Numerical Relativity beyond General Relativity *Benasque, June 2018*

Waveforms in Scalar-Tensor Theory of Gravity - Part I

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Scalar-Tensor Gravity

- Alternate theory of gravity (ATG)
- Variable gravitational "constant" → scalar field

$$S = \frac{1}{16\pi}\int\left[\phi R - \frac{1}{\phi}\omega\left(\phi\right) g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi\right]\sqrt{-g}d^4x + \int \mathcal{L}\left(m, g_{\alpha\beta}\sqrt{-g}d^4x\right)$$

$$\Rightarrow T^{\alpha\beta}_{,\beta} = 0$$

- Obeys Einstein's Equivalence principle
- $\boxed{\text{Violates Strong Equivalence principle}}$

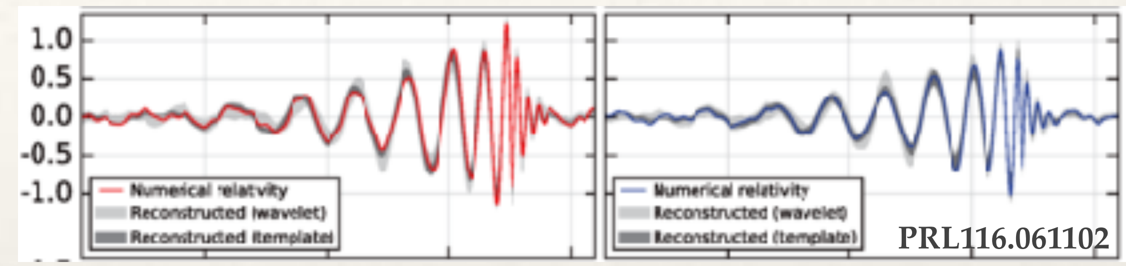
Internal Structure affects motion and GW emission

- Wave equations via the Landau-Lifshitz formalism

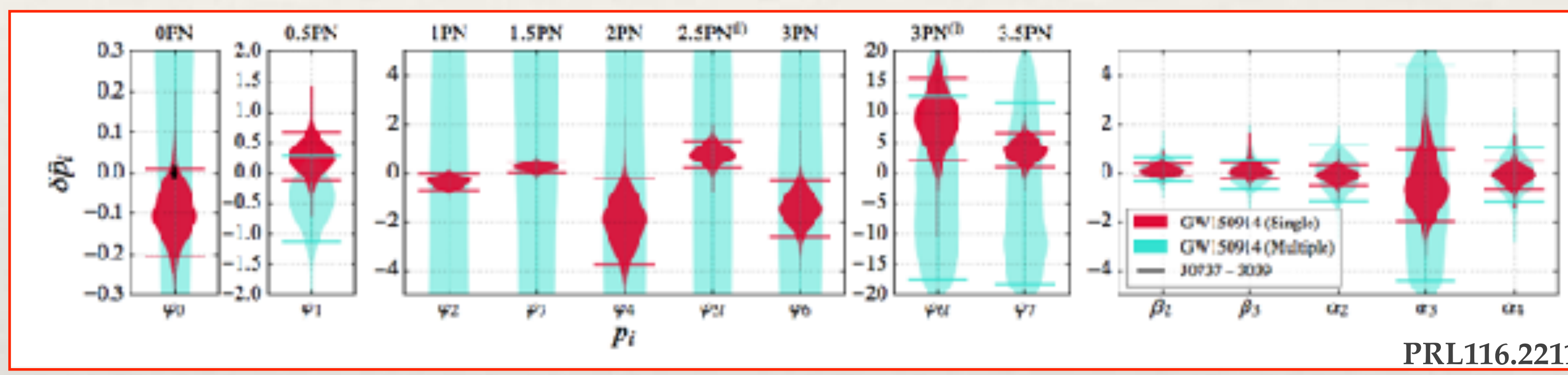
$$\Box_\eta \tilde{h}^{\alpha\beta} = -16\pi\tau^{\alpha\beta}, \qquad \Box_\eta \Psi = -8\pi\tau_s.$$

Motivation

- Seek to verify / constrain / discard ATG's ➔ New tool: Gravitational Wave Astronomy



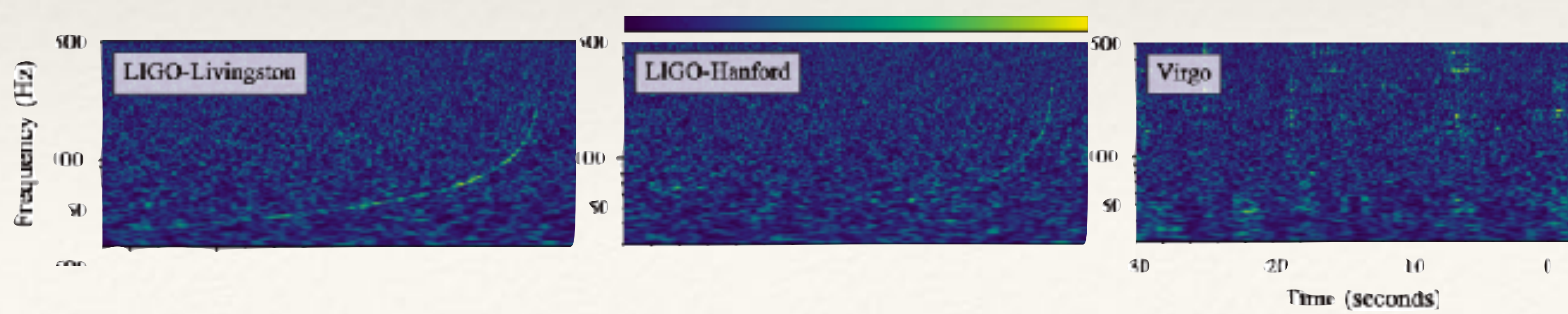
- Current Testing: Agnostic
  (See talks: Del Pozzo, Sennett)



- Why Scalar Tensor?
  - One of the simplest variations of GR
  - Encapsulates some of F(R) and Superstring theories

Warning

  - No-Hair Theorem
  - No problem: GW170817



To date …

- Necessary ingredients for 2PN waveforms:
  - Equations of Motion +0.5PN higher

3PN EOM

    - 2.5PN EOM (Mirshekari & Will, 2013)

  - Tensor and scalar fields (and resulting energy fluxes) to 2PN
    - 2PN Tensor gravitational waves and tensorial energy flux (Lang, 2014)

2.5PN Ψ

    - 1.5PN Scalar gravitational waves and 1PN scalar energy flux (Lang, 2015)
  - Ready to use waveforms, incomplete 2PN (Sennett et al., 2016)
- Culprit: Non-vanishing scalar dipole moment

$\Psi = \Psi_{-1/2} + \Psi_0 + \Psi_{1/2} + \Psi_1 + \Psi_{3/2},$ $\Rightarrow$We require $\Psi_{n+1/2}$ for $\dot{E}_n$,

$\dot{E}_S \propto \dot{\Psi}^2 \Rightarrow \dot{E} = \dot{E}_{-1} + \dot{E}_0 + \dot{E}_{1/2} + \dot{E}_1.$ $\Rightarrow$ We require $\mathrm{EOM}_{(n+1)}$ for $\Psi_{n+1/2}$.

- 3PN EOM (Laura Bernard, 2018 - paper 1 of 2)

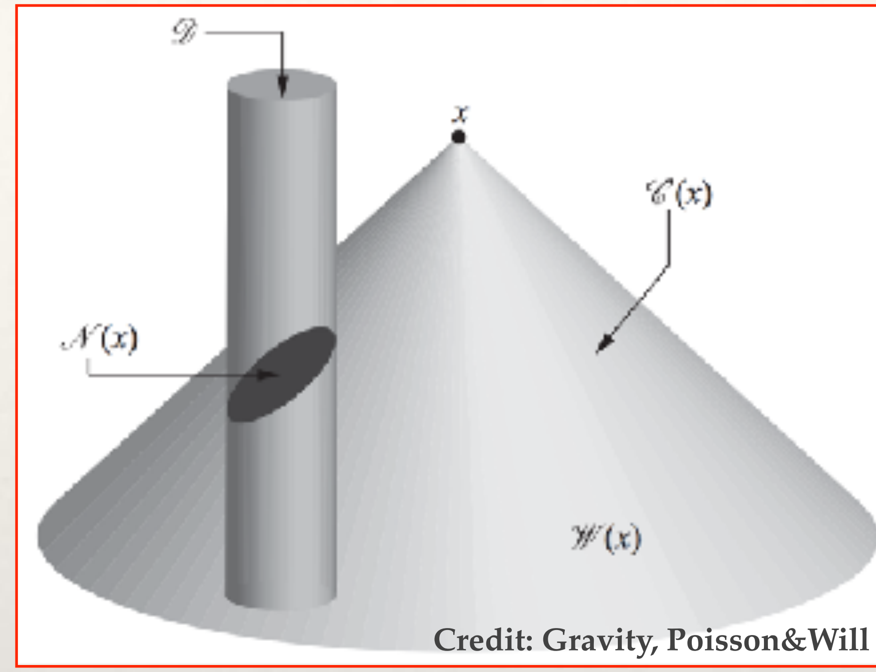
Direct Integration of Relaxed Einstein Equations

- Zones: Near zone, wave zone

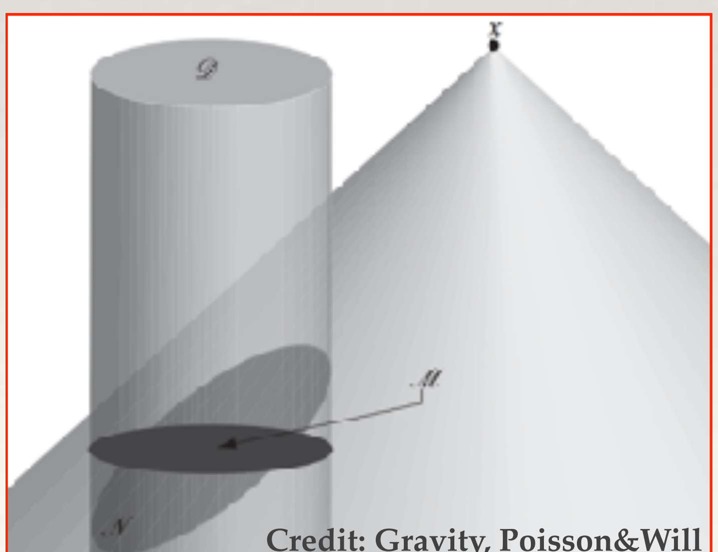
$$\Psi = \Psi_{\mathcal{N}} + \Psi_{\mathcal{W}}, \quad h^{\alpha\beta} = h^{\alpha\beta}_{\mathcal{N}} + h^{\alpha\beta}_{\mathcal{W}}$$

$$\tilde{h}^{\alpha\beta}_{(\mathcal{N}/\mathcal{W})}(t,x) = 4\int_{\mathcal{N}/\mathcal{W}} \frac{\tau^{\alpha\beta}(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|} d^4x',$$

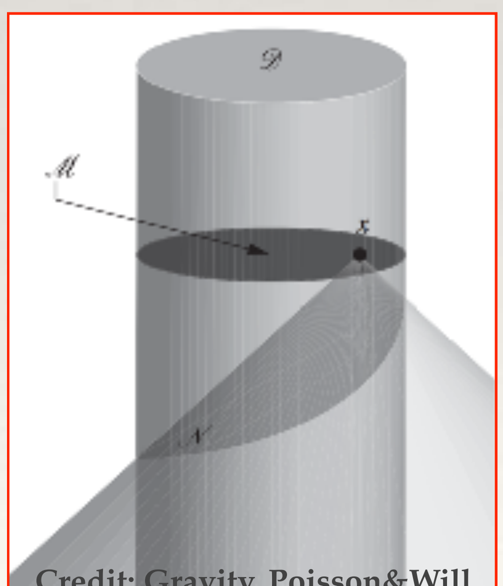
$$\psi_{(\mathcal{N}/\mathcal{W})}(t,x) = 2\int_{\mathcal{N}/\mathcal{W}} \frac{\tau_s(t',x')\,\delta(t'-t+|x-x'|)}{|x-x'|} d^4x'.$$



- Near zone field

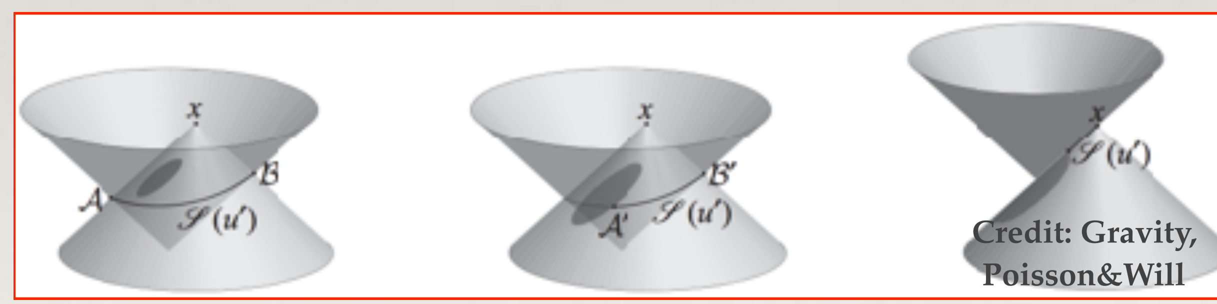


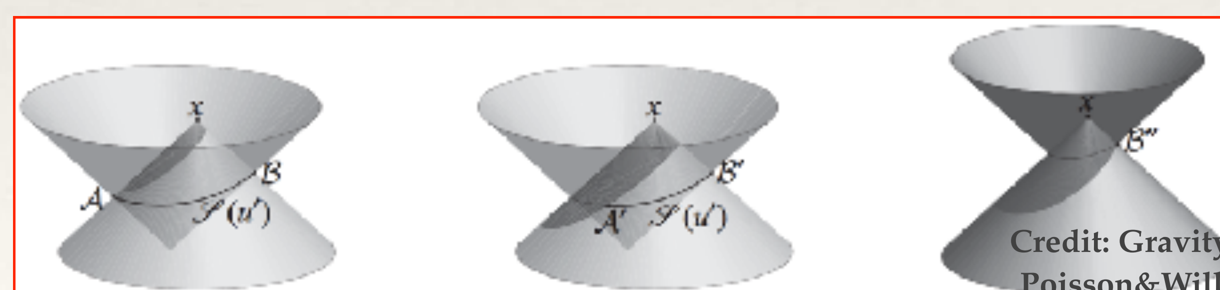
$\Rightarrow x'\, small$



$\Rightarrow |x-x'|\, small$

- Far zone field





Direct Integration of Relaxed Einstein Equations

- Solve simultaneously for source terms $(\tau^{\alpha\beta}, \tau_s)$ and fields $(g^{\alpha\beta}, \Psi)$

3PN

Flat metric
$g^{\alpha\beta} \approx \eta^{\alpha\beta}$

$\sigma \equiv T^{00} + T^{ii}$

$\sigma_s \equiv -T + 2\psi \frac{\partial T}{\partial \psi}$

Calculate 0PN source term

$$\tau^{00} = \frac{\sigma}{\phi_0} + \mathcal{O}(\epsilon\rho)\,,$$
$$\tau^{0a} = \tau^{ab} = \tau^{aa} = \mathcal{O}(\epsilon\rho)\,,$$
$$\tau_s = G\zeta\sigma_s + \mathcal{O}(\epsilon\rho)\,.$$

Add perturbation to fields

$$g^{\alpha\beta}_{(n+1/2)} = g^{\alpha\beta}_{(n)} + \epsilon^{n+1/2} h^{\alpha\beta}_{(n+1/2)}$$
$$\Psi_{s(n+1/2)} = \Psi_{s(n)} + \epsilon^{n+1/2} \Psi_{s(n+1/2)}$$

Calculate next +0.5PN source terms
$(\tau^{\alpha\beta}_{(n+1/2)}, \tau_{s(n+1/2)})$

Solve for next +0.5PN field terms
$(g^{\alpha\beta}_{(n+1/2)}, \Psi_{s(n+1/2)})$

- Use source terms to calculate multipole moments

$$\mathcal{I}^Q_{(s)} \equiv \int \tau_{(s)} x^Q d^3x$$

$G_1, G_{1s}, G_2, G_{2s}, G_3, G_{3s}, G_4, G_5, G_{6s}, v.G_7, H, H_s, H^s{}_s, H^s, U^3, U_s{}^3, U_s U^2, v^2 U^2, v^2 U_s{}^2, v^a v^b P_{2ab}, v^a v^b P_{2sab}, V^2, v^2 v.V, v.V_2, \overset{(4)}{Y}, \overset{(4)}{Y}_s, v^2 \ddot{\chi}, \ddot{\chi}_1, \ddot{\chi}_2, v.\ddot{\chi}, v^2 \ddot{\chi}_s, \ddot{\chi}^s{}_2, \ddot{\chi}^s{}_{2s}, \Sigma_s(\ddot{\chi}),$
$\Sigma_s(\ddot{\chi}_s), \Sigma_s(\Phi_1), \Sigma_s(\Phi^s{}_2), \Sigma_s(\Phi^s{}_{2s}), \Sigma(\ddot{\chi}), \Sigma(\Phi_1), \Sigma(\Phi^s{}_{2s}), v^2 \Phi_1, v^a v^b \Phi_{1ab}, v^2 \Phi_2, v.\Phi_2, P(\dot{U}_s \nabla U_s.v), v^6, v.V^s{}_{2s}, \ddot{\chi}^s{}_1, \ddot{\chi}(U_s a_s), \Sigma_a{}^a(U), \Sigma(a_s \Sigma(U_s a_s)), \Sigma_s(U^2), \Sigma_s(U_s{}^2), \Sigma_s(U_s v^2),$
$\Sigma_s(U U_s), \Sigma_s(U v^2), \Sigma_s(v^4), \Sigma_s(v.V), \Sigma_s(\Sigma(U_s a_s)), \Sigma_s(\Phi_2), \Sigma_s(\Phi^s{}_1), \Sigma(U_s{}^2), \Sigma(U^2), \Sigma(U_s{}^2 a_s), \Sigma(U_s{}^2 b_s), v^2 \Sigma(U_s a_s), \Sigma(U_s v^2 a_s), \Sigma(U U_s a_s), \Sigma(U v^2), v.\Sigma(v^2 v), \Sigma(v^4), \Sigma(v.V), \Sigma(\ddot{\chi}_s a_s),$
$\Sigma(\Phi_2), \Sigma(\Phi^s{}_1 a_s), \Sigma(\Phi^s{}_2 a_s), \Sigma(\Phi^s{}_{2s} a_s), v^2 \Phi^s{}_1, v^2 \Phi^s{}_2, v^2 \Phi^s{}_2, v^2 \Phi^s{}_{2s}, U U_s{}^2, v^2 U U_s, v.\nabla U, U \ddot{\chi}, U \ddot{\chi}_s, U \Phi_1, U \Phi_2, v^4 U, U \Sigma(U_s a_s), U \Phi^s{}_1, U \Phi^s{}_2, U \Phi^s{}_{2s}, U_s v.V, U_s \ddot{\chi}, U_s \ddot{\chi}_s,$
$U_s \Phi_1, U_s \Phi_2, v^4 U_s, U_s \Sigma(U_s a_s), U_s \Phi^s{}_1, U_s \Phi^s{}_2, U_s \Phi^s{}_{2s}.$

ρ' coefficients

$a_s G_{2s}, a_s G_{3s}, a_s G_{6s}, a_s H^s{}_s, a_s H^s, c_s U_s{}^3, b_s U_s{}^3, a_s U_s{}^3, a_s U_s U^2, b_s v^2 U_s{}^2, a_s v^2 U_s, a_s \overset{(4)}{Y}_s, a_s v^2 \ddot{\chi}_s, a_s \ddot{\chi}^s{}_2, a_s \ddot{\chi}^s{}_{2s}, a_s \Sigma_s(\ddot{\chi}), a_s \Sigma_s(\ddot{\chi}_s), a_s \Sigma_s(\Phi_1), a_s \Sigma_s(\Phi^s{}_2), a_s \Sigma_s(\Phi^s{}_{2s}), a_s \ddot{\chi}^s{}_1,$
$a_s \ddot{\chi}(U_s a_s), a_s \Sigma(a_s \Sigma(a_s U_s)), a_s \Sigma_s(U^2), a_s \Sigma_s(U_s{}^2), a_s \Sigma_s(U_s v^s), a_s \Sigma_s(U U_s), a_s \Sigma_s(U v^2), a_s \Sigma_s(v^4), a_s \Sigma_s(v.V), a_s \Sigma_s(\Sigma(U_s a_s)), a_s \Sigma_s(\Phi_2), a_s \Sigma_s(\Phi^s{}_1), a_s \Sigma(U_s{}^2 a_s), a_s \Sigma(U_s{}^2 a_s),$
$a_s \Sigma(U_s{}^2 b_s), a_s v^2 \Sigma(U a_s), a_s \Sigma(U_s v^2 a_s), a_s \Sigma(U U_s a_s), a_s \Sigma(\ddot{\chi}_s a_s), a_s \Sigma(\Phi^s{}_1 a_s), a_s \Sigma(\Phi^s{}_2 a_s), a_s \Sigma(\Phi^s{}_{2s} a_s), v^2 \Phi^s{}_1, a_s v^2 \Phi^s{}_2, a_s v^2 \Phi^s{}_2, a_s v^2 \Phi^s{}_{2s}, b_s U U_s{}^2, a_s U U_s{}^2, a_s v^2 U U_s,$
$a_s U \ddot{\chi}_s, a_s U \Sigma(U_s a_s), a_s U \Phi^s{}_1, a_s U \Phi^s{}_2, a_s U \Phi^s{}_{2s}, a_s v.V U_s, a_s U_s \ddot{\chi}, b_s U_s \ddot{\chi}_s, a_s U_s \ddot{\chi}_s, a_s U_s \Phi_1, a_s U_s \Phi_2, b_s U_s \Sigma(U_s a_s), b_s U_s \Phi^s{}_1, a_s v^4 U_s, a_s U_s \Sigma(U_s a_s), a_s U_s \Phi^s{}_1, b_s U_s \Phi^s{}_2,$
$a_s U_s \Phi^s{}_2, b_s U_s \Phi^s{}_{2s}, a_s U_s \Phi^s{}_{2s}.$

$r_{s,F1}$

$U \ddot{U}_s{}^2, \dot{U}_s \overset{(3)}{\chi}_s, \dot{U}_s \dot{\Sigma}(U_s a_s), \dot{U}_s \dot{\Phi}^s{}_1, \dot{U}_s \dot{\Phi}^s{}_2, \dot{U}_s \dot{\Phi}^s{}_{2s}, \dot{U}_s V.\nabla U_s, U^2 (\nabla U_s)^2, U_s{}^2 (\nabla U_s)^2, \Phi_2 (\nabla U_s)^2, \Sigma(U_s a_s)(\nabla U_s)^2, \Phi^s{}_1 (\nabla U_s)^2, \nabla\Sigma_s(U^2).\nabla U_s, \nabla\Sigma_s(U_s{}^2).\nabla U_s, \nabla\Sigma_s(U_s v^2).\nabla U_s,$
$\nabla\Sigma_s(U U_s).\nabla U_s, \nabla\Sigma_s(U v^2).\nabla U_s, \nabla\Sigma_s(\Sigma(U_s a_s)).\nabla U_s, \nabla\Sigma_s(\Phi_1).\nabla U_s, \nabla\Sigma_s(\Phi_2).\nabla U_s, \nabla\Sigma_s(\Phi_{2s}).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_1).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_2).\nabla U_s, \nabla\Sigma_s(\Phi^s{}_{2s}).\nabla U_s, \nabla\Sigma(U_s{}^2 a_s).\nabla U_s, \nabla\Sigma(U_s U a_s).\nabla U_s,$
$\nabla\Sigma(a_s \Sigma(U_s a_s)).\nabla U_s, \nabla\Sigma_s(v^4).\nabla U_s, \nabla\Sigma_s(v.V).\nabla U_s, \nabla\Sigma_s(\ddot{\chi}).\nabla U_s, \nabla\Sigma_s(\ddot{\chi}_s).\nabla U_s, \nabla\Sigma(U_s{}^2 b_s).\nabla U_s, \nabla\Sigma(U_s v^2 a_s).\nabla U_s, \nabla\Sigma(\ddot{\chi}_s a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_1 a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_2 a_s).\nabla U_s, \nabla\Sigma(\Phi^s{}_{2s} a_s).\nabla U_s,$
$\nabla G_{2s}.\nabla U_s, \nabla G_{3s}.\nabla U_s, \nabla G_{6s}.\nabla U_s, \nabla H^s{}_s.\nabla U_s, \nabla H^s.\nabla U_s, \ddot{\chi}_s (\nabla U_s)^2, (\nabla \ddot{\chi}_s)^2, \nabla\Sigma(U_s a_s).\nabla\ddot{\chi}_s, \nabla\Phi^s{}_1.\nabla\ddot{\chi}_s, \nabla\Phi^s{}_2.\nabla\ddot{\chi}_s, \nabla\Phi^s{}_{2s}.\nabla\ddot{\chi}_s, \nabla U_s.\nabla\overset{(4)}{Y}_s, \nabla U_s.\nabla\ddot{\chi}^s{}_2, \nabla U_s.\nabla\ddot{\chi}^s{}_{2s},$
$\nabla U_s.\nabla\ddot{\chi}^s{}_1, \nabla U_s.\nabla\ddot{\chi}(U_s a_s), (\nabla\Sigma(U_s a_s))^2, \nabla\Phi^s{}_1.\nabla\Sigma(U_s a_s), (\nabla\Phi^s{}_1)^2, \nabla\Phi^s{}_2.\nabla\Phi^s{}_{2s}, \nabla\Phi^s{}_2.\nabla\Sigma(U_s a_s), \nabla\Phi^s{}_2.\nabla\Phi^s{}_1, (\nabla\Phi^s{}_2)^2, \nabla\Phi^s{}_{2s}.\nabla\Sigma(U_s a_s), \nabla\Phi^s{}_{2s}.\nabla\Phi^s{}_1, (\nabla\Phi^s{}_{2s})^2, P_2{}^{ab} \nabla_a U_s \nabla_b U_s,$
$P_{2s}{}^{ab} \nabla_a U_s \nabla_b U_s, \Phi_1{}^{ab} \nabla_a U_s \nabla_b U_s, \Phi^s{}_2 (\nabla U_s)^2, \Phi^s{}_{2s} (\nabla U_s)^2, U_s \ddot{U}_s{}^2, U_s \nabla U_s.\nabla\Sigma(U_s a_s), U_s \nabla U_s.\nabla\Phi^s{}_1, U_s \nabla U_s.\nabla\Phi^s{}_2, U_s \nabla U_s.\nabla\Phi^s{}_{2s}, U_s \nabla U_s.\nabla\ddot{\chi}_s$

$r_{s,F2}$

$\rho_s G_1, \rho_s G_4, \rho_s G_5, \rho_s U^3, \rho_s U_s{}^3, \rho_s V^2, \rho_s \ddot{\chi}_2, \rho_s \ddot{\chi}^s{}_{2s}, \rho_s \Sigma_s(V^a), \rho_s \Sigma_s(\ddot{\chi}_s), \rho_s \Sigma_s(\Phi^s{}_2), \rho_s \Sigma_s(\Phi^s{}_{2s}), \rho_s \Sigma_s(U_s{}^2), \rho_s \Sigma_s(U_s U), \rho_s \Sigma_s(U_s v^2), \rho_s \Sigma_s(\Sigma(U_s a_s)), \rho_s \Sigma_s(\Phi^s{}_1), \rho_s \Sigma(U_s U), \rho_s \Sigma(\ddot{\chi}),$
$\rho_s \Sigma(\Phi_{2s}), \rho_s \Sigma(U^2), \rho_s \Sigma(U_s{}^2 a_s), \rho_s \Sigma(U v^2), \rho_s \Sigma(\Phi_1), \rho_s \Sigma(\Phi_2), \rho_s \Sigma(\Phi_{2s}), \rho_s \Sigma(\Phi^s{}_{2s}), \rho_s U U_s{}^2, \rho_s U \ddot{\chi}, \rho_s U \Phi_2, \rho_s U \Phi^s{}_{2s}, \rho_s U \Phi_{2s}, \rho_s U \Phi_1, \rho_s U^2 U_s, \rho_s U_s \ddot{\chi}_s, \rho_s U_s \Phi_2, \rho_s U_s \Phi^s{}_2,$
$\rho_s U_s \Phi^s{}_{2s}, \ddot{U}_s \ddot{\chi}, \ddot{U}_s \Phi_1, G_7.\nabla\dot{U}_s, P(\dot{U}_s \nabla U_s).\nabla\dot{U}_s, V_2.\nabla\dot{U}_s, V^s{}_{2s}.\nabla\dot{U}_s, \ddot{\chi}.\nabla\dot{U}_s, \Sigma(v^2 v).\nabla\dot{U}_s, \Phi_2.\nabla\dot{U}_s, V.\nabla\dot{\Sigma}(U_s a_s), V.\nabla\dot{\Phi}^s{}_1, V.\nabla\dot{\Phi}^s{}_2, V.\nabla\dot{\Phi}^s{}_{2s}, V.\nabla\dot{U}_s \dot{U}_s, V.\nabla\overset{(3)}{\chi}_s, P_{3s}{}^{ab} \nabla_a \nabla_b U_s,$
$P^S{}_{s3s}{}^{ab} \nabla_a \nabla_b U_s, P^S{}_{s3}{}^{(ab)} \nabla_a \nabla_b U_s, P(\nabla^{(a} U V^{b)}) \nabla_a \nabla_b U_s, P(\nabla^{(a} U \nabla^{b)} \ddot{\chi}) \nabla_a \nabla_b U_s, P(\nabla^{(a} U_s \nabla^{b)} \ddot{\chi}_s) \nabla_a \nabla_b U_s, S_2{}^{ab} \nabla_a \nabla_b U_s, S_{2s}{}^{ab} \nabla_a \nabla_b U_s, \ddot{\chi}^{ab} \nabla_a \nabla_b U_s, \Sigma^{ab}(U) \nabla_a \nabla_b U_s, \Sigma^{ab}(U_s) \nabla_a \nabla_b U_s,$
$P_3{}^{(ab)} \nabla_a \nabla_b U_s, P_{3s}{}^{(ab)} \nabla_a \nabla_b U_s, P_{s3s}{}^{(ab)} \nabla_a \nabla_b U_s, P^S{}_{s3}{}^{(ab)} \nabla_a \nabla_b U_s, P^S{}_{s3s}{}^{(ab)} \nabla_a \nabla_b U_s, P(\nabla^{(a} U_s \nabla^{b)} \Sigma(U_s a_s)) \nabla_a \nabla_b U_s, P(\nabla^{(a} U_s \nabla^{b)} \Phi^s{}_1) \nabla_a \nabla_b U_s, P(\nabla^{(a} U \nabla^{b)} \Phi_1) \nabla_a \nabla_b U_s,$
$\Sigma_s(U_s v^a v^b) \nabla_a \nabla_b U_s, \Sigma(U_s v^a v^b) \nabla_a \nabla_b U_s, \Sigma(U v^a v^b) \nabla_a \nabla_b U_s, \Sigma(v^2 v^a v^b) \nabla_a \nabla_b U_s, \Phi_1{}^{ab} \nabla_a \nabla_b \Sigma(U_s a_s), \Phi_1{}^{ab} \nabla_a \nabla_b \Phi^s{}_1, \Phi_1{}^{ab} \nabla_a \nabla_b \Phi^s{}_2, \Phi_1{}^{ab} \nabla_a \nabla_b \Phi^s{}_{2s}, P_2{}^{ab} \nabla_a \nabla_b \Sigma(U_s a_s),$
$P_2{}^{ab} \nabla_a \nabla_b \Phi^s{}_1, P_2{}^{ab} \nabla_a \nabla_b \Phi^s{}_2, P_2{}^{ab} \nabla_a \nabla_b \Phi^s{}_{2s}, P_{2s}{}^{ab} \nabla_a \nabla_b \Sigma(U_s a_s), P_{2s}{}^{ab} \nabla_a \nabla_b \Phi^s{}_1, P_{2s}{}^{ab} \nabla_a \nabla_b \Phi^s{}_2, P_{2s}{}^{ab} \nabla_a \nabla_b \Phi^s{}_{2s}, \Phi_1{}^{ab} \nabla_a \nabla_b \ddot{\chi}_s, P_2{}^{ab} \nabla_a \nabla_b \ddot{\chi}_s, P_{2s}{}^{ab} \nabla_a \nabla_b \ddot{\chi}_s, U \overset{(4)}{\chi}_s, U U_s \ddot{U}_s,$
$U \ddot{\Sigma}(U_s a_s), U \ddot{\Phi}^s{}_1, U \ddot{\Phi}^s{}_2, U \ddot{\Phi}^s{}_{2s}, U \dot{U}_s{}^2, U \nabla\dot{U}_s.V, U^2 \ddot{U}_s, U^2 (\nabla U_s)^2, \Phi_2 \ddot{U}_s, \Phi_2 (\nabla U_s)^2, \Phi^s{}_{2s} \ddot{U}_s, \Phi^s{}_{2s} (\nabla U_s)^2, U_s{}^2 (\nabla U_s)^2, P_2{}^{ab} \nabla_a U_s \nabla_b U_s, P_{2s}{}^{ab} \nabla_a U_s \nabla_b U_s, \Phi_1{}^{ab} \nabla_a U_s \nabla_b U_s,$
$P(V^{[c.a]} V_{[c}{}^{.b]}) \nabla_a \nabla_b U_s, U_s V.\nabla\dot{U}_s, P_2{}^{ab} \nabla_a \nabla_b U_s, P_{2s}{}^{ab} \nabla_a \nabla_b U_s, \Phi_1{}^{ab} \nabla_a \nabla_b U_s$

$\rho U^2 U_s a_s, \rho_s U^2 v^2, \rho_s U^3, \rho_s U^2 U_s, \rho \Phi_2 U_s a_s, \rho_s \Phi_2 v^2, \rho_s \Phi_2 U, \rho_s \Phi_2 U_s, \rho \Phi^s{}_{2s} U_s a_s, \rho_s \Phi^s{}_{2s} v^2, \rho_s \Phi^s{}_{2s} U, \rho_s \Phi^s{}_{2s} U_s, \rho U_s{}^3 a_s, \rho_s U_s{}^2 v^2, \rho_s U_s{}^2 U, \rho_s U_s{}^3, \rho_s U_s \Sigma(U_s a_s),$
$\rho_s U_s \Phi^s{}_1$

Direct Integration of Relaxed Einstein Equations

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3PN

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$\sigma_s \equiv -T + 2\psi \frac{\partial T}{\partial \psi}$

Calculate 0PN source term

$$\tau^{00} = \frac{\sigma}{\phi_0} + \mathcal{O}(\epsilon\rho)\,,$$
$$\tau^{0a} = \tau^{ab} = \tau^{aa} = \mathcal{O}(\epsilon\rho)\,,$$
$$\tau_s = G\zeta\sigma_s + \mathcal{O}(\epsilon\rho)\,.$$

Add perturbation to fields

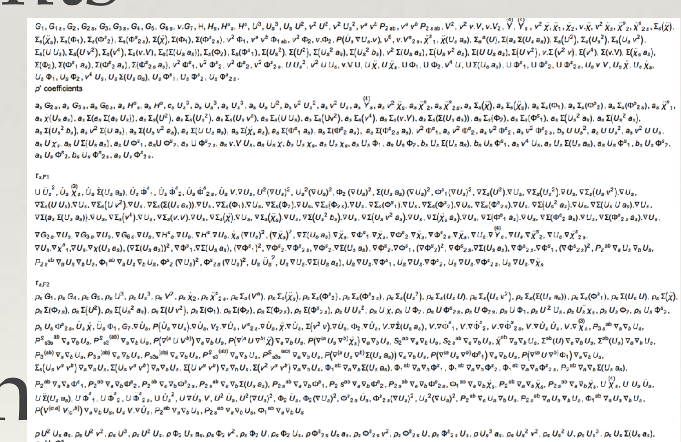
$$g^{\alpha\beta}_{(n+1/2)} = g^{\alpha\beta}_{(n)} + \epsilon^{n+1/2} h^{\alpha\beta}_{(n+1/2)}$$
$$\Psi_{s(n+1/2)} = \Psi_{s(n)} + \epsilon^{n+1/2}\Psi_{s(n+1/2)}$$

Calculate next +0.5PN source terms
$(\tau^{\alpha\beta}_{(n+1/2)}, \tau_{s(n+1/2)})$

Solve for next +0.5PN field terms
$(g^{\alpha\beta}_{(n+1/2)}, \Psi_{s(n+1/2)})$

Use source terms to calculate multipole moments

$$\mathcal{I}^{Q}_{(s)} \equiv \int \tau_{(s)} x^Q d^3x$$



- Terms like $\dddot{\mathcal{I}}$ require 3PN equations of motion

Tricky in Will, Wiseman &Pati   Solution: Laura Bernard!

And now to Laura .....