Non-Abelian geometric transformations in a cold Fermionic strontium gas

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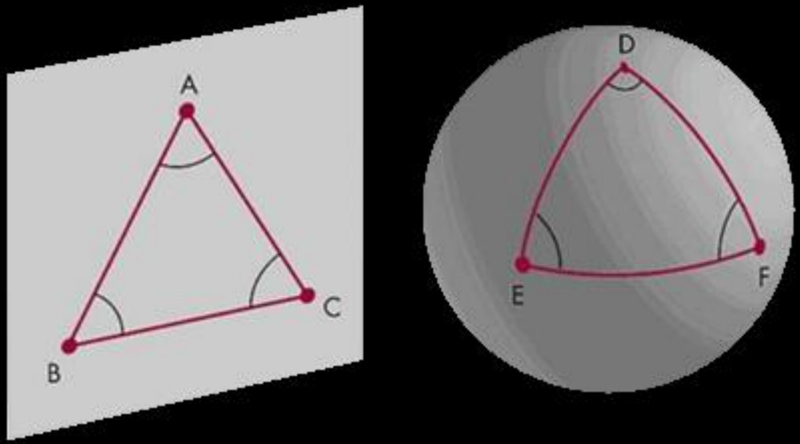
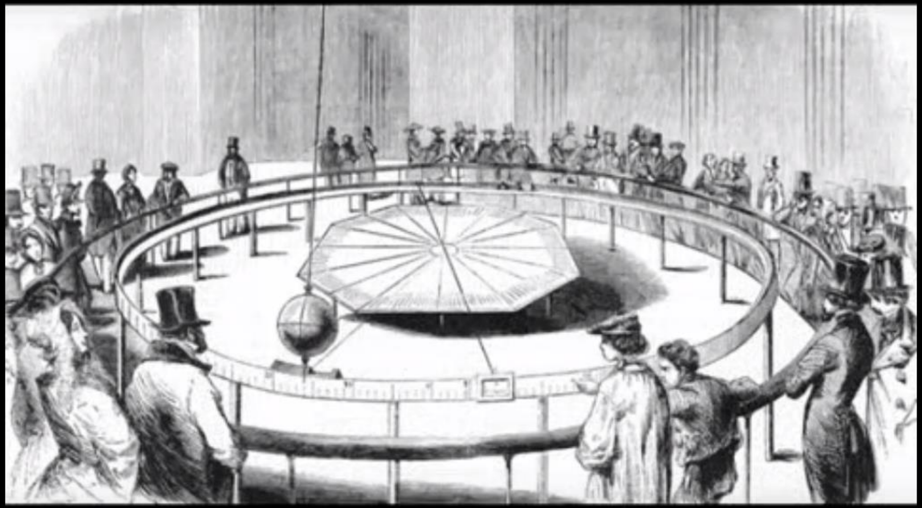
Benasque, May 16th 2019



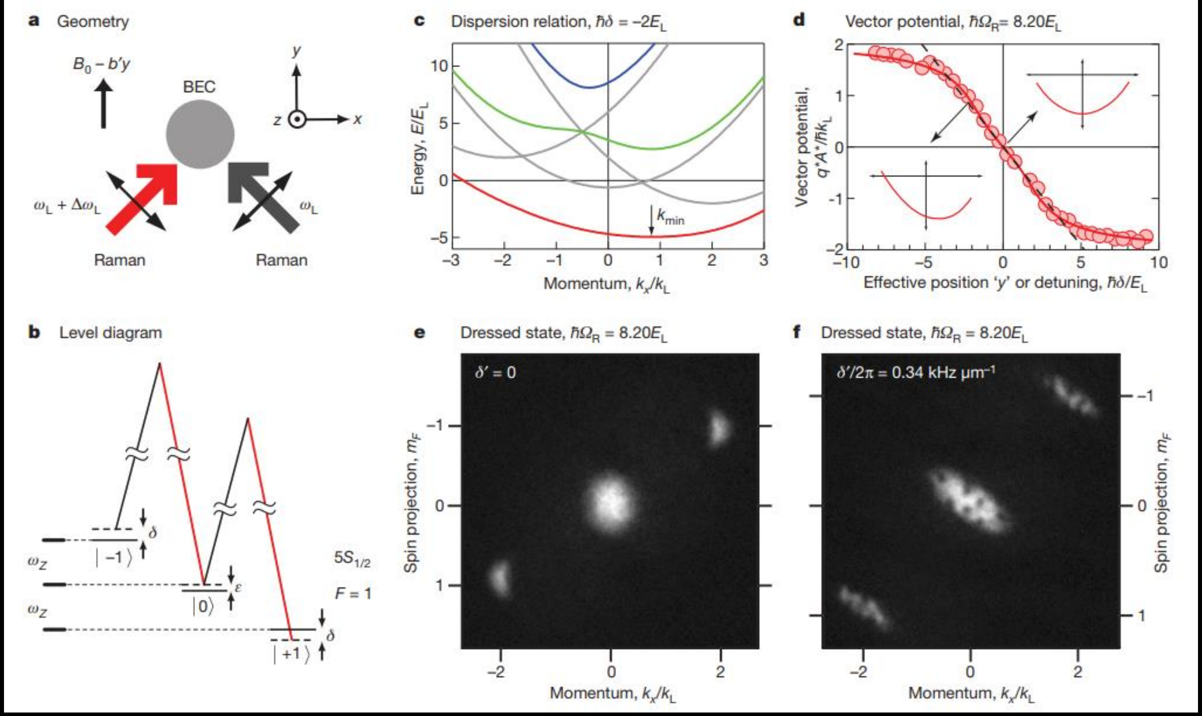
Triangle on flat/curve space



Foucault Pendulum (Paris, 1851)

Synthetic gauge fields in ultracold gases

Synthetic magnetic field: Y.-J. Lin et al, *Nature* **462**, 628 (2009)
Edge state on lattice strip: M. Mancini et al, *Science* **349**, 1510 (2015)
Quantum Hall: M. Aidelsburger et al, *Nat. Phys.* **11**, 162 (2015)



Extracted from: Y.-J. Lin et al, *Nature* **462**, 628 (2009)

Quantum computing

Topologically protected Quantum state

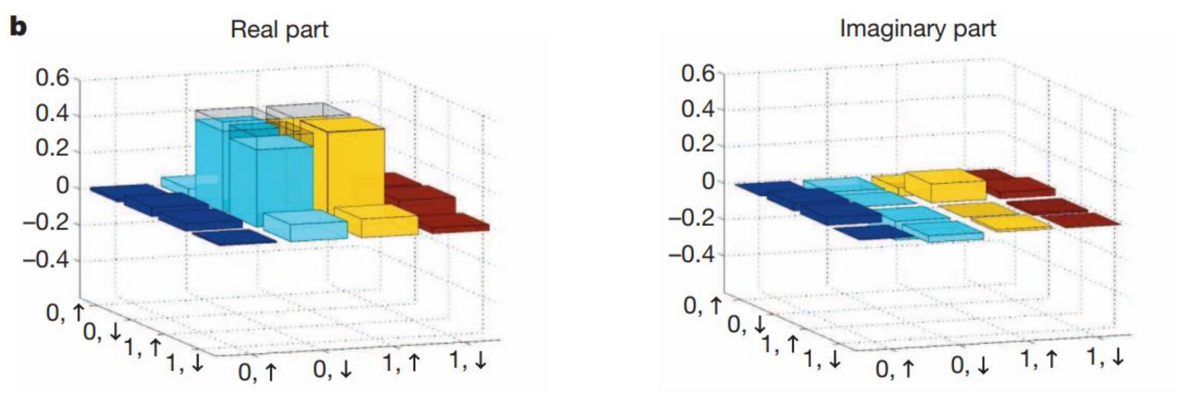
Fractional hall effect ($\nu = 5/2$): S. Das Sarma et al, *PRL* **94**, 166802 (2005)

Geometrical quantum gate

NV centers: C. Zu et al, *Nature* **514**, 72 (2014)
Superconducting circuit: A. Abdumalikov Jr et al, *Nature* **496**, 482 (2013)

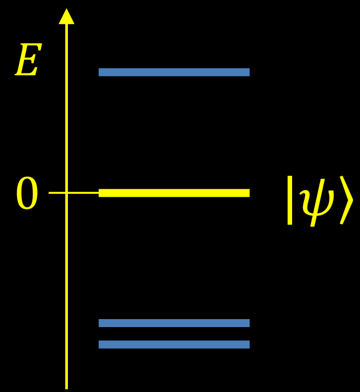
a

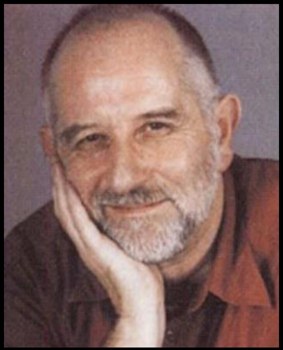
Two-qubit geometric CNOT gate						
Initial state	$\|0,\uparrow\rangle$	$\|0,\downarrow\rangle$	$\|1,\uparrow\rangle$	$\|1,\downarrow\rangle$	$\|0\rangle(\|\uparrow\rangle+\|\uparrow\rangle)$	$\|0\rangle(\|\uparrow\rangle+i\|\downarrow\rangle)$
Ideal final state	$\|1,\uparrow\rangle$	$\|0,\downarrow\rangle$	$\|0,\uparrow\rangle$	$\|1,\downarrow\rangle$	$\|1,\uparrow\rangle+\|0,\downarrow\rangle$	$\|1,\uparrow\rangle+i\|0,\downarrow\rangle$
Measured fidelity	0.99(1)	0.97(1)	0.87(1)	0.94(2)	0.90(3)	0.86(4)

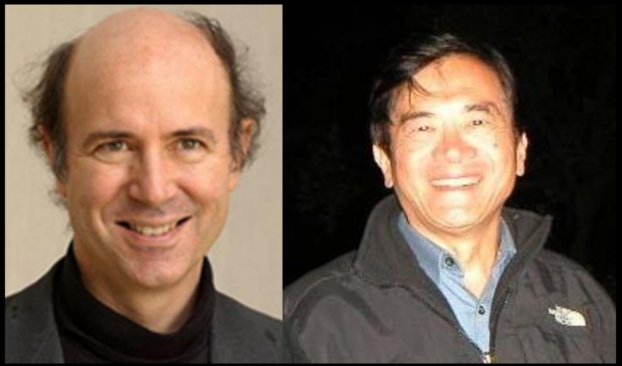


Extracted from: C. Zu et al, *Nature* **514**, 72 (2014)

Geometric gate: Degenerated states or not



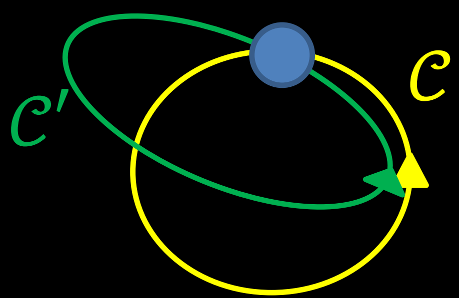




F. Wilczek and A. Zee PRL **52**, 2111 (1984)

M. V. Berry Proc. R. Soc. Lond. A **392**, 45 (1984)

Non degenerated states

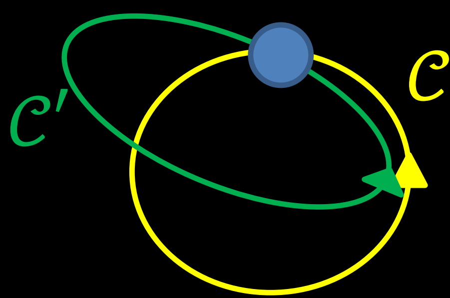


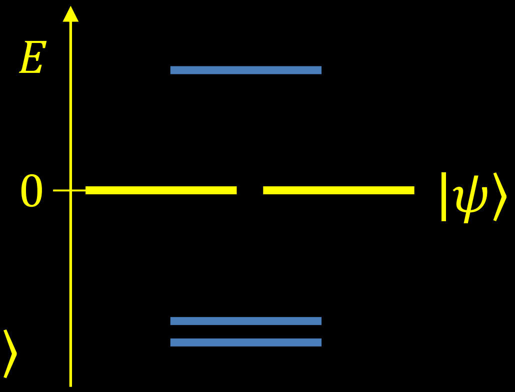
$$|\psi(\mathcal{C})\rangle = e^{i\gamma}|\psi(0)\rangle$$

$$e^{i\gamma}e^{i\gamma'} = e^{i\gamma'}e^{i\gamma}$$

Abelian transformation

Degenerated states



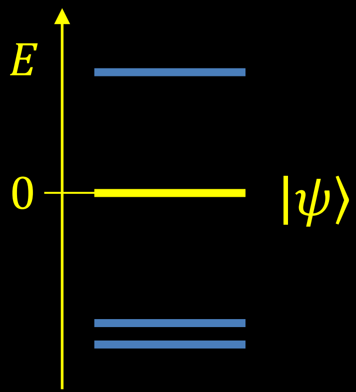


$$|\psi(\mathcal{C})\rangle = U|\psi(0)\rangle$$

$UU' \neq U'U$: Path ordering sensitivity

Non-Abelian transformation

Geometric gate: Degenerated states or not

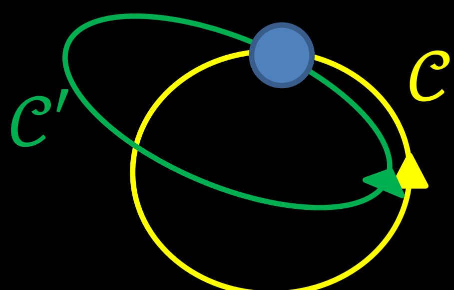


$$U = \wp\exp\left(-\oint_C \vec{A}\mathrm{d}\vec{\lambda}\right)$$

$\vec{\lambda}$: parameters
C: close loop

$\vec{A}$: vector potential or Berry connection.
$\wp$: path ordering operator.
$\vec{\Phi} = \vec{\nabla} \times \vec{A} + \vec{A} \times \vec{A}$: Berry curvature.

Non degenerated states

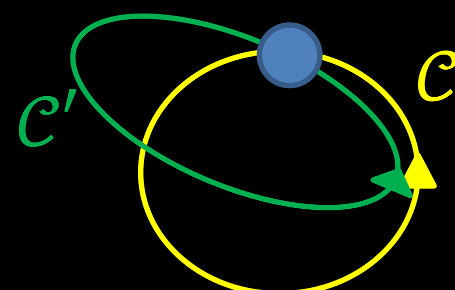


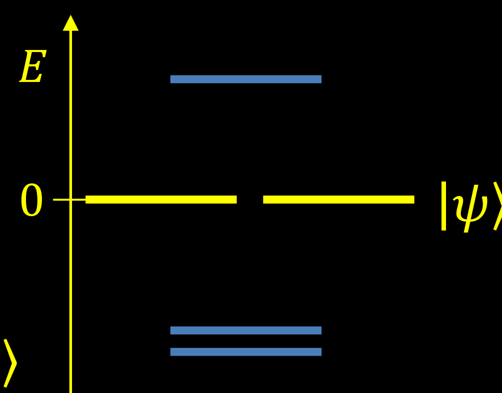
$|\psi(\mathcal{C})\rangle = e^{i\gamma}|\psi(0)\rangle$

$e^{i\gamma}e^{i\gamma'} = e^{i\gamma'}e^{i\gamma}$

Abelian transformation

Degenerated states



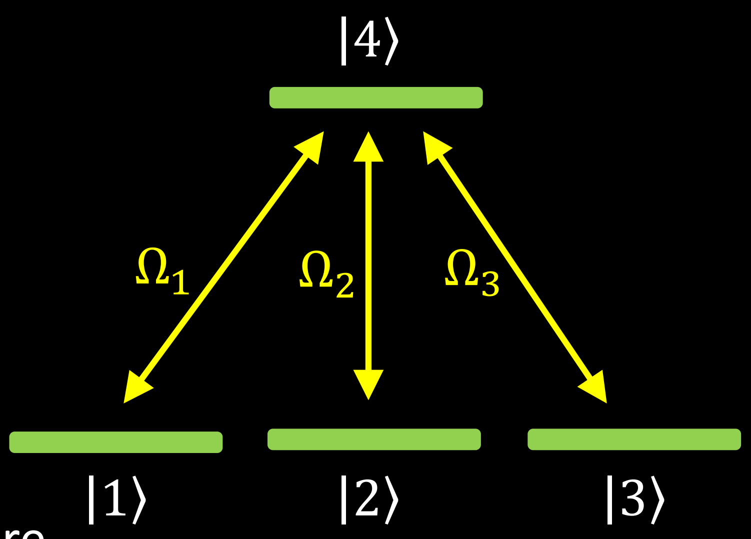


$|\psi(\mathcal{C})\rangle = U|\psi(0)\rangle$

$UU' \neq U'U$: Path ordering sensitivity

Non-Abelian transformation

Content



Tripod system

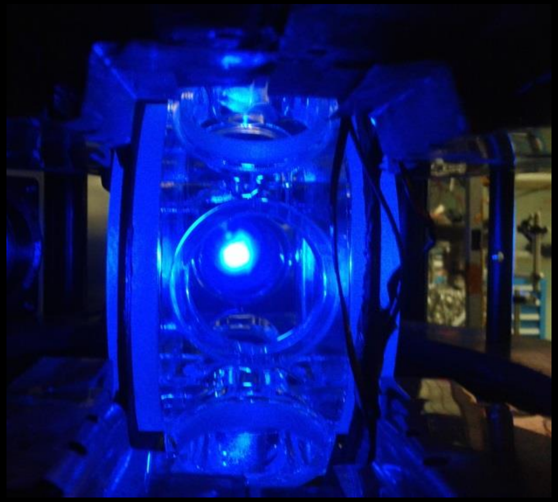
Implementation on a strontium ultracold gas

Interferometric measurement of the temperature

Geometric quantum gate

Path sensitive measurement and non-Abelianicity

Conclusion and perspective for atomtronics



Tripod and degenerated dark states

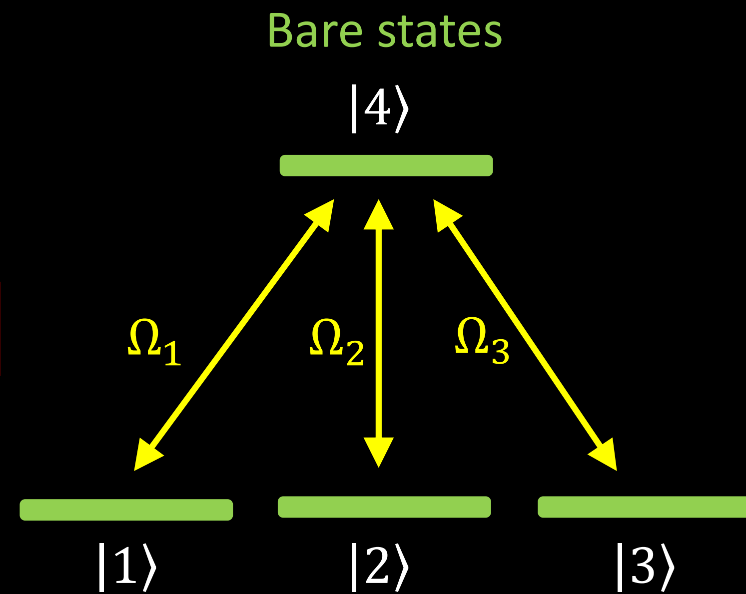
$$
\begin{array}{c}
\begin{array}{cccc} |1\rangle & |2\rangle & |3\rangle & |4\rangle \end{array} \\
H_I = \hbar \begin{pmatrix} 0 & 0 & 0 & \Omega_1^* \\ 0 & 0 & 0 & \Omega_2^* \\ 0 & 0 & 0 & \Omega_3^* \\ \Omega_1 & \Omega_2 & \Omega_3 & 0 \end{pmatrix}
\end{array}
$$

with $\Omega_i = |\Omega_1| e^{i\Phi_i}$

$$\boxed{\Phi_i = \vec{k}_i \vec{r} + \boldsymbol{\phi}_i(t)}$$

External control

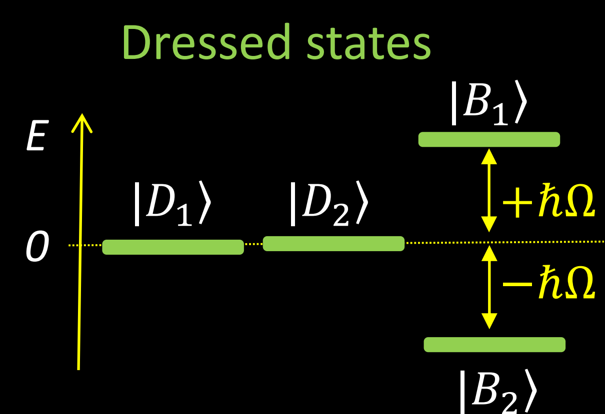
$$|\Omega_1| = \Omega \sin\alpha \cos\beta$$
$$|\Omega_2| = \Omega \sin\alpha \sin\beta$$
$$|\Omega_3| = \Omega \cos\alpha$$



Diagonalization of the Hamiltonian

$$|D_1\rangle = \begin{pmatrix} \sin\beta\, e^{i\Phi_{31}} \\ -\cos\beta\, e^{i\Phi_{32}} \\ 0 \\ \boxed{0} \end{pmatrix} \quad |D_2\rangle = \begin{pmatrix} \cos\alpha \cos\beta\, e^{i\Phi_{31}} \\ \cos\alpha \sin\beta\, e^{i\Phi_{32}} \\ -\sin\alpha \\ \boxed{0} \end{pmatrix}$$

Excited state population is zero

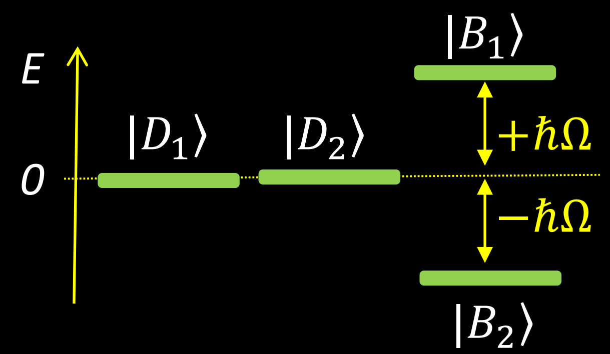


$$\Phi_{ij} = \Phi_i - \Phi_j$$

Tripod: SU(2) non-Abelian transformation

The non-Abelian structure appears in the $\{|D_1\rangle, |D_2\rangle\}$ subspace

$$|D_1\rangle = \begin{pmatrix} \sin\beta\, e^{i\Phi_{31}} \\ -\cos\beta\, e^{i\Phi_{32}} \\ 0 \end{pmatrix} \qquad |D_2\rangle = \begin{pmatrix} \cos\alpha\cos\beta\, e^{i\Phi_{31}} \\ \cos\alpha\sin\beta\, e^{i\Phi_{32}} \\ -\sin\alpha \end{pmatrix}$$



Geometric Gauge field: $\vec{A}_{ij} = i\hbar\langle D_i|\vec{\nabla}D_j\rangle$

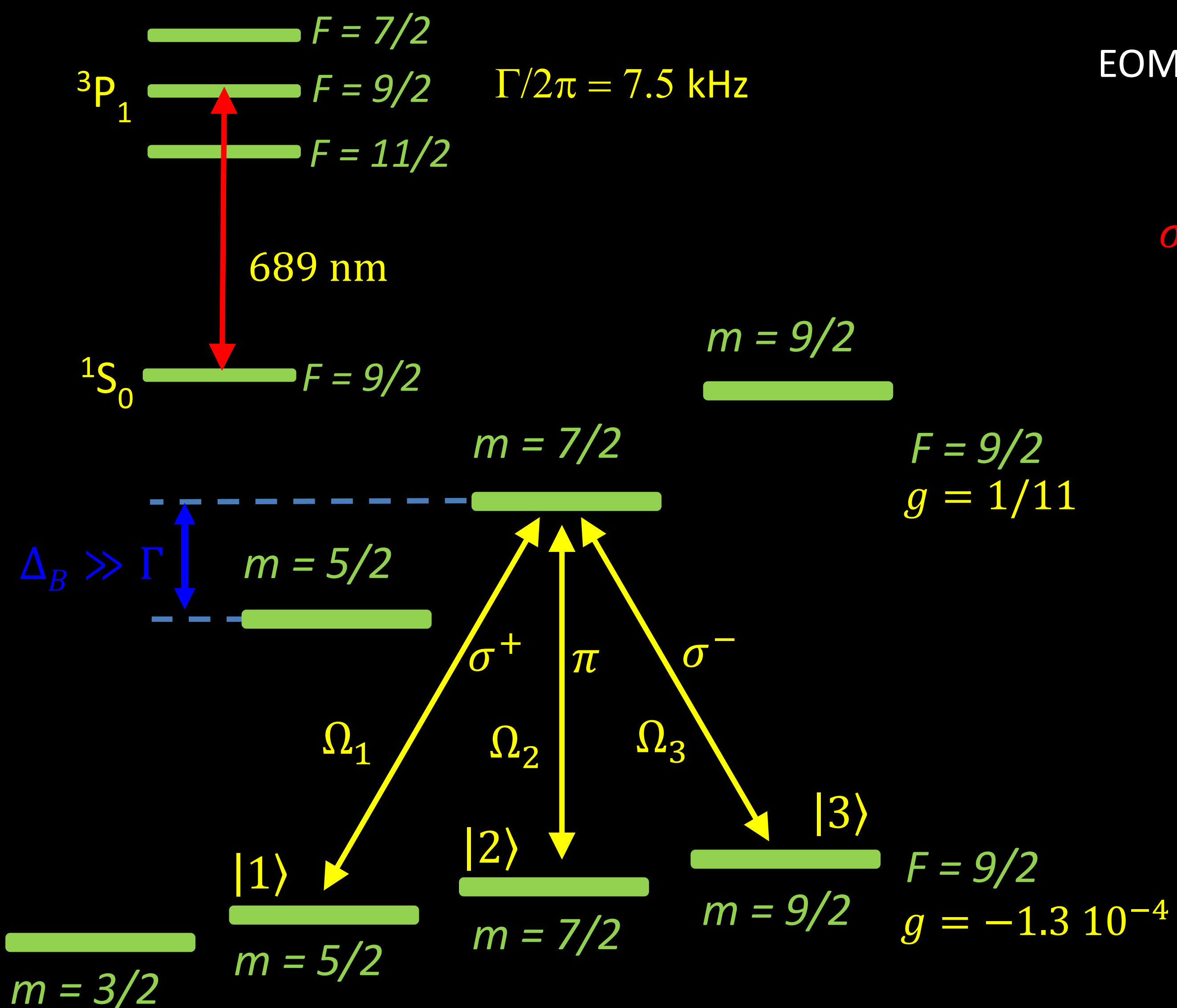
We get:

$$\vec{A}_{11} = \hbar\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$
$$\vec{A}_{12} = \hbar\cos\alpha\left(\frac{1}{2}\sin(2\beta)\,\vec{\nabla}\Phi_{12} - i\vec{\nabla}\beta\right)$$
$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

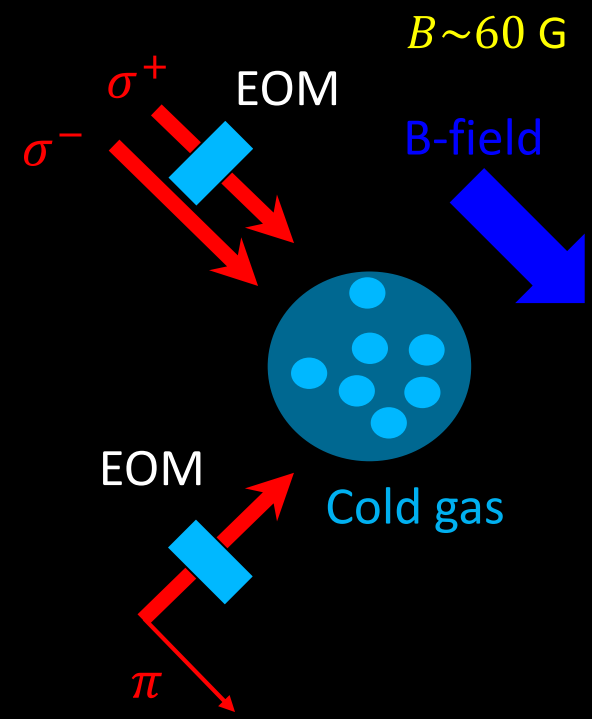
In general the components of $\vec{A}$ do not commute

Tripod: Implementation

Fermionic isotope on the $^1S_0 \rightarrow {}^3P_1$ intercombination line



EOM to control the laser phases



Abelian or non-Abelian ?

We have:

$$\vec{A}_{11} = \hbar\left(\cos^2\beta\,\underline{\vec{\nabla}\Phi_{23}} + \sin^2\beta\,\underline{\vec{\nabla}\Phi_{13}}\right)$$

$$\vec{A}_{12} = \hbar\cos\alpha\left(\frac{1}{2}\sin(2\beta)\,\underline{\vec{\nabla}\Phi_{12}} - \underline{i\vec{\nabla}\beta}\right)$$

$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\,\vec{\nabla}\Phi_{23} + \sin^2\beta\,\vec{\nabla}\Phi_{13}\right)$$

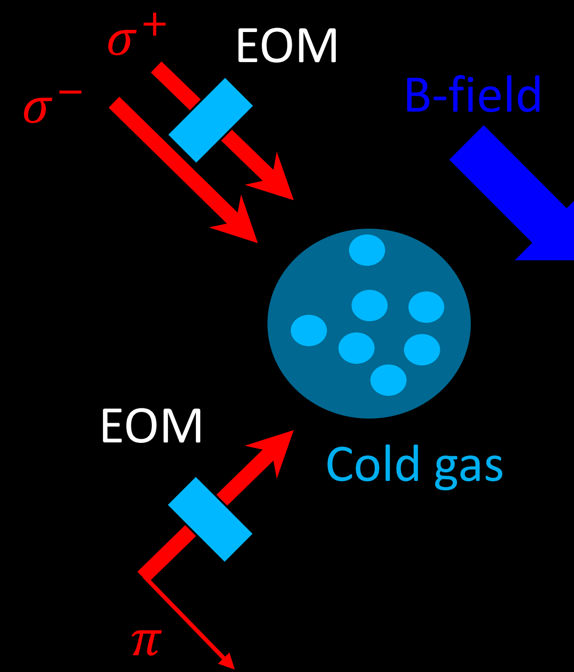
$$\boxed{\Phi_i = \vec{k}_i\vec{r} + \cancel{\phi_i(t)}}$$

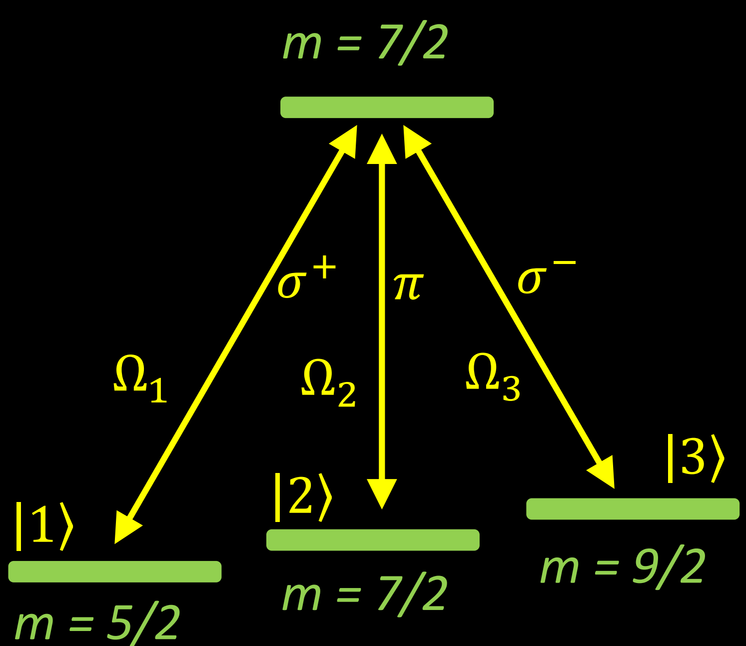
$$\Phi_{ij} = \Phi_i - \Phi_j$$

$$\vec{\nabla}\beta = 0,\ \Phi_1 = \Phi_3 \rightarrow \vec{\nabla}\Phi_{13} = 0$$

$$\vec{\nabla}\Phi_{12} = -\vec{\nabla}\Phi_{23} = \vec{k}_1 - \vec{k}_2$$

$\rightarrow$ Abelian gauge field





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We have:

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$$\vec{A}_{22} = \hbar\cos^2\alpha\left(\cos^2\beta\vec{\nabla}\Phi_{23} + \sin^2\beta\vec{\nabla}\Phi_{13}\right)$$

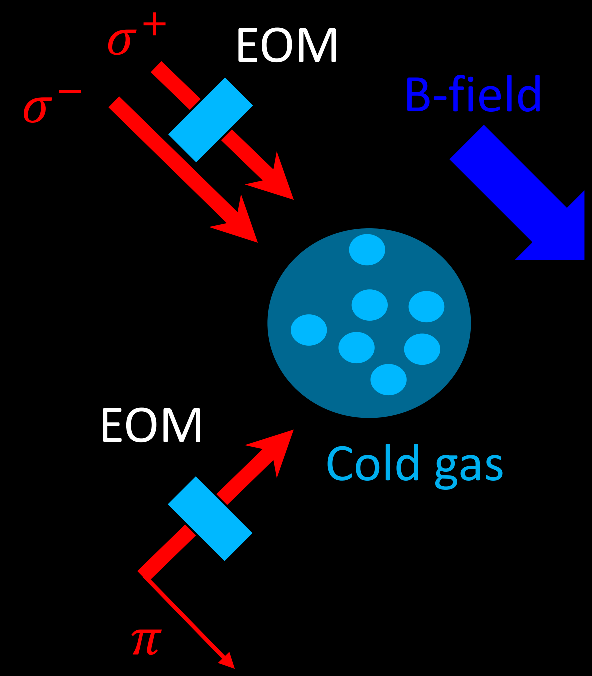
$$\boxed{\Phi_i = \cancel{\vec{k}_i\vec{r}} + \boldsymbol{\phi_i}(t)}$$

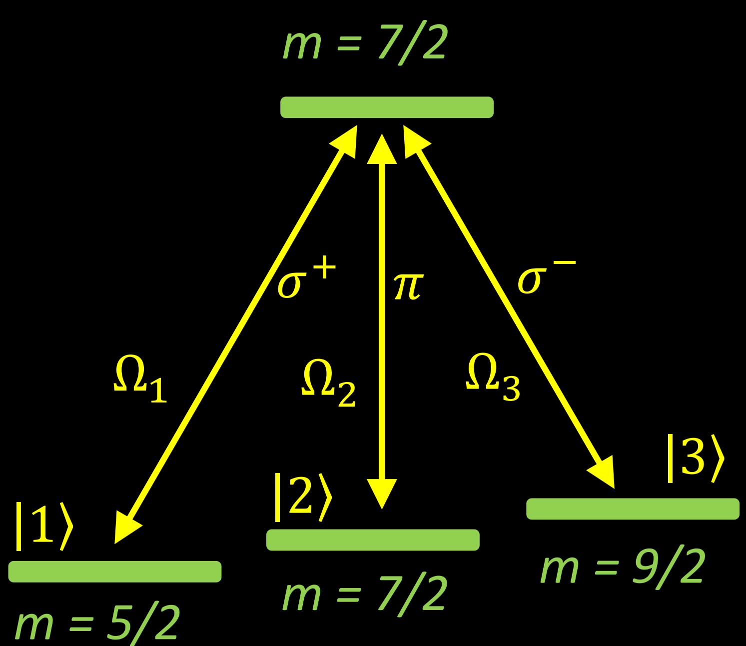
$$\Phi_{ij} = \Phi_i - \Phi_j$$

$\vec{\nabla}\beta = 0$, Φ_3 is constant (pinned atom)

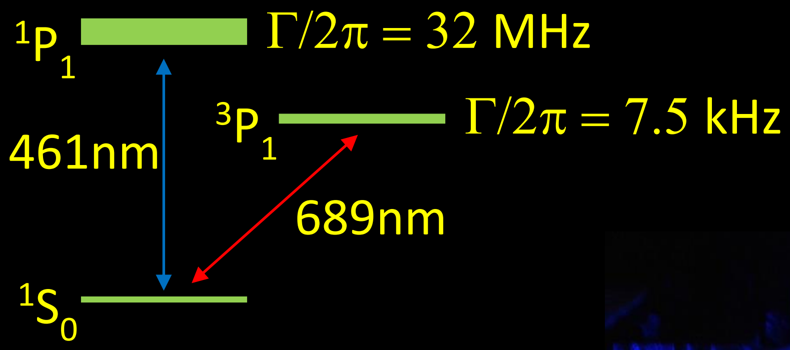
Φ_1 and Φ_2 independant

→ Non-Abelian transformation





Experimental platform: cold strontium atomic gas

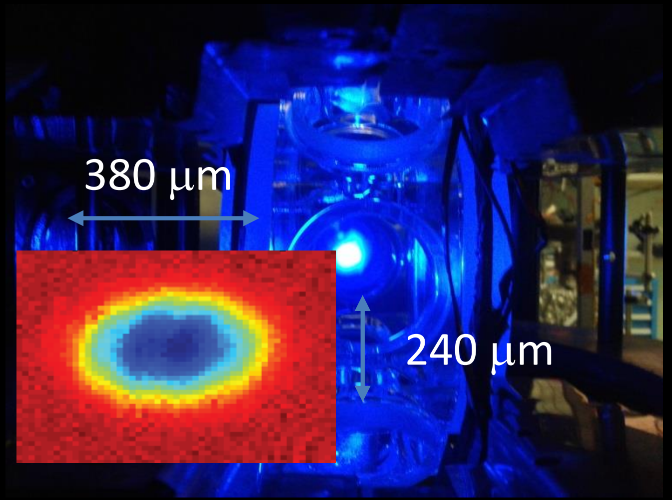


Temperature ~ Γ

$T \sim 1$ mK @ 461 nm

Magneto optical Trap

On the intercombination line at 689 nm



$T \sim 500$ nK @ 689 nm

$T_R = \hbar^2 k^2/2Mk_B = 230$ nK

The thermal energy dominates !!!

T. Yang, et *al*, Eur. Phys. J. D. **69**, 226 (2015)
C. Chalony et *al*, PRL **107** 243002 (2011)

Preparation of the dark state $\left|D_2\right\rangle$

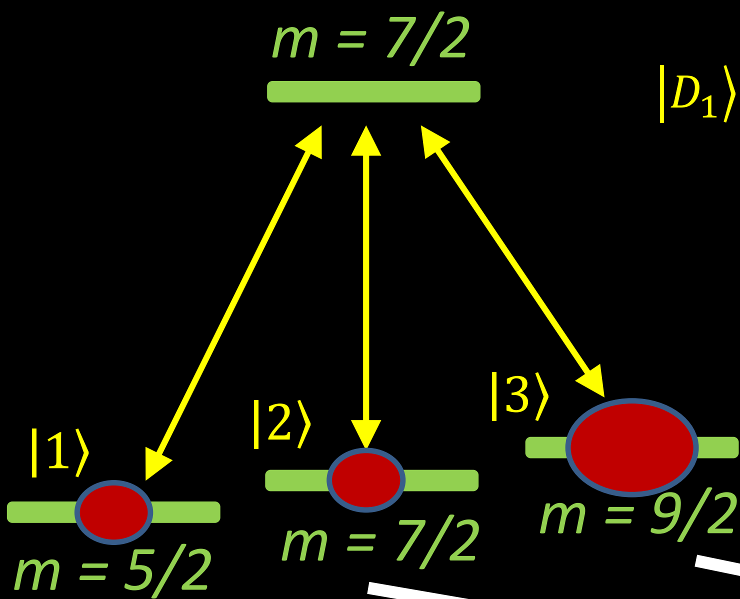
$$\left|D_1\right\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\Phi_{31}} \\ -e^{i\Phi_{32}} \\ 0 \end{pmatrix}$$

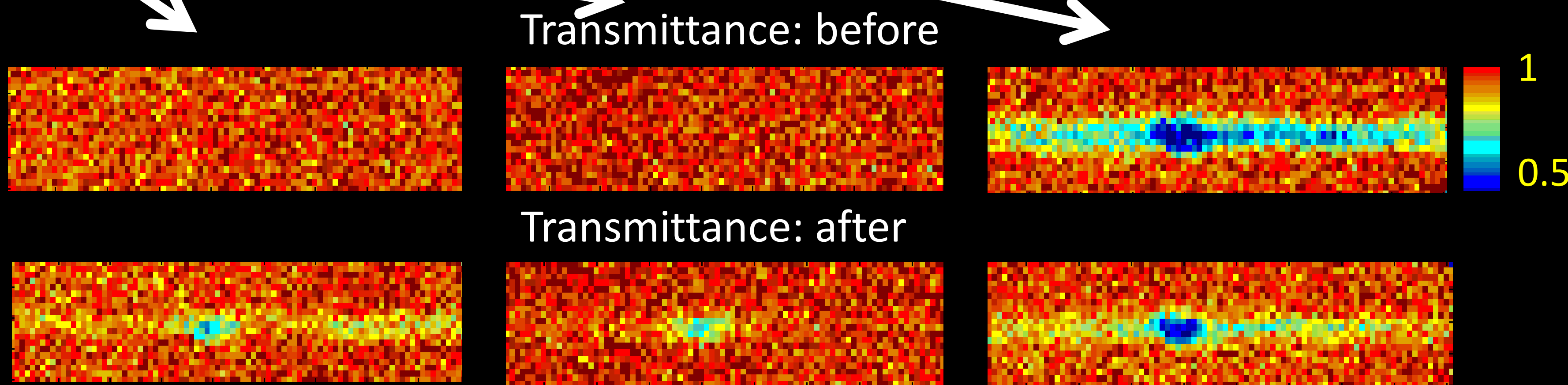
$$\boxed{\left|D_2\right\rangle = \frac{1}{\sqrt{6}}\begin{pmatrix} e^{i\Phi_{31}} \\ e^{i\Phi_{32}} \\ -2 \end{pmatrix}}$$

Then: $|\langle 1|D_2\rangle|^2 = 1/6$

$|\langle 2|D_2\rangle|^2 = 1/6$

$|\langle 3|D_2\rangle|^2 = 4/6$

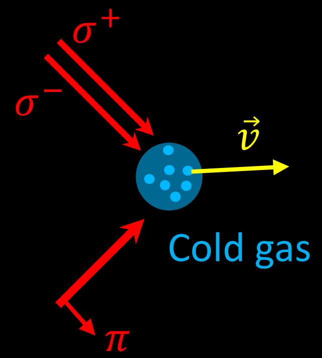




$P_1 = 0.24(0.04)$ $P_2 = 0.17(0.04)$ $P_3 = 0.59(0.04)$

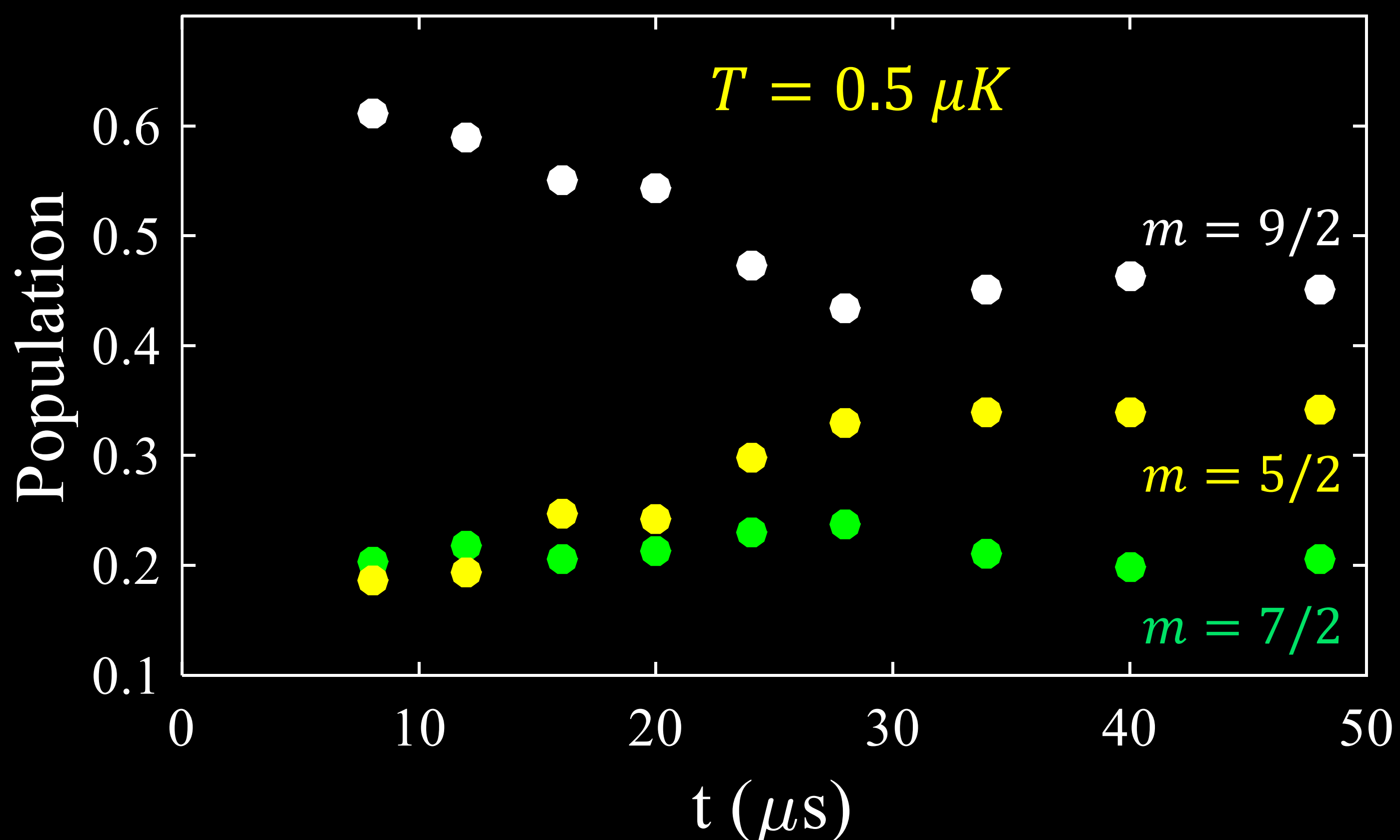
Fidelity ~95%

Temperature effect



Atom velocity

$$\Phi_i = \vec{k}_i\vec{r} + \cancel{\phi_i(t)}$$



Temperature effect: Comparison

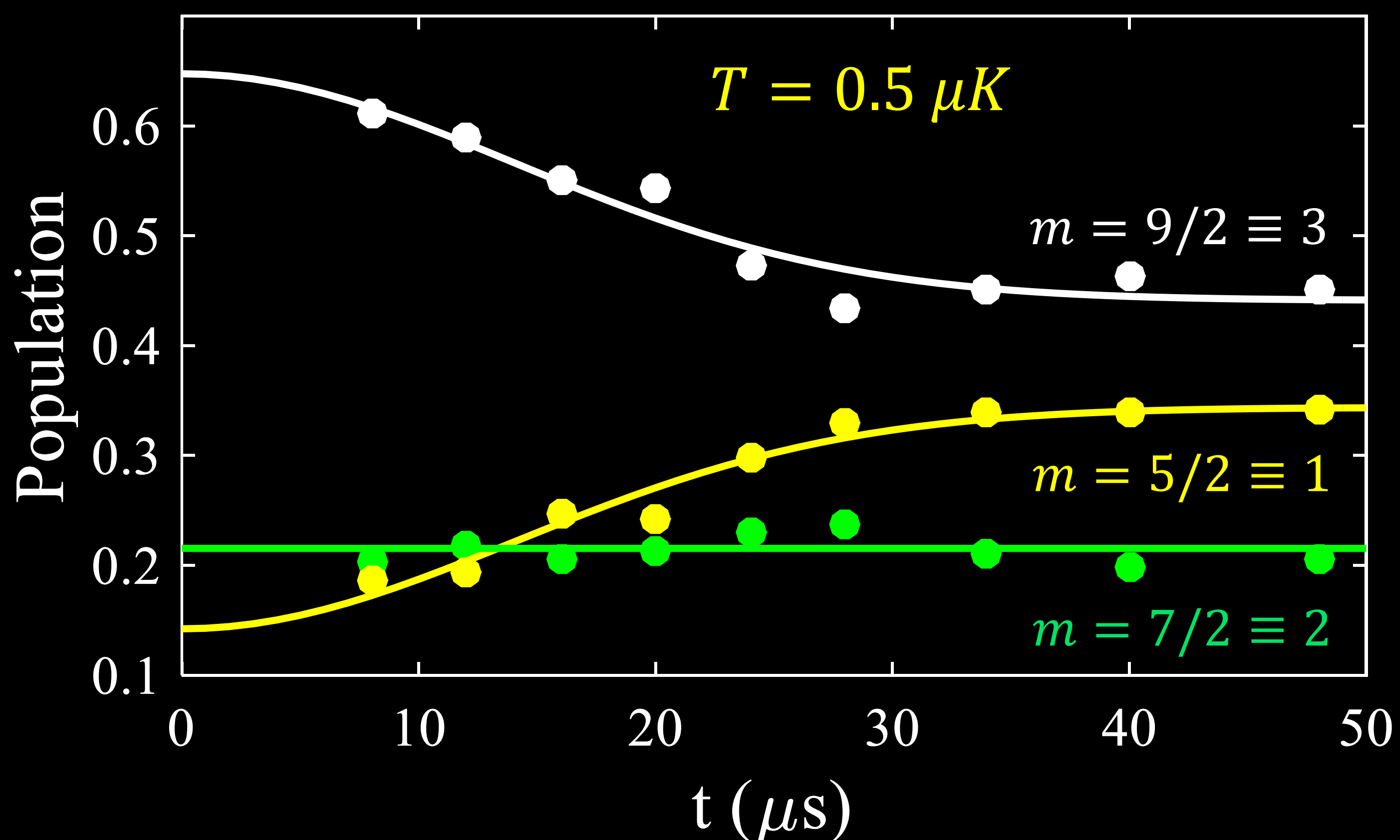
$$\overline{|\langle 1|\psi(t)\rangle|^2} = f_1 - g_1 \cos\left(\frac{4}{3}\omega_R t\right)\exp\left[-\frac{4}{9}(k\bar{v}t)^2\right]$$

$$\overline{|\langle 2|\psi(t)\rangle|^2} = f_2$$

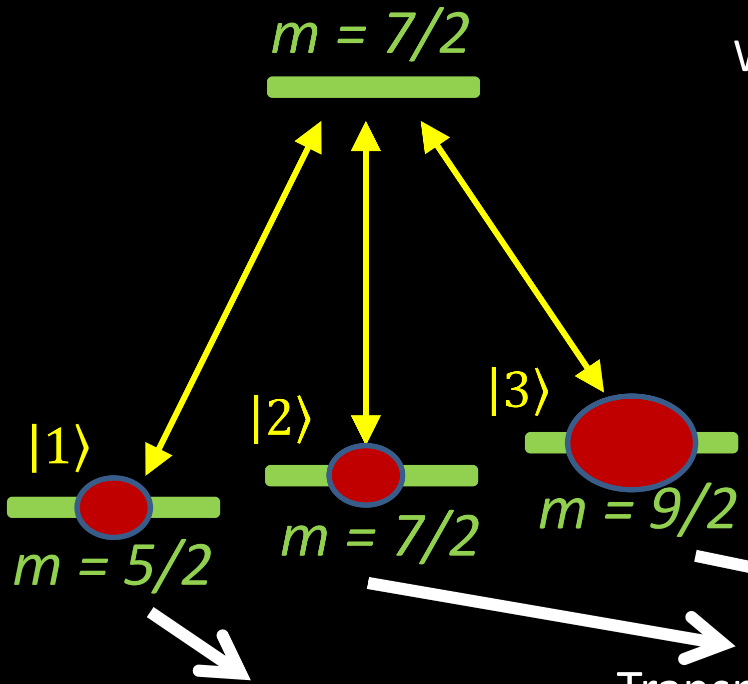
$$\overline{|\langle 3|\psi(t)\rangle|^2} = f_3 + g_3 \cos\left(\frac{4}{3}\omega_R t\right)\exp\left[-\frac{4}{9}(k\bar{v}t)^2\right]$$

For a thermal gas
$\bar{v}$: Thermal velocity

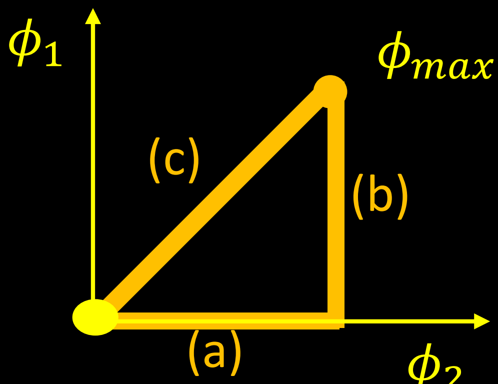
f_i and g_i
depend on the Rabi frequencies.

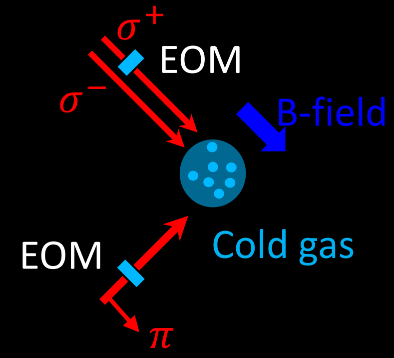


Geometric transformation: Closed loop of the lasers phases



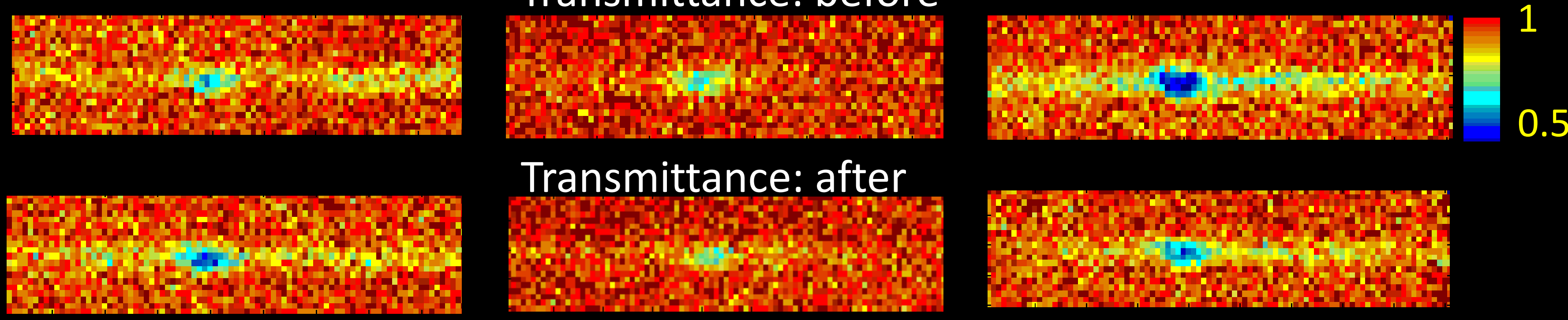
We perform:



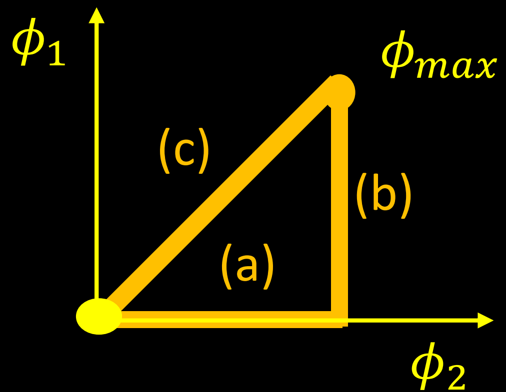


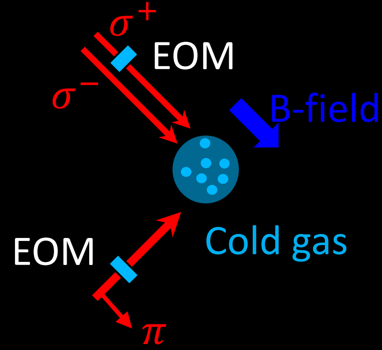
Transmittance: before

Transmittance: after



Geometric Q-Gate: Bare state populations

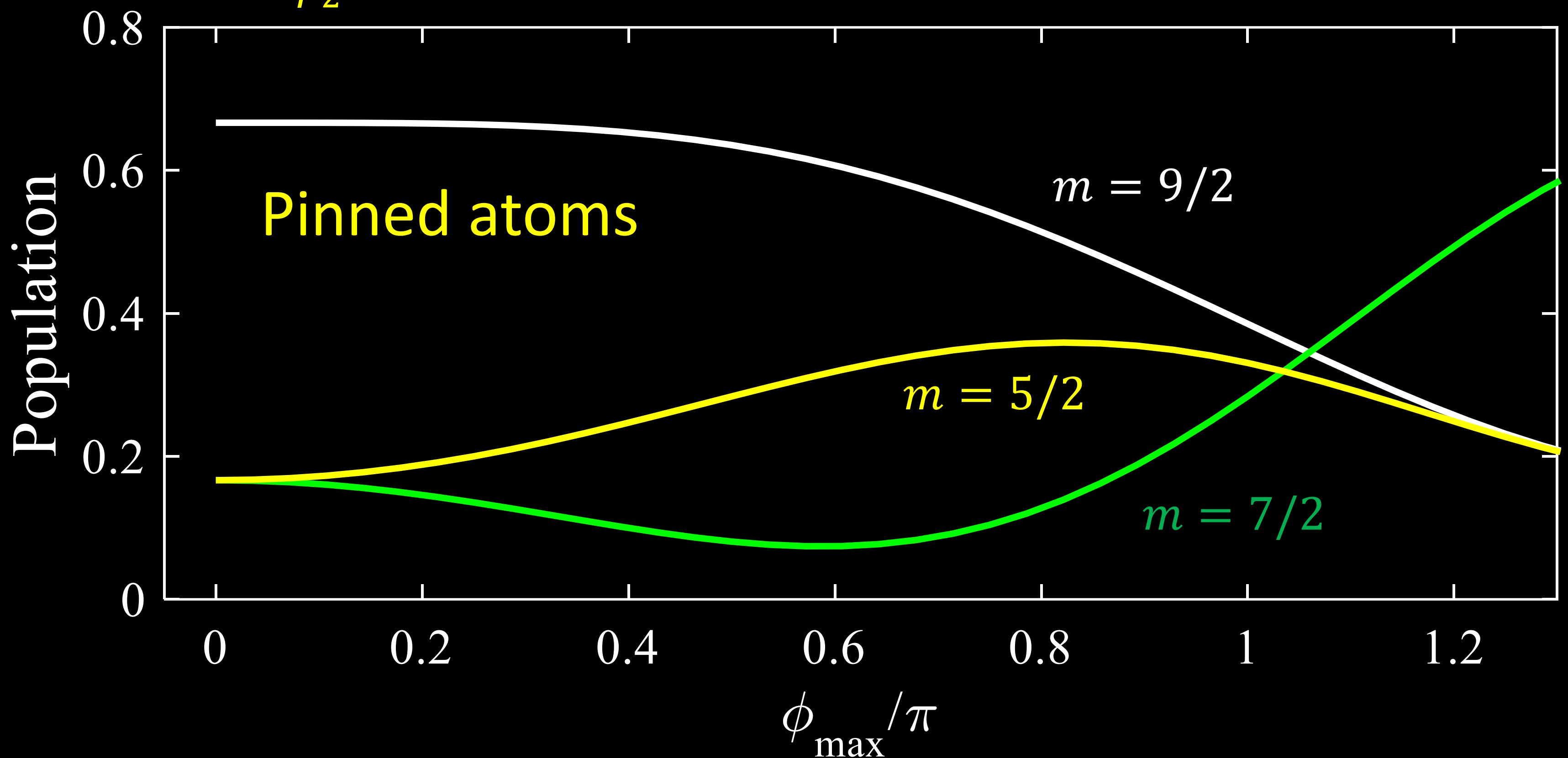




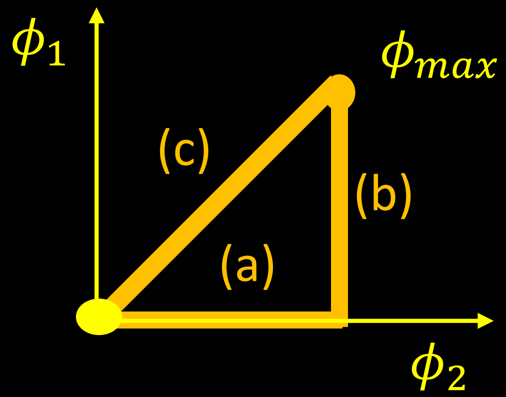
$$|\psi\rangle_f = U|D_2\rangle$$

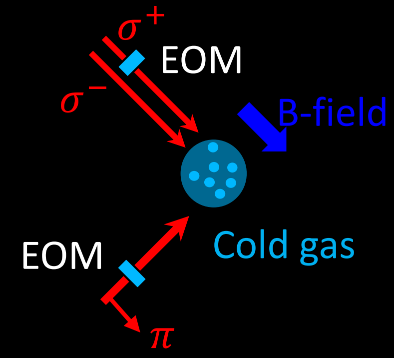
$$U = \wp\exp\left(-\oint_C \vec{A}\mathrm{d}\vec{\lambda}\right)$$

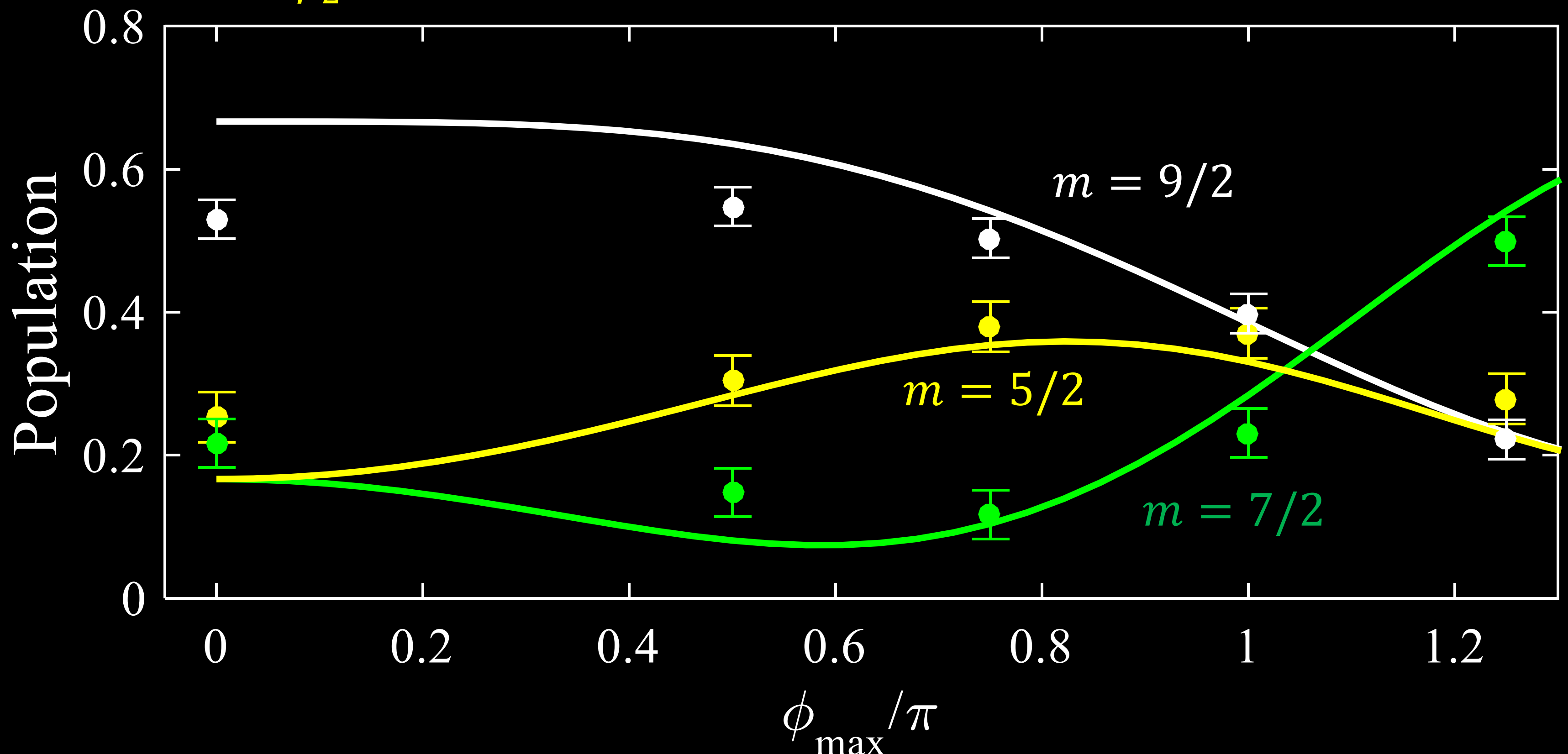
Bare state populations



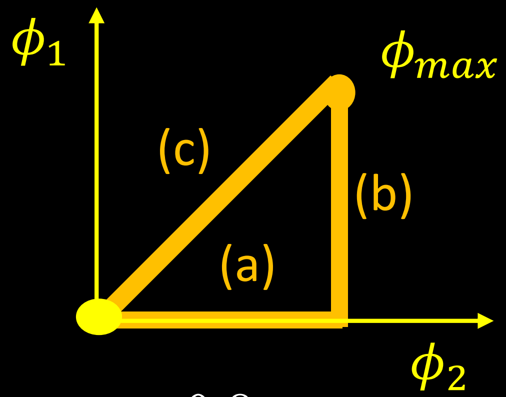
Geometric Q-Gate: Theory vs experiment





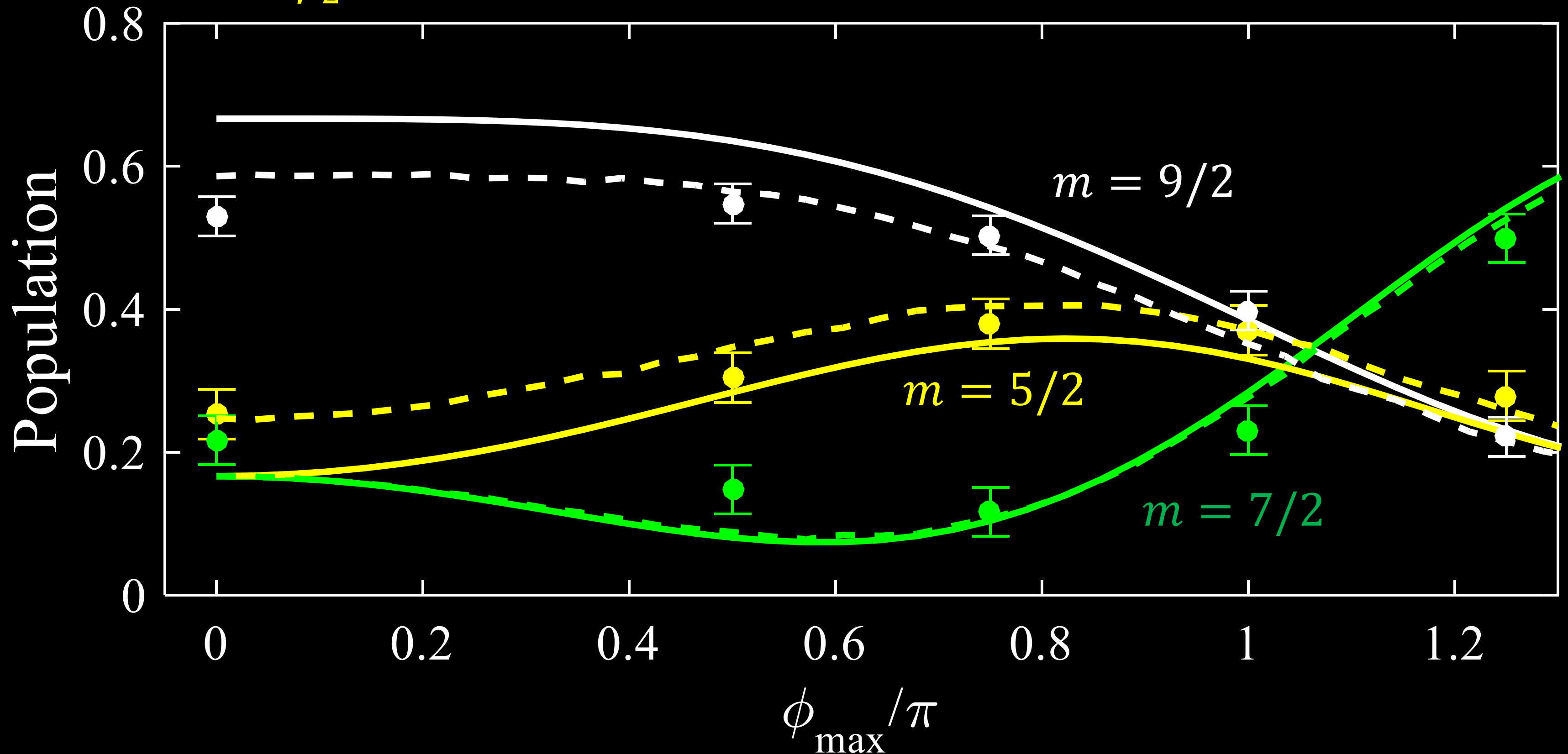


Geometric Q-gate: Finite temperature



Finite temperature contribution $\Phi_i = \vec{k}_i \vec{r} + \boldsymbol{\phi_i}(t)$

$T = 0.5\ \mu K$



State Tomography

Chosen dark state basis $|D_1\rangle = \begin{pmatrix} \sin\beta\, e^{i\Phi_{31}} \\ -\cos\beta\, e^{i\Phi_{32}} \\ 0 \end{pmatrix}$ $|D_2\rangle = \begin{pmatrix} \cos\alpha\cos\beta\, e^{i\Phi_{31}} \\ \cos\alpha\sin\beta\, e^{i\Phi_{32}} \\ -\sin\alpha \end{pmatrix}$

$$|\psi\rangle_f = U|D_2\rangle = (|d_1||D_1\rangle + |d_2|e^{i\varphi}|D_2\rangle)e^{i\varphi_g}$$

We measure the bare state population: $|\langle i|\psi\rangle|^2$

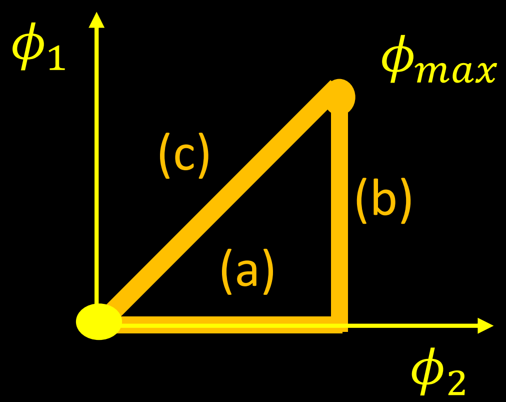
$$|d_2| = \frac{\sqrt{|\langle 3|\psi\rangle|^2}}{|\sin\alpha|} \qquad |d_1| = \sqrt{1-|d_2|^2}$$

$$\cos\varphi = \frac{|\langle 1|\psi\rangle|^2 - |\langle 2|\psi\rangle|^2 + (d_1^2 - d_2^2\cos^2\alpha)\cos(2\beta)}{|d_1||d_2|\cos\alpha\sin(2\beta)}$$

The sign of φ is missing

The reconstruction does not give φ_g

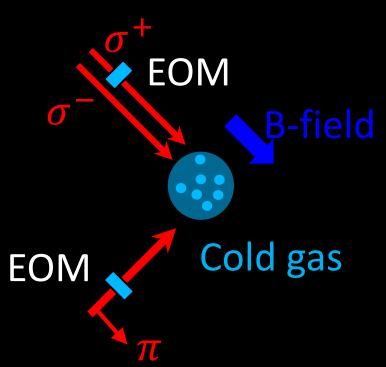
Geometric transformation: Dark state manifold

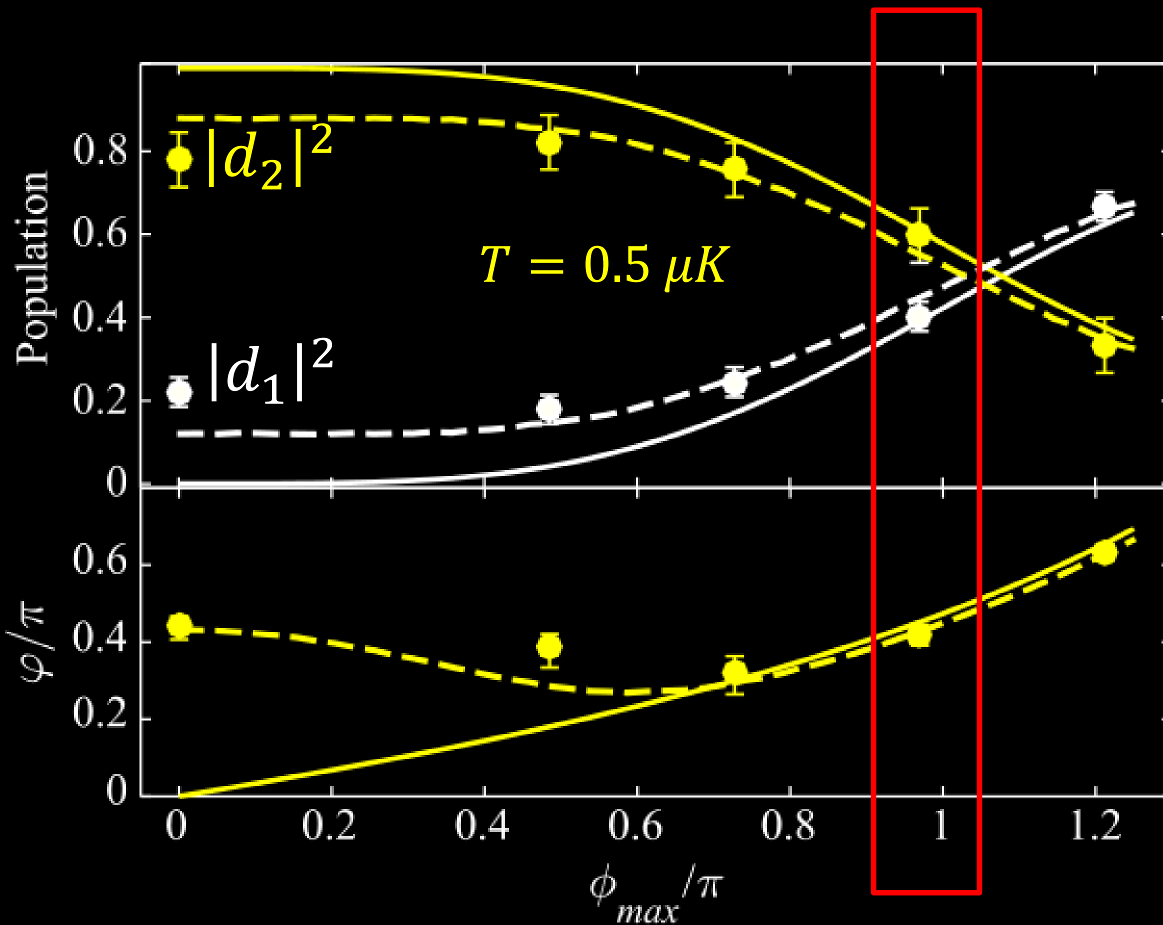


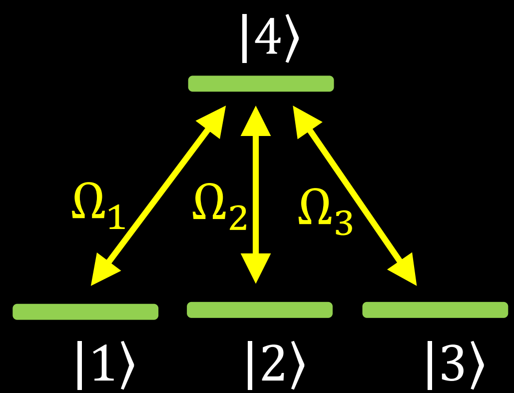
Bare state populations

$\Rightarrow \; |\psi\rangle_f = |d_1| \, |D_1\rangle + |d_2| e^{i\varphi} |D_2\rangle$

Finite temperature contribution $\Phi_i = \vec{k}_i \vec{r} + \boldsymbol{\phi}_i(t)$







Quenching of thermal dephasing if $\phi_{max} > k\bar{v}\Delta t \simeq 0.7\pi$

Loop time: $\Delta t = 12\ \mu s$

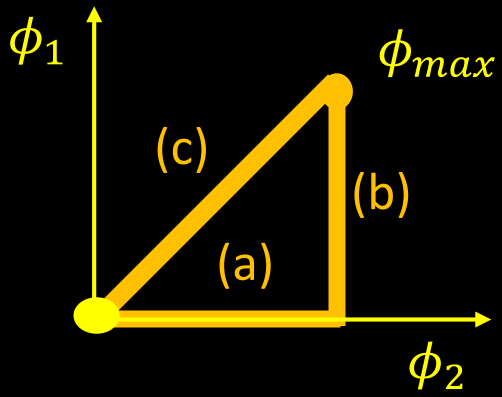
Unitary operator reconstruction

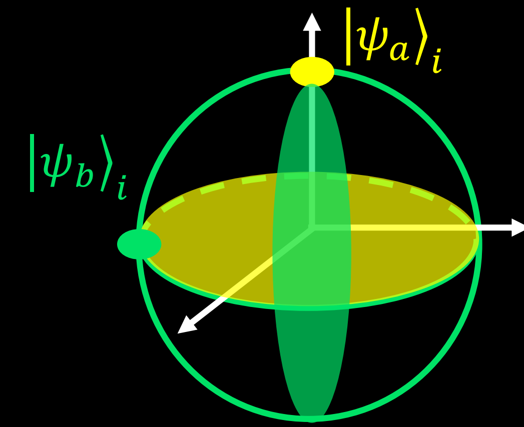
$|\psi\rangle_f = U|\psi\rangle_i = \left(|d_1||D_1\rangle + |d_2|e^{i\varphi}|D_2\rangle\right)e^{i\varphi_g}$

The state tomography does not give φ_g

U cannot be reconstructed

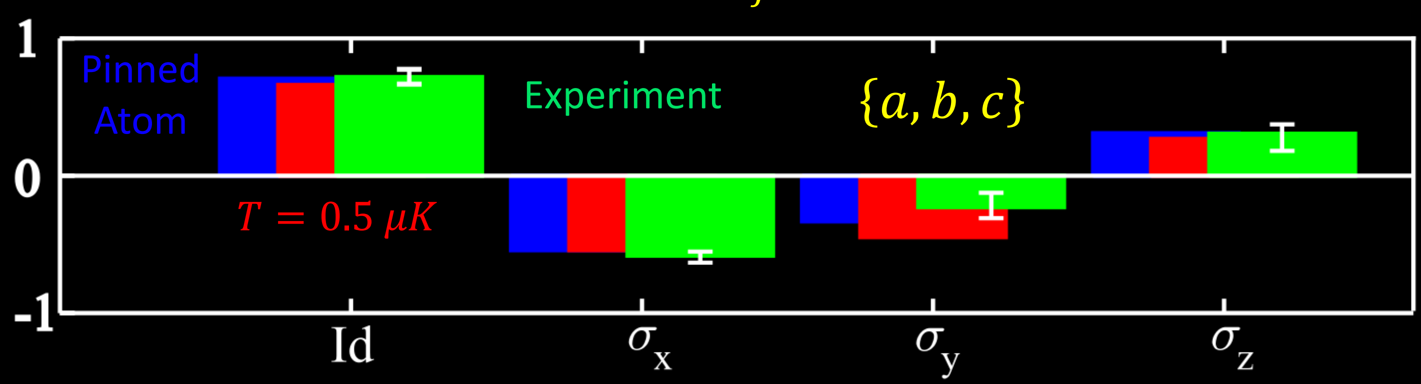
We use two initial (non orthogonal) states: $|\psi_a\rangle_f = U|\psi_a\rangle_i$ and $|\psi_b\rangle_f = U|\psi_b\rangle_i$





We use the decomposition: $U = \alpha_0 \mathrm{Id} + i\sum_j \alpha_j \sigma_j$

σ_j: Pauli matrices

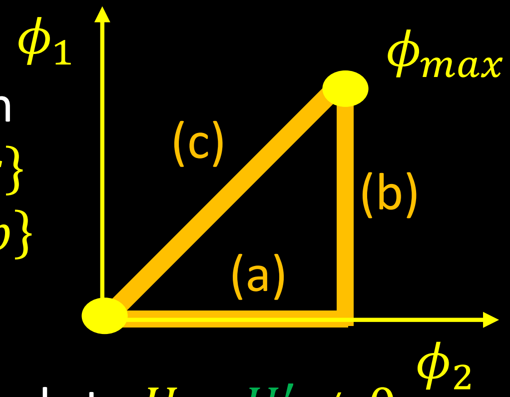


Path sensitive ordering

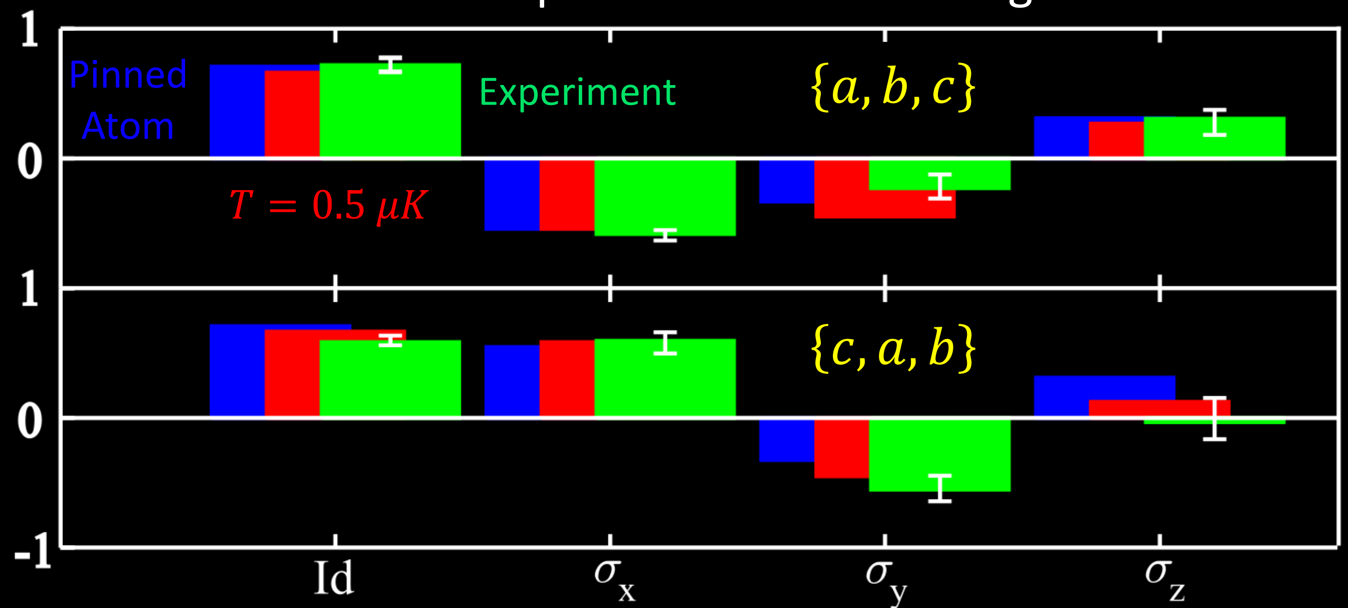
Practical implementation

$$U = U_c U_b U_a \equiv \{a, b, c\}$$

$$U' = U_b U_a U_c \equiv \{c, a, b\}$$



The path sensitive ordering leads to $U - U' \neq 0$



The Frobenius distance: $d = \sqrt{\sum_j (\alpha_j(a) - \alpha_j(b))^2}$

$d = 1.27\ (25)$, $d = 1.14$ and $d = 1.09$

Conclusion and perspective

F. Leroux, K. Pandey, R. Rebhi, F. Chevy, C. Miniatura, B. Gremaud, DW, Nature Communications **9**, 3580 (2018).

We showed the path sensitivity on a non-Abelian transformation

- Universal geometric single Qubit gate
  Geometrical quantum computing ?

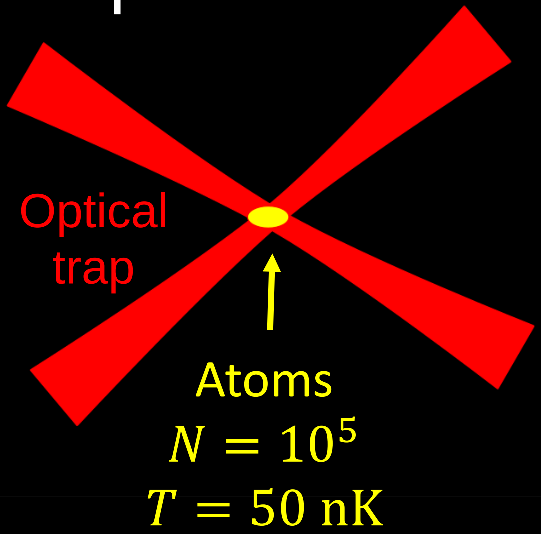
We used the underlying Abelian gauge field as a thermometer

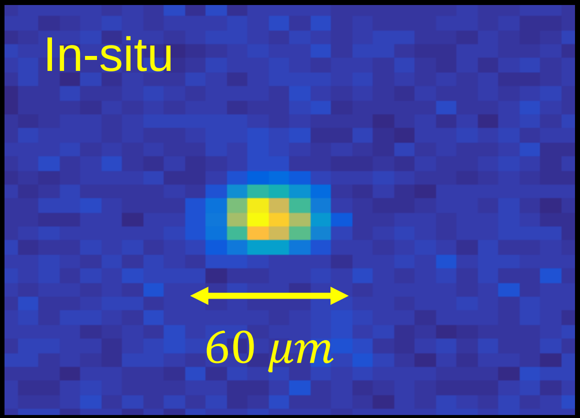
- Synthetic non-Abelian gauge field in bulk ultracold gases
  2D spin-orbit coupling
  Manipulation of matter wave in non-Abelian gauge fields
  Atomtronics

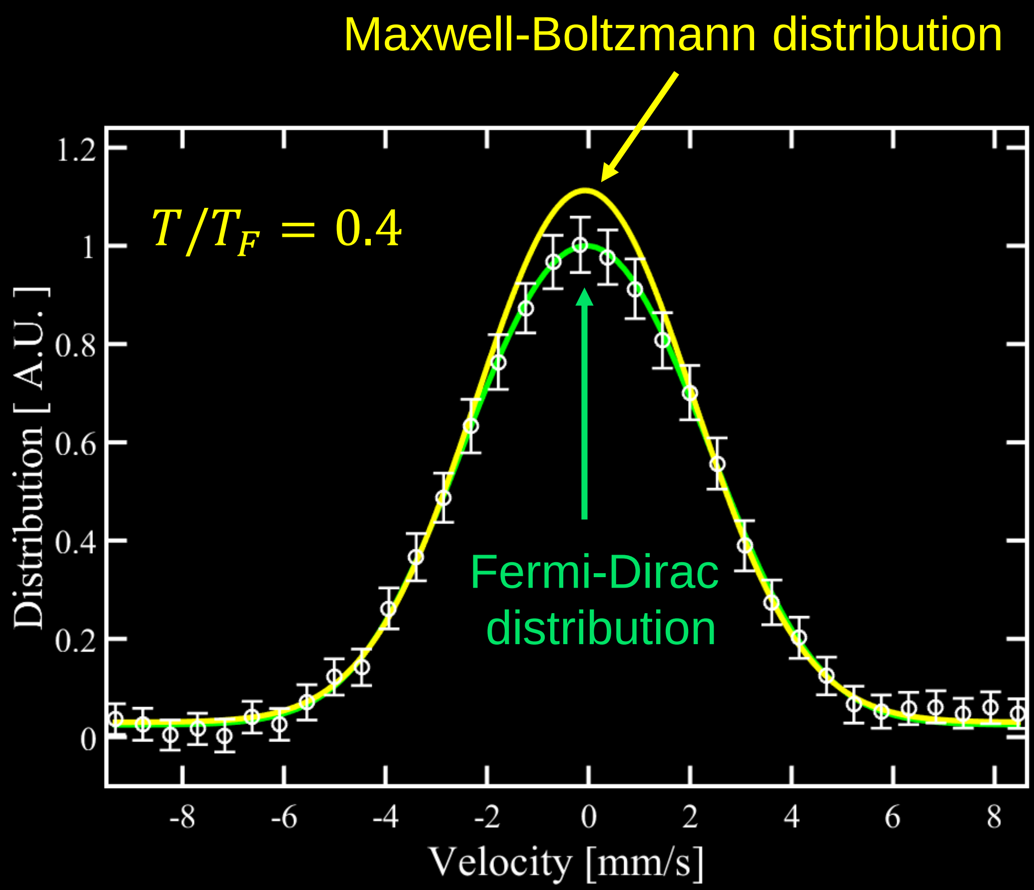
But we need to be colder, i.e. $T < T_R = \hbar^2 k^2 / 2 k_B \mathrm{M} = 230\ \mathrm{nK}$

Fermi-Dirac distribution

Evaporative cooling



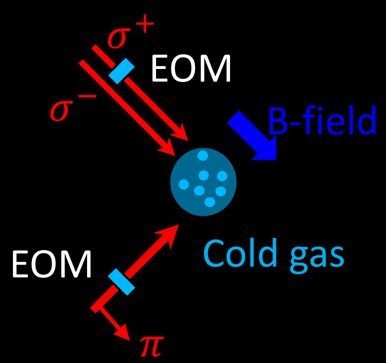


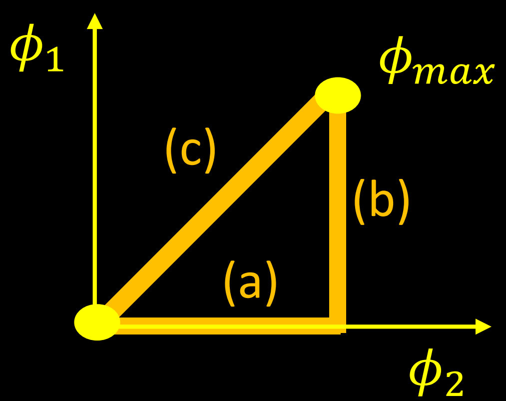


Atomtronics perspectives

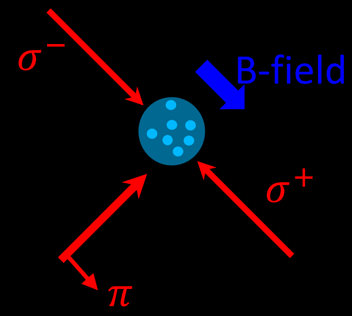
Atomtronics = Flow of atoms in optical circuit

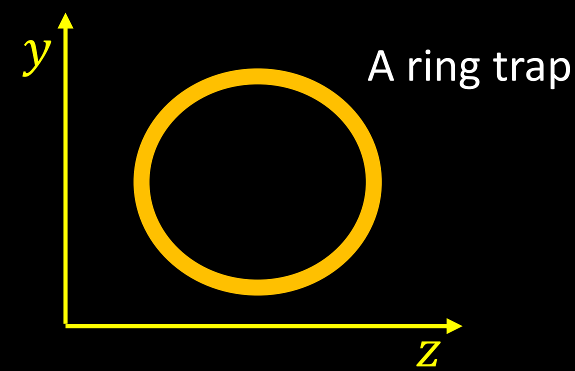
What we did … No Atomtronics





What we shall do





Coll with Rainer and Kwek

Persistent Non-Abelian current ?

Dynamic of an atomic wave-packet

$\vec{A}$: vector potential or a Berry connection.

Abelian case

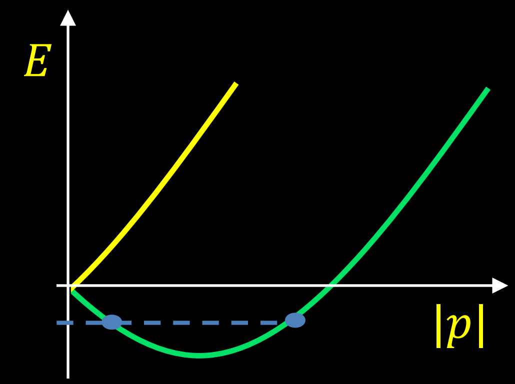
$\vec{\Phi} = \vec{\nabla} \times \vec{A}$: Berry curvature

If translational invariance of $\vec{A}$, no force

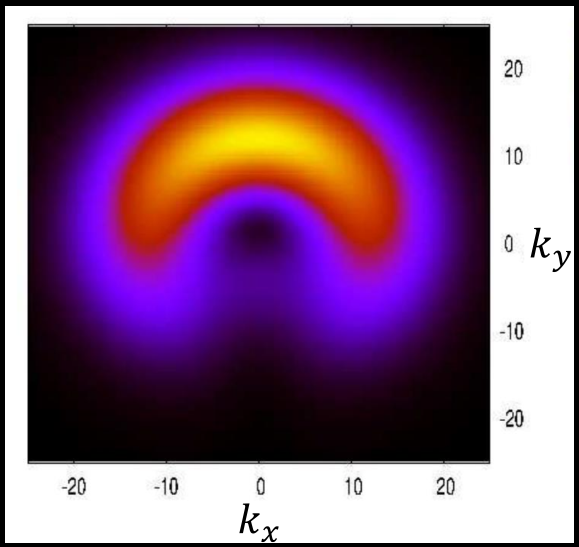
Non-abelian case

$\vec{\Phi} = \vec{\nabla} \times \vec{A} + \vec{A} \times \vec{A}$: Berry curvature

If translational invariance of $\vec{A}$, a force exists

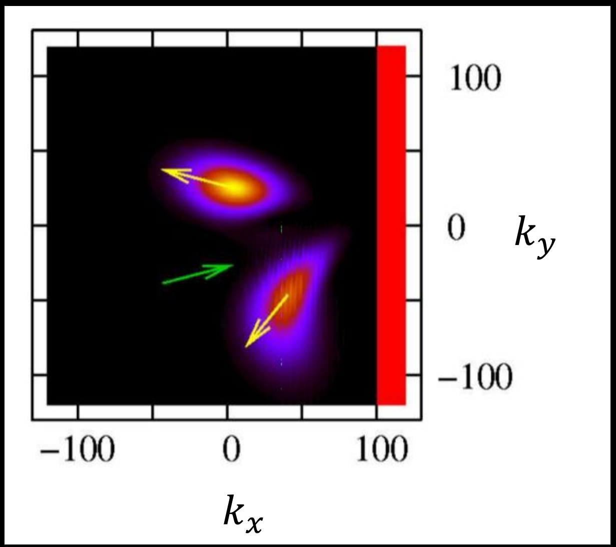


Anisotropic ballistic expansion



Form: Jacob et *al*, APB **89**, 439 (2007)

Anomalous reflection



Form: Juzeliunas et *al*, PRL **100**, 200405 (2008)

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