Repulsive BEC stirred with a rotating Dirac delta

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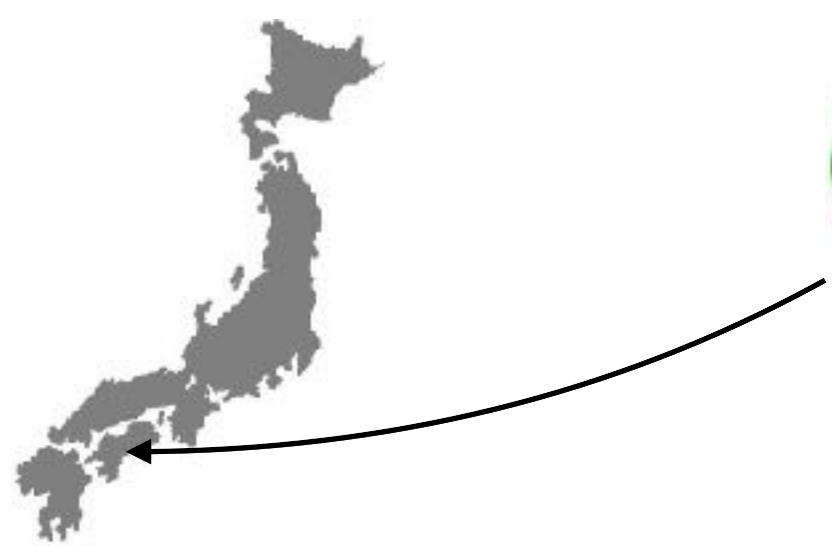
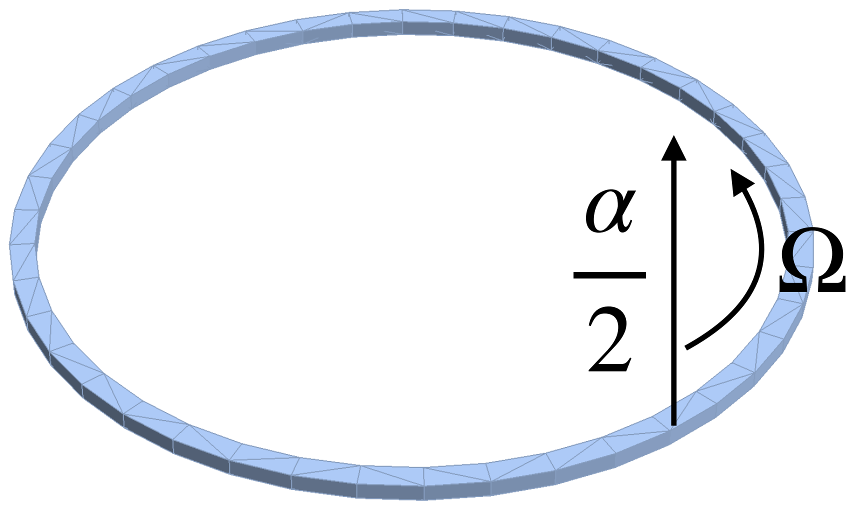
Kochi University of Technology, Japan

Atomtronics, Benasque, May 14th, 2019



1D BEC in a ring
g>0



$$i\hbar\,\partial_{t_L}\psi_L = -\frac{\hbar^2}{2MR^2}\,\partial^2_{\theta_L}\psi_L + g\,|\psi_L|^2\psi_L + \frac{\alpha}{2}\,\delta(\theta_L - \Omega\,t_L)\psi_L$$

- Ring experiments

C. Ryu et al., PRL 99, 260401 (2007); S. Moulder et al., PRA 86, 013629 (2012); S. Beattie et al., PRL 110, 025301 (2013); K. C. Wright et al., PRL 110, 025302 (2013)

- No rotation, no delta α=Ω=0:

L. D. Carr, C. W. Clark, and W. P. Reinhardt, PRA 62, 063610 (2000); PRA 62, 063611 (2000).

- Only rotation Ω:

R. Kanamoto, L. D. Carr, and M. Ueda, PRA 79, 063616 (2009).

- Only defect:

B. T. Seaman, L. D. Carr, and M. J. Holland, PRA 71, 033609 (2005).

- Rotating delta/defect:

S. Baharian and G. Baym, PRA 87, 013619 (2013) (1D);
M. Cominotti et al., PRL 113, 025301 (2014); (1D);
A. Munñoz et al., PRA 91, 063625 (2015); (1D/3D);
M. Kunimi and Y. Kato, PRA 91, 053608 (2015); (2D).

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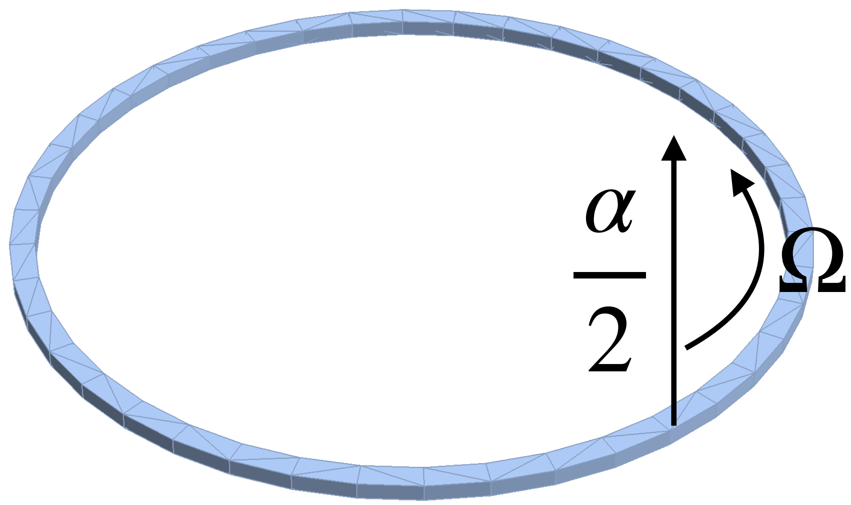

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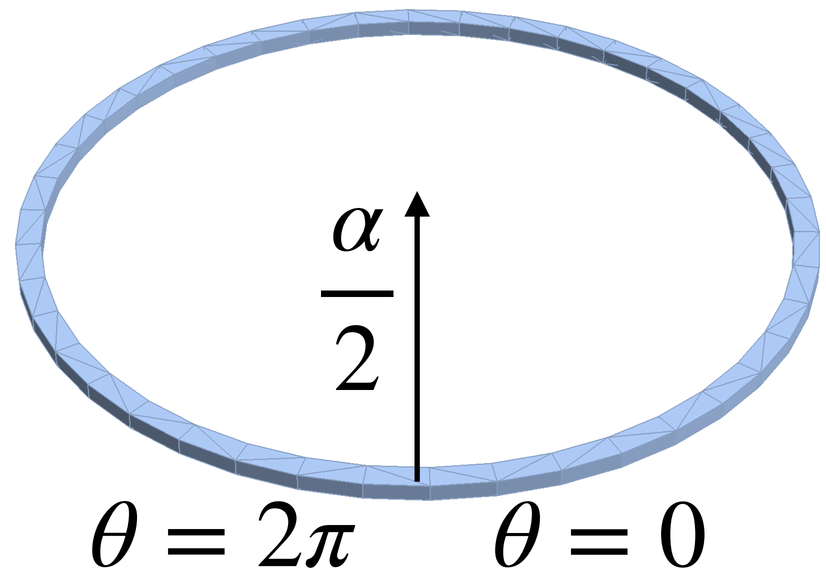
- Natural units: $\hbar = R = M = 1$

  (natural scales: $\tilde{g} = \frac{\hbar^2}{MR^2}, \tilde{\alpha} = \frac{\hbar^2}{MR^2}, \tilde{\Omega} = \frac{\hbar}{MR^2}$)

- Rotating frame: $\theta = \theta_L - \Omega\, t_L$

- Stationary solution: $\psi(\theta, t) = e^{-i\,\mu t}\phi(\theta)$

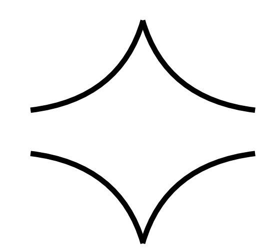
In the comoving frame



$$-\frac{1}{2}\phi''(\theta) + g\,|\,\phi(\theta)\,|^2\,\phi(\theta) = \mu\,\phi(\theta)$$

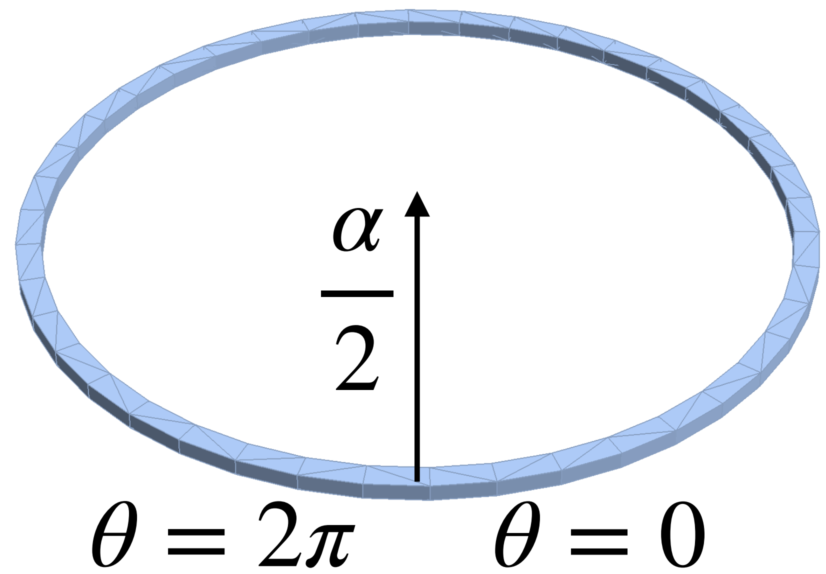
$$\left.\begin{aligned}&\phi(0) - e^{-i2\pi\Omega}\phi(2\pi) = 0\\&\phi'(0) - e^{-i2\pi\Omega}\phi'(2\pi) = \alpha\,\phi(0)\end{aligned}\right\}$$

attractive, $\alpha < 0$
repulsive, $\alpha > 0$



$$\int_0^{2\pi} d\theta\,|\,\phi(\theta)\,|^2 = 1$$

In the comoving frame



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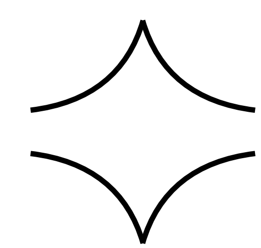

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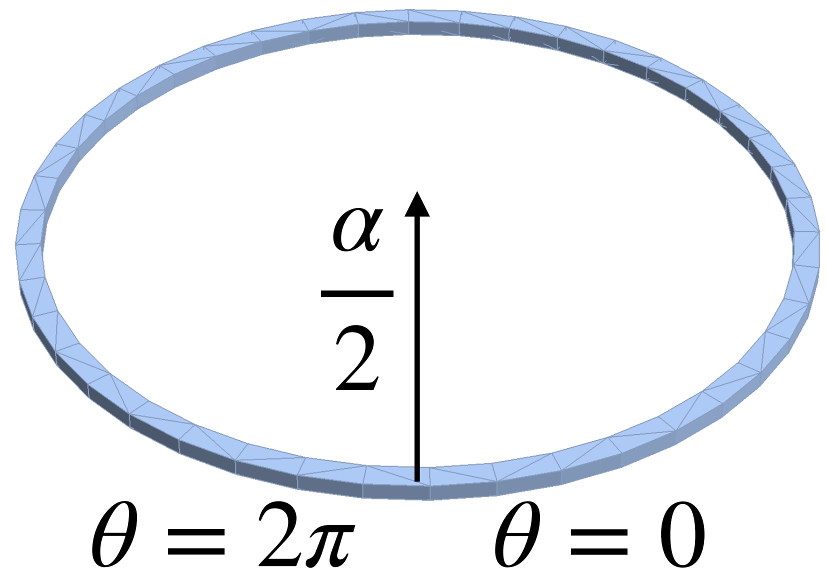

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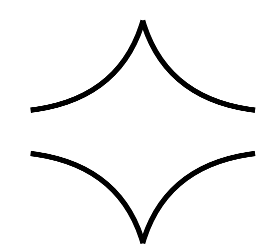

$$\int_0^{2\pi} d\theta\,|\phi(\theta)|^2 = 1$$

boosts: $\Omega \to \Omega + integer$

parity $(\theta \to 2\pi - \theta)$: $\Omega \to -\Omega$

$$-\frac{1}{2}\phi''(\theta) + g\,|\,\phi(\theta)\,|^2\,\phi(\theta) = \mu\,\phi(\theta)$$

General solution: in terms of Jacobi elliptic functions

$$\phi(\theta) = r(\theta)e^{i\,\beta(\theta)} \quad \begin{cases} r_J^2(\theta) = A + B\,J^2(k(\theta-\theta_0), m) \\ \beta_J'(\theta) = \dfrac{\gamma_J}{r_J^2(\theta)} \end{cases}$$

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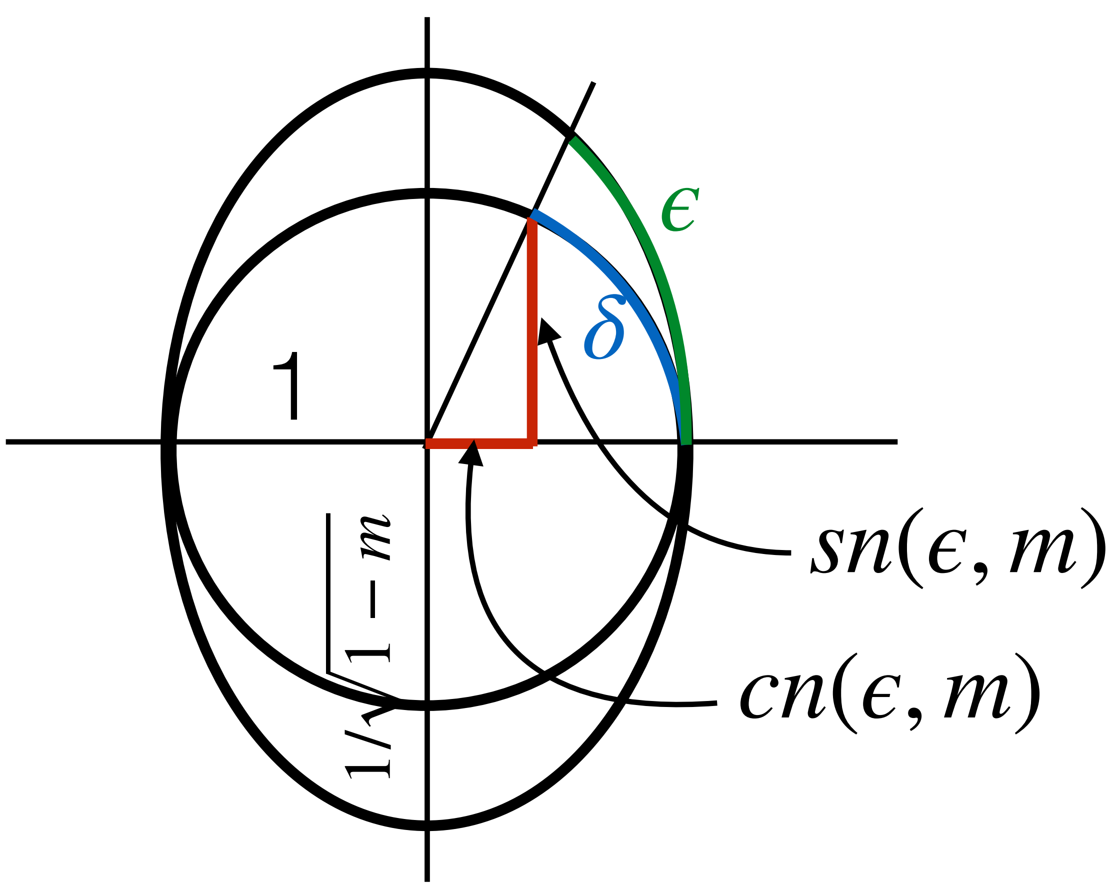

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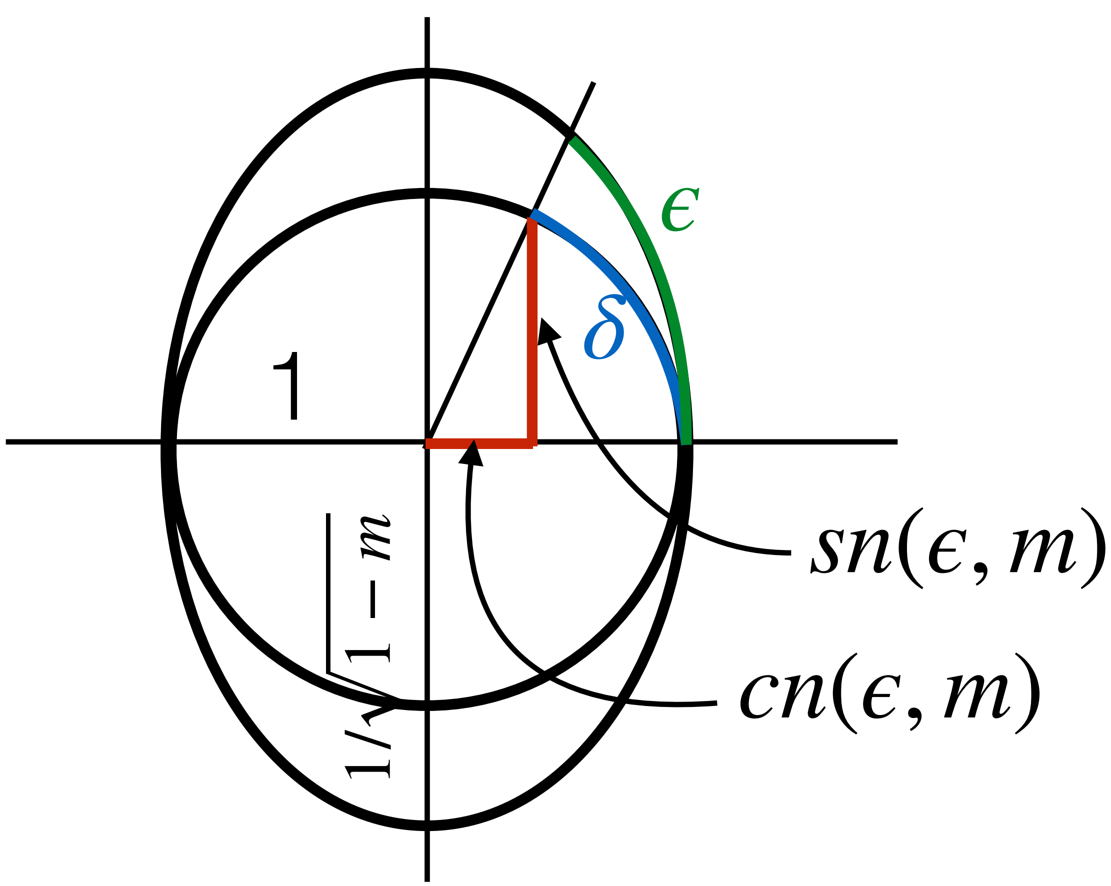


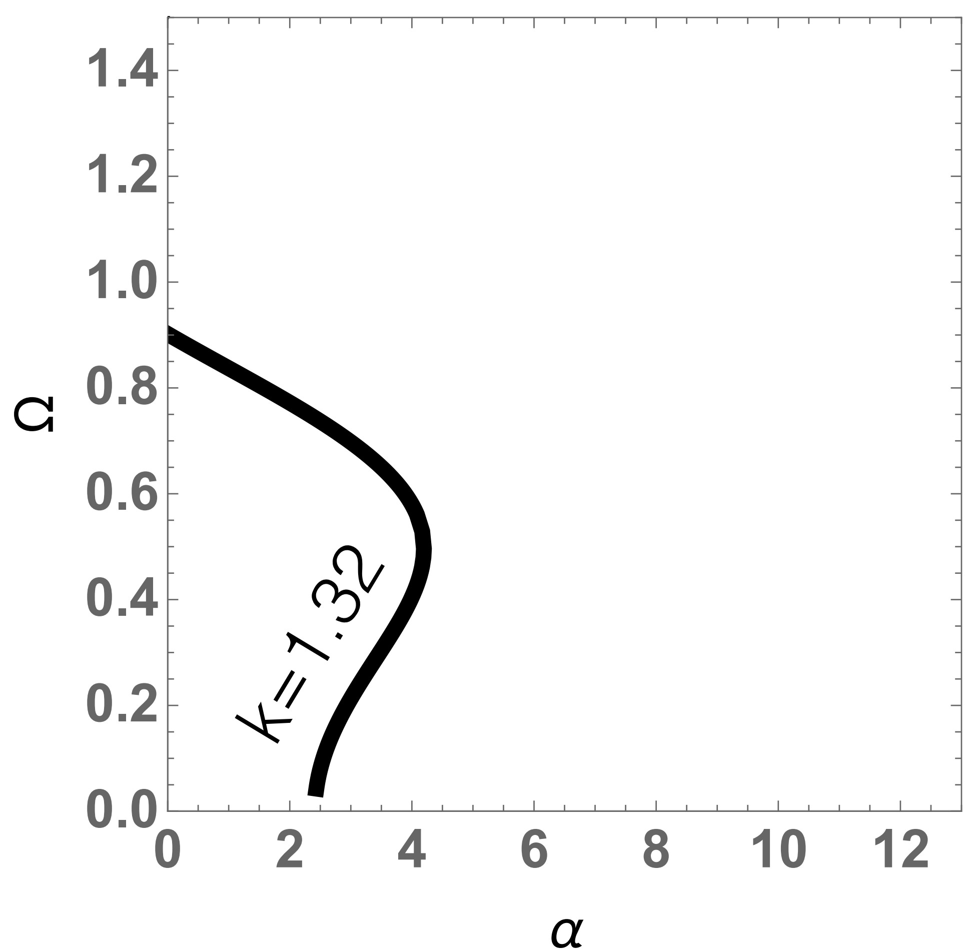
$$\alpha_J(k,m) = \frac{r_J'(0) - r_J'(2\pi)}{r_J(0)}$$

$$\Omega_J(k,m) = \frac{1}{2\pi}[\,\beta_J(2\pi) - \beta_J(0)\,]$$

$$\mu_J(k,m) = \frac{-1/2\phi''(0) + g\,|\,\phi(0)\,|^2\,\phi(0)}{\phi(0)}$$

Compute energy levels by running k and m: g=10

$$\big(\alpha(k,m),\ \Omega(k,m)\big)$$



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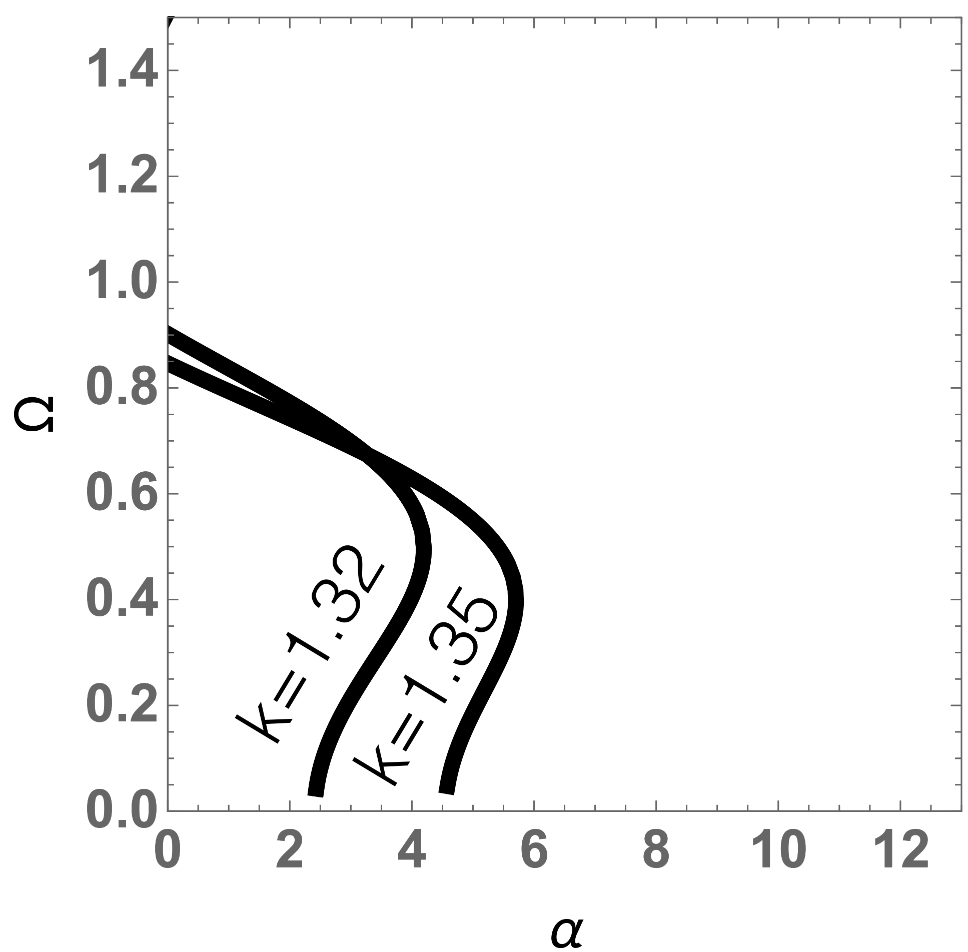

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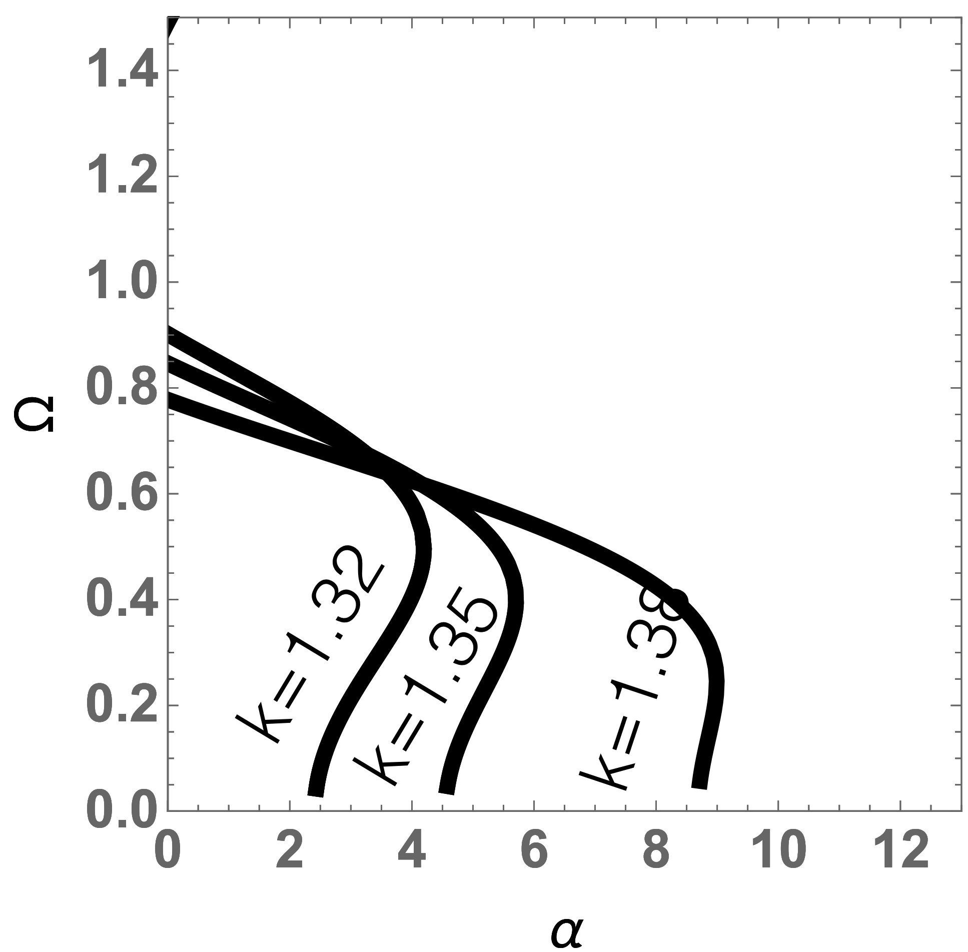

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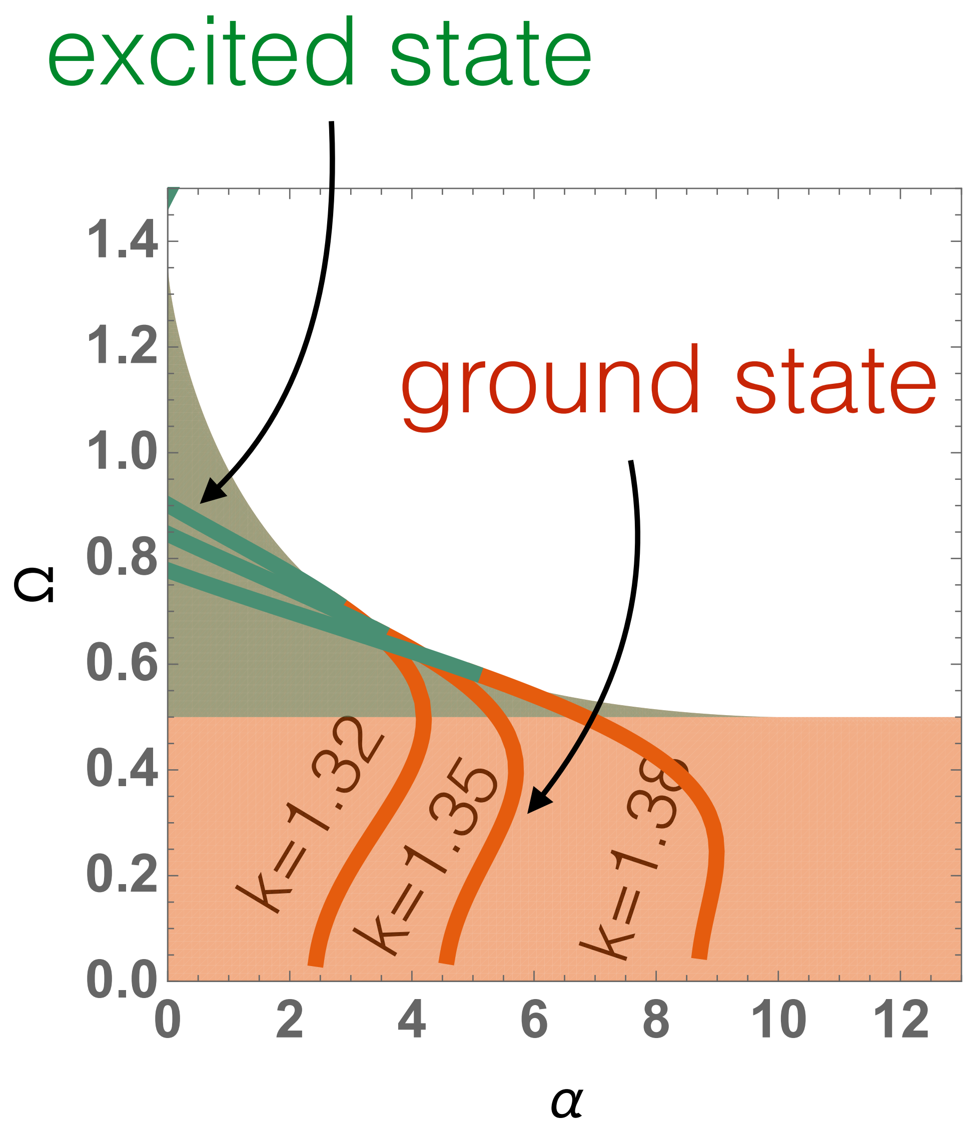

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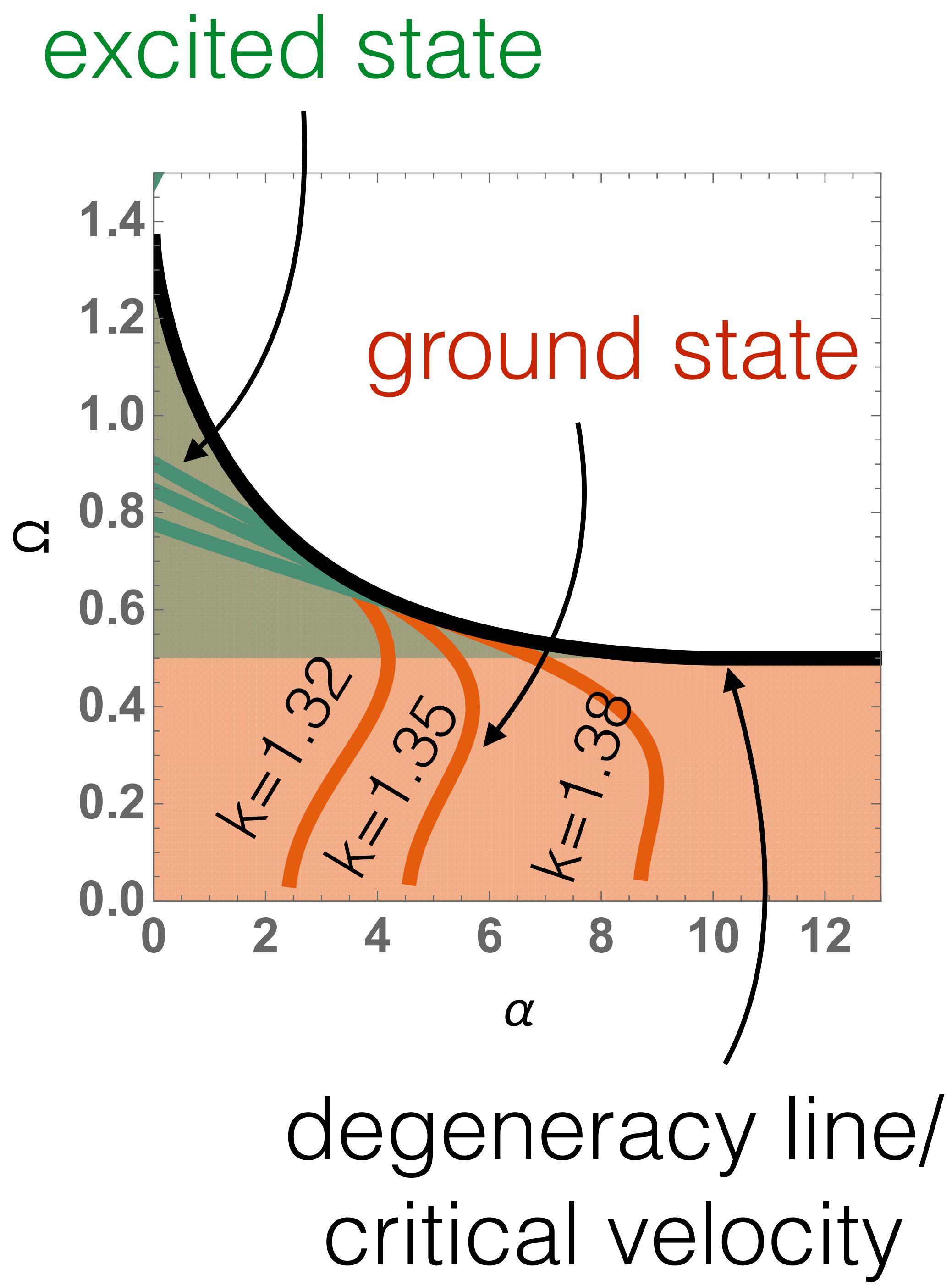

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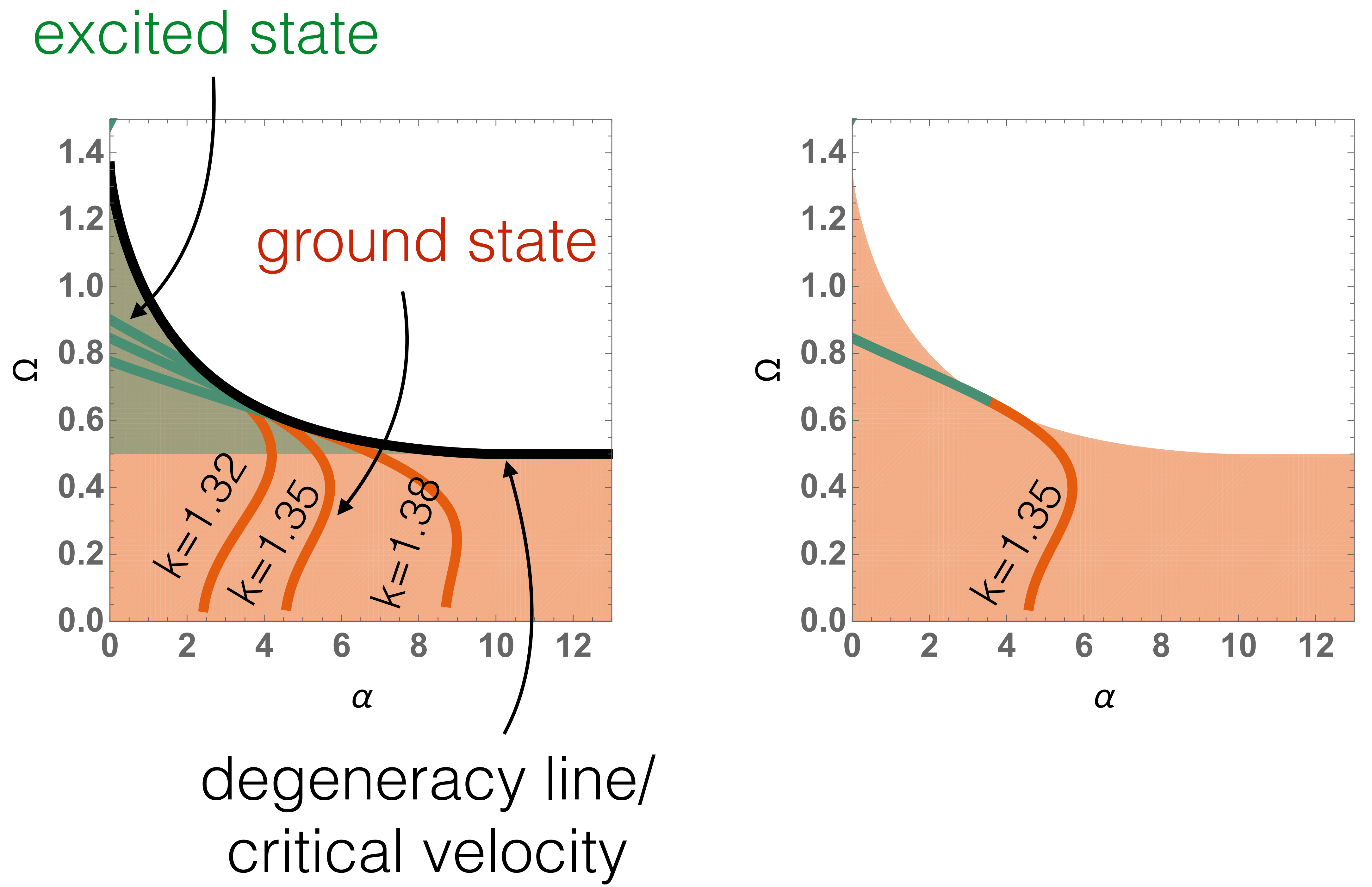

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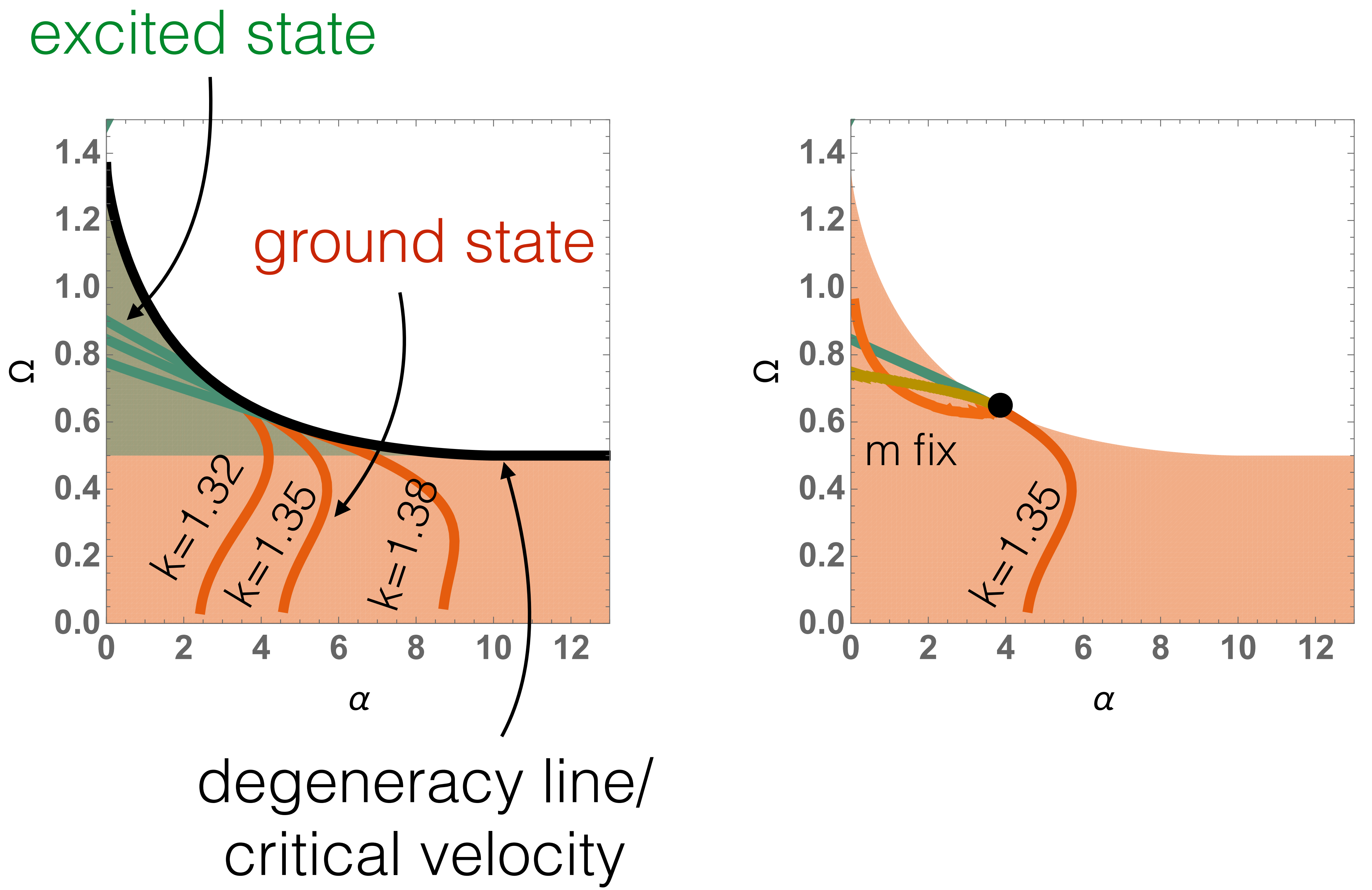

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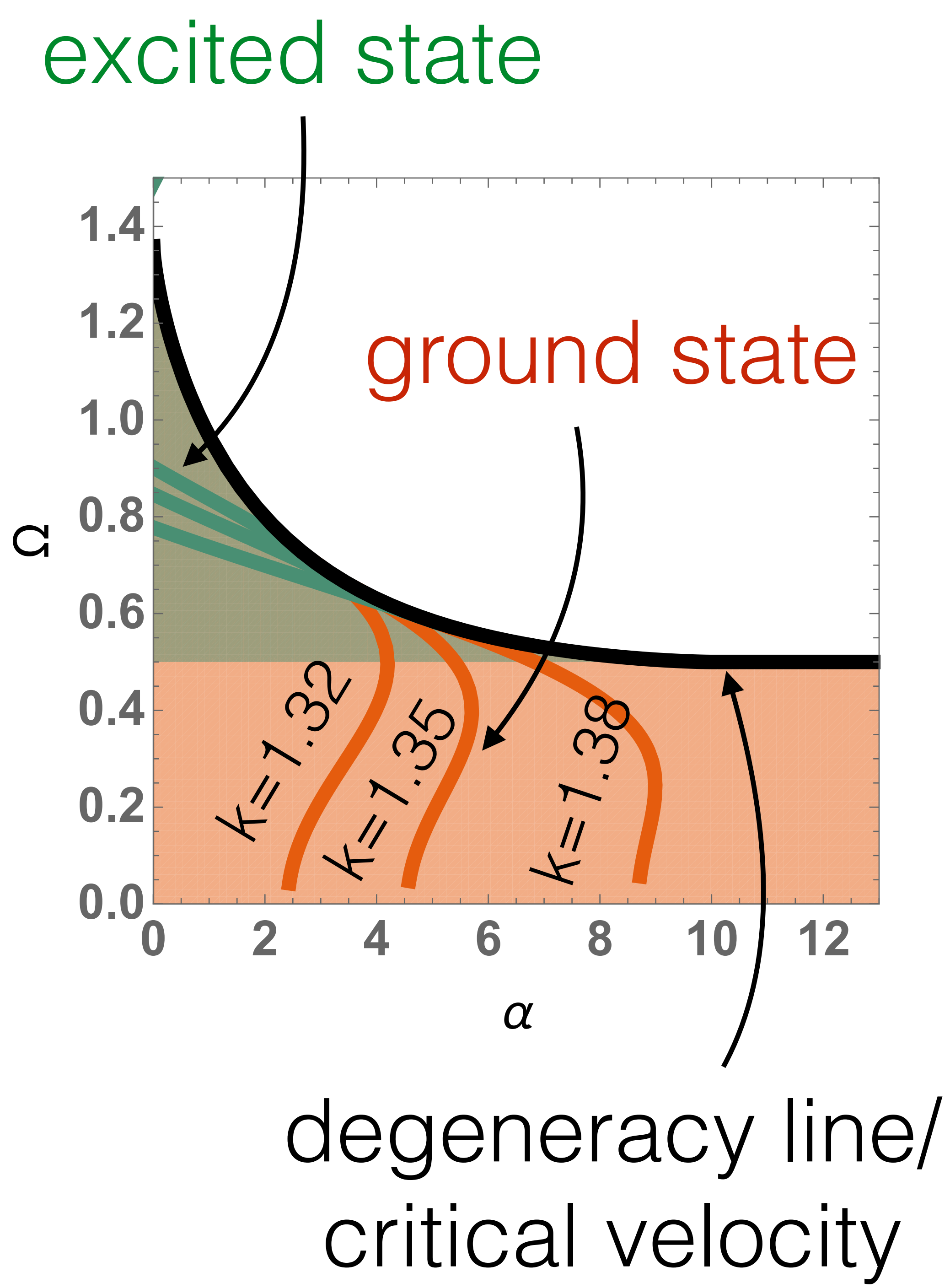
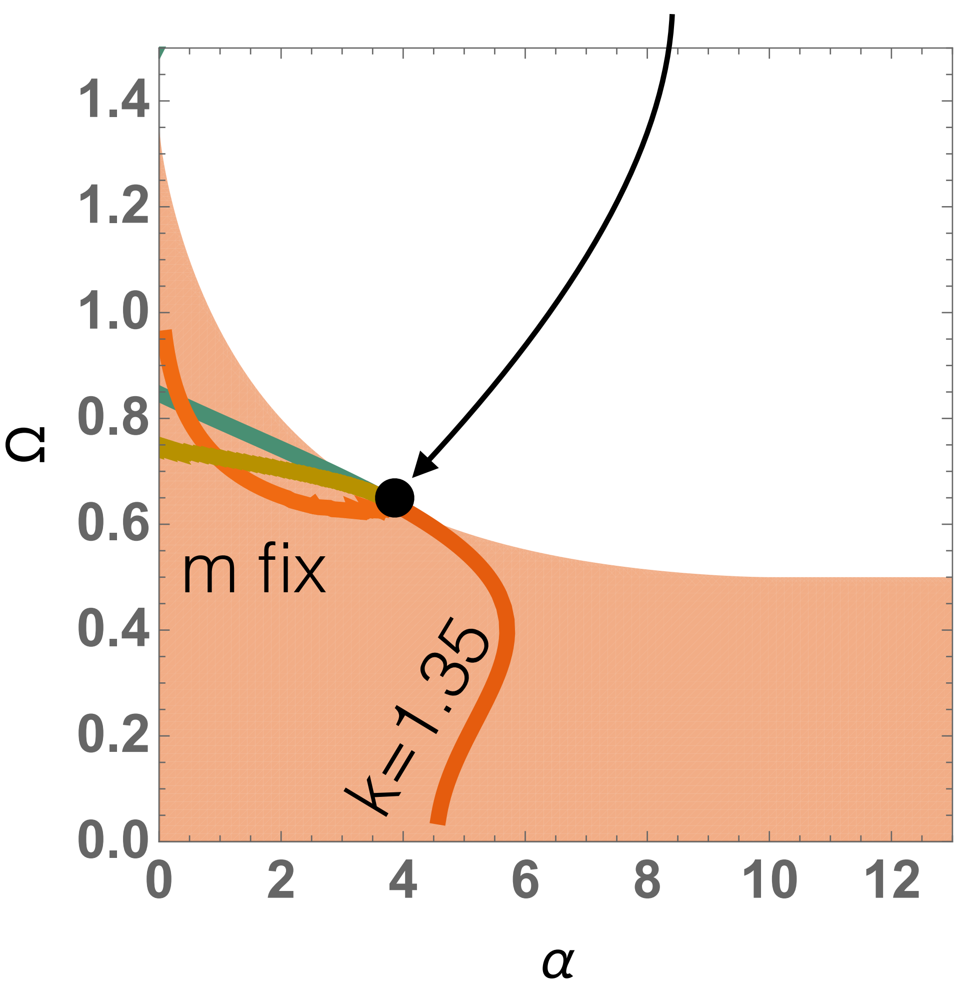


$$\frac{\partial}{\partial k}\left(\alpha(k,m),\Omega(k,m)\right) \propto \frac{\partial}{\partial m}\left(\alpha(k,m),\Omega(k,m)\right)$$



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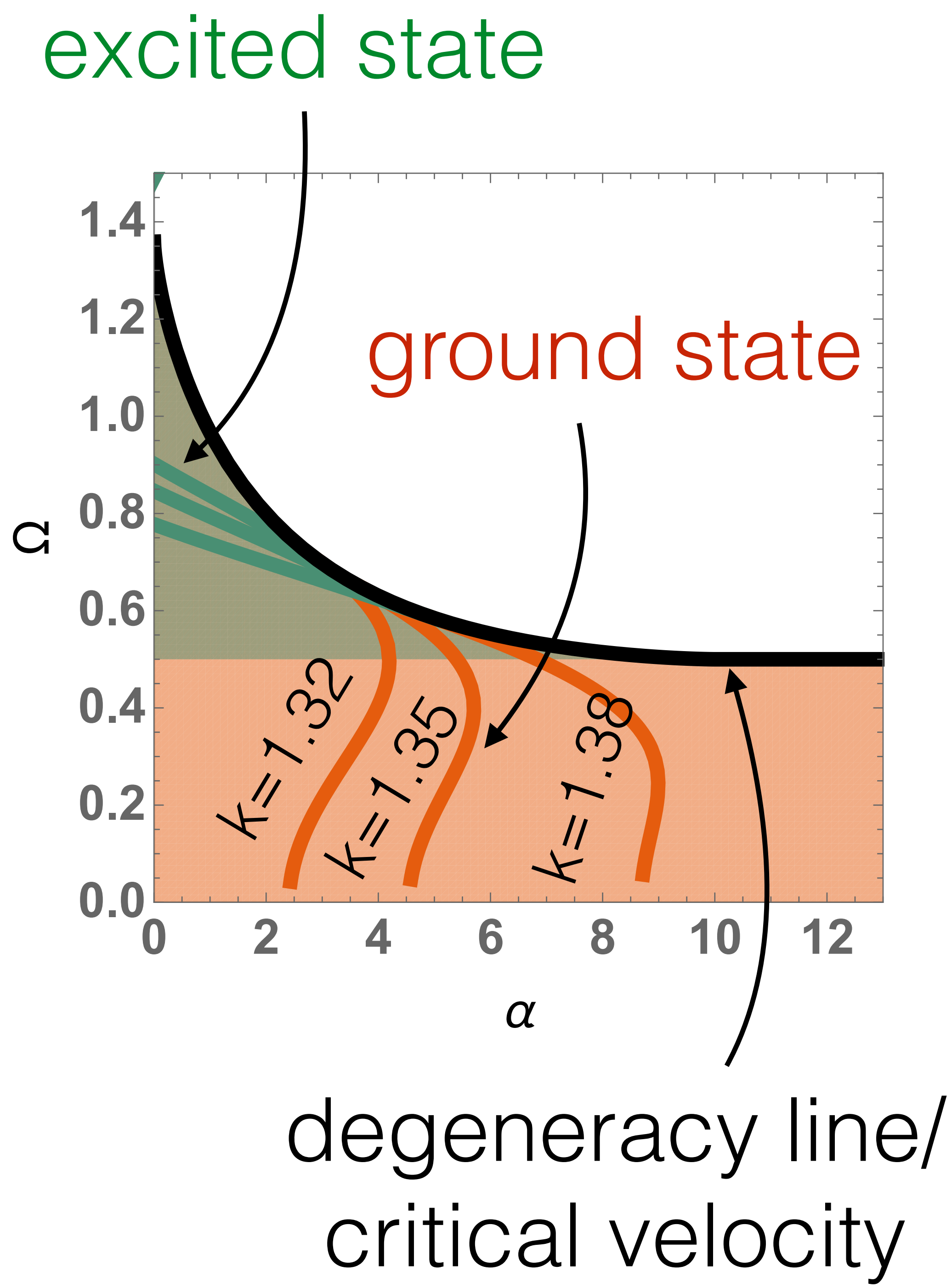

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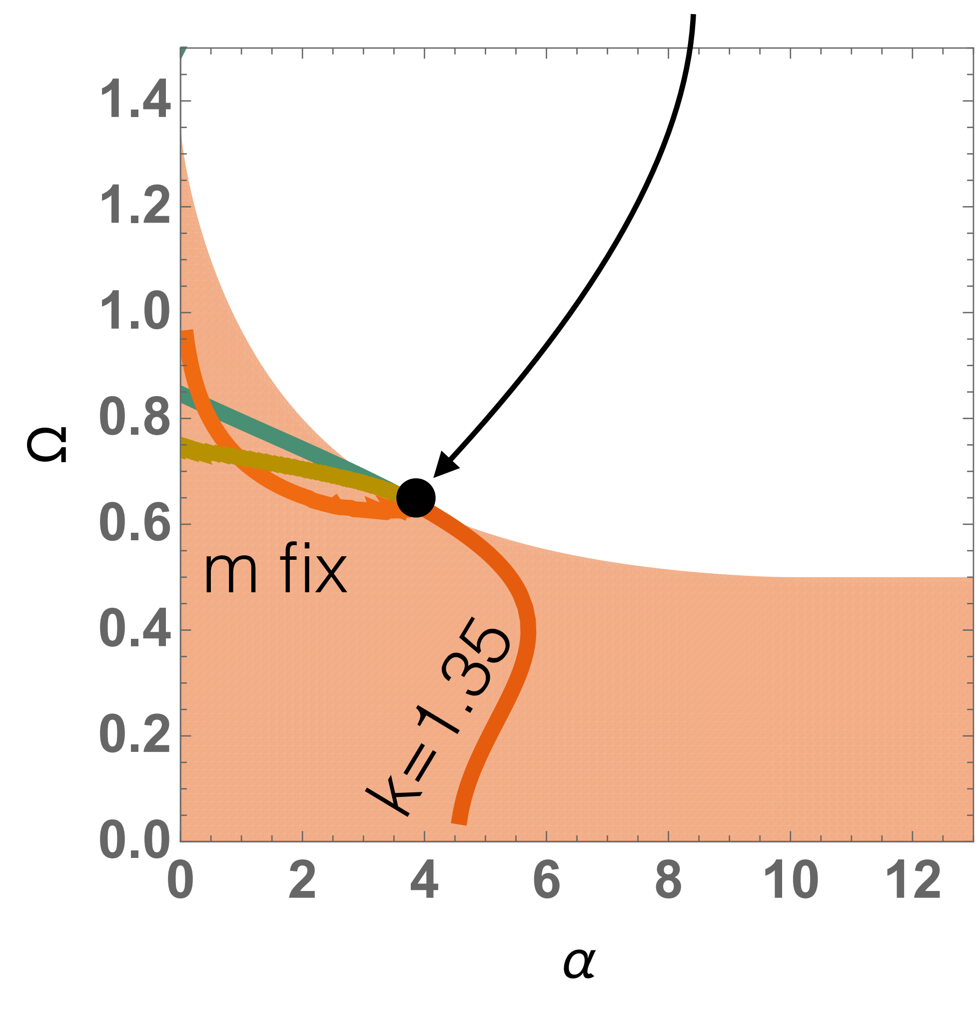
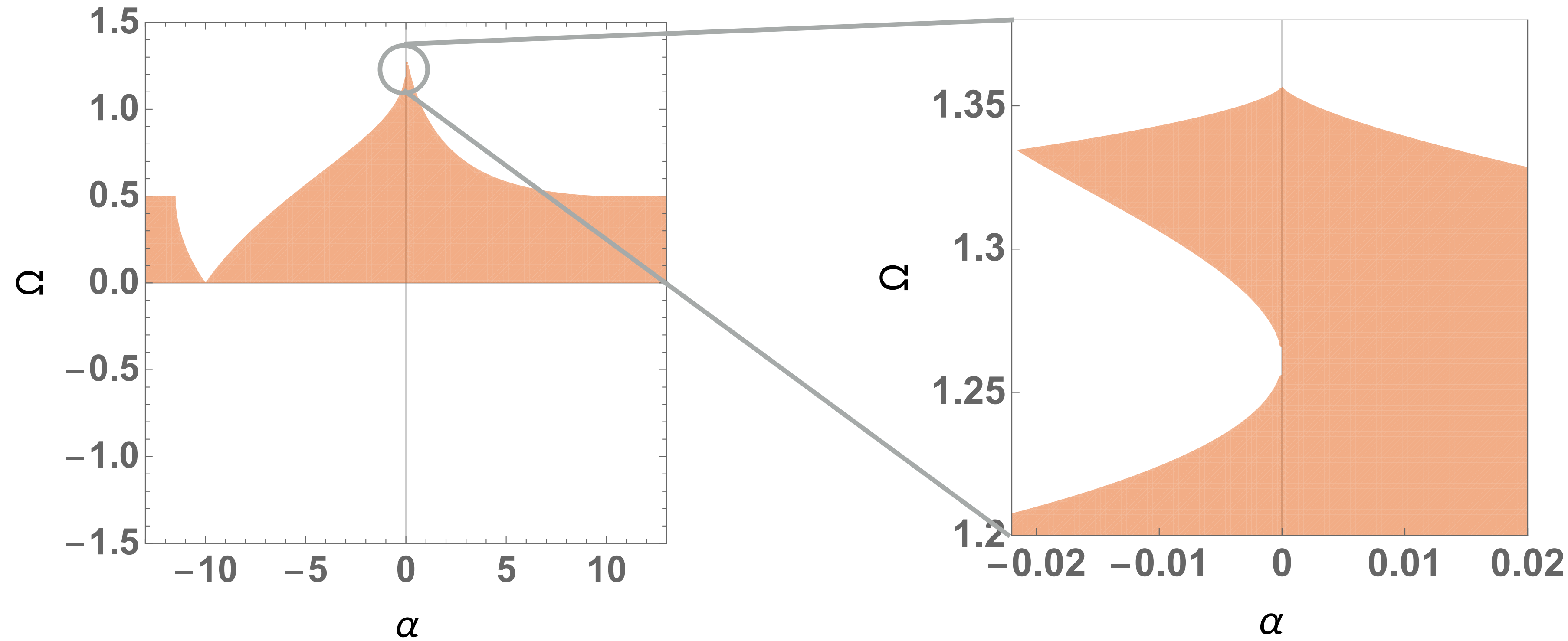


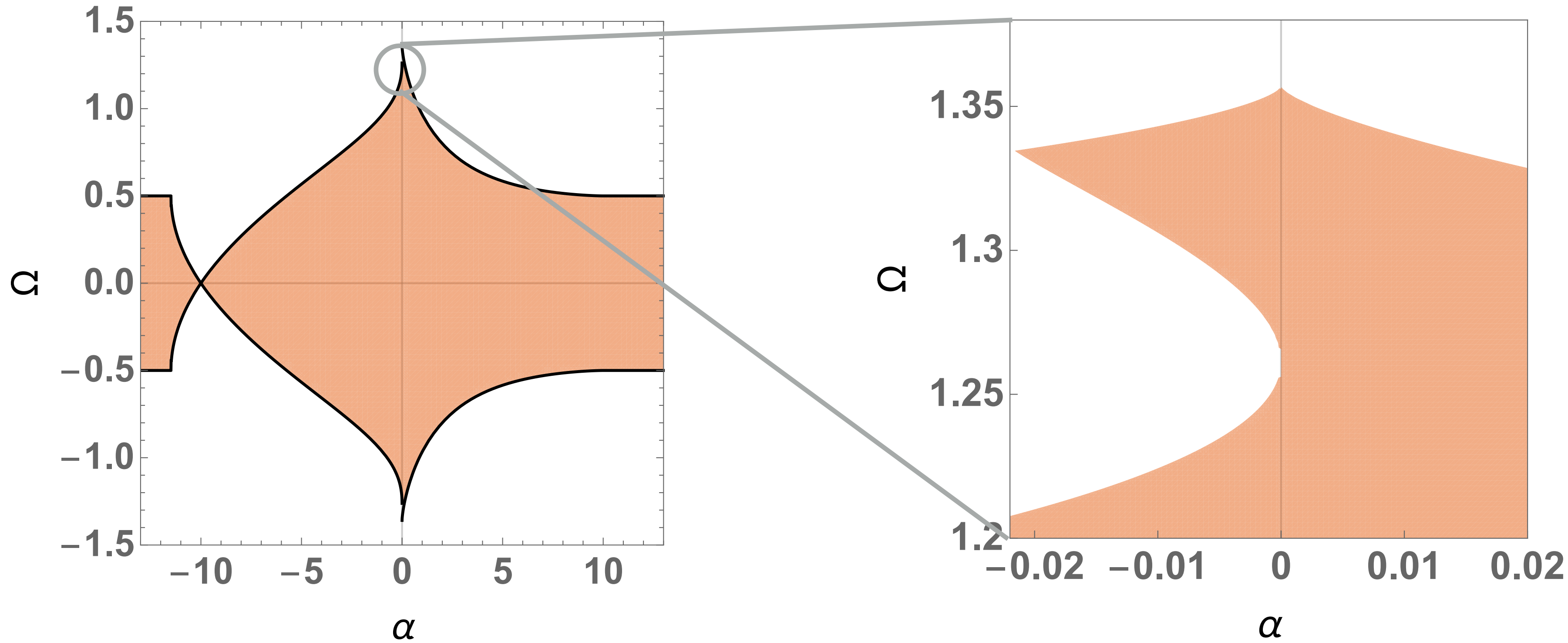
$$\Longrightarrow \frac{\partial\Omega(k,m)}{\partial k}\frac{\partial\alpha(k,m)}{\partial m} = \frac{\partial\Omega(k,m)}{\partial m}\frac{\partial\alpha(k,m)}{\partial k}$$

Stationary solutions: ground and first excited states



Stationary solutions: ground and first excited states

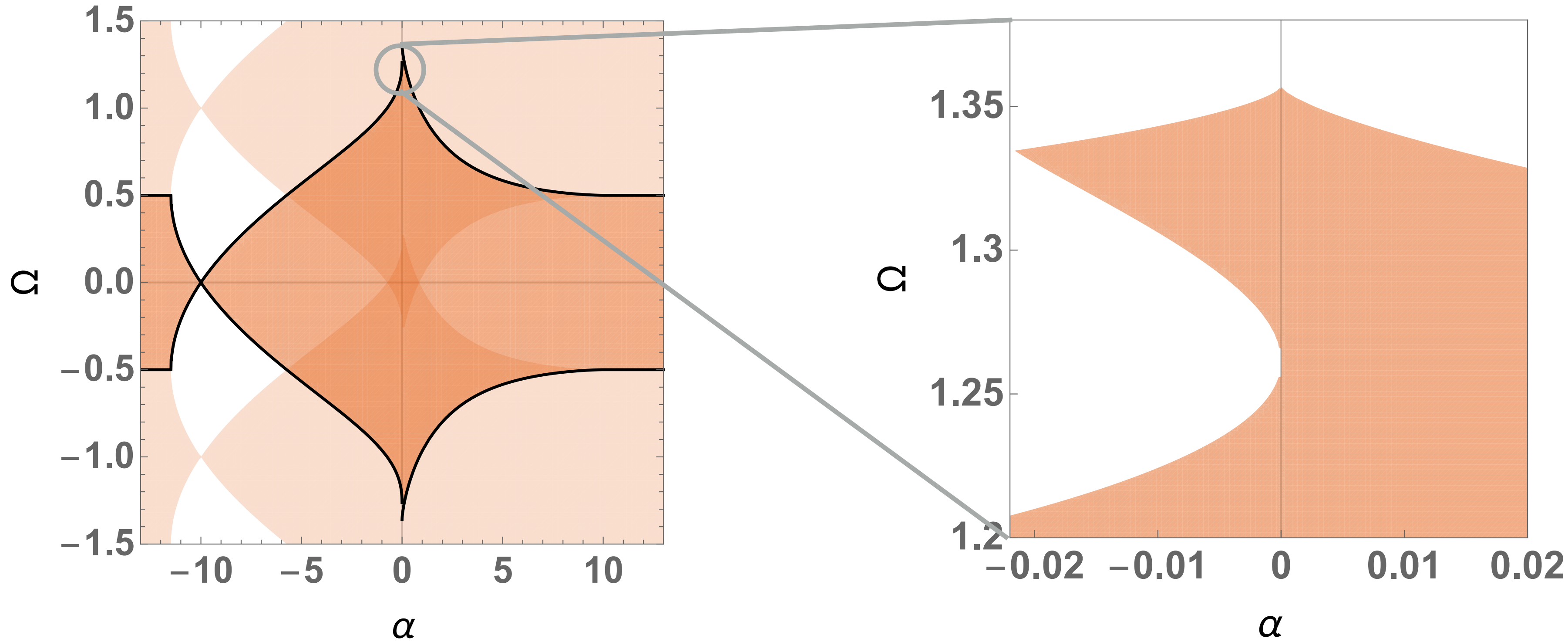
$\Omega \rightarrow -\Omega$



Stationary solutions: ground and first excited states

$\Omega \to -\Omega$

$\Omega \to \Omega + n$



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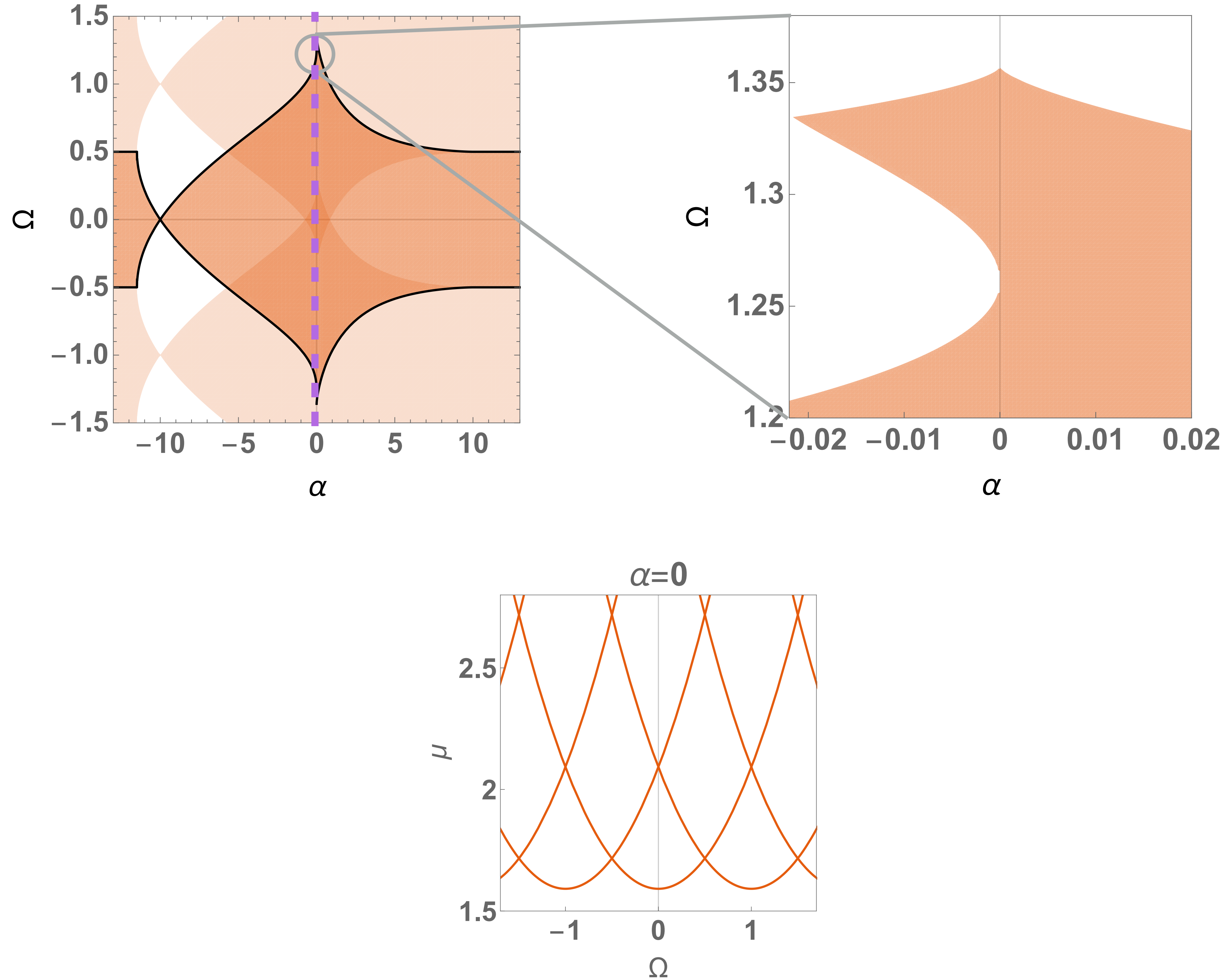

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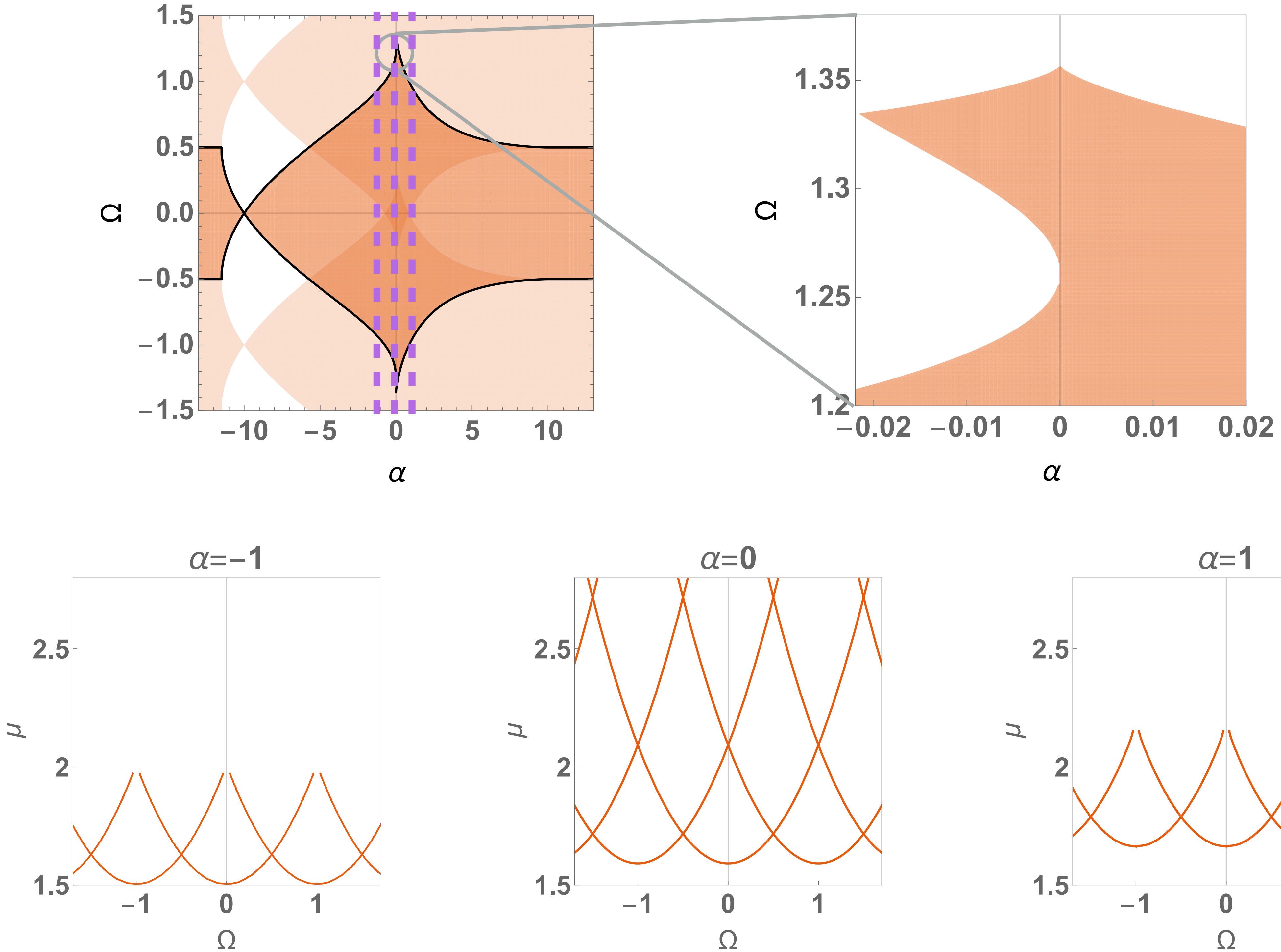

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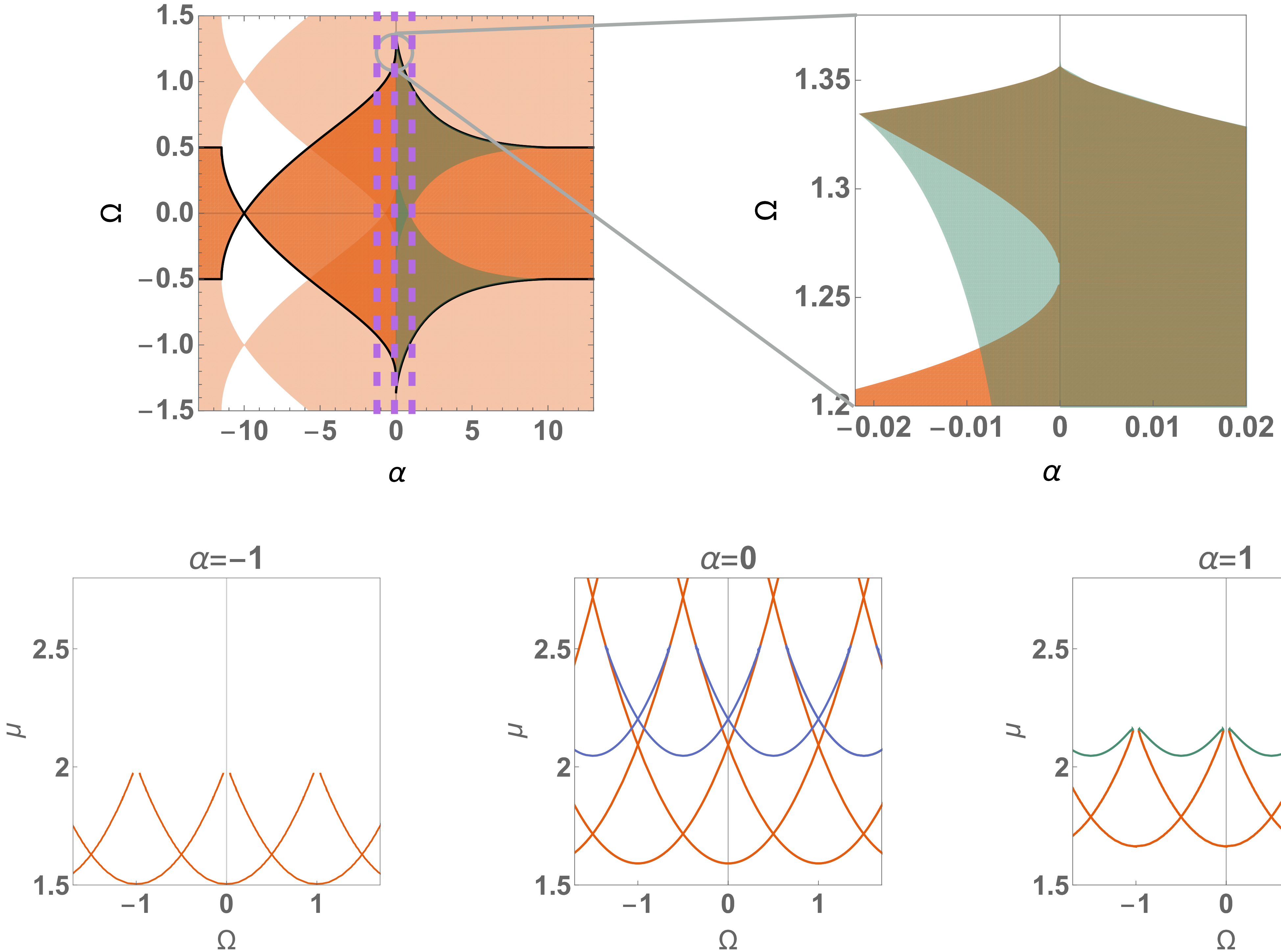

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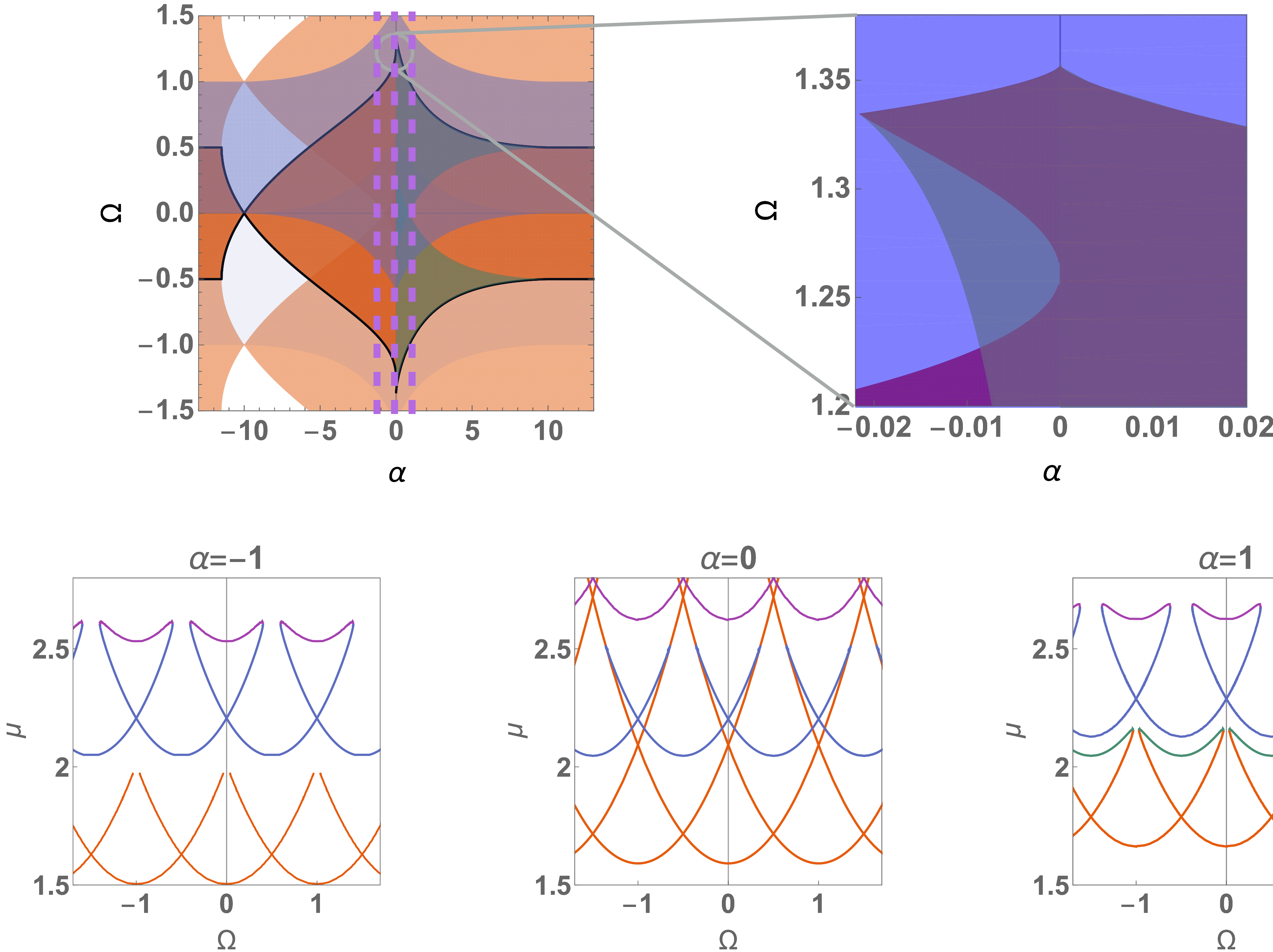

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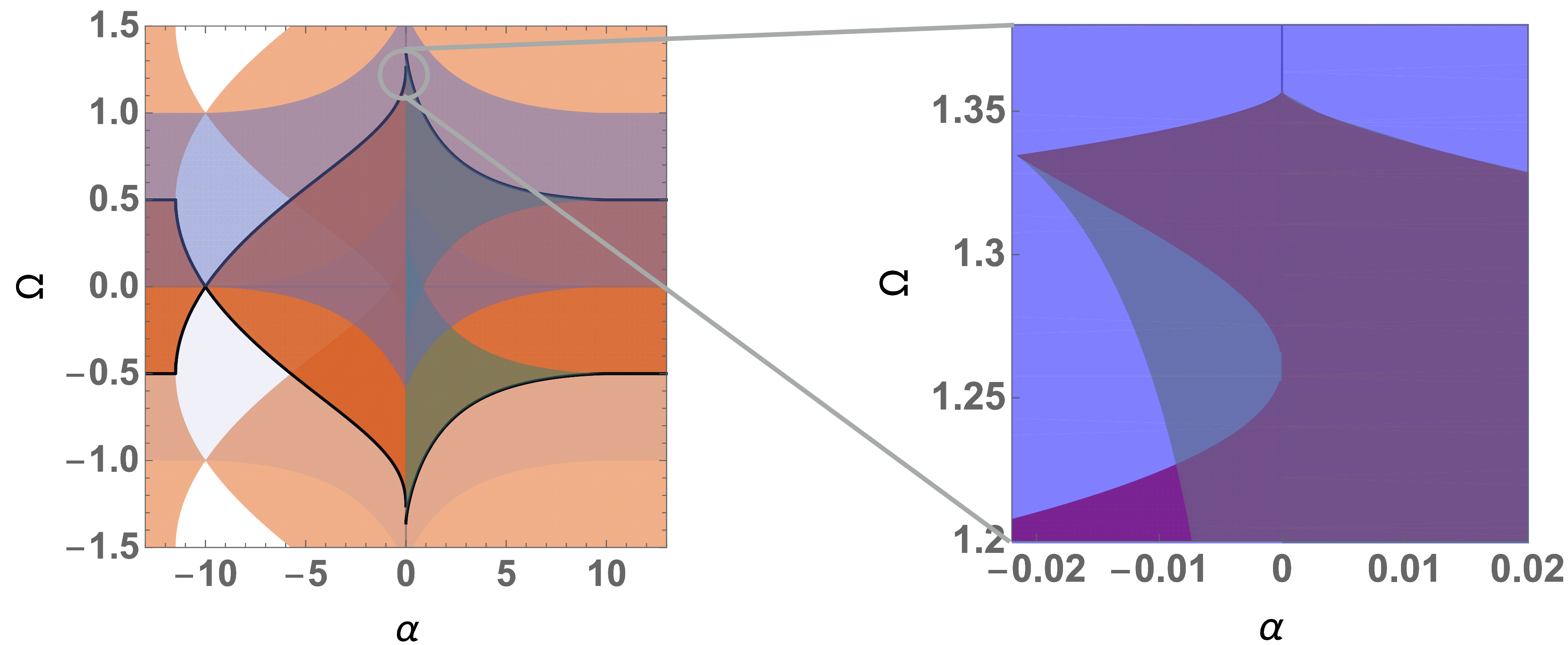

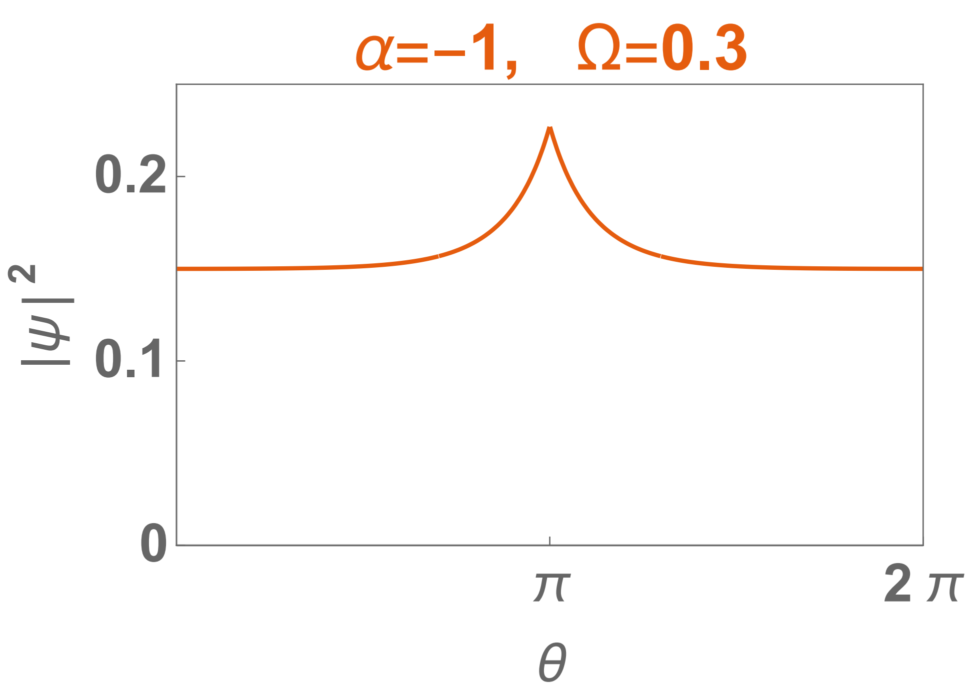

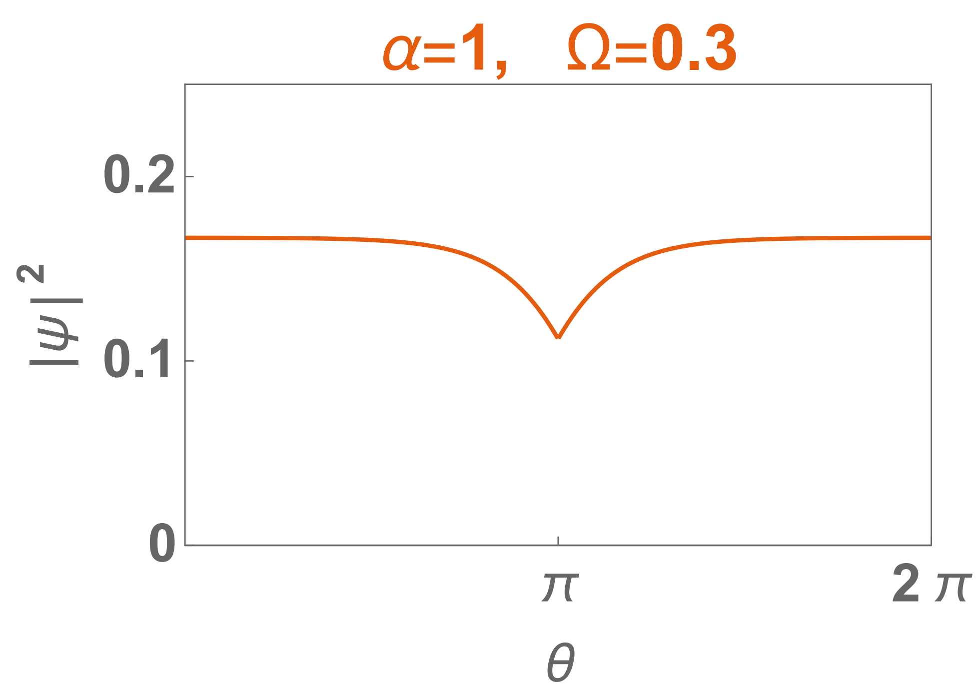

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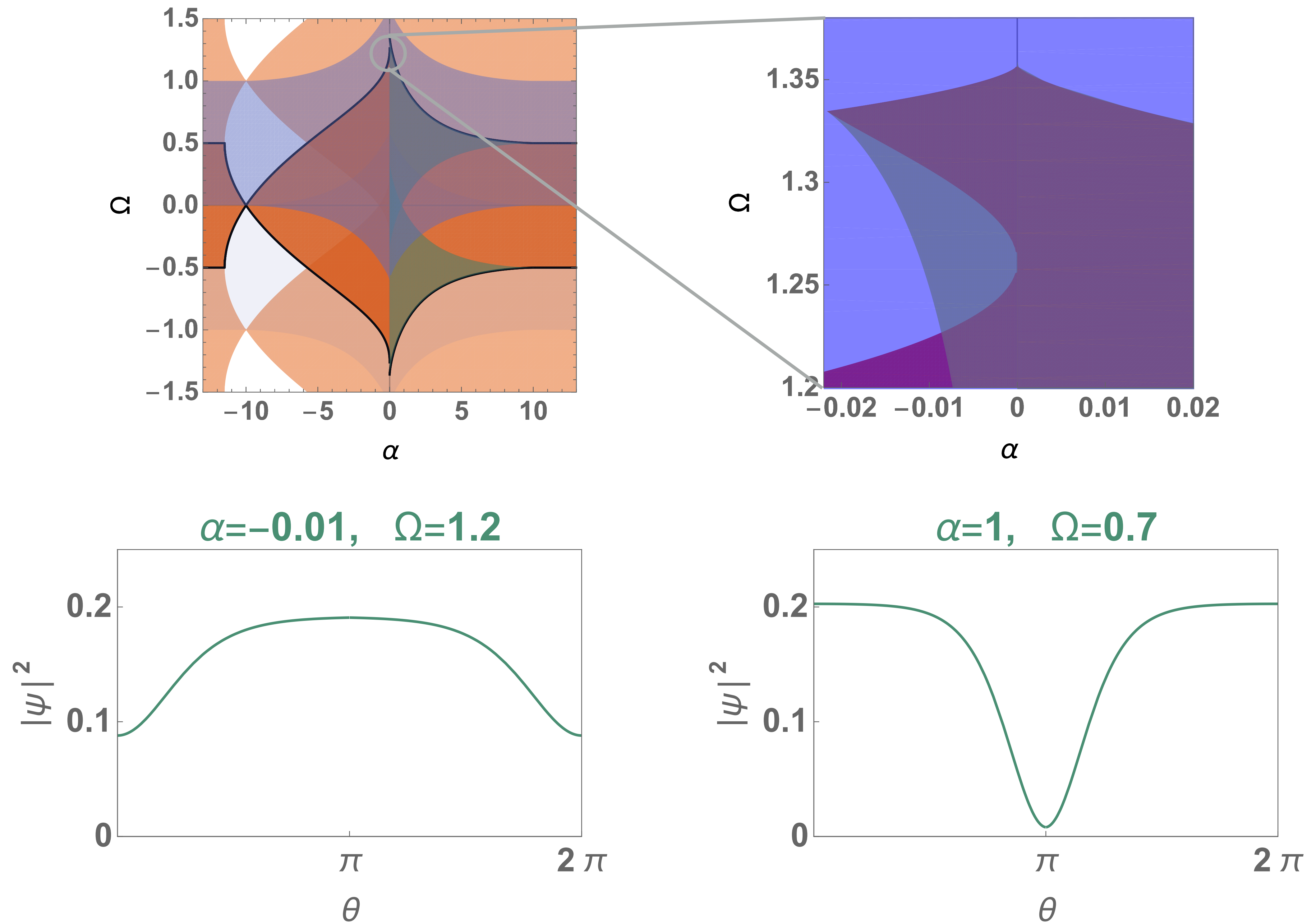

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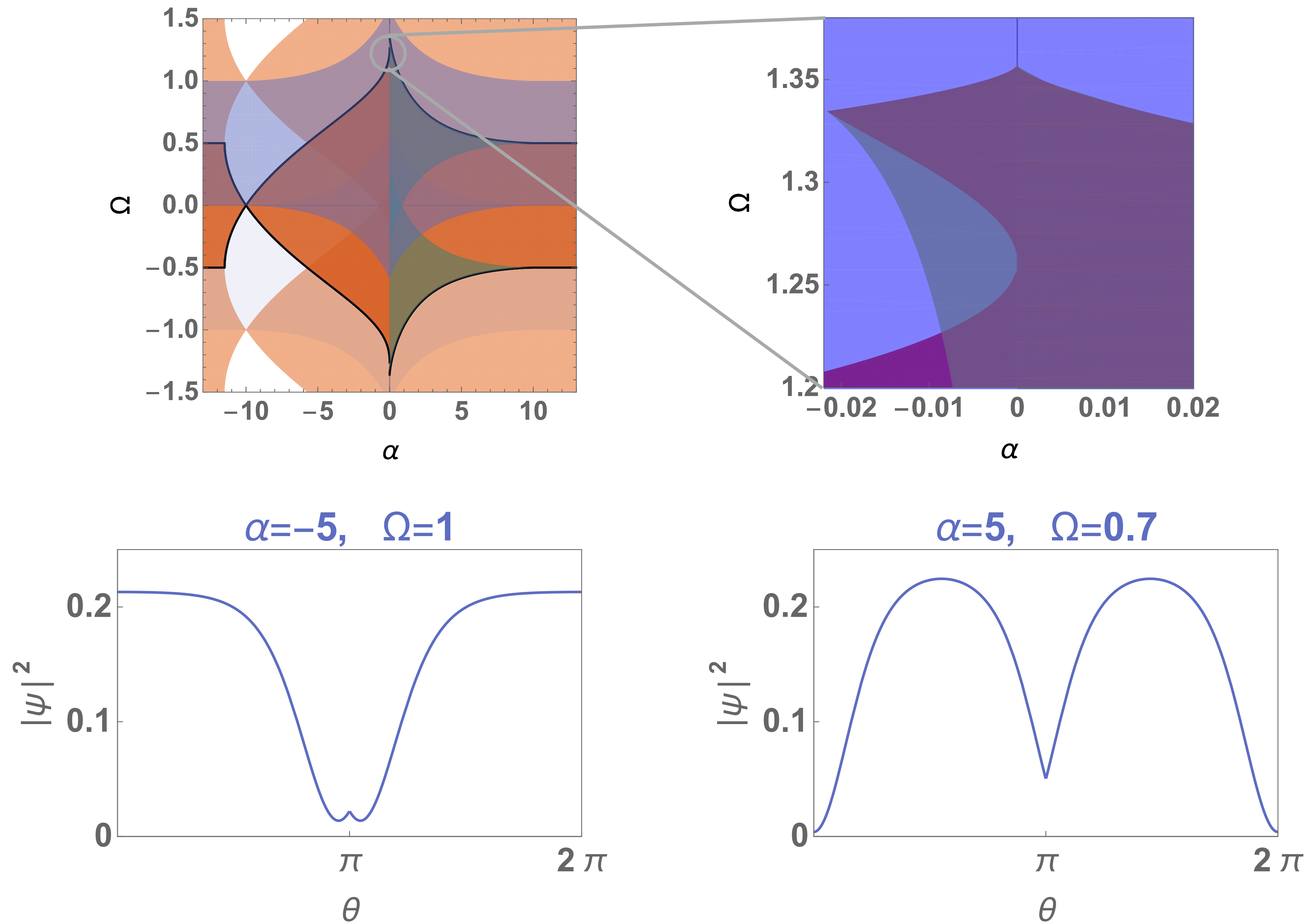

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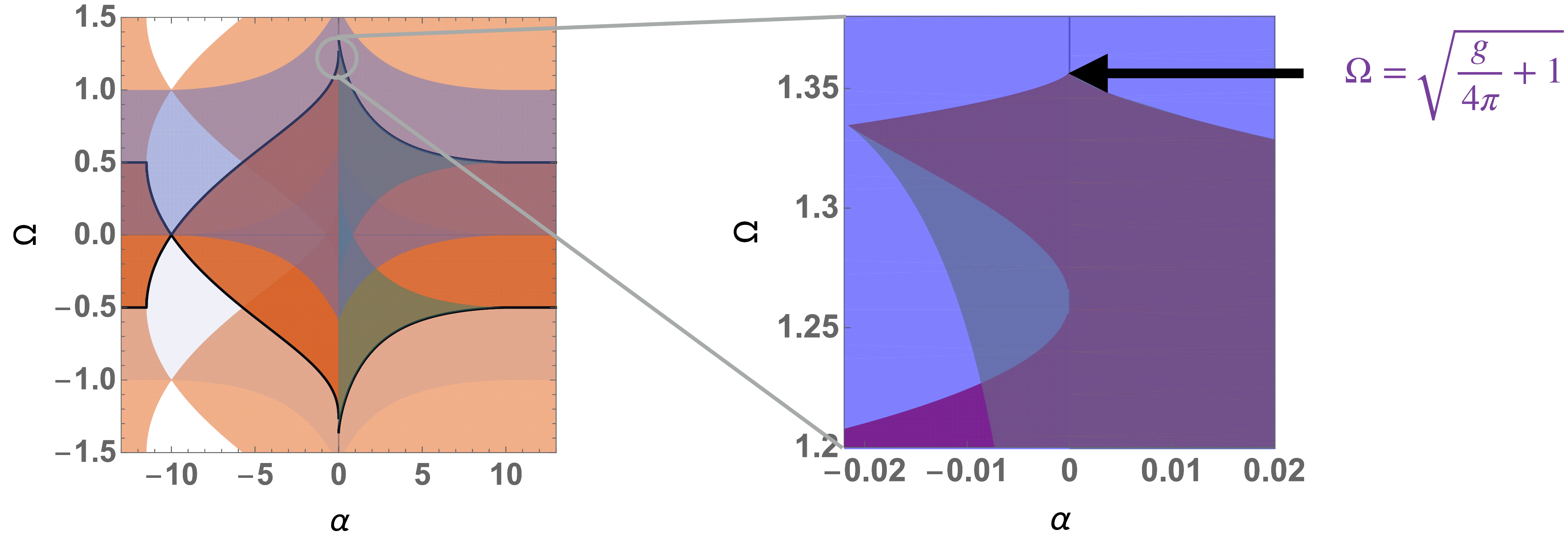
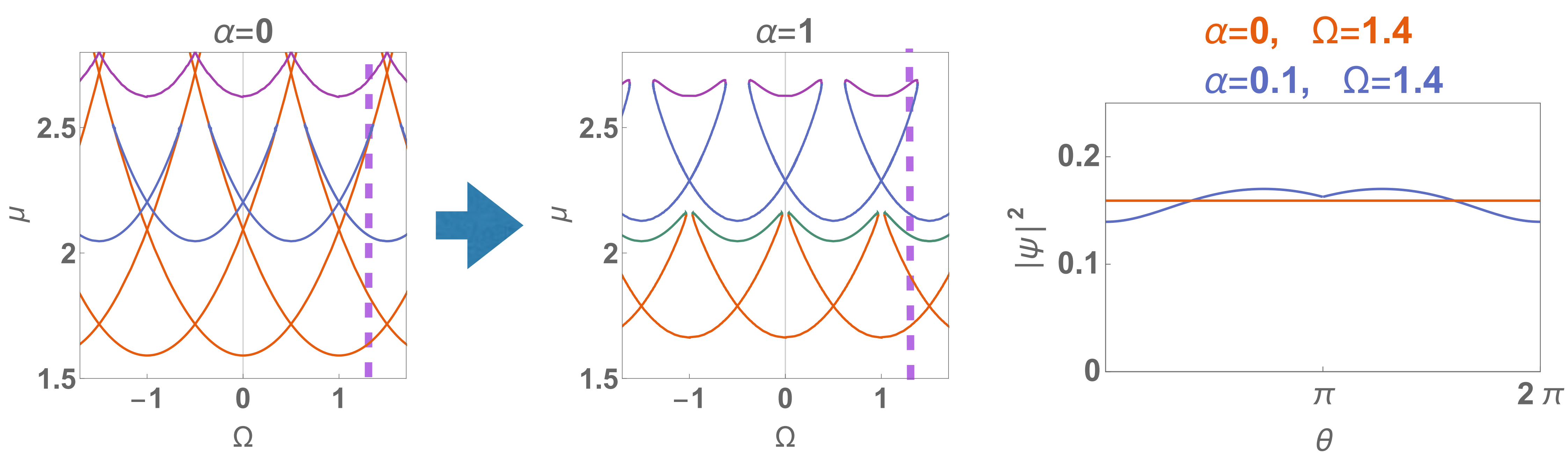
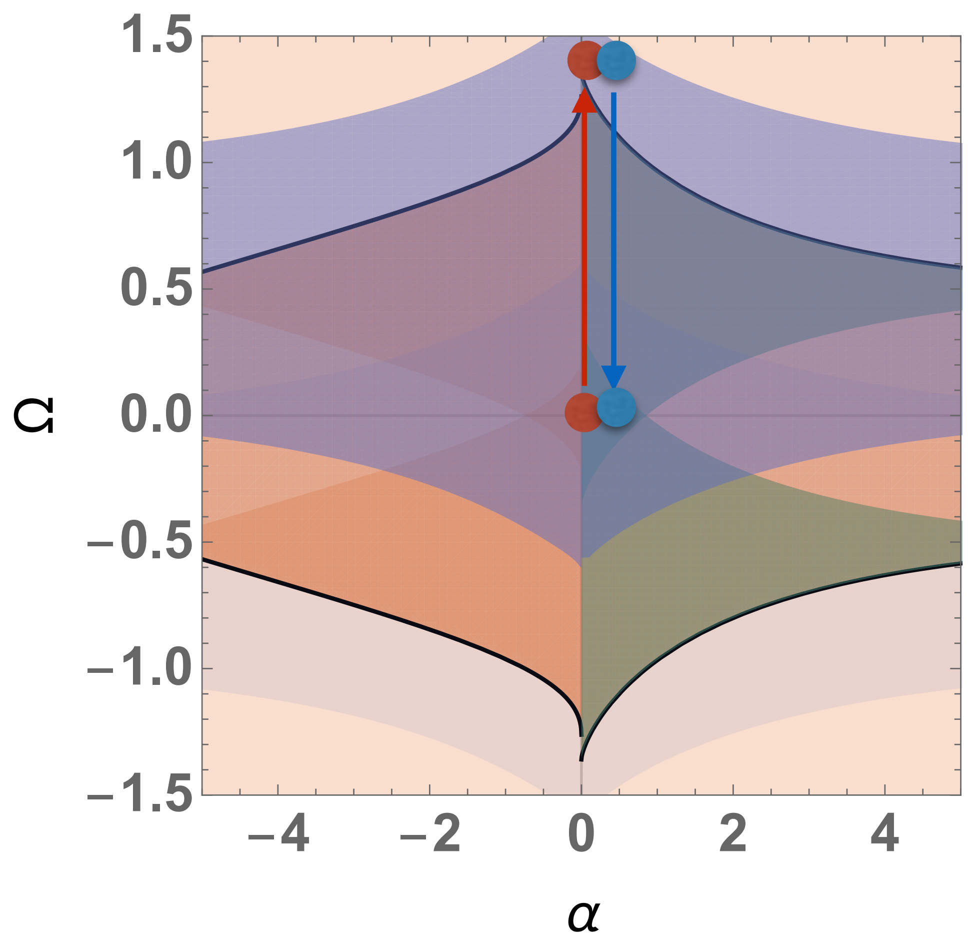
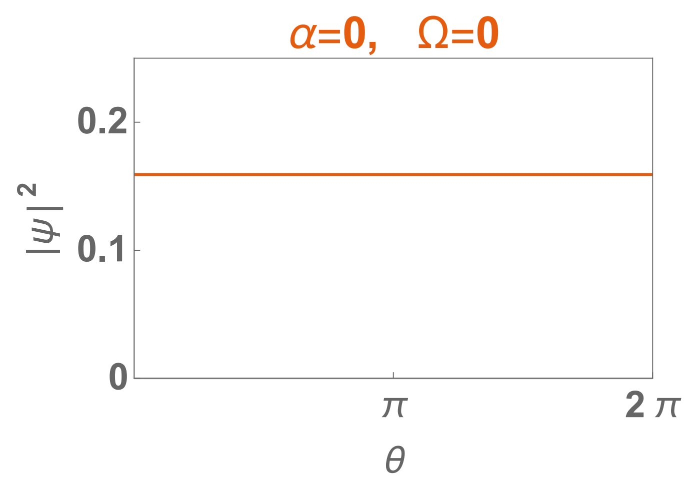




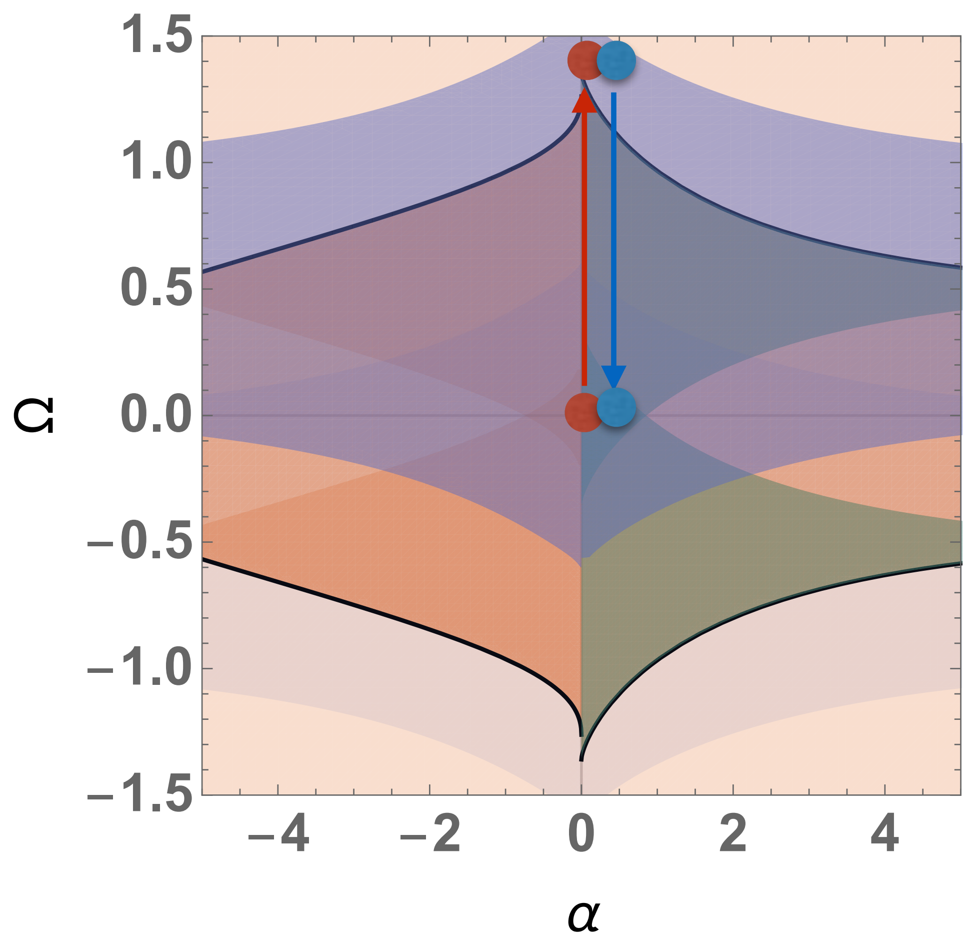
Adiabatic cycle

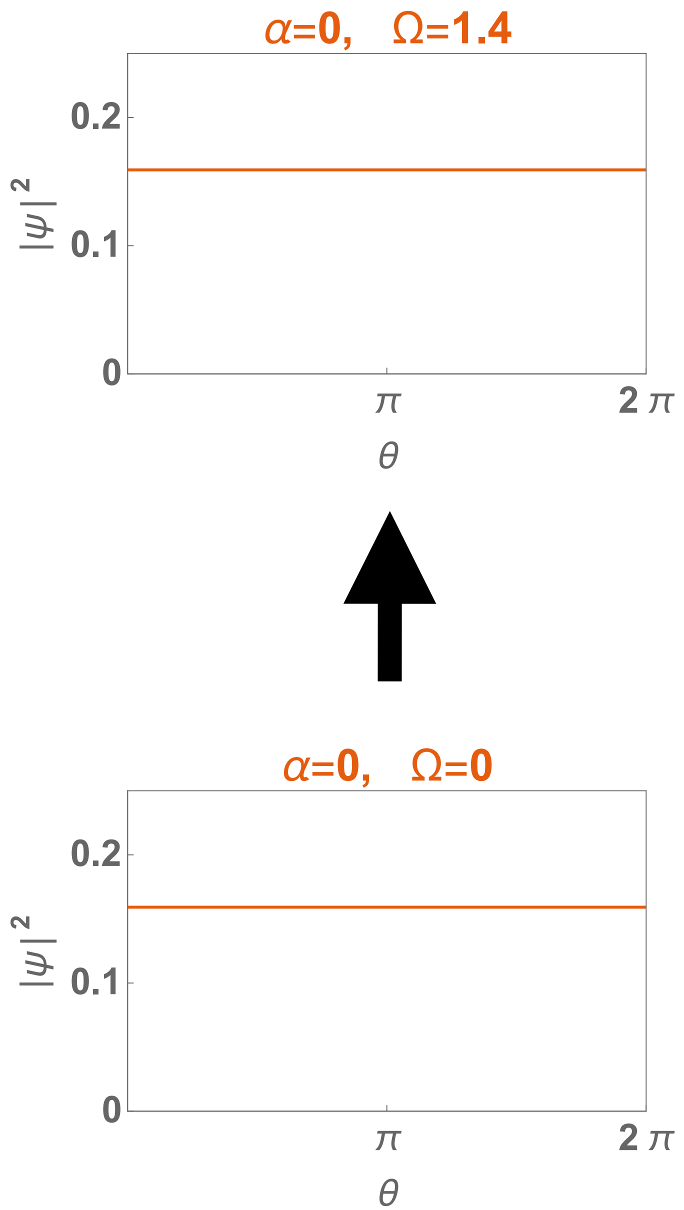




Adiabatic cycle

$$\sqrt{\frac{g}{4\pi}+1} < \Omega < \sqrt{\frac{g}{4\pi}+\frac{9}{4}}$$





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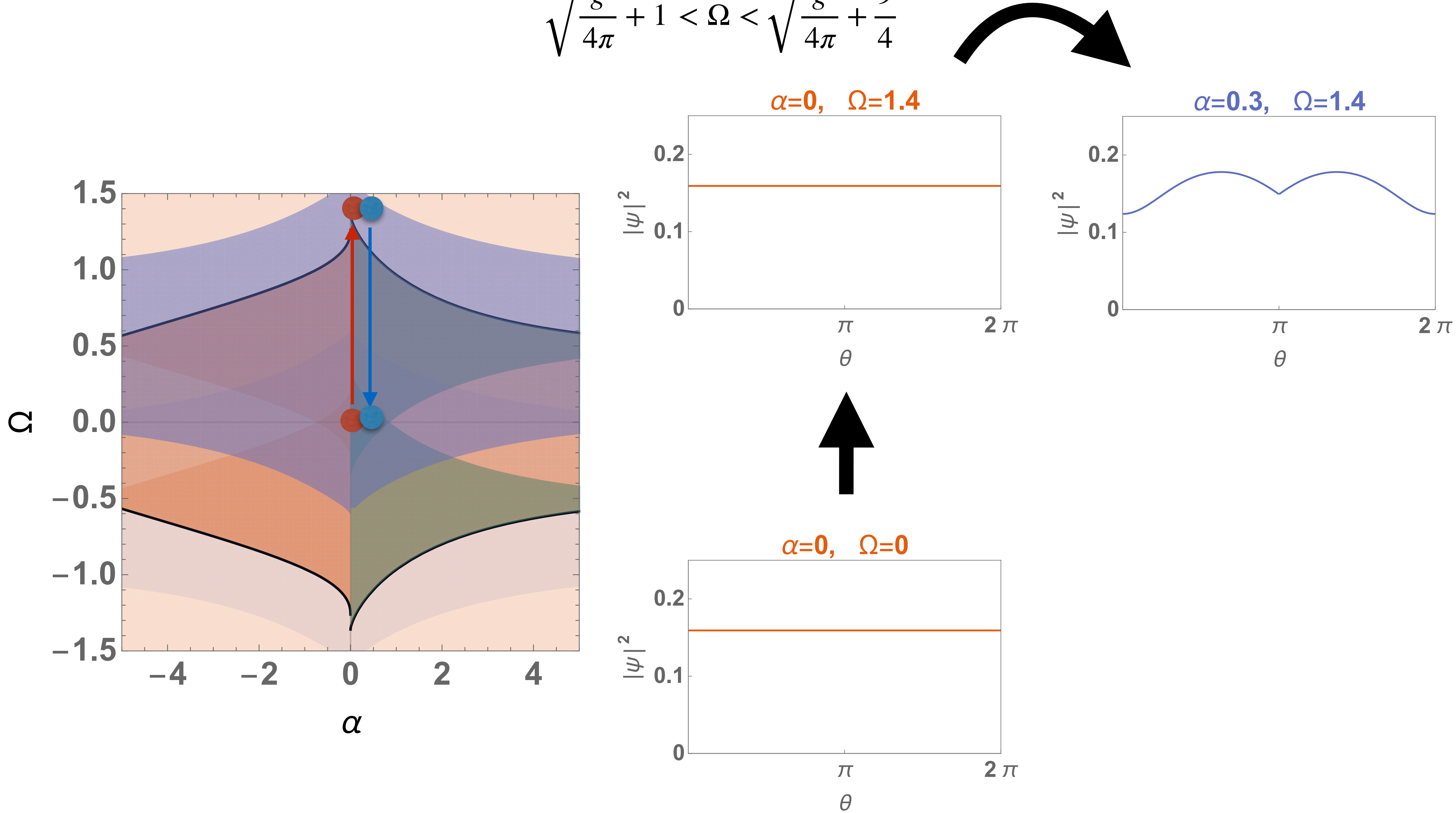

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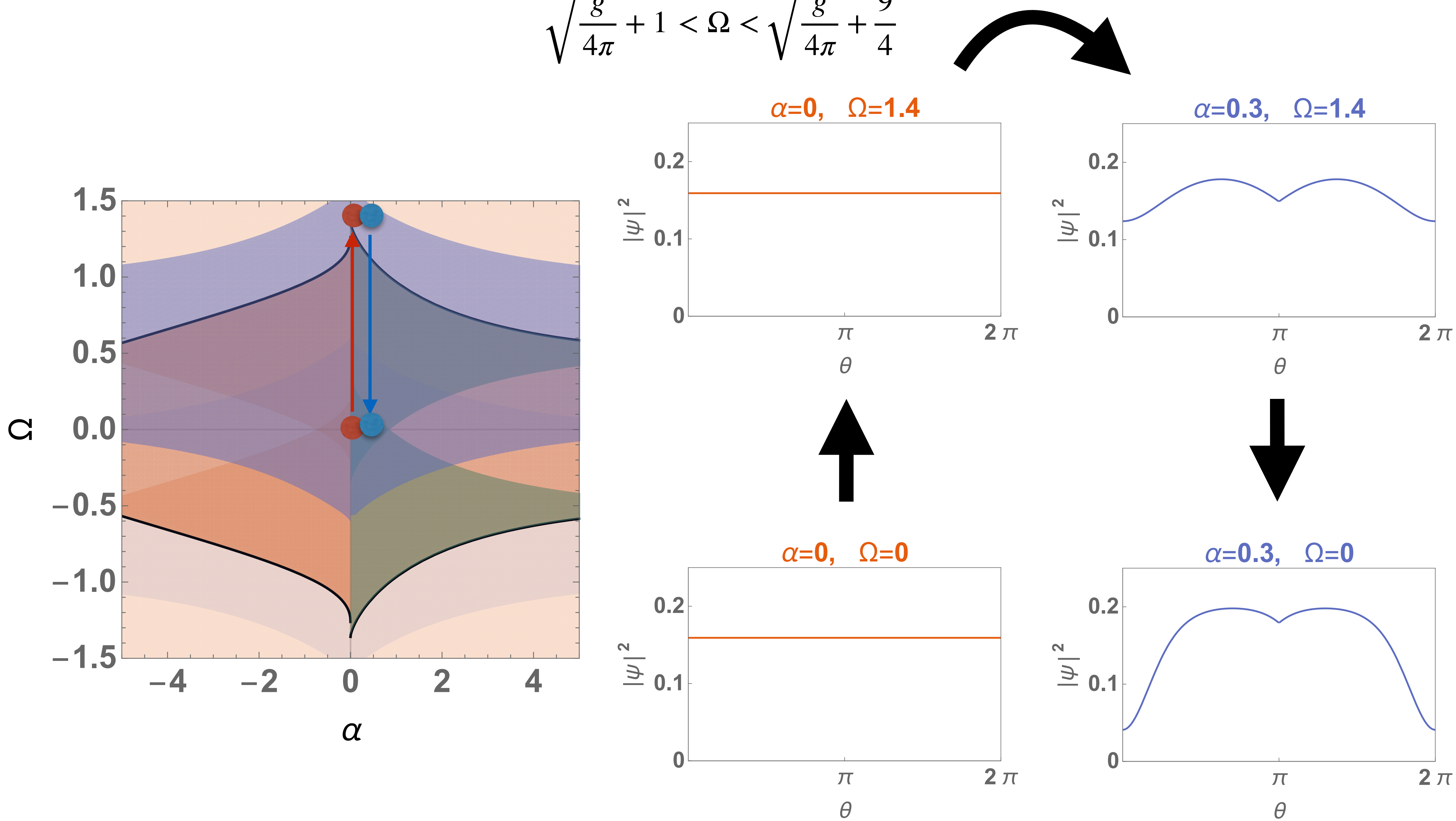

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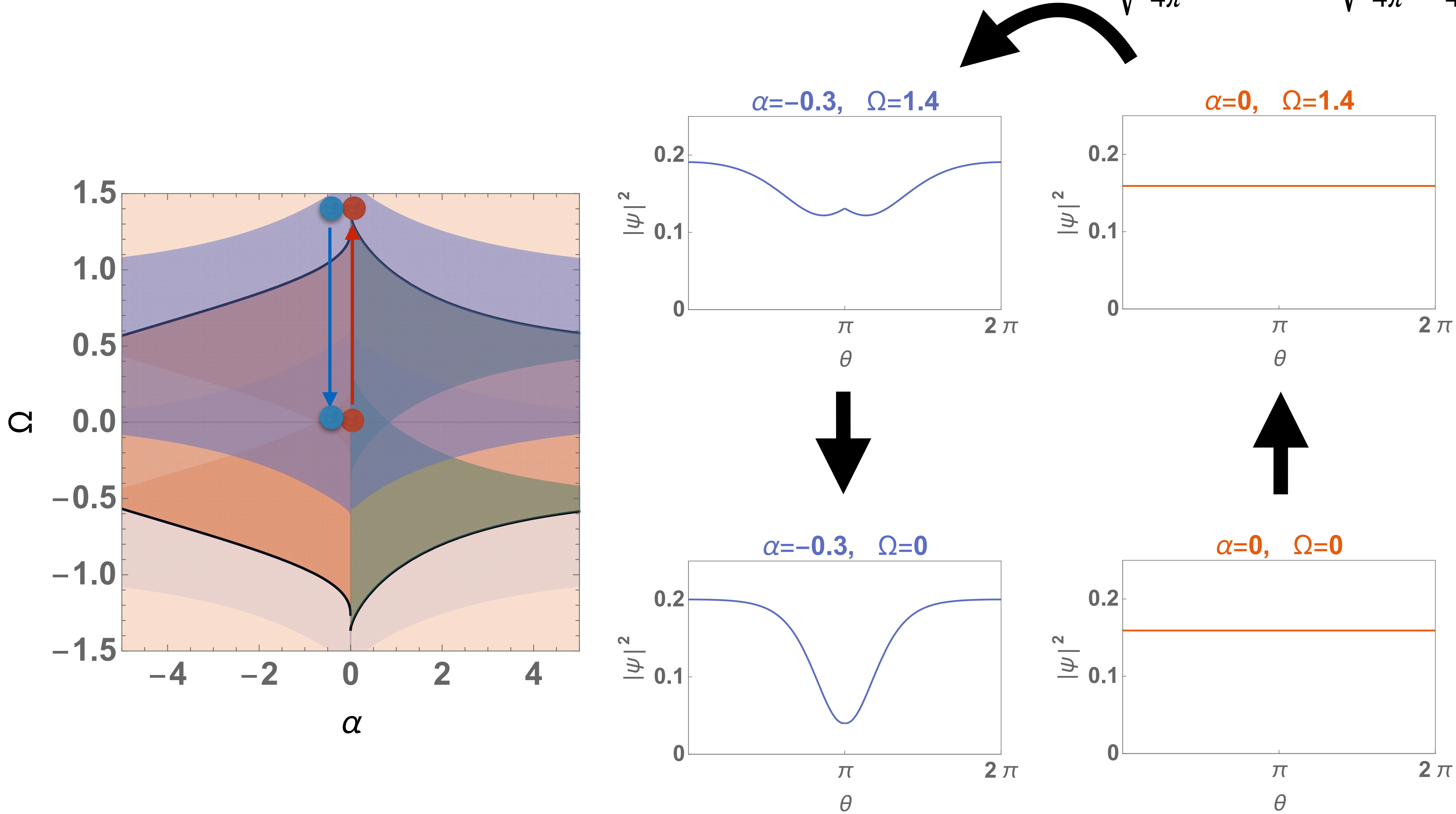
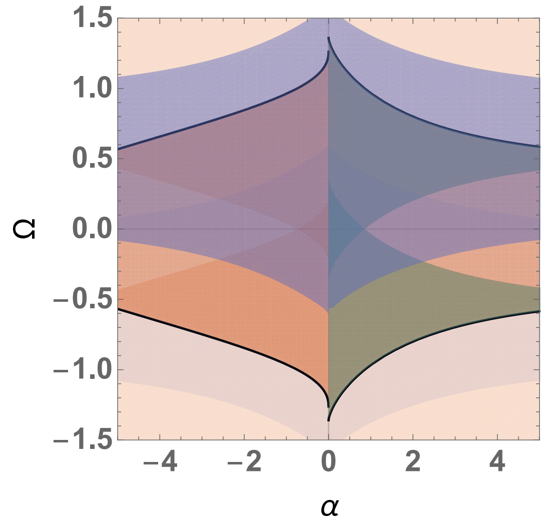
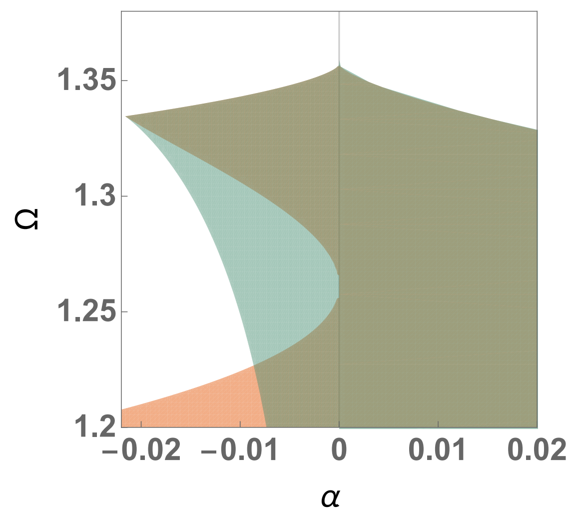
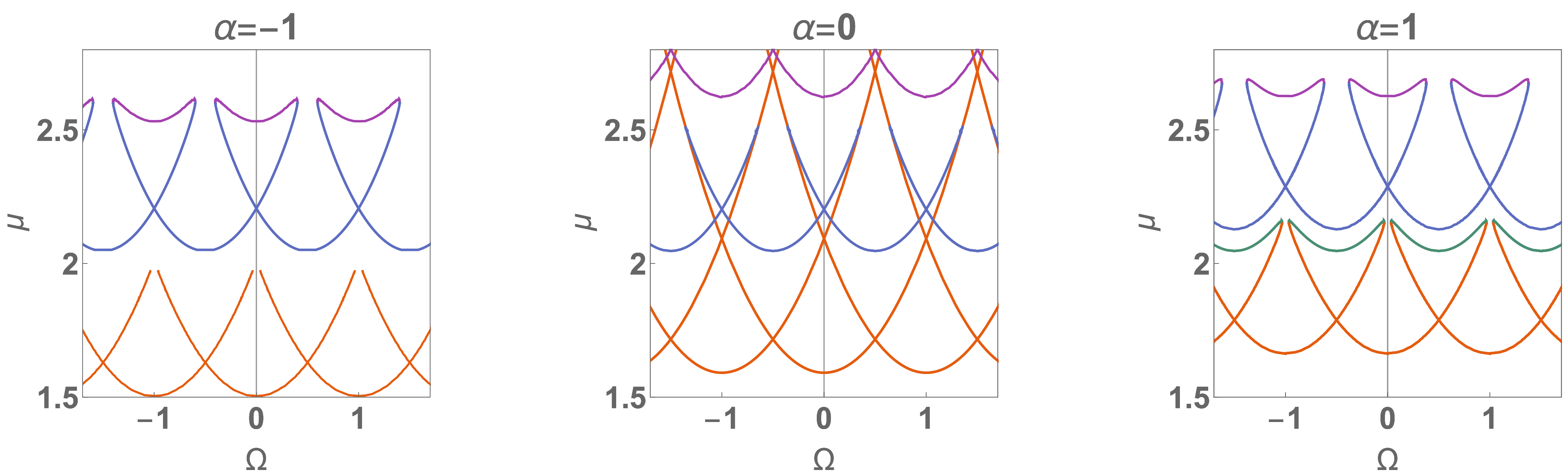


Stability: Bogoliubov analysis





- Add perturbation to solution & linearize GP
- Expand in basis that satisfies delta conditions
- Solve matrix eigenvalue problem



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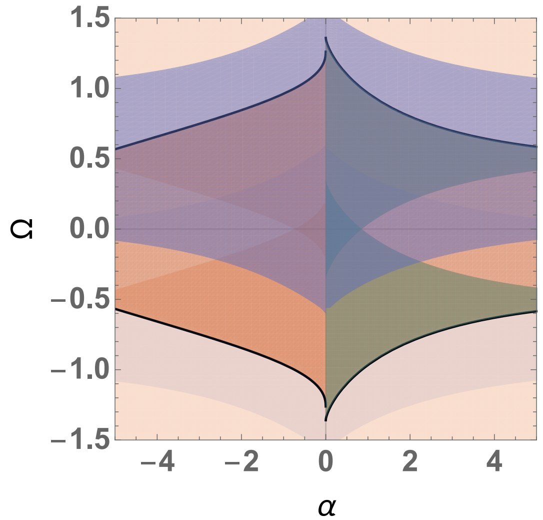

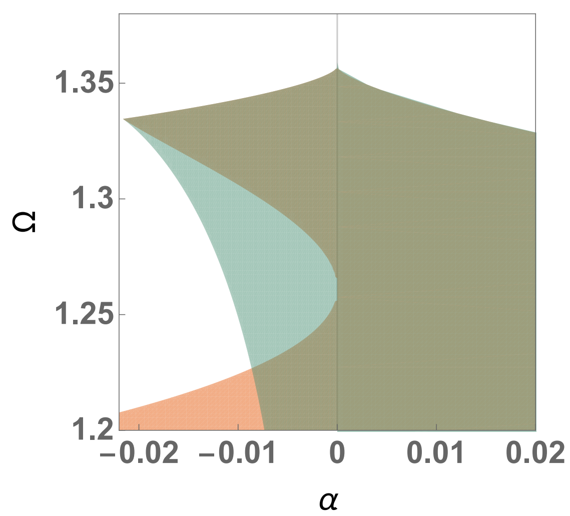

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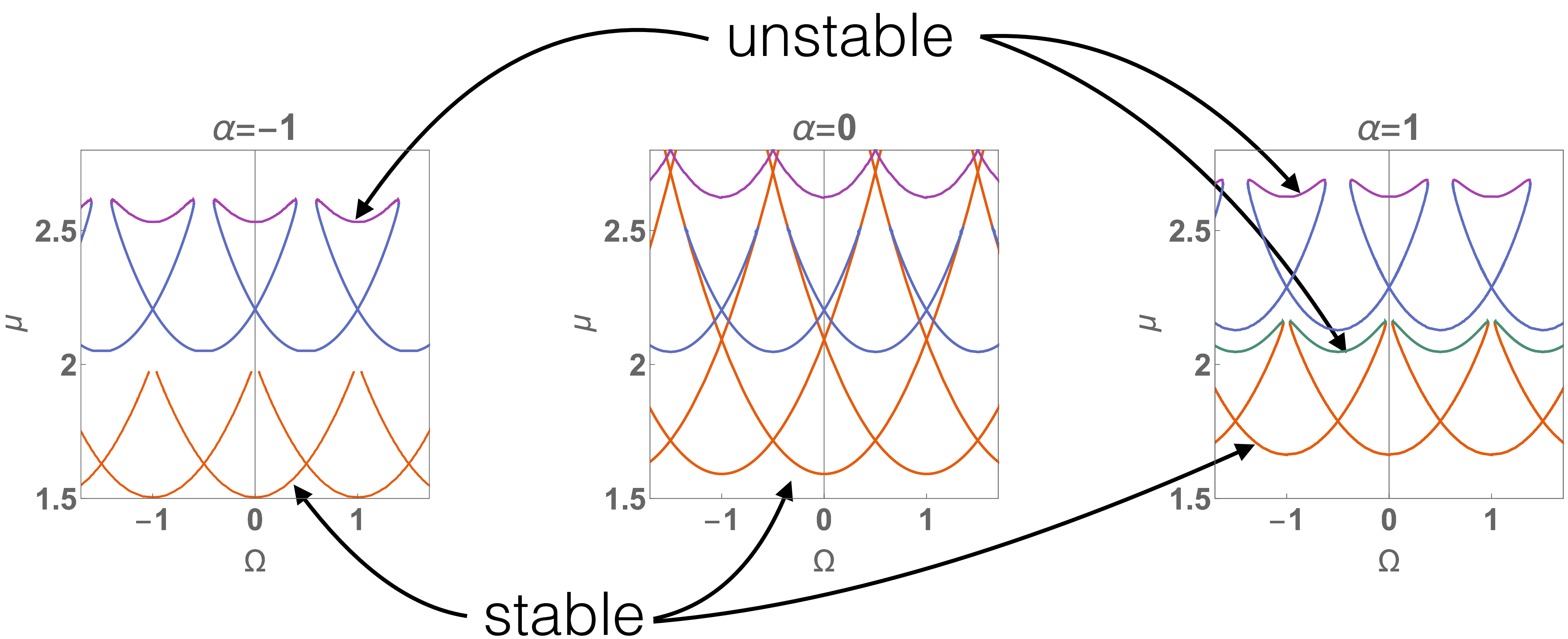

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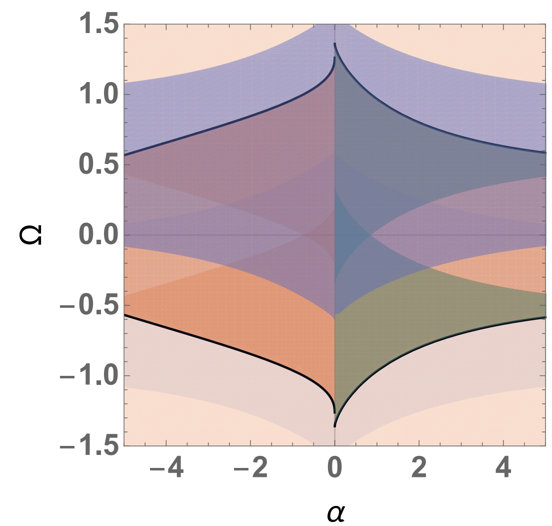

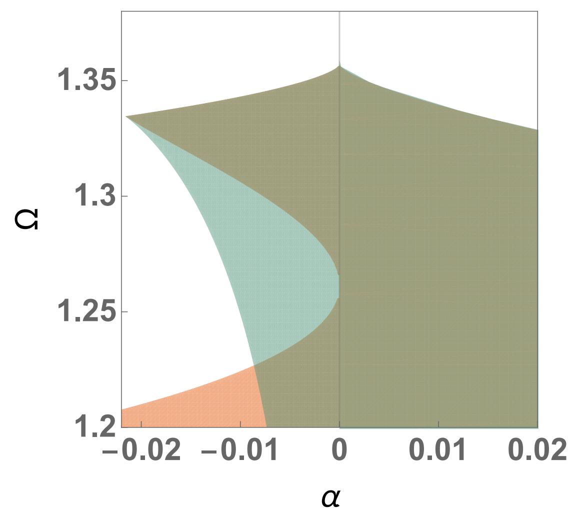

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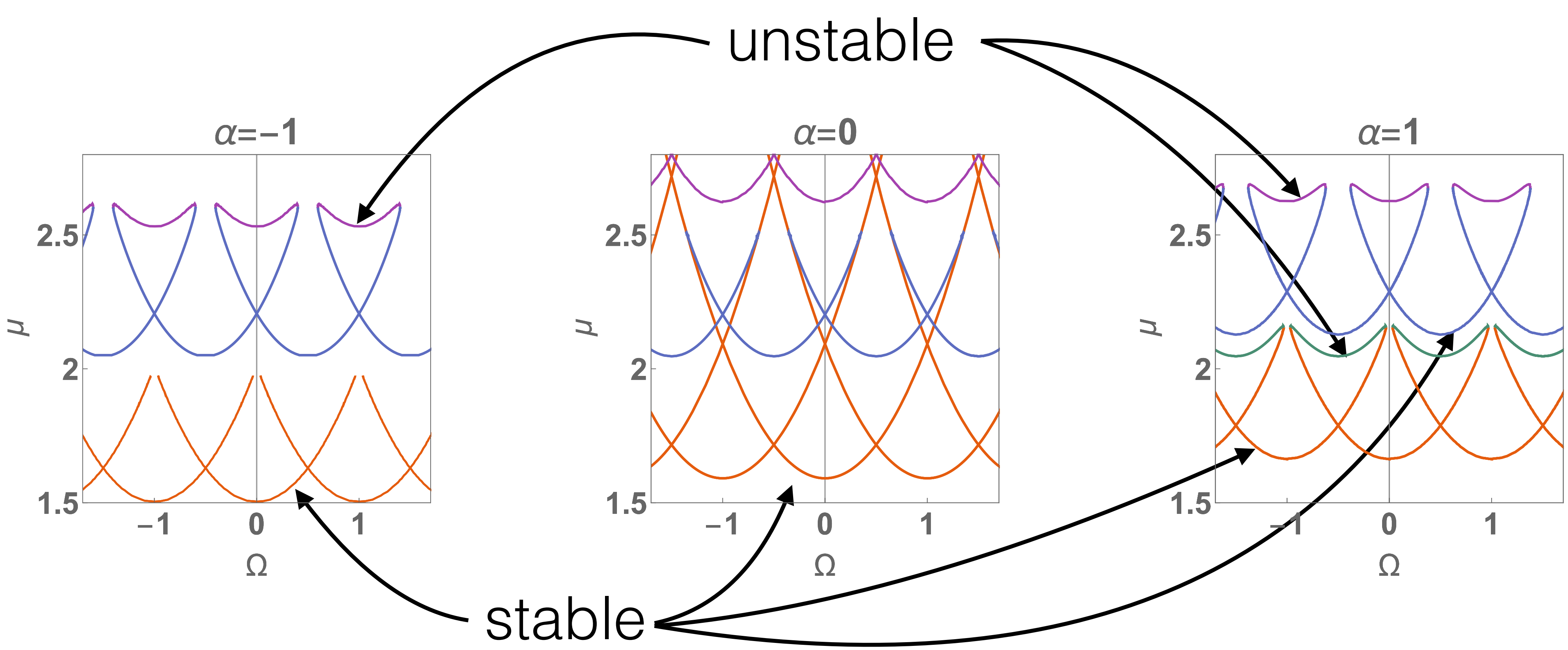

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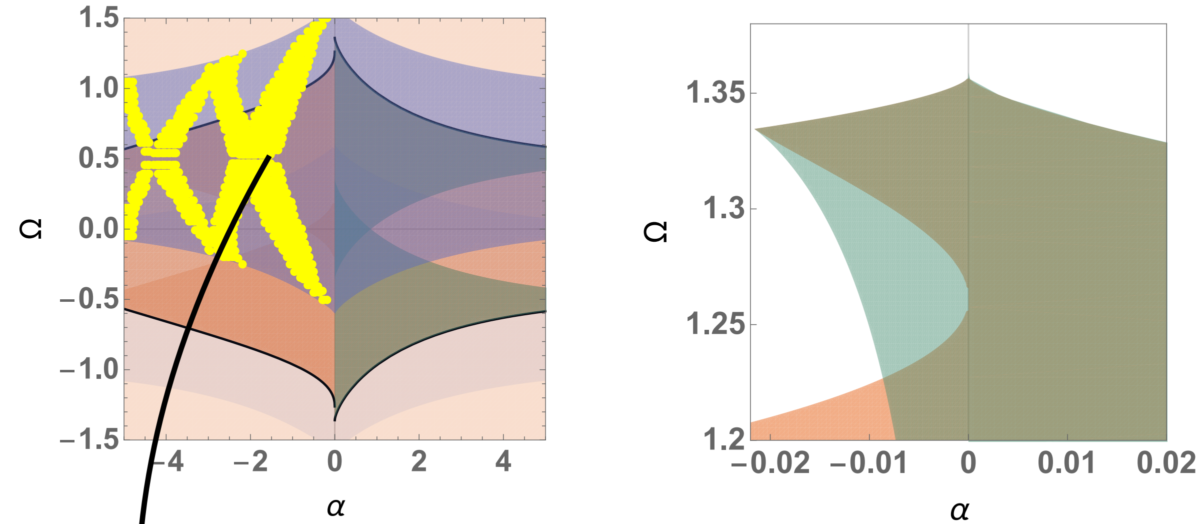

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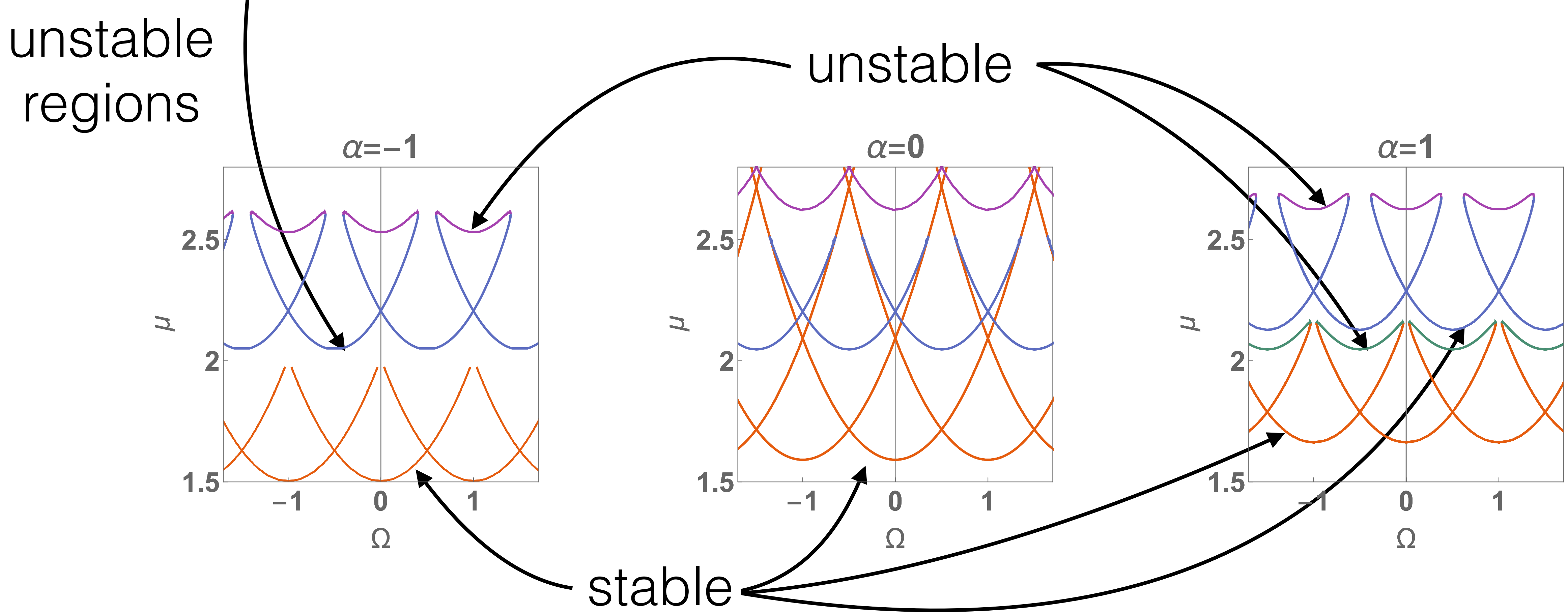
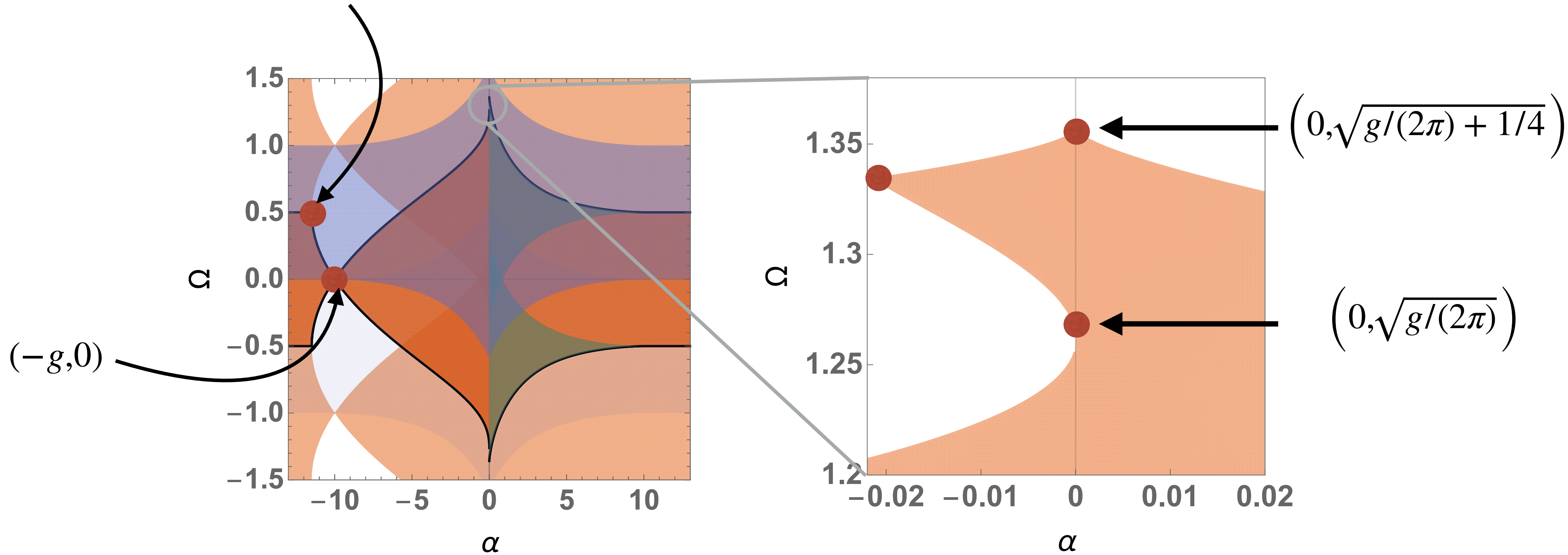
- Expand in basis that satisfies delta conditions
- Solve matrix eigenvalue problem



Dependence on g:

$$\left(-\frac{8\pi k^3 \tan(\pi k)}{g\cos(2\pi k)+g+4\pi k^2-2k\sin(2\pi k)}, \frac{1}{2}\right) \text{ with } g+2\pi k^2-2k\tan(k\pi)=0$$



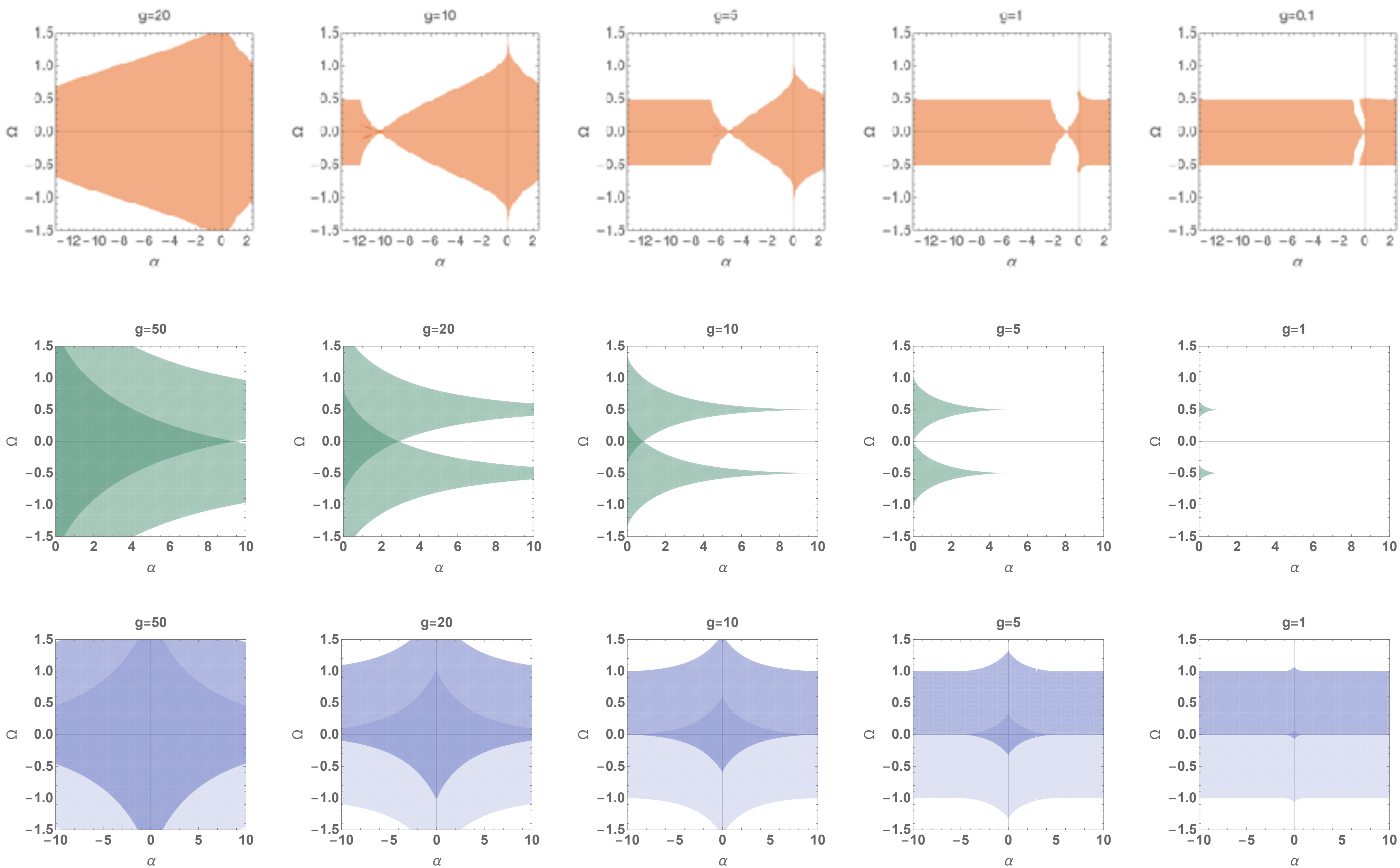
linear limit: g=0

Jacobi functions → trigonometric functions

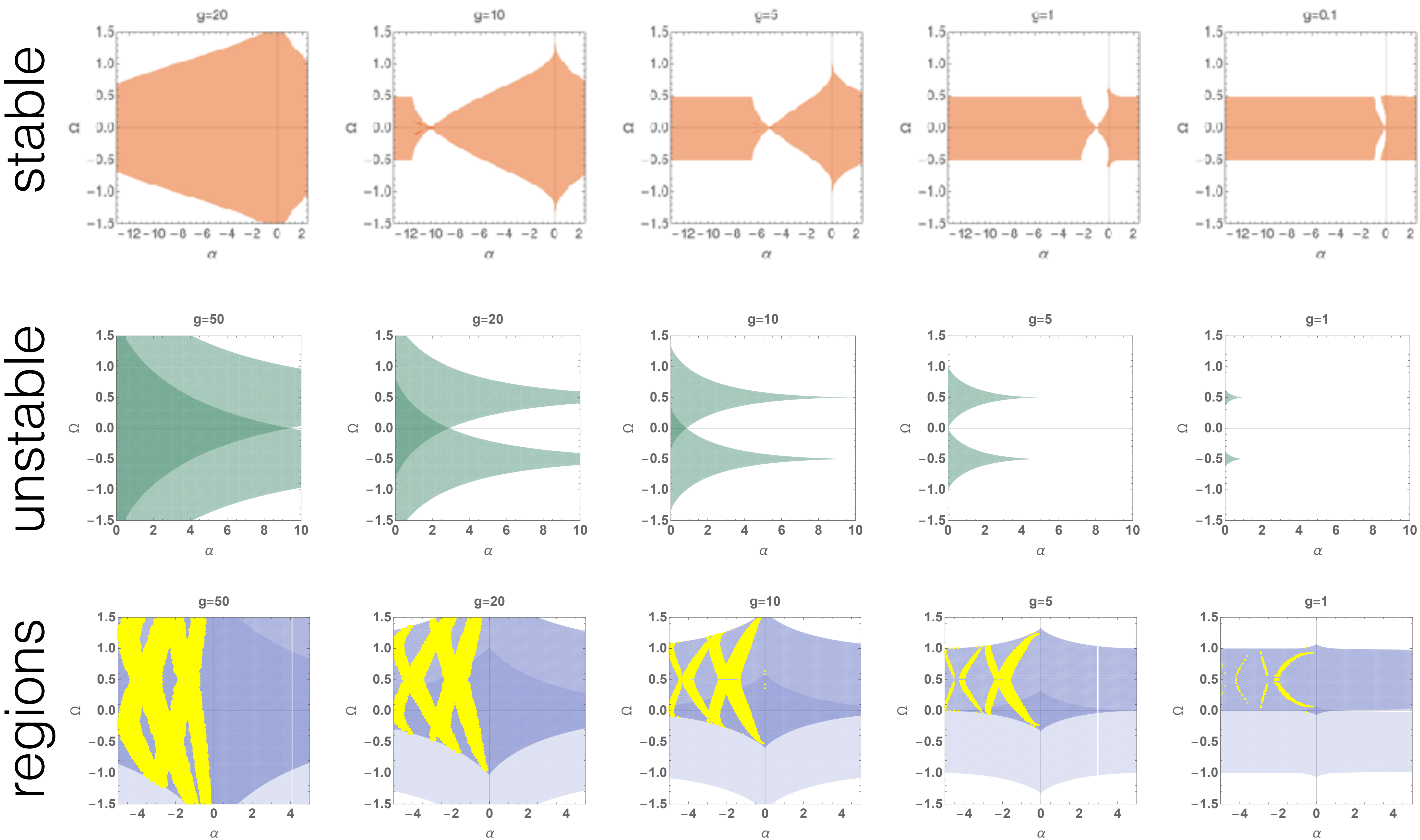
$$r_c^2 = A_c\left[1+B_c\cos(k(\theta-\pi))^2\right]$$

$$r_{ch}^2 = A_{ch}\left[1+B_{ch}\cosh(k(\theta-\pi))^2\right]$$

Dependence on g:
ground & first excited levels



Dependence on g:
ground & first excited levels



Summary

- **Spectrum of GP with rotating delta:** 3D swallow tail structure, degeneracy lines, solitonic trains
- **Bogoliubov analysis:** stable & unstable levels, regions
- **Adiabatic cycles:** excite the BEC with rotating delta
- **Dependence of g:** various g computed, linear limit

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