The 2N+1 body problem: an impurity immersed in a spin ½ fermionic superfluid

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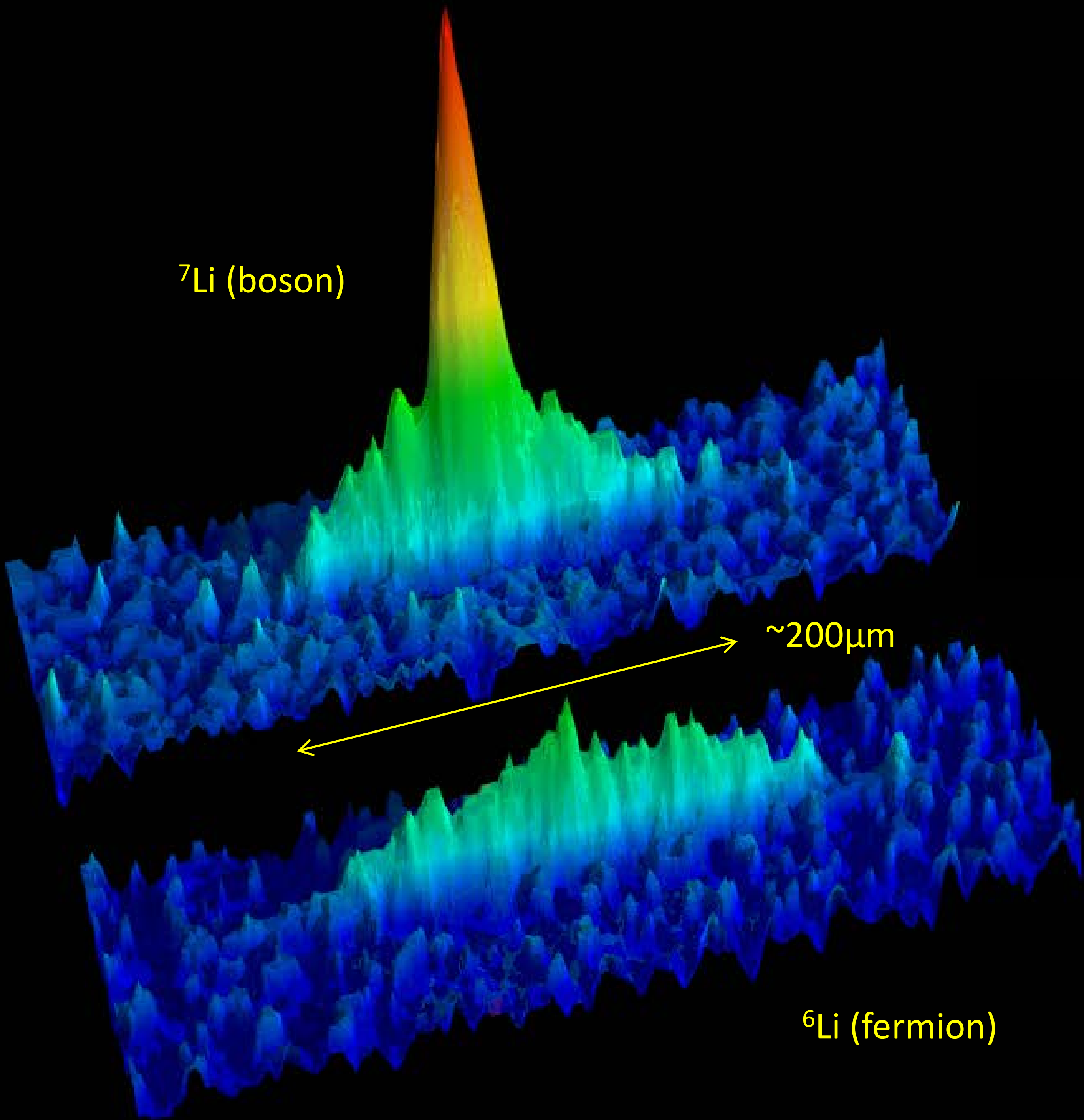
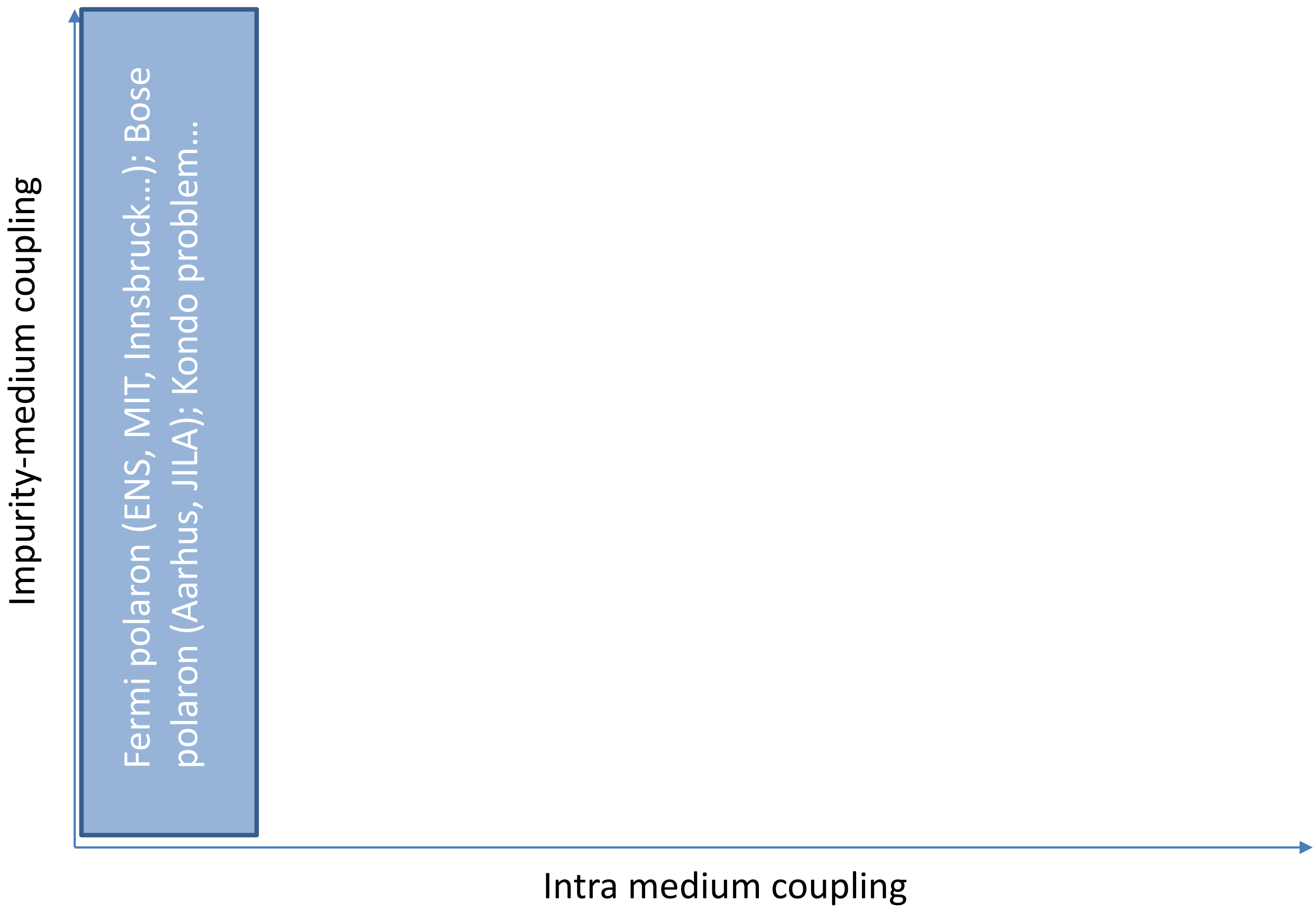
Also: I. Ferrier-Barbut, S. Laurent, M. Delehaye, X. Leyronas, F. Werner & Y. Castin

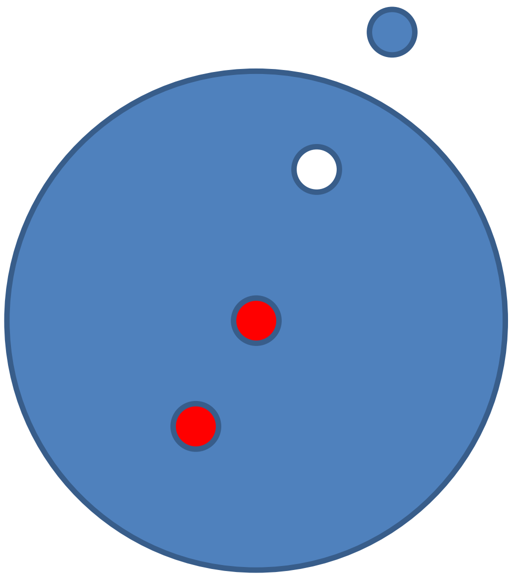

^{7}Li (boson)
~200µm
^{6}Li (fermion)

POLARON PHASE DIAGRAM



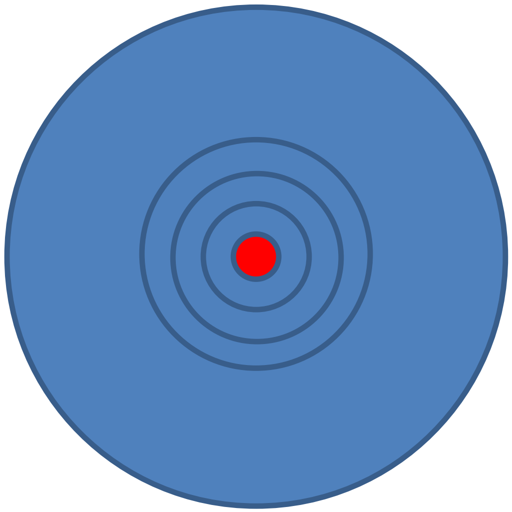
Fermi and Bose polarons

Fermi polaron: impurity immersed in an ideal gas of spin polarized fermions



Stable for any $k_F a$ (Moser & Seringer, 2016)

Bose polaron: impurity immersed in an weakly interacting Bose-Einstein condensate



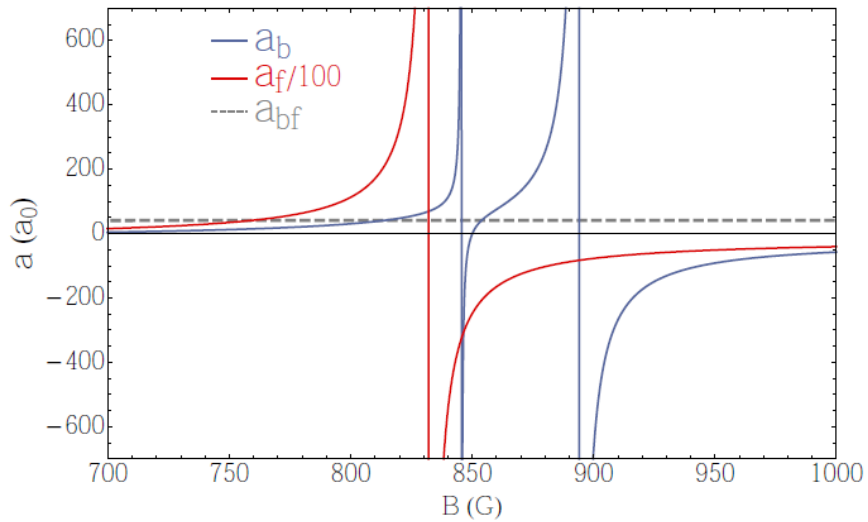
Efimov effect: polaron unstable for large na^3

See Landau, Frölich, Prokof'ev, Svistunov, Bruun, Parish, Levinsen,Massignan,Recati, Giorgini, Lobo…

Polaron phase diagram

Impurity-medium coupling

Fermi polaron (ENS, MIT, Innsbruck…); Bose polaron (Aarhus, JILA); Kondo problem…



Impurity weakly coupled to a strongly correlated medium (ENS, Seattle, Tokyo, Shanghai…)

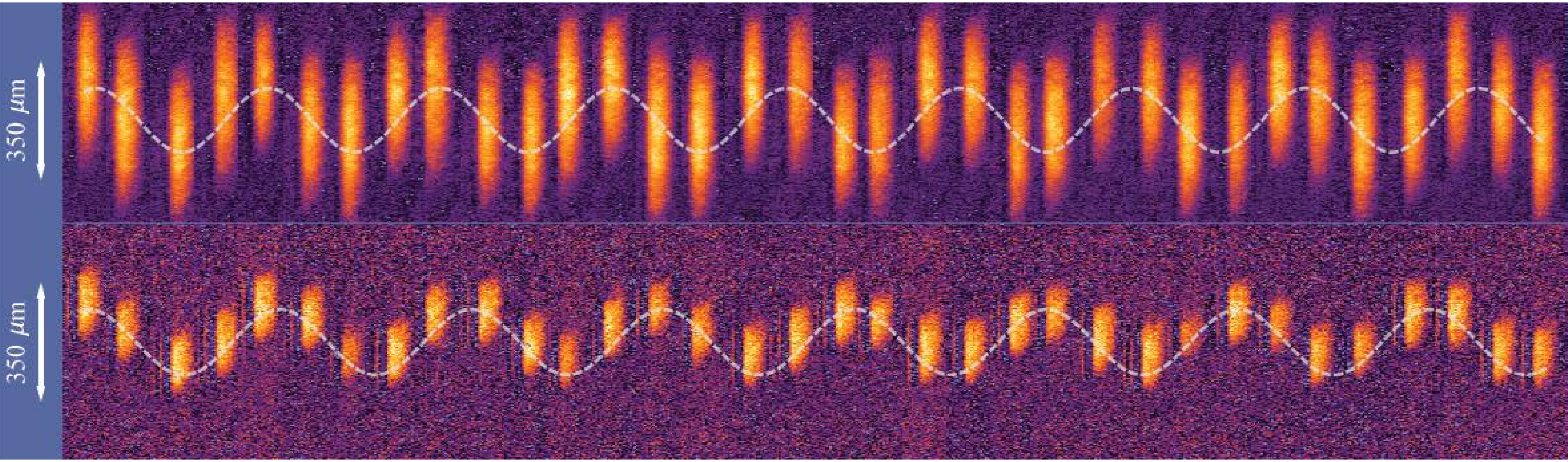
Intra medium coupling

Dynamics of the impurity

I. Ferrier-Barbut *et al.*, Science **345**, 1035 (2014)

In collaboration with Y. Castin (ENS), S. Stringari (Trento), A. Recati (Trento), I. Danaila (U. Rouen), P. Parnaudeau (U. Poitier)

DYNAMICS OF THE MIXTURES



$$\omega_6 = 2\pi \times 16.80(2)\text{Hz} \qquad \tilde{\omega}_6 = 2\pi \times 16.80(1)\text{Hz}$$

$$\omega_7 = 2\pi \times 15.27(2)\text{Hz} \qquad \tilde{\omega}_7 = 2\pi \times 15.00(1)\text{Hz}$$

Single Superfluid
Ratio = $(7/6)^{1/2} = (m_7/m_6)^{1/2}$

Coupled Superfluids

See also C. Hammer *et al* Phys. Rev. Lett. **106**, 065302 (2011) for boson-boson superfluid counterflow

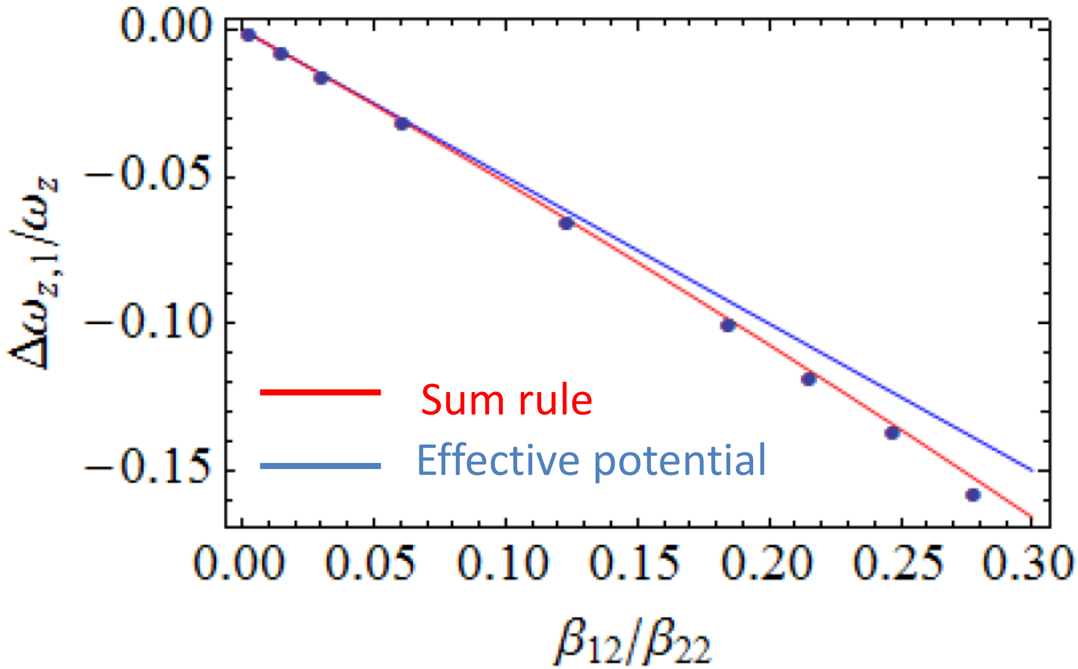
FREQUENCY SHIFT

$$V_{\text{eff},7} = V(\mathbf{r}) + g_{67} n_6(\mu_6(\mathbf{r})) \qquad \mu_6(\mathbf{r}) = \mu_6^0 - V(\mathbf{r})$$ (Local Density Approximation)

$$\approx g_{67} n_6(\mu_6^0) + V(\mathbf{r}) \left(1 - g_{67} \frac{\partial n_6}{\partial \mu_6} \right)$$

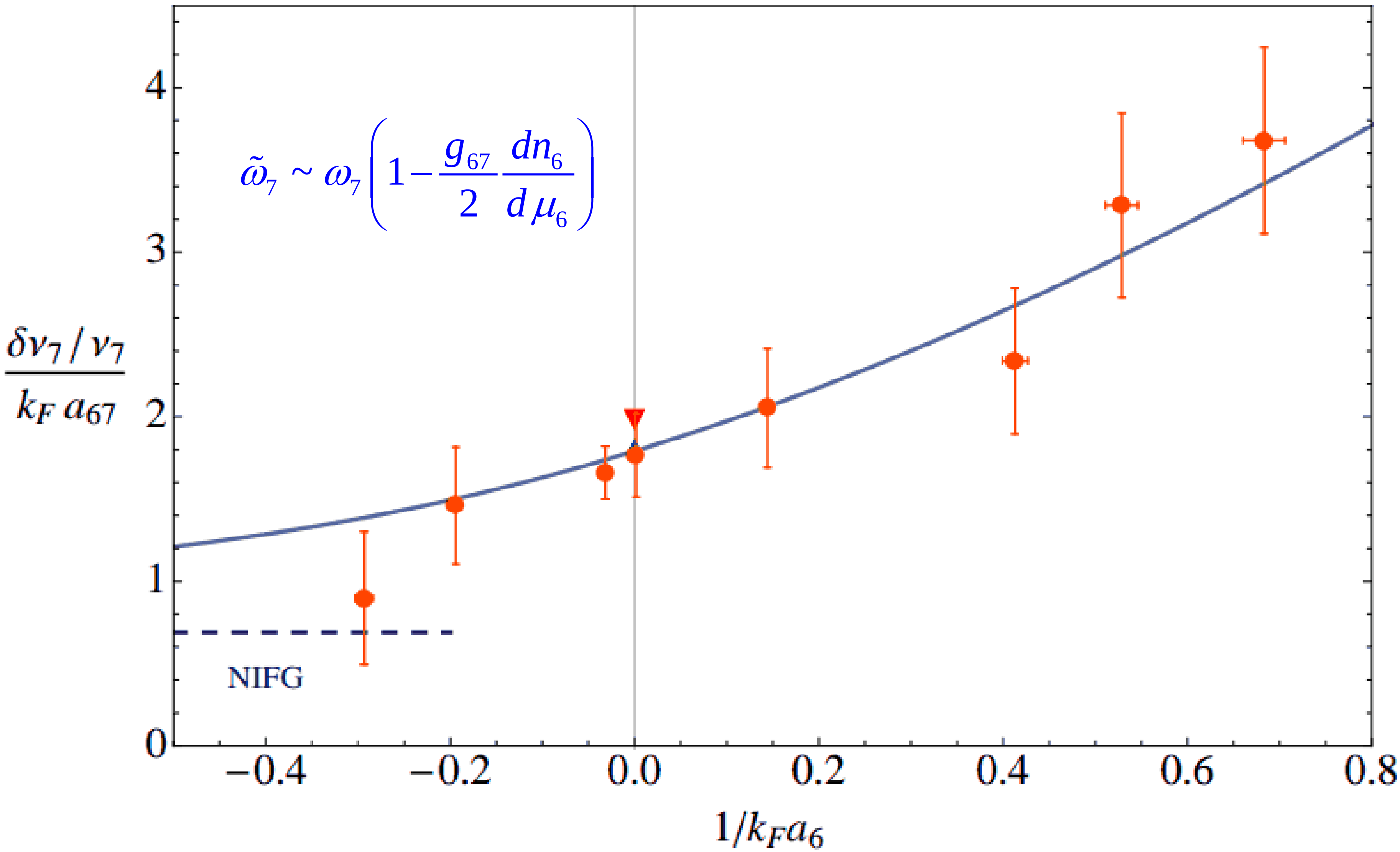
Harmonic trap: $$\frac{\Delta\omega_7}{\omega_7} \approx -\frac{g_{67}}{2} \frac{\partial n_6}{\partial \mu_6}$$

Benchmark: Numerical solution of GPE (with P. Parnaudeau, I. Danaila, arXiv:1904.07040)



Oscillation Frequency of the BEC

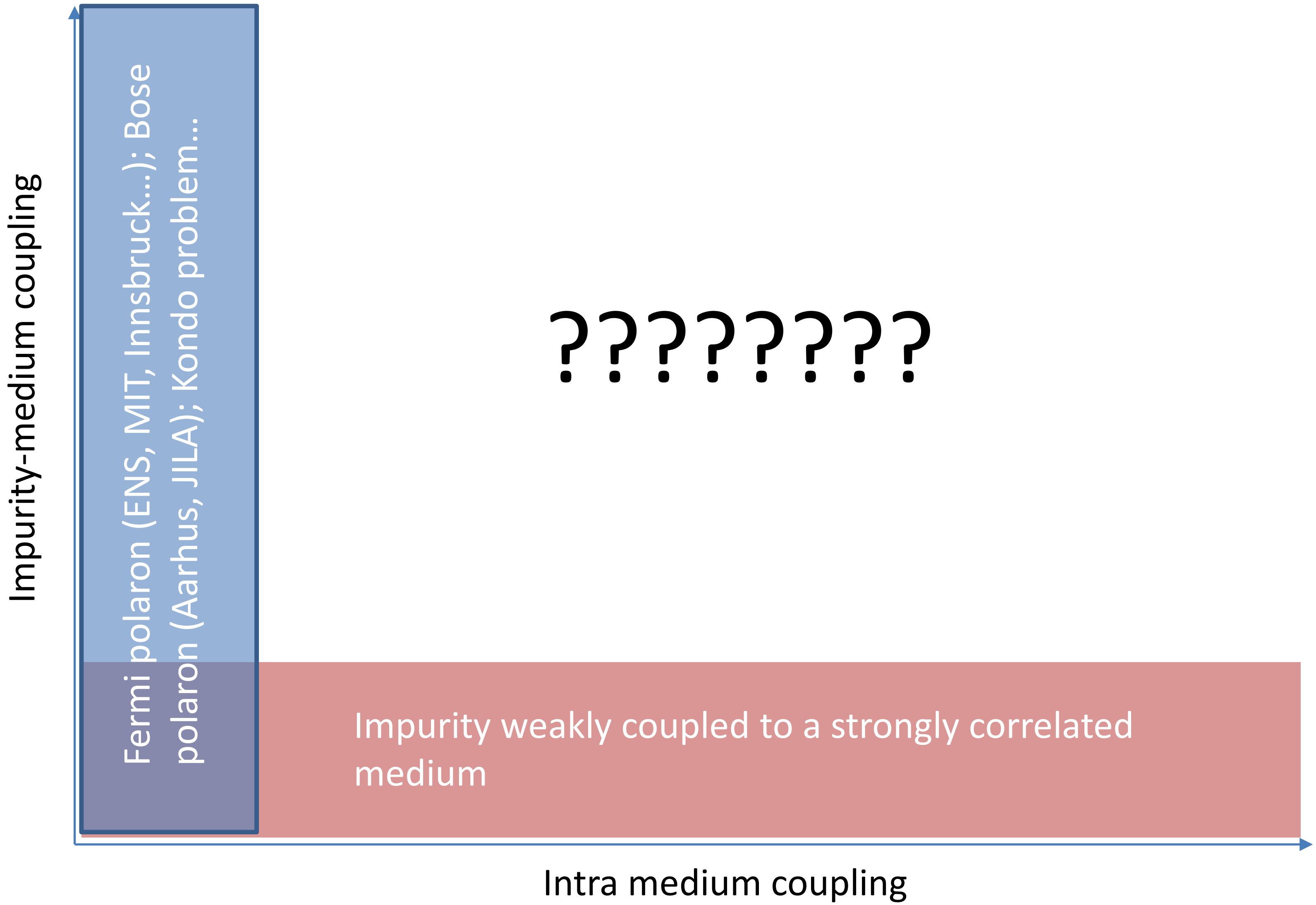
Weak frequency shift (few percents) of the bosons due to the fermions



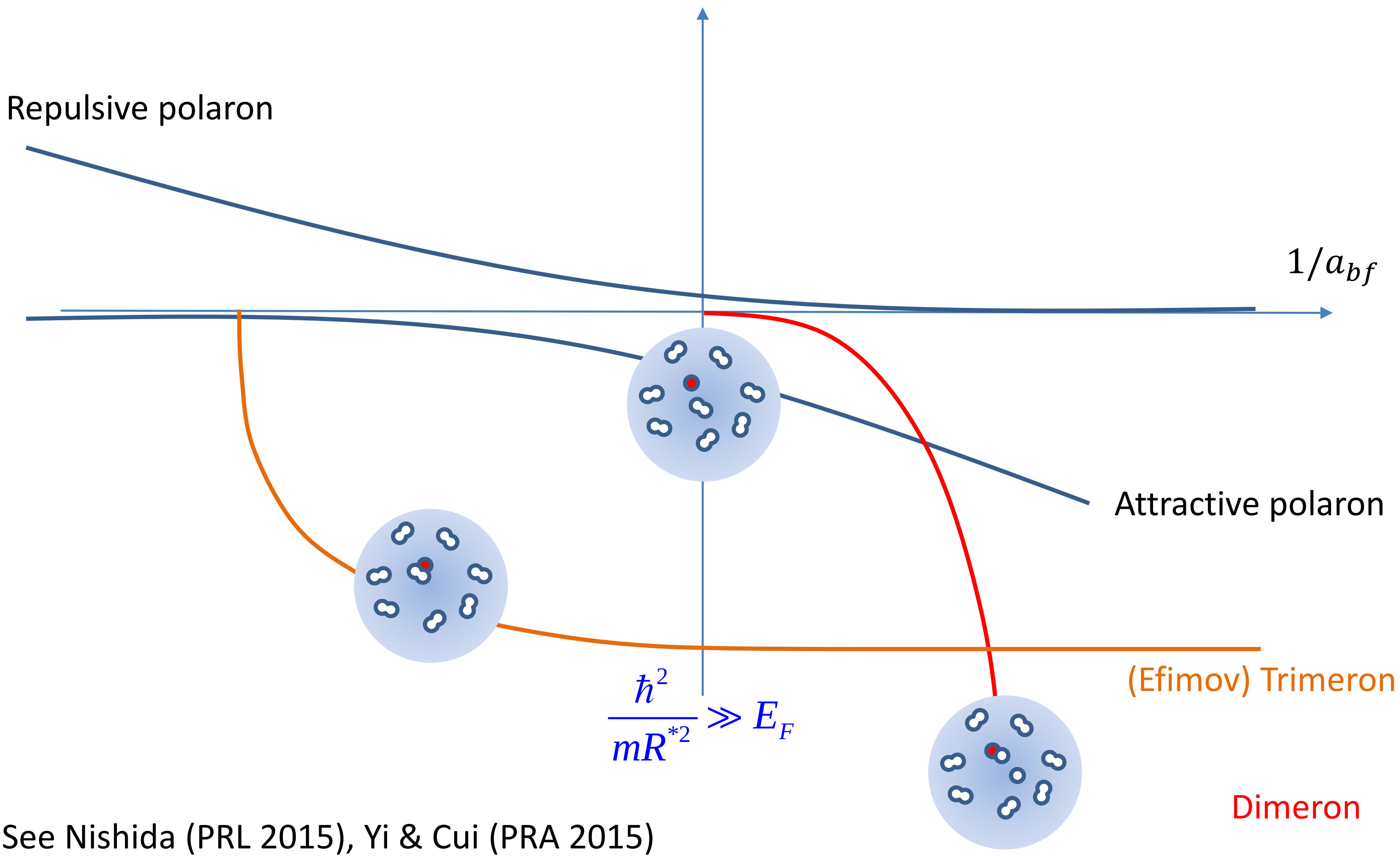
Beyond weak coupling

M. Pierce *et al.* arXiv:1903.01108

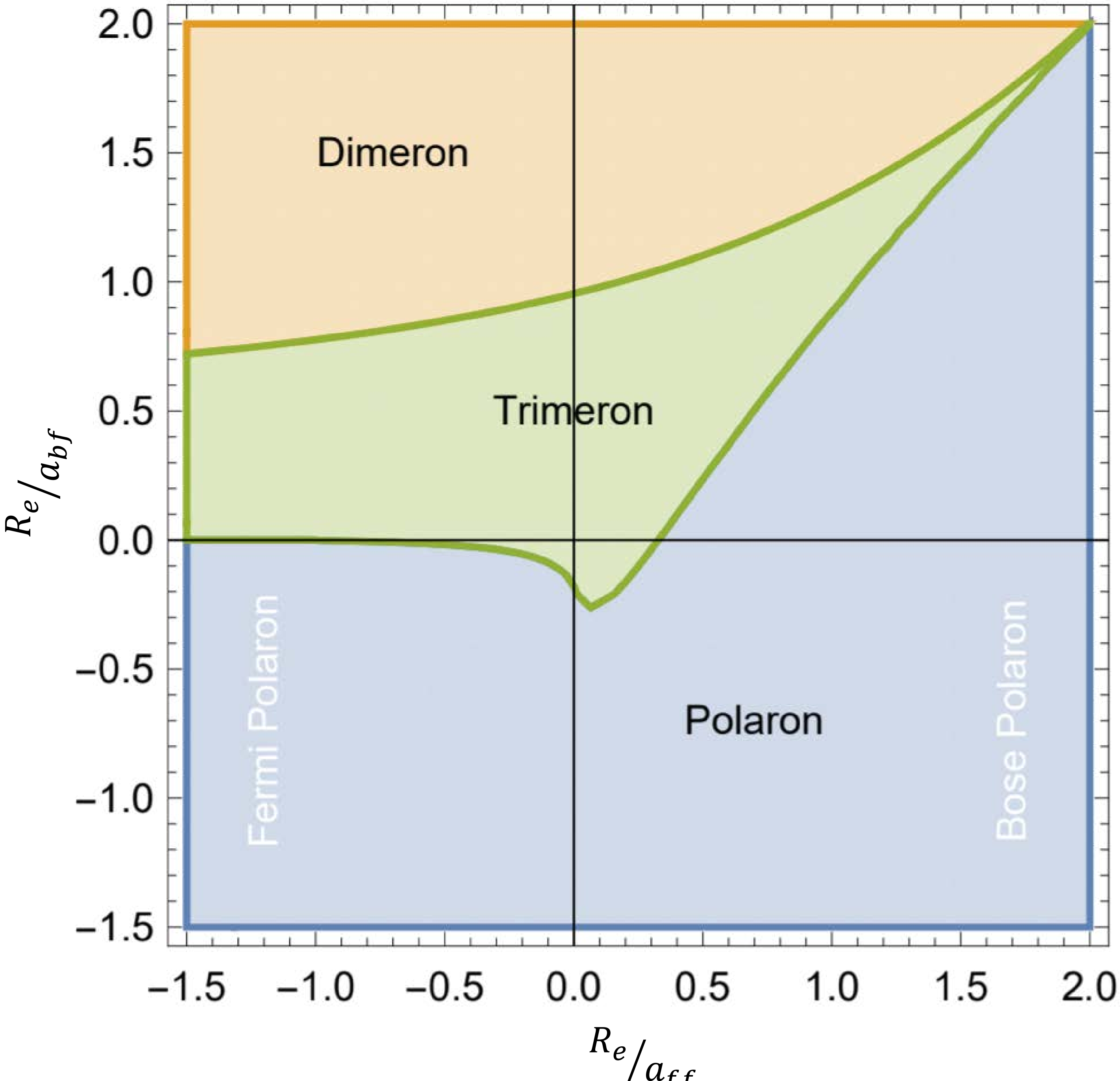
POLARON PHASE DIAGRAM



Polaron, dimeron, trimeron



See Nishida (PRL 2015), Yi & Cui (PRA 2015)


Dimeron
Trimeron
Polaron
Fermi Polaron
Bose Polaron
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
R_e/a_{bf}
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
R_e/a_{ff}

Energy of the polaron: beyond mean-field contributions

Contact interaction $V(r) = g'_{BF}\,\delta(\mathbf{r})$

$$g'_{BF} = g_{BF} + \frac{g_{BF}^2}{\Omega}\sum_{q<\Lambda}\frac{1}{2\varepsilon_q} + \ldots$$

Second-order perturbation theory

$$\Delta E = g_{BF} n_F + \frac{g_{BF}^2 n_F}{\Omega}\sum_q \left[\frac{1}{2\varepsilon_q} - \chi(q,\varepsilon_q)\right]$$

UV (log) divergent!

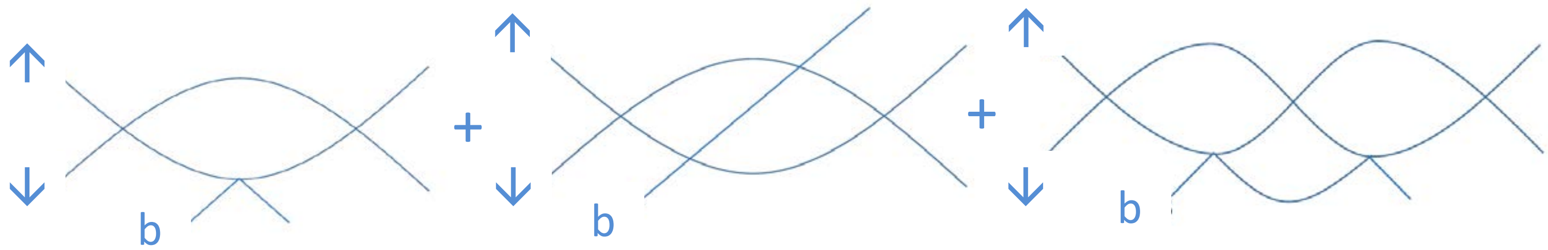
(see for instance BCS)

Dynamical compressibility of the superfluid

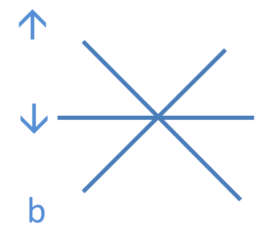
Back to the three-body problem

Same divergence in the 3-body scattering (≈Wu beyond mean-field correction for BECs)

$\Gamma =$

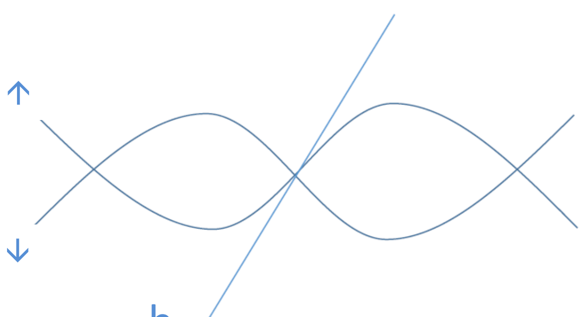


Cure the divergence by adding an explicit 3-body contact interaction to regularize the 3-body scattering amplitude (Braaten & Nieto '99)



$= g_3(\Lambda)$

$\Gamma_{3b} =$



$$g_3(\Lambda)\left(\frac{1}{\Omega}\sum_{k<\Lambda}\frac{1}{2\varepsilon_k}\right)^2 = -g_{BF}^2\,\kappa\left(\frac{m_B}{m_F}\right)\log(\Lambda R_{3b})$$

$$\kappa(1) = \frac{4}{9\sqrt{3}} + \frac{1}{3\pi} - \frac{\sqrt{3}}{2\pi^2}$$

Curing the polaron energy

Treat the three-body interaction perturbatively for the polaron

$$E = g_{\mathrm{BF}} n_{\mathrm{F}} \left[1 + k_{\mathrm{F}} a_{\mathrm{BF}} F\left(\frac{1}{k_{\mathrm{F}} a_{\mathrm{FF}}}\right) + \frac{a_{\mathrm{BF}} C_{\mathrm{f}}}{N} \kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}}\right) \ln\left(k_{\mathrm{F}} R_{3\mathrm{b}}\right) + \ldots \right]$$

$$F\left(\frac{1}{k_F a_{FF}}\right) = \frac{4\pi}{k_F}\left[\frac{\hbar^2}{\mu}\int_{q<\Lambda} \frac{d^3\mathbf{q}}{(2\pi)^3}\left(\frac{1}{2\varepsilon_q} - \chi(q,\varepsilon_q)\right) + \frac{C_{\mathrm{f}}}{N}\kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}}\right)\ln(\Lambda / k_{\mathrm{F}})\right]$$

Expression does not make any assumption on the properties of the many-body background

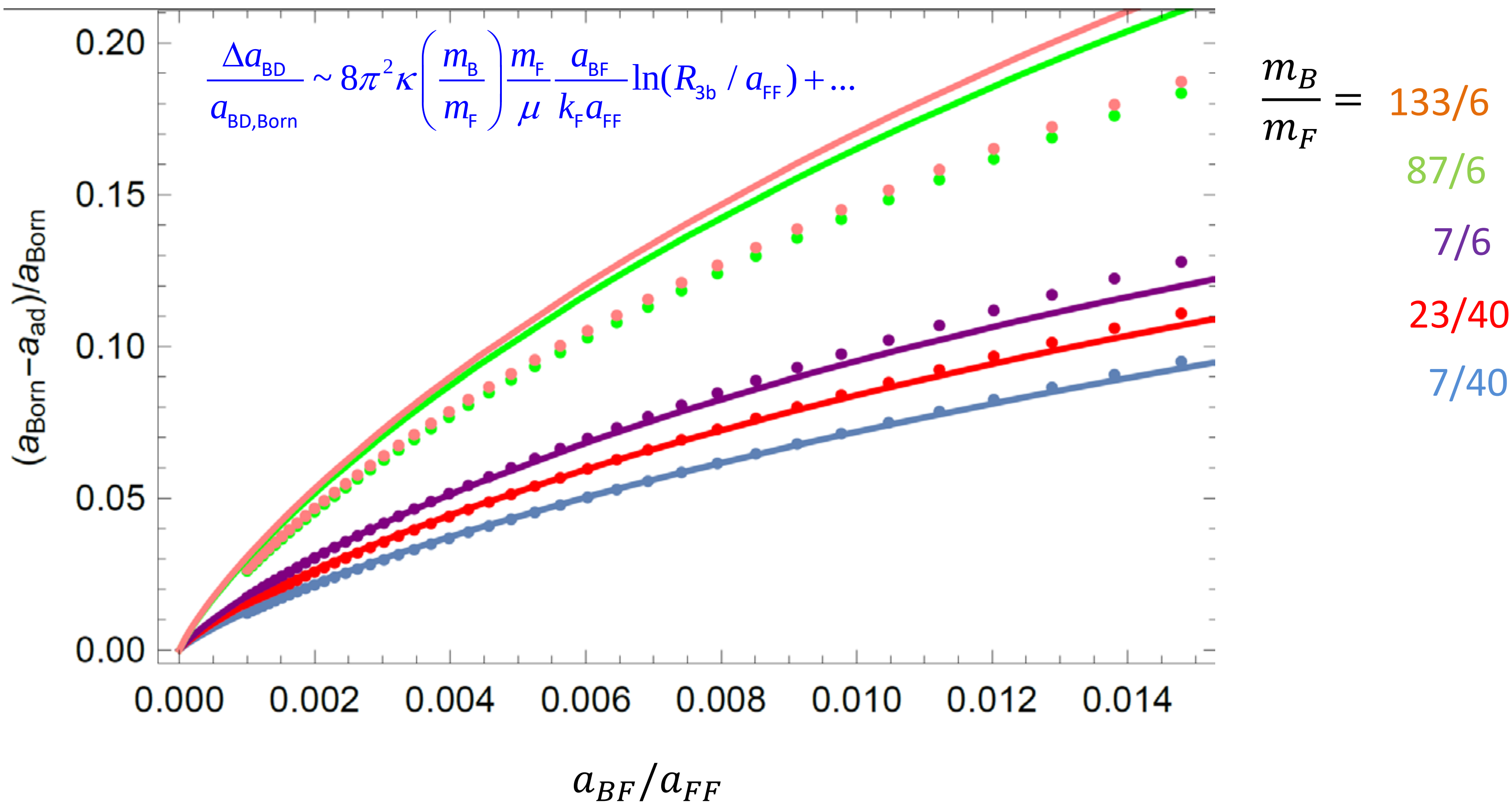
Asymptotic expressions

$a_{\mathrm{FF}} \to 0^- \Rightarrow$ Fermi polaron problem $\Rightarrow F(-\infty) = \dfrac{3}{4\pi}$

$a_{\mathrm{FF}} \to 0^+ \Rightarrow$ Bose polaron problem $\Rightarrow E = g_{\mathrm{BD}} n_{\mathrm{D}}$

$$\Rightarrow F(+\infty) \sim 8\pi^2 \kappa\left(\frac{m_{\mathrm{B}}}{m_{\mathrm{F}}}\right)\frac{m_{\mathrm{F}}}{\mu}\frac{\ln(k_{\mathrm{F}} a_{\mathrm{FF}})}{k_{\mathrm{F}} a_{\mathrm{FF}}}$$

Cross-check: impurity-dimer scattering length

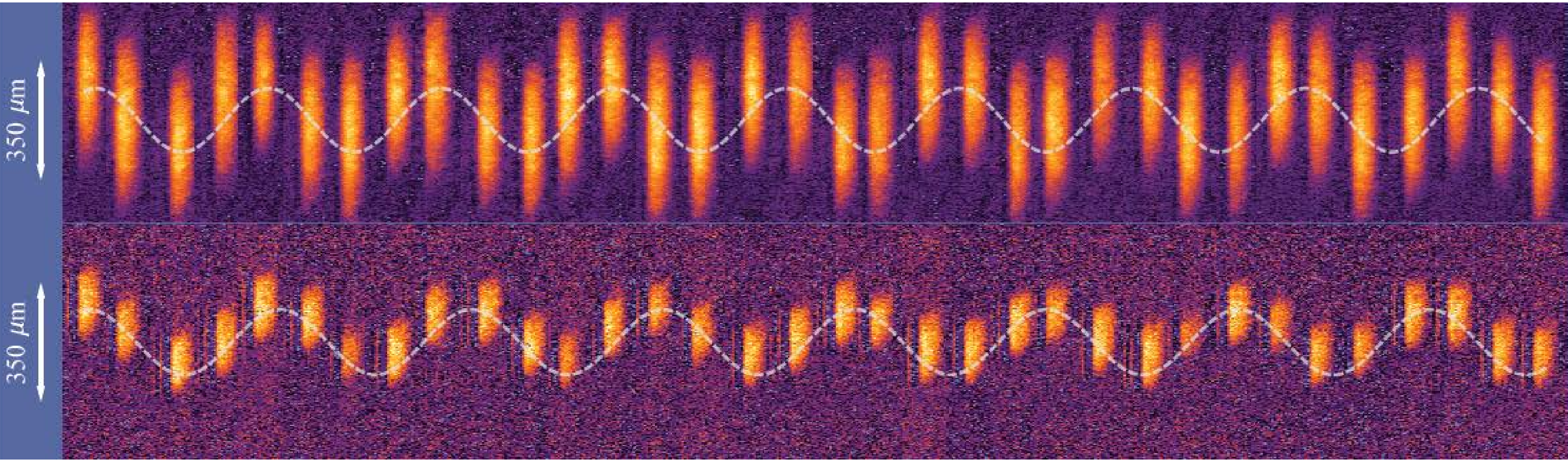


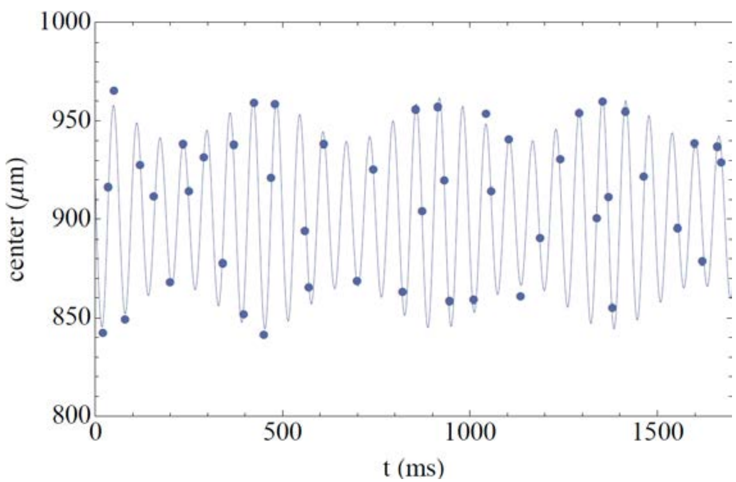
Zhang et al. (2014): Skornyakov Ter-Martirosyan

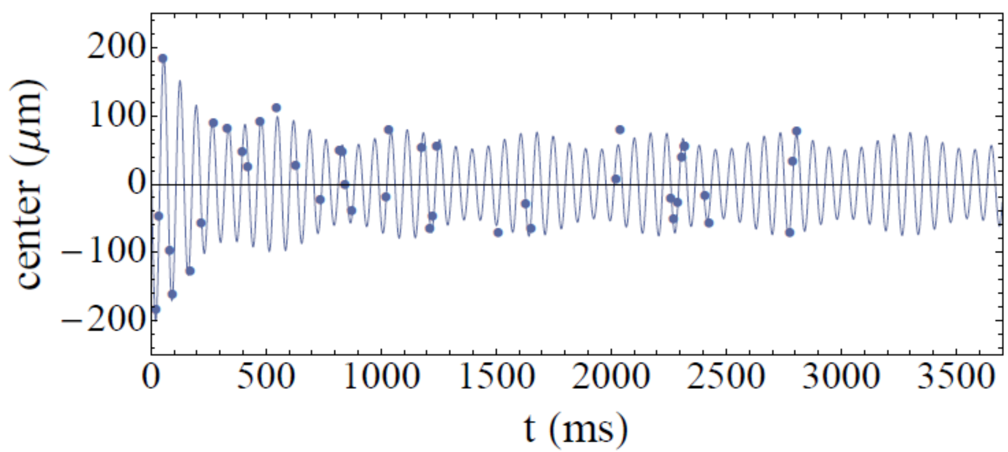
Dissipative polaron transport

(Delehaye *et al.* Phys. Rev. Lett. **115**, 265303 (2015); Laurent *et al.*, arXiv:1904.07040)

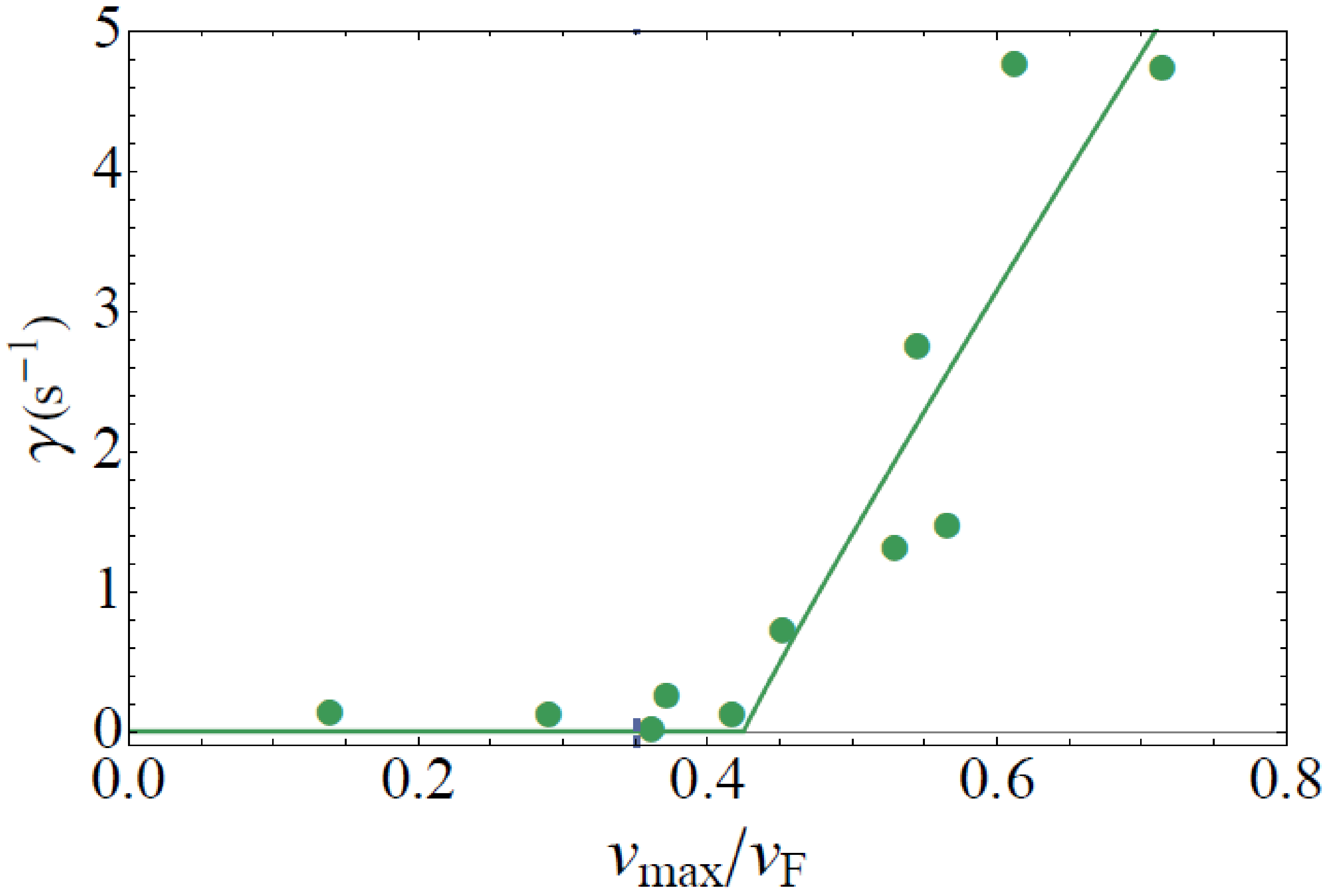
Dynamics of the Mixtures







Critical velocity



Landau's Criterion

Momentum Conservation : $M\mathbf{V} = M\mathbf{V}' + \hbar\mathbf{k}$

Energy Conservation : $MV^2/2 = MV'^2/2 + \varepsilon_\mathbf{k}$

$$\hbar k V \geq \hbar\mathbf{k}.\mathbf{V} = \varepsilon_k + \hbar^2 k^2/2m \geq \varepsilon_k$$

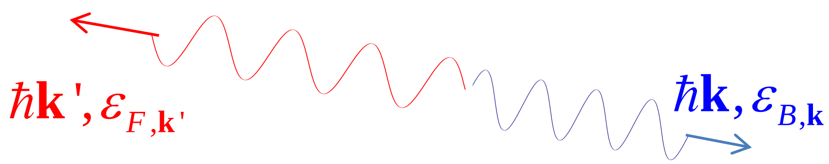
The motion of the impurity is damped by the creation of elementary excitations if

$$V \geq V_c = \min_k \left(\frac{\varepsilon_k}{\hbar k} \right)$$

= sound velocity for a linear excitation spectrum $\varepsilon = \hbar k c$

Landau criterion for a superfluid Mixture

(Castin *et al.* Comptes Rendus Physique **16,** 241 (2015) **arXiv:1408.1326**)



1 Excitation in the bosonic superfluid $\qquad E_{\mathrm{B},\mathbf{k}} = \varepsilon_{\mathrm{B},\mathbf{k}} + \hbar \mathbf{k} \cdot \mathbf{V}_{\mathrm{B}}$

1 Excitation in the fermionic superfluid $\qquad E_{\mathrm{F},\mathbf{k}} = \varepsilon_{\mathrm{F},\mathbf{k}'} + \hbar \mathbf{k}' \cdot \mathbf{V}_{\mathrm{F}}$

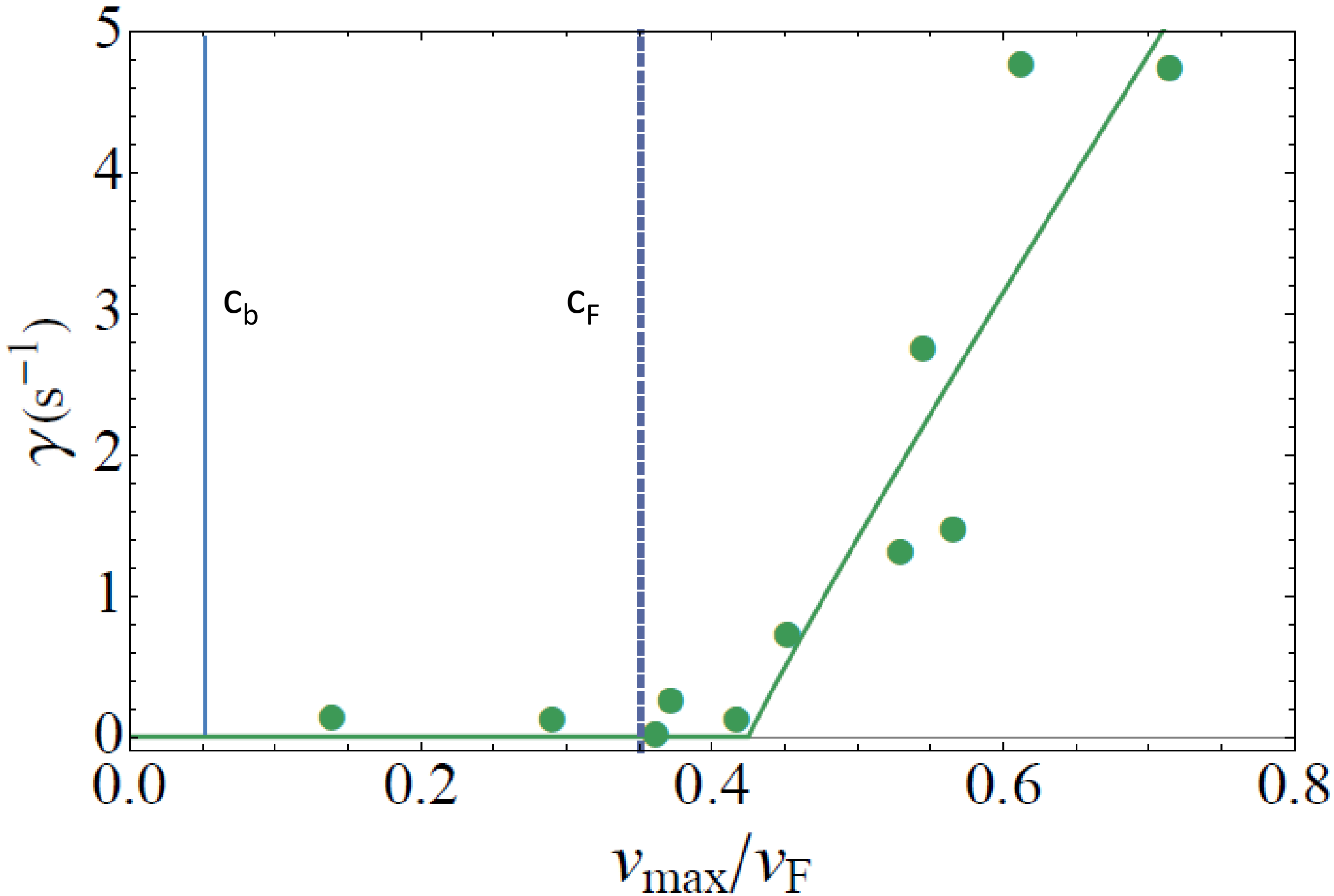
Energy-momentum conservation: $\qquad E_{\mathrm{B},\mathbf{k}} + E_{\mathrm{F},\mathbf{k}'} = 0 \qquad \mathbf{k} + \mathbf{k}' = 0$

$$|\mathbf{V}_B - \mathbf{V}_F| \geq \min_k \left(\frac{\varepsilon_{B,k} + \varepsilon_{F,-k}}{\hbar k} \right)$$

Acoustic Modes: $V_c = \cancel{c_{\mathrm{B}}} + c_{\mathrm{F}}$

See also Abbad et al. EPJD 69, 126 (2015), F. Chevy PRA **91**, 063606 (2015)

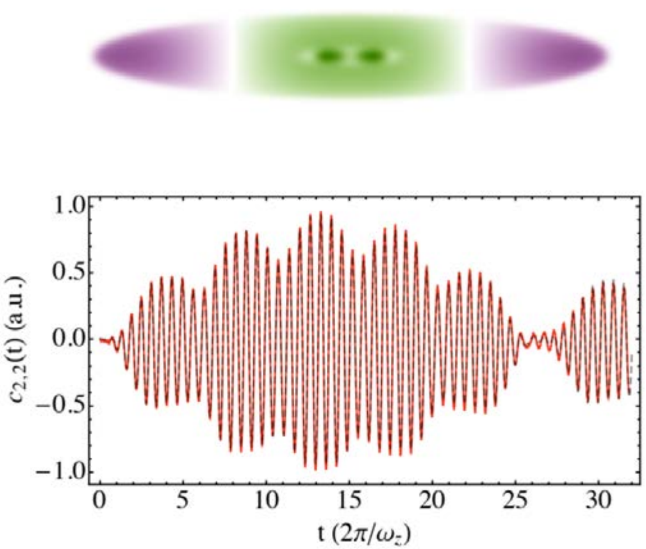
Critical velocity

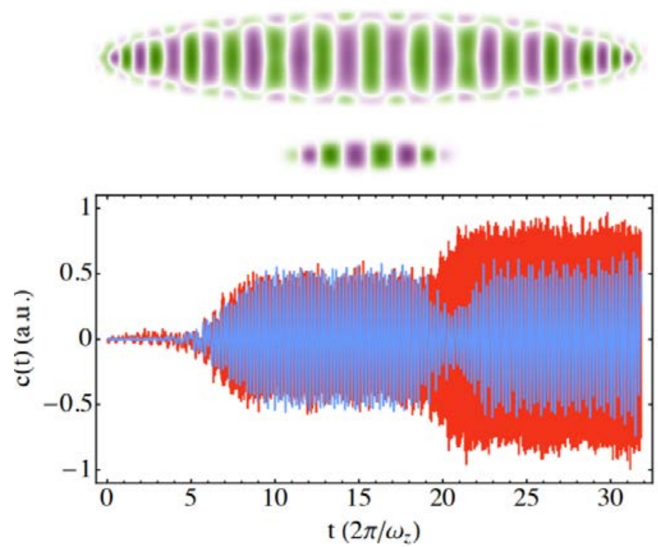


Landau vs generalized Landau competition

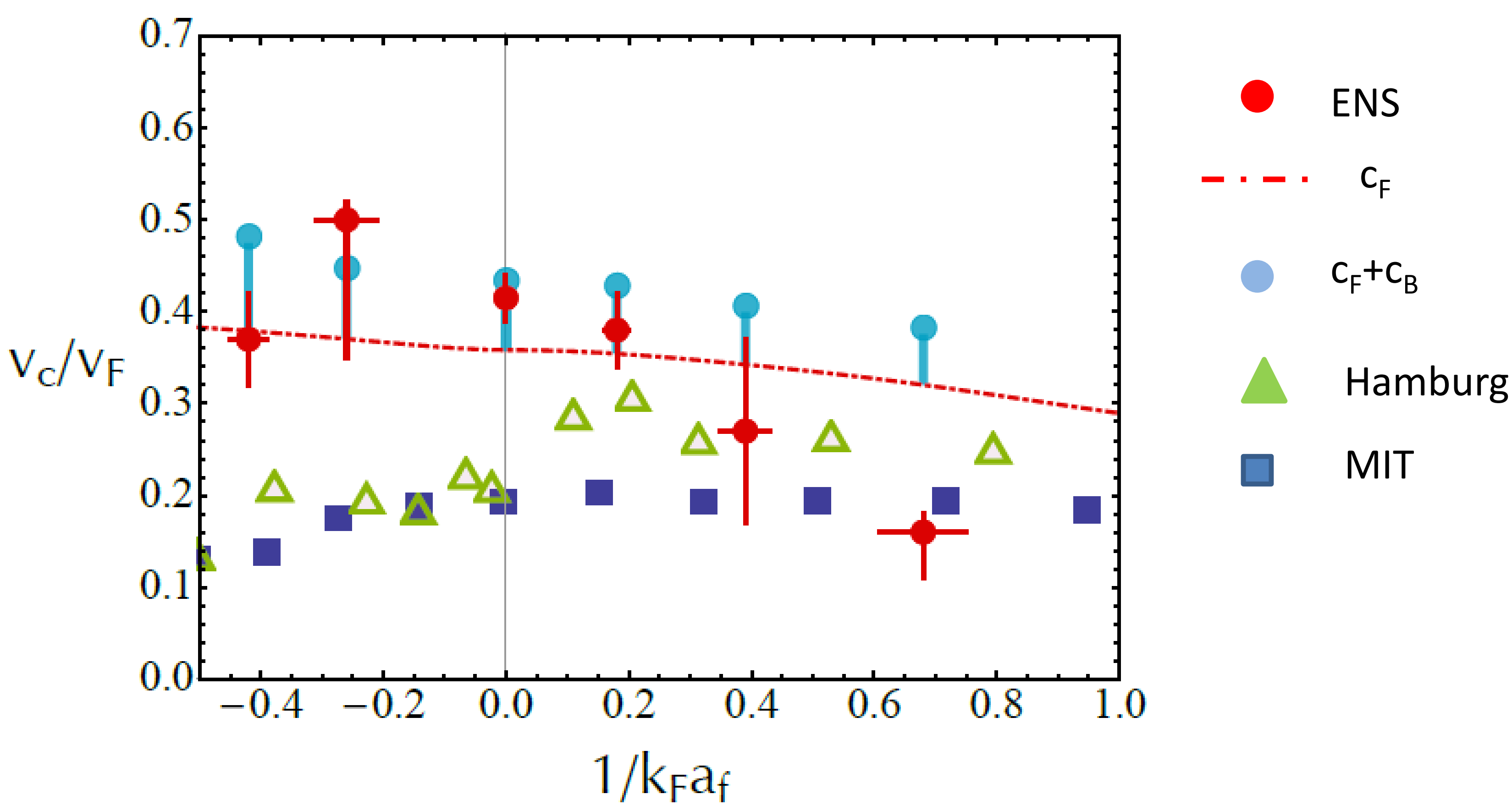
- Landau's original argument does not apply to a homogeneous mixture (breaks galilean invariance)
- Competition between the two mechanisms in a trap.

Simulation of GPE for a mixture of two BEC highlights this competition,
Principle Component Analysis of the profile





Critical Velocity



CONCLUSION AND OUTLOOK

- **Experimental challenges**:
  - Find a spin/species mixture **with joint Feshbach resonances**
  - **Stability of the mixture** near the interspecies Feshbach resonances
- **Theoretical challenges**:
  - Go **beyond the perturbative approach**.
  - Is it possible to describe the polaron/trimeron transition using a **variational scheme** including **Efimov physics** and **independent of a-priori assumptions** about the properties of the many-body background.
  - Clarify the dominant damping mechanism.

Validity of Landau's argument?

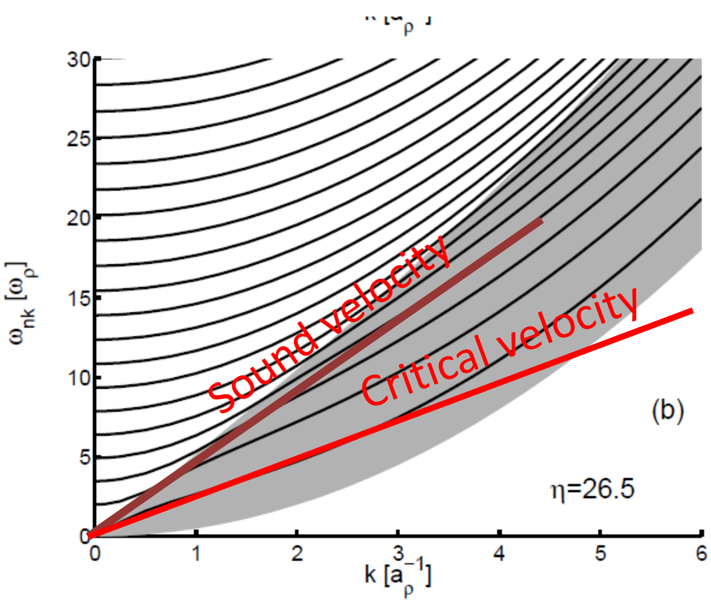
- Argument valid for a **constant** velocity in an **homogeneous** medium.
- But:

Transverse trapping potential

Back-bending of the dispersion relation of axial phonons (« roton-like» behaviour

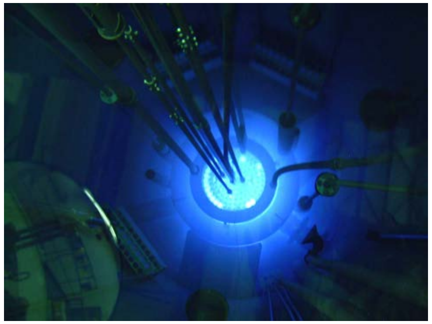
Mean-field BEC:
Fedichev & Shlyapnikov (PRA 2001);
Cozzo & Dalfovo, NJP (2003)

Hydrodynamic superfluid:
Crépin, Leyronas, Chevy (EPL 2016)

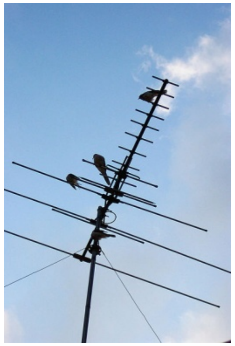


Validity of Landau's argument (II)?

Axial trapping: Oscillatory motion



vs



Hydrodynamic response to an oscillating potential: **No Landau critical velocity!**
(S. Jin et al., EPJST 2018)