Numerical Relativity beyond GR: Motivation and Challenges

Thomas P. Sotiriou

COST EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

Science & Technology Facilities Council

The University of Nottingham
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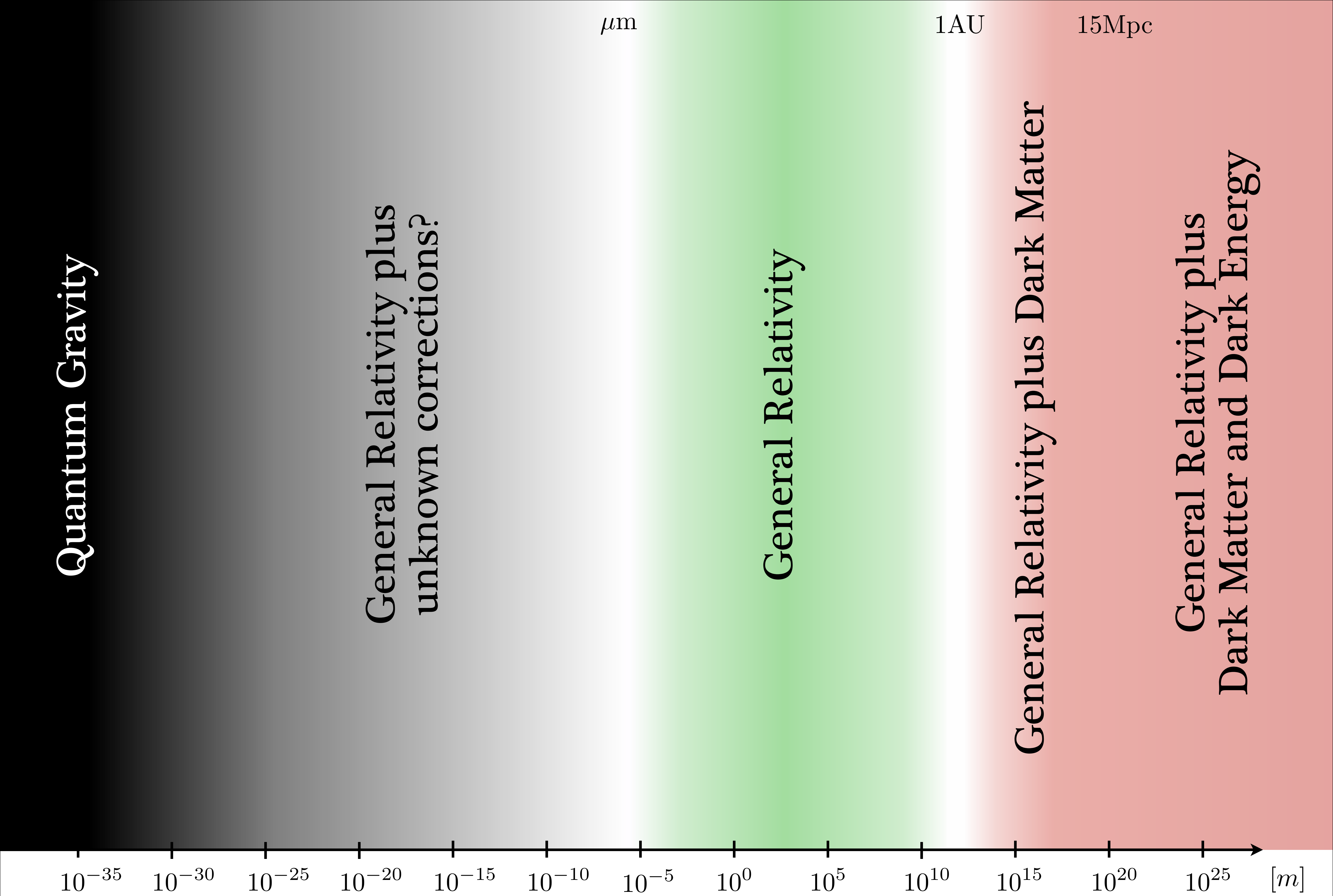
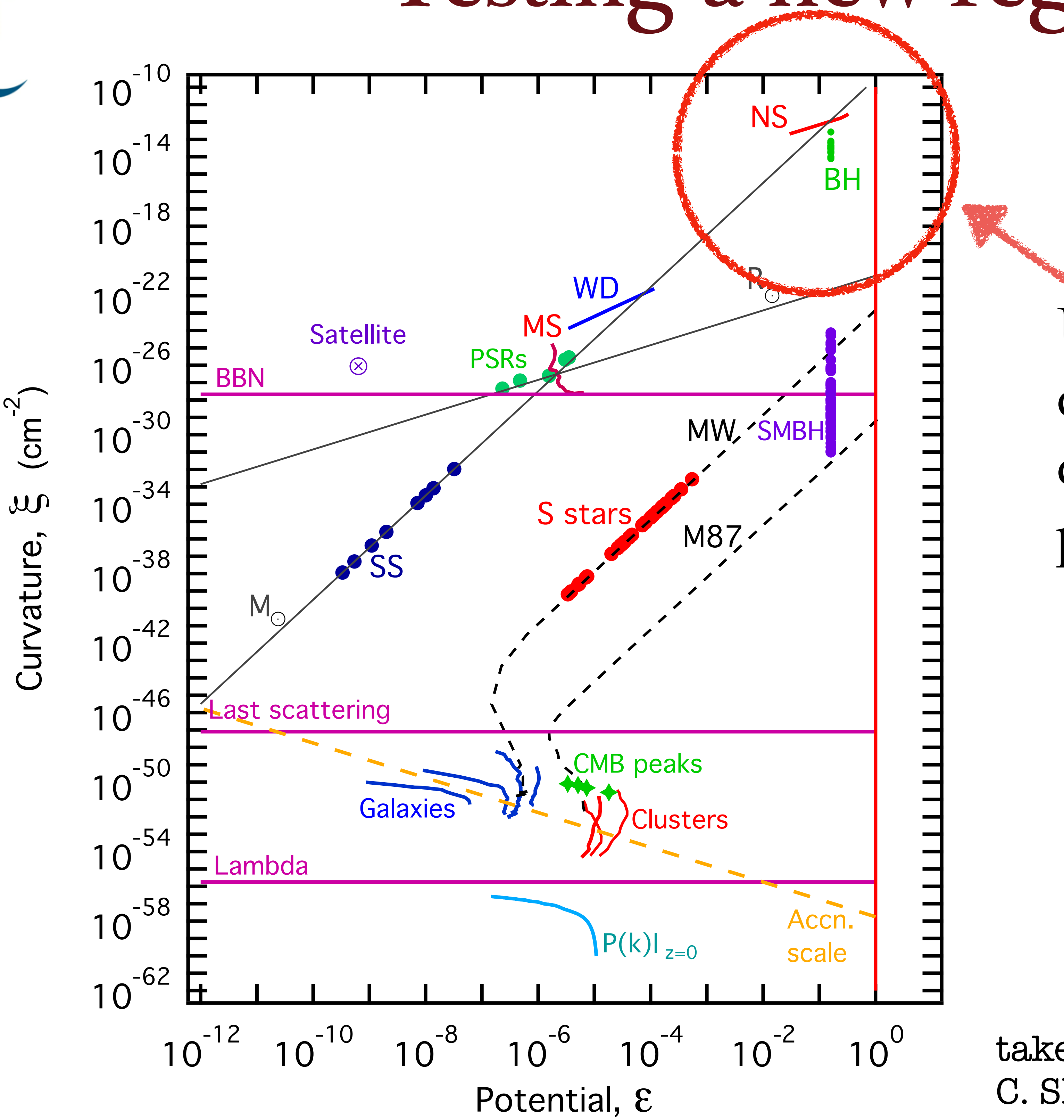


μm
1AU
15Mpc
Quantum Gravity
General Relativity plus unknown corrections?
General Relativity
General Relativity plus Dark Matter
General Relativity plus Dark Matter and Dark Energy
10^{-35} 10^{-30} 10^{-25} 10^{-20} 10^{-15} 10^{-10} 10^{-5} 10^{0} 10^{5} 10^{10} 10^{15} 10^{20} 10^{25} $[m]$

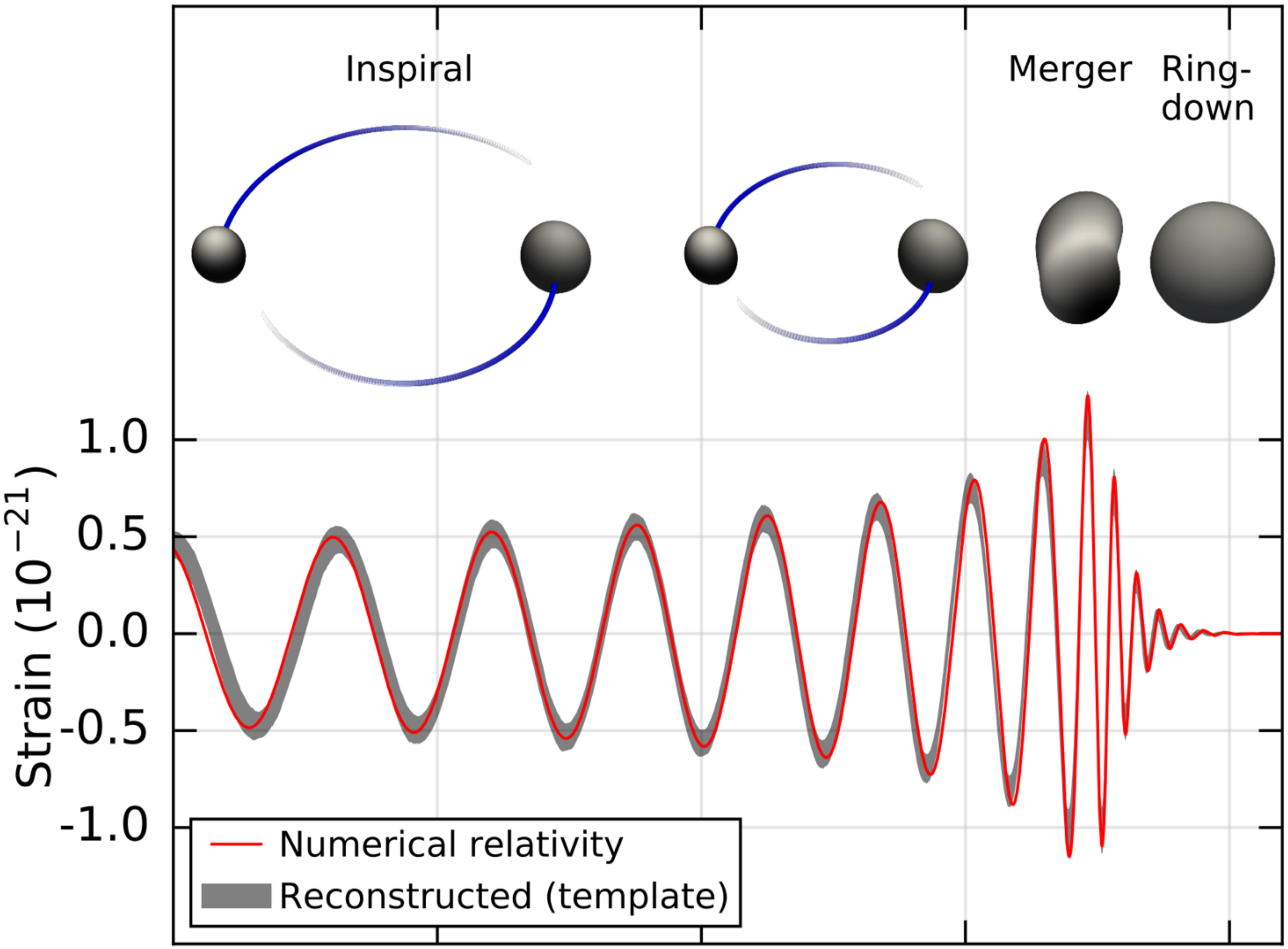
Testing a new regime



Untested combination of curvature and potential!

taken from T. Baker, D. Psaltis, C. Skordis, ApJ 802, 63 (2015)

Waveform



taken from B. P. Abbott et al. (LIGO -Virgo) Phys. Rev. Lett. 116, 061102 (2016)

Modelling new physics

To be tested with GW it

- has to leave an imprint on BHs/NSs
- has to persist in the classical regime
- to be modelled! (i.e. we need equations!)

We can test

- deviations from GR
- extensions of the standard model that couple non-minimally to gravity

In both cases, we are looking for new fields!

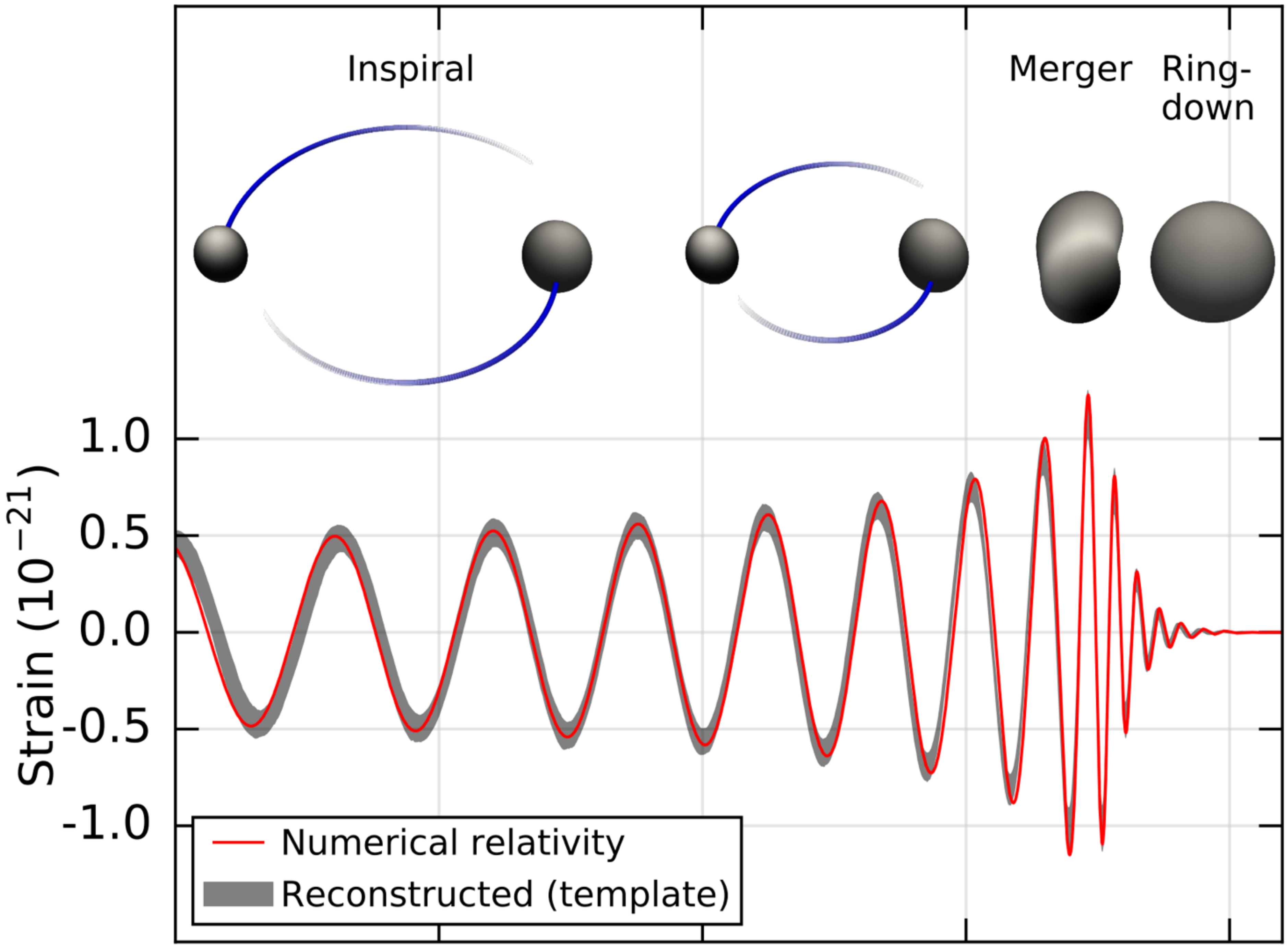
Lovelock and GR

Lovelock's theorem leads to GR under assumptions:

- 4 dimensions
- Covariance
- Second order equations
- No extra fields
- Locality

Not all of them are equally important for phenomenology!

Waveform



taken from B. P. Abbott et al. (LIGO -Virgo) Phys. Rev. Lett. 116, 061102 (2016)

Extracting new physics

Step-by-step guide for your favourite candidate:

- Study compact objects and determine their properties
- Model the inspiral (post-Newtonian); model the ringdown (perturbation theory)
- Do full-blown numerics to get the merger (requires initial value formulation!)

Propagation effects

$$E^2 = m_g^2 \pm M_1 p + c_g^2 p^2 \pm \frac{p^3}{M_3} \pm \frac{p^4}{M_4^2} + \dots$$

- Strong bound on the mass of the graviton, M_1, M_3
- But marginally interesting from a theory perspective
- Weak bounds on M_4 in eV range
- Strong constraint from BNS and EM

$$-2 \times 10^{-15} \geq \Delta c_g/c \geq 7 \times 10^{-16}$$

This rules out several dark energy models that predict $c_g \neq c$

But we can do better in constraining Lorentz violations by looking for other polarisations!

T.P.S., PRL 120, 041104 (2018);
A. E. Gumrukcuoglu, M. Saravani and T.P.S., PRD 97, 024032 (2018).

Parametrizations vs. theories

Advantages of parametrizations:

- We do not need to know the theory!

Disadvantages of parametrizations:

- They only get us half way there - they need interpretation in terms of a theory
- They give us a false sense of achievement - constraints can be meaningless or not independent
- They have limited range of validity

We need theory-specific tests as well!

NR beyond GR: challenges

- Establishing well-posedness: Existence, uniqueness and continuous dependence on initial data

- Interpreting well/ill-posedness in the context of effective field theory (EFT)

- Numerical challenges associated with the above and with having extra fields

Scalar-tensor theory

Jordan frame action:

$$S_{\rm st} = \int d^4x\sqrt{-g}\Big(\varphi R - \frac{\omega(\varphi)}{\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - V(\varphi) + L_m(g_{\mu\nu},\psi)\Big)$$

Redefinitions:

$$\hat g_{\mu\nu} = \varphi g_{\mu\nu} = A^2(\phi) g_{\mu\nu} \qquad 4\sqrt{\pi}\varphi d\phi = \sqrt{2\omega(\varphi)+3}\, d\varphi$$

Einstein frame action:

$$S_{\rm st} = \int d^4x\sqrt{-\hat g}\Big(\frac{\hat R}{16\pi} - \frac{1}{2}\hat g^{\nu\mu}\partial_\nu\phi\partial_\mu\phi - U(\phi)\Big) + S_m(g_{\mu\nu},\psi)$$

Scalar fields in BH spacetimes

The equation
$$\Box\phi = 0$$

admits only the trivial solution in a BH spacetime that is

- stationary, as the endpoint of collapse
- asymptotically flat, i.e. isolated

S.W. Hawking, Comm. Math. Phys. 25, 152 (1972).

The same is true for the equation

$$\Box\phi = U'(\phi)$$

with the additional assumption of local stability

$$U''(\phi_0) > 0$$

T. P. S. and V. Faraoni, Phys. Rev. Lett. 108, 081103 (2012)