

Stellar core collapse in scalar-tensor theory with massive fields

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in collaboration with

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Introduction

- GR has passed all tests successfully (even in the strong regime), so why bother?
- Many physical concepts left unexplained by GR:
 - Dark energy
 - Dark matter
 - Quantum Field Theory etc.
- Scalar-tensor theories show interesting behaviour: e.g. spontaneous scalarization, dynamical scalarization.
- They show deviations from GR which would be flag in their detection or a way to constrain them.
- It's one of the few modified gravity theories where we know it's well posed and that we can work with.

Spontaneous scalarization

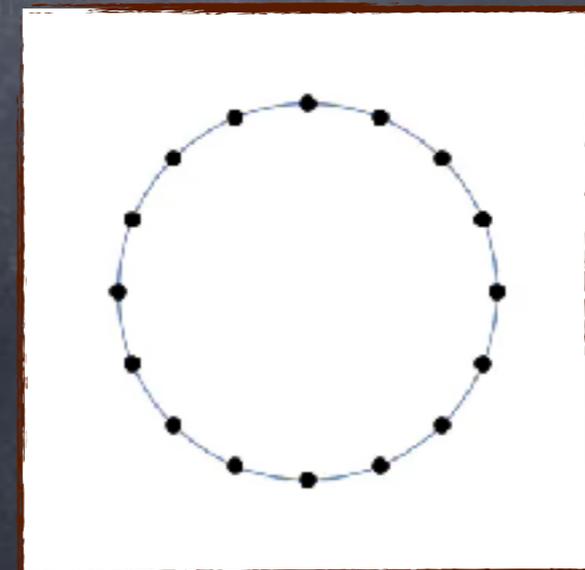
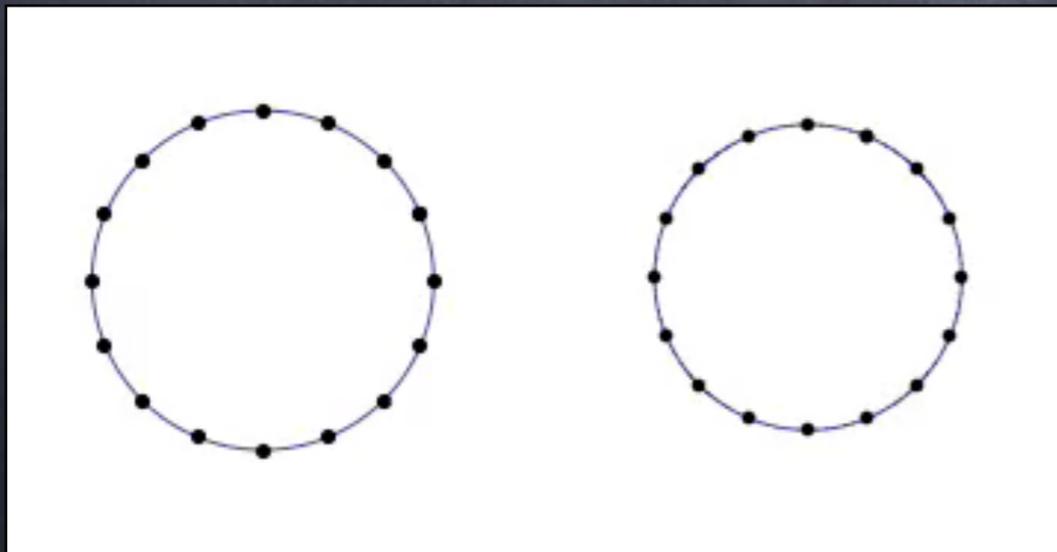
For certain parameter space of the coupling function between the scalar fields and the matter fields, the GR solution ("weak field" solution) becomes unstable and the "strong field" solutions in which the scalar field has a non-trivial value become the stable solutions.

see [Damour and Esposito-Farèse PRL 1993](#)

Monopolar scalar mode ("breathing mode")

The gravitational wave strain is:
$$h_o(t) = \frac{2}{D} \alpha_0 r (\varphi - \varphi_0)$$

see [Damour and Esposito-Farèse CQG 1992](#)



Formalism

Action for the scalar tensor theory with a potential in the Jordan-Fierz Frame:

$$S = \int dx^4 \sqrt{-g} \left[\frac{F(\phi)}{16\pi G} R - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right] + S_m(\psi_m, g_{\mu\nu})$$

Tensor equations the Jordan frame are:

$$G_{\alpha\beta} = \frac{8\pi}{F} \left(T_{\alpha\beta}^F + T_{\alpha\beta}^\phi + T_{\alpha\beta} \right) \quad T_{\alpha\beta}^\phi = \partial_\alpha \phi \partial_\beta \phi - g_{\alpha\beta} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) \right]$$
$$\nabla^\mu \nabla_\mu \phi = -\frac{1}{16\pi} F_{,\phi} R + V_{,\phi} \quad T_{\alpha\beta}^F = \frac{1}{8\pi} (\nabla_\alpha \nabla_\beta F - g_{\alpha\beta} \nabla^\mu \nabla_\mu F)$$

Action for the scalar tensor theory with a potential in the Einstein Frame:

$$S = \frac{1}{16\pi G} \int dx^4 \sqrt{-\bar{g}} \left[\bar{R} - 2\bar{g}^{\mu\nu} (\partial_\mu \varphi)(\partial_\nu \varphi) - 4W(\varphi) \right] + S_m[\psi_m, \frac{\bar{g}_{\mu\nu}}{F(\varphi)}]$$

where

$$g_{\alpha\beta} = \frac{1}{F} \bar{g}_{\alpha\beta} \quad \frac{\partial \varphi}{\partial \phi} = \sqrt{\frac{3}{4} \frac{F_{,\phi}^2}{F^2} + \frac{4\pi}{F}} \quad V(\phi) = \frac{F^2}{4\pi} W(\varphi)$$

Energy-momentum tensor

$$\bar{T}^{\alpha\beta} \equiv \frac{2}{\sqrt{-\bar{g}}} \frac{\delta S_m}{\delta \bar{g}_{\alpha\beta}} = \frac{1}{F(\varphi)^3} \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g_{\alpha\beta}} \equiv \frac{1}{F(\varphi)^3} T^{\alpha\beta}$$

Tensor equations the Einstein frame are:

$$\bar{G}_{\alpha\beta} = 2\partial_\alpha\varphi\partial_\beta\varphi - \bar{g}_{\alpha\beta}\bar{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + 8\pi G\bar{T}_{\alpha\beta} \quad \alpha(\varphi) = -\frac{1}{2} \frac{\partial \ln F}{\partial \varphi}$$

$$\bar{\square}\varphi = -4\pi\alpha(\varphi)\bar{T}$$

Spherical symmetry:

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = -\nu^2 dt^2 + X^2 dr^2 + \frac{r^2}{F} d\Omega^2$$

Perfect fluid:

$$T_{\alpha\beta} = \rho h u_\alpha u_\beta + P g_{\alpha\beta}$$

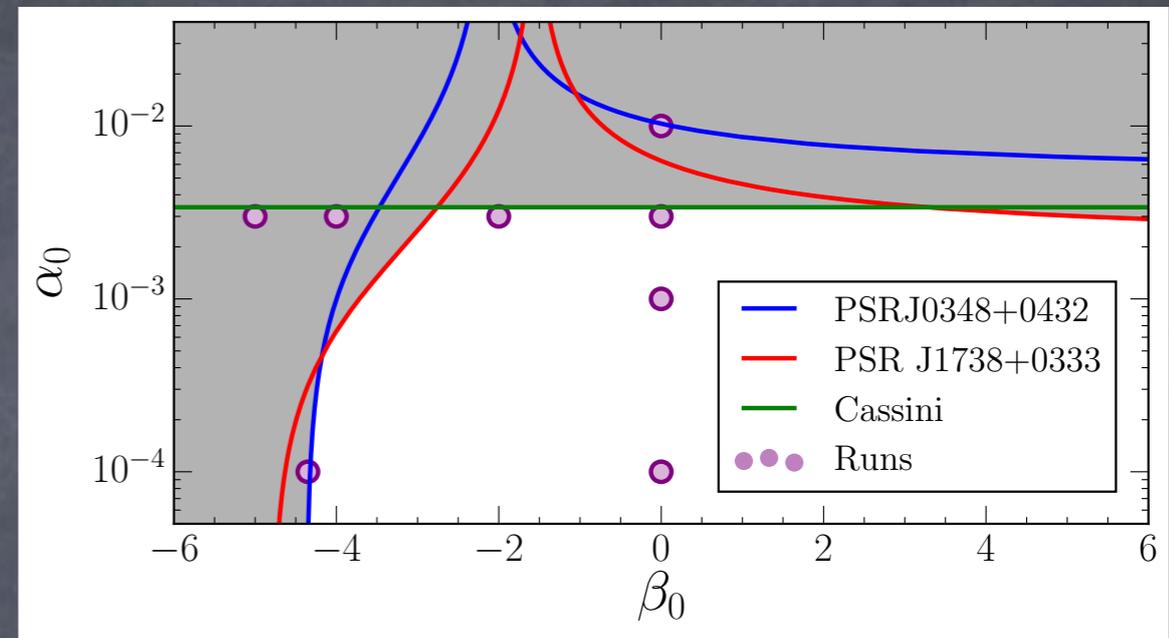
where

$$u^\alpha = \frac{1}{\sqrt{1-v^2}} \left[\frac{1}{\nu}, \frac{v}{X}, 0, 0 \right]$$

Full equations: Eqs. (2.21 – 23), (2.26 – 28) and (2.33) from [D. Gerosa et al CQG 2016 \(arXiv:1602.06952v2\)](#) with some extra terms linear in W.

Constraints

In [Damour and Esposito-Farèse CQG 1992](#) and [Damour and Esposito-Farèse PRD 1996](#), it has been shown that all weak-field deviations from GR can be expressed in terms of the asymptotic value of $\alpha(\varphi)$ at spatial infinity and of its scalar field derivatives.



[Freire, Wex, Esposito-Farèse et al MNRAS 2012](#)

$$\alpha_0 = \alpha(\varphi_0), \beta_0 = \partial\alpha(\varphi_0)/\partial\varphi$$

φ_0 - value of φ at spatial infinity

Massless case:

Cassini space mission ([Bertotti B et al Nature 2003](#))

$$\alpha_0 < 3.4 \times 10^{-3}$$

Observations from binary pulsars ([Damour and Esposito-Farèse PRL 1996](#) and [Freire, Wex, Esposito-Farèse et al MNRAS 2012](#))

$$\beta_0 > -4.5$$

Massive case: ([F.M. Ramazanoglu and F. Pretorius PRD 2016](#))

$$\mu \in [10^{-15}, 10^{-9}] \text{ eV}$$

$$W = \frac{\mu^2 \varphi^2}{2}$$

$$3 \lesssim -\beta_0 \lesssim 10^3$$

Core-collapse scenario

- Massive stars $M_{\text{ZAMS}} = 8 \dots 100 M_{\odot}$
- Core compressed from $\sim 1500 \text{ km}$ to $\sim 15 \text{ km}$
 $\sim 10^{10} \text{ g/cm}^3$ to $\gtrsim 10^{15} \text{ g/cm}^3$
- Released gravitational energy: $O(10^{53}) \text{ erg}$
 $\sim 99\%$ in neutrinos, $\sim 10^{51} \text{ erg}$ in outgoing shock, explosion
- Explosion mechanism: still uncertainties...
- Failed explosions lead to BH formation

All of this is handled for us by [Woosley and Heger Phys. Rept 2007](#)

→ Initial precollapse profile

Equation of state

Pressure: "cold" + "thermal" contribution:

$$P = P_c + P_{th}$$

Hybrid EOS for cold part:

$$P_c = \begin{cases} K_1 \rho^{\Gamma_1}, & \text{if } \rho \leq \rho_{\text{nuc}} \\ K_2 \rho^{\Gamma_2}, & \text{if } \rho > \rho_{\text{nuc}} \end{cases}$$

Internal energy from 1st law:

$$\varepsilon_c = \begin{cases} \frac{K_1}{\Gamma_1 - 1} \rho^{\Gamma_1 - 1}, & \text{if } \rho \leq \rho_{\text{nuc}} \\ \frac{K_2}{\Gamma_2 - 1} \rho^{\Gamma_2 - 1} + \text{constant}, & \text{if } \rho > \rho_{\text{nuc}} \end{cases}$$

Thermal pressure:

$$P_{th} = (\Gamma_{th} - 1) \rho \varepsilon_{th} = \rho (\Gamma_{th} - 1) (\varepsilon - \varepsilon_c)$$

$$K_1 = 4.9345 \times 10^{14} [\text{cgs}], \rho_{\text{nuc}} = 2 \times 10^{14} \text{g cm}^{-3}$$

$$\Gamma_1 = \{1.28, 1.30, 1.32\}, \Gamma_2 = \{2.5, 3.0\}, \Gamma_{th} = \{1.35, 1.50\}$$

Coupling function

$$F = e^{-2\alpha_0 \varphi - \beta_0 \varphi^2}$$

Code and convergence test

Code used: built on top of GR1D developed by [E. O'Connor and C. D. Ott](#)
([arXiv:0912.2393](#))

Convergence between first and second order

For $\mu = 10^{-14}$ eV,

$$\alpha_0 = 10^{-4}, \beta_0 = -20$$

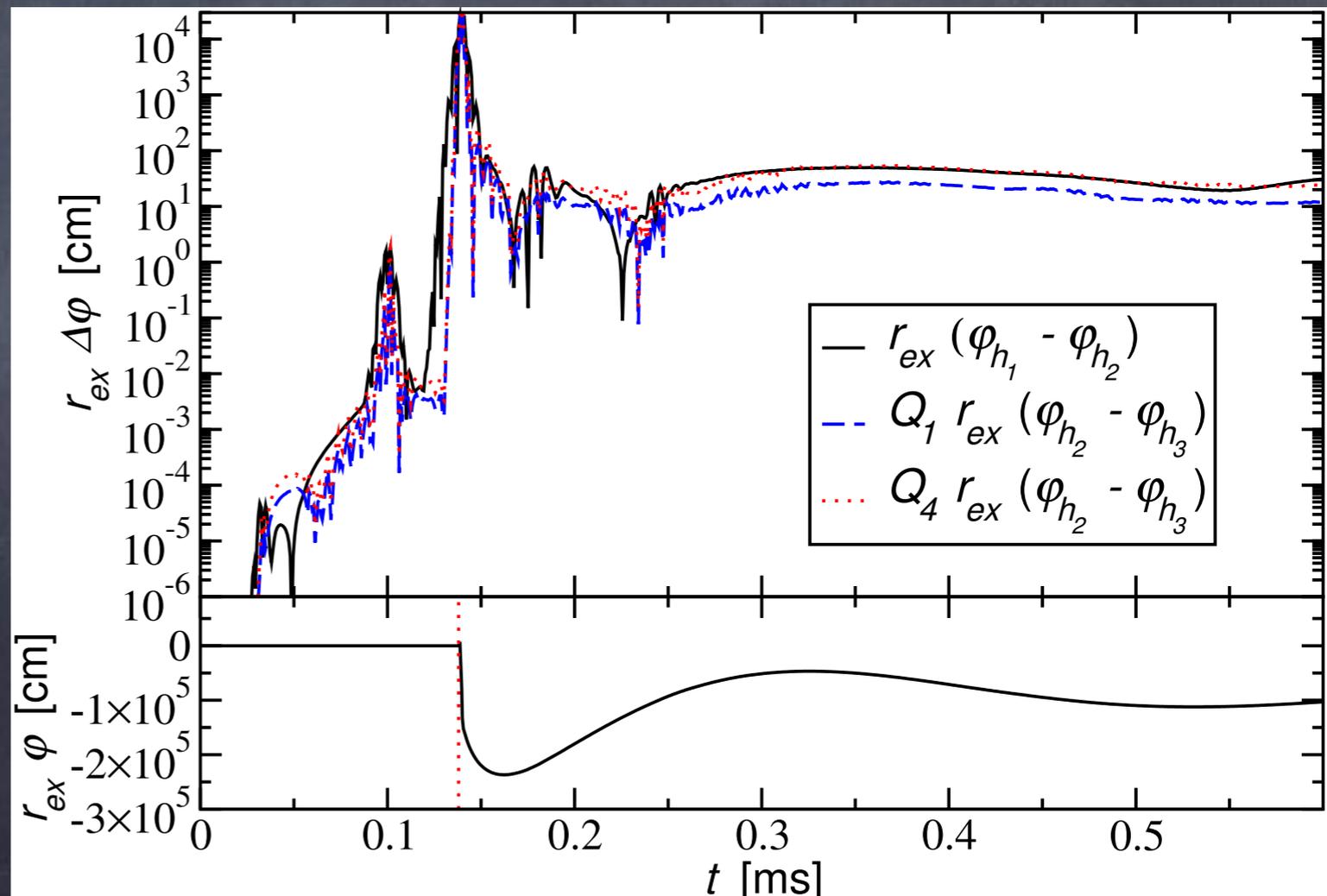
$$\Gamma_1 = 1.3, \Gamma_2 = 2.5, \Gamma_{th} = 1.35$$

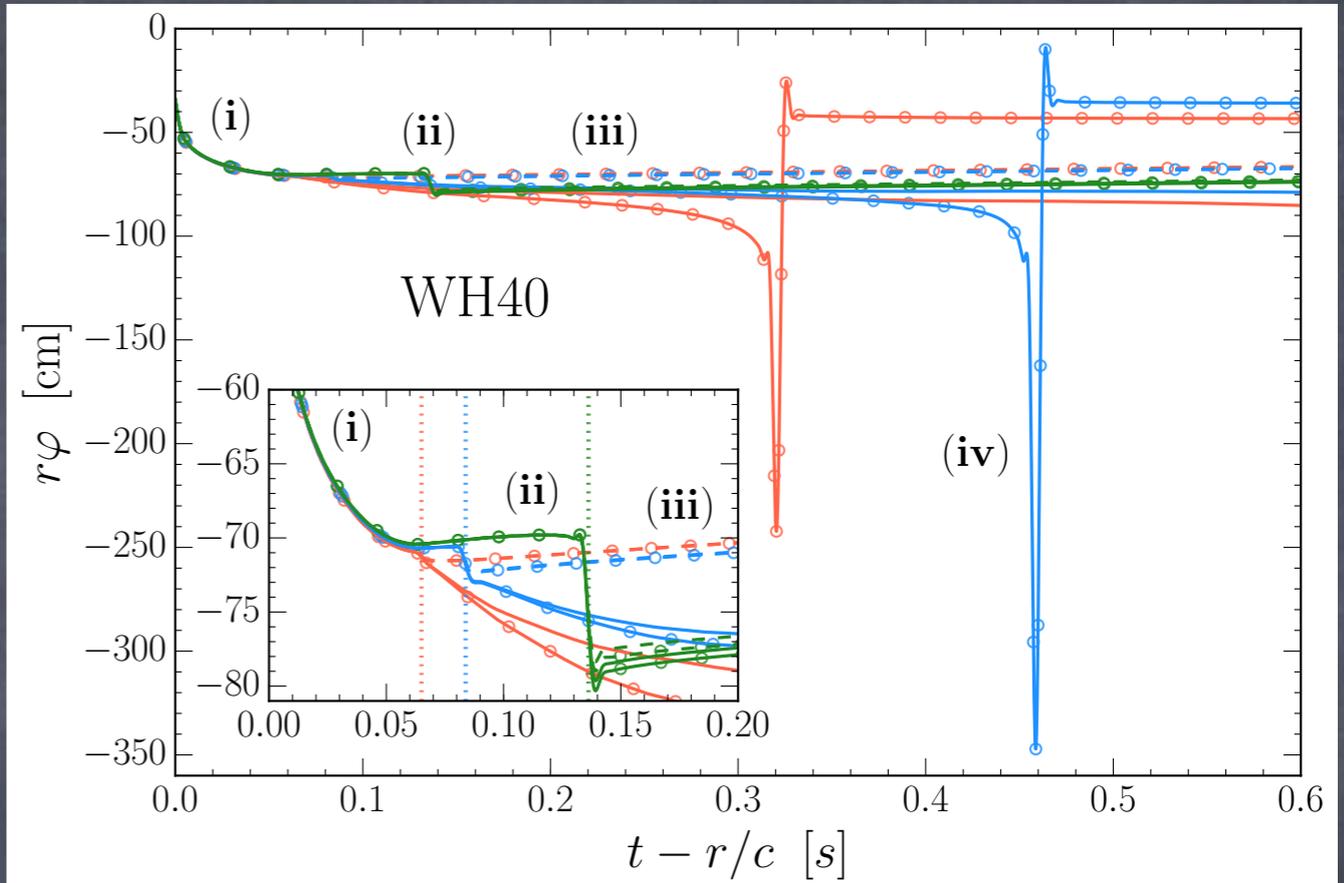
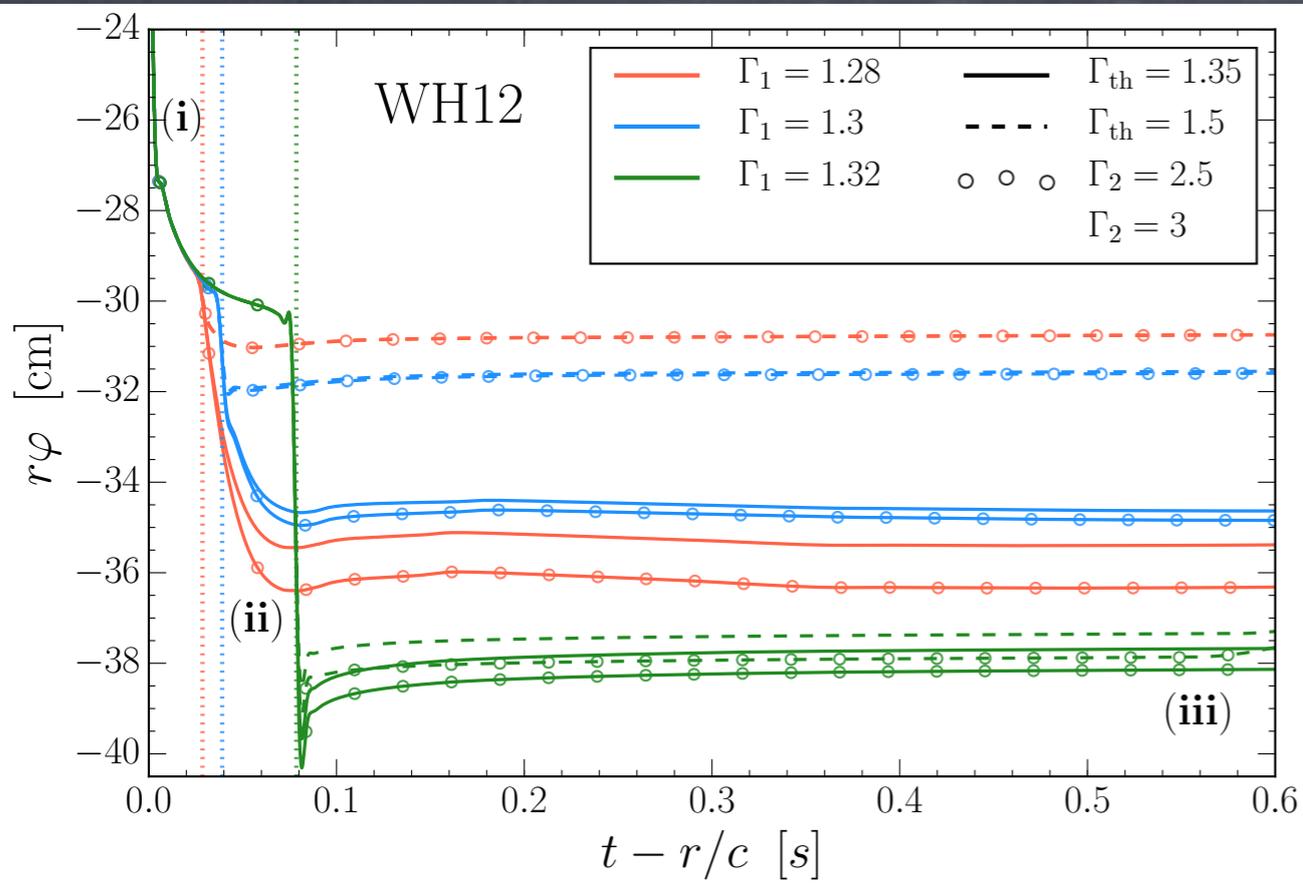
Runs performed using

$$N_1 = 5000, N_2 = 10000,$$

$$N_3 = 20000 \text{ points}$$

Discretization error $\sim 5\%$





D. Gerosa et al CQG 2016 (arXiv: 1602.06952v2)

Signals from 12 and 40 solar mass

stars of various equations of state and scalar parameters

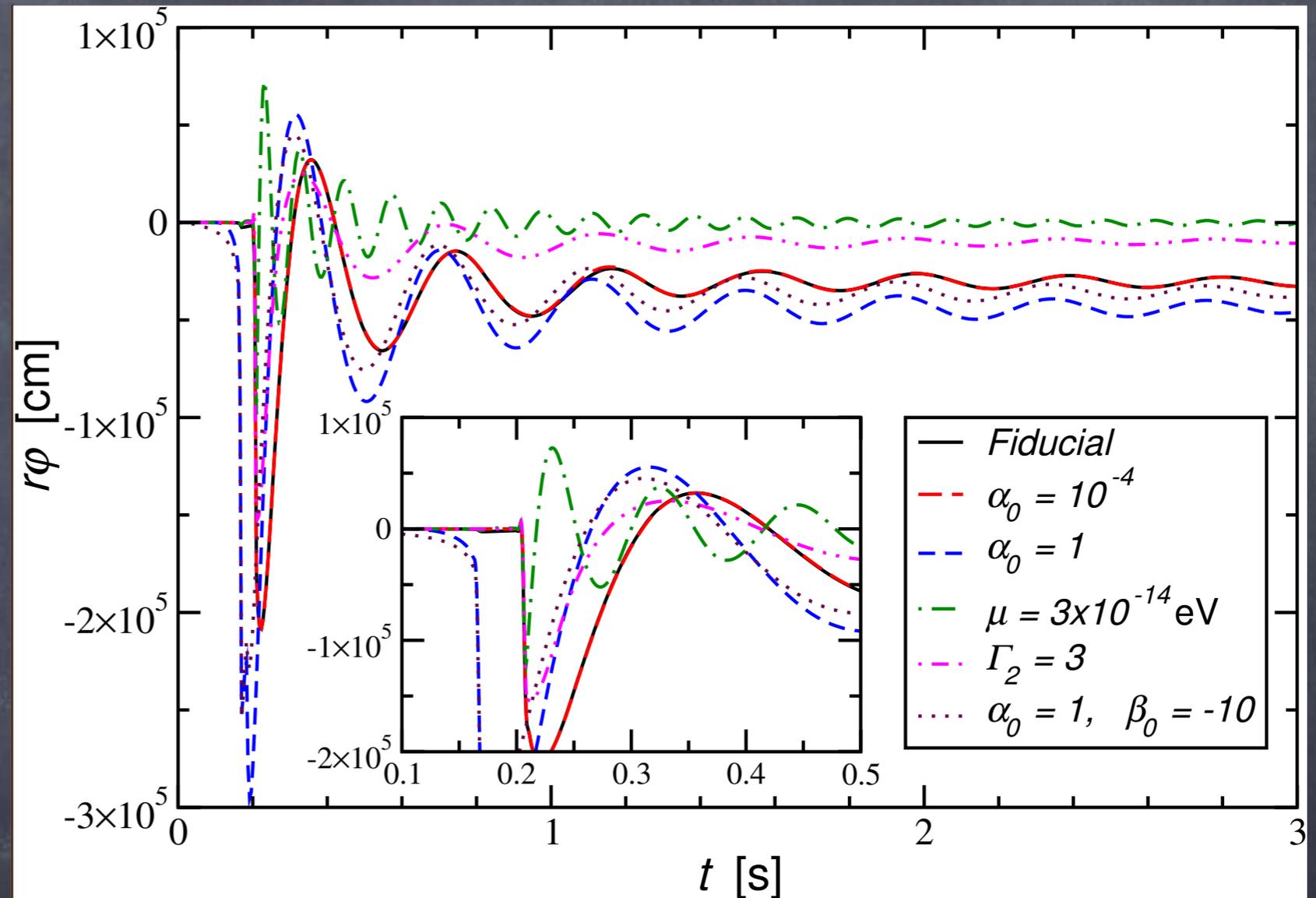
$$\alpha_0 = 10^{-4}, \beta_0 = -4.35.$$

Results

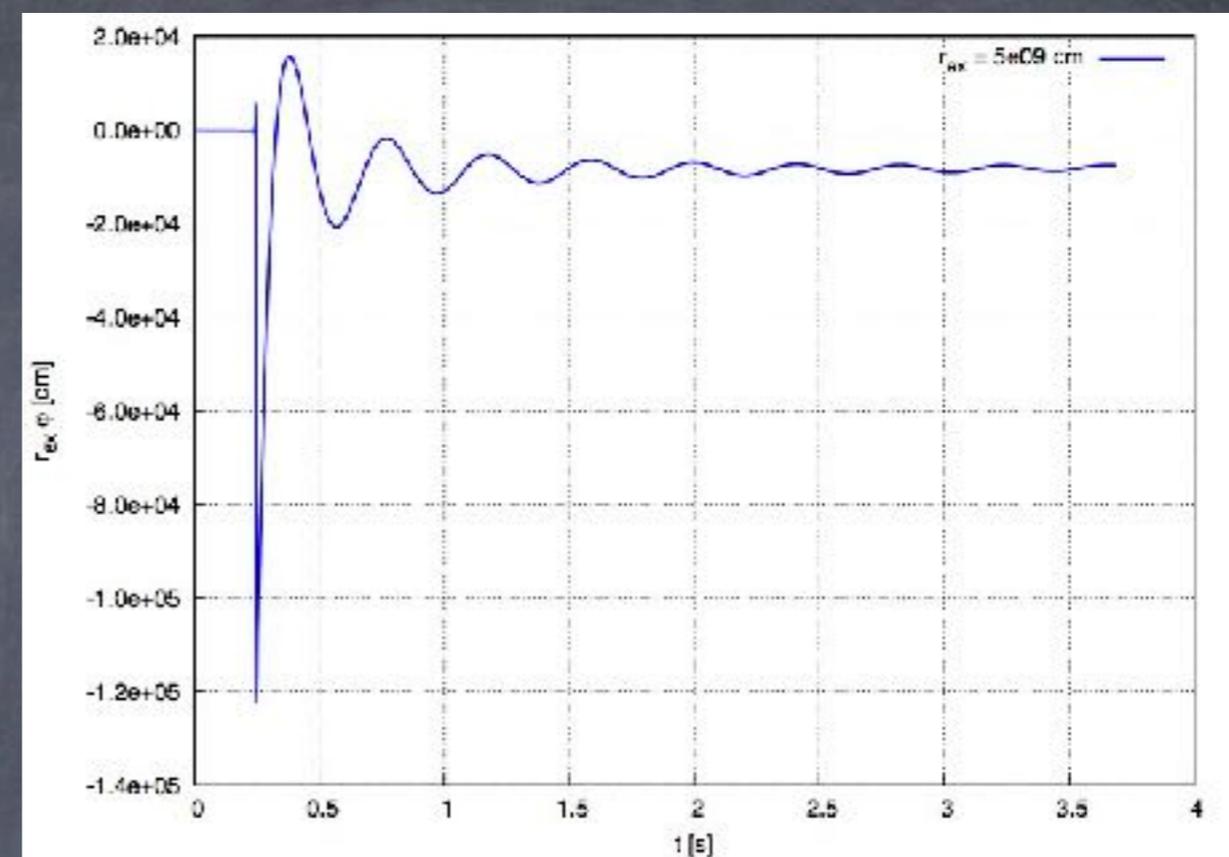
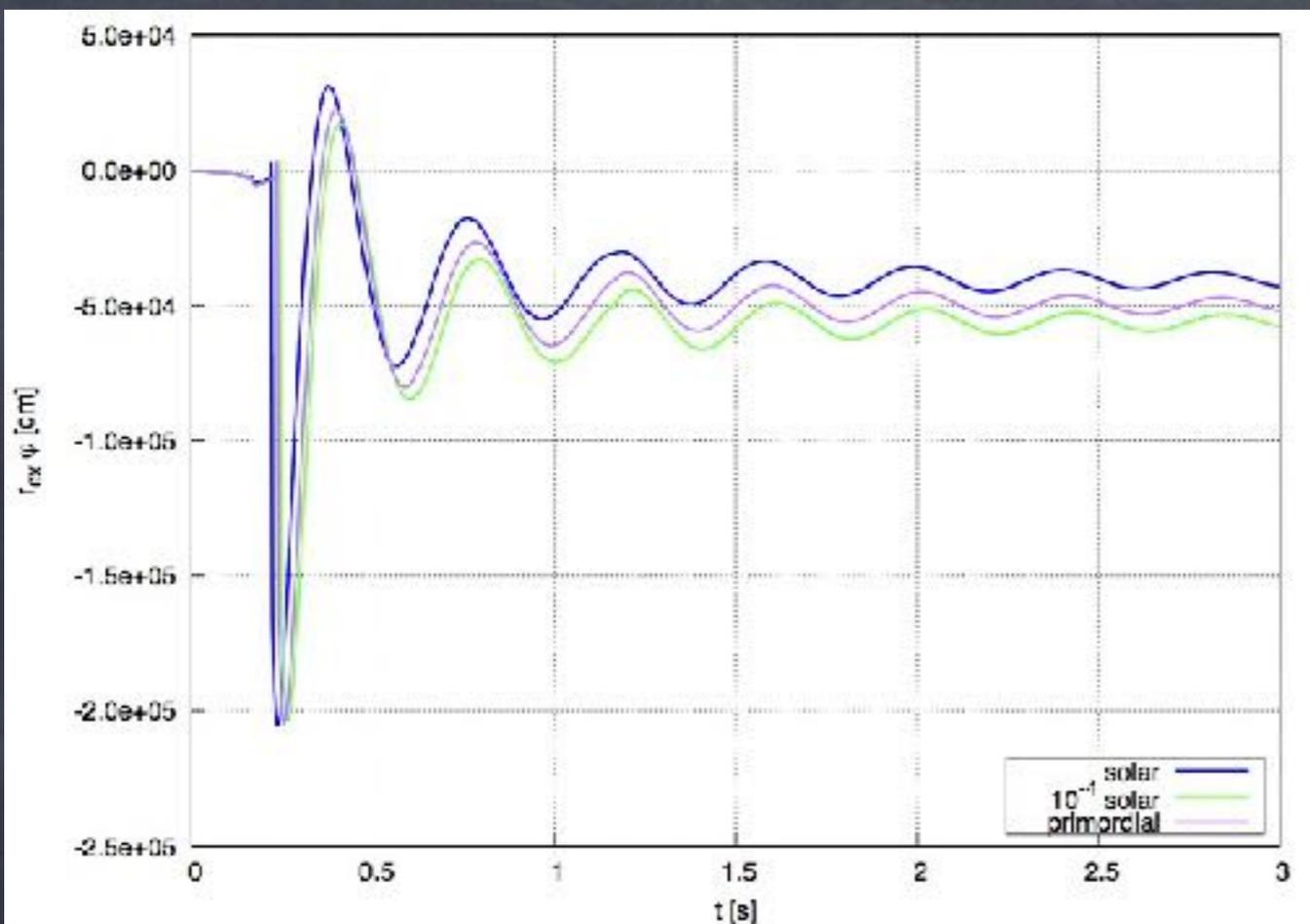
Signal much stronger than
in the massless case

Parameters are
less constrained
in this case

The signal is dispersed!



Waveforms extracted at 5×10^4 km. The legend lists deviations from the fiducial parameters $\mu = 10^{-14}$ eV, $\alpha_0 = 10^{-2}$, $\beta_0 = -20$, $\Gamma_1 = 1.3, \Gamma_2 = 2.5, \Gamma_{th} = 1.35$.



Top left

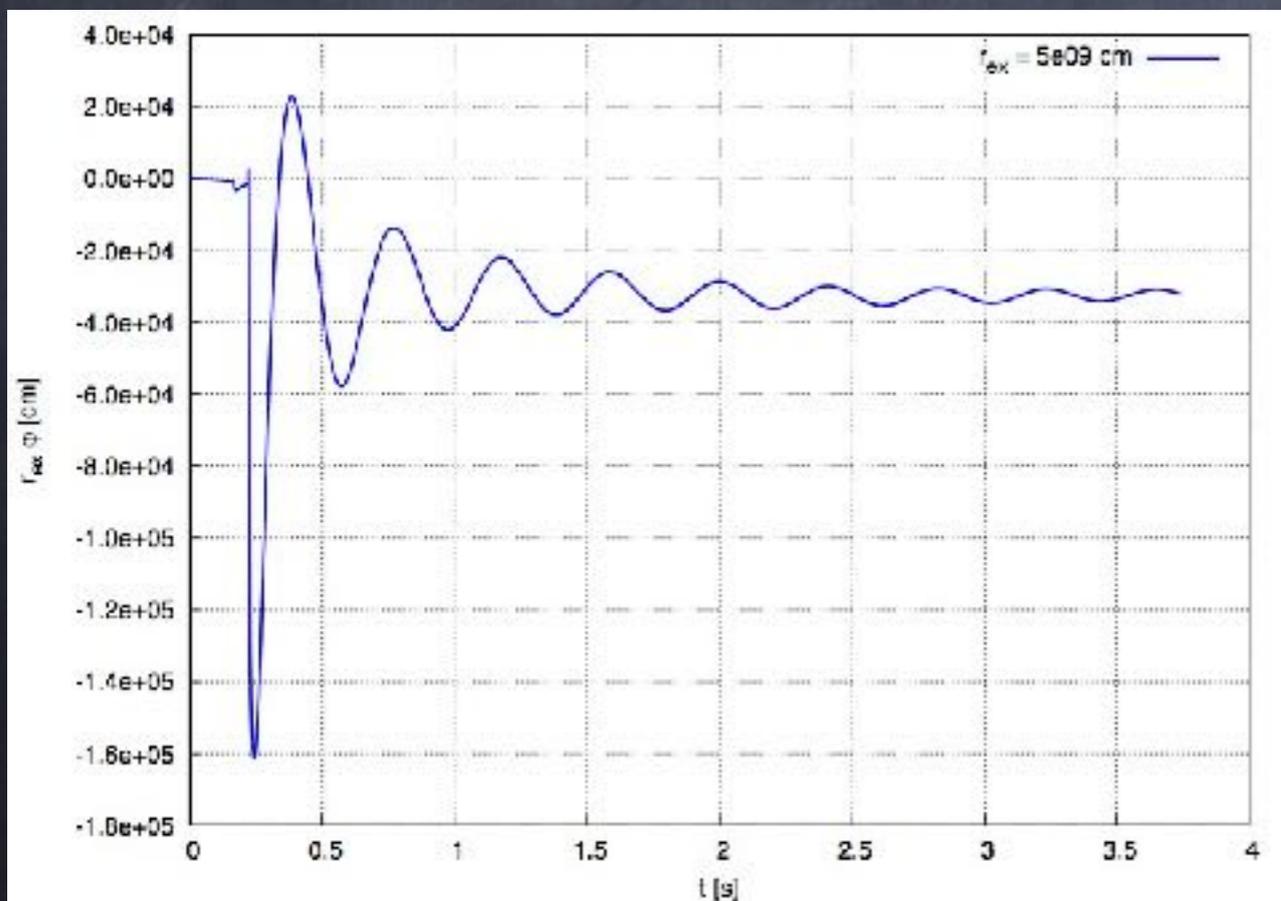
$$31 M_{\odot}, \Gamma_{th} = 1.35, \Gamma_1 = 1.3, \Gamma_2 = 2.5, \\ \mu = 10^{-14} \text{ eV}, \alpha_0 = 10^{-2}, \beta_0 = -20$$

Top right

$$12 M_{\odot}, \Gamma_{th} = 1.35, \Gamma_1 = 1.32, \Gamma_2 = 2.5, \\ \mu = 10^{-14} \text{ eV}, \alpha_0 = 10^{-4}, \beta_0 = -9$$

Bottom left

$$75 M_{\odot}, \Gamma_{th} = 1.35, \Gamma_1 = 1.3, \Gamma_2 = 2.5, \\ \mu = 10^{-14} \text{ eV}, \alpha_0 = 10^{-2}, \beta_0 = -15$$



Waveforms "far from" the source

LIGO will observe the above scalar profiles after they have propagated large distances

In the massless case things are trivial: $\varphi(t, r) = \frac{1}{r} \varphi(t - r, r_{\text{extracted}})$

In the massive case things are more complicated because of dispersion

Far from the source, scalar dynamics are governed by the flat-space Klein-Gordon wave equation:

$$\partial_t^2 \varphi - \nabla^2 \varphi + \omega_*^2 \varphi = 0$$

Group/phase velocity:

$$v_g = \left[1 - (\omega_*^2 / \omega^2) \right]^{1/2}$$

The scalar field mass
has a natural frequency

$$\omega_* = c^2 m_\varphi / \hbar$$

- Low frequencies are suppressed
- High frequency power spectrum is unaffected
- Signal spreads out in time
- High frequencies arrive earlier than low frequencies
- Signal becomes increasingly oscillatory

Easier to work with:

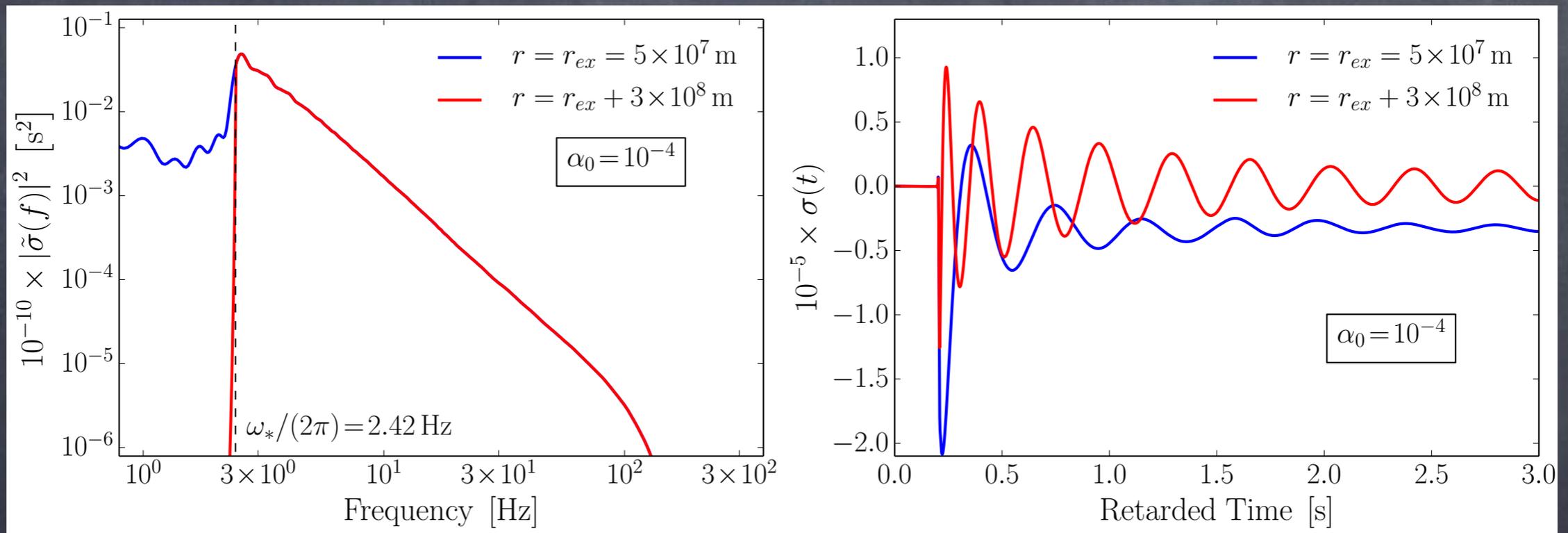
$$\sigma \equiv r\varphi$$

$$\partial_t^2 \sigma - \partial_r^2 \sigma + \mu^2 \sigma = 0$$

Analytical solution in the frequency domain:

$$\tilde{\sigma}(\omega, r) = \tilde{\sigma}(\omega, r_{ex}) \begin{cases} e^{-i\sqrt{\omega^2 - \omega_*^2}(r - r_{ex})} & \text{for } \omega < -\omega_* \\ e^{+i\sqrt{\omega^2 - \omega_*^2}(r - r_{ex})} & \text{for } \omega > -\omega_* \end{cases}$$

- Signals become more oscillatory as they propagate outwards
- In the large distance limit the **stationary phase approximation** applies, we obtain an analytic expression for the time-domain signal
- Signals have a characteristic “inverse chirp” lasting many years



SPA frequency as function of time (**inverse chirp**)

$$\sigma(t, r) = A(t, r) \cos \phi(t, r) \quad A(t, r) = \sqrt{\frac{2}{\pi}} \frac{(F^2 - \omega_*^2)^{3/4}}{\omega_* (r - r_{\text{ex}})^{1/2}} \text{Abs}[\tilde{\sigma}(F, r_{\text{ex}})]$$

$$F(t) = \frac{\omega_*}{2\pi} \frac{1}{\sqrt{1 - (d/t)^2}} \quad \phi(t, r) = \sqrt{F^2 - \omega_*^2} (r - r_{\text{ex}}) - Ft - \frac{\pi}{4} + \text{Arg}[\tilde{\sigma}(F, r_{\text{ex}})]$$

Detection with aLIGO-Virgo

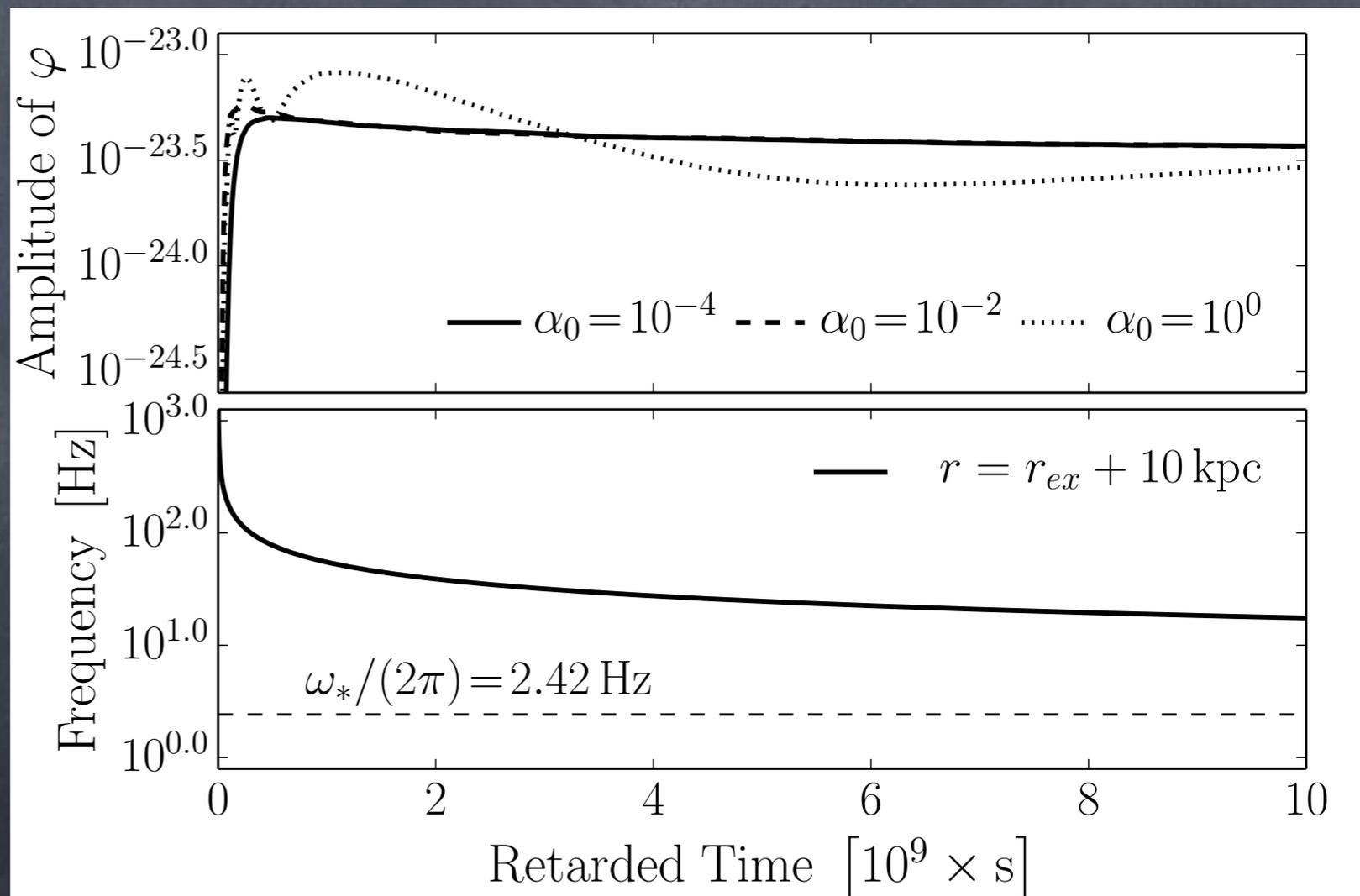
GWs from core-collapse in ST gravity may fall into 3 classes

- **Burst signals:** Hybrid EOS for light scalars (scalar mass $< 10^{-20}$ eV) and short distances (10 kpc), the pulse does not disperse significantly: will look like a < 1 s burst
- **Continuous wave signals:** for heavier scalars, long dispersion turns pulse into a quasi-monochromatic signal \rightarrow capture using standard targeted CW searches (assuming EM counterpart)
- **Stochastic background (work in progress):**
 - many quiet sources + very long duration (superposed)
 - cosmological redshift + mass variation \rightarrow smeared out low-f cutoff
 - well in reach for aLIGO/AdVirgo stochastic searches

Detection of Continuous waves

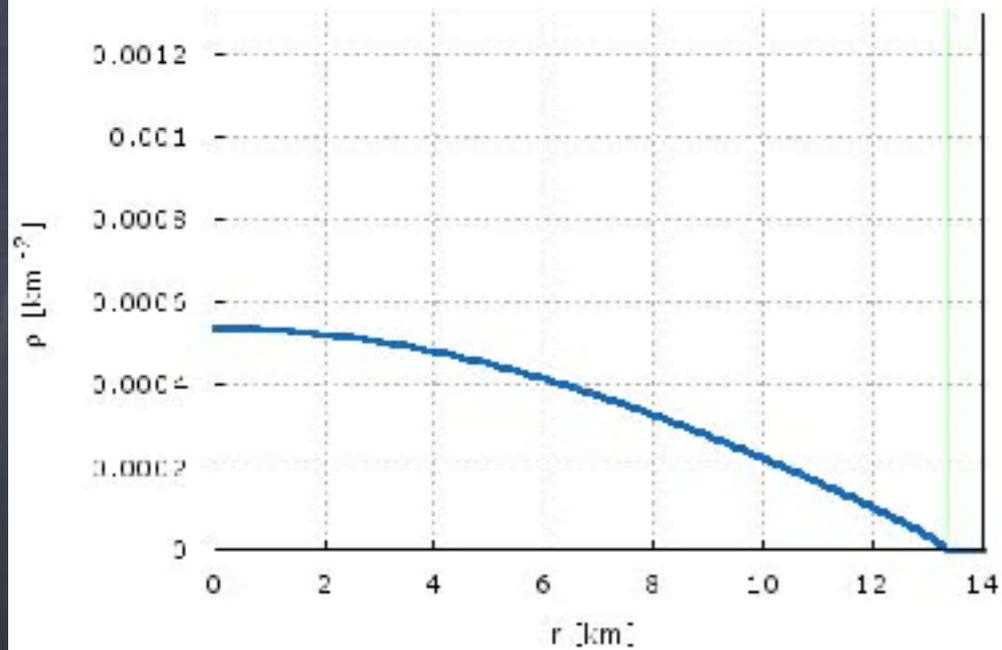
If $\omega_{\min} < \omega_*$ then GW can last for very long time
(from months to centuries).

- SPA gives characteristic $F(t)$ curve (inverse chirp)
- For large scalar mass, instantaneous frequency changes very slowly : quasi-monochromatic
- Targeted CW searches may be able to detect SN signals, even decades after the EM event was observed.



Conclusions

- We have simulated core collapse in massive scalar-tensor theories
- We have explored combined parameter space of EOS and ST theory parameters
- Spontaneous scalarization occurs as in massless case, but the effect can be even more dramatic because the scalar mass “screens” the effect of the mass allowing larger values of beta to be compatible with binary pulsar observations
- Signals propagate with dispersion, signals can last for years to decades at kilo parsec distances
- Signals can show up in aLIGO burst, CW, or stochastic searches

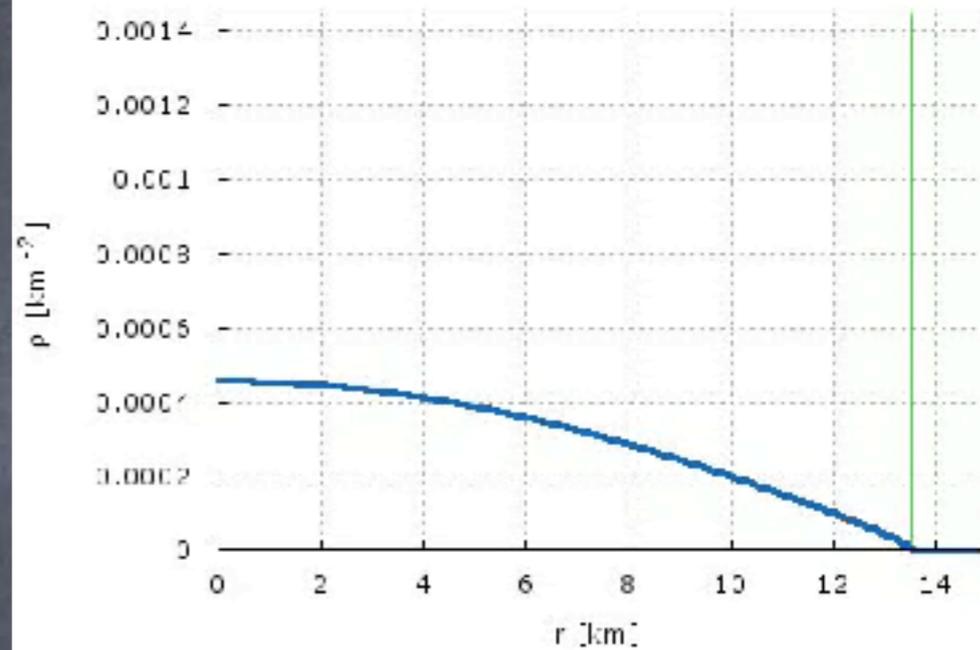


Left

$$\beta_0 = -5.0$$

$$\alpha_0 = 0.0001$$

$$m_\varphi = 4.8e^{-13} \text{ eV}$$

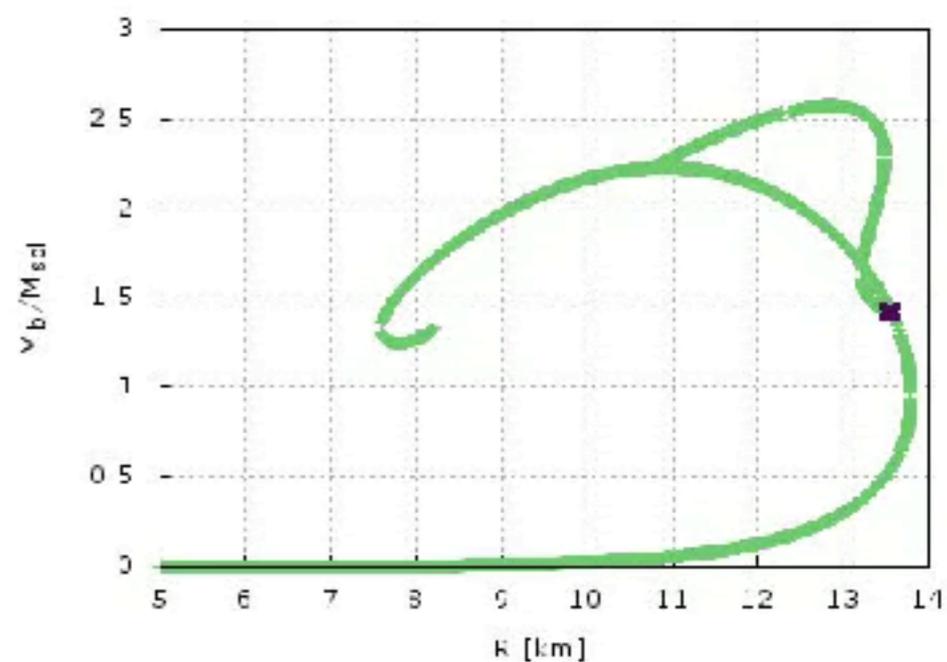
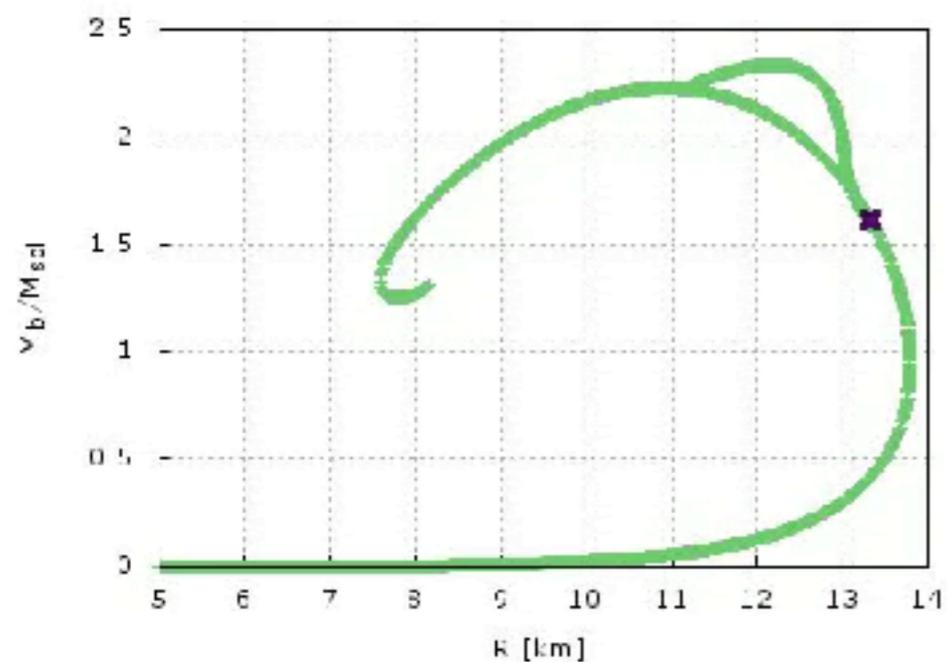
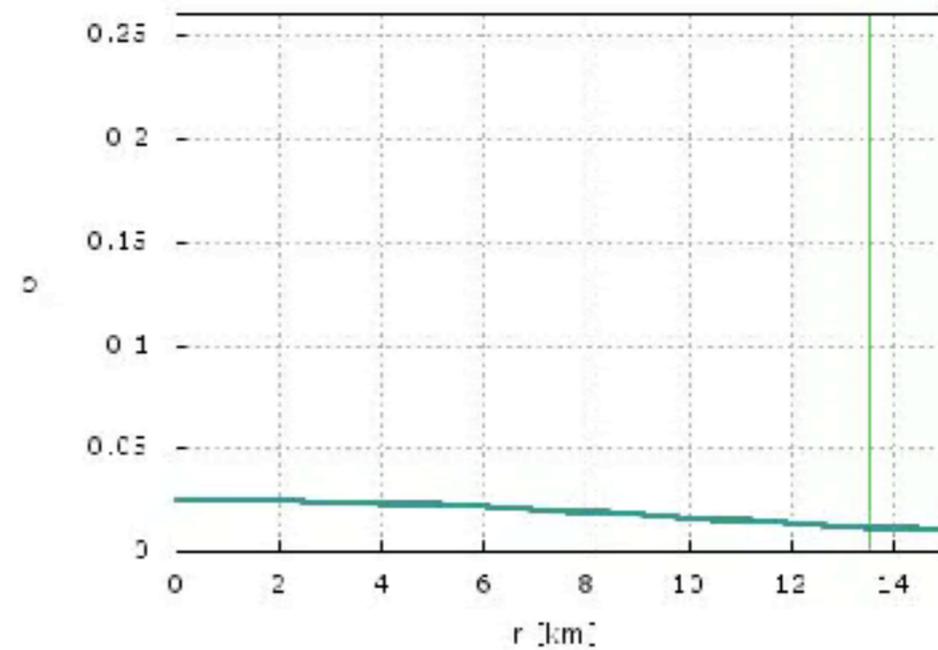
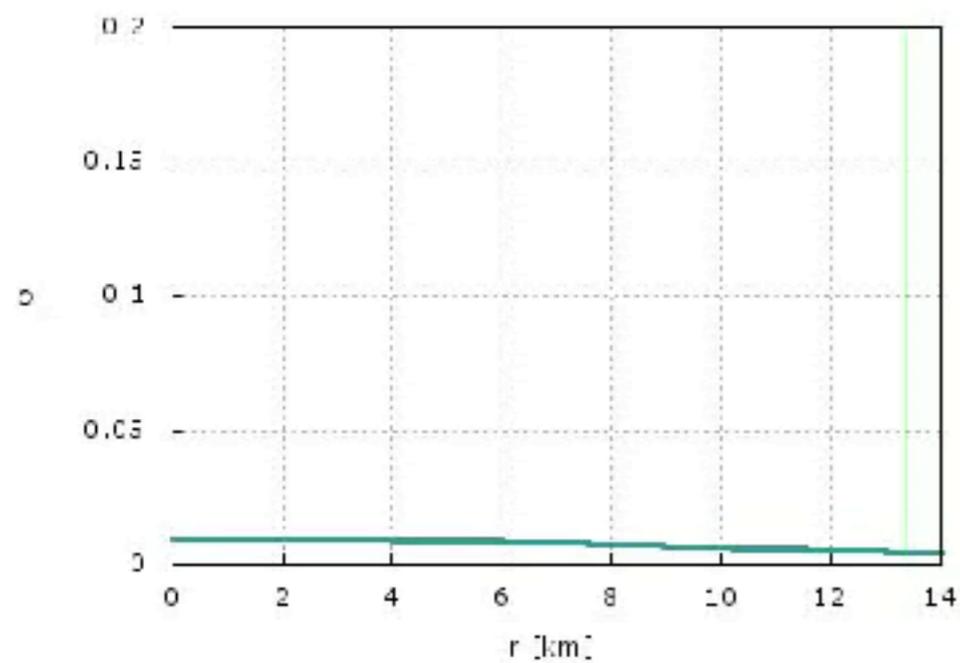


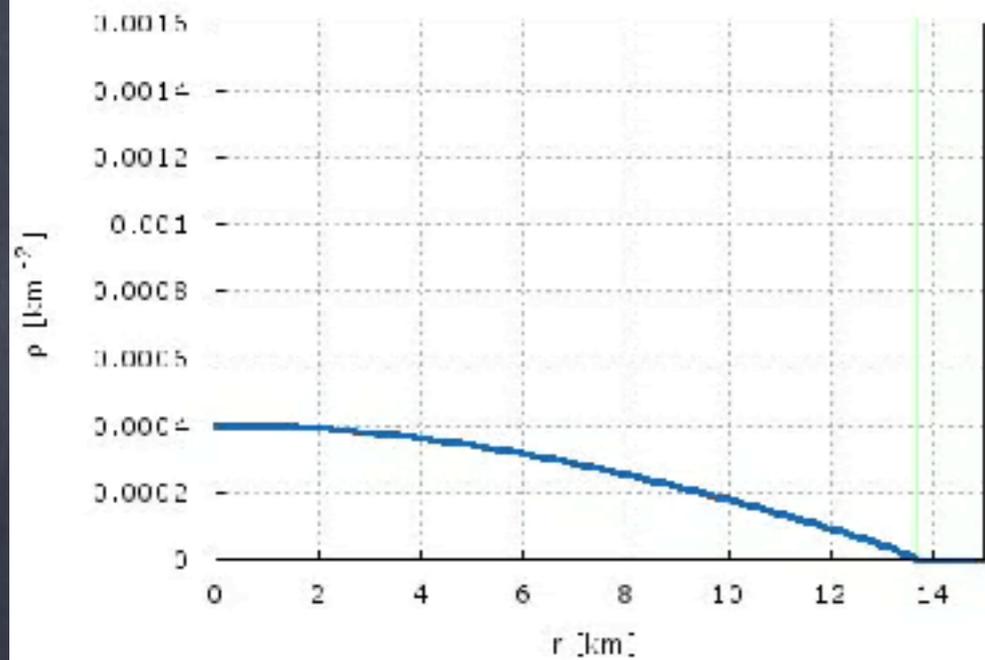
Right

$$\beta_0 = -5.5$$

$$\alpha_0 = 0.0001$$

$$m_\varphi = 4.8e^{-13} \text{ eV}$$



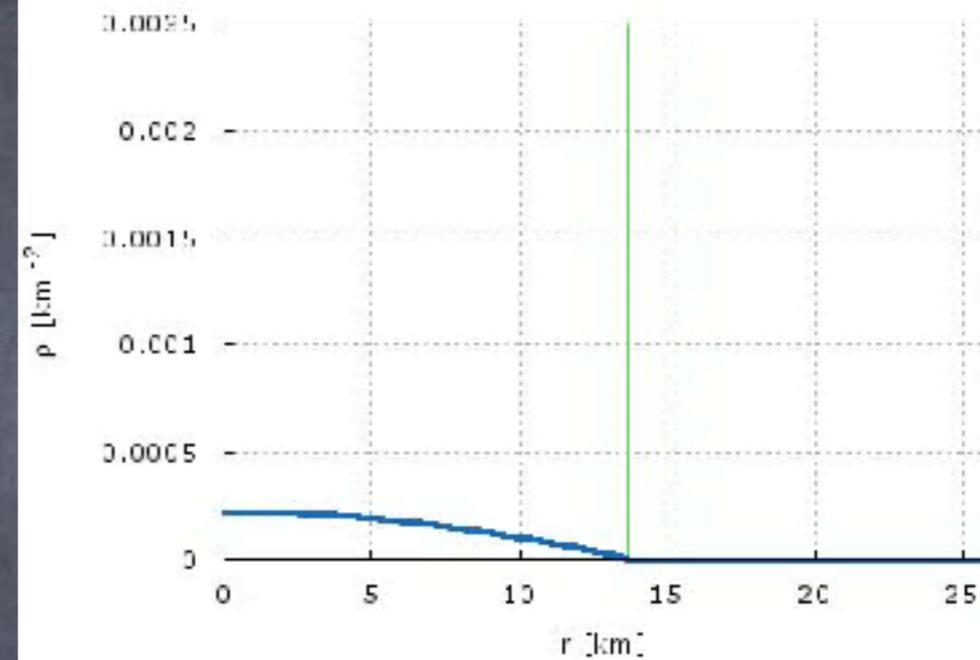


Left

$$\beta_0 = -6$$

$$\alpha_0 = 0.0001$$

$$m_\varphi = 4.8e^{-13} \text{ eV}$$



Right

$$\beta_0 = -10$$

$$\alpha_0 = 0.0001$$

$$m_\varphi = 4.8e^{-13} \text{ eV}$$

