Goldstone bosons in different PT-regimes of non-Hermitian scalar quantum field theory

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Department of Mathematics, City, University of London

7th International Workshop on New Challenges in Quantum Mechanics: Integrability and Supersymmetry 2019

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based on: Fring, A. and Taira, T., 2019. arXiv preprint arXiv:1906.05738.

Plan

Introduction

Motivation/Key observations

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Key observation (what we will talk about)

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Key observation (what we will talk about)

(1) Physical region in the parameter space (section 2)

(2) Explicit forms of Goldstone fields at exceptional points (section 3)

Standard Goldstone theorem

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Model

Our model

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$$\mathcal{L}_n = \sum_{i=1}^{n} |\partial\phi_i|^2 + c_i m_i^2 |\phi_i| - \frac{g_i}{4}(|\phi_i|^2)^2$$

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- Generalisation of [Alexandre, J.,Ellis,J.,Millington,P., Seynaeve,D.(2018).PhysRevD,98(4),045001.], [Mannheim,P.D.(2019).PhysRevD,99(4),045006.]

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Another proof of Goldstone theorem for Non-Hermitian model is in [Alexandre, J.,Ellis,J.,Millington,P., Seynaeve,D]

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- From $\mathcal{PT}-QM$, $\eta H\eta^{-1} = h$ where $h^{\dagger} = h$, same energy spectrum.
- We use generalisation of η in [Mannheim]
  $\eta = \prod_{n=2,4,...} e^{\frac{\pi}{2}\int d^3x \Pi_n^{\varphi}(\vec{x},0)\varphi_n(\vec{x},0)} e^{\frac{\pi}{2}\int d^3x \Pi_n^{\chi}(\vec{x},0)\chi_n(\vec{x},0)}$.
- Π^{ϕ} is a canonical momenta of ϕ.
- Need to assume equal time commutation relation.

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- Where $\frac{1}{\sqrt{2}}\Phi \equiv (\varphi_1, \chi_2, \ldots, \chi_1, \varphi_2, \ldots)^T$ and $\mathcal{I} = diag(1, -1, 1, 1, -1, 1)$.

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- equations of motion: $\Box \mathcal{I} \Phi + \mathcal{I} M^2 \Phi = 0 \implies \Box \Phi + M^2 \Phi = 0$

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- **Question**: Positive region (Physical region) in parameter space exist?(not done before). Can our model be empty? (i.e no physical region)

Parameter space

Recall our procedure

Complex **Real**

$$\mathcal{L}_n(\varphi_i, \chi_i) \quad \rightarrow \quad \mathcal{L}_{n\mathcal{R}} \quad \xrightarrow[\textbf{of motion}]{\textbf{Equations}} \quad \Box\Phi + M^2\Phi = 0$$

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Complex		**Real**		
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(3) Parameters of $\mathcal{L}_3$ are $\{m_1, m_2, m_3, \mu_1, \mu_2, g, c_1, c_2, c_3, k_1, k_2\}$. We will fix all of them except for μ_2 when plotting the eigenvalues.

U(1)-invariant vacuum

Let's take the trivial vacuum $\phi^{(0)} = 0$ (This is the only $U(1)$ invariant vacuum). At this vacuum we have 3 degenerate eigenvalues (Two 3×3 Jordan blocks with same eigenvalues).

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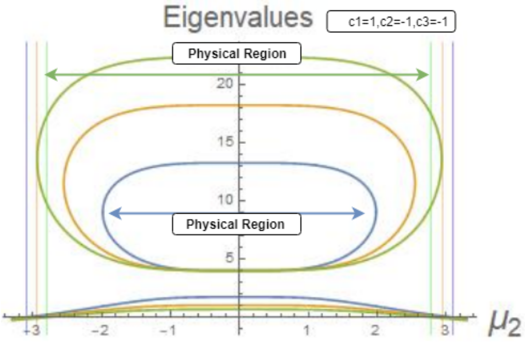

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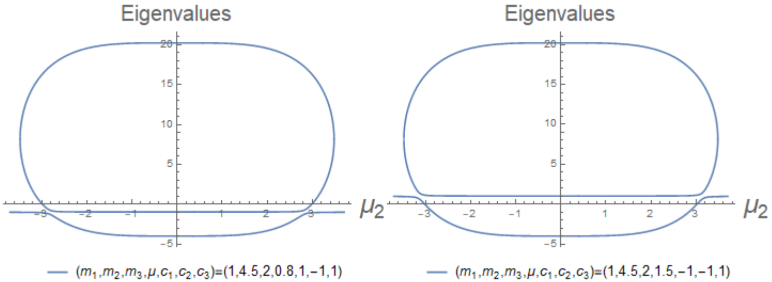

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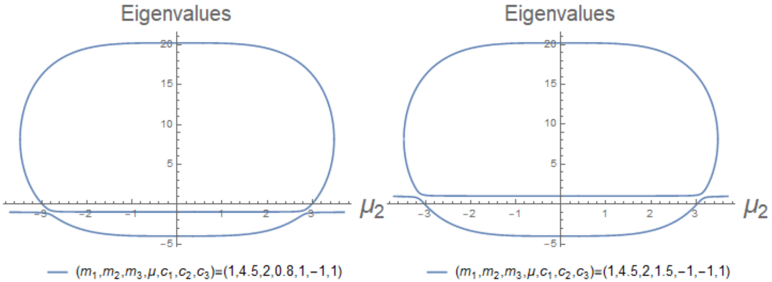

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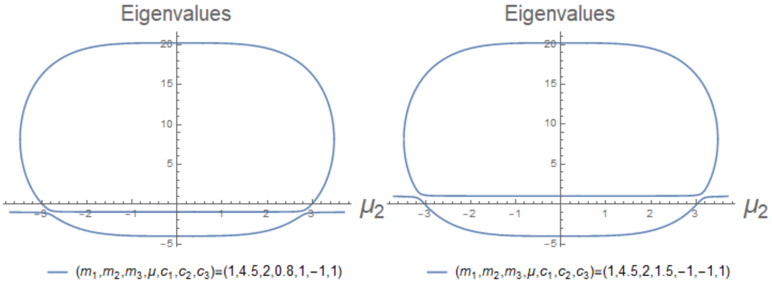

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- Where are physical regions?

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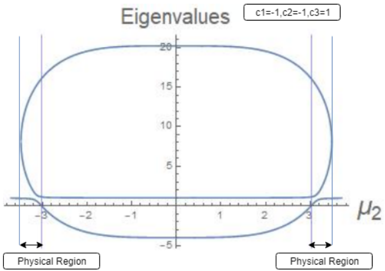

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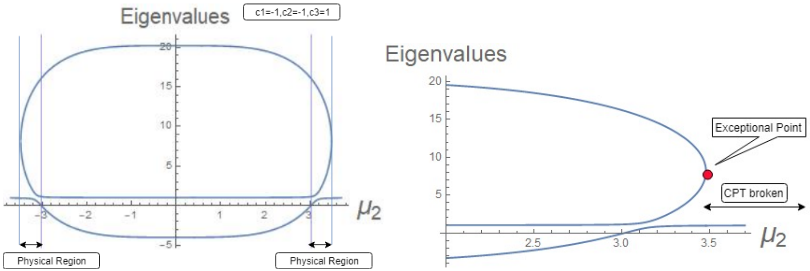

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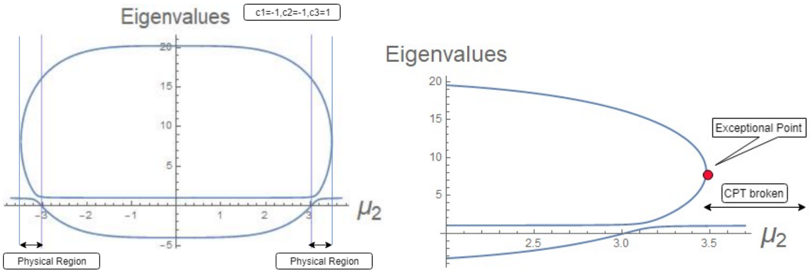

- So at exceptional point and beyond, $[\mathcal{PT}, M^2] \neq 0$ or $\mathcal{PT} v \neq e^{i\alpha} v$

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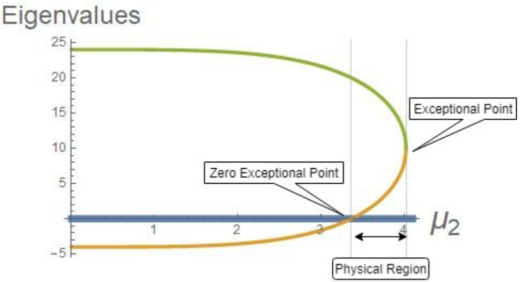

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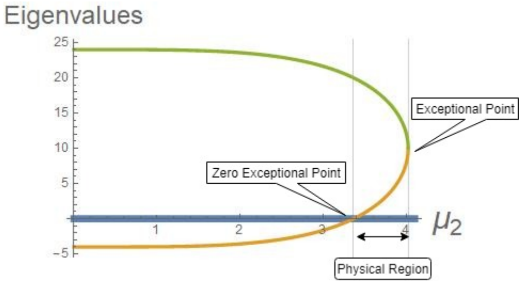

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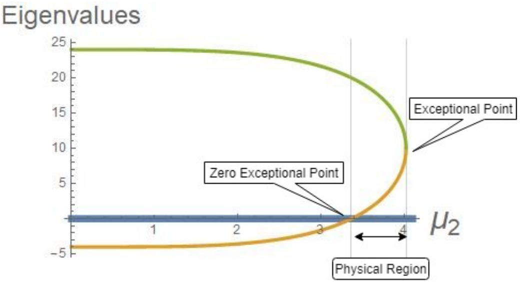

- $\mathcal{CPT}$ is broken at ZeroEp, EP and beyond EP.
- Mass matrix is NOT diagonalisable at EP and Zero EP, This will give us headache in section 3

U(1) broken vacuum

Next we look at $U(1)$ broken vacuum.

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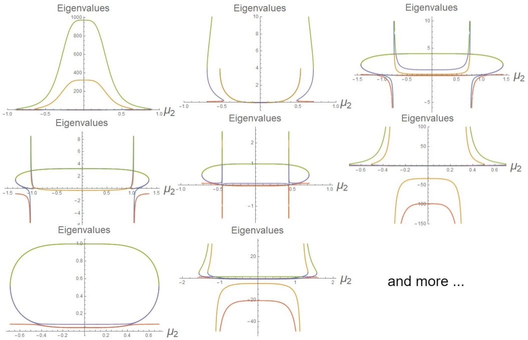

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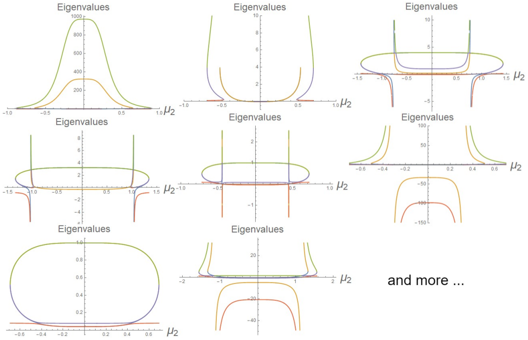

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Let's focus on one.

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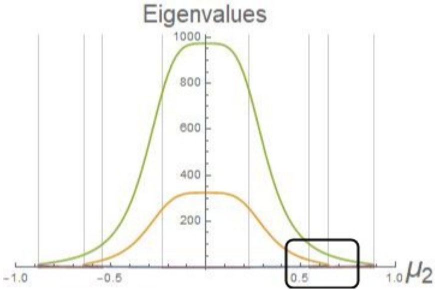

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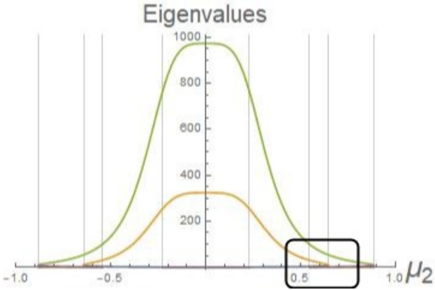

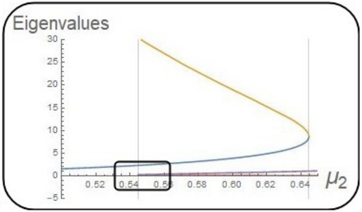

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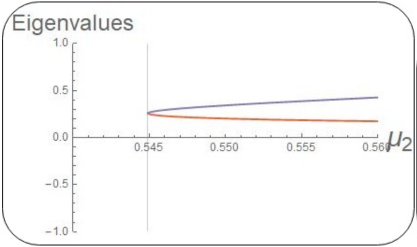

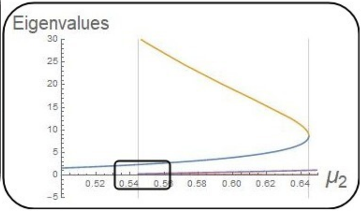

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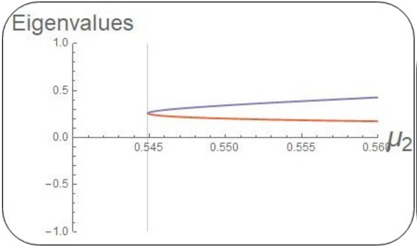

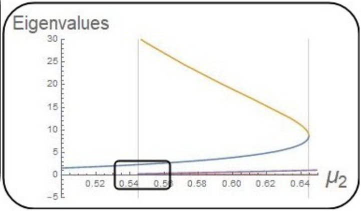

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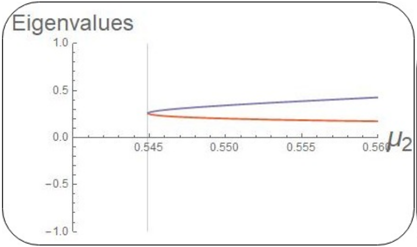

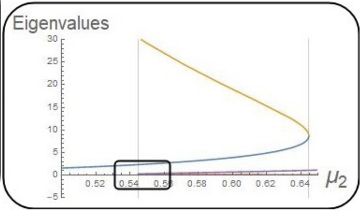

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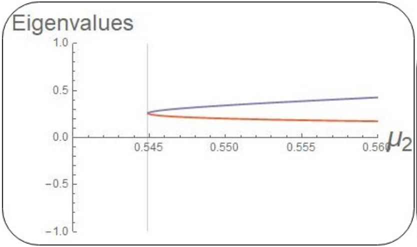

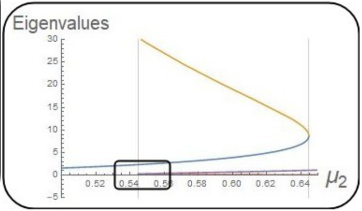

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- **Question**: Can we define Goldstone field everywhere in the parameter space?

Goldstone bosons at exceptional points

Diagonalising mass matrix

Recall that the action after similarity transformation and expansion around some vacuum looks like

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Diagonalising mass matrix

Recall that the action after similarity transformation and expansion around some vacuum looks like

$$\begin{aligned} S &= \int \frac{1}{2}\partial_\mu \Phi^T \mathcal{I} \partial^\mu \Phi - \frac{1}{2}\Phi^T \mathcal{I} M^2 \Phi + \mathcal{O}(\Phi^3) \\ &= \int -\frac{1}{2}\Phi^T \mathcal{I}(\Box + M^2)\Phi + \dots \end{aligned}$$

Let's say we can write the mass matrix like $M^2 = TJT^{-1}$ then

$$\phi^T \mathcal{I}(\Box + M^2)\phi = \phi^T \mathcal{I} T(\Box + J) T^{-1}\phi \equiv (\psi^L)^T (\Box + J)\psi^R$$

Forms of J, ψ^R and ψ^L at (zero) exceptional points are

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So we can define $\psi_{Gb} \equiv \sqrt{\psi_1^R \psi_1^L}$ everywhere except at zero exceptional point! recall $\Psi_{Gb} = (\delta\Phi^{(0)})^T \mathcal{I} \Phi$ from proof of Goldstone theorem

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PT symmetric	$Det(T) = k_2\lambda_+\lambda_-(\lambda_+ - \lambda_-)\mu_1^4\mu_2^2$

Where $\kappa = c_2 c_3 m_2^2 m_3^2 + \nu^4$ and μ_e is a value of μ when $\lambda_+ = \lambda_- \equiv \lambda_e$.

- ψ_{Gb} is ill-defined when $\lambda_+ = 0$,$\lambda_- = 0$,$\lambda_+ = \lambda_-$ $\mu_1 = 0$ or $\mu_2 = 0$

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- ψ_{Gb}^e is ill-defined when $\kappa = 0$, $m_3 = 0$ or $\lambda_e = 0$
- These Goldstone fields are related to symmetry generators $\psi_{Gb} = \frac{\kappa}{2N}(\delta\Phi^{(0)})^T\mathcal{I}\Phi$.

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Good News:

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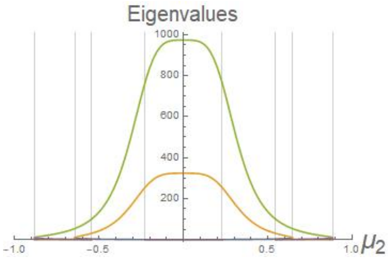

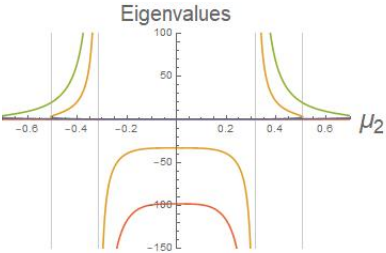

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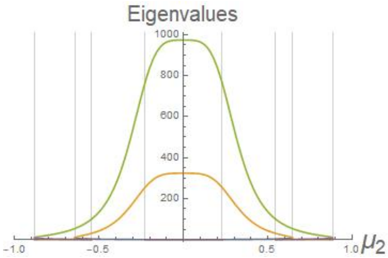

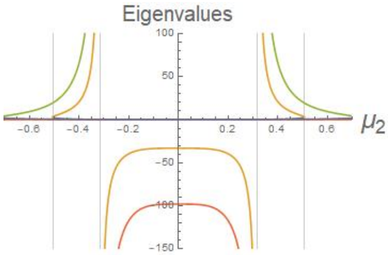

There is no zero exceptional point within the physical region so for this theory we can define Goldstone everywhere (even at edges) in the physical region.

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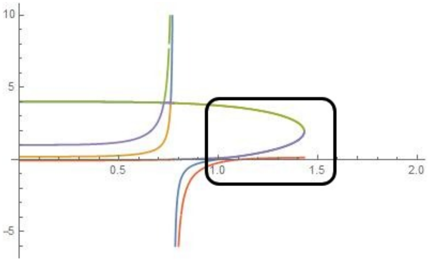

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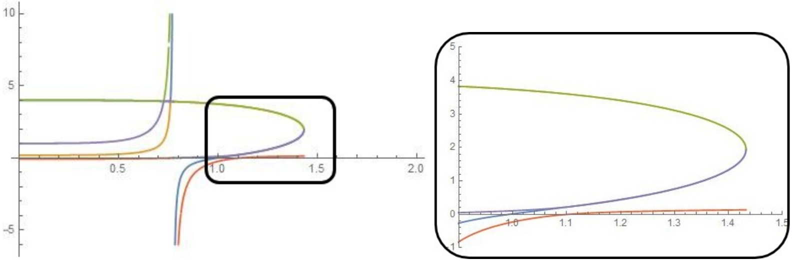

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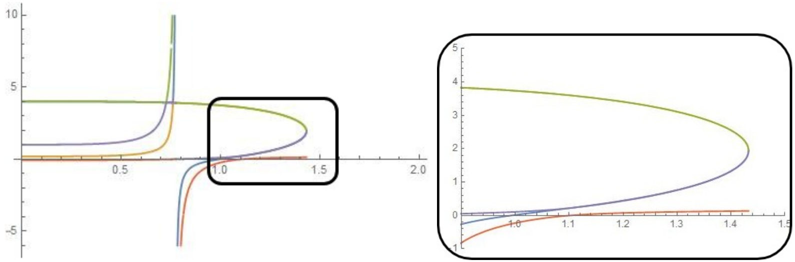

In this theory we have zero exceptional point within the physical region (at the edge). Eigenvalues at $\mu_2 = 1.2$ are $\{0.441699, 3.40854, 3.40857, 0.0704341, 0.431723\}$

Conclusion/Future work

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Conclusion

(1) We found that the physical region in the parameter space exist in the non-Hermitian theory but they are usually bounded between (zero) exceptional points and singularity.

(2) Goldstone fields can be defined in the $\mathcal{CPT}$-regime and at the exceptional point but not at the zero exceptional point.

(3) We can avoid zero exceptional point in some physical region.

Future work

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(6) And many more ...

Thank you

Thank you for your attention
Gracias