^0 - \overline{L_L^0}\ \Pi_2\Phi_2\ell_R^0\ + \text{h.c.}$$

$$\Big(\ - \overline{L_L^0}\ \Sigma_1\tilde{\Phi}_1\nu_R^0 - \overline{L_L^0}\ \Sigma_2\tilde{\Phi}_2\nu_R^0 + \text{h.c.}\ \Big)$$

$$\Big(\ \frac{1}{2}{\nu_R^0}^T C^{-1} M_R \nu_R^0 + \text{h.c.}\ \Big)$$

Expansion around the vev's

$$\Phi_j = \left(\begin{array}{c} \phi_i^+ \\ \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + \eta_j) \end{array} \right), \qquad j = 1, 2$$

We perform the following transformation:

$$\left(\begin{array}{c} H^0 \\ R \end{array} \right) = U \left(\begin{array}{c} \rho_1 \\ \rho_2 \end{array} \right); \quad \left(\begin{array}{c} G^0 \\ I \end{array} \right) = U \left(\begin{array}{c} \eta_1 \\ \eta_2 \end{array} \right); \quad \left(\begin{array}{c} G^+ \\ H^+ \end{array} \right) = U \left(\begin{array}{c} \phi_1^+ \\ \phi_2^+ \end{array} \right),$$

$$U = \frac{1}{v} \left(\begin{array}{cc} v_1 e^{-i\alpha_1} & v_2 e^{-i\alpha_2} \\ v_2 e^{-i\alpha_1} & -v_1 e^{-i\alpha_2} \end{array} \right); \qquad v = \sqrt{v_1^2 + v_2^2} = (\sqrt{2}G_F)^{-\frac{1}{2}} \simeq 246\text{GeV}$$

U singles out

H^0 **with couplings to quarks proportional to mass matrices**

G^0 **neutral pseudo-Goldstone boson**

G^+ **charged pseudo-Goldstone boson**

Physical neutral fields are combinations of $H^0 \quad R \quad I$

Neutral and charged Higgs Interactions for the quark sector

$$\begin{aligned}
\mathcal{L}_Y(\text{quark, Higgs}) = & -\overline{d_L^0}\frac{1}{v}\left[M_d H^0 + N_d^0 R + i N_d^0 I\right] d_R^0 \\
& -\overline{u_L^0}\frac{1}{v}\left[M_u H^0 + N_u^0 R + i N_u^0 I\right] u_R^0 \\
& -\frac{\sqrt{2}H^+}{v}\left(\overline{u_L^0} N_d^0\, d_R^0 - \overline{u_R^0} N_u^{0\dagger}\, d_L^0\right) + \text{h.c.}
\end{aligned}$$

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\theta}\Gamma_2), \quad N_u^0 = \frac{1}{\sqrt{2}}(v_2\Delta_1 - v_1 e^{-i\theta}\Delta_2).$$

Flavour structure of quark sector of 2HDM characterised by:

$$\text{four matrices } M_d,\ M_u,\ N_d^0,\ N_u^0.$$

Likewise for Leptonic sector, Dirac neutrinos:

$$M_\ell,\ M_\nu,\ N_\ell^0,\ N_\nu^0.$$

Yukawa Couplings in terms of quark mass eigenstates

for H^+, H^0, R, I

$$\mathcal{L}_Y(\text{quark, Higgs}) =$$

$$- \frac{\sqrt{2}H^+}{v}\bar{u}\left(VN_d\gamma_R - N_u^\dagger\, V\gamma_L\right)d + \text{h.c.} - \frac{H^0}{v}\left(\bar{u}D_u u + \bar{d}D_d\, d\right) -$$

$$- \frac{R}{v}\left[\bar{u}(N_u\gamma_R + N_u^\dagger\gamma_L)u + \bar{d}(N_d\gamma_R + N_d^\dagger\gamma_L)\ d\right] +$$

$$+ i\frac{I}{v}\left[\bar{u}(N_u\gamma_R - N_u^\dagger\gamma_L)u - \bar{d}(N_d\gamma_R - N_d^\dagger\gamma_L)\ d\right]$$

$$\gamma_L = (1 - \gamma_5)/2 \qquad\qquad \gamma_R = (1 + \gamma_5)/2$$

$$V = V_{CKM}$$

Flavour changing neutral currents controlled by:

$$N_d = \frac{1}{\sqrt{2}} U_{d_L}^{\dagger} \left(v_2 \Gamma_1 - v_1 e^{i\alpha} \Gamma_2 \right) U_{d_R}$$

$$N_u = \frac{1}{\sqrt{2}} U_{u_L}^{\dagger} \left(v_2 \Delta_1 - v_1 e^{-i\alpha} \Delta_2 \right) U_{u_R}$$

For generic two Higgs doublet models N_u, N_d non-diagonal arbitrary

For definiteness rewrite N_d :

$$N_d = \frac{v_2}{v_1} D_d - \frac{v_2}{\sqrt{2}} \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) U_{d_L}^{\dagger} e^{i\alpha} \Gamma_2 U_{d_R}$$

conserves flavour

leads to FCNC

Up till here everything is perfectly general for 2HDM

The flavour structure of Yukawa couplings is not constrained by gauge invariance

All flavour changing transitions in SM are mediated by charged weak currents with flavour mixing controlled by V_{CKM}

MFV essentially requires flavour and CP violation linked to known structures of Yukawa couplings

[all new flavour changing transitions are controlled by the CKM matrix]

About Minimal Flavour Violation

Buras, Gambino, Gorbahn, Jager, Silvestrini (2001)
D'Ambrosio, Giudice, Isidori, Strumia (2002)

leptonic sector

Cirigliano, Grinstein, Isidori, Wise (2005)

$G_F = U(3)^5$ largest symmetry of the gauge sector
flavour violation completely determined by Yukawa couplings

Our framework

- multi-Higgs models
- no Natural Flavour Conservation
- must obey above condition (one of the defining ingredients of MFV framework)

In order to obtain a structure for Γ_i, Δ_i such that FCNC at tree level strength completely controlled V_{CKM}

Branco, Grimus, Lavoura imposed symmetry

$$Q^0_{L_j} \to \exp(i\tau) Q^0_{L_j} \; ; \; u^0_{R_j} \to \exp(2i\tau) u^0_{R_j} \; ; \; \Phi_2 \to \exp(i\tau) \Phi_2 \, , \; \tau \neq 0, \pi$$

$$\Gamma_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} ; \; \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix} ; \; \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \; \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$j = 3$

Both Higgs have non-zero Yukawa couplings in the up and down secto

Special WB chosen by the symmetry

FCNC in down sector

if instead of $u^0_{R_j} \to \exp(2i\tau) u^0_{R_j}$ impose $d^0_{R_j} \to \exp(2i\tau) d^0_{R_j}$

then FCNC in up sector

Six different BGL models

$$(N_d)_{rs} = \frac{v_2}{v_1}(D_d)_{rs} - \left(\frac{v_2}{v_1}+\frac{v_1}{v_2}\right)\underbrace{(V_{CKM}^{\dagger})_{r3}(V_{CKM})_{3s}}_{MFV}(D_d)_{sp}$$

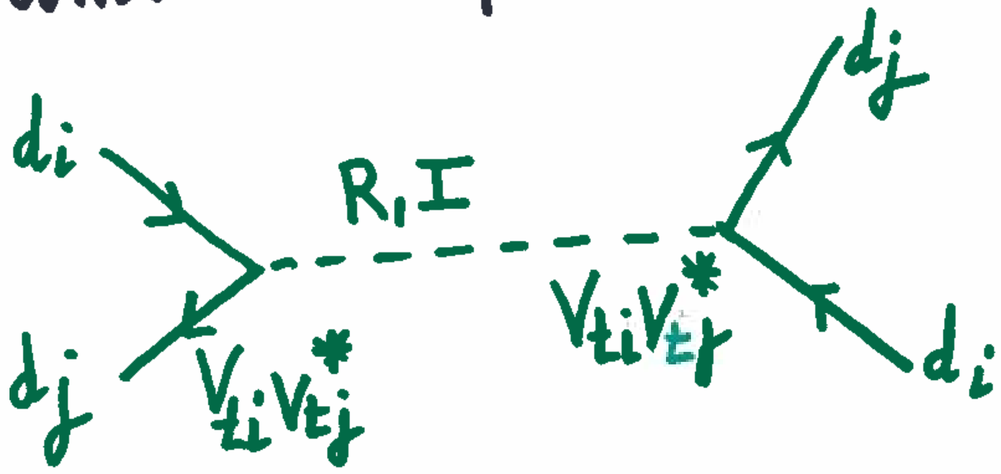
$$j = 3$$

$$N_u = -\frac{v_1}{v_2}\,\mathrm{diag}(0, 0, m_t) + \frac{v_2}{v_1}\,\mathrm{diag}(m_u, m_c, 0)$$

FCNC only in the down sector
suppression by the 3rd row of V_{CKM}
dependence on V_{CKM} and $\tan\beta$ only

Strong and Natural suppression of the most constrained processes

e.g. $|V_{td}V_{ts}^*|^2 \sim \lambda^{10}$



Neutral couplings in BGL models

$$N_u = -\frac{v_1}{v_2}\mathrm{diag}(0,0,m_t) + \frac{v_2}{v_1}\mathrm{diag}(m_u,m_c,0)$$

Explicitely

$$N_d = \frac{v_2}{v_1}\begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} \quad -$$

$$\left(\frac{v_1}{v_2}+\frac{v_2}{v_1}\right)\begin{pmatrix} m_d|V_{31}|^2 & m_s V^*_{31}V_{32} & m_b V^*_{31}V_{33} \\ m_d V^*_{32}V_{31} & m_s|V_{32}|^2 & m_b V^*_{32}V_{33} \\ m_d V^*_{33}V_{31} & m_s V^*_{33}V_{32} & m_b|V_{33}|^2 \end{pmatrix}$$

It all comes from the symmetry

What is the necessary condition for N_d^0, N_u^0 to be of MFV type?

Should be functions of M_d, M_u no other flavour dependence

Furthermore, N_d^0, N_u^0 should transform appropriately under WB

$$Q_L^0 \to W_L Q_L^0, \quad d_R^0 \to W_R^d d_R^0, \quad u_R^0 \to W_R^u u_R^0$$

$$M_d \to W_L^\dagger M_d W_R^d, \quad M_u \to W_L^\dagger M_u W_R^u$$

N_d^0, N_u^0 transform as M_d, M_u

$$N_d^0 \propto M_d \; ; \; (M_d M_d^\dagger) M_d \; ; \; (M_u M_u^\dagger) M_d$$

$$Y_d \; ; \; (Y_d Y_d^\dagger) Y_d \; ; \; (Y_u Y_u^\dagger) Y_d \quad \text{Yukawa}$$

see previous references

What is particular about BGL models in MFV context?

$$M_d M_d^\dagger \equiv H_d \quad ; \quad U_{d_L}^\dagger M_d U_{d_R} = D_d \quad ; \quad U_{d_L}^\dagger H_d U_{d_L} = D_d^2$$

$$D_d^2 = \mathrm{diag}(m_d^2, m_s^2, m_b^2) = m_d^2 \underbrace{\begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix}}_{P_1} + m_s^2 \underbrace{\begin{pmatrix} 0 & & \\ & 1 & \\ & & 0 \end{pmatrix}}_{P_2} + m_b^2 \underbrace{\begin{pmatrix} 0 & & \\ & 0 & \\ & & 1 \end{pmatrix}}_{P_3}$$

$$D_d^2 = \sum_i m_{d_i}^2 P_i$$

$$H_d = U_{d_L} D_d^2 U_{d_L}^\dagger = \sum_i m_{d_i}^2 U_{d_L} P_i U_{d_L}^\dagger = \sum_i m_{d_i}^2 P_i^{d_L}$$

$U_{d_L} P_i U_{d_L}^\dagger$ rather than $Y_d Y_d^\dagger$ are the minimal building blocks to be used in the expansion of N_d^0, N_u^0 conforming to the MFV requirement

Botella, Nebot, Vives 2004

WB covariant definition for BGL models

$$N_d^0 = \frac{v_2}{v_1} M_d - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^\gamma M_d$$

$$N_u^0 = \frac{v_2}{v_1} M_u - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2}\right) \mathcal{P}_j^\gamma M_u$$

together with

$$\mathcal{P}_j^\gamma \Gamma_2 = \Gamma_2 \quad , \quad \mathcal{P}_j^\gamma \Gamma_1 = 0$$

$$\mathcal{P}_j^\gamma \Delta_2 = \Delta_2 \quad , \quad \mathcal{P}_j^\gamma \Delta_1 = 0$$

γ stands for u (up) or d (down)

$\mathcal{P}_j^\gamma$ are projection operators   Botella, Nebot, Vives 2004

$$\mathcal{P}_j^u = U_{uL} P_j U_{uL}^\dagger \qquad \mathcal{P}_j^d = U_{dL} P_j U_{dL}^\dagger$$

$$(P_j)_{\ell k} = \delta_{j\ell}\delta_{jk}$$

e.g. $P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

BGL is the only implementation of models where Higgs FCNC are a function of V_{CKM} only (together with v_1, v_2) which are based on an Abelian symmetry obeying the sufficient conditions of having Mu block diagonal together with the existence of a matrix P such that

$$P\Gamma_2 = \Gamma_2 \quad ; \quad P\Gamma_1 = 0$$

Ferreira, Silva arXiv: 1012287

How to recognize a BGL model when written in arbitrary WB

Necessary and sufficient conditions for BGL

$\Delta_1^\dagger \Delta_2 = 0 \; ; \quad \Delta_1 \Delta_2^\dagger = 0 \; ; \quad \Gamma_1^\dagger \Delta_2 = 0 \; ; \quad \Gamma_2^\dagger \Delta_1 = 0$

Higgs mediated FCNC in the down sector

Imply existence of WB where these matrices can be cast in the form given before

Possible generalisation of BGL models

MFV expansion for $N_d^0, \; N_u^0$

$$N_d^0 = \lambda_1 \; M_d + \lambda_{2i} \; U_{dL} P_i U_{dL}^\dagger \; M_d + \lambda_{3i} \; U_{uL} P_i U_{uL}^\dagger \; M_d + \ldots$$

$$N_u^0 = \tau_1 \; M_u + \tau_{2i} \; U_{uL} P_i U_{uL}^\dagger \; M_u + \tau_{3i} \; U_{dL} P_i U_{dL}^\dagger \; M_u + \ldots$$

In the quark mass eigenstate basis $N_d^0, \; N_u^0$ become:

$$N_d = \lambda_1 \; D_d + \lambda_{2i} \; P_i \; D_d + \lambda_{3i} \; (V_{CKM})^\dagger \; P_i \; V_{CKM} \; D_d + \ldots$$

$$N_u = \tau_1 \; D_u + \tau_{2i} \; P_i \; D_u + \tau_{3i} \; V_{CKM} \; P_i \; (V_{CKM})^\dagger \; D_u + \ldots$$

At this stage lambda and tau coefficients appear as free parameters

Need for symmetries in order to constrain these coefficients

Alternative MFV implementations in 2HDM

Dery, Efrati, Hiller, Hochberg, Nir (2013)

$Y^U = \frac{\sqrt{2}\,M^U}{v}$, $Y^D = \frac{\sqrt{2}\,M^D}{v}$, $Y^E = \frac{\sqrt{2}\,M^E}{v}$; Y_S^F , $S = h, H, A$

e.g. leptonic sector $G^{\ell}_{global} = SU(3)_L \times SU(3)_E$

Definition leptonic MFV, only one spurion breaks G^{ℓ}_{global}

$\hat{Y} \sim (3, \bar{3})$

In the most general case, each Yukawa matrix Y_1, Y_2 is a power series in this spurion

$$Y_i = [a_i + b_i \hat{Y}\hat{Y}^{\dagger} + c_i (\hat{Y}\hat{Y}^{\dagger})^2 + \dots]\hat{Y} \qquad i = 1, 2$$

For each sector $F = U, D, E$ there are two Yukawa matrices $Y^F_{1,2}$

- Is there a loss of generality when we choose as basic spurion one over the other?
- Can we choose the mass matrices $(\sqrt{2}/v)\,M^F$ to play the role of spurions?

New: gBGL allowing for HFCNC both in up and down sectors

1703.03796, Alves, Botella, Branco, Cornet-Gomez, Nebot

Symmetry:

$$Q_{L_3} \mapsto -Q_{L_3},$$
$$d_R \mapsto d_R, \qquad \Phi_1 \mapsto \Phi_1,$$
$$u_R \mapsto u_R, \qquad \Phi_2 \mapsto -\Phi_2.$$

- drastic reduction in number of free parameters

-no NFC

one may say that *the principle leading to gBGL constrains the Yukawa couplings so that each line of* Γ_j, Δ_j *couples only to one Higgs doublet.*

$$\Gamma_1 = \begin{pmatrix} \times & \times & \gamma_{13} \\ \times & \times & \gamma_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma_{31} & \gamma_{32} & \times \end{pmatrix},$$
$$\Delta_1 = \begin{pmatrix} \times & \times & \delta_{13} \\ \times & \times & \delta_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \delta_{31} & \delta_{32} & \times \end{pmatrix},$$

- renormalisable;

- FCNC both in up and down sectors;

- no longer of MFV type, four additional flavour parameters;

- both up and down type BGL appear as special limits;

gBGL verify:

$$\Gamma_2^\dagger \Gamma_1 = 0, \quad \Gamma_2^\dagger \Delta_1 = 0,$$
$$\Delta_2^\dagger \Delta_1 = 0, \quad \Delta_2^\dagger \Gamma_1 = 0.$$

The leptonic sector

Required for completeness

- study of experimental implications
- study of stability under RGE

Models with two Higgs doublets with FCNC

- controlled by V_{CKM} in the quark sector
- controlled by V_{PMNS} in the leptonic sector

Case of Dirac neutrinos, straightforward

Same flavour structure

Six different BGL-type models

Similarly, for the leptonic sector,

In the leptonic sector, with Dirac type neutrinos, there is perfect analogy with the quark sector. The requirement that FCNC at tree level have strength completely controlled by the Pontecorvo – Maki – Nakagawa – Sakata (PMNS) matrix, U is enforced by one of the following symmetries. Either

$$L^0_{Lk} \to \exp{(i\tau)}\ L^0_{Lk}\ , \qquad \nu^0_{Rk} \to \exp{(i2\tau)}\nu^0_{Rk}\ , \qquad \Phi_2 \to \exp{(i\tau)}\Phi_2\ ,$$

or

$$\tau \neq 0, \pi$$

$$L^0_{Lk} \to \exp{(i\tau)}\ L^0_{Lk}\ , \qquad \ell^0_{Rk} \to \exp{(i2\tau)}\ell^0_{Rk}\ , \qquad \Phi_2 \to \exp{(-i\tau)}\Phi_2\ ,$$

which imply

$$\begin{aligned} \mathcal{P}^\beta_k \Pi_2 &= \Pi_2\ , \qquad \mathcal{P}^\beta_k \Pi_1 = 0\ , \\ \mathcal{P}^\beta_k \Sigma_2 &= \Sigma_2\ , \qquad \mathcal{P}^\beta_k \Sigma_1 = 0\ , \end{aligned}$$

where β stands for neutrino (ν) or for charged lepton (ℓ) respectively. In this case

$$\mathcal{P}^\ell_k = U_{\ell L} P_k U^\dagger_{\ell L}\ , \qquad \mathcal{P}^\nu_k = U_{\nu L} P_k U^\dagger_{\nu L}\ ,$$

where $U_{\nu L}$ and $U_{\ell L}$ are the unitary matrices that diagonalize the corresponding square mass matrices

$$\begin{aligned} U^\dagger_{\ell L} M_\ell M^\dagger_\ell U_{\ell L} &= \operatorname{diag}\left(m^2_e, m^2_\mu, m^2_\tau\right)\ , \\ U^\dagger_{\nu L} M_\nu M^\dagger_\nu U_{\nu L} &= \operatorname{diag}\left(m^2_{\nu_1}, m^2_{\nu 2}, m^2_{\nu 3}\right)\ , \end{aligned}$$

$$M_\ell = \frac{1}{\sqrt{2}}(v_1 \Pi_1 + v_2 e^{i\theta} \Pi_2)\ , \quad M_\nu = \frac{1}{\sqrt{2}}(v_1 \Sigma_1 + v_2 e^{-i\theta} \Sigma_2)\ .$$

Scalar Potential

The softly broken Z_2 symmetric 2HDM potential

$$V(\Phi_1,\Phi_2) = m_{11}^2\,\phi_1^\dagger\phi_1 + m_{22}^2\,\phi_2^\dagger\phi_2 - \left(m_{12}^2\,\phi_1^\dagger\phi_2 + h.c.\right) + \frac{1}{2}\lambda_1\left(\phi_1^\dagger\phi_1\right)^2 + \frac{1}{2}\lambda_2\left(\phi_2^\dagger\phi_2\right)^2 +$$

$$+ \lambda_3\left(\phi_1^\dagger\phi_1\right)\left(\phi_2^\dagger\phi_2\right) + \lambda_4\left(\phi_1^\dagger\phi_2\right)\left(\phi_2^\dagger\phi_1\right) + \frac{1}{2}\left[\lambda_5\left(\phi_1^\dagger\phi_2\right)^2 + h.c.\right]$$

$$\phi_1 \to \phi_1 \quad , \quad \phi_2 \to -\phi_2$$

in our case $\phi_1 \to \phi_1$, $\phi_2 \to e^{iz}\,\phi_2$, $z \neq 0, \pi$ no λ_5 term

V does not violate CP neither explicitly nor spontaneously

7 free parameters : m_h, m_H, m_A, $m_{H^\pm}$, $v = \sqrt{v_1^2 + v_2^2}$, $\tan\beta$, α (H^0, R)

soft symmetry breaking prevents ungauged accidental continuous symmetry

In BGL models the Higgs potential is constrained by the imposed symmetry to be of the form:

$$V_\Phi = \mu_1 \Phi_1^\dagger \Phi_1 + \mu_2 \Phi_2^\dagger \Phi_2 - \left(m_{12} \Phi_1^\dagger \Phi_2 + \text{ h.c. } \right) + 2\lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right)$$
$$+ 2\lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) + \lambda_1 \left(\Phi_1^\dagger \Phi_1 \right)^2 + \lambda_2 \left(\Phi_2^\dagger \Phi_2 \right)^2 ,$$

Hermiticity would allow the coefficient m_{12} to be complex, unlike the other coefficients of the scalar potential. However, freedom to rephase the scalar doublets allows to choose without loss of generality all coefficients real. As a result, V_Φ does not violate CP explicitly. It can also be easily shown that it cannot violate CP spontaneously. In the absence of CP violation the scalar field I does not mix with the fields R and H^0, therefore I is already a physical Higgs and the mixing of R and H^0 is parametrized by a single angle. There are two important rotations that define the two parameters, $\tan\beta$ and α, widely used in the literature:

$$\begin{pmatrix} H^0 \\ R \end{pmatrix} = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ -v_2 & v_1 \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

$$\begin{pmatrix} H \\ h \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}$$

Our analysis:

Approximation of no mixing between R and H^0

We identify H^0 with the recently discovered Higgs field

This limit corresponds to $\beta - \alpha = \pi/2$

$v \equiv \sqrt{v_1^2 + v_2^2}$, $\tan\beta \equiv v_2/v_1$, **the quantity v is of course fixed by experiment**

Electroweak precision tests and in particular the T and S parameters lead to constraints relating the masses of the new Higgs fields among themselves

Grimus, Lavoura, Ogreid, Osland 2008

The bounds on T and S together with direct mass limits significantly restrict the masses of the new Higgs particles once the mass of charged Higgs is fixed

It is instructive to plot our results in terms of $m_{H^\pm}$ versus $\tan\beta$,

since in this context there is not much freedom left

	BGL - 2HDM				SM	
	Charged $H^{\pm}$		Neutral R, I		Tree	Loop
	Tree	Loop	Tree	Loop		
$M \to \ell\bar{\nu}, M'\ell\bar{\nu}$	✓	✓		✓	✓	✓
Universality	✓	✓		✓	✓	✓
$M^0 \to \ell_1^+\ell_2^-$		✓	✓	✓		✓
$M^0 \rightleftarrows \bar{M}^0$		✓	✓	✓		✓
$\ell_1^- \to \ell_2^-\ell_3^+\ell_4^-$		✓	✓	✓		✓
$B \to X_s\gamma$		✓		✓		✓
$\ell_j \to \ell_i\gamma$		✓		✓		✓
EW Precision		✓		✓		✓

Summary of relevant constraints

This table indicates possible new contributions but for each specific model type some of them will be absent

$\|g_\mu/g_e\|^2$ $\|g^S_{RR,\tau e}\|$	$1.0018(14)$ < 0.70	$\|g^S_{RR,\tau\mu}\|$ $\|g^S_{RR,\mu e}\|$	< 0.72 < 0.035
$\mathrm{Br}(B^+ \to e^+\nu)$ $\mathrm{Br}(B^+ \to \mu^+\nu)$ $\mathrm{Br}(B^+ \to \tau^+\nu)$	$< 9.8 \cdot 10^{-7}$ $< 1.0 \cdot 10^{-6}$ $1.15(23) \cdot 10^{-4}$	$\mathrm{Br}(D_s^+ \to e^+\nu)$ $\mathrm{Br}(D_s^+ \to \mu^+\nu)$ $\mathrm{Br}(D_s^+ \to \tau^+\nu)$	$< 1.2 \cdot 10^{-4}$ $5.90(33) \cdot 10^{-3}$ $5.43(31) \cdot 10^{-2}$
$\mathrm{Br}(D^+ \to e^+\nu)$ $\mathrm{Br}(D^+ \to \mu^+\nu)$ $\mathrm{Br}(D^+ \to \tau^+\nu)$	$< 8.8 \cdot 10^{-6}$ $3.82(33) \cdot 10^{-4}$ $< 1.2 \cdot 10^{-3}$		
$\frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}$ $\frac{\Gamma(K^+ \to e^+\nu)}{\Gamma(K^+ \to \mu^+\nu)}$	$1.230(4) \cdot 10^{-4}$ $2.488(12) \cdot 10^{-5}$	$\frac{\Gamma(\tau^- \to \pi^-\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}$ $\frac{\Gamma(\tau^- \to K^-\nu)}{\Gamma(K^+ \to \mu^+\nu)}$	$9703(54)$ $469(7)$
$\frac{\Gamma(B \to D\tau\nu)_{\mathrm{NP}}}{\Gamma(B \to D\tau\nu)_{\mathrm{SM}}}$ $\frac{\Gamma(B \to D^*\tau\nu)_{\mathrm{NP}}}{\Gamma(B \to D^*\tau\nu)_{\mathrm{SM}}}$		$\log C \ (K \to \pi\ell\nu)$	$0.194(11)$

Tree level $H^\pm$ mediated processes

$\mathrm{Br}(\tau^- \to e^- e^- e^+)$	$< 2.7 \cdot 10^{-8}$	$\mathrm{Br}(\tau^- \to \mu^- \mu^- \mu^+)$	$< 2.1 \cdot 10^{-8}$
$\mathrm{Br}(\tau^- \to e^- e^- \mu^+)$	$< 1.5 \cdot 10^{-8}$	$\mathrm{Br}(\tau^- \to e^- \mu^- e^+)$	$< 1.8 \cdot 10^{-8}$
$\mathrm{Br}(\tau^- \to \mu^- \mu^- e^+)$	$< 1.7 \cdot 10^{-8}$	$\mathrm{Br}(\tau^- \to \mu^- e^- \mu^+)$	$< 2.7 \cdot 10^{-8}$
$\mathrm{Br}(\mu^- \to e^- e^- e^+)$	$< 1 \cdot 10^{-12}$		
$\mathrm{Br}(K_L \to \mu^\pm e^\mp)$	$< 4.7 \cdot 10^{-12}$	$\mathrm{Br}(\pi^0 \to \mu^\pm e^\mp)$	$< 3.6 \cdot 10^{-10}$
$\mathrm{Br}(K_L \to e^- e^+)$	$< 9 \cdot 10^{-12}$		
$\mathrm{Br}(K_L \to \mu^- \mu^+)$	$< 6.84 \cdot 10^{-9}$		
$\mathrm{Br}(D^0 \to e^- e^+)$	$< 7.9 \cdot 10^{-8}$	$\mathrm{Br}(B^0 \to e^+ e^-)$	$< 8.3 \cdot 10^{-8}$
$\mathrm{Br}(D^0 \to \mu^\pm e^\mp)$	$< 2.6 \cdot 10^{-7}$	$\mathrm{Br}(B^0 \to \tau^\pm e^\mp)$	$< 2.8 \cdot 10^{-5}$
$\mathrm{Br}(D^0 \to \mu^- \mu^+)$	$< 1.4 \cdot 10^{-7}$	$\mathrm{Br}(B^0 \to \mu^- \mu^+)$	$3.6(1.6) \cdot 10^{-10}$
$\mathrm{Br}(B_s^0 \to e^+ e^-)$	$< 2.8 \cdot 10^{-7}$	$\mathrm{Br}(B^0 \to \tau^\pm \mu^\mp)$	$< 2.2 \cdot 10^{-5}$
$\mathrm{Br}(B_s^0 \to \mu^\pm e^\mp)$	$< 2 \cdot 10^{-7}$	$\mathrm{Br}(B^0 \to \tau^+ \tau^-)$	$< 4.1 \cdot 10^{-3}$
$\mathrm{Br}(B_s^0 \to \mu^- \mu^+)$	$2.9(0.7) \cdot 10^{-9}$		

Tree level R, I mediated processes (I)

$2\|M_{12}^K\|$	$< 3.5 \cdot 10^{-15}$ GeV	$2\|M_{12}^D\|$	$< 9.47 \cdot 10^{-15}$ GeV
$\|\epsilon_K\|_{NP} \Delta m_K$	$< 7.8 \cdot 10^{-19}$ GeV		
$\mathrm{Re}(\Delta_d)$	$0.823(143)$	$\mathrm{Re}(\Delta_s)$	$0.965(133)$
$\mathrm{Im}(\Delta_d)$	$-0.199(62)$	$\mathrm{Im}(\Delta_s)$	$0.00(10)$

Tree level R, I mediated processes (II)

$\mathrm{Br}(\mu \to e\gamma)$	$< 5.6 \cdot 10^{-13}$	$\mathrm{Br}(B \to X_s\gamma)^{\mathrm{NNLO}}_{\mathrm{SM}}$	$3.15(23) \cdot 10^{-4}$
$\mathrm{Br}(\tau \to e\gamma)$	$< 3.3 \cdot 10^{-8}$	$\mathrm{Br}(B \to X_s\gamma)$	$3.55(35) \cdot 10^{-4}$
$\mathrm{Br}(\tau \to \mu\gamma)$	$< 4.4 \cdot 10^{-8}$		
ΔT	$0.02(11)$	$F_{Zb\bar{b}}$	< 0.0024 GeV^{-1}
ΔS	$0.00(12)$		

Loop level R, I, $H^\pm$ mediated processes

Each of the thirty six models labelled by the pair $(\gamma^{i}_{j}, \beta_k)$

j, k refer to projectors $P_{j,k}$ in each sector γ^{i}, β

Example: $(up_3, \ell_2) = (t, \mu)$

will have no tree level NFC couplings (neutral flavour changing) in the up quark and charged lepton sectors, neutral HFC couplings in the down quark and neutrino sector controlled by

$V_{td_i} V^*_{td_j}$ and $U_{\mu\nu_a} U^*_{\mu\nu_b}$

Analysis of implications, 36 BGL models

Decays mediated by charged currents

i) pure leptonic of type $\ell_i \to \ell_j \bar{\nu}_j \nu_i$

$m_{H^\pm} <$ LEP bound ~ 80GeV irrelevant

FCNC if present always negligible

ii) leptonic decays of pseudoscalar mesons $M \to \ell\nu$

eg $B^+ \to \tau^+\nu$ $D_s^+ \to \mu^+\nu$ $D_s^+ \to \tau^+\nu$

helicity suppressed in SM; new physics contributions more relevant in the case of heavy pseudoscalar mesons, dependence $m_M^2/m_{H^\pm}^2$

iii) semileptonic processes of the form $\ell \to M\nu$ eg $\tau^- \to \pi^-\nu$

iv) semileptonic decays of pseudoscalar mesons $M \to M'\ell\nu$

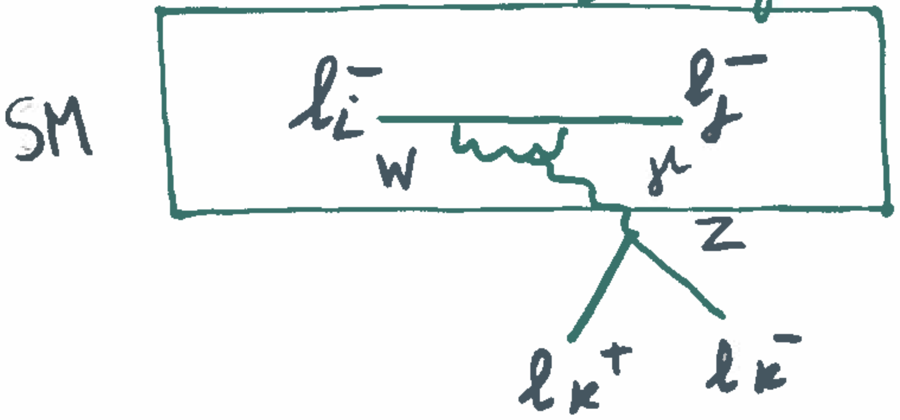
eg $B \to D\tau\nu$, $B \to D^+\tau\nu$

Analysis of implications , BGL models

Flavour changing neutral currents at tree level

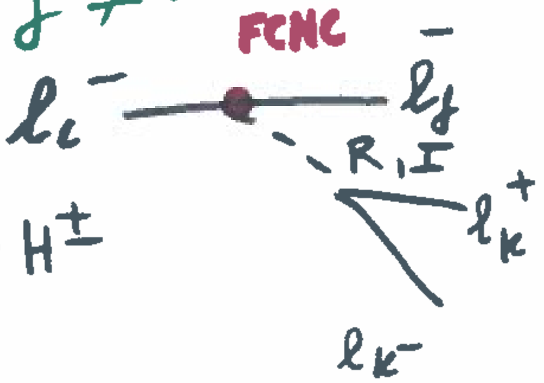
i) $\ell_i^- \to \ell_j^- \ell_k^- \ell e^+$

a) $\ell_i^- \to \ell_j^- \ell_k^- \ell_k^+$ $j = k$ or $j \neq k$



BGL

$(\mu \to e\gamma)$ $W^\pm \to H^\pm$



tree level diagram are irrelevant in this case, mass suppression

b) $\ell_i^- \to \ell_j^- \ell_j^- \ell_k^+$ $k \neq j$

boxes $W^\pm, H^\pm$ SM diagram above with R, I

tree level with two FCNC vertices

very suppressed

ii) decays of pseudoscalar meson into charged lepton

$B^0 \to \mu^+\mu^-$ $B_s \to \mu^+\mu^-$ SM helicity suppression

iii) neutral meson mixing

SM loop level, BGL models there are tree level cont. which might lead to stringent constraints $j=3$ FCNC up quark, FCNC ν sector

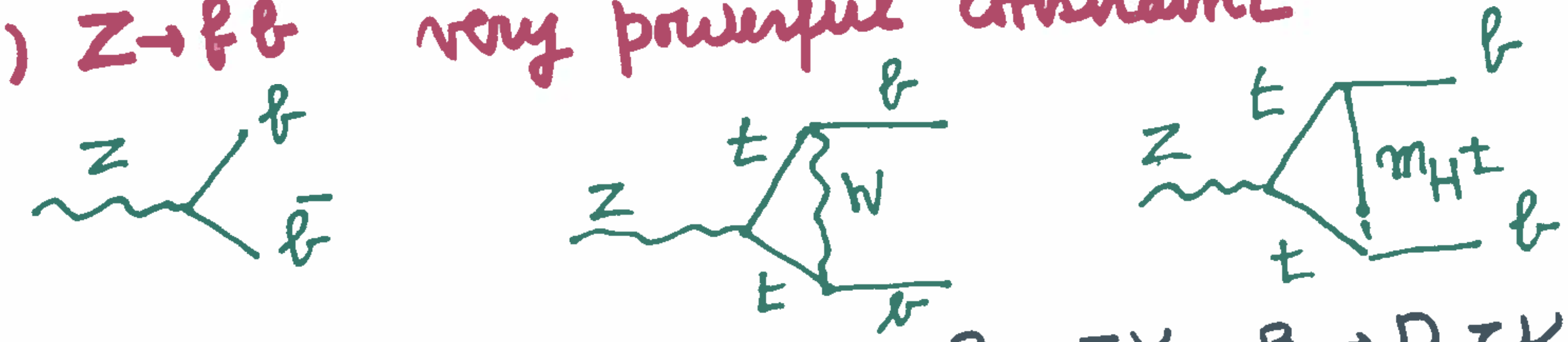
Analysis of implications, 36 BGL models

Loop induced processes

i) radiative leptonic decays of the form $\ell_1 \to \ell_2 \gamma$, $\mu \to e\gamma$

ii) $b \to s\gamma$ no trivial translation, important

iii) $Z \to b\bar{b}$ very powerful constraint



eliminates the region where $B \to \tau\nu$, $B \to D\tau\nu$, $B \to D^*\tau\nu$ are improved

Oblique Parameters and Direct searches

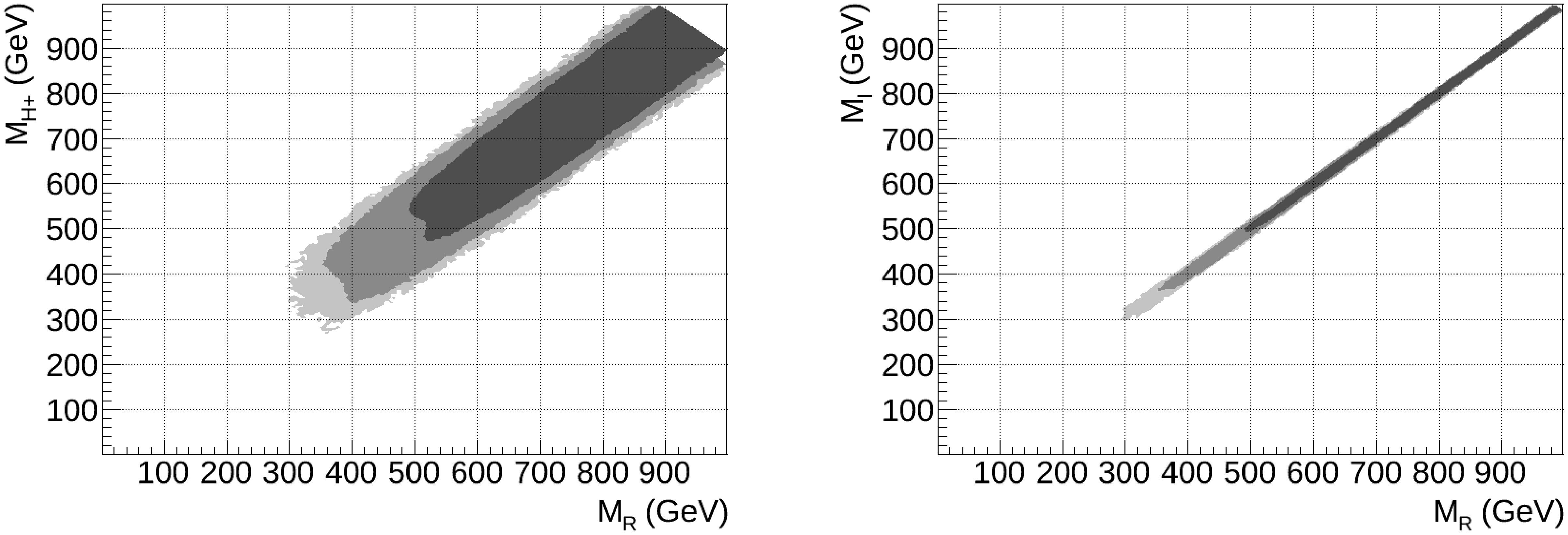
S, U in 2HDM tend to be small corrections

T receives corrections can be sizable

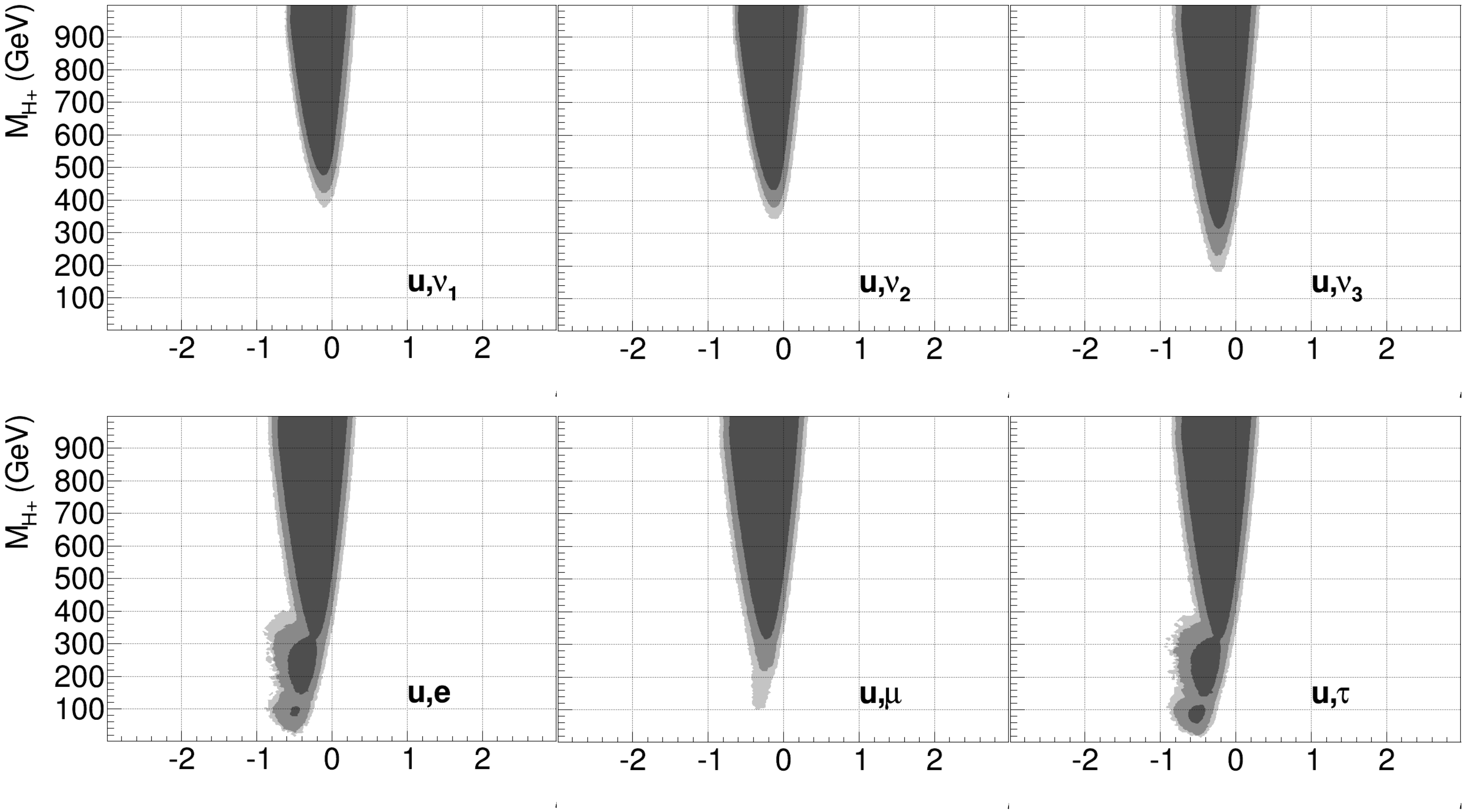
Grimus, Lavoura, Ogreid, Osland (200

$m_{H^\pm}$, m_H, m_A not very different

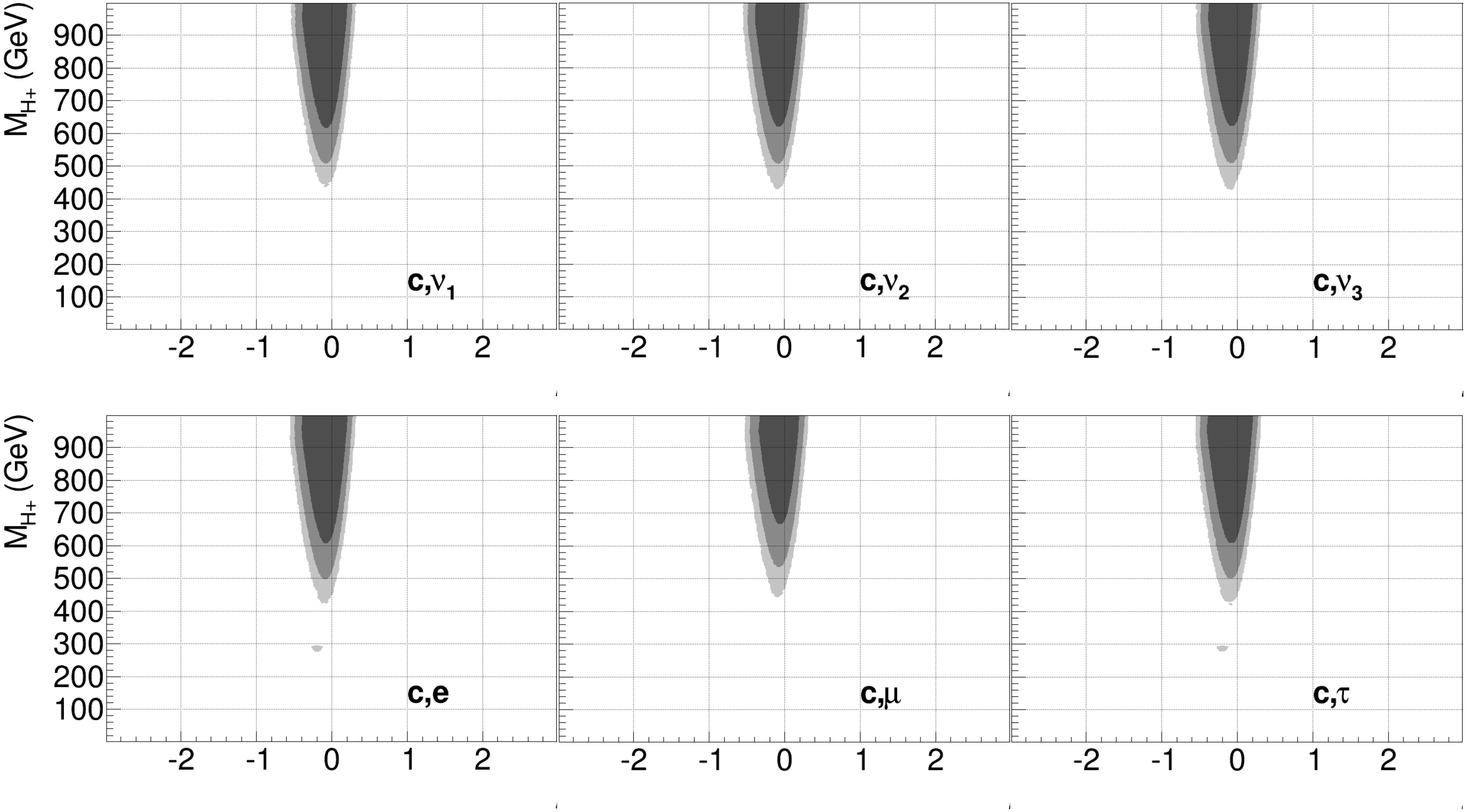
Results in $m_{H^\pm}$, $\tan\beta$ plane



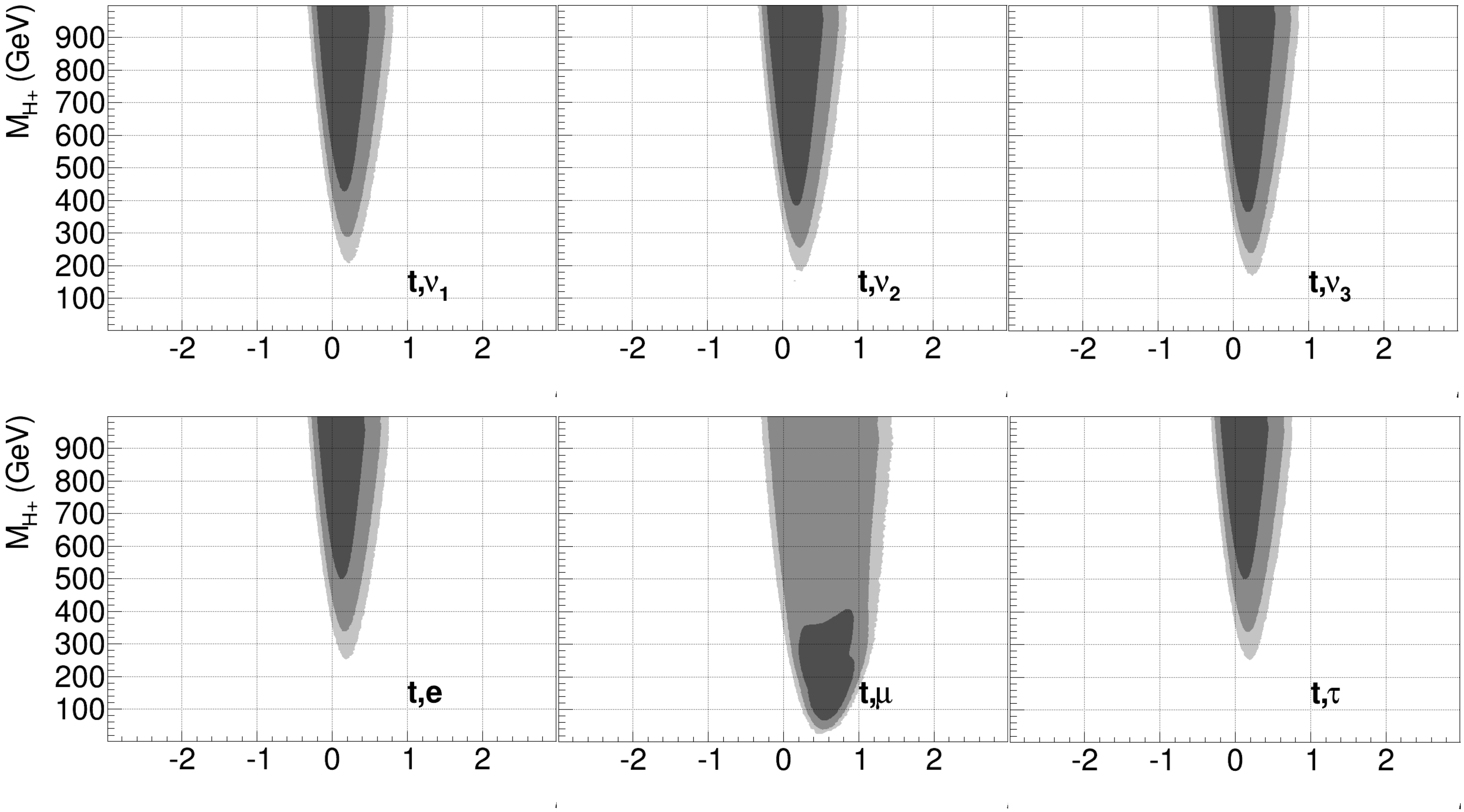
Effect of the oblique parameters constraints in model (t, τ)



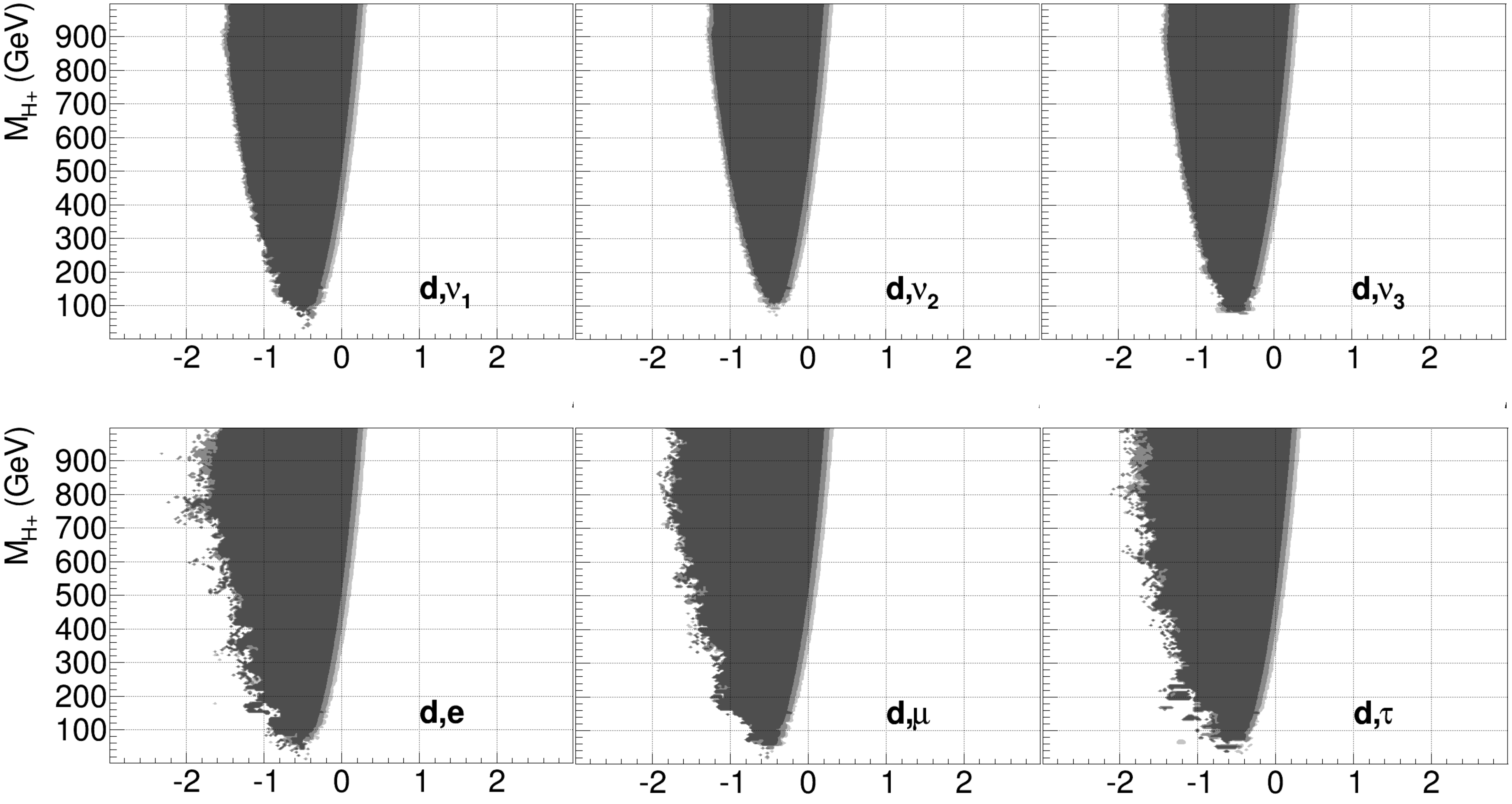
M_{H^+} vs. $\log_{10}(\tan\beta)$, u models



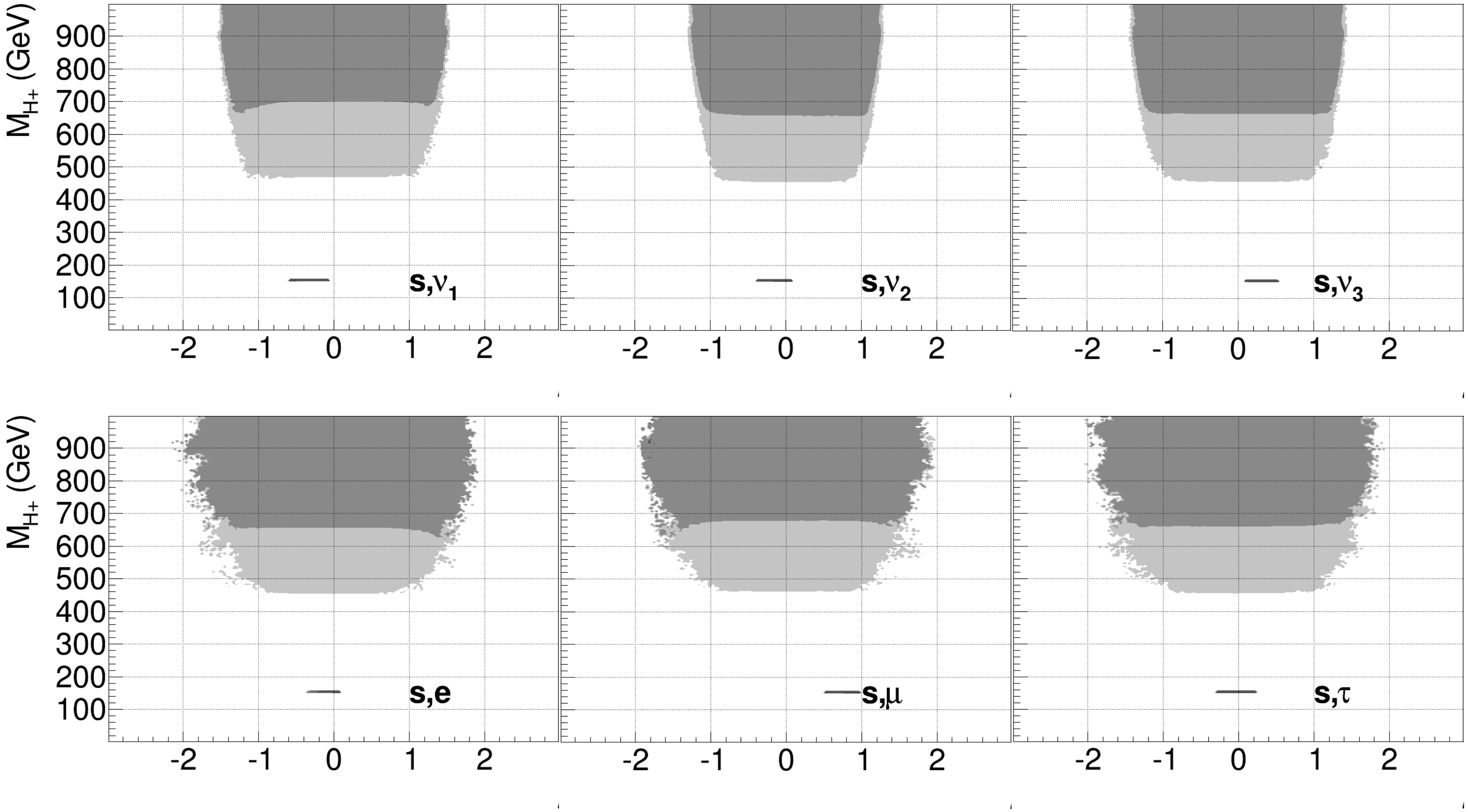
M_{H^+} vs. $\log_{10}(\tan\beta)$, c models



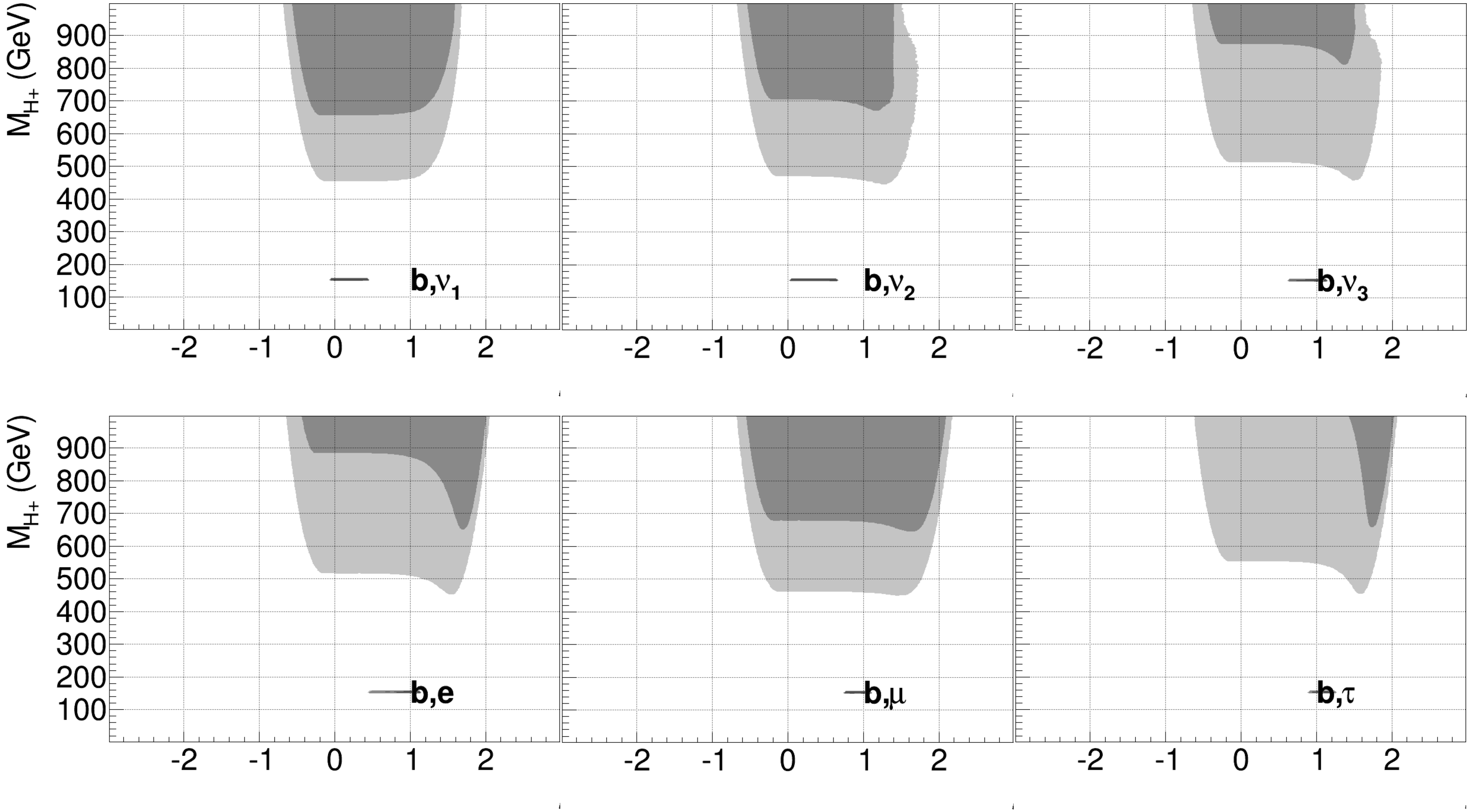
M_{H^+} vs. $\log_{10}(\tan\beta)$, t models



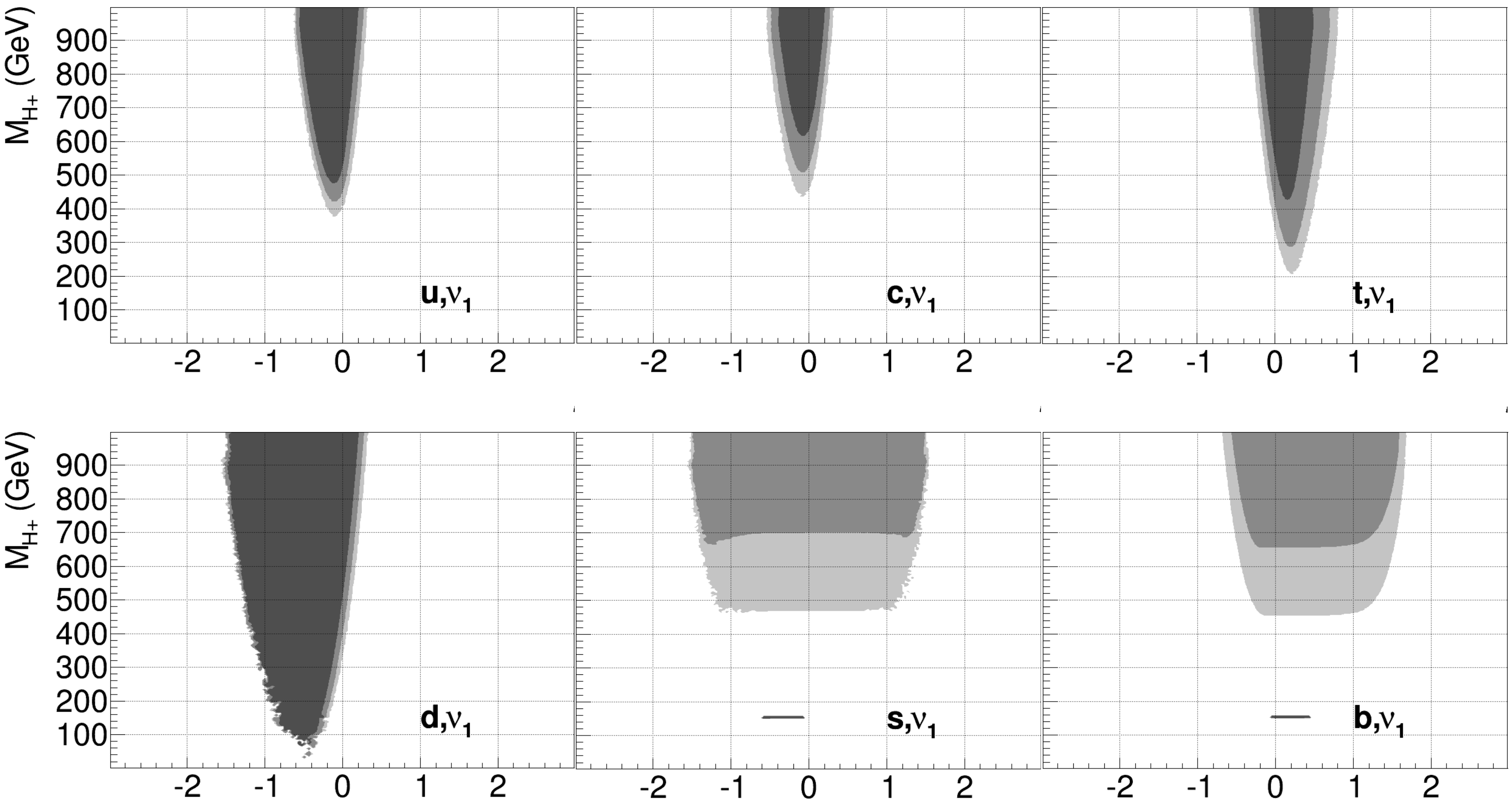
M_{H^+} vs. $\log_{10}(\tan\beta)$, d models



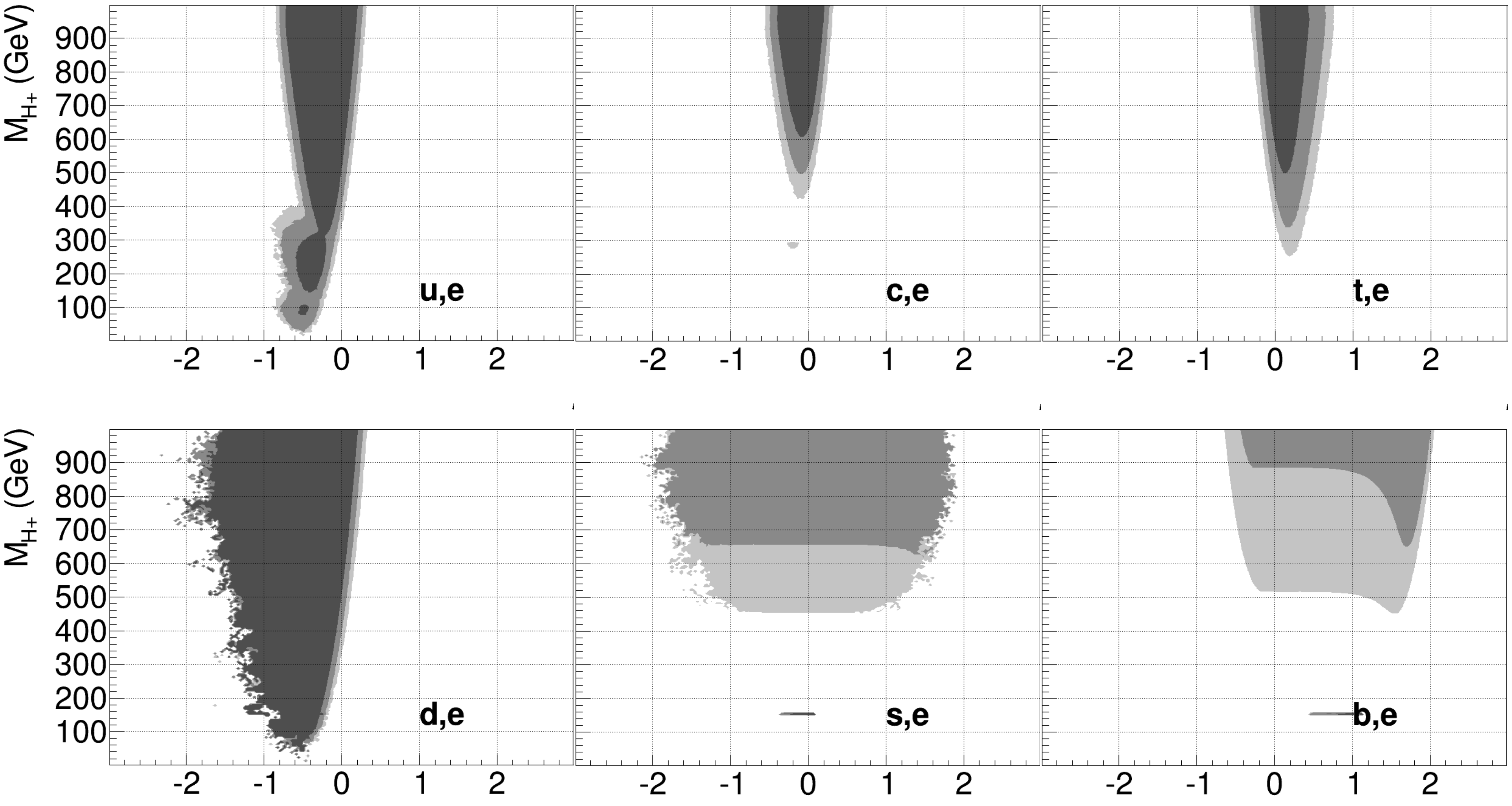
M_{H^+} vs. $\log_{10}(\tan\beta)$, s models



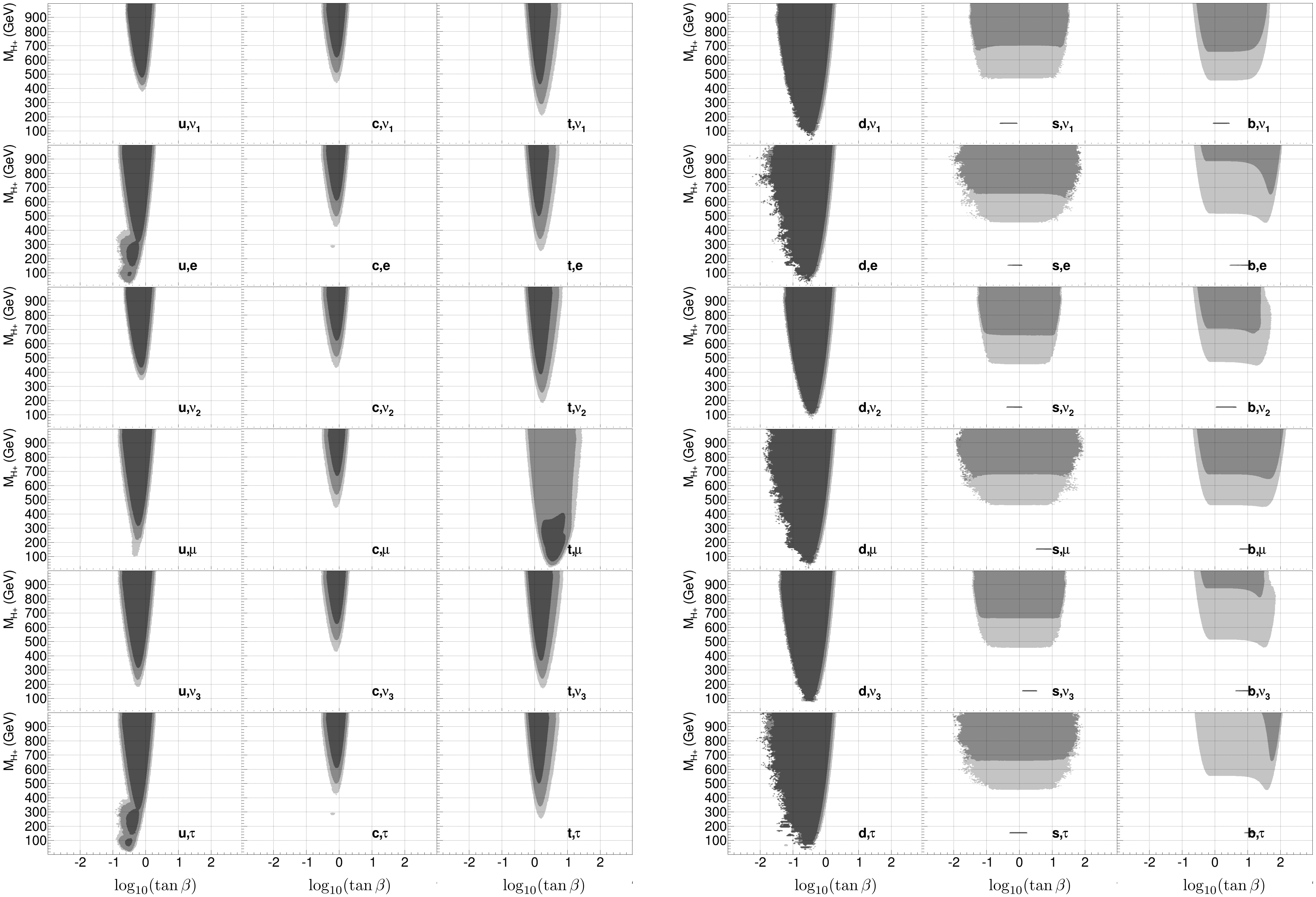
M_{H^+} vs. $\log_{10}(\tan\beta)$, b models



M_{H^+} vs. $\log_{10}(\tan\beta)$, ν_1 models



M_{H^+} vs. $\log_{10}(\tan\beta)$, e models


M_{H+} (GeV)
100
200
300
400
500
600
700
800
900
u,ν1
c,ν1
t,ν1
d,ν1
s,ν1
b,ν1
u,e
c,e
t,e
d,e
s,e
b,e
u,ν2
c,ν2
t,ν2
d,ν2
s,ν2
b,ν2
u,μ
c,μ
t,μ
d,μ
s,μ
b,μ
u,ν3
c,ν3
t,ν3
d,ν3
s,ν3
b,ν3
u,τ
c,τ
t,τ
d,τ
s,τ
b,τ
-2
-1
0
1
2
log10(tan β)

$m_{H^\pm} > 380$ GeV from $b \to s\gamma$ in type II 2HD

In BGL several of the models allow

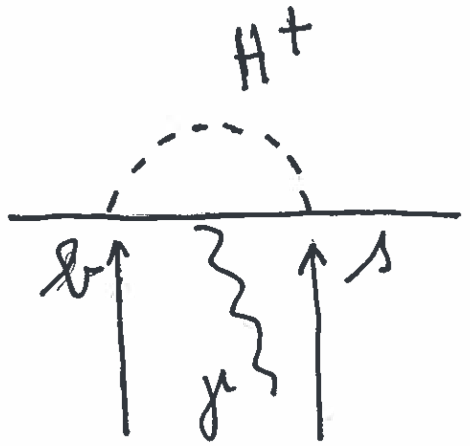
$m_{H^\pm} < 380$ GeV

In BGL $H^\pm$ dominates NP

$\tan\beta$ dependence $\{-1, \tan^2\beta, 1/\tan^2\beta\}$

$$\begin{pmatrix} t & & \\ & t & \\ & & -\frac{1}{t} \end{pmatrix}$$

in different positions



$t \qquad t \sim t^2$

$t \qquad -1/t \sim -1$ flat

$-\frac{1}{t} \qquad -1/t \sim 1/t^2$

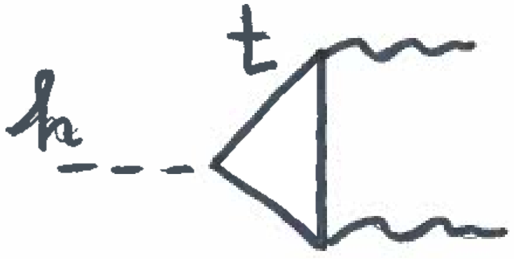
neutral scalars

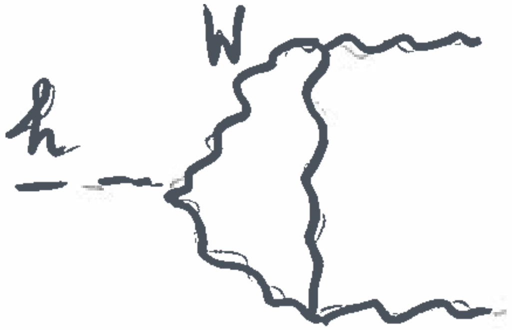
- most cases, negligible contribution from R, I
- otherwise these two contributions tend to cancel out

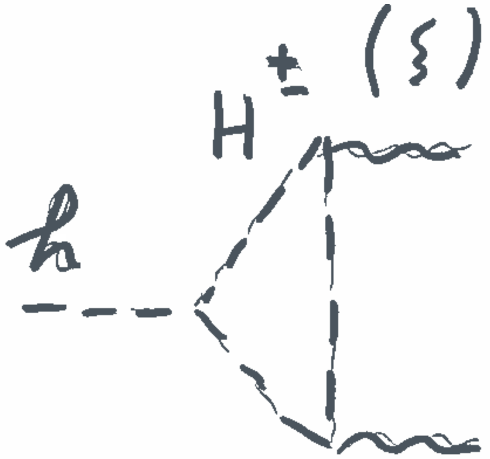
Study of charged Higgs contribution to $h \to \gamma\gamma$, $h \to Z\gamma$

$\beta - \alpha = \frac{\pi}{2}$ $m_h = 125\,\text{GeV}$ $m_\xi > 100\,\text{GeV}$

unitarity of scattering amplitudes
global stability of the potential
oblique electroweak T parameter







$$\mu_{\gamma\gamma} = \frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{SM}(h \to \gamma\gamma)} \qquad \mu_{Z\gamma} = \frac{\Gamma(h \to Z\gamma)}{\Gamma^{SM}(h \to Z\gamma)}$$

$m_\xi - \mu_{\gamma\gamma}$ plane $\mu_{\gamma\gamma}$ versus $\mu_{Z\gamma}$

Bhattacharyya, Das, Pal, MNR (2013)

h mediated FCNC (arXiv:1508.05101)

Flavour changing decays of top quarks

$$Y^U_{qt}(d_\rho) = -V_{q\rho}V^*_{t\rho}\,\frac{m_t}{v}\,c_{\beta\alpha}(t_\beta + t_\beta^{-1})\,, \quad q = u, c\,.$$

Model	$t \to hu$	$t \to hc$
d	$\lvert V_{ud}V_{td}\rvert^2\,(\sim \lambda^6) = 7.51 \cdot 10^{-5}$	$\lvert V_{cd}V_{td}\rvert^2\,(\sim \lambda^8) = 4.01 \cdot 10^{-6}$
s	$\lvert V_{us}V_{ts}\rvert^2\,(\sim \lambda^6) = 8.20 \cdot 10^{-5}$	$\lvert V_{cs}V_{ts}\rvert^2\,(\sim \lambda^4) = 1.53 \cdot 10^{-3}$
b	$\lvert V_{ub}V_{tb}\rvert^2\,(\sim \lambda^6) = 1.40 \cdot 10^{-5}$	$\lvert V_{cb}V_{tb}\rvert^2\,(\sim \lambda^4) = 1.68 \cdot 10^{-3}$

$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| \lesssim 4.9$ for b and s type models

Flavour changing Higgs decays

The decays $h \to \ell\tau$ $(\ell = \mu, e)$

$$Y^\ell_{\mu\tau}(\nu_\rho) = \frac{1}{v}\,c_{\beta\alpha}\left(N_\ell^{(\nu_\sigma)}\right)_{\mu\tau} = -c_{\beta\alpha}(t_\beta + t_\beta^{-1})\,U_{\mu\sigma}U^*_{\tau\sigma}\,\frac{m_\tau}{v}$$

Model	$h \to e\mu$	$h \to e\tau$	$h \to \mu\tau$
ν_1	$\lvert U_{e1}U_{\mu1}\rvert^2(\sim \frac{1}{9}) = 0.105$	$\lvert U_{e1}U_{\tau1}\rvert^2(\sim \frac{1}{9}) = 0.118$	$\lvert U_{\mu1}U_{\tau1}\rvert^2(\sim \frac{1}{36}) = 0.028$
ν_2	$\lvert U_{e2}U_{\mu2}\rvert^2(\sim \frac{1}{9}) = 0.089$	$\lvert U_{e2}U_{\tau2}\rvert^2(\sim \frac{1}{9}) = 0.126$	$\lvert U_{\mu2}U_{\tau2}\rvert^2(\sim \frac{1}{9}) = 0.115$
ν_3	$\lvert U_{e3}U_{\mu3}\rvert^2 = 0.0128$	$\lvert U_{e3}U_{\tau3}\rvert^2 = 0.0097$	$\lvert U_{\mu3}U_{\tau3}\rvert^2(\sim \frac{1}{4}) = 0.234$

$|c_{\beta\alpha}(t_\beta + t_\beta^{-1})| \sim 1$ to produce $\mathrm{Br}(h \to \mu\bar{\tau} + \tau\bar{\mu})$ of order 10^{-2}

Flavour changing Higgs decays

The flavour changing decays $\boldsymbol{h \to bq}$ $(\boldsymbol{q = s, d})$

$$Y^D_{qb}(u_k) = -c_{\beta\alpha}(t_\beta + t_\beta^{-1})\, V^*_{kq} V_{kb}\, \frac{m_b}{v}\,, \;\; q \neq b, \;\text{no sum in}\; k$$

Model	$h \to bd$	$h \to bs$
u	$\lvert V_{ud}V_{ub}\rvert^2\,(\sim \lambda^6) = 1.33 \cdot 10^{-5}$	$\lvert V_{us}V_{ub}\rvert^2\,(\sim \lambda^8) = 7.14 \cdot 10^{-7}$
c	$\lvert V_{cd}V_{cb}\rvert^2\,(\sim \lambda^6) = 8.52 \cdot 10^{-5}$	$\lvert V_{cs}V_{cb}\rvert^2\,(\sim \lambda^4) = 1.59 \cdot 10^{-3}$
t	$\lvert V_{td}V_{tb}\rvert^2\,(\sim \lambda^6) = 7.90 \cdot 10^{-5}$	$\lvert V_{ts}V_{tb}\rvert^2\,(\sim \lambda^4) = 1.61 \cdot 10^{-3}$

- in models c and t,

$$\mathrm{Br}(h \to \bar{b}s + b\bar{s}) \sim c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,\lambda^4 \;\sim 10^{-3}\,c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,,$$

- in model u,

$$\mathrm{Br}(h \to \bar{b}s + b\bar{s}) \sim c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,\lambda^8 \;\sim 10^{-7}\,c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,,$$

- in all u, c and t models,

$$\mathrm{Br}(h \to \bar{b}d + b\bar{d}) \sim c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,\lambda^6 \;\sim 10^{-5}\,c^2_{\beta\alpha}(t_\beta + t_\beta^{-1})^2\,.$$

Important correlations among Observables

BGL and the Cheng and Sher ansatz

$$|Y_{\mu\tau}| \leq \sqrt{m_\mu m_\tau}/v$$

neutrino type k model in BGL:

$$\begin{aligned} Y_{\mu\tau} &= -c_{\alpha\beta}\left(t+t^{-1}\right) U_{\mu k} U^*_{\tau k} \frac{m_\tau}{v} \\ Y_{\tau\mu} &= -c_{\alpha\beta}\left(t+t^{-1}\right) U_{\tau k} U^*_{\mu k} \frac{m_\mu}{v} \end{aligned}$$

$$|Y_{\tau\mu} Y_{\mu\tau}| = \left|c_{\alpha\beta}\left(t+t^{-1}\right)\right|^2 \left|U_{\mu k} U^*_{\tau k}\right| \left|U_{\tau k} U^*_{\mu k}\right| \frac{m_\mu m_\tau}{v^2}$$

BGL meets CS criterium provided:

$$\left|c_{\alpha\beta}\left(t+t^{-1}\right)\right|^2 \left|U_{\mu k} U^*_{\tau k}\right| \left|U_{\tau k} U^*_{\mu k}\right| \leq 1$$

$$\left|c_{\alpha\beta}\left(t+t^{-1}\right)\right| \lesssim 3$$

2HDM with NFC or flavour alignment have no HFCNC but have tree level charged Higgs mediated processes

Pich, Tuzon, Phys.Rev.D80:091702,2009

Mahmoud, Stal, Phys.Rev.D81:035016,2010

Enomoto, Watanabe,arXiv:1509.00491

2HDM of type III, i.e, models models where the Cheng-Sher Ansatz is assumed for the FC couplings:

$$\xi_{ij} = \lambda_{ij} \sqrt{m_i m_j} \frac{\sqrt{2}}{v},$$

where the λ_{ij} are of order one.

allow for scalar masses well below the TeV scale

Crivellin, KoKulu,Greub, Phys. Rev. D 87, 094031 (2013)

Gaitan,Garces,Martinez, de Oca, arXiv:1503:04391

Altunkaynak, Hou, Kao, Kohda, McCoy, arXiv:1506.00651

Arhrib, Benbrik, Chen, Gomez-Bock, Semlali, arXiv:1508.06490

Kim, Yoon, Yuan, arXiv:1509.00491

Benbrik, Chen, Nomura,Phys. Rev. D 93, 095004 (2016)

Conclusions

HFCNC at tree level are not ruled out even allowing for scalar masses of the order of a few hundred GeV

There are several promising scenarios within the 36 models that were presented.

Bhattacharyya, Das, Kundu 2014

The LHC may bring us interesting surprises!

BGL and the leptonic sector

Minimal Flavour Violation with Majorana neutrinos

Low energy effective theory and stability

$$\mathcal{L}_{\text{Majorana}} = \frac{1}{2} \nu_L^{0T} C^{-1} m_\nu \nu_L^0 + h.c.$$

generated from effective dimension five operator

$$\mathcal{O} = \sum_{i,j=1}^{2} \sum_{\alpha,\beta=e,\mu,\tau} \sum_{a,b,c,d=1}^{2} \left(L_{L\alpha a}^T \kappa_{\alpha\beta}^{(ij)} C^{-1} L_{L\beta c} \right) \left(\varepsilon^{ab} \phi_{ib} \right) \left(\varepsilon^{cd} \phi_{jd} \right)$$

$$\mathcal{L}_{Y_\ell} = - \overline{L_L^0}\, \Pi_1 \phi_1 \ell_R^0 - \overline{L_L^0}\, \Pi_2 \phi_2 \ell_R^0 + h.c.$$

Π_1, Π_2, κ^{11}, κ^{12}, κ^{21}, κ^{22} $\quad (\kappa^{(ij)})$

$L_{Lj}^0 \rightarrow \exp(i\alpha) L_{Lj}^0$, $\phi_2 \rightarrow \exp(i\alpha)\, \phi_2$

$\alpha = \pi/2$, Z_4 symmetry

Imposing this Z_4 symmetry implies: $(j=3)$

$$\kappa^{(12)} = \kappa^{(21)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\kappa^{(11)} = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \kappa^{(22)} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}$$

$\alpha = \pi/2$
ensures
$\kappa^{(22)}_{33} \neq 0$

$$\frac{1}{2} m_\nu = \frac{1}{2} v_1^2 \kappa^{(11)} + \frac{1}{2} v_2^2 e^{2i\theta} \kappa^{(22)}$$

$$\Pi_1 = \begin{bmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{bmatrix}$$

Higgs FCNC in the charged sector

Stability: $\kappa^{(12)} = \kappa^{(21)} = 0$

$\kappa^{(11)} P_3^\nu = 0$ $\quad$ $\kappa^{(22)} P_3^\nu = \kappa^{(22)}$

$P_3^\nu \Pi_1 = 0$ $\quad$ $P_3^\nu \Pi_2 = \Pi_2$

stable under renormalization

W. Grimus, L. Lavoura 2005

Seesaw framework

$$\mathcal{L}_Y + \text{mass} = -\bar{L}_L^0 \Pi_1 \phi_1 \ell_R^0 - \bar{L}_L^0 \Pi_2 \phi_2 \ell_R^0 -$$

$$- \bar{L}_L^0 \Sigma_1 \tilde{\phi}_1 \nu_R^0 - \bar{L}_L^0 \Sigma_2 \tilde{\phi}_2 \nu_R^0 +$$

$$+ \frac{1}{2} \nu_R^{0T} C^{-1} M_R \nu_R^0 + h.c.$$

$$m_\ell = \frac{1}{\sqrt{2}} \left(v_1 \Pi_1 + v_2 e^{i\theta} \Pi_2 \right) , \quad m_D = \frac{1}{\sqrt{2}} \left(v_1 \Sigma_1 + v_2 e^{-i\theta} \Sigma_2 \right)$$

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}} W_\mu^+ \bar{\ell}_L^0 \gamma^\mu \nu_L^0 + h.c$$

$$\mathcal{L}_Y (\text{neutral, lepton}) = -\bar{\ell}_L^0 \frac{1}{v} \left[m_\ell H^0 + N_\ell^0 R + i N_\ell^0 I \right] \ell_R^0 -$$

$$- \bar{\nu}_L^0 \frac{1}{v} \left[m_D H^0 + N_\nu^0 R + i N_\nu^0 I \right] \nu_R^0 + h.c.$$

$$N_\ell^0 = \frac{v_2}{\sqrt{2}} \Pi_1 - \frac{v_1}{\sqrt{2}} e^{i\theta} \Pi_2$$

$$N_\nu^0 = \frac{v_2}{\sqrt{2}} \Sigma_1 - \frac{v_1}{\sqrt{2}} e^{-i\theta} \Sigma_2$$

$$\mathcal{L}_{mass} = - \bar{\ell}_L^0 m_\ell \ell_R^0 + \frac{1}{2} \left(\nu_L^{0T} , (\nu_R^0)^{cT} \right) C^{-1} \mathcal{M}^* \begin{pmatrix} \nu_L^0 \\ (\nu_R^0)^c \end{pmatrix} + hc$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix} \qquad (\psi_L)^c \equiv C \gamma_0^T (\psi_L)^*$$

BGL type example , Z_4 symmetry

$L_{L3}^0 \rightarrow \exp(i\alpha) L_{L3}^0$, $\nu_{R3}^0 \rightarrow \exp(i 2\alpha) \nu_{R3}^0$, $\phi_2 \rightarrow \exp(i\alpha) \phi_2$

$\alpha = \frac{\pi}{2}$

$$\Pi_1 \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix} , \quad \Pi_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix} , \quad \Sigma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix} , \quad M_R = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}$$

New feature m_{ν_i} from $m_{eff} \equiv - m_D \frac{1}{M_R} m_D^T$ $\qquad M_{33} \neq 0$

Flavour structure (quark sector)

M_d, M_u, N_d^0, N_u^0

Freedom of choice of WB

Zero textures are WB dependent

Symmetries are only apparent in particular WB

WB transformations do not change the physics

Symmetries have physical implications

Above four matrices encode breaking of flavour symmetry present in gauge sector

large redundancy of parameters

WB invariants are very useful to study flavour

Three light neutrinos ν_i , plus heavy neutrinos N_j

light-light , light-heavy, heavy-heavy couplings

H^0, R, I couplings

$U^\dagger m_{eff} U^* = d$, $m_D \frac{1}{D} m_D^T = - U d U^T$ (WB M_D diag)

$m_D = i\, U \sqrt{d}\, O^c \sqrt{D}$ Casas and Ibarra, 2001

$$(N_\ell)_{ij} = \frac{v_2}{v_1} (D_\ell)_{ij} - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (U_\nu^\dagger)_{i3} (U_\nu)_{3j} (D_\ell)_{jj}$$

light-light neutral couplings : diag , d

light-heavy neutral couplings : sensitive to O^c, d, D

heavy-heavy neutral couplings: diag , sensitive to O^c, d, D

H^+ couplings

$$\frac{\sqrt{2} H^+}{v} \left(\bar{\nu}_L^0 N_\ell^0 \ell_R - \bar{\nu}_R^0 N_\nu^{0\dagger} \ell_L^0 \right) + h.c.$$

Examples of WB invariants

$\mathrm{tr}\,(H_u H_d)$, $\mathrm{tr}\,(H_u H_d^2)$

$\mathrm{tr}\,(H_u^2 H_d)$, $\mathrm{tr}\,(H_u^2 H_d^2)$

↓

V_{CKM} ambiguity in sign $\mathrm{Im}\, Q$

$$Q_{\alpha i \beta j} = V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \quad \begin{matrix}(\alpha \neq \beta)\\(i \neq j)\end{matrix}$$

Branco, Lavoura, 1988

WB also very useful to study CP violation

$$I_1^{CP} \equiv \mathrm{tr}\,[H_u, H_d]^3 = 6i\,(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \times$$

$$\times\,(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\ \mathrm{Im}\ Q_{uscb}$$

Bernabeu, Branco, Gronau 1986

det $[H_u, H_d]$ Jarlskog, 1985 3 generations

One can check predictions of flavour model comparing invariant quantities with their corresponding experimental values

In 2HDM one can build new WB invariants which do not arise SM

Special WB's M_d diagonal, $N_d^0 \equiv N_d$

or M_u diagonal, $N_u^0 \equiv N_u$

$$I_1 \equiv \mathrm{tr}\,(M_d N_d^{0\dagger}) = m_d (N_d^*)_{11} + m_s (N_d^*)_{22} + m_b (N_b^*)_{33}$$

not sensitive to HFCNC

$\mathrm{Im}\, I_1$ probes phases of $(N_d)_{jj}$ (electric dipole moment d quarks

$$I_2 \equiv \mathrm{tr}\,[M_d N_d^0, M_d M_d^\dagger]^2$$ sensitive to off-diag elements N_d

$$I_1^{CP} \propto \mathrm{Im}\, Q_{uscb}\ , \quad V_{CKM} = U_{u_L}^\dagger U_{d_L}$$

$U_{u_L} \neq U_{d_L}$ misalignment of the matrices H_d, H_u

analogously

$$I_3^{CP} \equiv \mathrm{tr}\,[H_d, H_{N_d^0}]^3 = 6i\, \Delta_d\, \Delta_{N_d}\, \mathrm{Im}\, Q_3\ , \quad V_3 \equiv U_{d_L}^\dagger U_{N_d^0 L}$$

$$H_{N_d^0} = N_d^0 N_d^{0\dagger}$$

$$I_2^{CP} \equiv \mathrm{tr}\,[H_u, H_{N_d^0}]^3 = 6i\, \Delta_u\, \Delta_{N_d}\, \mathrm{Im}\, Q_2\ , \quad V_2 = U_{u_L}^\dagger U_{N_d^0 L}$$

$$I_6^{CP} \equiv \mathrm{tr}\,[H_{N_d^0}, H_{N_u^0}]^3$$

and many more

V_{CKM}, V_2, V_3 signal misalignment in flavour space of Hermitian matrices constructed in the framework of 2HDM

So far, we have only written invariants which are sensitive to left-handed mixings

One can construct analogous invariants which are sensitive to right-handed mixings, like:

$$I_7^{CP} \equiv \text{Tr}\left[H_d' , H_{N_d^0}' \right]^3 = 6i\, \Delta_d\, \Delta_{N_d}\, \text{Im}\, Q_7$$

$$H_d' = M_d^\dagger M_d \quad , \quad H_{N_d^0}' = N_d^{0\dagger} N_d^0$$

Q_7 rephasing invariant quartet of $U_{dR} U_{N_d R}^{0\dagger}$

and again many more

The Minimal Flavour Violation case

Lowest invariant sensitive to CP violation

$$I_q^{CP} \equiv \operatorname{Im} \operatorname{tr} \left[M_d N_d^{0\dagger} M_d M_d^\dagger M_u M_u^\dagger M_d M_d^\dagger \right]$$

must contain flavour matrices from the up and down sector

lower order in powers of mass than SM case ($\operatorname{tr} [H_u, H_d]^3 \propto 12$)

BGL type models have richer flavour structure parametrized by four matrices

$$I_q^{CP} (\gamma = u, i=3) = - \left(\frac{v_2}{v_1} + \frac{v_1}{v_2} \right) (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2) \times$$

$$\times (m_c^2 - m_u^2) \operatorname{Im} \left(V_{22}^* V_{32} V_{33}^* V_{23} \right)$$

FCNC in down sector, P_3

I_q^{CP} controlled by V_{CKM} (BGL)

$I_q^{CP} \neq 0$ even if $m_t = m_c$ or $m_t = m_u$ since discrete symmetry singles out top quark

I_q^{CP} can be related to baryon asymmetry generated at EW phase transition