

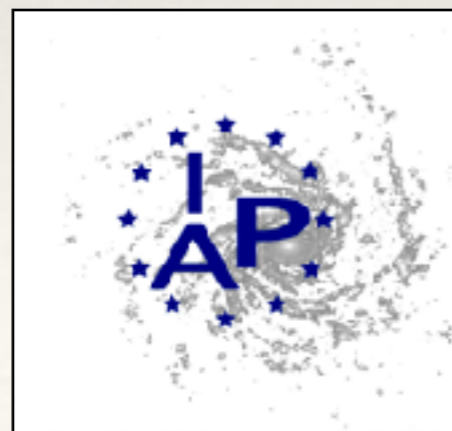


Numerical Relativity beyond General Relativity

Benasque, June 2018

Waveforms in Scalar-Tensor Theory of Gravity - Part I

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Scalar-Tensor Gravity

- ❖ Alternate theory of gravity (ATG)
- ❖ Variable gravitational “constant” → scalar field

$$S = \frac{1}{16\pi} \int \left[\phi R - \frac{1}{\phi} \omega(\phi) g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi \right] \sqrt{-g} d^4x + \int \mathcal{L}(m, g_{\alpha\beta}) \sqrt{-g} d^4x$$

$\Rightarrow T_{,\beta}^{\alpha\beta} = 0$

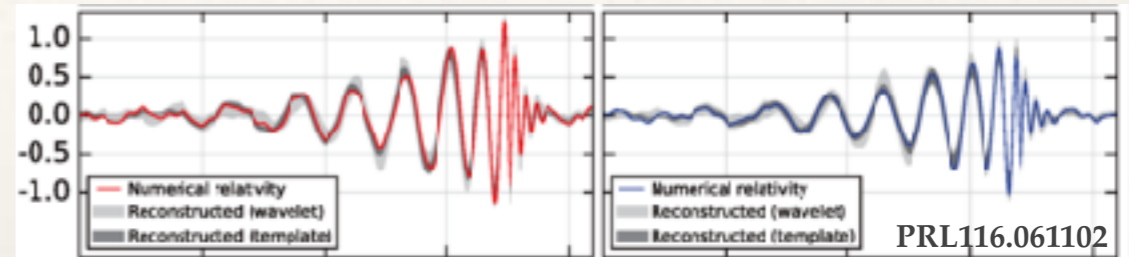
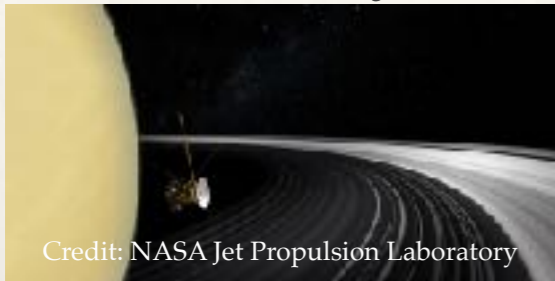
- ❖ Obeys Einstein’s Equivalence principle
- ❖ Violates Strong Equivalence principle
- ❖ Wave equations via the Landau-Lifshitz formalism

Internal Structure
affects motion and
GW emission

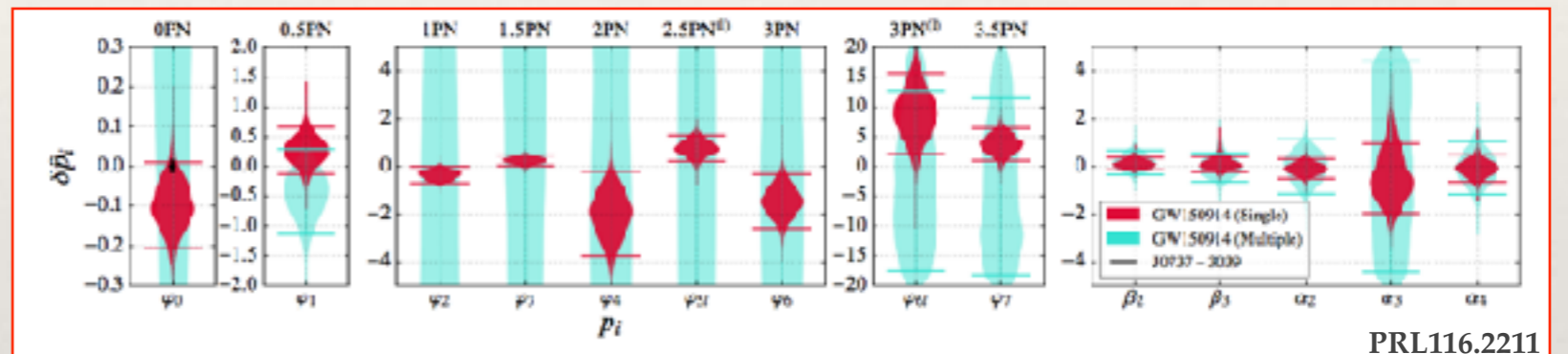
$$\square_\eta \tilde{h}^{\alpha\beta} = -16\pi \tau^{\alpha\beta}, \quad \square_\eta \Psi = -8\pi \tau_s.$$

Motivation

- ❖ Seek to verify / constrain / discard ATG's → New tool: Gravitational Wave Astronomy



- ❖ Current Testing: Agnostic (See talks: Del Pozzo, Sennett)



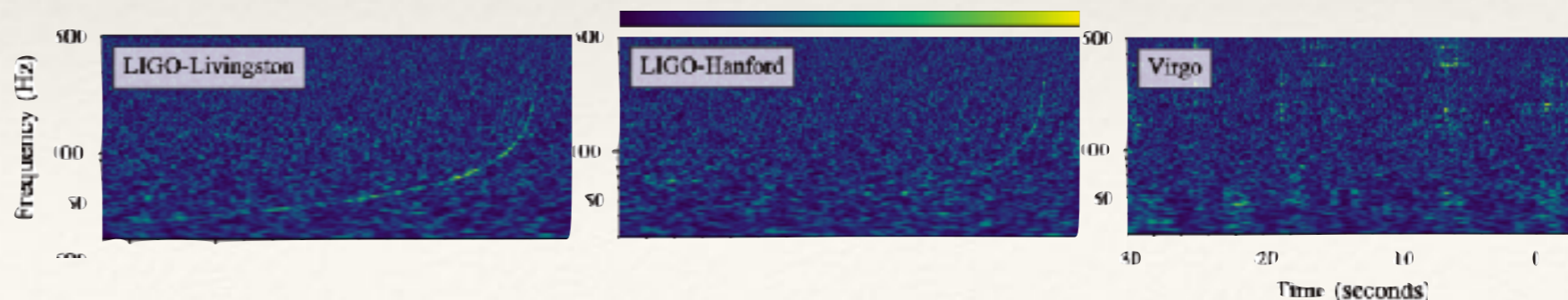
- ❖ Why Scalar Tensor?

- ❖ One of the simplest variations of GR
- ❖ Encapsulates some of F(R) and Superstring theories

Warning

- ❖ No-Hair Theorem

No problem: GW170817



To date ...

- ❖ Necessary ingredients for 2PN waveforms:

- ❖ Equations of Motion +0.5PN higher

3PN
EOM

- ❖ 2.5PN EOM (Mirshekari & Will, 2013)



- ❖ Tensor and scalar fields (and resulting energy fluxes) to 2PN

- ❖ 2PN Tensor gravitational waves and tensorial energy flux (Lang, 2014)

2.5PN
 Ψ

- ❖ 1.5PN Scalar gravitational waves and 1PN scalar energy flux (Lang, 2015)

- ❖ Ready to use waveforms, incomplete 2PN (Sennett et al., 2016)

- ❖ **Culprit: Non-vanishing scalar dipole moment**

$$\Psi = \Psi_{-1/2} + \Psi_0 + \Psi_{1/2} + \Psi_1 + \Psi_{3/2}, \Rightarrow \text{We require } \Psi_{n+1/2} \text{ for } \dot{E}_n,$$

$$\dot{E}_S \propto \dot{\Psi}^2 \Rightarrow \dot{E} = \dot{E}_{-1} + \dot{E}_0 + \dot{E}_{1/2} + \dot{E}_1. \Rightarrow \text{We require EOM}_{(n+1)} \text{ for } \Psi_{n+1/2}.$$

- ❖ 3PN EOM (Laura Bernard, 2018 - paper 1 of 2)

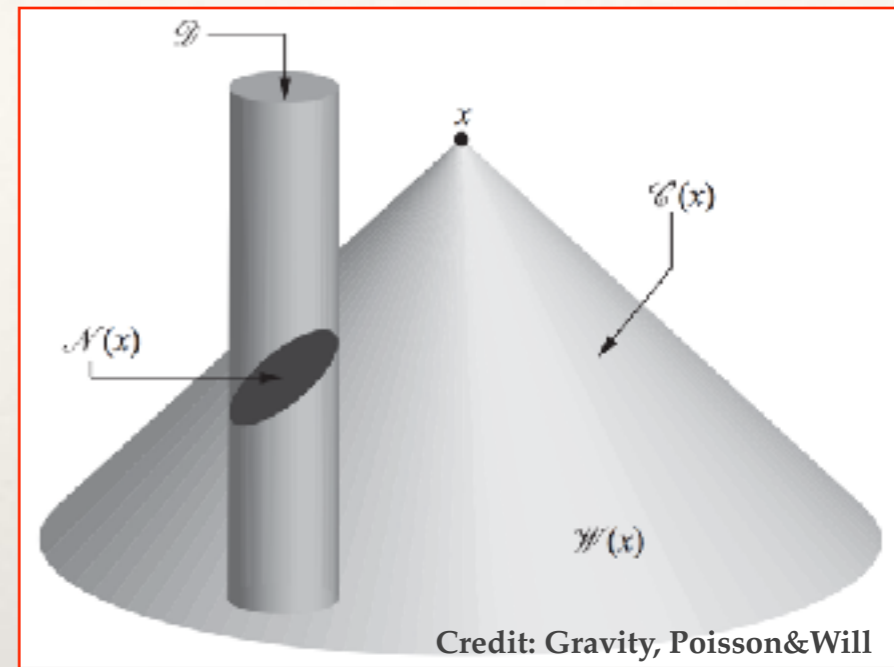
Direct Integration of Relaxed Einstein Equations

- ❖ Zones: Near zone, wave zone

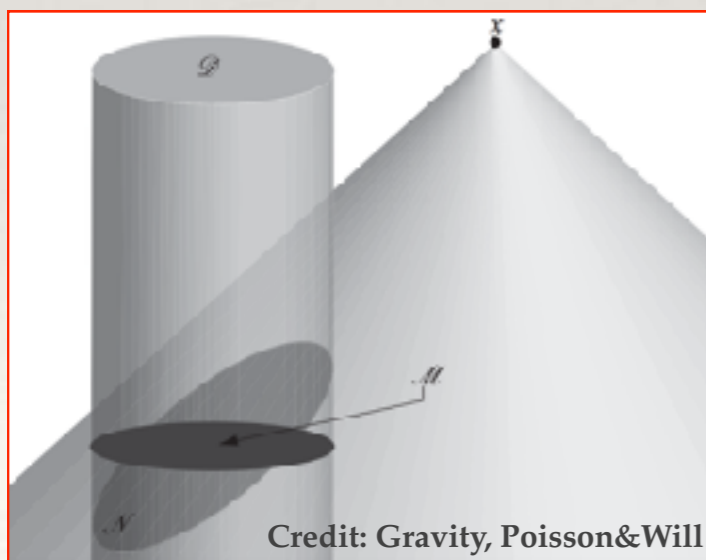
$$\Psi = \Psi_{\mathcal{N}} + \Psi_{\mathcal{W}}, \quad h^{\alpha\beta} = h_{\mathcal{N}}^{\alpha\beta} + h_{\mathcal{W}}^{\alpha\beta}$$

$$\tilde{h}_{(\mathcal{N}/\mathcal{W})}^{\alpha\beta}(t, x) = 4 \int_{\mathcal{N}/\mathcal{W}} \frac{\tau^{\alpha\beta}(t', x') \delta(t' - t + |x - x'|)}{|x - x'|} d^4 x',$$

$$\psi_{(\mathcal{N}/\mathcal{W})}(t, x) = 2 \int_{\mathcal{N}/\mathcal{W}} \frac{\tau_s(t', x') \delta(t' - t + |x - x'|)}{|x - x'|} d^4 x'.$$

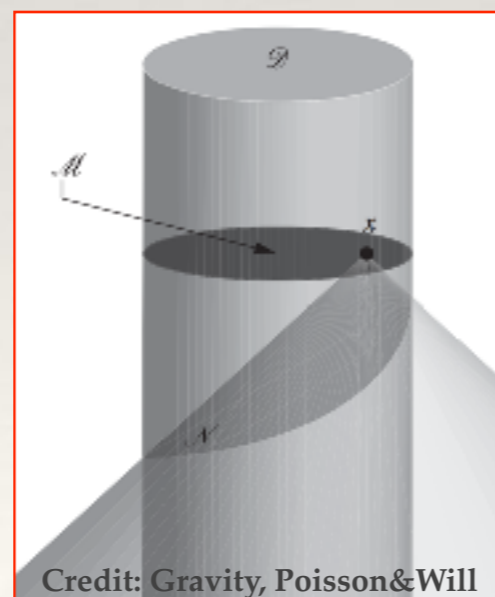


- ❖ Near zone field

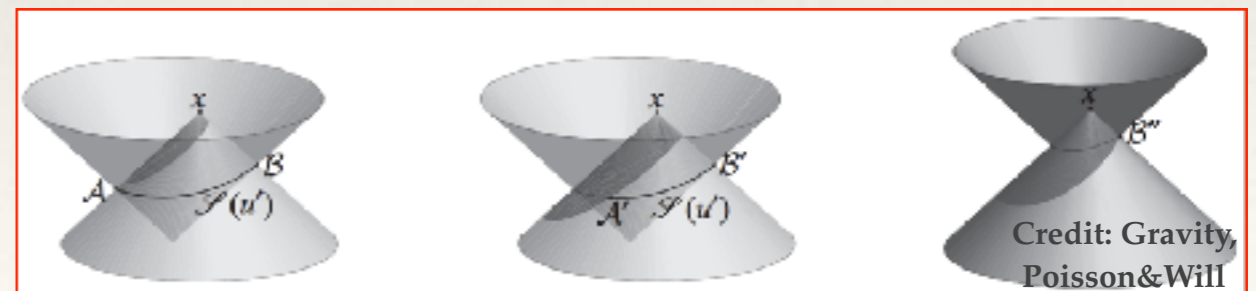
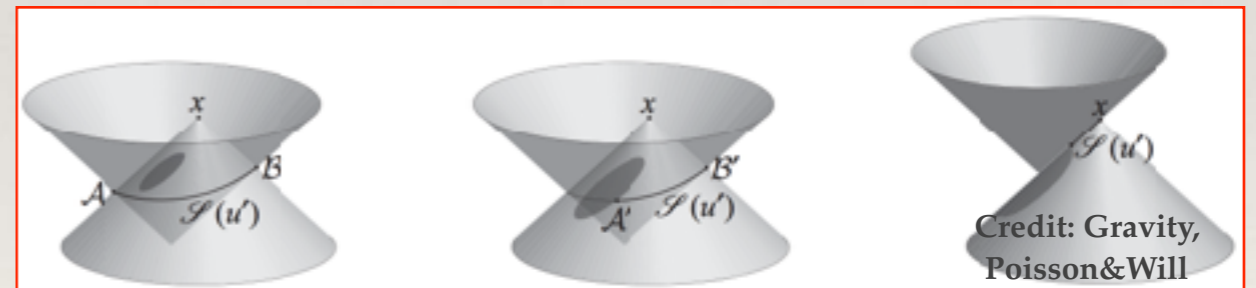


$\Rightarrow x'$ small

- ❖ Far zone field

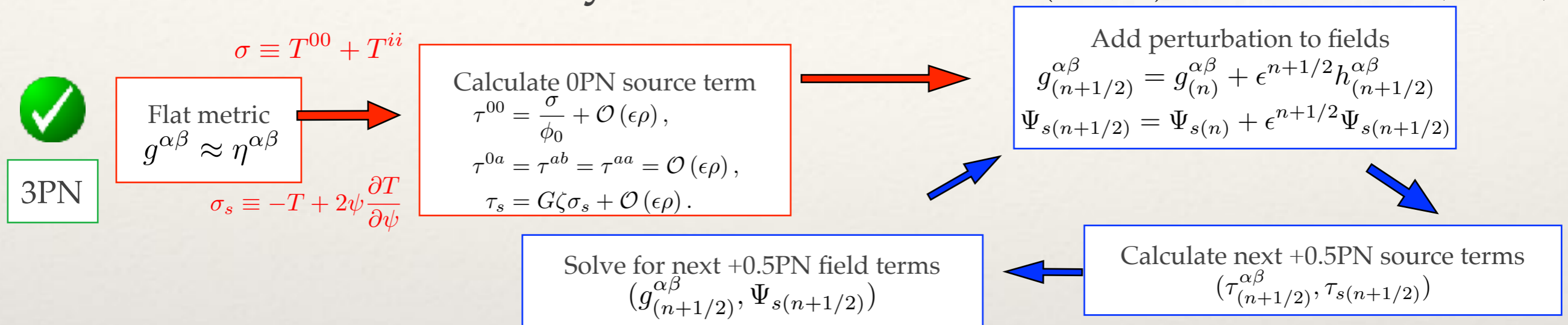


$\Rightarrow |x - x'|$ small



Direct Integration of Relaxed Einstein Equations

- ❖ Solve simultaneously for source terms $(\tau^{\alpha\beta}, \tau_s)$ and fields $(g^{\alpha\beta}, \Psi)$



- ❖ Use source terms to calculate multipole moments

$$\mathcal{I}_{(s)}^Q \equiv \int \tau_{(s)} x^Q d^3x$$

$G_1, G_{1s}, G_2, G_{2s}, G_3, G_{3s}, G_4, G_5, G_{6s}, v.G_7, H, H_s, H^s, H^s, U^3, U_s^3, U_s U^2, v^2 U^2, v^2 U_s^2, v^a v^b P_{2ab}, v^a v^b P_{2sab}, V^2, v^2 v.V, v.V_2, \overset{(4)}{Y}, \overset{(4)}{Y}_s, v^2 \ddot{\chi}, \ddot{\chi}_1, \ddot{\chi}_2, v.\ddot{\chi}, v^2 \ddot{\chi}_s, \ddot{\chi}^2, \ddot{\chi}^s_{2s}, \Sigma_s(\ddot{\chi}),$
 $\Sigma_s(\ddot{\chi}_s), \Sigma_s(\Phi_1), \Sigma_s(\Phi^2_2), \Sigma_s(\Phi^s_{2s}), \Sigma(\ddot{\chi}), \Sigma(\Phi_1), \Sigma(\Phi^2_2), v^2 \Phi_1, v^a v^b \Phi_{1ab}, v^2 \Phi_2, v.\Phi_2, P(\dot{U}_s \nabla U_s v), v^e, v.V^e_{2s}, \ddot{\chi}^s_1, \ddot{\chi}(U_s a_s), \Sigma^a(U), \Sigma(a_s \Sigma(U_s a_s)), \Sigma(U^2), \Sigma(U_s^2), \Sigma(U_s v^2),$
 $\Sigma_s(U U_s), \Sigma_s(U v^2), \Sigma_s(v^4), \Sigma_s(v.V), \Sigma_s(\Sigma(U_s a_s)), \Sigma_s(\Phi_2), \Sigma_s(\Phi^s_1), \Sigma(U_s^2), \Sigma(U^2), \Sigma(U_s^2 a_s), \Sigma(U_s^2 b_s), v^2 \Sigma(U_s a_s), \Sigma(U_s v^2 a_s), \Sigma(U U_s a_s), \Sigma(U v^2), v.\Sigma(v^2 v), \Sigma(v^4), \Sigma(v.V), \Sigma(\ddot{\chi}_s a_s),$
 $\Sigma(\Phi_2), \Sigma(\Phi^s_1 a_s), \Sigma(\Phi^s_2 a_s), \Sigma(\Phi^s_{2s} a_s), v^2 \Phi^s_1, v^2 \Phi^s_2, v^2 \Phi^s_2, v^2 \Phi^s_{2s}, U U_s^2, v^2 U U_s, v.V U, U \ddot{\chi}, U \ddot{\chi}_s, U \Phi_1, U \Phi_2, v^4 U, U \Sigma(U_s a_s), U \Phi^s_1, U \Phi^s_2, U \Phi^s_{2s}, U_s v.V, U_s \ddot{\chi}, U_s \ddot{\chi}_s,$
 $U_s \Phi_1, U_s \Phi_2, v^4 U_s, U_s \Sigma(U_s a_s), U_s \Phi^s_1, U_s \Phi^s_2, U_s \Phi^s_{2s}.$

ρ^s coefficients

$a_s G_{2s}, a_s G_{3s}, a_s G_{6s}, a_s H^s, a_s H^s, c_s U_s^3, b_s U_s^3, a_s U_s^3, a_s U_s U^2, b_s v^2 U_s^2, a_s v^2 U_s, a_s \overset{(4)}{Y}_s, a_s v^2 \ddot{\chi}_s, a_s \ddot{\chi}^s_2, a_s \ddot{\chi}^s_{2s}, a_s \Sigma_s(\ddot{\chi}), a_s \Sigma_s(\ddot{\chi}_s), a_s \Sigma_s(\Phi_1), a_s \Sigma_s(\Phi^2_2), a_s \Sigma_s(\Phi^s_{2s}), a_s \ddot{\chi}^s_1,$
 $a_s \ddot{\chi}(U_s a_s), a_s \Sigma(a_s \Sigma(U_s a_s)), a_s \Sigma_s(U^2), a_s \Sigma_s(U_s^2), a_s \Sigma_s(U_s v^2), a_s \Sigma_s(U U_s), a_s \Sigma_s(U v^2), a_s \Sigma_s(v^4), a_s \Sigma_s(v.V), a_s \Sigma_s(\Sigma(U_s a_s)), a_s \Sigma_s(\Phi_2), a_s \Sigma_s(\Phi^s_1), a_s \Sigma(U_s^2 a_s), a_s \Sigma(U_s^2 a_s),$
 $a_s \Sigma(U_s^2 b_s), a_s v^2 \Sigma(U a_s), a_s \Sigma(U_s v^2 a_s), a_s \Sigma(U U_s a_s), a_s \Sigma(\ddot{\chi}_s a_s), a_s \Sigma(\Phi^s_1 a_s), a_s \Sigma(\Phi^s_2 a_s), a_s \Sigma(\Phi^s_{2s} a_s), v^2 \Phi^s_1, a_s v^2 \Phi^s_2, a_s v^2 \Phi^s_2, a_s v^2 \Phi^s_{2s}, b_s U U_s^2, a_s U U_s^2, a_s v^2 U U_s,$
 $a_s U \ddot{\chi}_s, a_s U \Sigma(U_s a_s), a_s U \Phi^s_1, a_s U \Phi^s_2, a_s U \Phi^s_{2s}, a_s v.V U_s, a_s U_s \ddot{\chi}, b_s U_s \ddot{\chi}_s, a_s U_s \ddot{\chi}_s, a_s U_s \Phi_1, a_s U_s \Phi_2, b_s U_s \Sigma(U_s a_s), b_s U_s \Phi^s_1, a_s v^4 U_s, a_s U_s \Sigma(U_s a_s), a_s U_s \Phi^s_1, b_s U_s \Phi^s_2,$
 $a_s U_s \Phi^s_2, b_s U_s \Phi^s_{2s}, a_s U_s \Phi^s_{2s}.$

$\Gamma_{s,F1}$

$U \ddot{U}_s^2, \dot{U}_s \overset{(3)}{\ddot{\chi}}_s, \dot{U}_s \dot{\Sigma}(U_s a_s), \dot{U}_s \dot{\Phi}^s_1, \dot{U}_s \dot{\Phi}^s_2, \dot{U}_s \dot{\Phi}^s_{2s}, \dot{U}_s v.\nabla U_s, U^2(\nabla U_s)^2, U_s^2(\nabla U_s)^2, \Phi_2(\nabla U_s)^2, \Sigma(U_s a_s)(\nabla U_s)^2, \Phi^s_1(\nabla U_s)^2, \nabla \Sigma_s(U^2).\nabla U_s, \nabla \Sigma_s(U_s^2).\nabla U_s, \nabla \Sigma_s(U_s v^2).\nabla U_s,$
 $\nabla \Sigma_s(U U_s).\nabla U_s, \nabla \Sigma_s(U v^2).\nabla U_s, \nabla \Sigma_s(\Sigma(U_s a_s)).\nabla U_s, \nabla \Sigma_s(\Phi_1).\nabla U_s, \nabla \Sigma_s(\Phi_2).\nabla U_s, \nabla \Sigma_s(\Phi_{2s}).\nabla U_s, \nabla \Sigma_s(\Phi^s_1).\nabla U_s, \nabla \Sigma_s(\Phi^s_2).\nabla U_s, \nabla \Sigma_s(\Phi^s_{2s}).\nabla U_s, \nabla \Sigma(U_s^2 a_s).\nabla U_s, \nabla \Sigma(U_s U a_s).\nabla U_s,$
 $\nabla \Sigma(a_s \Sigma(U_s a_s)).\nabla U_s, \nabla \Sigma_s(v^4).\nabla U_s, \nabla \Sigma_s(v.V).\nabla U_s, \nabla \Sigma_s(\ddot{\chi}).\nabla U_s, \nabla \Sigma_s(\ddot{\chi}_s).\nabla U_s, \nabla \Sigma(U_s^2 b_s).\nabla U_s, \nabla \Sigma(U_s v^2 a_s).\nabla U_s, \nabla \Sigma(\ddot{\chi}_s a_s).\nabla U_s, \nabla \Sigma(\Phi^s_1 a_s).\nabla U_s, \nabla \Sigma(\Phi^s_2 a_s).\nabla U_s, \nabla \Sigma(\Phi^s_{2s} a_s).\nabla U_s,$
 $\nabla G_{2s}.\nabla U_s, \nabla G_{3s}.\nabla U_s, \nabla G_{6s}.\nabla U_s, \nabla H^s.\nabla U_s, \nabla H^s.\nabla U_s, \ddot{\chi}_s(\nabla U_s)^2, (\nabla \ddot{\chi}_s)^2, \nabla \Sigma(U_s a_s).\nabla \ddot{\chi}_s, \nabla \Phi^s_1.\nabla \ddot{\chi}_s, \nabla \Phi^s_2.\nabla \ddot{\chi}_s, \nabla \Phi^s_{2s}.\nabla \ddot{\chi}_s, \nabla U_s.\nabla \overset{(4)}{Y}_s, \nabla U_s.\nabla \ddot{\chi}^s_2, \nabla U_s.\nabla \ddot{\chi}^s_{2s},$
 $\nabla U_s.\nabla \ddot{\chi}^s_1, \nabla U_s.\nabla \ddot{\chi}(U_s a_s), (\nabla \Sigma(U_s a_s))^2, \nabla \Phi^s_1.\nabla \Sigma(U_s a_s), (\nabla \Phi^s_1)^2, \nabla \Phi^s_2.\nabla \Phi^s_{2s}, \nabla \Phi^s_2.\nabla \Sigma(U_s a_s), \nabla \Phi^s_2.\nabla \Phi^s_1, (\nabla \Phi^s_2)^2, \nabla \Phi^s_{2s}.\nabla \Sigma(U_s a_s), \nabla \Phi^s_{2s}.\nabla \Phi^s_1, (\nabla \Phi^s_{2s})^2, P_2^{ab} \nabla_a U_s \nabla_b U_s,$
 $P_{2s}^{ab} \nabla_a U_s \nabla_b U_s, \Phi_1^{ab} \nabla_a U_s \nabla_b U_s, \Phi^s_2(\nabla U_s)^2, \Phi^s_{2s}(\nabla U_s)^2, U_s \ddot{U}_s^2, U_s \nabla U_s.\nabla \Sigma(U_s a_s), U_s \nabla U_s.\nabla \Phi^s_1, U_s \nabla U_s.\nabla \Phi^s_2, U_s \nabla U_s.\nabla \Phi^s_{2s}, U_s \nabla U_s.\nabla \ddot{\chi}_s$

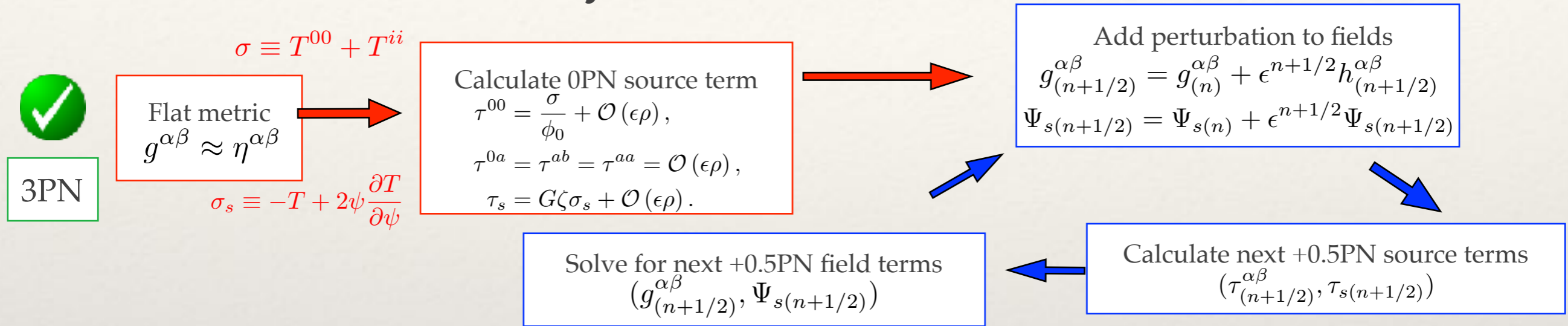
$\Gamma_{s,F2}$


$\rho_s G_1, \rho_s G_4, \rho_s G_5, \rho_s U^3, \rho_s U_s^3, \rho_s V^2, \rho_s \ddot{\chi}_2, \rho_s \ddot{\chi}^s_{2s}, \rho_s \Sigma_s(V^a), \rho_s \Sigma_s(\ddot{\chi}_s), \rho_s \Sigma_s(\Phi^2_2), \rho_s \Sigma_s(\Phi^s_{2s}), \rho_s \Sigma_s(U_s^2), \rho_s \Sigma_s(U_s U), \rho_s \Sigma_s(U_s v^2), \rho_s \Sigma_s(\Sigma(U_s a_s)), \rho_s \Sigma_s(\Phi^s_1), \rho_s \Sigma(U_s U), \rho_s \Sigma(\ddot{\chi}),$
 $\rho_s \Sigma(\Phi_{2s}), \rho_s \Sigma(U^2), \rho_s \Sigma(U_s^2 a_s), \rho_s \Sigma(U v^2), \rho_s \Sigma(\Phi_1), \rho_s \Sigma(\Phi_2), \rho_s \Sigma(\Phi_{2s}), \rho_s \Sigma(\Phi^s_{2s}), \rho_s U U_s^2, \rho_s U \ddot{\chi}, \rho_s U \Phi_2, \rho_s U \Phi^s_{2s}, \rho_s U \Phi_{2s}, \rho_s U \Phi_1, \rho_s U^2 U_s, \rho_s U_s \ddot{\chi}_s, \rho_s U_s \Phi_2, \rho_s U_s \Phi^s_{2s},$
 $\rho_s U_s \Phi^s_{2s}, \ddot{U}_s \ddot{\chi}, \ddot{U}_s \Phi_1, G_7.\nabla \dot{U}_s, P(\dot{U}_s \nabla U_s).\nabla \dot{U}_s, v_2.\nabla \dot{U}_s, v^e_{2s}.\nabla \dot{U}_s, \ddot{\chi}.\nabla \dot{U}_s, \Sigma(v^2 v).\nabla \dot{U}_s, \Phi_2.\nabla \dot{U}_s, v.\nabla \dot{\Sigma}(U_s a_s), v.\nabla \dot{\Phi}^s_1, v.\nabla \dot{\Phi}^s_2, v.\nabla \dot{\Phi}^s_{2s}, v.\nabla \dot{U}_s \dot{U}_s, v.\nabla \overset{(3)}{\ddot{\chi}}_s, P_{3s}^{ab} \nabla_a \nabla_b U_s,$
 $P_{3s}^{ab} \nabla_a \nabla_b U_s, P_{3s}^{(ab)} \nabla_a \nabla_b U_s, P(\nabla^a U \nabla^b) \nabla_a \nabla_b U_s, P(\nabla^a U \nabla^b) \ddot{\chi} \nabla_a \nabla_b U_s, P(\nabla^a U_s \nabla^b) \ddot{\chi}_s \nabla_a \nabla_b U_s, S_2^{ab} \nabla_a \nabla_b U_s, S_{2s}^{ab} \nabla_a \nabla_b U_s, \ddot{\chi}^{ab} \nabla_a \nabla_b U_s, \Sigma^{ab}(U) \nabla_a \nabla_b U_s, \Sigma^{ab}(U_s) \nabla_a \nabla_b U_s,$
 $P_3^{(ab)} \nabla_a \nabla_b U_s, P_{3s}^{(ab)} \nabla_a \nabla_b U_s, P_{3s}^{(ab)} \nabla_a \nabla_b U_s, P_{3s}^{(ab)} \nabla_a \nabla_b U_s, P_{3s}^{(ab)} \nabla_a \nabla_b U_s, P(\nabla^a U_s \nabla^b) \Sigma(U_s a_s) \nabla_a \nabla_b U_s, P(\nabla^a U_s \nabla^b) \Phi^s_1 \nabla_a \nabla_b U_s, P(\nabla^a U \nabla^b) \Phi_1 \nabla_a \nabla_b U_s,$
 $\Sigma_s(U_s v^a v^b) \nabla_a \nabla_b U_s, \Sigma(U_s v^a v^b) \nabla_a \nabla_b U_s, \Sigma(U v^a v^b) \nabla_a \nabla_b U_s, \Sigma(v^2 v^a v^b) \nabla_a \nabla_b U_s, \Phi_1^{ab} \nabla_a \nabla_b \Sigma(U_s a_s), \Phi_1^{ab} \nabla_a \nabla_b \Phi^s_1, \Phi_1^{ab} \nabla_a \nabla_b \Phi^s_2, \Phi_1^{ab} \nabla_a \nabla_b \Phi^s_{2s}, P_2^{ab} \nabla_a \nabla_b \Sigma(U_s a_s),$
 $P_2^{ab} \nabla_a \nabla_b \Phi^s_1, P_2^{ab} \nabla_a \nabla_b \Phi^s_2, P_2^{ab} \nabla_a \nabla_b \Phi^s_{2s}, P_{2s}^{ab} \nabla_a \nabla_b \Sigma(U_s a_s), P_{2s}^{ab} \nabla_a \nabla_b \Phi^s_1, P_{2s}^{ab} \nabla_a \nabla_b \Phi^s_2, P_{2s}^{ab} \nabla_a \nabla_b \Phi^s_{2s}, \Phi_1^{ab} \nabla_a \nabla_b \ddot{\chi}_s, P_2^{ab} \nabla_a \nabla_b \ddot{\chi}_s, P_{2s}^{ab} \nabla_a \nabla_b \ddot{\chi}_s, U \ddot{\chi}_s, U U_s \ddot{U}_s,$
 $U \ddot{\Sigma}(U_s a_s), U \dot{\Phi}^s_1, U \dot{\Phi}^s_2, U \dot{\Phi}^s_{2s}, U \dot{U}_s^2, U \nabla \dot{U}_s v, U^2 \dot{U}_s, U^2(\nabla U_s)^2, \Phi_2 \dot{U}_s, \Phi_2(\nabla U_s)^2, \Phi^s_{2s} \dot{U}_s, \Phi^s_{2s}(\nabla U_s)^2, U_s^2(\nabla U_s)^2, P_2^{ab} \nabla_a U_s \nabla_b U_s, P_{2s}^{ab} \nabla_a U_s \nabla_b U_s, \Phi_1^{ab} \nabla_a U_s \nabla_b U_s,$
 $P(v^{[c} v^{a]} v^{b]}) \nabla_a \nabla_b U_s, U_s v.\nabla \dot{U}_s, P_2^{ab} \nabla_a \nabla_b U_s, P_{2s}^{ab} \nabla_a \nabla_b U_s, \Phi_1^{ab} \nabla_a \nabla_b U_s$

$\rho U^2 U_s a_s, \rho_s U^2 v^2, \rho_s U^3, \rho_s U^2 U_s, \rho \Phi_2 U_s a_s, \rho_s \Phi_2 v^2, \rho_s \Phi_2 U, \rho_s \Phi_2 U_s, \rho \Phi^s_{2s} U_s a_s, \rho_s \Phi^s_{2s} v^2, \rho_s \Phi^s_{2s} U, \rho_s \Phi^s_{2s} U_s, \rho U_s^3 a_s, \rho_s U_s^2 v^2, \rho_s U_s^2 U, \rho_s U_s^3, \rho_s U_s \Sigma(U_s a_s),$
 $\rho_s U_s \Phi^s_1$

Direct Integration of Relaxed Einstein Equations

- ❖ Solve simultaneously for source terms $(\tau^{\alpha\beta}, \tau_s)$ and fields $(g^{\alpha\beta}, \Psi)$



 Use source terms to calculate multipole moments

$$\mathcal{I}_{(s)}^Q \equiv \int \tau_{(s)} x^Q d^3x$$

- ❖ Terms like $\ddot{\mathcal{I}}$ require 3PN equations of motion

$G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9, G_{10}, G_{11}, G_{12}, G_{13}, G_{14}, G_{15}, G_{16}, G_{17}, G_{18}, G_{19}, G_{20}, G_{21}, G_{22}, G_{23}, G_{24}, G_{25}, G_{26}, G_{27}, G_{28}, G_{29}, G_{30}, G_{31}, G_{32}, G_{33}, G_{34}, G_{35}, G_{36}, G_{37}, G_{38}, G_{39}, G_{40}, G_{41}, G_{42}, G_{43}, G_{44}, G_{45}, G_{46}, G_{47}, G_{48}, G_{49}, G_{50}, G_{51}, G_{52}, G_{53}, G_{54}, G_{55}, G_{56}, G_{57}, G_{58}, G_{59}, G_{60}, G_{61}, G_{62}, G_{63}, G_{64}, G_{65}, G_{66}, G_{67}, G_{68}, G_{69}, G_{70}, G_{71}, G_{72}, G_{73}, G_{74}, G_{75}, G_{76}, G_{77}, G_{78}, G_{79}, G_{80}, G_{81}, G_{82}, G_{83}, G_{84}, G_{85}, G_{86}, G_{87}, G_{88}, G_{89}, G_{90}, G_{91}, G_{92}, G_{93}, G_{94}, G_{95}, G_{96}, G_{97}, G_{98}, G_{99}, G_{100}, G_{101}, G_{102}, G_{103}, G_{104}, G_{105}, G_{106}, G_{107}, G_{108}, G_{109}, G_{110}, G_{111}, G_{112}, G_{113}, G_{114}, G_{115}, G_{116}, G_{117}, G_{118}, G_{119}, G_{120}, G_{121}, G_{122}, G_{123}, G_{124}, G_{125}, G_{126}, G_{127}, G_{128}, G_{129}, G_{130}, G_{131}, G_{132}, G_{133}, G_{134}, G_{135}, G_{136}, G_{137}, G_{138}, G_{139}, G_{140}, G_{141}, G_{142}, G_{143}, G_{144}, G_{145}, G_{146}, G_{147}, G_{148}, G_{149}, G_{150}, G_{151}, G_{152}, G_{153}, G_{154}, G_{155}, G_{156}, G_{157}, G_{158}, G_{159}, G_{160}, G_{161}, G_{162}, G_{163}, G_{164}, G_{165}, G_{166}, G_{167}, G_{168}, G_{169}, G_{170}, G_{171}, G_{172}, G_{173}, G_{174}, G_{175}, G_{176}, G_{177}, G_{178}, G_{179}, G_{180}, G_{181}, G_{182}, G_{183}, G_{184}, G_{185}, G_{186}, G_{187}, G_{188}, G_{189}, G_{190}, G_{191}, G_{192}, G_{193}, G_{194}, G_{195}, G_{196}, G_{197}, G_{198}, G_{199}, G_{200}, G_{201}, G_{202}, G_{203}, G_{204}, G_{205}, G_{206}, G_{207}, G_{208}, G_{209}, G_{210}, G_{211}, G_{212}, G_{213}, G_{214}, G_{215}, G_{216}, G_{217}, G_{218}, G_{219}, G_{220}, G_{221}, G_{222}, G_{223}, G_{224}, G_{225}, G_{226}, G_{227}, G_{228}, G_{229}, G_{230}, G_{231}, G_{232}, G_{233}, G_{234}, G_{235}, G_{236}, G_{237}, G_{238}, G_{239}, G_{240}, G_{241}, G_{242}, G_{243}, G_{244}, G_{245}, G_{246}, G_{247}, G_{248}, G_{249}, G_{250}, G_{251}, G_{252}, G_{253}, G_{254}, G_{255}, G_{256}, G_{257}, G_{258}, G_{259}, G_{260}, G_{261}, G_{262}, G_{263}, G_{264}, G_{265}, G_{266}, G_{267}, G_{268}, G_{269}, G_{270}, G_{271}, G_{272}, G_{273}, G_{274}, G_{275}, G_{276}, G_{277}, G_{278}, G_{279}, G_{280}, G_{281}, G_{282}, G_{283}, G_{284}, G_{285}, G_{286}, G_{287}, G_{288}, G_{289}, G_{290}, G_{291}, G_{292}, G_{293}, G_{294}, G_{295}, G_{296}, G_{297}, G_{298}, G_{299}, G_{300}, G_{301}, G_{302}, G_{303}, G_{304}, G_{305}, G_{306}, G_{307}, G_{308}, G_{309}, G_{310}, G_{311}, G_{312}, G_{313}, G_{314}, G_{315}, G_{316}, G_{317}, G_{318}, G_{319}, G_{320}, G_{321}, G_{322}, G_{323}, G_{324}, G_{325}, G_{326}, G_{327}, G_{328}, G_{329}, G_{330}, G_{331}, G_{332}, G_{333}, G_{334}, G_{335}, G_{336}, G_{337}, G_{338}, G_{339}, G_{340}, G_{341}, G_{342}, G_{343}, G_{344}, G_{345}, G_{346}, G_{347}, G_{348}, G_{349}, G_{350}, G_{351}, G_{352}, G_{353}, G_{354}, G_{355}, G_{356}, G_{357}, G_{358}, G_{359}, G_{360}, G_{361}, G_{362}, G_{363}, G_{364}, G_{365}, G_{366}, G_{367}, G_{368}, G_{369}, G_{370}, G_{371}, G_{372}, G_{373}, G_{374}, G_{375}, G_{376}, G_{377}, G_{378}, G_{379}, G_{380}, G_{381}, G_{382}, G_{383}, G_{384}, G_{385}, G_{386}, G_{387}, G_{388}, G_{389}, G_{390}, G_{391}, G_{392}, G_{393}, G_{394}, G_{395}, G_{396}, G_{397}, G_{398}, G_{399}, G_{400}, G_{401}, G_{402}, G_{403}, G_{404}, G_{405}, G_{406}, G_{407}, G_{408}, G_{409}, G_{410}, G_{411}, G_{412}, G_{413}, G_{414}, G_{415}, G_{416}, G_{417}, G_{418}, G_{419}, G_{420}, G_{421}, G_{422}, G_{423}, G_{424}, G_{425}, G_{426}, G_{427}, G_{428}, G_{429}, G_{430}, G_{431}, G_{432}, G_{433}, G_{434}, G_{435}, G_{436}, G_{437}, G_{438}, G_{439}, G_{440}, G_{441}, G_{442}, G_{443}, G_{444}, G_{445}, G_{446}, G_{447}, G_{448}, G_{449}, G_{450}, G_{451}, G_{452}, G_{453}, G_{454}, G_{455}, G_{456}, G_{457}, G_{458}, G_{459}, G_{460}, G_{461}, G_{462}, G_{463}, G_{464}, G_{465}, G_{466}, G_{467}, G_{468}, G_{469}, G_{470}, G_{471}, G_{472}, G_{473}, G_{474}, G_{475}, G_{476}, G_{477}, G_{478}, G_{479}, G_{480}, G_{481}, G_{482}, G_{483}, G_{484}, G_{485}, G_{486}, G_{487}, G_{488}, G_{489}, G_{490}, G_{491}, G_{492}, G_{493}, G_{494}, G_{495}, G_{496}, G_{497}, G_{498}, G_{499}, G_{500}, G_{501}, G_{502}, G_{503}, G_{504}, G_{505}, G_{506}, G_{507}, G_{508}, G_{509}, G_{510}, G_{511}, G_{512}, G_{513}, G_{514}, G_{515}, G_{516}, G_{517}, G_{518}, G_{519}, G_{520}, G_{521}, G_{522}, G_{523}, G_{524}, G_{525}, G_{526}, G_{527}, G_{528}, G_{529}, G_{530}, G_{531}, G_{532}, G_{533}, G_{534}, G_{535}, G_{536}, G_{537}, G_{538}, G_{539}, G_{540}, G_{541}, G_{542}, G_{543}, G_{544}, G_{545}, G_{546}, G_{547}, G_{548}, G_{549}, G_{550}, G_{551}, G_{552}, G_{553}, G_{554}, G_{555}, G_{556}, G_{557}, G_{558}, G_{559}, G_{560}, G_{561}, G_{562}, G_{563}, G_{564}, G_{565}, G_{566}, G_{567}, G_{568}, G_{569}, G_{570}, G_{571}, G_{572}, G_{573}, G_{574}, G_{575}, G_{576}, G_{577}, G_{578}, G_{579}, G_{580}, G_{581}, G_{582}, G_{583}, G_{584}, G_{585}, G_{586}, G_{587}, G_{588}, G_{589}, G_{590}, G_{591}, G_{592}, G_{593}, G_{594}, G_{595}, G_{596}, G_{597}, G_{598}, G_{599}, G_{600}, G_{601}, G_{602}, G_{603}, G_{604}, G_{605}, G_{606}, G_{607}, G_{608}, G_{609}, G_{610}, G_{611}, G_{612}, G_{613}, G_{614}, G_{615}, G_{616}, G_{617}, G_{618}, G_{619}, G_{620}, G_{621}, G_{622}, G_{623}, G_{624}, G_{625}, G_{626}, G_{627}, G_{628}, G_{629}, G_{630}, G_{631}, G_{632}, G_{633}, G_{634}, G_{635}, G_{636}, G_{637}, G_{638}, G_{639}, G_{640}, G_{641}, G_{642}, G_{643}, G_{644}, G_{645}, G_{646}, G_{647}, G_{648}, G_{649}, G_{650}, G_{651}, G_{652}, G_{653}, G_{654}, G_{655}, G_{656}, G_{657}, G_{658}, G_{659}, G_{660}, G_{661}, G_{662}, G_{663}, G_{664}, G_{665}, G_{666}, G_{667}, G_{668}, G_{669}, G_{670}, G_{671}, G_{672}, G_{673}, G_{674}, G_{675}, G_{676}, G_{677}, G_{678}, G_{679}, G_{680}, G_{681}, G_{682}, G_{683}, G_{684}, G_{685}, G_{686}, G_{687}, G_{688}, G_{689}, G_{690}, G_{691}, G_{692}, G_{693}, G_{694}, G_{695}, G_{696}, G_{697}, G_{698}, G_{699}, G_{700}, G_{701}, G_{702}, G_{703}, G_{704}, G_{705}, G_{706}, G_{707}, G_{708}, G_{709}, G_{710}, G_{711}, G_{712}, G_{713}, G_{714}, G_{715}, G_{716}, G_{717}, G_{718}, G_{719}, G_{720}, G_{721}, G_{722}, G_{723}, G_{724}, G_{725}, G_{726}, G_{727}, G_{728}, G_{729}, G_{730}, G_{731}, G_{732}, G_{733}, G_{734}, G_{735}, G_{736}, G_{737}, G_{738}, G_{739}, G_{740}, G_{741}, G_{742}, G_{743}, G_{744}, G_{745}, G_{746}, G_{747}, G_{748}, G_{749}, G_{750}, G_{751}, G_{752}, G_{753}, G_{754}, G_{755}, G_{756}, G_{757}, G_{758}, G_{759}, G_{760}, G_{761}, G_{762}, G_{763}, G_{764}, G_{765}, G_{766}, G_{767}, G_{768}, G_{769}, G_{770}, G_{771}, G_{772}, G_{773}, G_{774}, G_{775}, G_{776}, G_{777}, G_{778}, G_{779}, G_{780}, G_{781}, G_{782}, G_{783}, G_{784}, G_{785}, G_{786}, G_{787}, G_{788}, G_{789}, G_{790}, G_{791}, G_{792}, G_{793}, G_{794}, G_{795}, G_{796}, G_{797}, G_{798}, G_{799}, G_{800}, G_{801}, G_{802}, G_{803}, G_{804}, G_{805}, G_{806}, G_{807}, G_{808}, G_{809}, G_{810}, G_{811}, G_{812}, G_{813}, G_{814}, G_{815}, G_{816}, G_{817}, G_{818}, G_{819}, G_{820}, G_{821}, G_{822}, G_{823}, G_{824}, G_{825}, G_{826}, G_{827}, G_{828}, G_{829}, G_{830}, G_{831}, G_{832}, G_{833}, G_{834}, G_{835}, G_{836}, G_{837}, G_{838}, G_{839}, G_{840}, G_{841}, G_{842}, G_{843}, G_{844}, G_{845}, G_{846}, G_{847}, G_{848}, G_{849}, G_{850}, G_{851}, G_{852}, G_{853}, G_{854}, G_{855}, G_{856}, G_{857}, G_{858}, G_{859}, G_{860}, G_{861}, G_{862}, G_{863}, G_{864}, G_{865}, G_{866}, G_{867}, G_{868}, G_{869}, G_{870}, G_{871}, G_{872}, G_{873}, G_{874}, G_{875}, G_{876}, G_{877}, G_{878}, G_{879}, G_{880}, G_{881}, G_{882}, G_{883}, G_{884}, G_{885}, G_{886}, G_{887}, G_{888}, G_{889}, G_{890}, G_{891}, G_{892}, G_{893}, G_{894}, G_{895}, G_{896}, G_{897}, G_{898}, G_{899}, G_{900}, G_{901}, G_{902}, G_{903}, G_{904}, G_{905}, G_{906}, G_{907}, G_{908}, G_{909}, G_{910}, G_{911}, G_{912}, G_{913}, G_{914}, G_{915}, G_{916}, G_{917}, G_{918}, G_{919}, G_{920}, G_{921}, G_{922}, G_{923}, G_{924}, G_{925}, G_{926}, G_{927}, G_{928}, G_{929}, G_{930}, G_{931}, G_{932}, G_{933}, G_{934}, G_{935}, G_{936}, G_{937}, G_{938}, G_{939}, G_{940}, G_{941}, G_{942}, G_{943}, G_{944}, G_{945}, G_{946}, G_{947}, G_{948}, G_{949}, G_{950}, G_{951}, G_{952}, G_{953}, G_{954}, G_{955}, G_{956}, G_{957}, G_{958}, G_{959}, G_{960}, G_{961}, G_{962}, G_{963}, G_{964}, G_{965}, G_{966}, G_{967}, G_{968}, G_{969}, G_{970}, G_{971}, G_{972}, G_{973}, G_{974}, G_{975}, G_{976}, G_{977}, G_{978}, G_{979}, G_{980}, G_{981}, G_{982}, G_{983}, G_{984}, G_{985}, G_{986}, G_{987}, G_{988}, G_{989}, G_{990}, G_{991}, G_{992}, G_{993}, G_{994}, G_{995}, G_{996}, G_{997}, G_{998}, G_{999}, G_{1000}$

 Tricky in Will, Wiseman & Pati  Solution: Laura Bernard!

And now to Laura