# Numerical approaches to the self-force problem

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based on work with:
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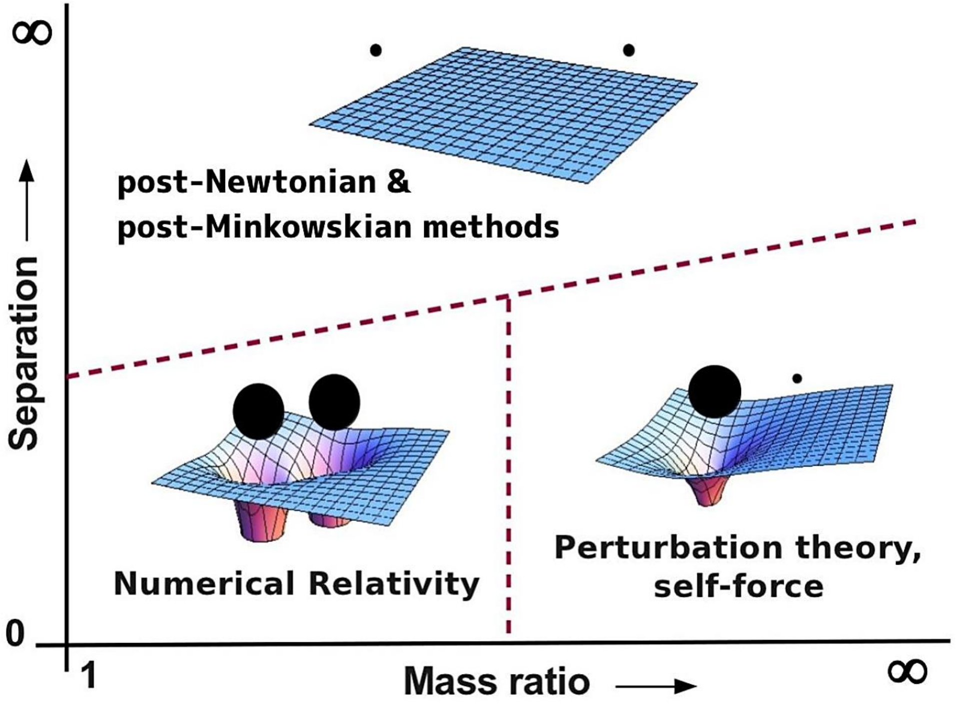
# Idea of this talk

- *What I will* ***try not*** *to do*: get into the dirty details (or propagandize) … too much.
- *What I will* ***try*** *to do*: describe the “basics” of the GSF and outline some approaches.
- *What I will unavoidably do*: be biased in the treatment.
  … mostly, I will follow the **Gralla-Wald approach** [Gralla & Wald, CQG **25**, 205009 (2008)].

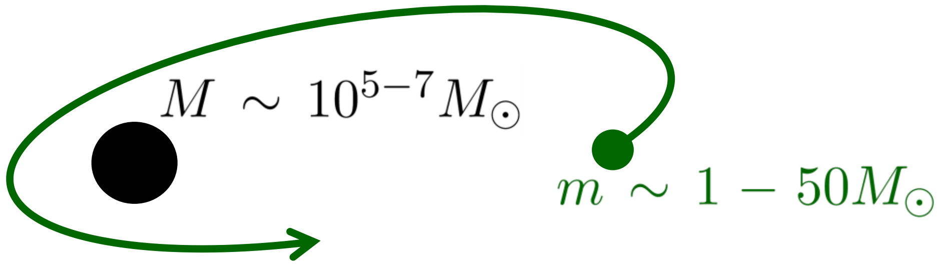
- **References**—some review papers:
  - Short and sweet: *Introduction to Gravitational Self-Force* [Wald, arXiv:0907.0412]
  - Long and “basic”: *Self-force and radiation reaction in general relativity* [Barack+, arXiv:1805.10385]
  - Longer and classic: *The Motion of Point Particles in Curved Spacetime* [Poisson+, LRR **14**, 7 (2011)]

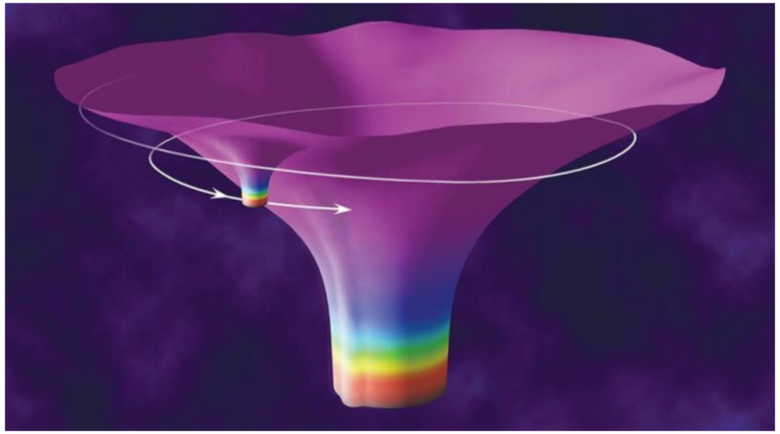
# Introduction and motivation: *extreme-mass-ratio inspirals (EMRIs)*

- The detection of GWs from EMRIs will require the computation of the **gravitational self-force (GSF)**.




[arXiv:1805.10385]

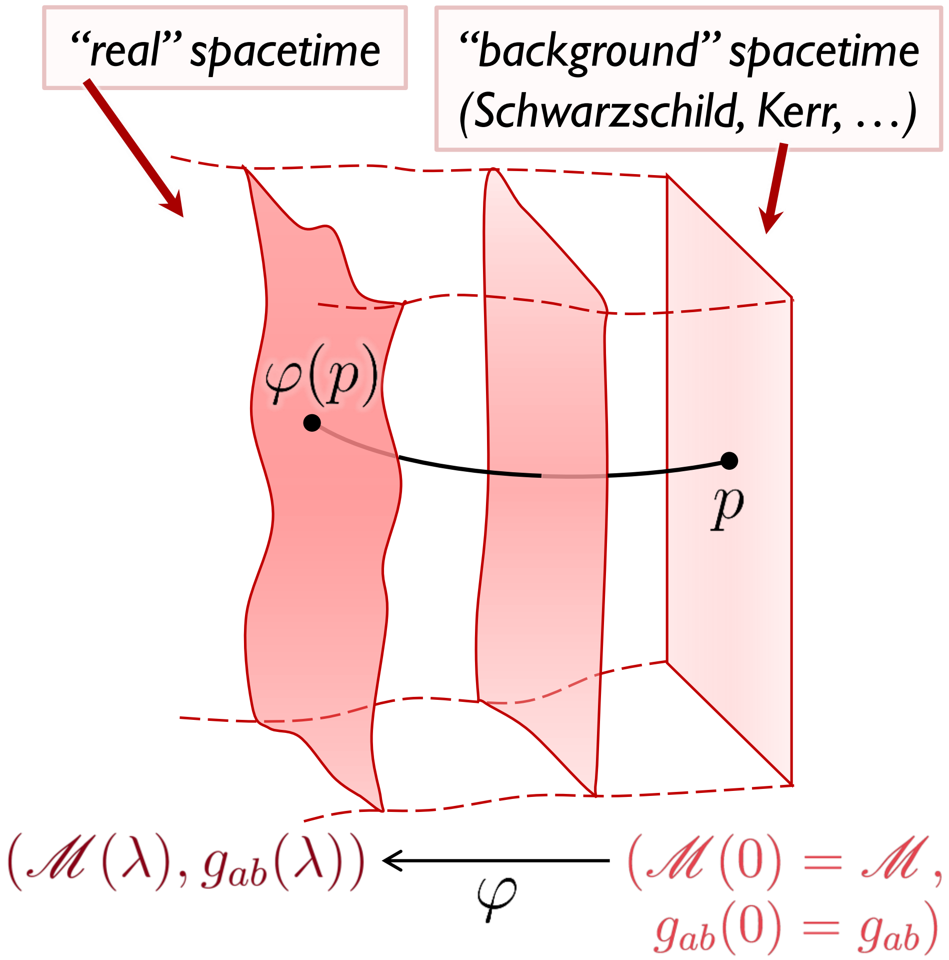


[arXiv:1305.5720]

# Basic setup: *perturbation theory*

[Bruni+, CQG **14**, 2585 (1997)]




- What do we mean when we say "$g_{ab} + h_{ab}$"?
  - There is a "real" spacetime somewhere and we think $g_{ab}$ is "close" (in some suitable sense).
- Define a *one-parameter family of metrics* $\{g_{ab}(\lambda)\}_{\lambda \geq 0}$ and **choose** a map (the **"perturbative gauge"**):

$$\boxed{\varphi : \mathscr{M} \to \mathscr{M}(\lambda)}$$

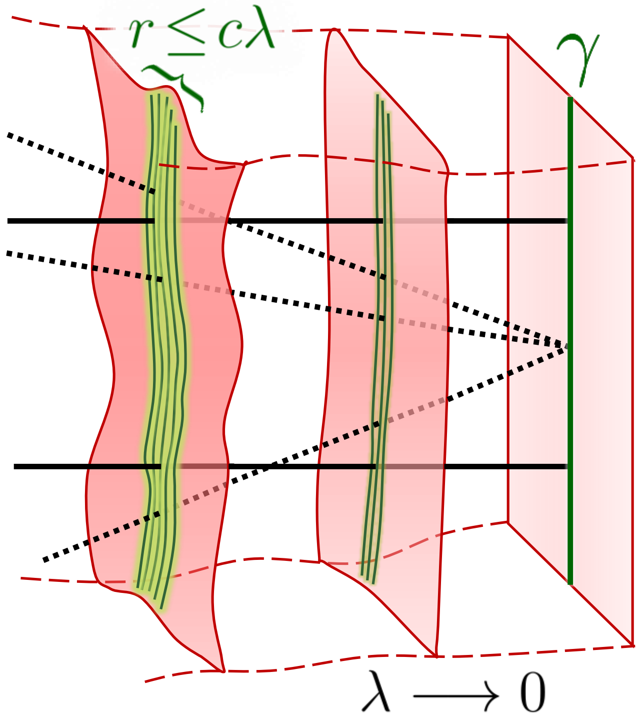
  - Then, the "perturbed" metric is

$$\boxed{\varphi_* g_{ab}(\lambda) = g_{ab} + \lambda h_{ab} + \mathcal{O}(\lambda^2)}$$

- Under a choice of a different "gauge" $\tilde{\varphi} : \mathscr{M} \to \mathscr{M}(\lambda)$ everything just gets mapped by $\tilde{\varphi}^{-1} \circ \varphi : \mathscr{M} \to \mathscr{M}$ (e.g., coordinates transform as $x^a \mapsto x^a - \lambda \xi^a + \mathcal{O}(\lambda^2)$).

# Gralla-Wald derivation of the GSF: *assumptions*

[Gralla & Wald, CQG **25**, 205009 (2008)]

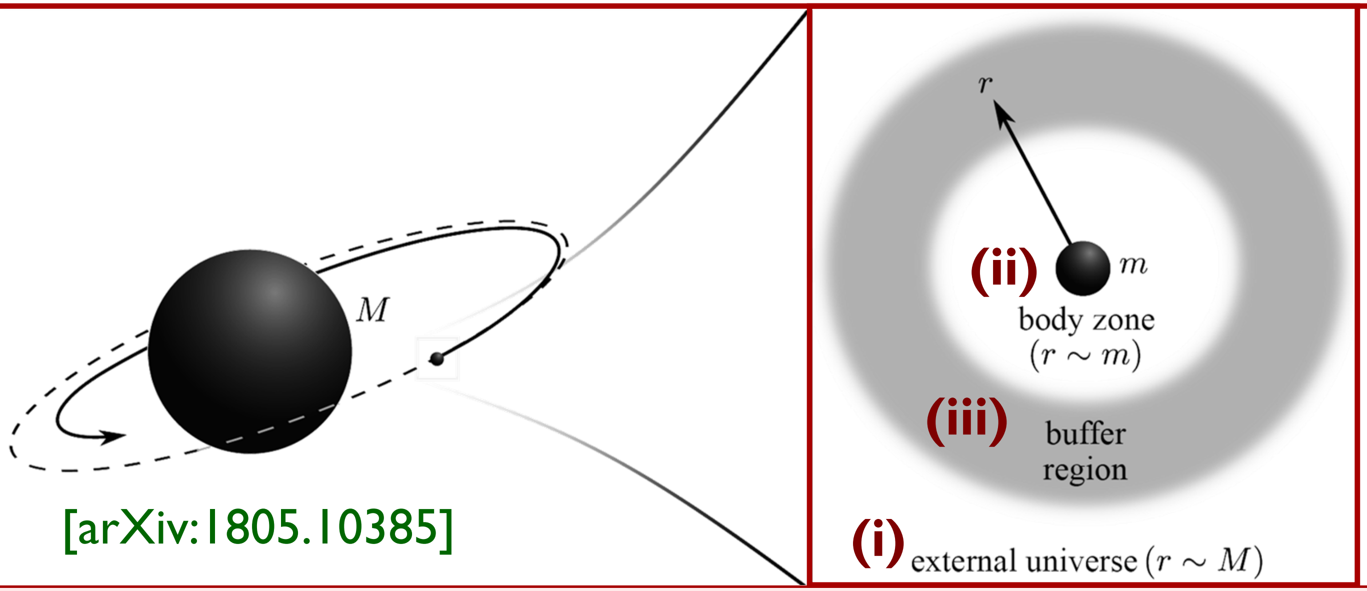



**Assumptions**: $\exists \{g_{ab}(\lambda)\}_{\lambda \geq 0}$ *vacuum such that:*

**(i)** $\exists$ *"ordinary limit" (the object shrinks down to a worldline* $\gamma$ *with its mass going to zero at least as fast as its radius).*

**(ii)** $\exists$ *"scaled limit" (it shrinks to zero size asymptotically "self-similarly").*

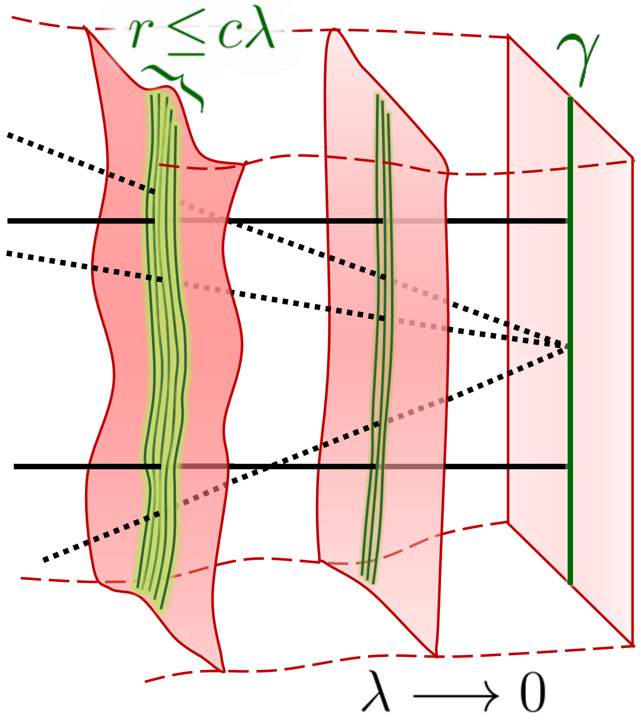
**(iii)** *"Uniformity condition" ($\nexists$ "bumps of curvature" in the family).*




[arXiv:1805.10385]

# Gralla-Wald derivation of the GSF: *consequences*

[Gralla & Wald, CQG **25**, 205009 (2008)]




**Consequences** of the assumptions:

- ZEROTH ORDER: *$\gamma$ is a* ***geodesic*** *in the background and the "scaled" ("body zone") metric is* ***stationary and asymptotically flat****.*
- FIRST ORDER: *the stress-energy tensor is that of a* ***"point particle"****,*

$$\boxed{\delta G_{ab}[h_{cd}] = T_{ab} = m\int_\gamma \mathrm{d}\tau\, u_a(\tau)u_b(\tau)\delta_4(x - z(\tau))}$$

*4-velocity*

*parametrization of $\gamma$.*

*and the* ***"correction to the motion"*** *is given by:*

*"deviation vector"*

$$\boxed{\nabla_u\nabla_u Z^a = \overbrace{L^{abc}h^{\text{tail}}_{bc}}^{\textit{GSF}} - \overbrace{R_{bcd}{}^{a}u^b Z^c u^d}^{\textit{geodesic deviation}}}$$

*first-order linear differential operator*

*"tail" integral (of Green's functions of $h_{ab}$)*

# Summary of the problem and alternative formulations

- **Basic picture**: we have to solve $$\begin{cases} \delta G_{ab}[h_{cd}] \sim m\delta \\ \mathcal{F}^a = \mathrm{D}^2_\tau Z^a = L^{abc} h^{\mathrm{tail}}_{bc} \end{cases}$$

*field equation*

*"equation of motion"*

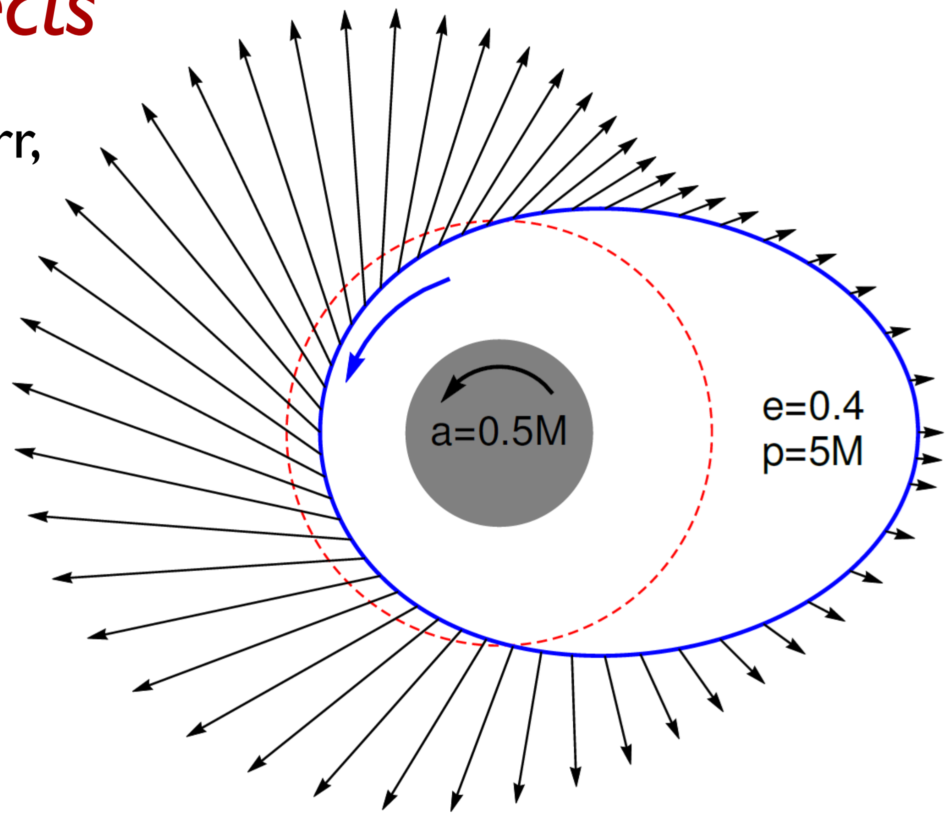
- **Alternative formulations** of the problem:
  - Regularization method (*Singular-Regular splitting*) [Detweiler & Whiting, PRD **67**, 024025 (2003)]: $\boxed{\mathcal{F}^a = L^{abc} h^{\mathrm{R}}_{bc}}$ where $h^{\mathrm{R}}_{ab} = h_{ab} - h^{\mathrm{S}}_{ab}$ (one subtracts a "Coulombian" field $h^{\mathrm{S}}_{ab} \sim m/r$ ).
    ⟹ (probably) the most widely used for numerics. .
  - Puncture method [Barack+, PRD **76**, 044020 (2007)]: one "regularizes" the field equation itself, $\delta G_{ab}[h^{\mathcal{R}}_{cd}] \sim S_{\mathrm{eff}}$ (non-distributional), and $\boxed{\mathcal{F}^a = L^{abc} h^{\mathcal{R}}_{bc}}$ ⟹ Also quite used in numerics.
  - Angle average method [Gralla, PRD **84**, 084050 (2011)]: $$\boxed{\mathcal{F}^a = \frac{1}{4\pi} \lim_{r\to 0} \int_{\mathbf{S}^2_{(r)}} \mathrm{d}\Omega\, L^{abc} h_{bc}}$$
    ⟹ Not appreciably explored so far… but we're working on it!
    [M.O., R. Epp, C. Sopuerta, A. Spallicci, R. Mann, *GSF from quasilocal conservation laws*, coming soon…]
  - Effective field theory method [Galley+, PRD **79**, 064002 (2009)]. ⟹ Not very much explored either.

# Numerical approaches

- Numerically, we typically have to solve linear (P)DEs $\boxed{L[u] = S}$ ← *distributional* $(\sim \delta + \delta' + \cdots)$
- What can we do to solve such equations? Here is a sample of **some** ideas:
  - Delta function approximation: e.g. as a narrow Gaussian [Lopez-Aleman+, CQG **20**, 3259 (2003)].
    $\Longrightarrow$ Not really typical.
  - Time-domain: e.g. finite-difference, integrate $S$ over its grid cells [Lousto+, PRD **56**, 6439 (1997)].
    $\Longrightarrow$ Useful for waveform extraction, but can be computationally costly.
  - Frequency-domain : e.g. "extended homogeneous solutions" [Barack+, PRD **78**, 084021 (2008)].
    $\Longrightarrow$ Less useful for waveform extraction, but computationally faster.
  - "Particle-without-Particle" method [M.O., C. Sopuerta, A. Spallicci, arXiv:1802.03405].
    $\Longrightarrow$ Pseudospectral collocation method, time or frequency domain.

# Conclusions: *status, problems and prospects*

- **State of the art**: GSF on generic bound geodesics in Kerr, frequency-domain [van de Meent, PRD **97**, 104033 (2018)].
- **Open problems**:
  - Gauge issue: there exists **no** formulation of the GSF that works (a priori) in an arbitrary gauge.
    (Perhaps a good case for more foundational work!)
  - Second-order problem: nonlinearly, one can no longer use (an analysis that yields) distributional sources!
    (Though it depends who you ask if we even need it…)
  - Improve the number-crunching: the road from here to waveforms will not be quick.
    (Advertisement to NR people: the codes are (probably) easier than what you are used to.)



# Thanks for your attention!

- *Introduction to Gravitational Self-Force* [Wald, arXiv:0907.0412]
- *Self-force and radiation reaction in general relativity* [Barack+, arXiv:1805.10385]
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