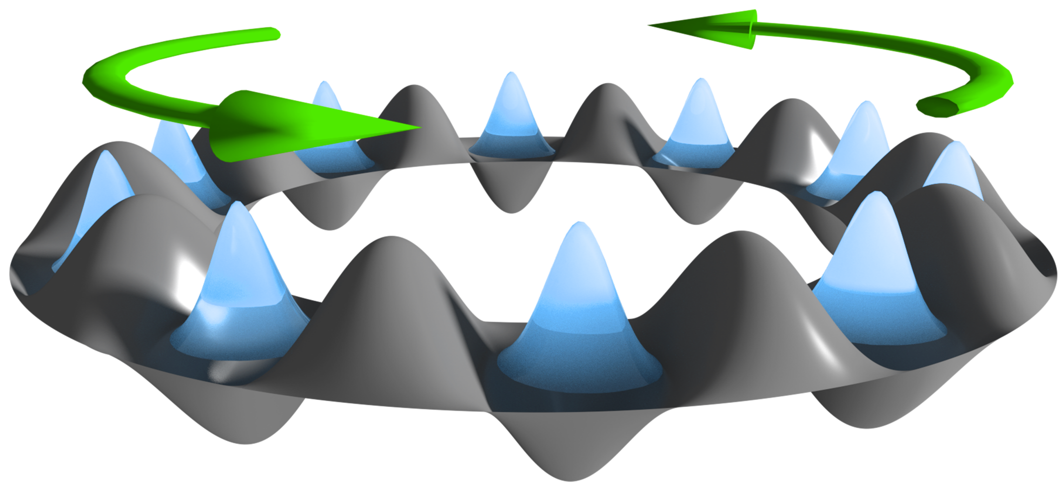
Angular momentum fractonalization for atttracting bosons in ring-shaped potentials



Piero Naldesi

LPMMC - Laboratoire de Physique et Modélisation des Milieux Condensés

Grenoble

Collaborators

PRL 122, 053001 (2019) *arxiv: 1901.09398*

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Boston

Maxim Olshanii

Boston

Hélène Perrin

Paris

Boris Malomed

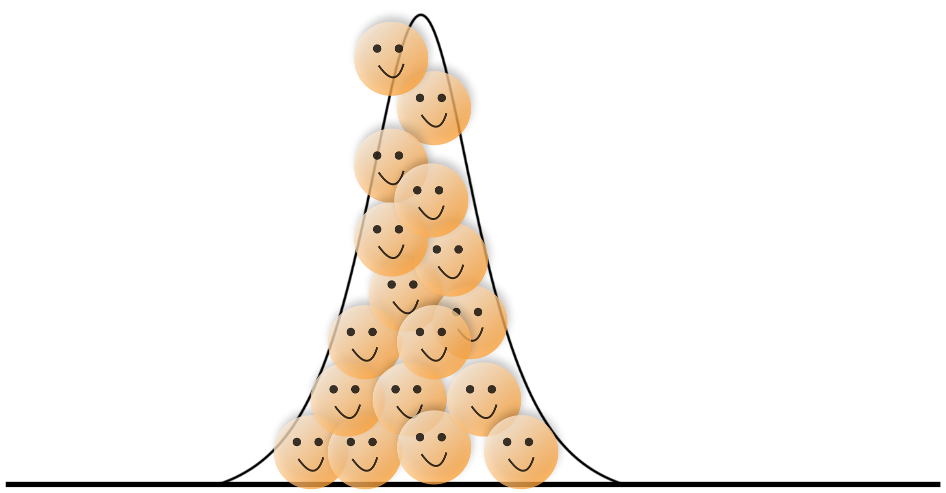
Tel Aviv

Outline

Atomtronic circuits with attractive bosons

1D optical ring

Bright soliton



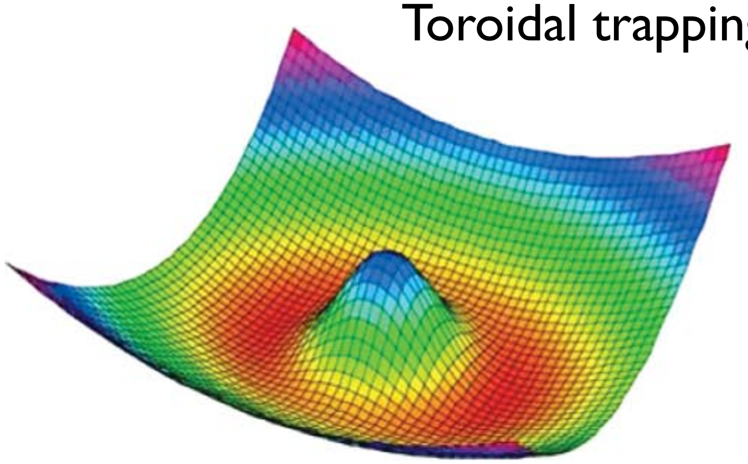
Possible applications:

Quantum simulations

Quantum sensing

ATOMIC CIRCUITS: many quantum particles in *ring-shaped optical lattices*

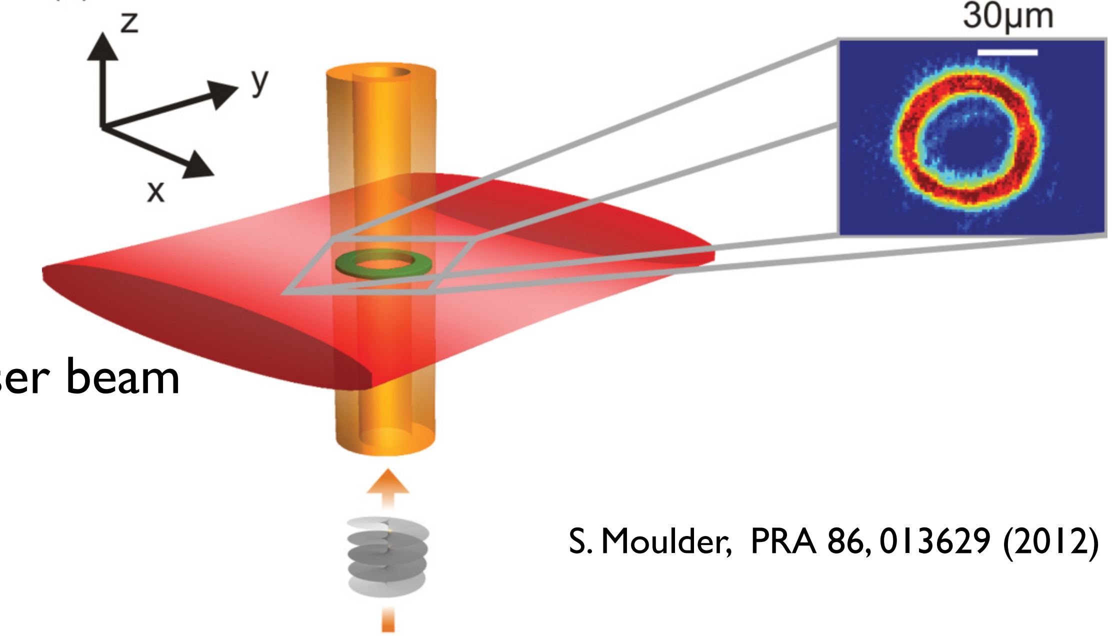
Toroidal trapping potential



C. Ryu, PRL 99, 260401 (2007)

vertical "tube" beam

density of particles

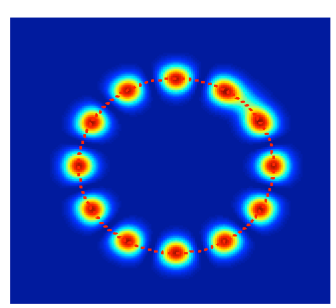


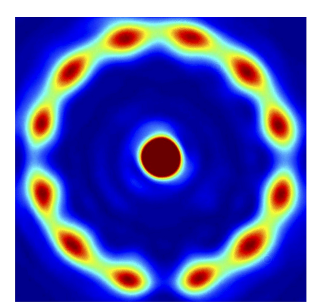
Sheet laser beam

S. Moulder, PRA 86, 013629 (2012)

Superimposing a spacial modulation: Lattice ring

Spatial Light Modulators



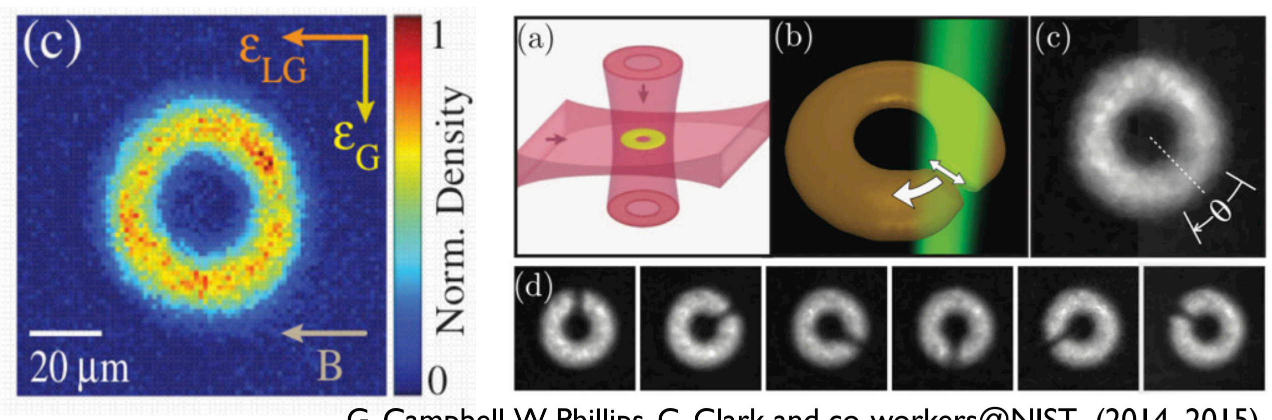


Amico, Aghamalyan, Aukstol, Crepatz, Kwek, Dumke SREP 2014.

Aghamalyan, Nguyen, Auksztol, Gan, Martinez Valado, Condylis, Kwek, Dumke, Amico NJP 2016

Aghamalyan, Cominotti, Rizzi, Rossini, Hekking, Minguzzi, Kwek, Amico NJP 2015.

The condensate can be put in motion!

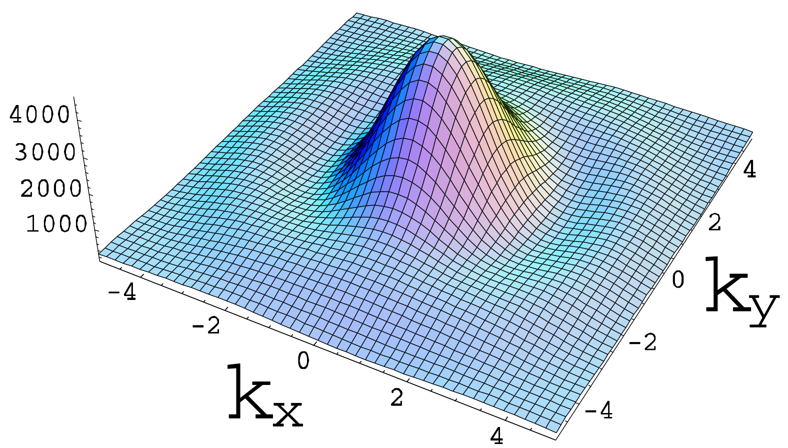


G. Campbell, W. Phillips, C. Clark and co-workers@NIST, (2014–2015)

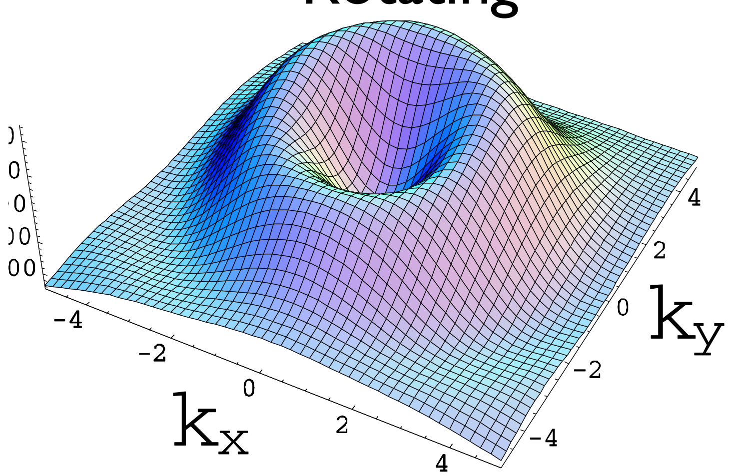
Performing absorption imaging (Invasive measurement), we can obtain the momentum distribution and ***rotation can be detected!***

Momentum Distribution

Non-Rotating



Rotating

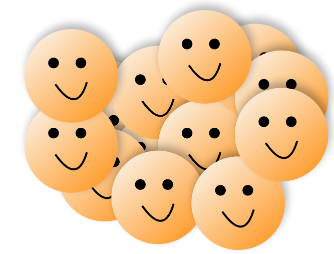


Amico, Osterloh, Cataliotti, PRL 2005.

Introduction - Solitons

Gross-Pitaevskii equation GPE: $$i\hbar\psi_t + \frac{\hbar^2}{2m}\psi_{xx} - V(x,t)\psi - g\,|\psi|^2\psi = 0$$

***Bosons** with **attractive** interaction*



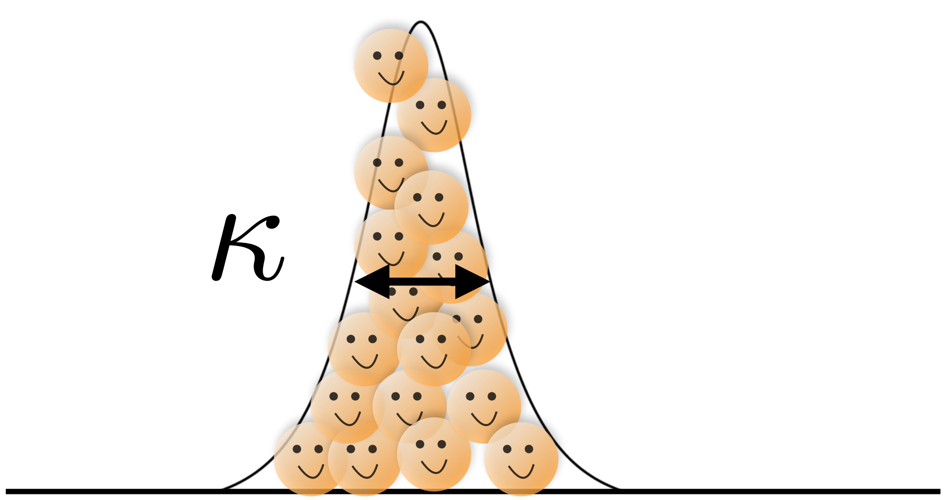
$$\psi(x,t) = \frac{a_r}{\sqrt{2|a_s|\kappa}}\,\mathrm{sech}\left(\frac{x-vt}{\kappa}\right)\exp\left[i\frac{mv}{\hbar}z - \frac{i}{\hbar}\left(\frac{mv^2}{2} - \frac{\hbar^2\kappa^2}{2m}\right)t\right]$$

$g = 2\hbar\omega_r a_s$

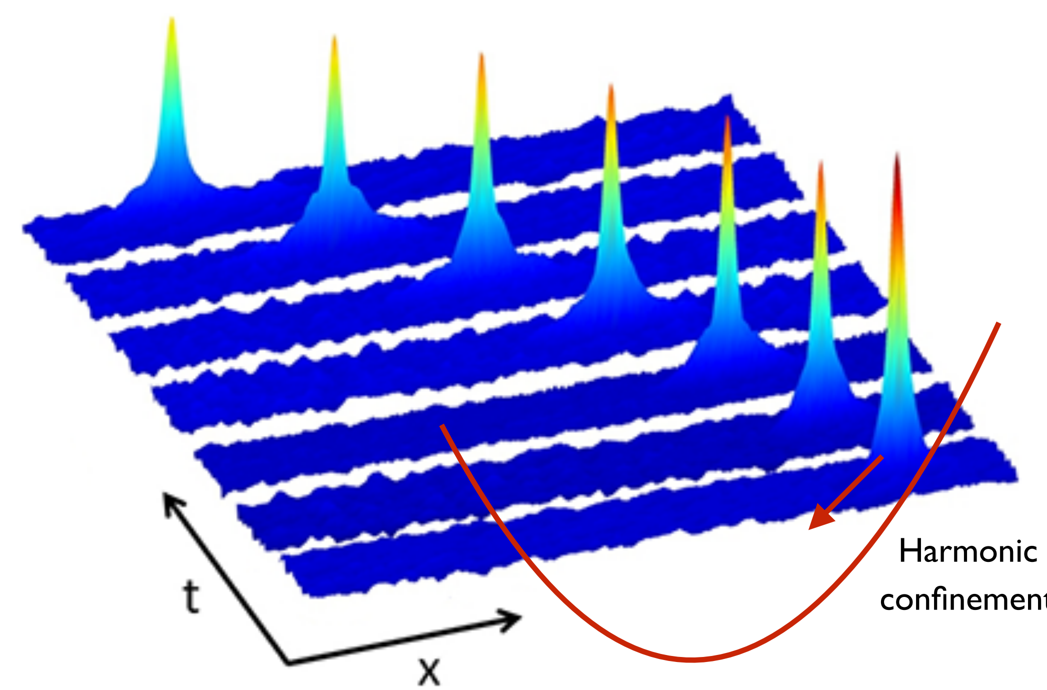
$\kappa = a_r^2/(|a_s|N)$

$a_r = \sqrt{\hbar/(m\omega r)}$

Bright soliton



S. Gardiner (2012), experiment with ultra-cold Rubidium[85]

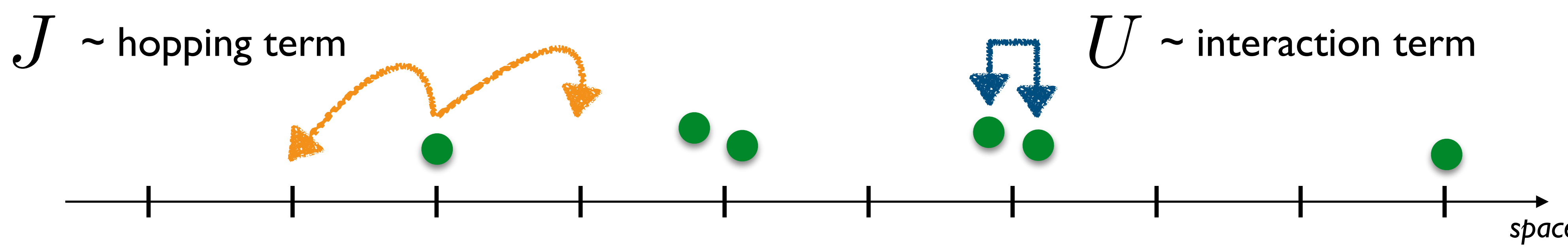


Bose-Hubbard Model

Bose-Hubbard model: interacting bosons in a lattice

$$\hat{H}(U) = -J \sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j \right) - \frac{|U|}{2} \sum_j n_j \left(n_j - 1 \right)$$

attractive interactions



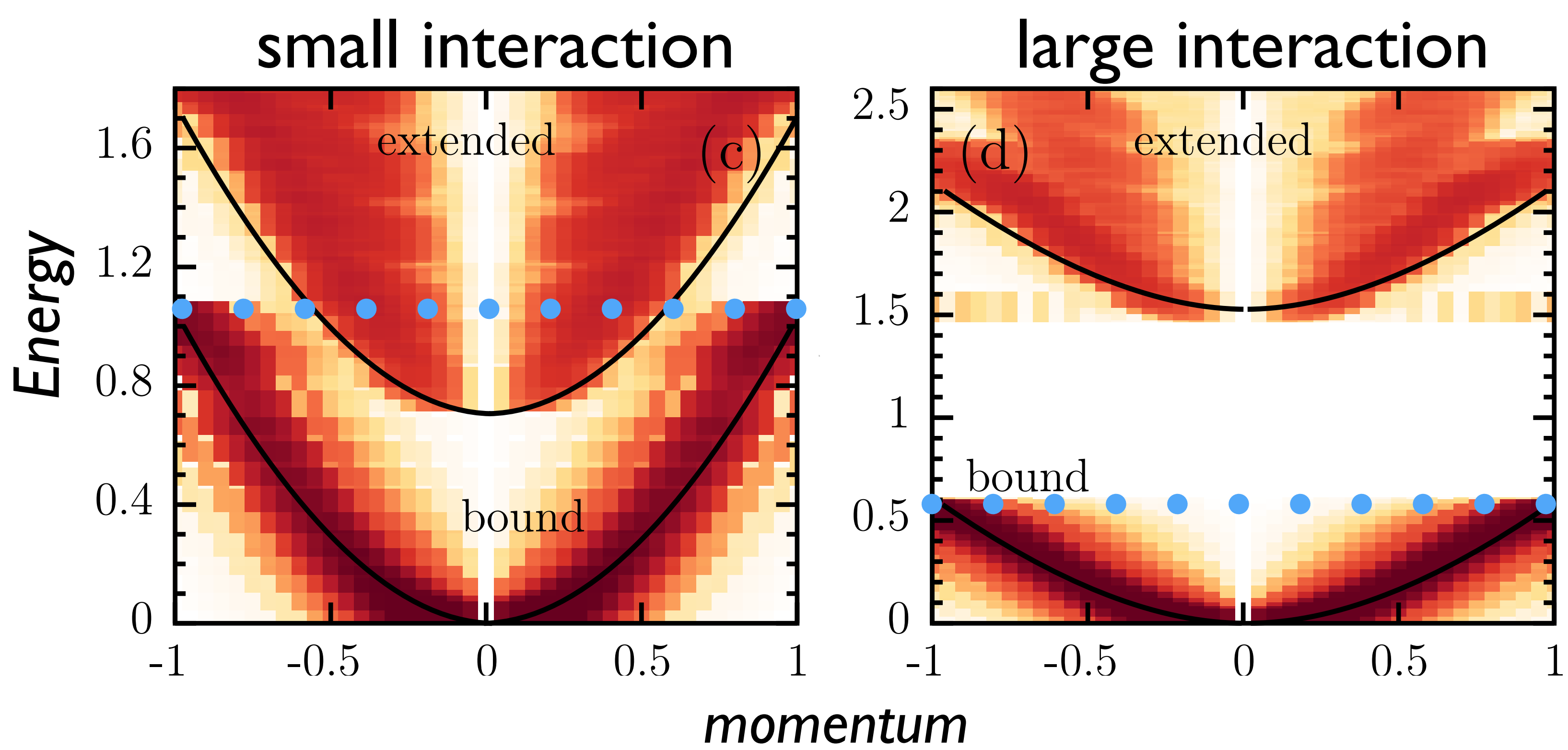
N-particles problem is not exactly solvable: still a lot of open questions!

Numerical Solution

Density Matrix Renormalization Group

S. R. White, PRL 69 2863 (1992) *A. Feiguin, S. R. White, PRB, 72, 020404 (2005)*

Bose-Hubbard Model: band structure and correlations

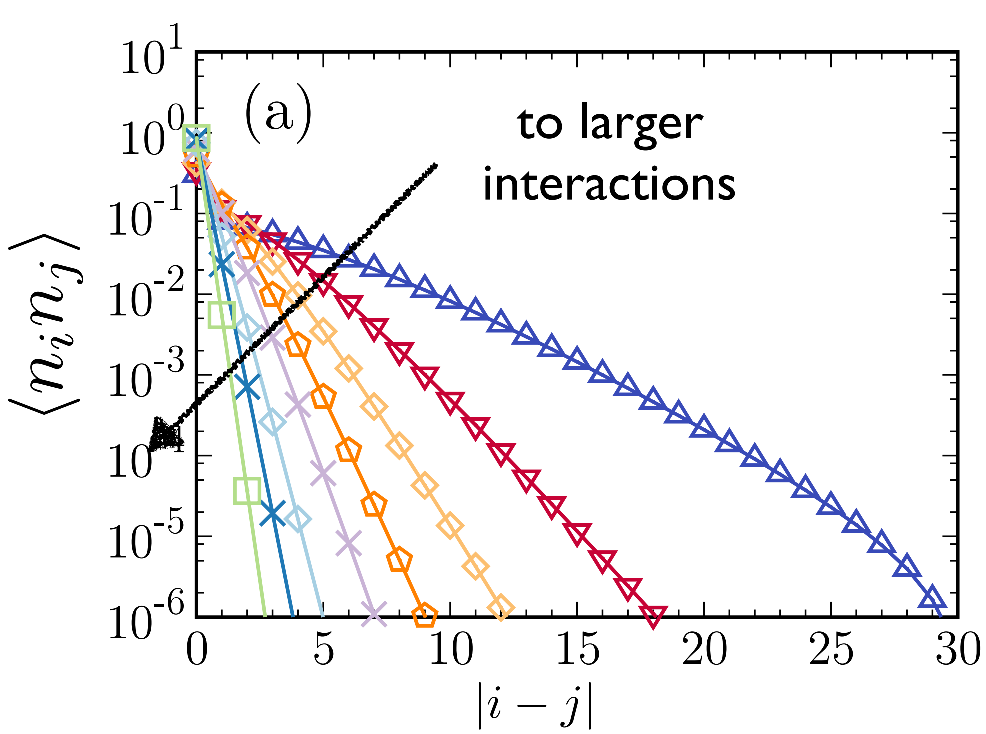


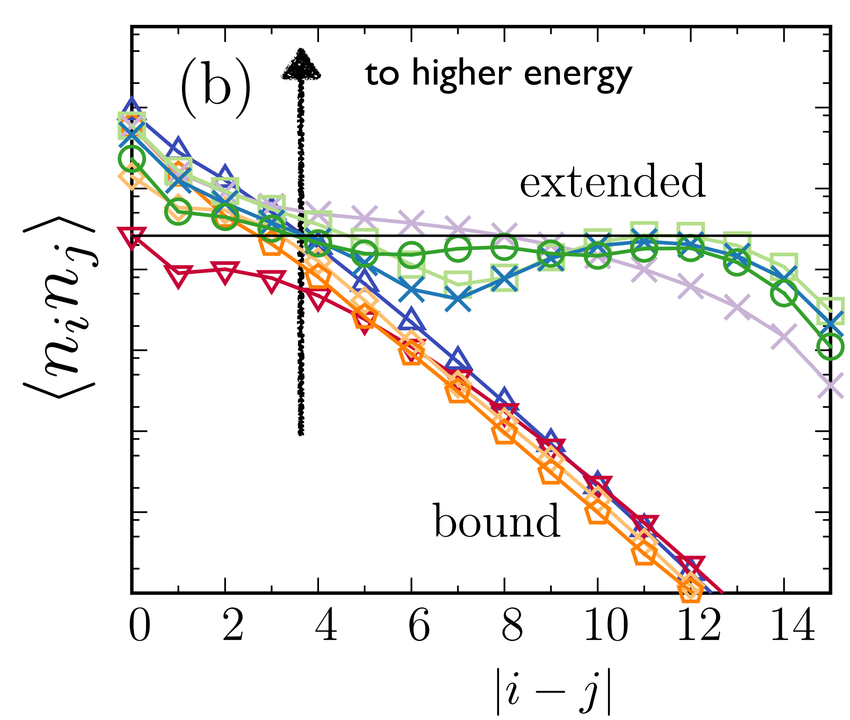
The critical interaction for which the two bands FULLY DETACH scales like 1/N

Bound and Scattering states can be characterised by Density-Density correlations.

Ground state always bound: exponential decay!

Extended states behave differently!

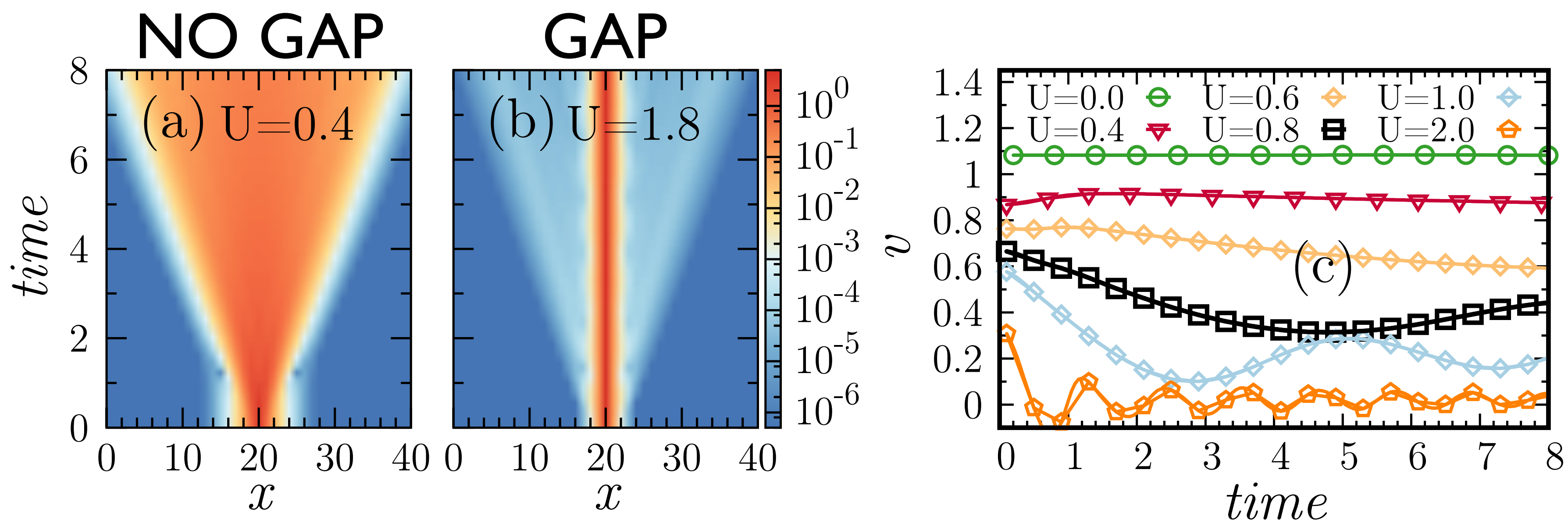




Bose-Hubbard Model

The band structure has important implications!!!

Short time dynamics: expansion of pinned solitons.

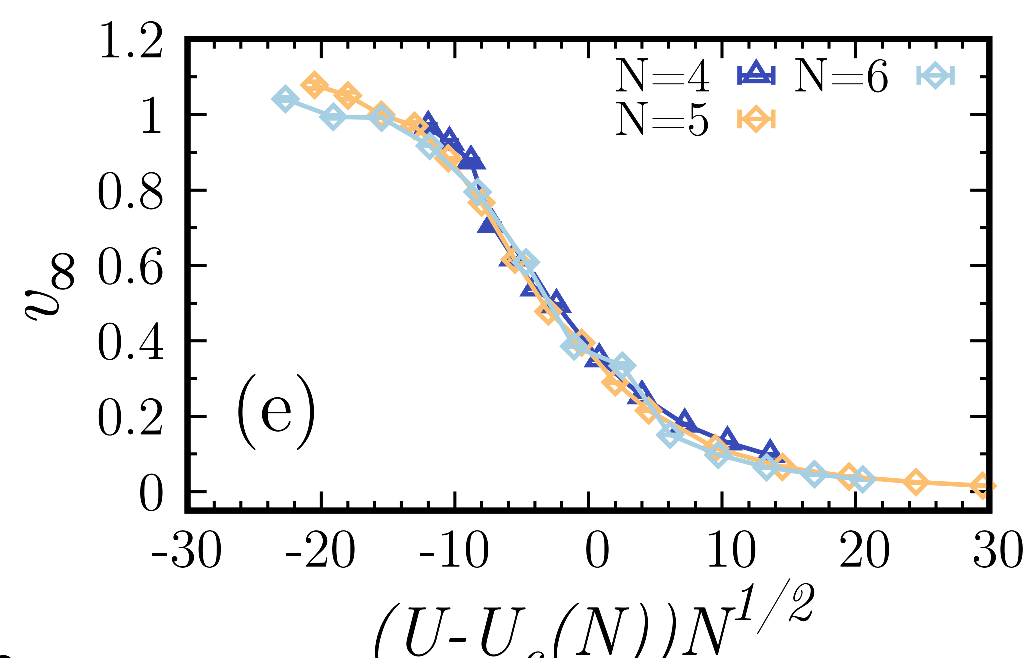


Expansion velocity

$$v(t) = \frac{d}{dt}\sqrt{R^2(t) - R^2(0)}$$

$$R^2(t) = \frac{1}{N}\sum_{i=1}^{L} n_i(t)\,(i - i_0)^2$$

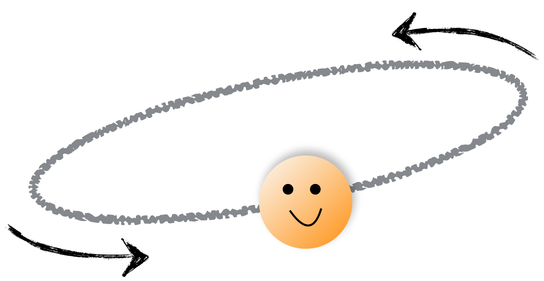
The asymptotic value of the expansion velocity acts as an "order parameter"!



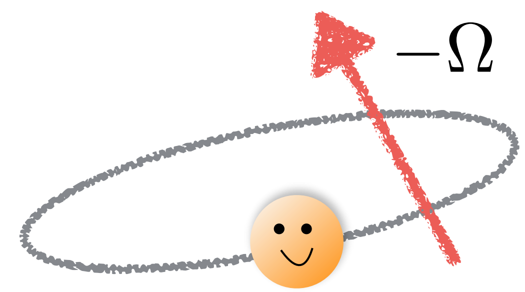
Introduction: One BOSON & rotation

Rotating bosons in a ring $\qquad \mathcal{H} = \mathcal{H}_0 + \Omega \hat{L}_Z$

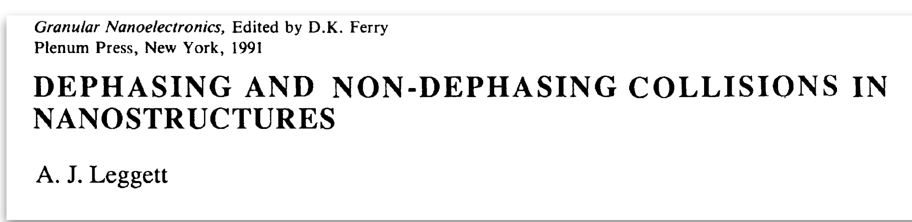
This mimes the presence of ROTATION or of any external FLUX.



$\approx$



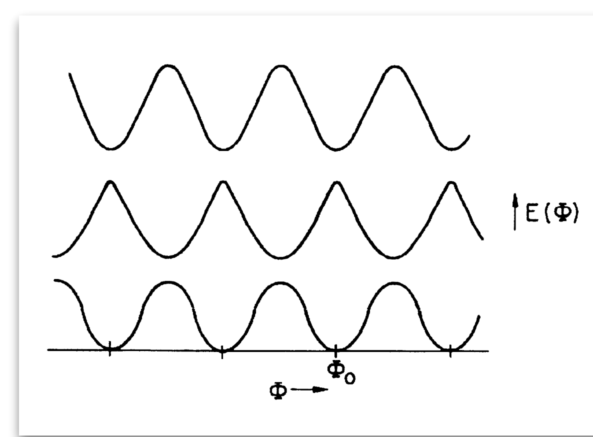
Energy bands in mesoscopic rings: Leggett Theorem



Granular Nanoelectronics, Edited by D.K. Ferry
Plenum Press, New York, 1991

DEPHASING AND NON-DEPHASING COLLISIONS IN NANOSTRUCTURES

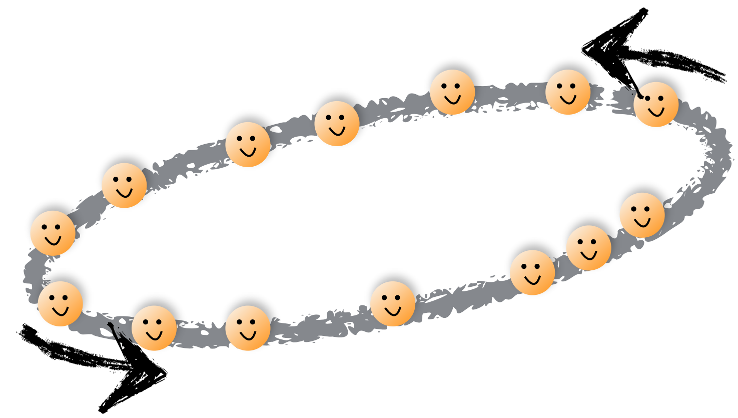
A. J. Leggett



The periodicity of the persistent current is given by flux quantum. It does not depend on the details of interaction or local disorder.

N particles with delta interaction: Lieb Liniger

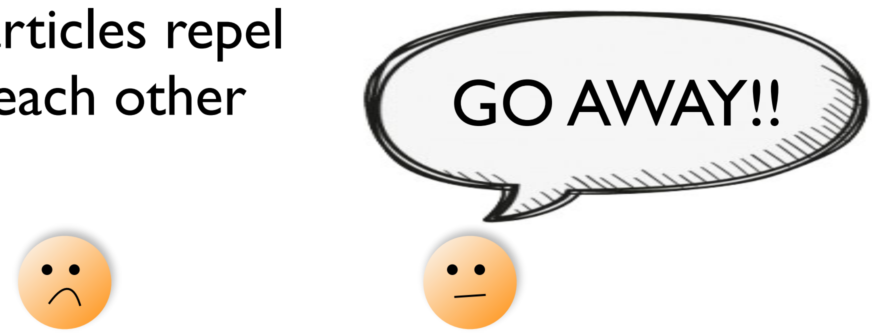
$$\mathcal{H} = \sum_{i=1}^{N} \frac{\hat{p}_i^{\,2}}{2m} + \Omega \hat{L}_{Z,i} \boxed{+ g \sum_{j<l} \delta(x_j - x_l)}$$



Lieb-Liniger model is integrable: exact solution through Bethe-Ansatz

$g > 0$ repulsive interactions

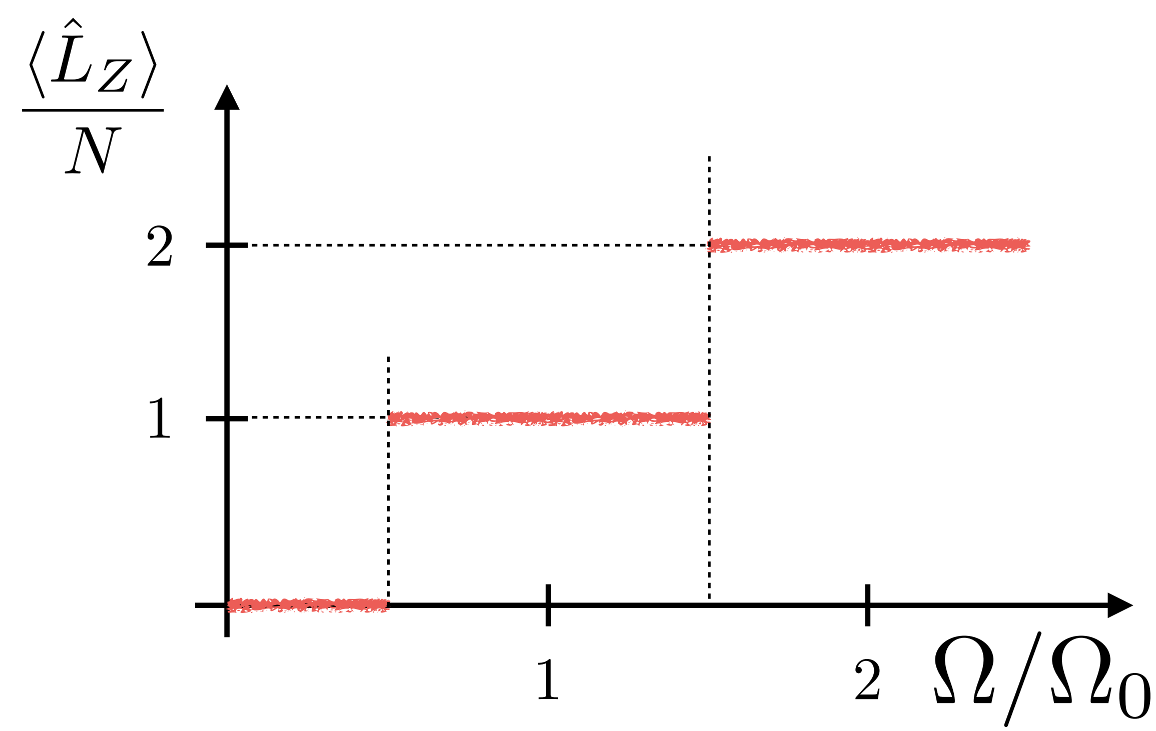
Particles repel each other



Energy $\quad E(\Omega) = \frac{\hbar^2}{2mR^2} N l^2 - N\hbar\,\Omega\, l + E^{+}_{int}$

Angular momentum per particle $\quad \frac{\langle L_z \rangle}{N} = \hbar l$

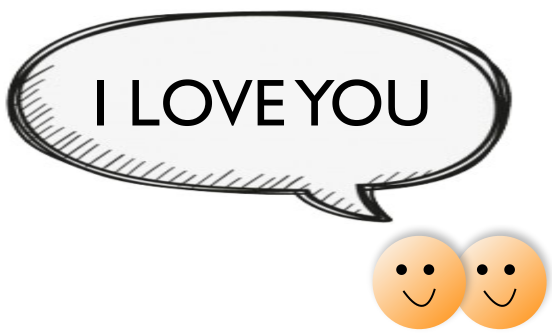
Periodicity of persistent current



N particles with delta interaction: Lieb Liniger

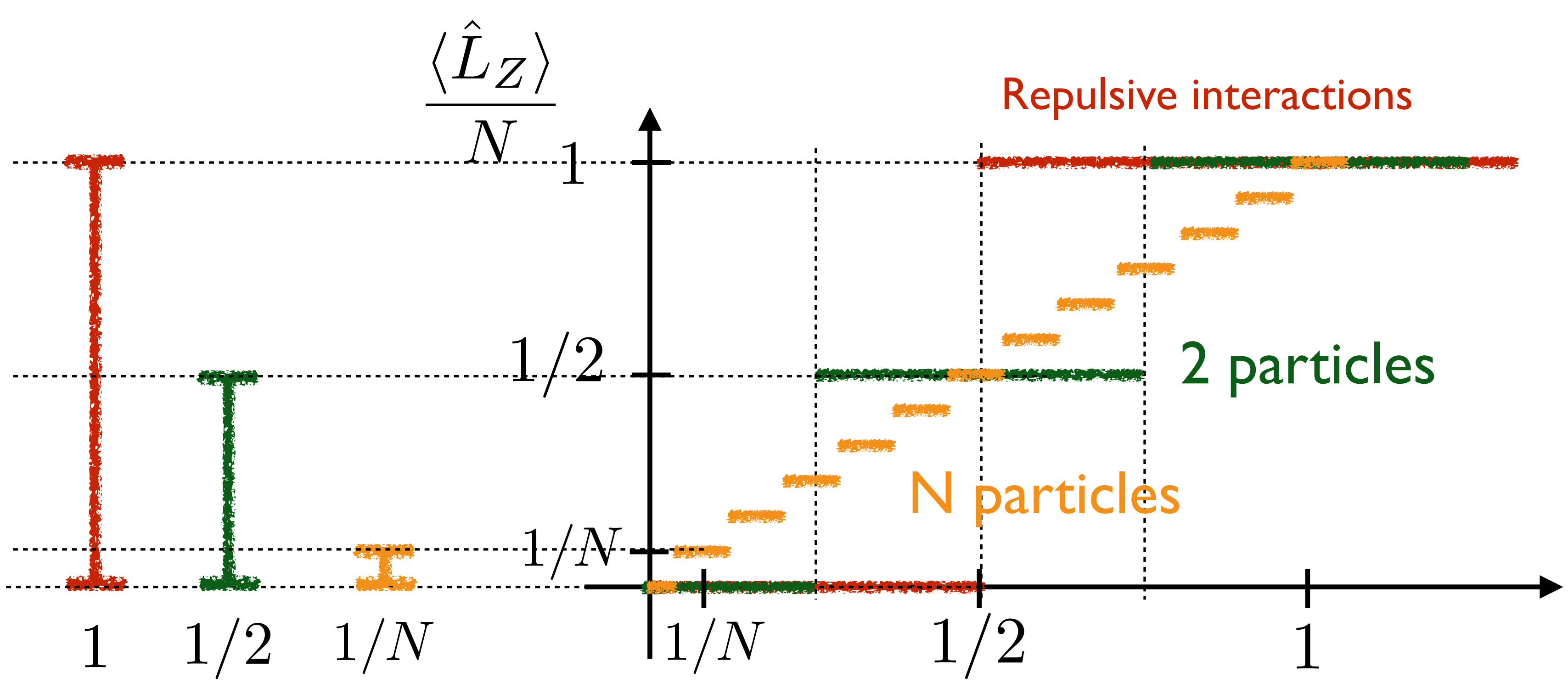
$g < 0$ attractive interactions

Particles cluster together generating "molecules"



Energy $$E(\Omega) = \frac{\hbar^2}{2mR^2}\frac{l^2}{N} - \hbar\,\Omega\,l + E^{-}_{int}$$

Angular momentum per particle $$\frac{\langle L_z \rangle}{N} = \frac{\hbar l}{N}$$



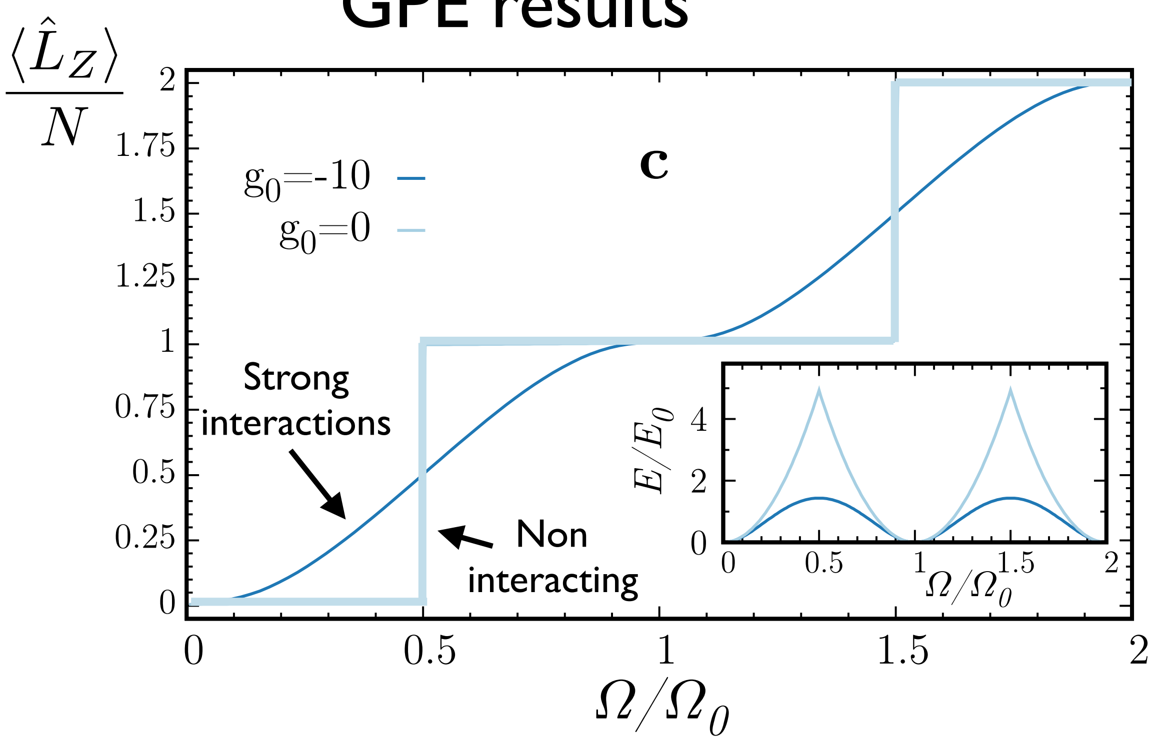
The Angular Momentum is ***FRACTIONALIZED***: it varies in unit of 1/N

Solitons in the GP equation

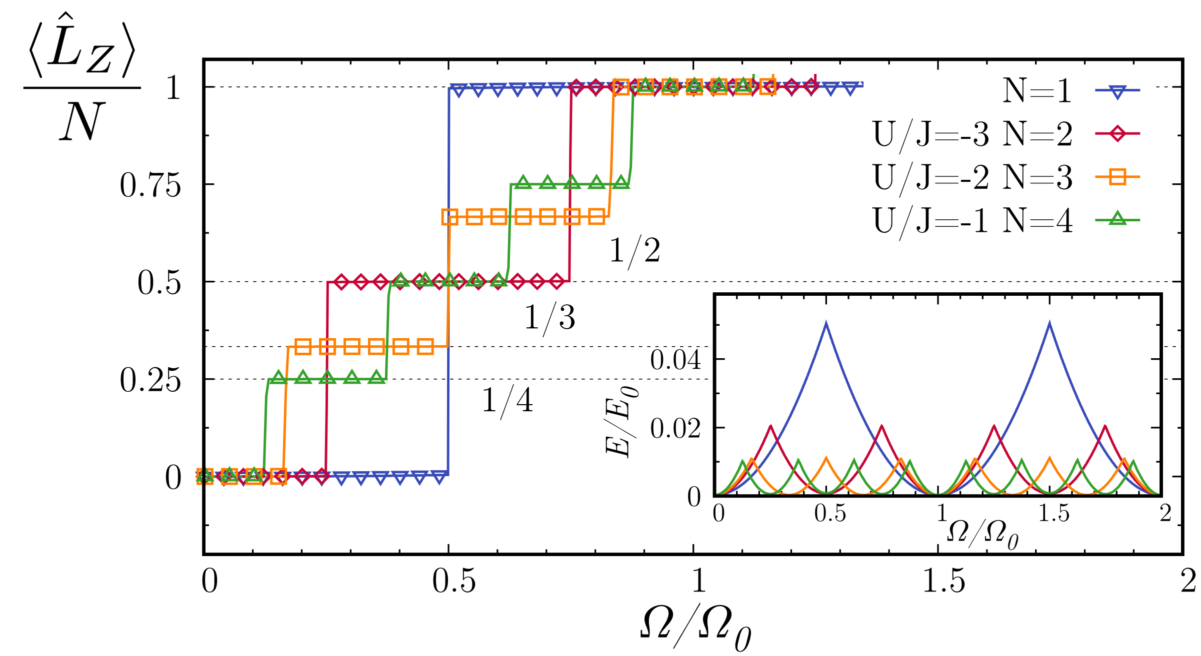
Mean field Gross-Pitaevskii description.

It's a good approximation for weak interactions!

GPE results



LIEB LINIGER results



The FRACTIONALIZATION is a ***pure quantum effect***
and it completely disappears in GPE analysis!!!

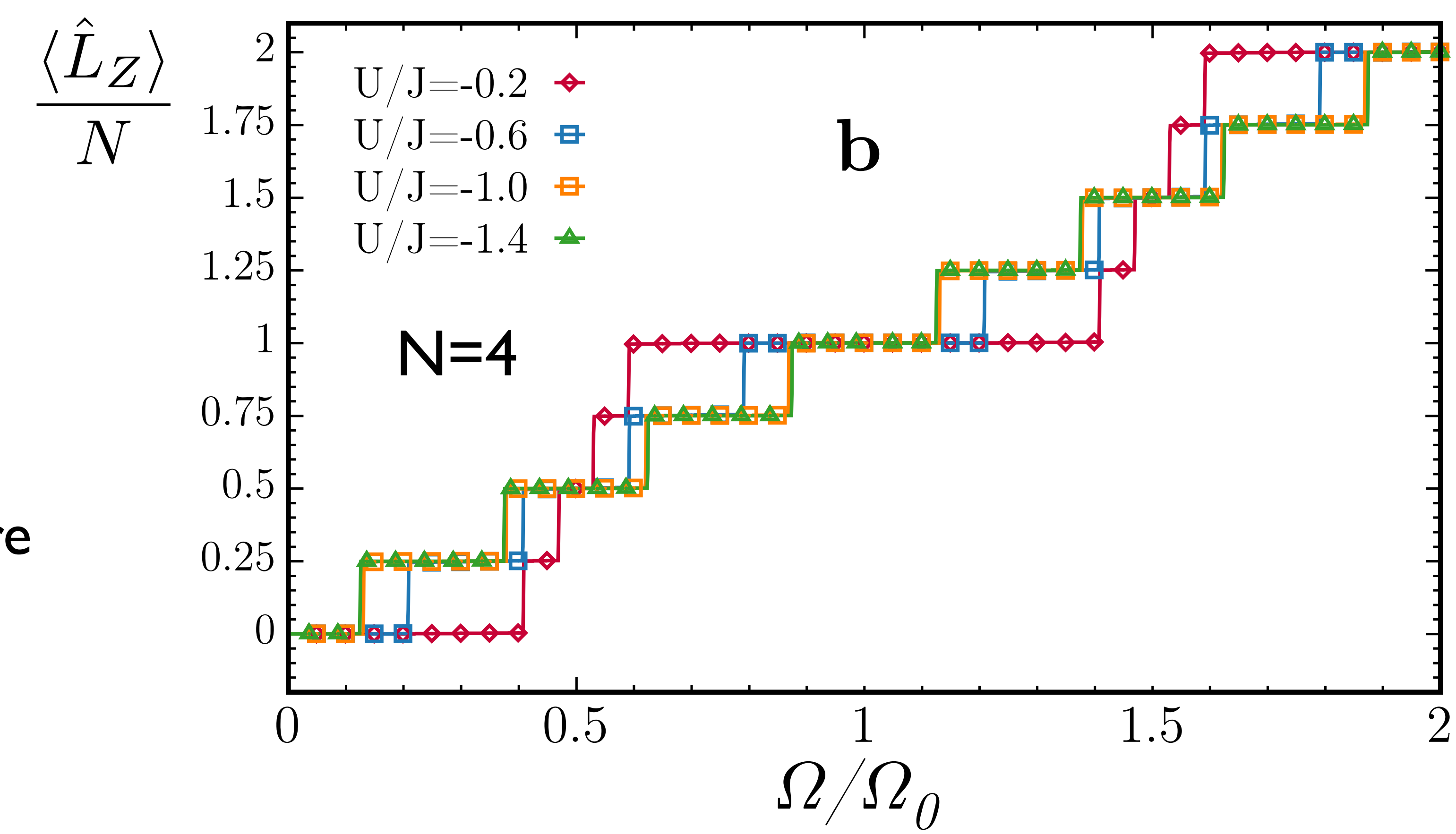
Fractionalization in the Bose-Hubbard Model

NON INTEGRABLE MODEL

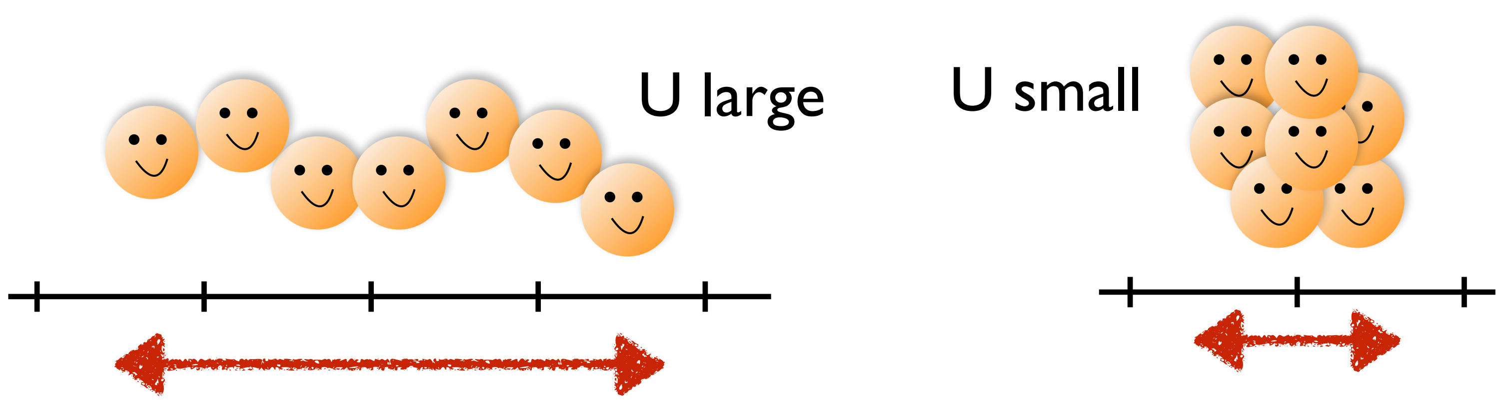
$$\mathcal{H} = -\sum_{l=1}^{L}\left(Je^{-2\pi i\Omega/L}\, b^{\dagger}_{l+1}b_l + h.c. + \frac{U}{2}n_j(n_j-1)\right)$$

… the periodicity depends on the value of the interaction!

For large interactions we restore the 1/N periodicity!!



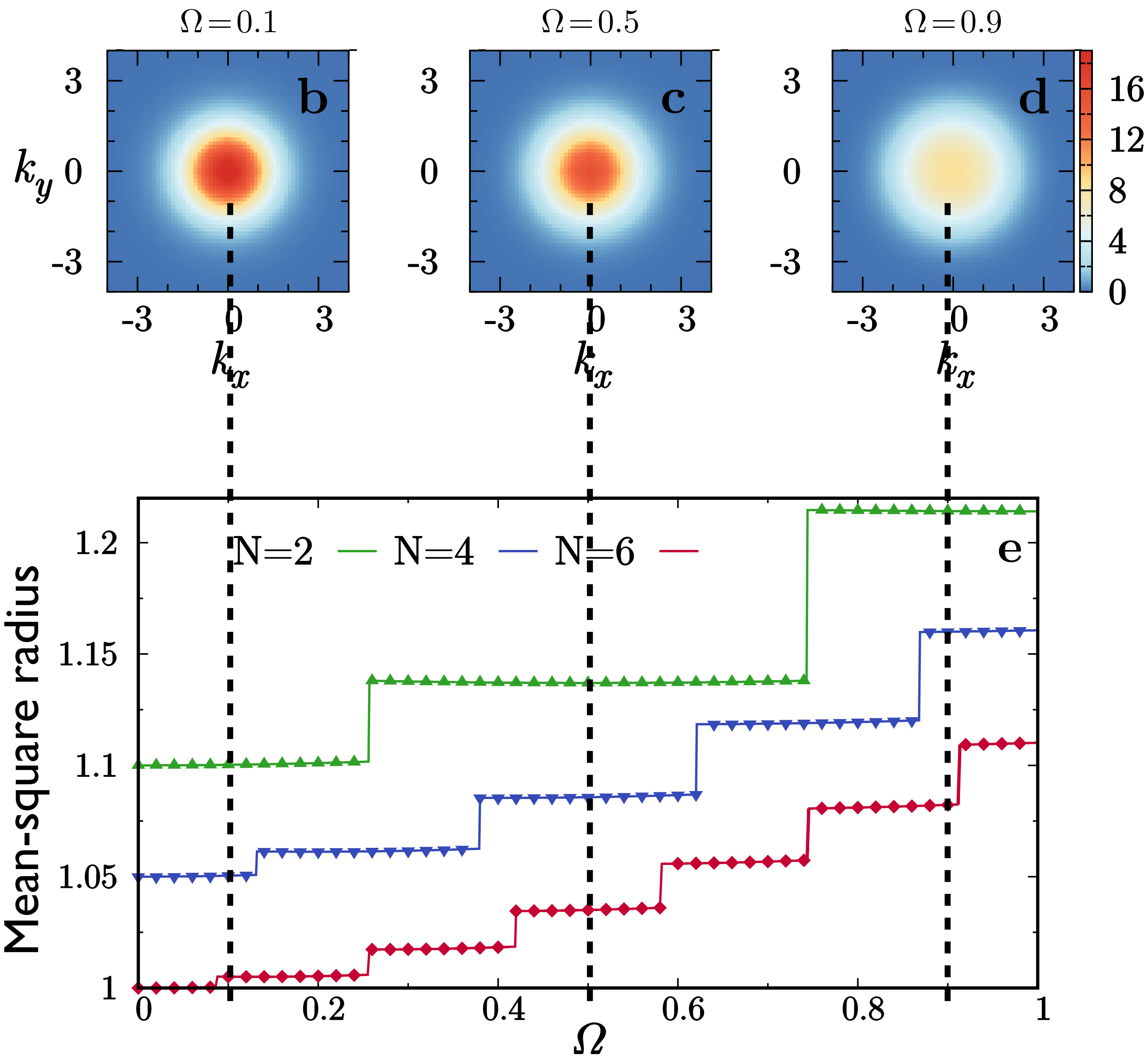
For large interactions, particles cluster together over a length comparable to the lattice spacing



Time of Flight (TOF)

Momentum distribution

$$n(\vec{k}) = |w(\vec{k})|^2 \sum_{j,l} e^{i\vec{k}\cdot(\vec{x}_j - \vec{x}_l)} \langle b_j^\dagger b_l \rangle$$



Each step in the mean-square radius corresponds to a step in L_z

TOF allows us to measure the angular momentum!!

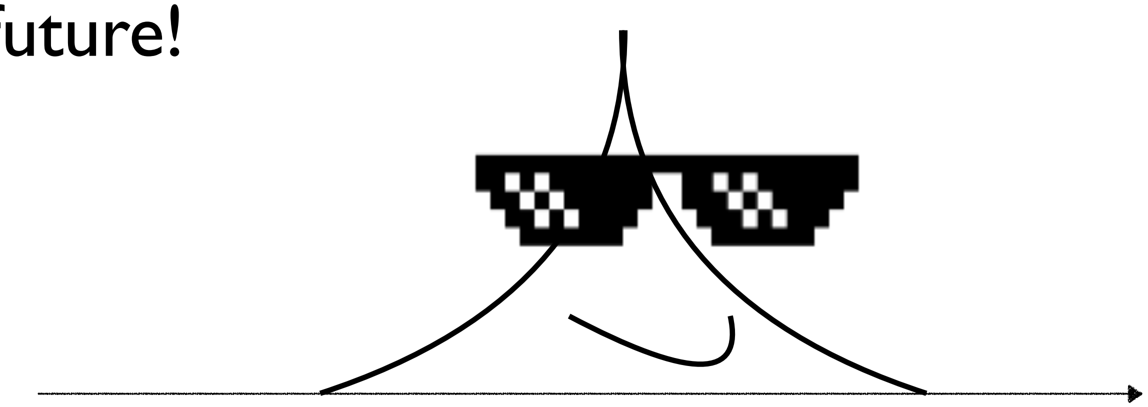
Quantum solitons behave better?

Can they outperform classical systems?

$$\frac{1}{N} \quad \text{vs} \quad \frac{1}{\sqrt{N}}$$

Thank you for your attention!

Other solitonic news are going to arrive in the next future!



Who am I?

PostDoc in LPMMC - Grenoble, FRANCE

Main Subjects of research: One dimensional strongly correlated systems

Spin chains with long range interactions → *PRL 120, 050401 (2018)*
PRB 95, 245111 (2017)

Disordered interacting Fermions → *SciPost 1, 010 (2016)*
arxiv: 1810.09779

Systems of attractive Bosons → *PRA 90, 043606 (2014)*
arxiv: 1804.10133
arxiv: 1901.09398

Numerical Simulations: Density Matrix Renormalization Group
Exact Diagonalization

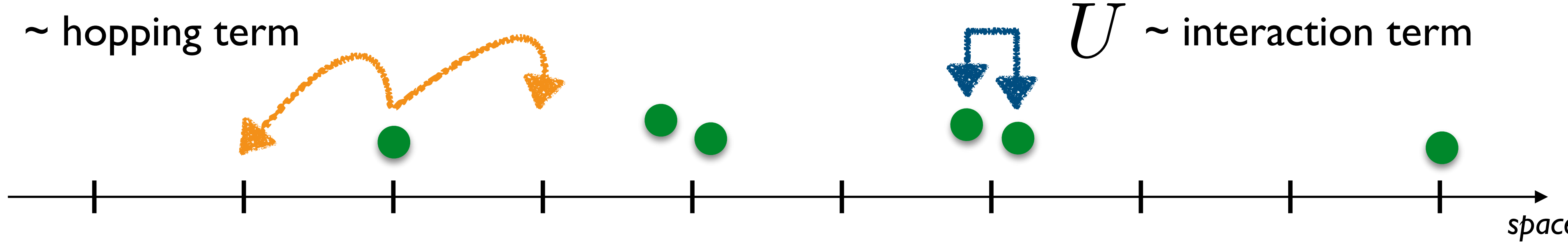
Bose-Hubbard Model $\longrightarrow$ attractive Bosons in a *lattice*.

$$\hat{H}(U) = -J\sum_j \left(b_j^\dagger b_{j+1} + b_{j+1}^\dagger b_j\right) - \frac{|U|}{2}\sum_j n_j\left(n_j - 1\right)$$

attractive interactions

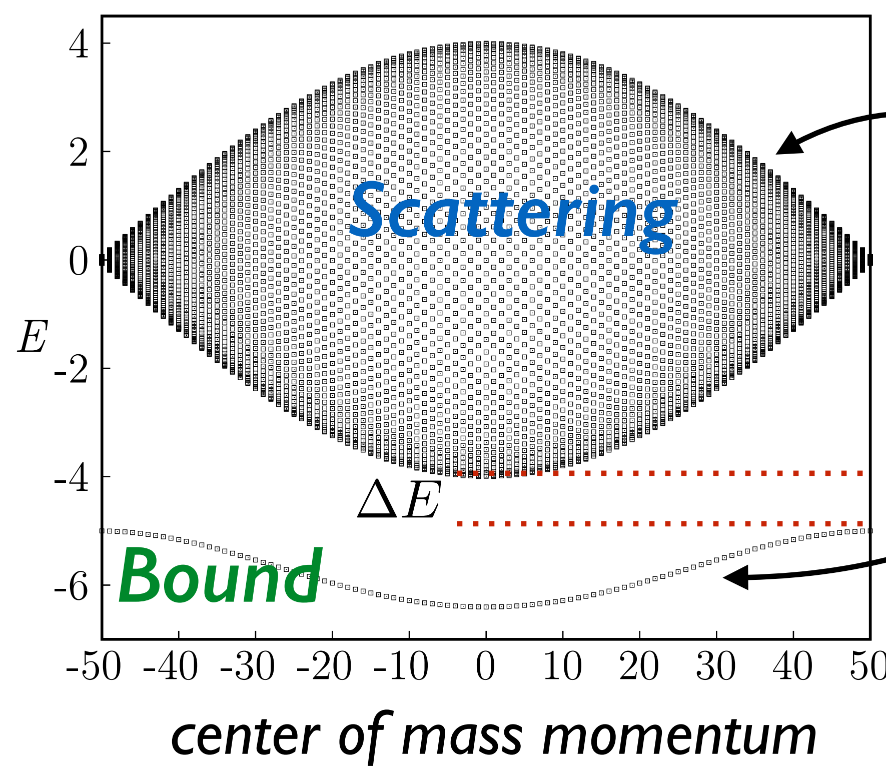
J ~ hopping term

U ~ interaction term



Exact solution: center of mass & relative frame

2 particle problem

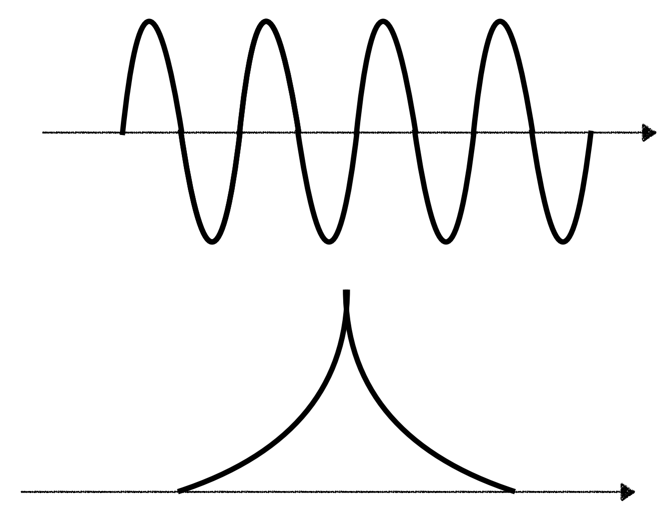


Scattering States

Bound State

L bound state

One bound state each lattice site



$$\boxed{\Delta E \neq 0 \ \textit{ if } \ |U| > 4}$$

Energy GAP between bands

Thermalization in presence of Solitons

LONG TIME DYNAMICS: THERMALIZATION

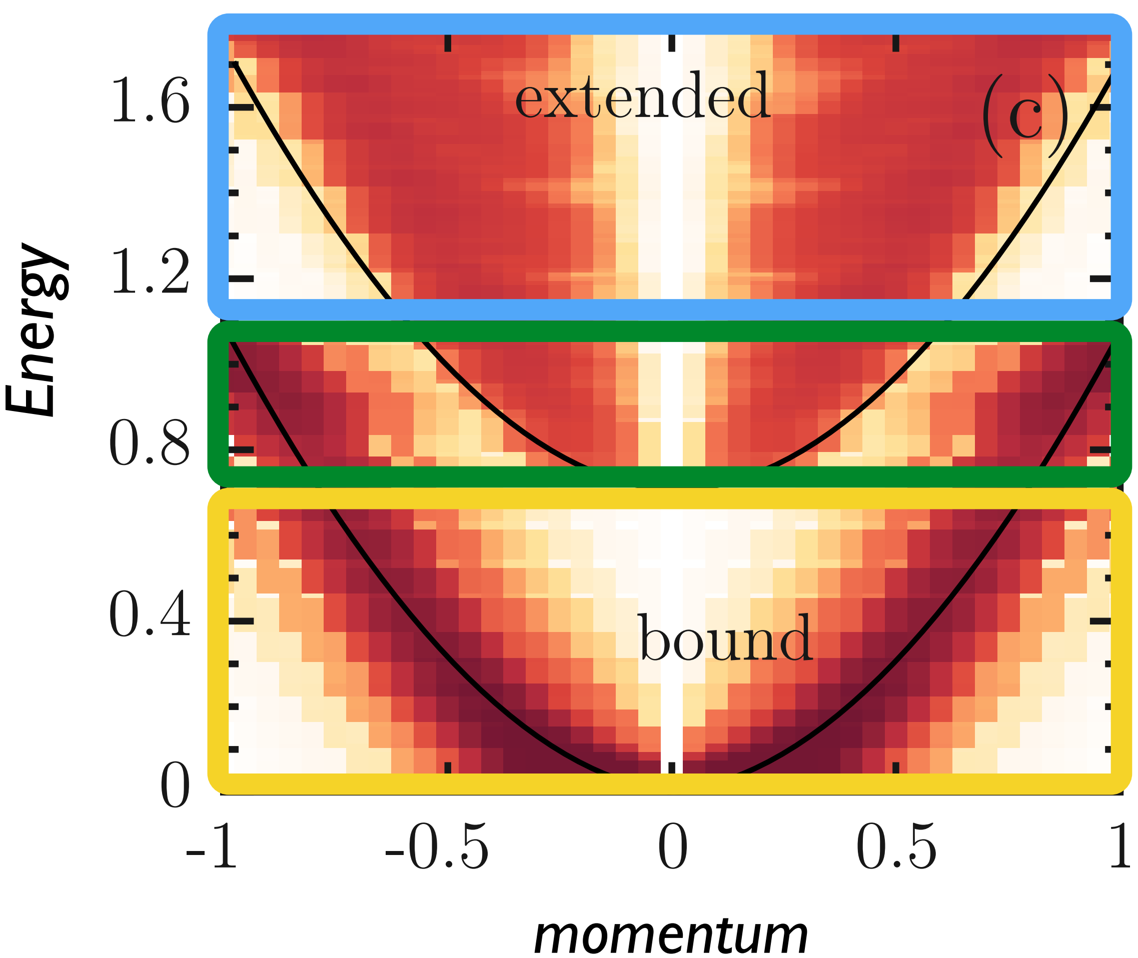
Ergodicity

(simple version)

→ Long time dynamics is determined by some MACROSCOPIC quantities of the system (temperature, number of particles….)!

→ During the dynamics the system looses information on the microscopic details of the initial state!

For interactions $0<U<U_c$ we have three regions in the spectrum:



ONLY EXTENDED

BOTH EXTENDED AND BOUND

ONLY BOUND

This regime is "mesoscopic": it disappears increasing the number of particles!

Thermalization in presence of Solitons

initial state

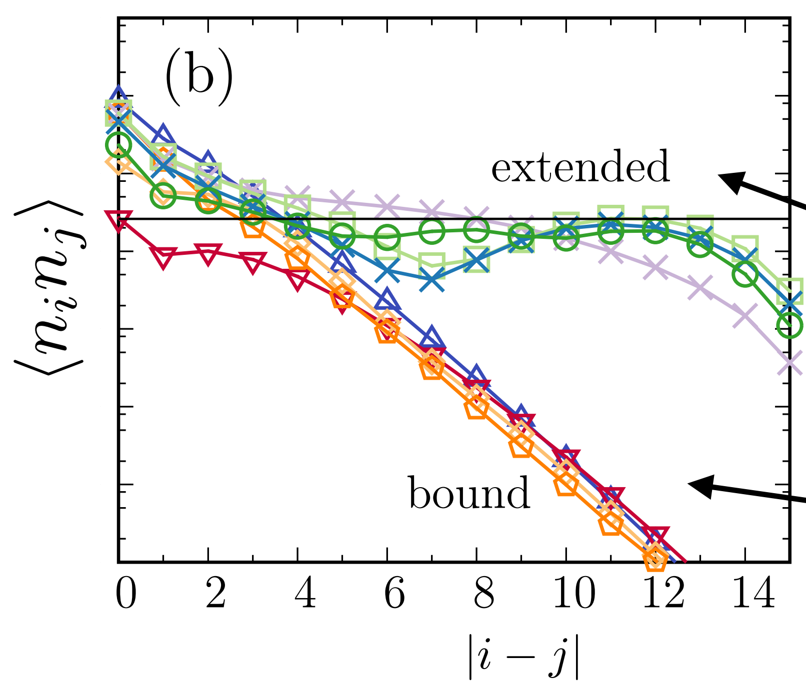
$$|\psi_0\rangle = \sum_S P_S\,|S\rangle + \sum_B P_B\,|B\rangle$$

time evolution

$$|\psi(t)\rangle = \sum_S e^{-iE_S t} P_S\,|S\rangle + \sum_B e^{-iE_B t} P_B\,|B\rangle$$

time evolution of density density correlations!

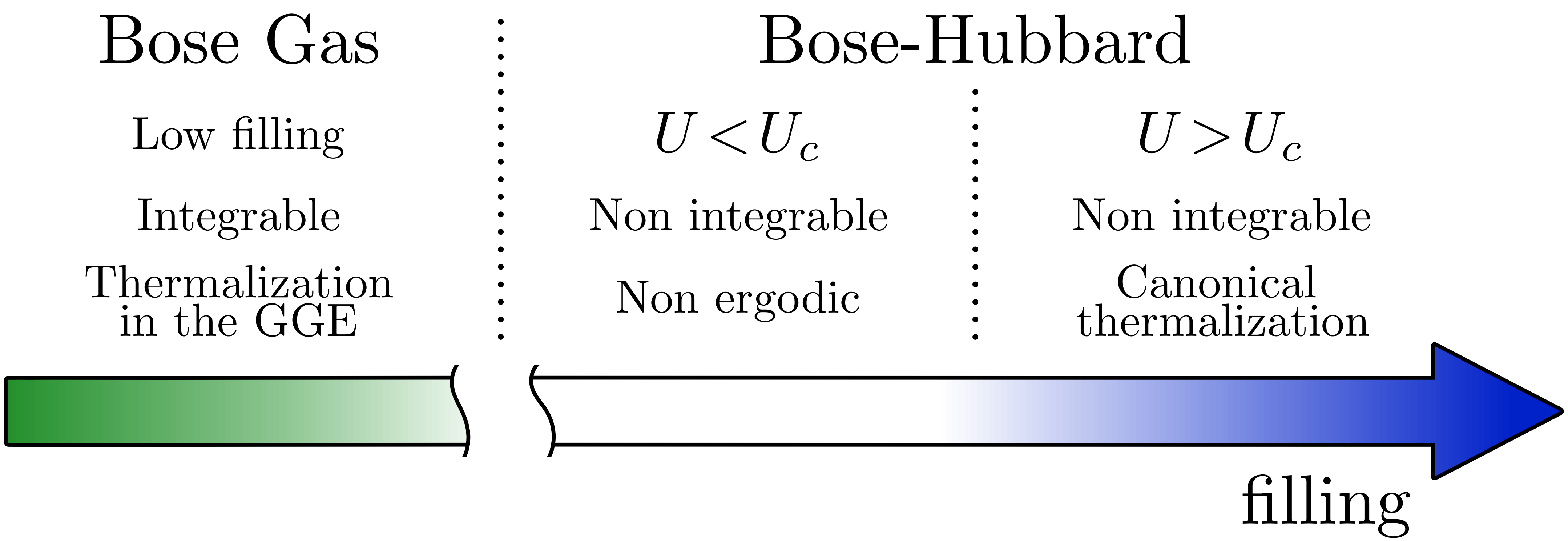
$$\begin{aligned}
C(r,t) &= \langle n_0(t)\, n_r(t)\rangle = \langle \psi(t)|n_0\, n_r|\psi(t)\rangle \\
&\cancel{= 2\sum_{S,B} P_S P_B e^{i(E_S-E_B)t}\langle S|n_0 n_r|B\rangle} \\
&\cancel{+ \sum_{S\neq S'} P_S P_{S'} e^{i(E_S-E_{S'})t}\langle S|n_0 n_r|S'\rangle} \\
&\cancel{+ \sum_{B\neq B'} P_B P_{B'} e^{i(E_B-E_{B'})t}\langle B|n_0 n_r|B'\rangle} \\
&+ \sum_S |P_S|^2\,\langle S|n_0 n_r|S\rangle \\
&+ \sum_B |P_B|^2\,\langle B|n_0 n_r|B\rangle
\end{aligned}$$



If we can initialize the system at the same energy but with different PB and PS we will have really different long time evolutions!!

On going research

Mesoscopic violation of ergodicity



DANGER - WORK IN PROGRESS!