CONTINUOUS OPTIMIZATION

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# From hot steel rolling to the design of pipe systems in Buildings

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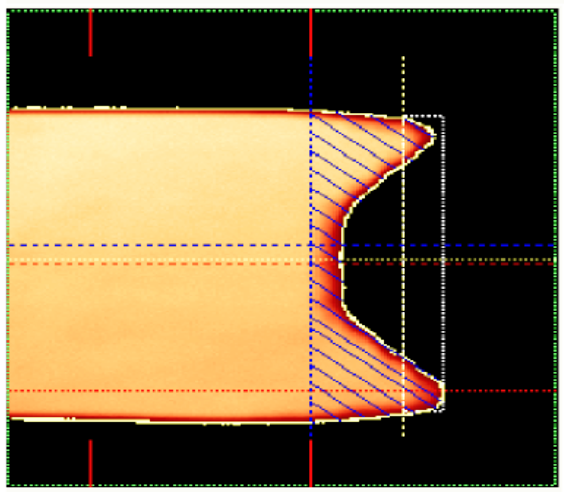
PDE problems VII, Benasque 2017

# Application example: hot rolling process



# Outline

**Optimal control of edger position in roughing mills**

- Formulation of state problem
- Formulation of optimization problem
- Sensitivity analysis with direct differentiation

**Numerical results**

**Conclusion and outlook**

# Hot rolling – governing equations

- modeling hot rolling process leads to nonlinear hyperbolic IBVP

$$\varrho^r \ddot{\boldsymbol{u}} - \nabla \cdot \boldsymbol{P}^\sigma(\boldsymbol{u}) = \boldsymbol{0} \qquad \text{in } \Omega \times (0,T)$$

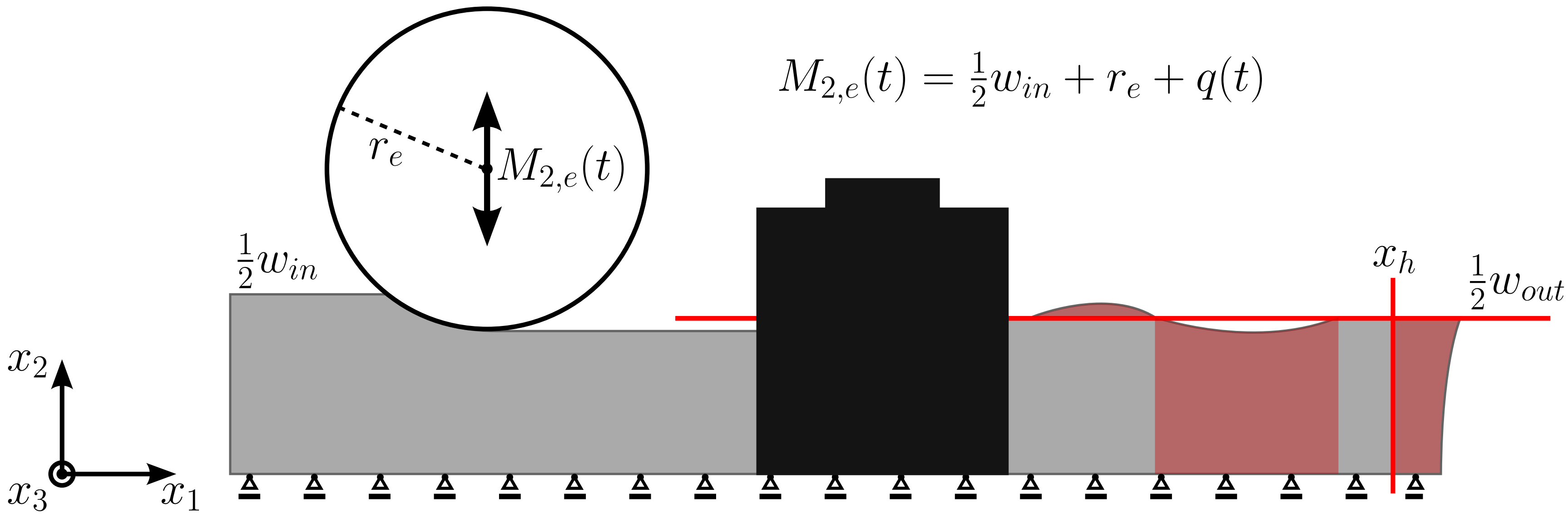
$$+ \text{ ICs } + \text{ symmetry BCs } + \text{ contact BCs}$$

- frictional contact between slab and rolls described by complementarity conditions or subdifferentials of indicator functions [Pietrzak, Curnier (1999)]
  - normal: impenetrability and compressive contact pressure
  - tangential: relative movement of contacting bodies restricted due to Coulomb friction law
- elasto-viscoplastic material model with nonlinear hardening [Simo (1988)]
  - based on multiplicative split of deformation gradient into elastic and viscoplastic parts
  - plastic yielding described by system of nonlinear ODEs and complementarity conditions
- weak form after Augmented Lagrangian regularization of contact forces

$$\frac{d^2}{dt^2}(\varrho^r \boldsymbol{u}(t)\,|\,\boldsymbol{\eta}) + F^\sigma(\boldsymbol{u}(t),\boldsymbol{\eta}) + \widetilde{F}^c(\boldsymbol{u}(t),\lambda_\nu(t),\boldsymbol{\lambda}_\tau(t),\boldsymbol{\eta}) = 0 \qquad \forall\, \boldsymbol{\eta} \in \boldsymbol{V}$$

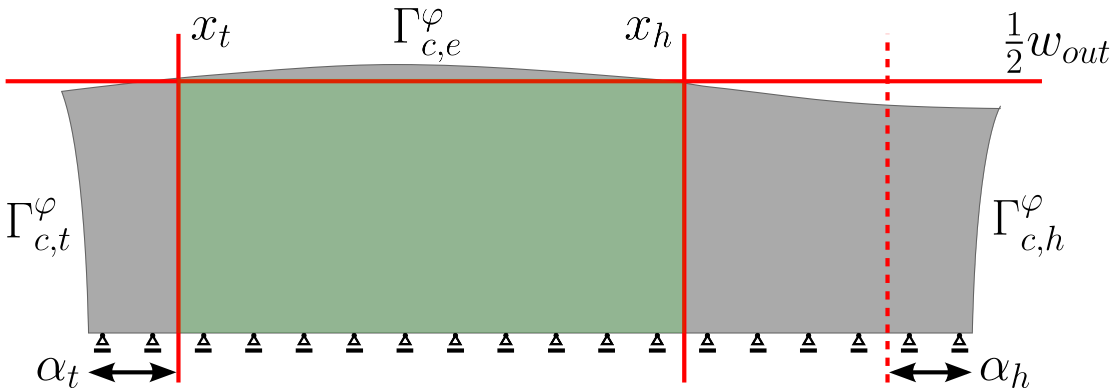
$$\boldsymbol{V} = \left\{ \boldsymbol{v} \in \left[H^1(\Omega)\right]^3 : \boldsymbol{v} \cdot \boldsymbol{n}^r = 0 \text{ on } \Gamma_s \right\}$$

# Optimal control of edger position in roughing mills




- slab head/tail will be cut off at $x_h/x_t$ after leaving the roughing mill
- slab shape should in the end be as rectangular as possible
- slab should have a prescribed end width $w_{out}$
- underwidth much more critical than overwidth
- control $q(t)$: $x_2$-position of edger over time interval $[0, T]$
- assumption: $q$ Lipschitz continuous, i.e. $q \in C^{0,1}(0, T)$

# Cost function and constraints




$$\min_{(q,x_t,x_h)\in \boldsymbol{Q}_{ad}} \mathcal{J}(q,x_t,x_h) = 4V_\Omega(\boldsymbol{\mathcal{S}}(q)) - (x_h - x_t)w_{out}h_{out} + \varrho_q R(q)$$

$$\boldsymbol{Q}_{ad} = \Big\{ (q,\, x_t,\, x_h) \in C^{0,1}(0,T)\times \mathbb{R}\times\mathbb{R} \; :$$

$\boldsymbol{u}(T) = \boldsymbol{\mathcal{S}}(q)$ solves the hot rolling problem,

$$x_1 + u_1(T)(\boldsymbol{x}) \le x_t - \alpha_t \qquad \forall \boldsymbol{x} \in \overline{\Gamma}_{c,t},$$

$$x_h + \alpha_h \le x_1 + u_1(T)(\boldsymbol{x}) \quad \forall \boldsymbol{x} \in \overline{\Gamma}_{c,h},$$

$$\tfrac{1}{2}w_{out} \le x_2 + u_2(T)(\boldsymbol{x}) \quad \forall \boldsymbol{x} \in \overline{\Gamma}_{c,e} : x_1 + u_1(T)(\boldsymbol{x}) \in [x_t, x_h],$$

$$-\alpha_s^- \le \dot{q} \le \alpha_s^+,$$

$$-\tfrac{1}{2}\Delta w_{max} \le q \le -\tfrac{1}{2}\Delta w_{min} \Big\}$$

# Regularization of non-differentiabilities

- gradient-based optimization algorithms require cost functional and constraints to be continuously differentiable with respect to the design variables
- problem: control-to-observation map $\boldsymbol{\mathcal{S}} : C^{0,1}(0,T) \to \boldsymbol{V}$ is not differentiable due to changes of state (elastic $\leftrightarrow$ plastic, separation $\leftrightarrow$ contact, stick $\leftrightarrow$ slip)
- solution: regularization of non-differentiabilities
  - similar approaches used in [Duvaut, Lions (1976)], [Eck (1996)], [Wachsmuth (2012)]
  - typical example: replace $\min\{c, \cdot\}$, $c = \mathrm{const.}$, by smooth regularization, e.g.

$$\min_\varepsilon(c; z) = \begin{cases} z & \text{if } z \le c - \varepsilon, \\ -\dfrac{1}{4\varepsilon}\left[z - c - \varepsilon\right]^2 + c & \text{if } z \in (c-\varepsilon, c+\varepsilon), \\ c & \text{if } z \ge c + \varepsilon \end{cases}$$

  - result: regularized hot rolling problem

$$\frac{d^2}{dt^2}\left(\varrho^r \boldsymbol{u}(t) \,|\, \boldsymbol{\eta}\right) + F^{\sigma,\varepsilon}(\boldsymbol{u}(t), \boldsymbol{\eta}) + \widetilde{F}^{c,\varepsilon}(\boldsymbol{u}(t), \lambda_\nu(t), \boldsymbol{\lambda}_\tau(t), \boldsymbol{\eta}) = 0 \qquad \forall\, \boldsymbol{\eta} \in \boldsymbol{V}$$

  - use regularized control-to-observation map $\boldsymbol{\mathcal{S}}^\varepsilon$ to replace $\boldsymbol{\mathcal{S}}$ in optimization problem

# Sensitivity analysis with direct differentiation

- # (design variables) $\ll$ # (constraints), path-dependent state problem
- consequence: Direct Differentiation Method (DDM), e.g. by [Kowalczyk (2006)], in this case more efficient and less memory-consuming than adjoint methods
- sensitivities are computed simultaneously with solution of state problem

$$\frac{d^2}{dt^2}(\varrho^r \boldsymbol{u}(t) \,|\, \boldsymbol{\eta}) + F^{\sigma,\varepsilon}(\boldsymbol{u}(t), \boldsymbol{\eta}) + \widetilde{F}^{c,\varepsilon}(\boldsymbol{u}(t), \lambda_\nu(t), \boldsymbol{\lambda}_\tau(t), \boldsymbol{\eta}) = 0 \qquad \forall\, \boldsymbol{\eta} \in \boldsymbol{V}$$

$\Downarrow$ (spatial & temporal discretization)

$$\frac{1-\alpha_m}{\beta_N (\Delta t_n)^2} \boldsymbol{M} \hat{\boldsymbol{u}}_n + (1-\alpha_f)\left[\boldsymbol{F}^{\sigma,\varepsilon}(\hat{\boldsymbol{u}}_n) + \widetilde{\boldsymbol{F}}^{c,\varepsilon}(\hat{\boldsymbol{u}}_n,\, \lambda_{\nu,n},\, \boldsymbol{\lambda}_{\tau,n})\right] = \widehat{\boldsymbol{F}}^{\alpha,\varepsilon}_{n-1} \quad \overset{\text{(linearization)}}{\Longrightarrow} \quad \ldots$$

$\Downarrow$ (direct differentiation)

$$\frac{1-\alpha_m}{\beta_N (\Delta t_n)^2} \boldsymbol{M} \frac{d\hat{\boldsymbol{u}}_n}{d\boldsymbol{q}} + (1-\alpha_f)\left[\frac{d\boldsymbol{F}^{\sigma,\varepsilon}}{d\boldsymbol{q}}(\hat{\boldsymbol{u}}_n) + \frac{d\widetilde{\boldsymbol{F}}^{c,\varepsilon}}{d\boldsymbol{q}}(\hat{\boldsymbol{u}}_n, \lambda_{\nu,n}, \boldsymbol{\lambda}_{\tau,n})\right] = \frac{d\widehat{\boldsymbol{F}}^{\alpha,\varepsilon}_{n-1}}{d\boldsymbol{q}}$$

- $\boldsymbol{q} = [q(t_1^q), \ldots, q(t_{n_q}^q)]^T$ results from piecewise linear time discretization of control $q$

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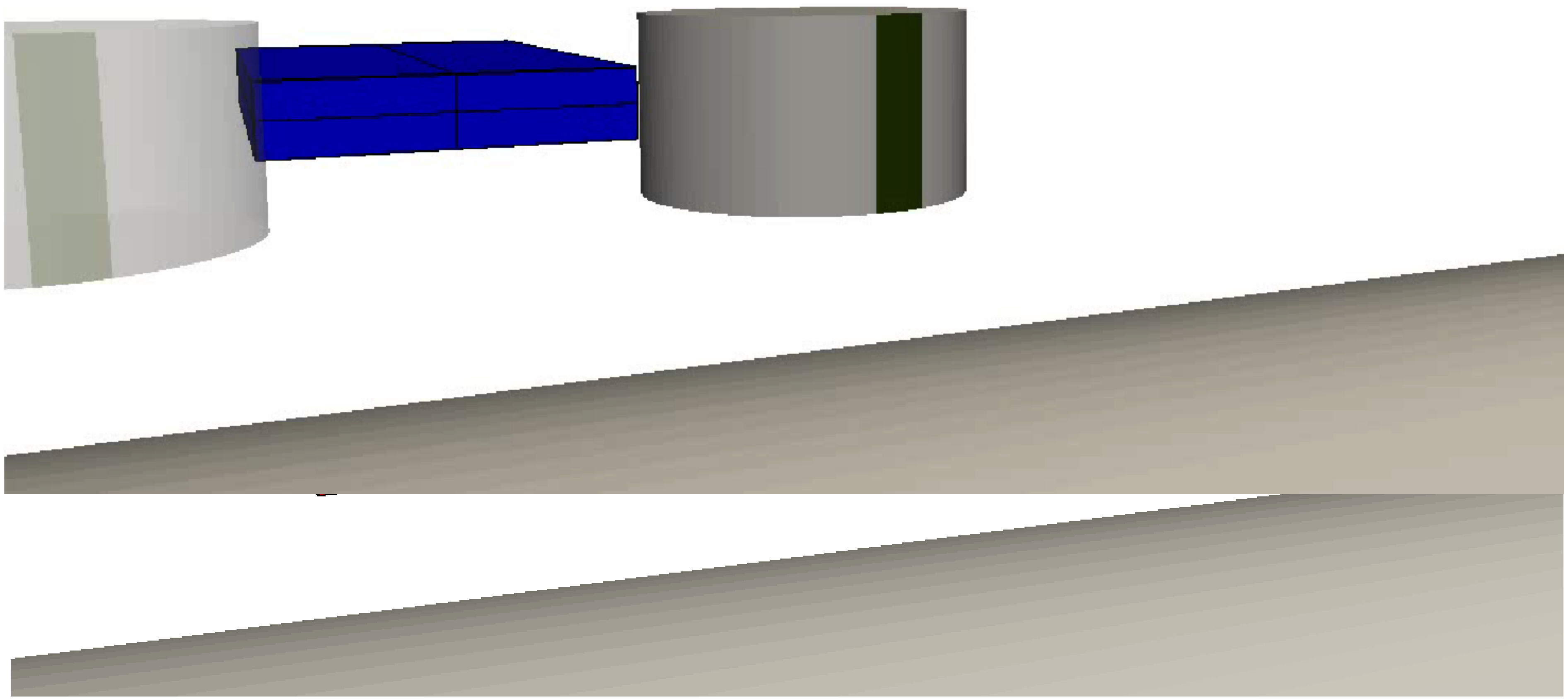
Optimal control of edger position in roughing mills

**Numerical results**

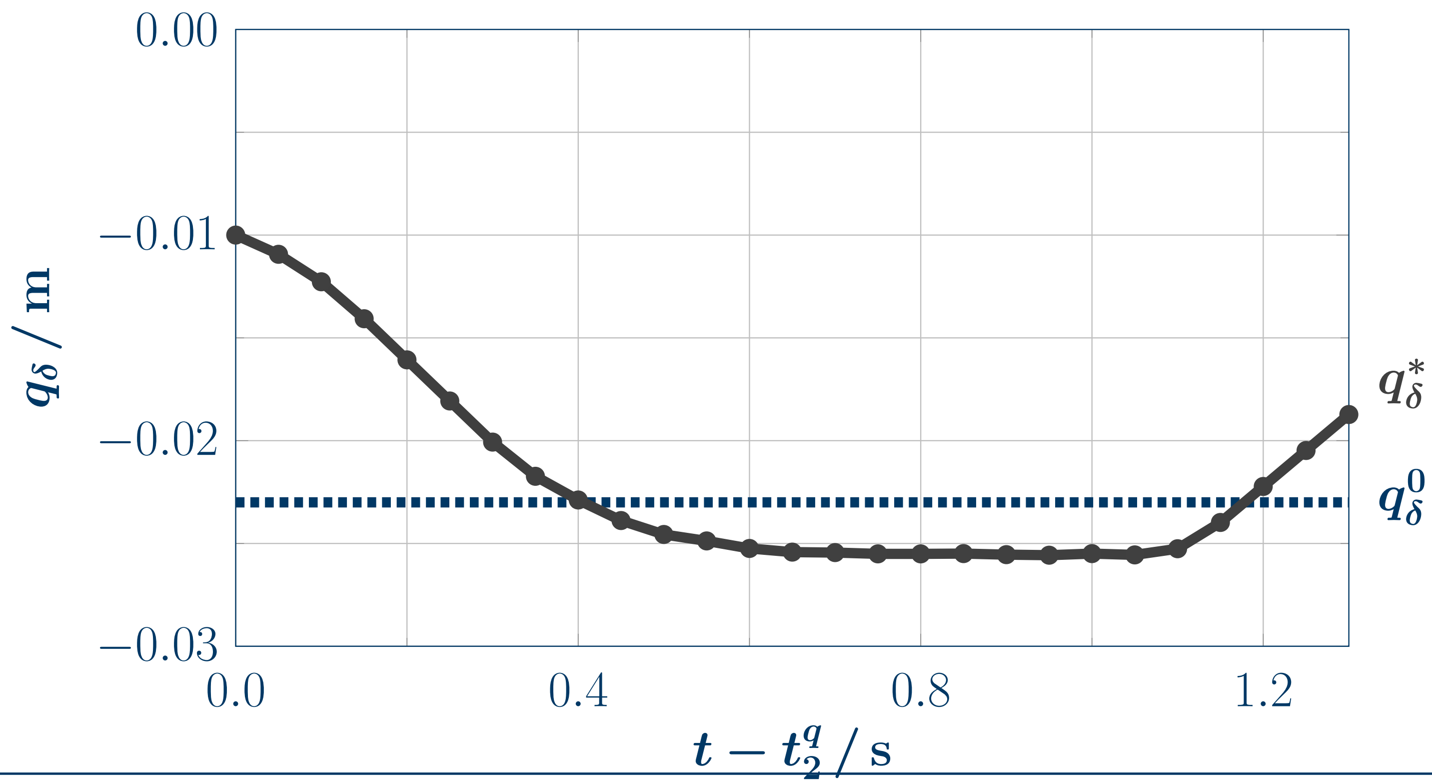
Conclusion and outlook

# Roughing mill – simulation



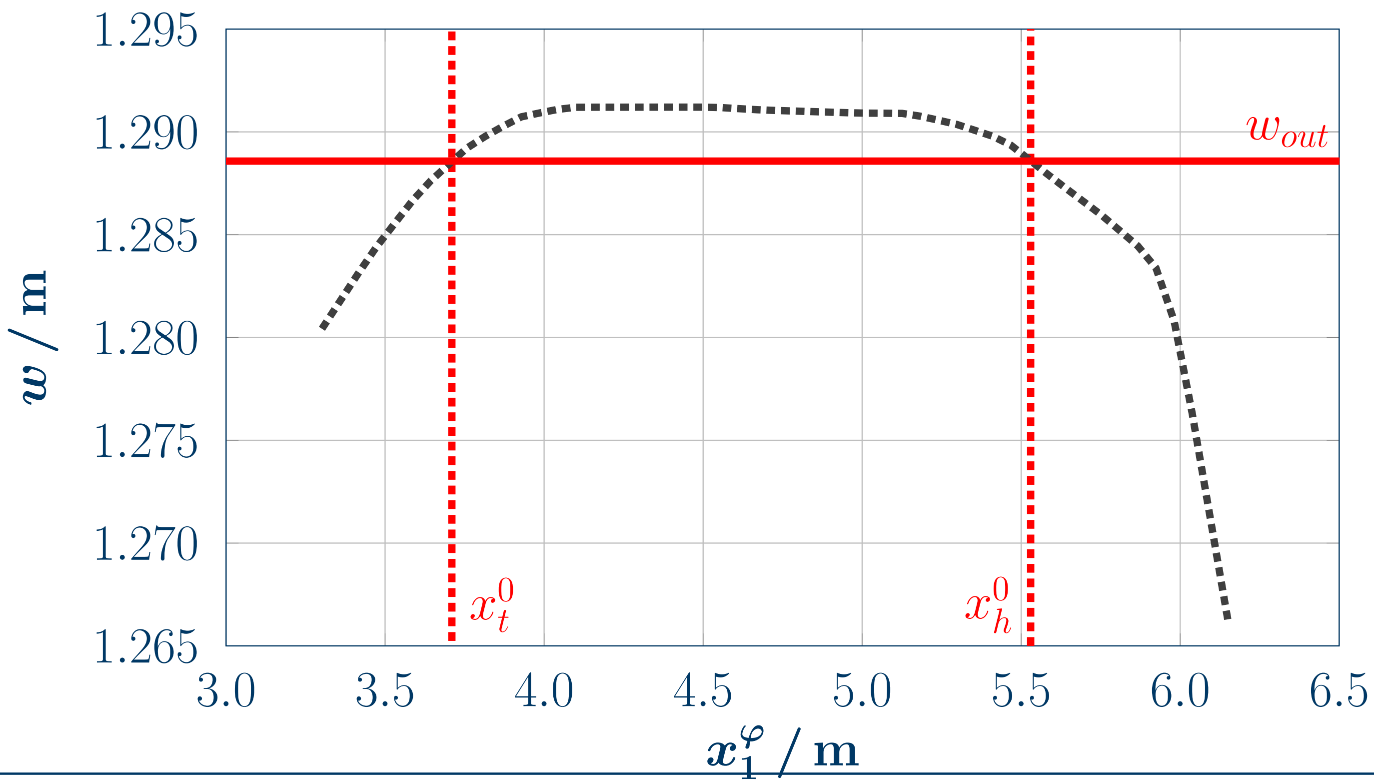
# Optimal control of edger position in roughing mills

- control: $x_2$-position of edgers, piecewise linear discretization ($n_q = 29$)
- curvature regularization: $\varrho_q = 0.01$
- optimizer: SNOPT (gradient-based SQP algorithm)
- goal: final width $w_{out} = 1.289\,\mathrm{m}$



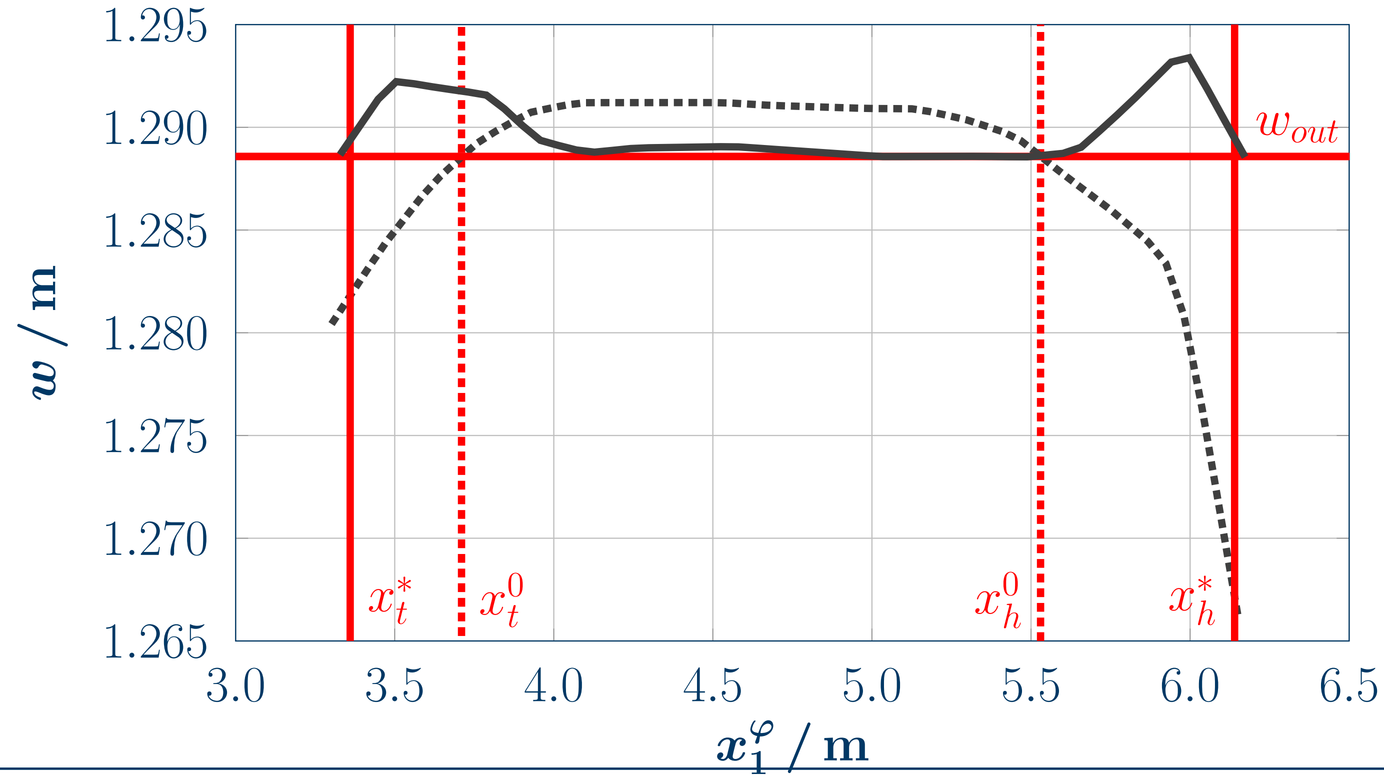
# Final slab shape with constant initial control

- control: $x_2$-position of edgers, piecewise linear discretization ($n_q = 29$)
- plot: minimal width of slab over position $x_1^\varphi$ at final time $T = 3.3\,\mathrm{s}$
- red lines: $x_1$-position of cutting planes, final width $w_{out} = 1.289\,\mathrm{m}$
- useable volume: $64.98\,\%$



# Final slab shape with optimal control

- control: $x_2$-position of edgers, piecewise linear discretization ($n_q = 29$)
- plot: minimal width of slab over position $x_1^\varphi$ at final time $T = 3.3\,\mathrm{s}$
- red lines: $x_1$-position of cutting planes, final width $w_{out} = 1.289\,\mathrm{m}$
- useable volume: $99.25\,\%$



# Outline

Optimal control of edger position in roughing mills

Numerical results

**Conclusion and outlook**

# Conclusion and outlook

**goals achieved:**

- application example for model-based control of dynamic frictional contact problems: optimization of slab shapes in hot rolling process
- mathematically consistent approach: regularization of non-differentiabilities before calculation of sensitivities with DDM
- implementation of solution schemes and numerical results for real-world examples

**useful extensions and open questions:**

- application of model reduction techniques to reduce high computation times
- relation between regularized and original state problem, regularity of regularized control-to-observation map $\mathcal{S}^{\varepsilon}$ and behavior for $\varepsilon \to 0$

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- Pietrzak G., Curnier A.: *Large deformation frictional contact mechanics: continuum formulation and augmented Lagrangian treatment.* Computer Methods in Applied Mechanics and Engineering 177 (1999)
- Simo J.C.: *A framework for finite strain elastoplasticity based on maximum plastic dissipation and the multiplicative decomposition.* Computer Methods in Applied Mechanics and Engineering 66 (1988)
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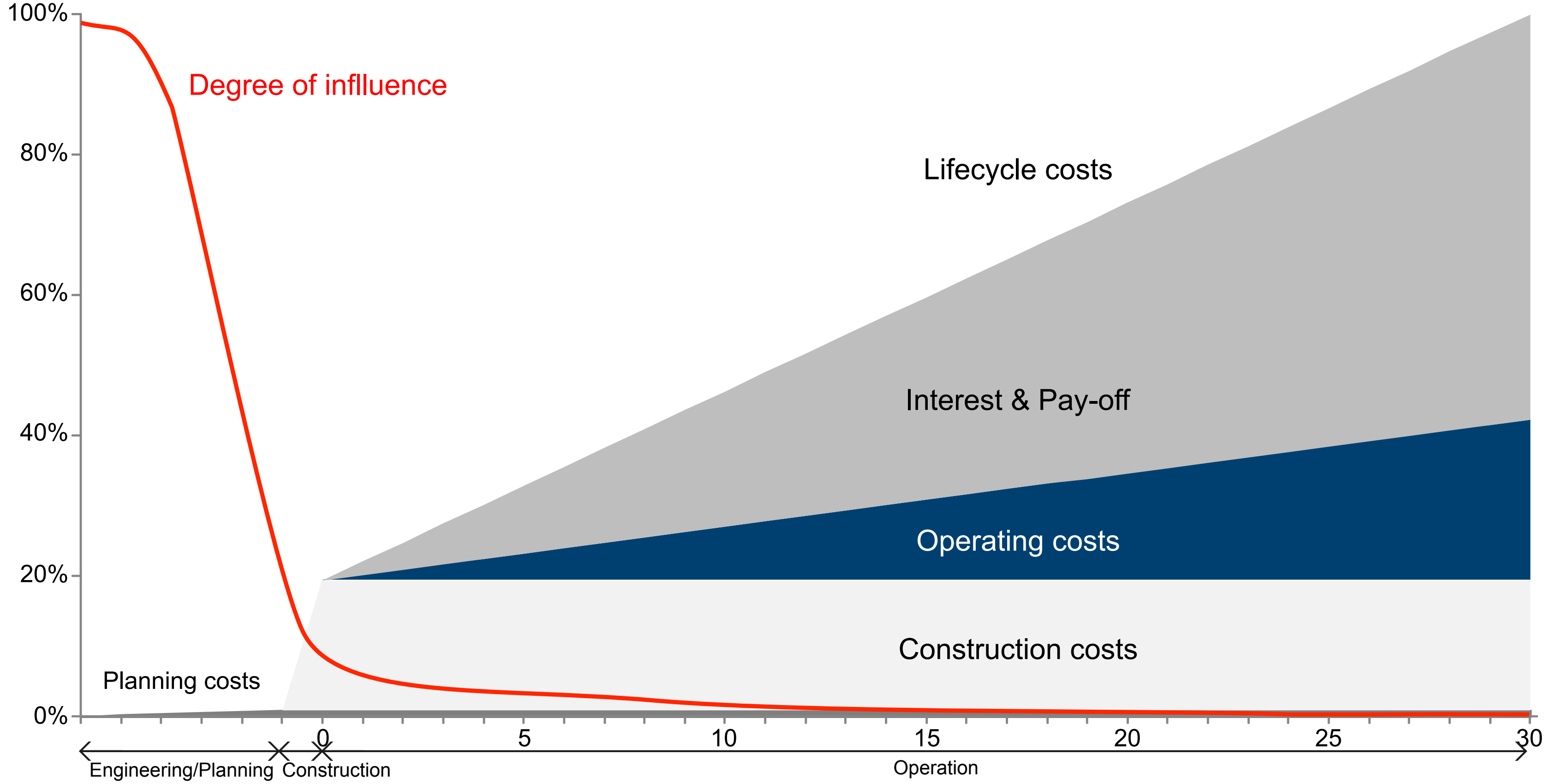
# Importance of the planning phase



# Complexity of the planning process

- Many degrees of freedom:
  - Geometry
  - Dimensions
  - Materials
  - ...
- Big leverage of the decisions
- Many side constraints and dependencies
- Current trends to extend the scope of planning:
  - Sustainability
  - Life cycle
  - ...

# Challenges in the planning phase

## Tasks

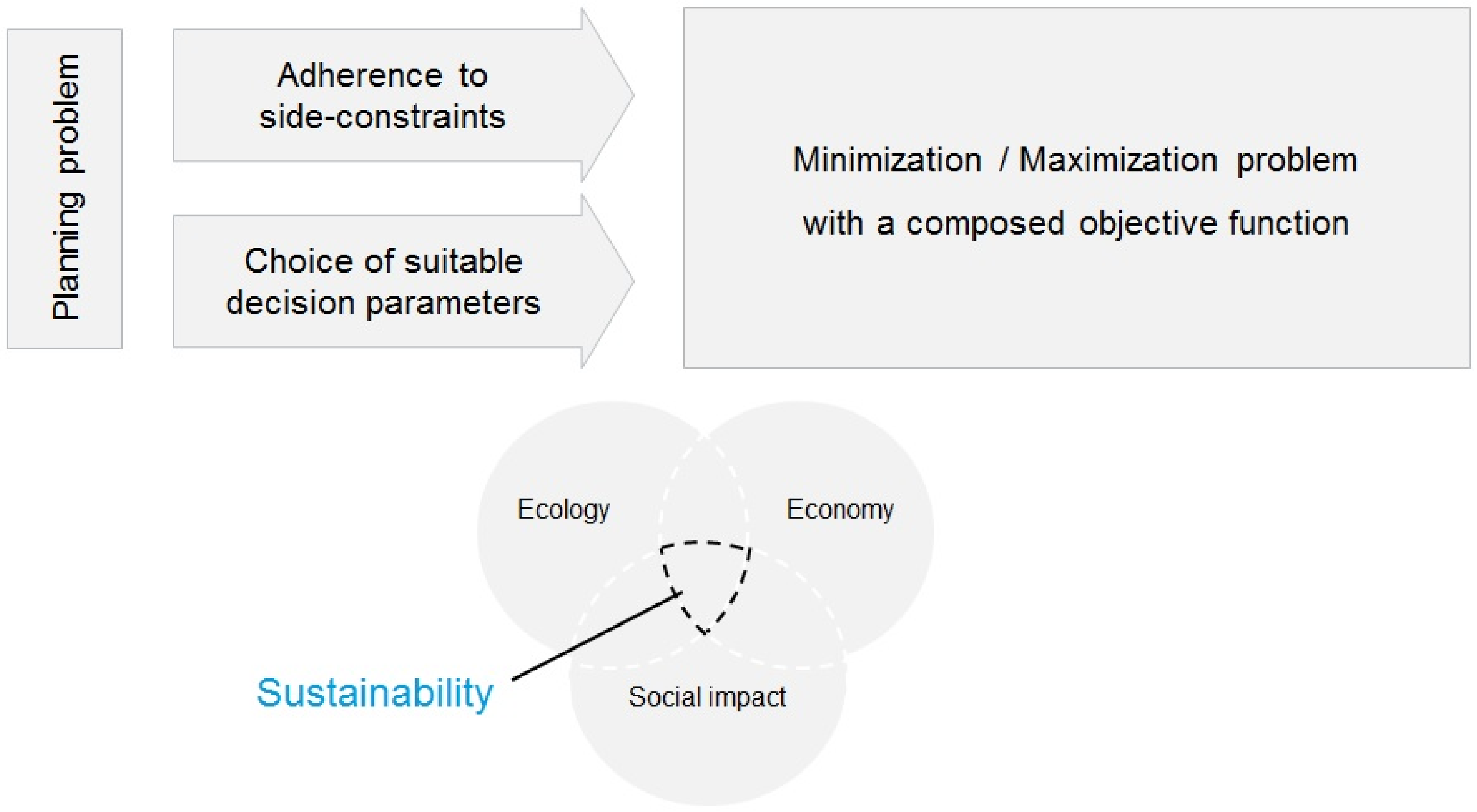
- Transformation of requirements into concrete planning results
- Optimization of
  - Design
  - Functionality
  - Equipment
  - Costs
  - Appointments

  over the life cycle
- Setting the course for optimal construction and operation

## Difficulties

- Many specialized planners with different know-how and focus
- High complexity:
  - Limited amount of investigated variants
  - Successive decision-making
  - Missing feedback loops
  - Optimization of subtasks
- Limited time
- Changing side constraints

# Optimization problem



# Goals and Vision of LeOpIn

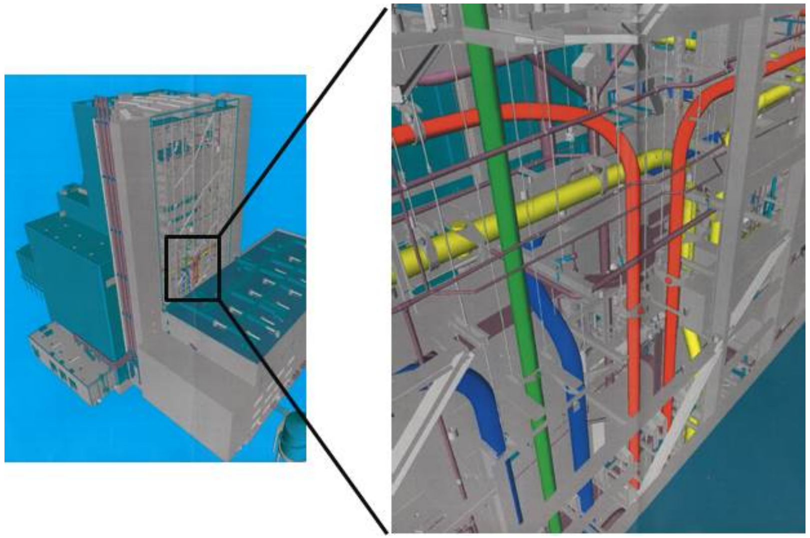
Advancement of the method to a tool mature for practical application:

- Holistic judgement of planning decisions
- Simple evidence for the impact of changes in the plan
- Integrated optimization of planning objects over the life cycle
  - Reduction of the investment costs
  - Considerable reduction of operation costs for the customer
- Acceleration of the planning process

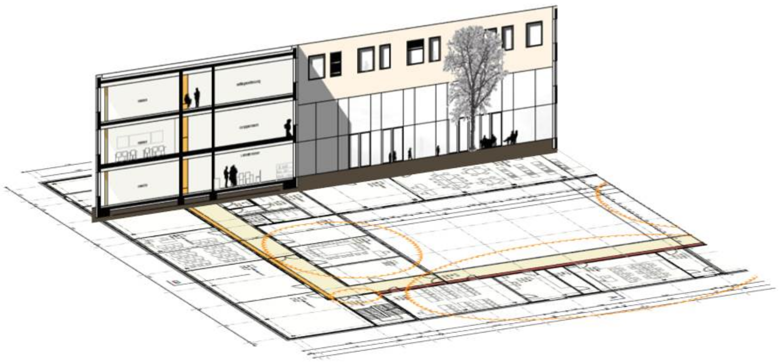
$\Longrightarrow$ The aim is not to replace the civil engineer but to assist him in finding holistic solutions!

# Strategy of LeOpIn

Development of the methodology based on specific application scenarios:



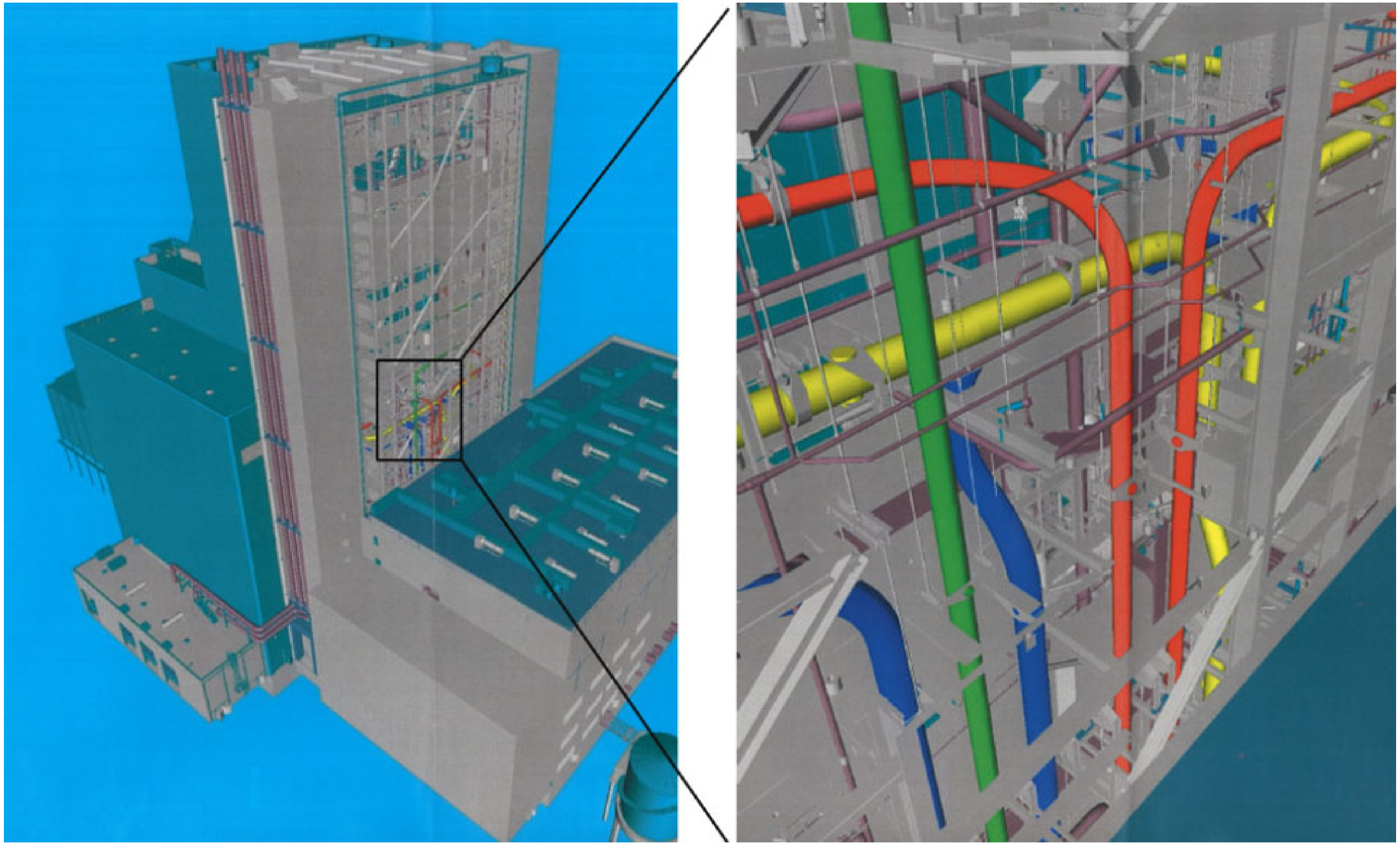
High-pressure pipe system



Building

# Application to buildings

# Application to power plants



# Characteristics of the application scenarios

## High-pressure pipe system

- Defined side constraints and parameters
- Defined physics
  - Structural mechanics
  - Fluid mechanics
  - Heat transmission
  - Material degradation
- Well-defined decision variables:
  - Line routing
  - Pipe thickness
  - Hanger location
  - Welding joints
  - Pipe bendings
  - ...
- Combination challenging

## Building

- Side constraints and parameters still unclear
- Easy physics:
  - Heat transmission
  - Lighting
  - Fluid mechanics
  - Structural mechanics of minor importance
- Complex, coupled decision variables:
  - Shape of the building
  - Room, area, element location
  - Choice of product for the façade
  - TGA
  - ...
- Abstraction challenging

# Challenges in LeOpIn

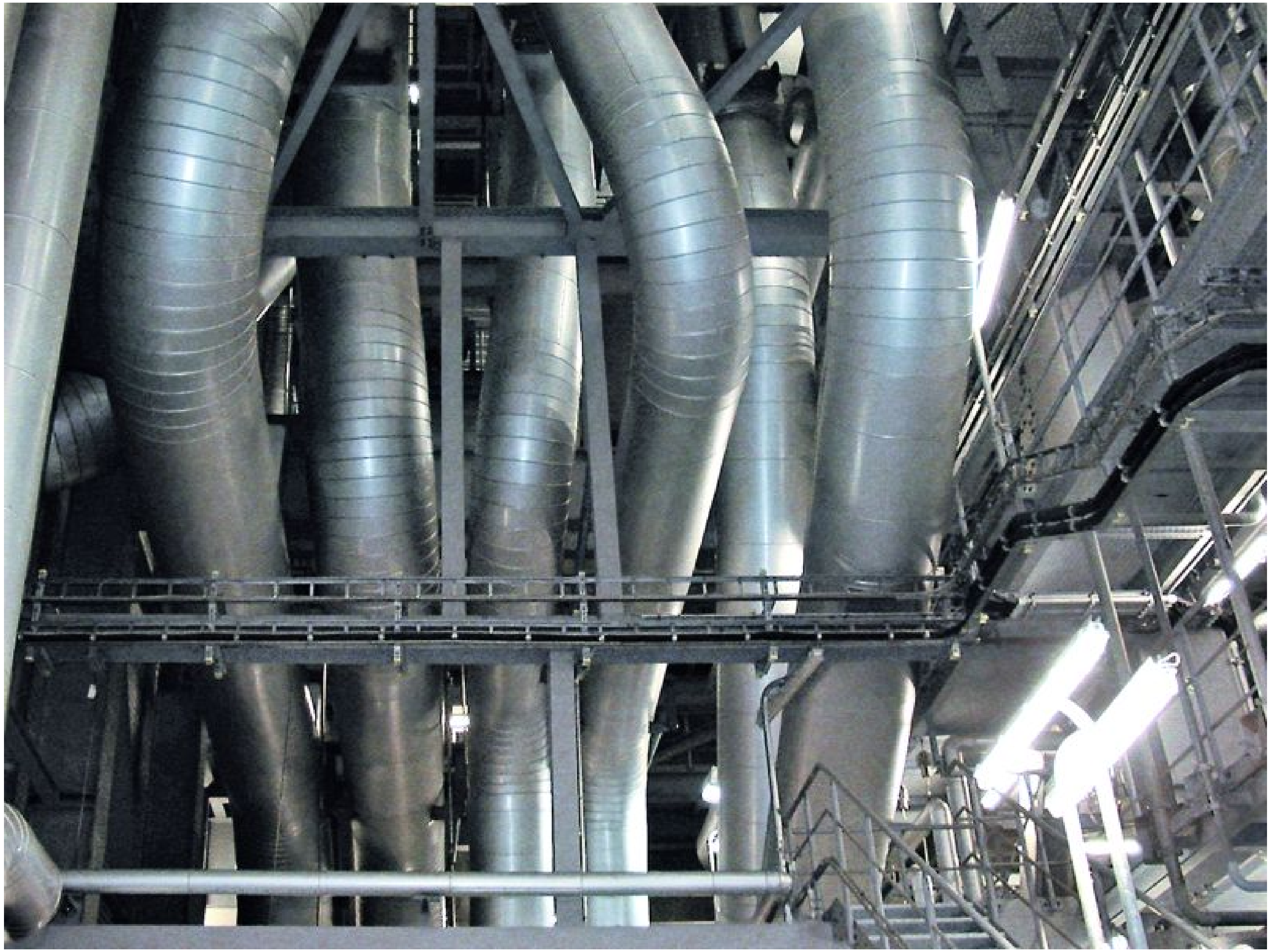
## Scienctific research groups

- Complex, unstructured problem
- Hierarchical approach with feedback function from lower levels
- Connection of optimization methods and physical modelling

## Bilfinger

- Reflection of the planning process
- Identification and weighting of side constraints, dependencies, influences
- Generalization to enable the application of mathematical optimization methods

# High Pressure Pipe System

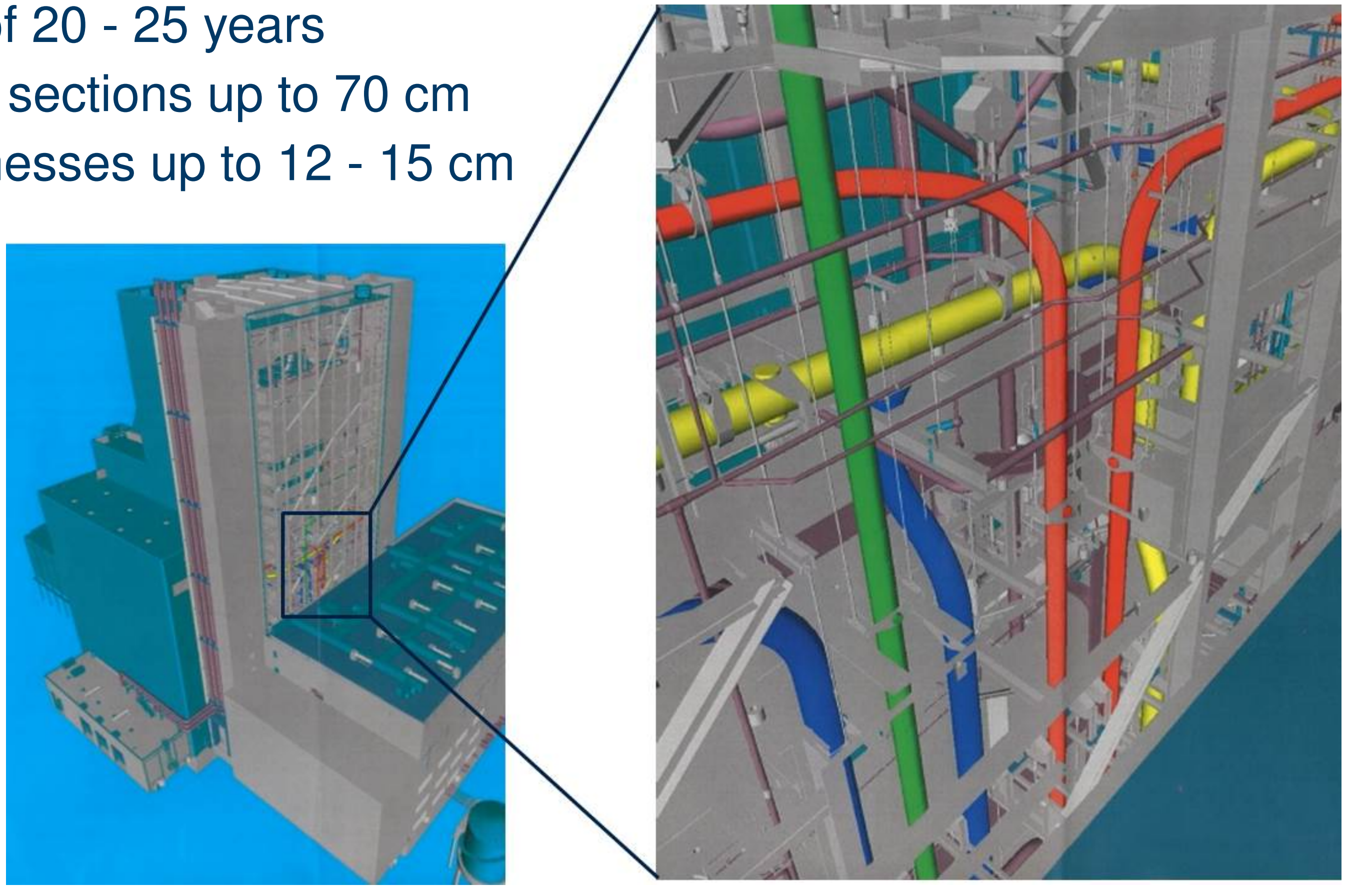


# High Pressure Pipe System

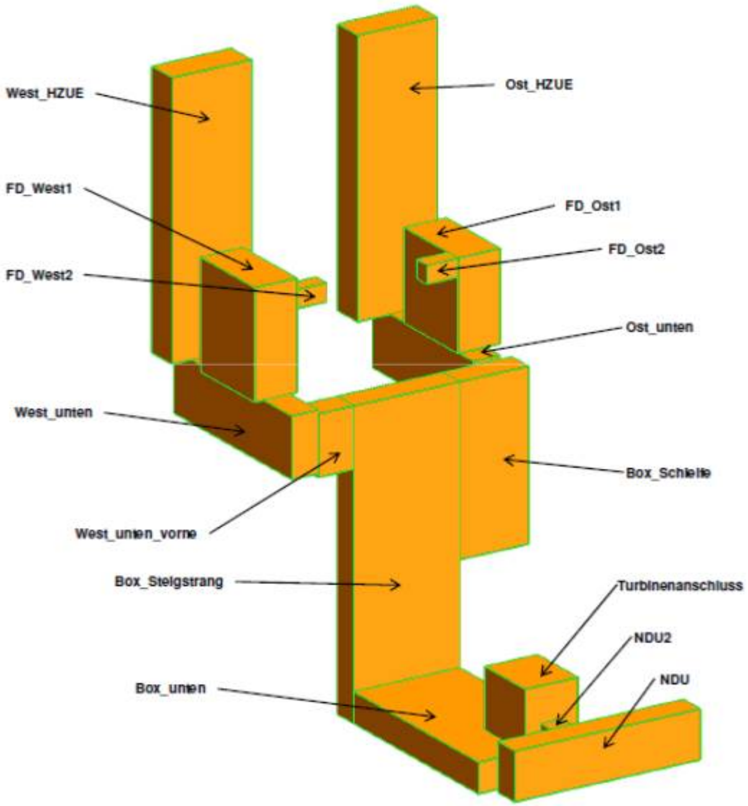
# High Pressure Pipe System

# Challenges in high-pressure pipe systems

- Temperatures above 600 degrees
- Pressures up to 300 bars
- Life span of 20 - 25 years
- Pipe cross sections up to 70 cm
- Pipe thicknesses up to 12 - 15 cm



# The task

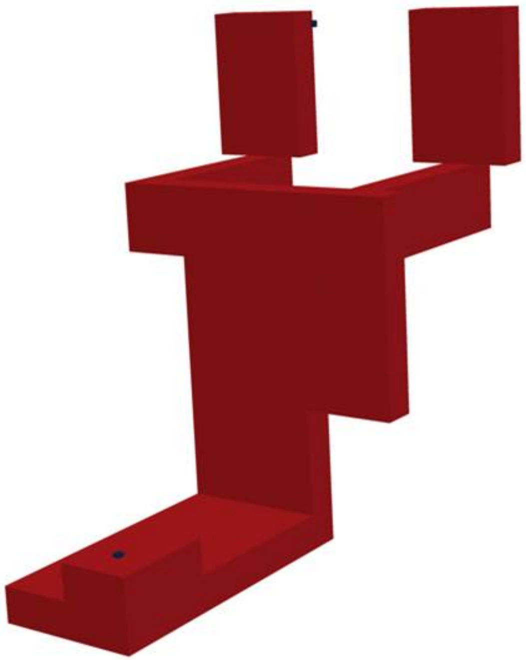


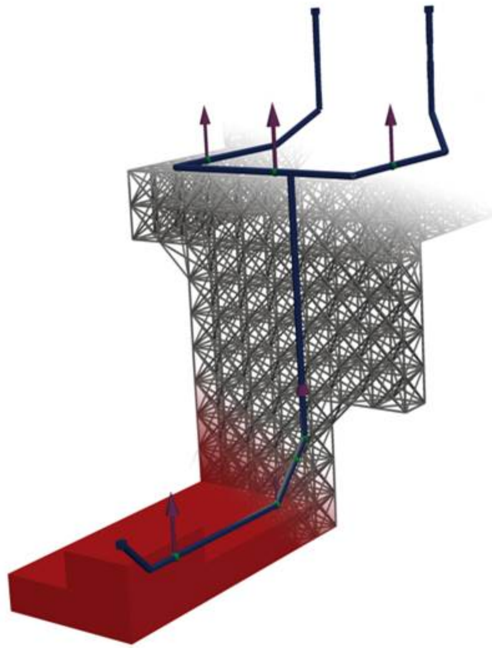

Goals:

- Optimal connection of the entry and exit points for a pipe system in a power plant
- Minimal cost over the life-cycle
- Minimal amount of $CO_2$-emissions
- Adherence to the side-constraints:
  - Geometry
  - Tensions
  - Transport restrictions
  - ...

# 1. Pipe routing (subproject B)

Goal: Coarse layout of the pipe routing and placement of hangers under simple physical side-constraints



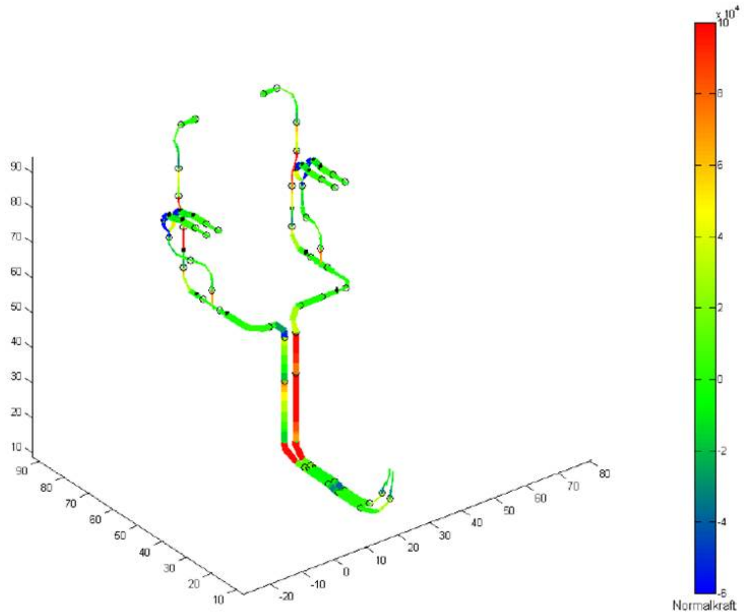


$\Longrightarrow$ Minimize the life-cycle costs

Objective function: $K_{total} = K_I + K_C + K_{CO_2}$

# 2. Topology optimization (subproject C)

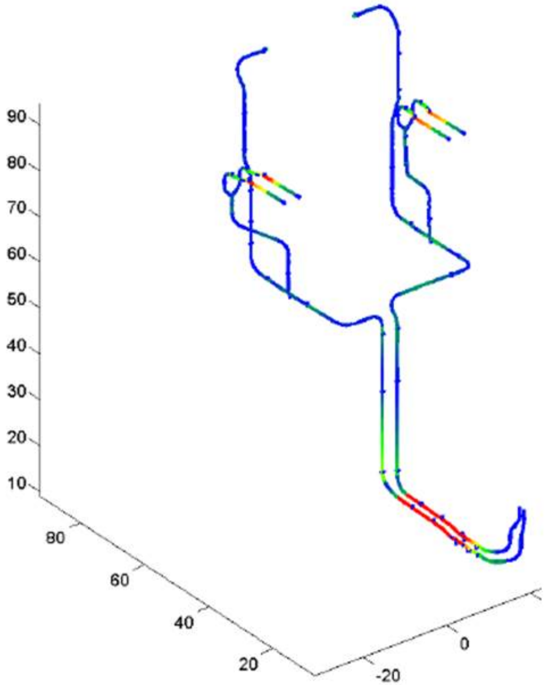
Goal: Optimal choice of material, pipe thicknesses, hanger positions, bending radii, etc., to optimize the pipe system while respecting all relevant side-constraints, e.g. tensions

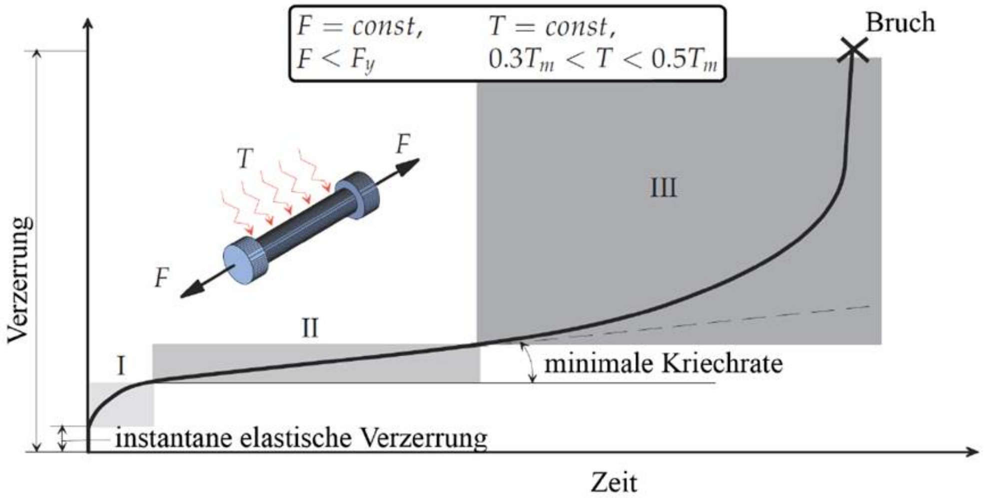


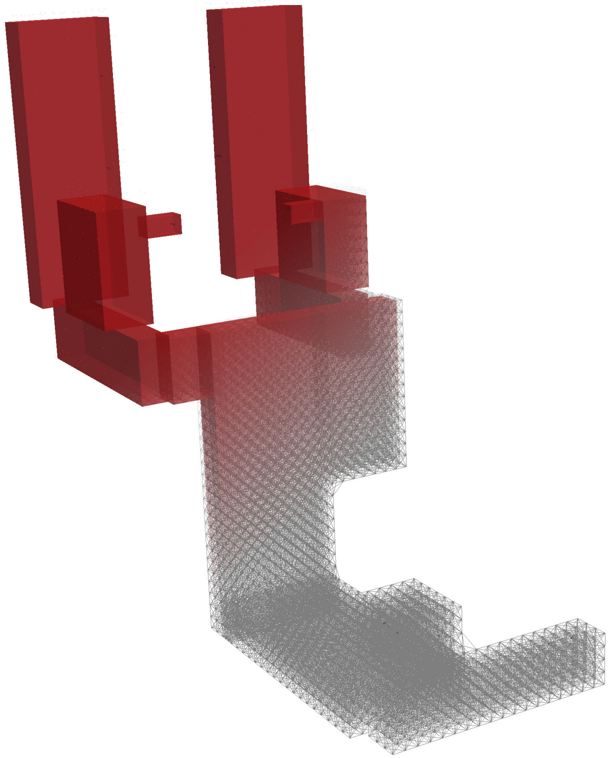

- Restrictions with respect to tensions: dimensioning for interior pressures $\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5$ according to DIN EN 13480
- Geometrical restrictions

# 3. Models for material degradation (subproject F)

Goal: Development of a physically improved modelling of nonlinear effects and the material degradation of pipe systems



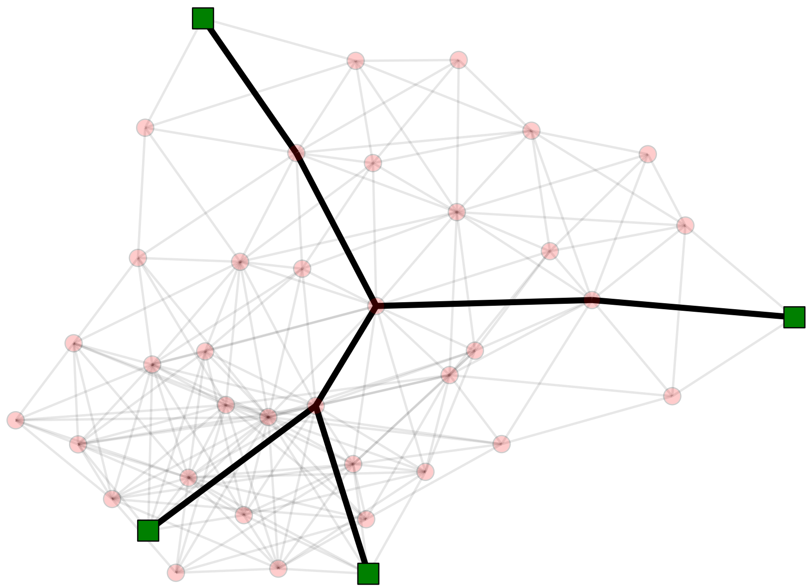


## Problem

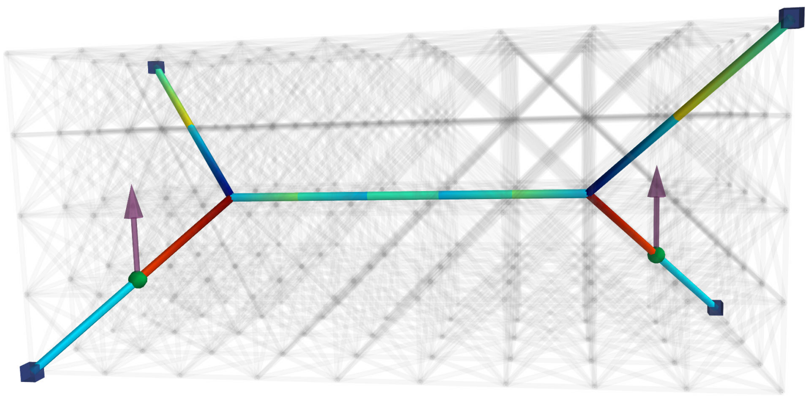
- Given a rough outline of a power plant
- Route a pipe through the plant considering physical constraints

# Problem's ingredients

Combinatorics: Path/Steiner tree



Nonlinearities: Mechanics



# Basic problem statement

$$
\begin{aligned}
\min \quad & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, u(x,y)) \\
\text{s.t.} \quad & \text{Steiner tree}(x) \\
& \text{pipe physics}(x, y, u(x,y)) \\
& \text{hangers}(x, y, u(x,y)) \\
& \text{industrial standards}(x, y, u(x,y))
\end{aligned}
$$

**Variables**

- $x$ Pipe variables
- $y$ Hanger variables
- $u$ Displacement variables (depend on $x$ and $y$)

# More than one inlet/outlet – Steiner tree

**Definition: Steiner tree problem**

Given graph $G = (V, E)$ with vertices $V$, edges $E$ with weights $c : E \to \mathbb{R}^+$ and a set of terminal nodes $T \subseteq V$.
Find the cheapest tree $S$ that includes all nodes in $T$.

- Huge catalogue of Steiner tree models available
- Usually few terminals in our application $\Rightarrow$ Use a flow formulation
- Computational study shows advantage over other models

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# Linear 3D Timoshenko beam equations

$$
\begin{aligned}
EAu'' + q_x &= 0 \\
kGA(v'' - \theta') + q_y &= 0 \\
kGA(w'' + \psi') + q_z &= 0 \\
GI_t\varphi'' + m_x &= 0 \\
EI_z\psi'' - kGA(w' + \psi) + m_y &= 0 \\
EI_y\theta'' + kGA(v' - \theta) + m_z &= 0
\end{aligned}
$$

$EA$: extensional stiffness
$kGA$: shear stiffness with factor
$GI_t$: torsional stiffness
$EI$: bending stiffness

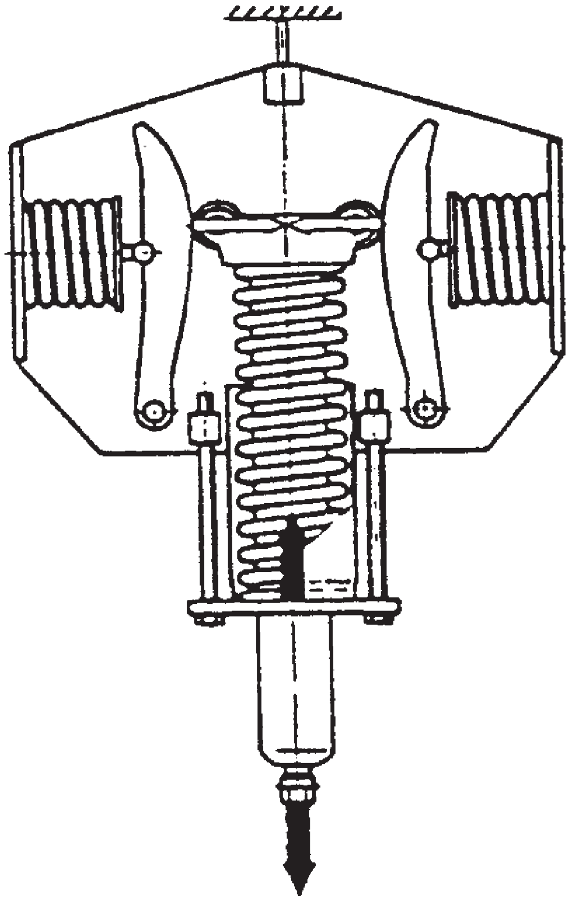
- Analytical solutions to homogeneous Timoshenko equations as ansatz functions give rise to proper stiffness matrices (Luo, 2008)
- Global stiffness matrix: $K(x) = \sum_{i=1}^{n} T_i^T K_i T_i x_i$

**Constraints: pipe physics$(x, y, \mathbf{u}(x, y))$**

For hot an cold scenario:

$$K(x)\mathbf{u} = \sum_{e \in \mathcal{E}} g_i x_i + \sum_{n \in \mathcal{V}_{\text{free}}} l_j h_j$$

# How to model the hangers?



Source: G. Wossog, Handbuch Rohrleitungsbau, p 594

# Solution approaches

**MINLP Model**

$$
\begin{aligned}
\min\ & c_{\text{pipe}}(x) + c_{\text{hangers}}(y, u(x,y)) \\
\text{s.t.}\ & \text{Steiner tree}(x) \\
& \text{pipe physics}(x, y, u(x,y)) \\
& \text{hangers}(x, y, u(x,y)) \\
& \text{industrial standards}(x, y, u(x,y))
\end{aligned}
$$

**MILP Model**

- Linearize non-convex terms $\xi_{il} = x_i u_l$
- Can be done completely or adaptive

**MISOCP Model**

- Replace industrial standards with substitute constraint
- Problem becomes convex

**Decomposition**

- Decompose in master- and subproblem
- Both problems are convex

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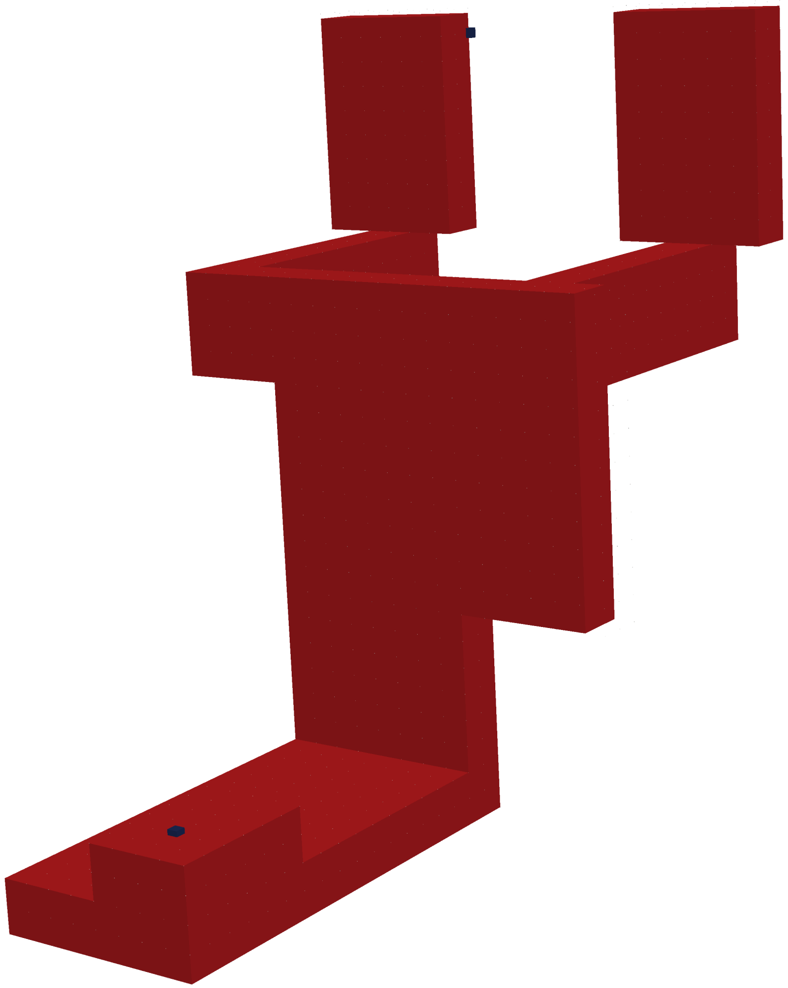
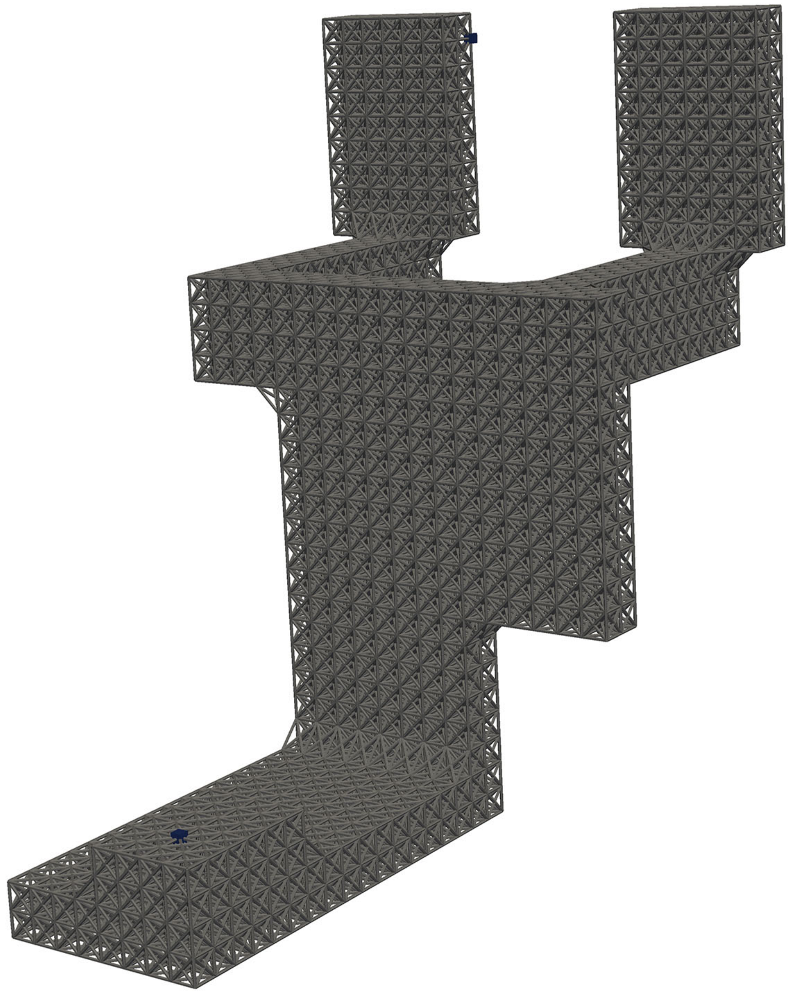
# Decomposition algorithm

Set $k = 0$, *best* = None, $\phi = 0$, $\theta = \infty$
**while** $|\theta - \phi| > \varepsilon$ **do**
  Solve Masterproblem for a lower bound $\phi$ and solution $x^k$.
  **if** Check for infeasibility returns False **then**
    Solve Subproblem with fixed $\hat{x} = x^k$
  **if** Subproblem$(\hat{x})$ was feasible **then**
    Get solution $y^k$ with costs $\gamma$ (includes costs for hangers and pipe).
    **if** $\gamma < \theta$ **then**
      $\theta = \gamma$
      *best* $= (x^k, y^k)$
    Add Cost-Cut to Masterproblem
  **else**
    Add No-Good-Cut to Masterproblem
  $k = k + 1$

# Examples – Complete optimization



# Examples – Complete optimization



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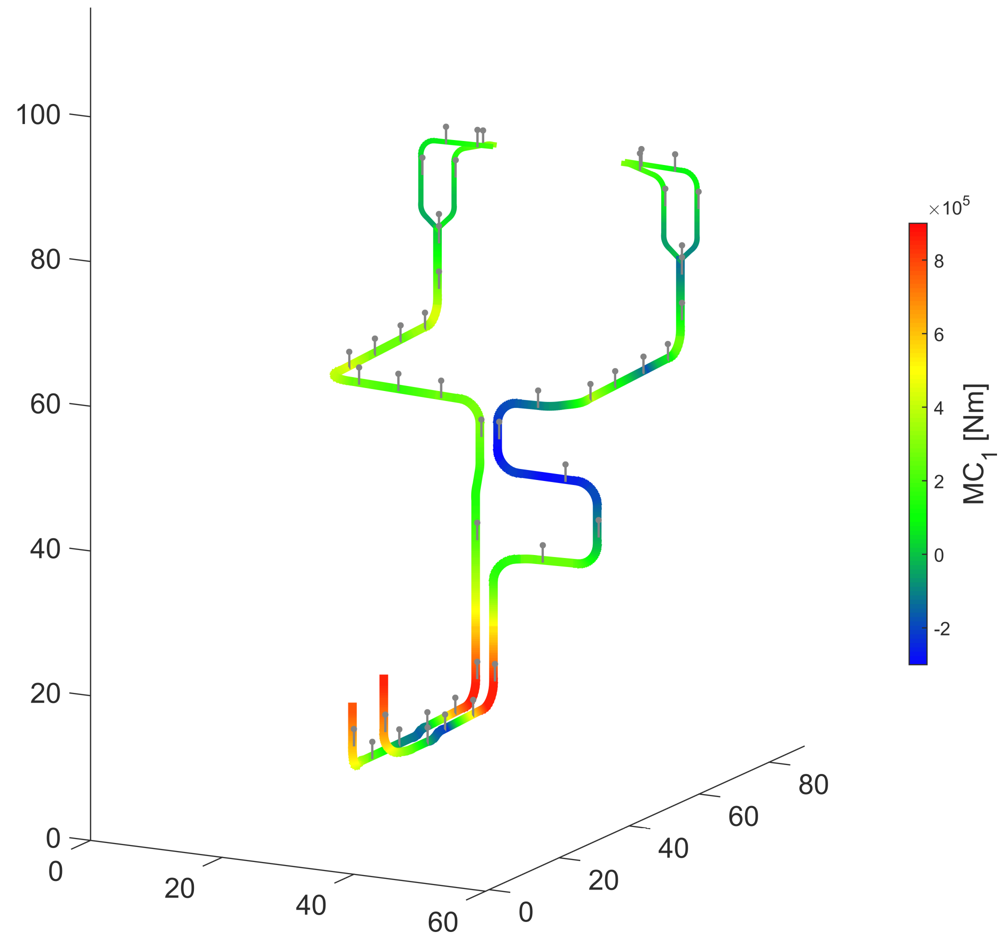
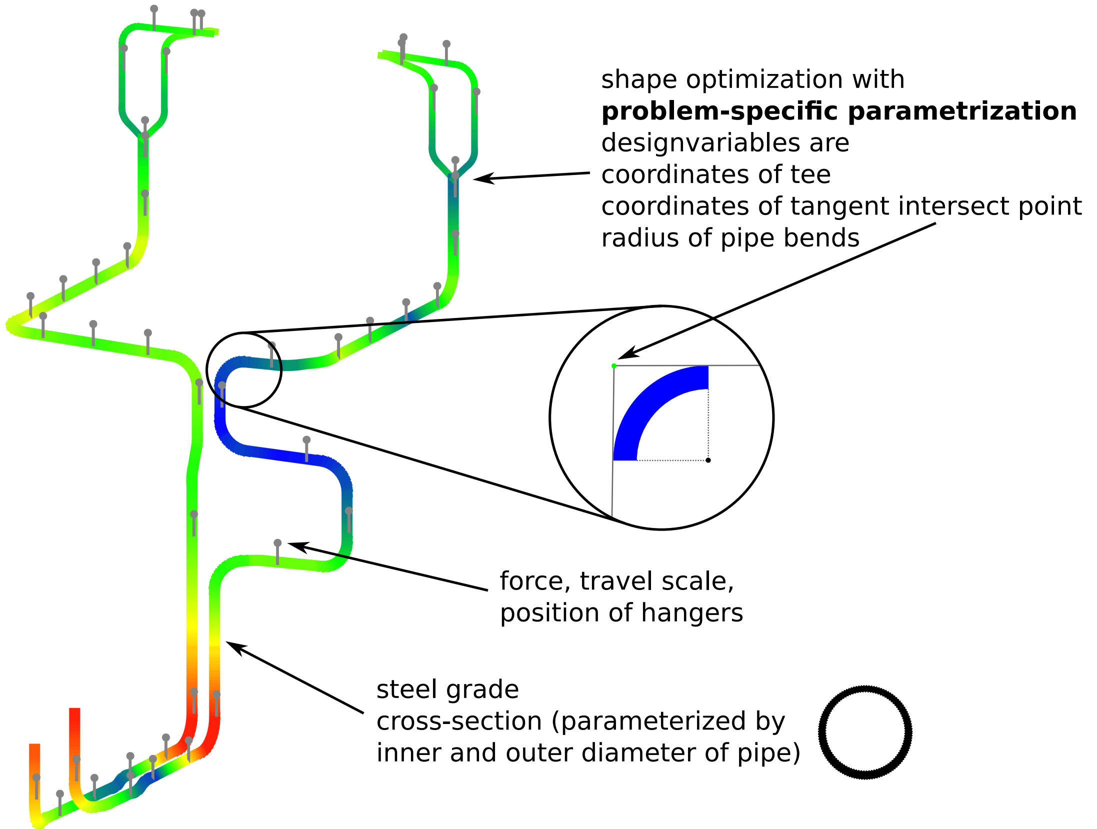
# Bilfinger piping system (initial)

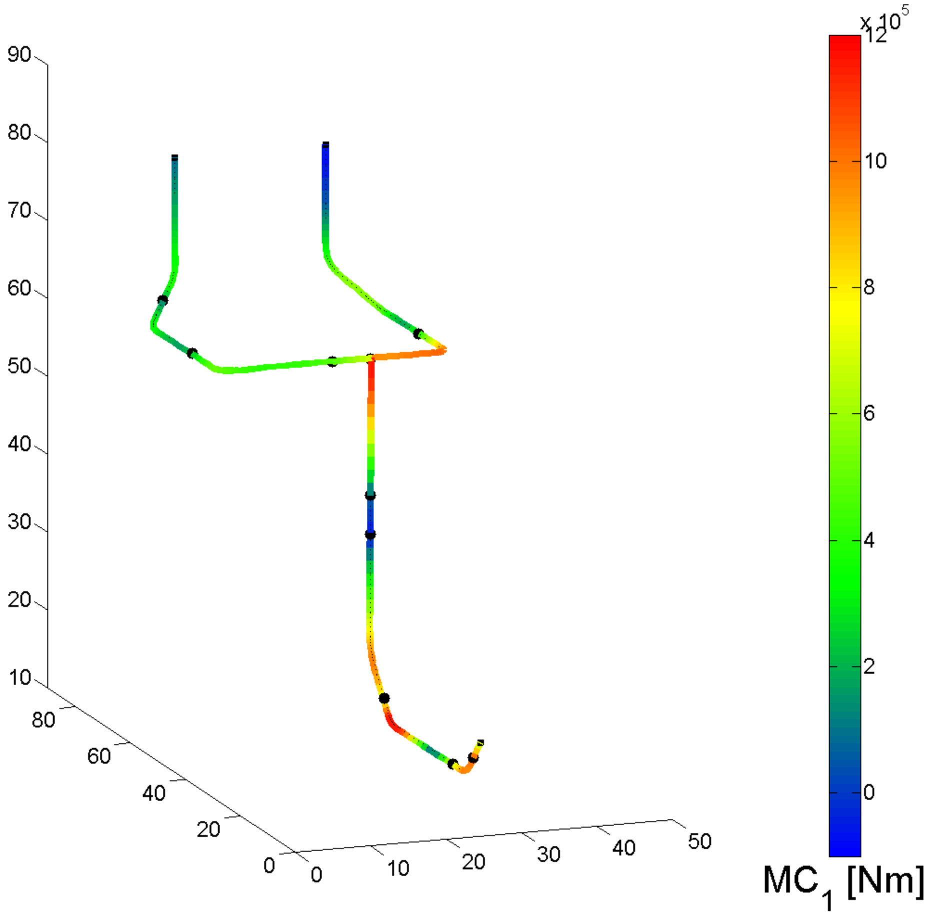


# Design Variables



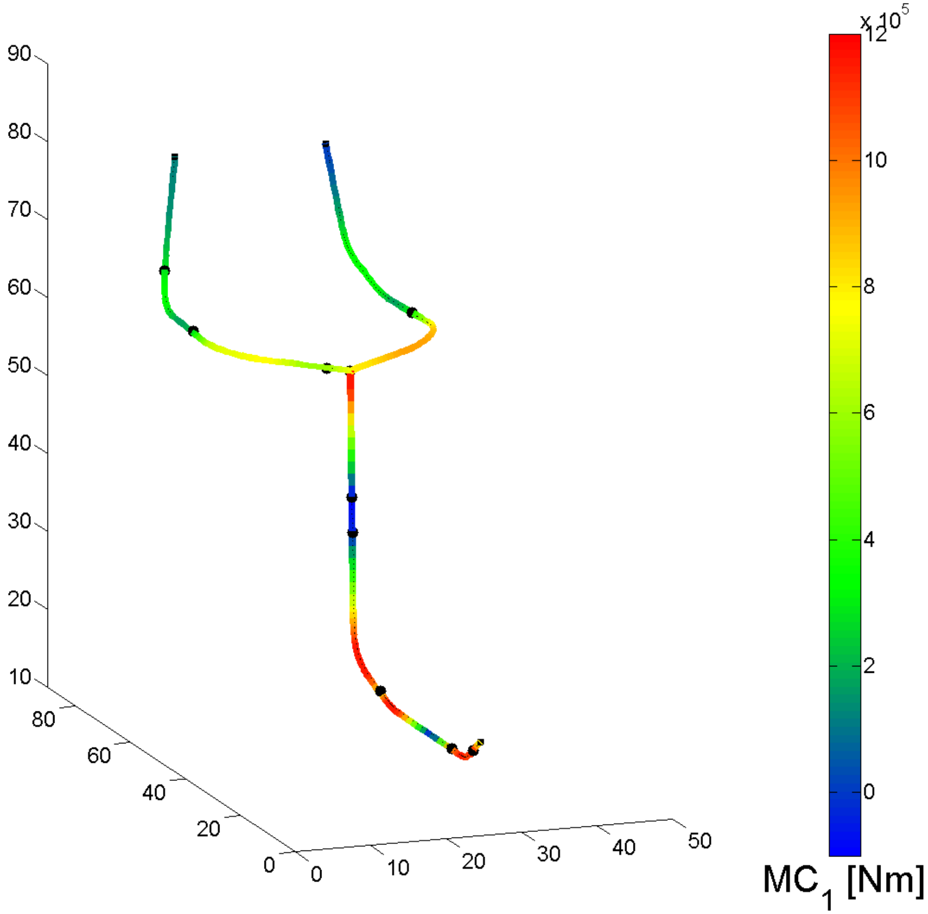
# Results - academic example

Piping system (initial), costs: 11.352.112 €



# Results - academic example

After shape-optimization, costs: 8.266.301 €



# Results - academic example

## Geometry-boxes