Eternal life of entropy in non-Hermitian quantum systems

Thomas Frith

City, University of London

7th International Workshop on New challenges in Quantum Mechanics: Integrability and Supersymmetry

Benasque

2nd September 2019

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A. Fring and T. Frith, Phys. Rev. A ,100, 010,102 (2019).

Outline

Introduction

Why Study Non-Hermitian Systems?

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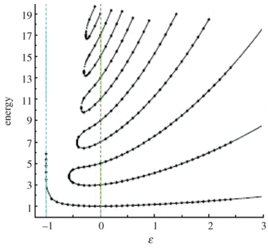

Figure: Bender and Boettcher, 1998

$$H = p^2 + (ix)^\epsilon$$

Theory

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$$h(t)\phi(t) = i\hbar\partial_t\phi(t), \quad H(t)\Phi(t) = i\hbar\partial_t\Phi(t).$$

Time dependent Dyson operator

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$\Longrightarrow$ Time dependent Dyson equation

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$\Longrightarrow$ Time dependent quasi-Hermiticity relation

$$H^{\dagger}(t)\rho(t) - \rho(t)H(t) = i\hbar\partial_t\rho(t).$$

where $\rho(t) = \eta^{\dagger}(t)\eta(t)$

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Then we have

$$\langle\phi(t)\,|o(t)\phi(t)\rangle = \langle\Psi(t)\,|\rho(t)\mathcal{O}(t)\Psi(t)\rangle \, .$$

$\rho\,(t) = \eta^{\dagger}\,(t)\,\eta\,(t)$ is vital!

Von Neumann entropy

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Hermitian

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Substitute time-dependent Dyson equation into von Neumann equation,

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$$S_H = -tr\left[\varrho_H \ln \varrho_H\right].$$

$$S_H = -\sum_i \lambda_i \ln \lambda_i = S_h.$$

A Simple Model

System/bath coupled harmonic oscillators

System/bath coupled harmonic oscillators

$$H = \nu a^\dagger a + \nu \sum_{n=1}^{N} q_n^\dagger q_n + (g+\kappa)\, a^\dagger \sum_{n=1}^{N} q_n + (g-\kappa)\, a \sum_{n=1}^{N} q_n^\dagger,$$

ν, g, $\kappa \in \mathcal{R}$ are time-independent parameters.

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$\mathcal{PT}$ symmetry spontaneously broken when $g < \kappa$

Calculating the metric

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Generators

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Generators

$$N_A = a^\dagger a, \quad N_Q = \sum_{n=1}^{N} q_n^\dagger q_n, \quad N_{AQ} = N_A - \frac{1}{N} N_Q - \frac{1}{N} \sum_{n \neq m} q_n^\dagger q_m,$$

$$A_x = \frac{1}{\sqrt{N}} \left(a^\dagger \sum_{n=1}^{N} q_n + a \sum_{n=1}^{N} q_n^\dagger \right), \quad A_y = \frac{i}{\sqrt{N}} \left(a^\dagger \sum_{n=1}^{N} q_n - a \sum_{n=1}^{N} q_n^\dagger \right).$$

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Algebra

$$[N_A, N_Q] = 0, \; [N_A, N_{AQ}] = 0, \; [N_A, A_x] = -iA_y, \; [N_A, A_y] = iA_y,$$
$$[N_Q, A_x] = iA_y, \; [N_Q, A_y] = -iA_x, \; [N_{AQ}, A_x] = -2iA_y, \; [N_{AQ}, A_y] = 2iA_x.$$

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$$\eta(t) = e^{\beta(t) A_y} e^{\alpha(t) N_{AQ}},$$

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Coupled differential equations

$$\dot{\alpha} = -\tanh(2\beta)\left[\sqrt{N}g\cosh(2\alpha) + \sqrt{N}\kappa\sinh(2\alpha)\right],$$

$$\dot{\beta} = \sqrt{N}\kappa\cosh(2\alpha) + \sqrt{N}g\sinh(2\alpha).$$

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$$\tanh\left(2\alpha\right) = \frac{-Ng\kappa + \dot{\beta}\sqrt{\dot{\beta}^2 + N\left(g^2 - \kappa^2\right)}}{Ng^2 + \dot{\beta}^2}.$$

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$$\sigma = \frac{c_1}{\sqrt{g^2 - \kappa^2}}\sin\left(2\sqrt{N}\sqrt{g^2 - \kappa^2}\left(t + c_2\right)\right),$$

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$$\mu(t) = \frac{(g^2 - \kappa^2)\sqrt{N}\sqrt{c_1^2 + g^2 - \kappa^2}}{c_1^2 + 2(g^2 - \kappa^2) - c_1^2 \cos\left(4\sqrt{N}\sqrt{g^2 - \kappa^2}(t + c_2)\right)}.$$

Three types of entropy evolution

Von Neumann entropy

Von Neumann entropy

Initial State

Von Neumann entropy

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$$|\phi(0)\rangle = \sin\gamma\,|1_a 0_q\rangle + \frac{\cos\gamma}{\sqrt{N}}\sum_{i=1}^{N}|0_a 1_i\rangle\,,$$

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State at time t

$$|\phi(t)\rangle = e^{-i\nu t}\left(\sin\gamma\sin\mu_I(t) + \cos\gamma\cos\mu_I(t)\right)|1_a 0_q\rangle$$
$$+\frac{e^{-i\nu t}}{\sqrt{N}}\left(\sin\gamma\cos\mu_I(t) - \cos\gamma\sin\mu_I(t)\right)\sum_{i=1}^{N}|0_a 1_i\rangle\,.$$

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$$\mu_I(t) = \int^t \mu(s)\,ds = \frac{1}{2}\arctan\left(\frac{\sqrt{c_1^2+g^2-\kappa^2}\tan\left(2\sqrt{N}\sqrt{g^2-\kappa^2}\,(t+c_2)\right)}{\sqrt{g^2-\kappa^2}}\right).$$

Von Neumann entropy

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Reduced density matrix

Von Neumann entropy

Reduced density matrix

$$\varrho_a(t) = Tr_q\left[\varrho_h(t)\right] = \begin{pmatrix} (\sin\gamma\sin\mu_I(t) + \cos\gamma\cos\mu_I(t))^2 & 0 \\ 0 & (\sin\gamma\cos\mu_I(t) - \cos\gamma\sin\mu_I(t))^2 \end{pmatrix}.$$

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Eigenvalues

$$\lambda_1(t) = \left(\sin\gamma\sin\mu_I(t) + \cos\gamma\cos\mu_I(t)\right)^2,$$

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$$S_{h,a}(t) = -\lambda_1(t)\log\left[\lambda_1(t)\right] - \lambda_2(t)\log\left[\lambda_2(t)\right] = S_{H,a}(t).$$

Unbroken regime

Unbroken regime

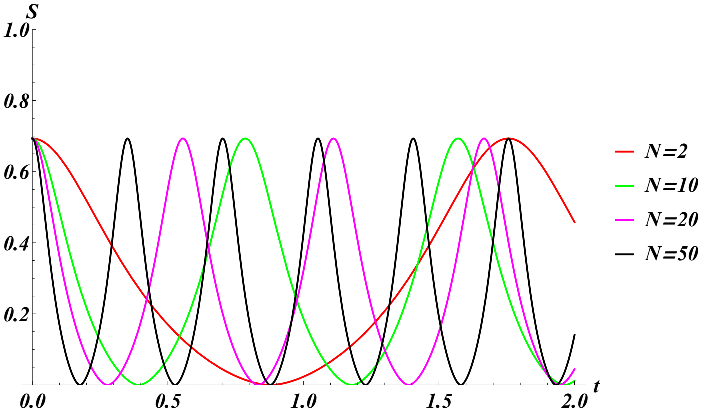

Figure: Von Neumann entropy as a function of time and varied bath size, with $c_1 = 1$, $g = 0.7$, $\kappa = 0.3$

Exceptional point

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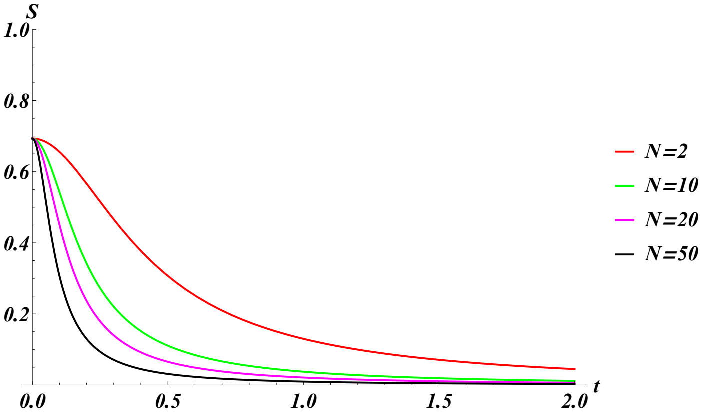

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Broken regime

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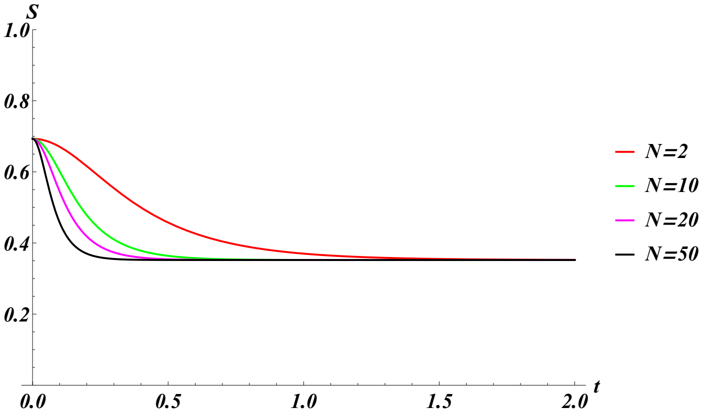

Figure: Von Neumann entropy as a function of time and varied bath size, with $c_1 = 1$, $g = 0.3$, $\kappa = 0.7$. The asymptote is at $S_{t\to\infty} \approx 0.3521$

Conclusion

- Framework for Von Neumann entropy

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- Three different types of entropy evolution $\rightarrow \mathcal{PT}$ regimes

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- Quantum computing- maintaining entanglement

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- Quantum computing- maintaining entanglement
- Further models

Questions?