Optimal control and applications in aerospace

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- Why aerospace problems ?

- or, if you are a girl, you may be the princess in your class: girl/(girl+boy)=0.1

Introduction

- Trajectory optimization :

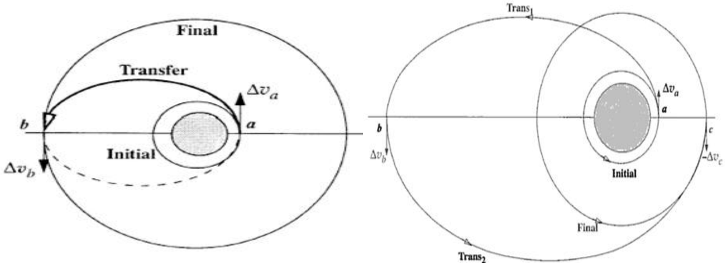
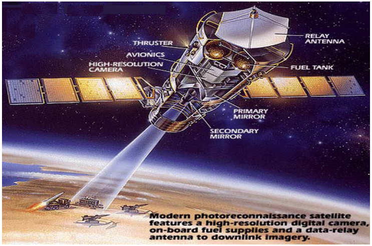


Figure: Hohmann transfer (left); Bi elliptic transfer (right)

- Optimal control problem : how to go from an initial trajectory (orbit) to another target one while minimizing the cost (fuel, mass, time, etc) ?
- Basic assumption: a spacecraft is a "point' mass'...

- However, it is not only a point ... $\Rightarrow$ Attitude reorientation



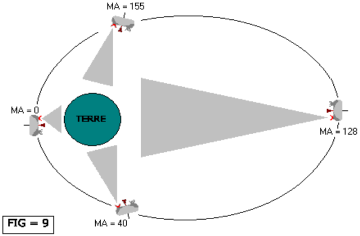


Figure: Hubble telescope (left); Communication satellite (right)

- Optimal control problem : how to go from an initial attitude to another target one while minimizing the cost (fuel, mass, time, etc) ?
- Trajectory and attitude problems are usually treated separately

Introduction

- Trajectory-Attitude coupled problem ?

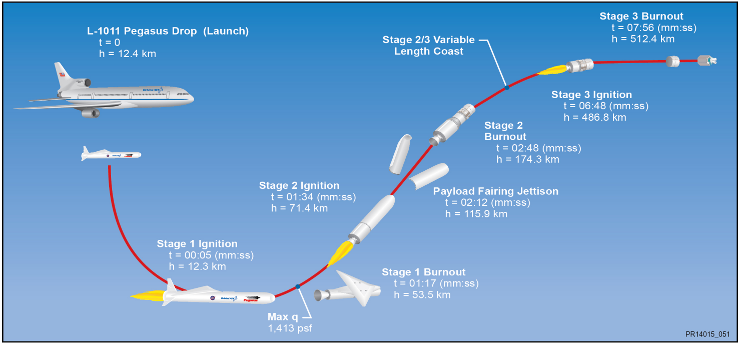
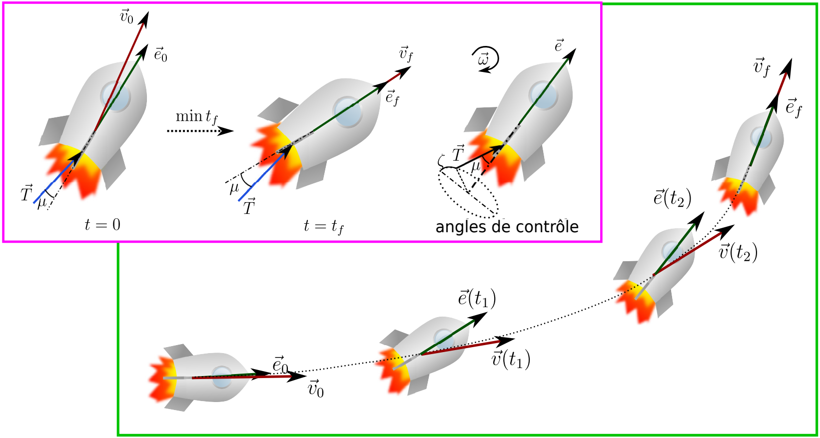


Figure: Airborne launcher

- Optimal control problem : how to go from an initial trajectory and attitude to another target one while minimizing the cost (fuel, mass, time, etc) ?

Problem statement

- Comes from aerospace industry (Airbus)



Problem statement

- Problem : minimizing the maneuver time such that the coupled system (a bi-input control affine system) satisfies initial and final conditions, and constraints on the control ($\|u\| \leqslant 1$) and on the state.

$$\min C = T$$

$$\dot{x} = f(x) + g(x)u, \quad x \in \mathbb{R}^{11}, \quad u \in \mathcal{U} \subset \mathbb{R}^2$$

$$\text{Terminal conditions}: \ x(0) = x_0, \quad \Phi(x(T)) = 0,$$

- Objectif : automatic software (efficient numerical resolution)

Numerical methods

- Classical methods : direct method & indirect method
  - **Direct method** (easy to implement) : discretize the state and the control $\Rightarrow$ finite dimensional nonlinear optimization problem
  - **Indirect method** (higher precision, but difficult to initialize) : Pontryagin Maximum Principle $\Rightarrow$ two boundary value problem

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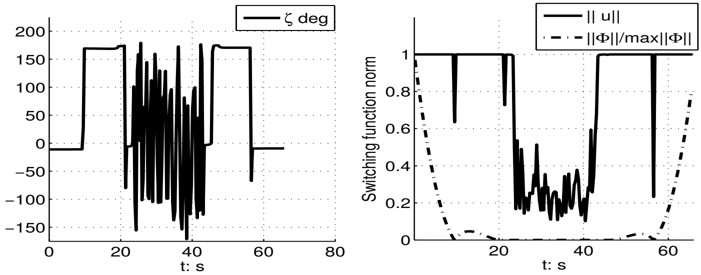

AMPL (différentiation automatique) + **IpOpt** (optim num non linéaire, C++)

Numerical methods

- Classical methods : direct method & indirect method
  - **Direct method** (easy to implement) : discretize the state and the control $\Rightarrow$ finite dimensional nonlinear optimization problem
  - However : not fast enough + the control oscillates $\Rightarrow$ **Indirect method ?**

  Emmanuel Trélat : "In the present aerospace applications, the use of shooting methods is privileged in general because of their very good numerical accuracy."

Numerical methods

- Classical methods : direct method & indirect method
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Optimal control problem

$$\min \int_0^{t_f} f_0(x,u)dt + g(x(t_f))$$
$$\dot{x} = f(x,u)$$
$$x(0) = x_0$$
$$x(t_f) = x_f$$

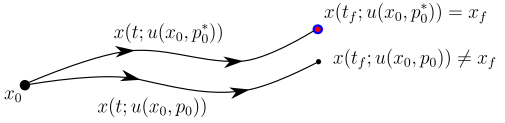
Pontryagin Maximum Principle (PMP) $\longrightarrow$

Two boundary value problem

$$\dot{x} = \frac{\partial H}{\partial p}(x,p,u(x,p))$$
$$\dot{p} = -\frac{\partial H}{\partial x}(x,p,u(x,p))$$
$$H = \langle x, f(x,u)\rangle - f_0(x,u)$$

Shooting method

Choose $p(0) = p_0$
s.t. the trajectoire
arrives at the
"target" point



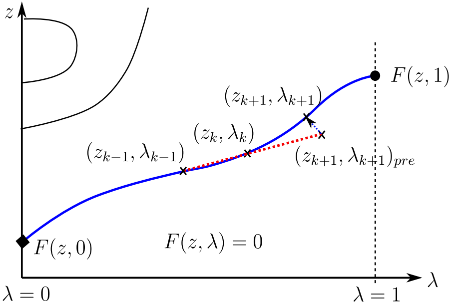
- Classical methods : direct method & indirect method
  - **Indirect method** (higher precision, but difficult to initialize) : Pontryagin Maximum Principle ⇒ two boundary value problem
  - Precise solutions, but : initialization ? ⇒ **Homotopy method**

- Classical methods : direct method & indirect method
- **Homotopy method** :
  - Difficulty : how to choose a proper "simple" problem $\Rightarrow$ deep comprehension of the problem $\Rightarrow$ analyze the extremals

Simple problem
(we know the solution)
(well chosen !)
$F(z,\lambda)=0 \qquad \lambda=0$

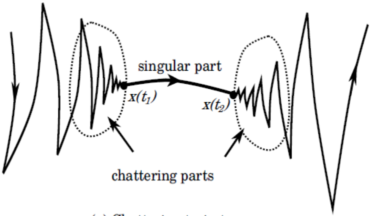
$\downarrow$

Difficult problem
(original)
$F(z,\lambda)=0 \qquad \lambda=1$

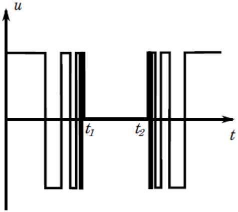


Geometric optimal control

- Geometric analysis of etremals $\Rightarrow$ deep comprehension of the problem
  - Theorem : sufficient conditions for the existence of chattering (bi-input control affine system)
  - $\Rightarrow$ violent oscillation of control
  - Tools (geometric optimal control) : PMP, higher order singular optimal control, higher order optimality conditions, Lie (and Poisson) brackets and configurations, etc.



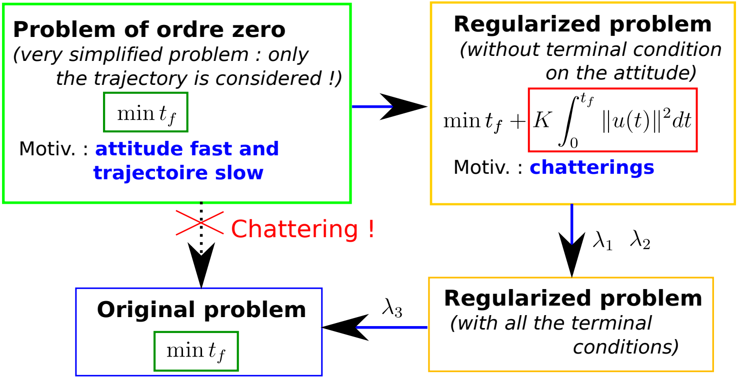
(a) Chattering trajectory



(b) Chattering control

- Geometric analysis of extremals $\Rightarrow$ deep comprehension of the problem
  - Theorem : sufficient conditions for the existence of chattering (bi-input control affine system)
  - Chattering phenomenon (called also Fuller's phenomenon) $\Rightarrow$ violent oscillation of control
  - Coupling of rapid (attitude) and slow (trajectory) dynamics
- These essential difficulties (properties) $\Rightarrow$ failure of classical methods

- Geometric analysis of extremals $\Rightarrow$ chattering + coupling
- Indirect method + Homotopy method



Development of numerical method

- Geometric analysis of extremals $\Rightarrow$ chattering + coupling
- Indirect method + Homotopy method
- Valorization: development of the software (rapid and preciser) and statistic tests for ensure the robustness
  (This step is very important in industry)

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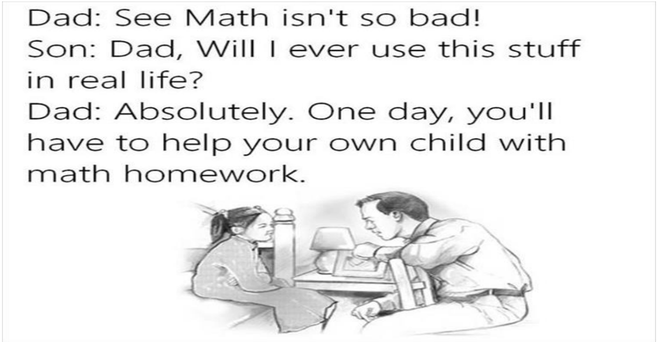
To see more details : J. Zhu, E. Trélat, M. Cerf, Geometric optimal control and applications to aerospace, Pac. J. Math. Ind. 9 (2017), 9:8.

Still numerous challenges ...

- Collecting space debris (a urgent challenge !!) : difficult mathematical problems combining optimal control, continuous / discrete / combinatorial optimization


- Planning low-cost missions to the Moon or interplanetary one, using the gravity corridors and other gravitational properties
- Inverse problems: reconstructing a thermic, acoustic, electromagnetic environment (coupling ODE's / PDE's)
- ...



My answer: to explain to engineers why their things work, and to understand how the things of mathematicians can be used.

Thank you for your attention !