EXPLORING TOPOLOGY AND SYMMETRY PROTECTION IN QUANTUM HALL NEMATICS

References : arXiv: 1809.09616 (review); 1807.10293 (theory)
Nature 566, 363 (2019) (experiments)
PRB 82, 035428; 88, 045133; 93, 014442.

x ——— x

Goal : explain some of the physics of QH states w/ broken symmetry + make contact with SPT ideas.

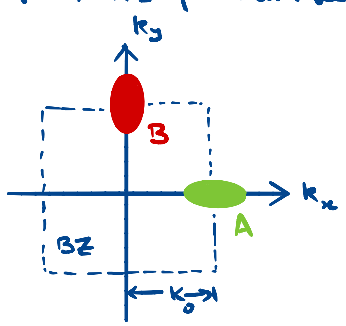
Plan : Lecture 1 : basic theory of the bulk + topological terms.
Lecture 2 : theory of domain walls + symmetry protection.

I. Introduction

Setting : Integer QH in "multivalley" semiconductors.

Minimal model : (~ AlAs quantum wells)

$$H = \sum_\alpha \frac{(p_x - K_x^\alpha - eA_x)^2}{2m_x^\alpha} + \frac{(p_y - K_y^\alpha - eA_y)^2}{2m_y^\alpha}$$



$m_x^A = m_y^B = M$; $m_x^B = m_y^A = m$

$\frac{m}{M} = \lambda^2 \neq 1$ in general ; $\vec{K}^A = (0, K_0)$
$\vec{K}^B = (K_0, 0)$.

$K_0 \sim 1/a$; $a \sim$ lattice spacing

Exercise: work out LL orbitals in each valley.

In "Landau gauge" $\vec{A} = (0, -Bx)$ we have

$$\psi^{(n)}_{X,\alpha}(\vec{r}) = \frac{e^{iXy/\ell_B^2}}{\sqrt{L_y \ell_B}} \phi_n(x - X; u_\alpha) \cdot e^{i\vec{K}_\alpha \cdot \vec{r}} \; ; \; E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega_c$$

$\ell_B = \left(\frac{\hbar}{eB}\right)^{1/2}$ ~ magnetic length ; $\omega_c = \frac{eB}{\sqrt{mM}}$;

ϕ_n ~ n^{th} harmonic oscillator wavefunction.

Landau levels (LLs) are each twofold degenerate b/c valley

$\Rightarrow$ without interactions, only predict QH plateaus at $\sigma_{xy} = 2n\frac{e^2}{h}$.

Interaction effects remove this degeneracy by spontaneously breaking valley symmetry.

Basic Idea is simple: think of Pauli exclusion:

$$\Psi_\alpha^{(n)}(\vec{r}_1, \ldots, \vec{r}_N) \equiv \langle \vec{r}_1 \ldots \vec{r}_N | \prod_X c^\dagger_{X,\alpha,n} | 0 \rangle$$

fill all states in LL in <u>same</u> valley.

For repulsive interactions: energy is <u>minimized</u> by this choice, since $\Psi_\alpha^{(n)}$ vanishes whenever two electrons come together

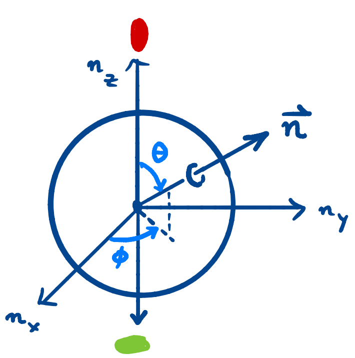
$\Rightarrow$ ground state is a QH <u>ferromagnet</u>. (originally developed for spins in GaAs)

QHFMs ~ combine <u>broken symmetry</u> + topological aspects.

<u>First</u>: what is the "effective action"?

<u>Basic idea</u>: "pseudospin" order parameter: parametrize on valley Bloch sphere

$$\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$



$$|\Psi(\vec{n})\rangle = \prod_X \left(\cos\frac{\theta}{2} c^\dagger_{X,A} + \sin\frac{\theta}{2} e^{i\phi} c^\dagger_{X,B}\right)|0\rangle$$

~ variational trial state (good approx at $\nu = 1$).
~ fix LL index
~ need to determine Hamiltonian by projecting to LL.

Can easily compute matrix elements of interactions in Landau basis.

$$V^{\gamma Y, \delta Y'}_{\beta X', \alpha X} = \int d^2r\, d^2r'\, \psi^*_{\alpha X}(\vec{r})\, \psi^*_{\beta X'}(\vec{r}')\, V(\vec{r} - \vec{r}')\, \psi_{\delta Y'}(\vec{r}')\, \psi_{\gamma Y}(\vec{r})$$

Assume Coulomb interactions: $V(\vec{r}) = \dfrac{e^2}{\epsilon|\vec{r}|}$ Coulomb.

Generically, expect:

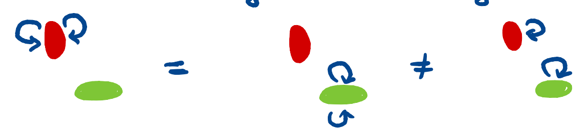
$$H = \frac{1}{2} \sum_{\substack{\alpha,\beta \\ \gamma,\delta}} \sum_{\substack{X,X' \\ Y,Y'}} V^{\alpha Y, \beta Y'}_{\beta X', \alpha X} \, c^\dagger_{\gamma Y} c^\dagger_{\delta Y'} c_{\beta X'} c_{\alpha X} + \text{background}.$$

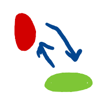
<u>But</u>: $V(q) = \dfrac{2\pi e^2}{\epsilon|\vec{q}|}$ ~ small for large momentum transfer.

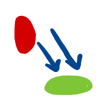
$\Rightarrow$ matrix elements between valleys suppressed ("small parameter ~ $1/k_0 \ell_B \sim a/\ell_B$)

<u>Hierarchy of scales</u>:

Label interactions in valley space by arrows starting at c, ending on $c^\dagger$.

0 $\sim \dfrac{e^2}{\epsilon \ell_B}$: set $\gamma = \alpha$, $\delta = \beta$ 

1 $\sim \dfrac{e^2}{\epsilon \ell_B} \cdot \left(\dfrac{a}{\ell_B}\right)^2$: set $\delta = \alpha \neq \beta = \gamma$  + h.c.

2 $\sim \dfrac{e^2}{\epsilon \ell_B} e^{-\ell_B/a}$ set $\delta = \gamma = A$, $\beta = \alpha = B$  + others } we'll ignore these terms.

For a first pass: only take H_0. (N.B. $\vec{n} \neq$ nematic director ~ discrete nematic).

$$\langle \Psi(\vec{n}) | \hat{H}_0 | \Psi(\vec{n}) \rangle \sim -(E_s + E_a n_z^2) \qquad E_{s,a} \sim O(1) \cdot \frac{e^2}{\epsilon \ell_B} > 0$$

* minimized for $n_z = \pm 1$ ~ all electrons fill one or other valley.

"Ising-like" order b/c $E_a > 0$.] This breaks $C_4 \rightarrow C_2$ ~ "nematic" order.

[N.B. Can have other possibilities: $\alpha, \beta \sim \uparrow, \downarrow$ spin $E_a = 0$ "Heisenberg"
~ layer index $E_a < 0$ "XY".]

II. Effective Theory

So far we've obtained the g.s. energy.

We'd also like an effective action for low-energy fluctuations.

Various ways to derive this:

1) Chern-Simons + Landau-Ginzburg theory.

2) "Guess" answer:

Split $\quad \mathcal{L} = \mathcal{L}_{kin}[\vec{n}] - \mathcal{E}[\vec{n}]$

Gradient expansion (but can compute coefficients exactly in Hartree-Fock + spin wave)

$$\mathcal{E}[\vec{n}] \simeq \frac{1}{2}\int d^2r \left\{ \rho_s (\nabla \vec{n})^2 - \beta n_z^2 + \dots \right\} \qquad (\rho_s \propto E_s,\ \beta \propto E_a).$$

$\mathcal{L}_{kin}$ is more subtle b/c need to capture (pseudo) spin precession.

* Simpler problem: single spin-s in B field. ($\hbar = 1$ units)

$$H = -\vec{B}\cdot\vec{S} \quad ; \quad \frac{d}{dt}\vec{S} = i[H,\vec{S}] = -\vec{B}\times\vec{S} \quad \text{(precession)}$$

Want to get this from $\quad \frac{d}{dt}\frac{\delta \mathcal{L}_K}{\delta \dot{n}^\mu} = \frac{\delta \mathcal{L}_K}{\delta n^\mu}$

General rule: for first-order-in time equation of motion, need a vector potential. ansatz:

$$\mathcal{L}_K = s\left\{ -\dot{n}_\mu A^\mu[\vec{n}] + B_\mu n^\mu \right\} \quad + \vec{n}^2 = 1 \text{ constraint.}$$

EOMs:

$$\frac{d}{dt}\left\{ -A^\mu[\vec{n}] \right\} = +B_\mu - \dot{n}_\nu \frac{\partial A^\nu[\vec{n}]}{\partial n_\mu}$$

$$-\frac{\partial A^\mu}{\partial n_\nu}\cdot \dot{n}^\nu = +B_\mu - \dot{n}_\nu \frac{\partial A^\nu}{\partial n_\mu}[\vec{n}].$$

$$s\,\mathcal{F}_{\mu\nu}[\vec{n}]\,\dot{n}^\nu = +B_\mu.$$

"flux in $\vec{n}$-space"

Choose $A_\mu[\vec{n}]$ so that $\quad F_{\mu\nu} = \frac{\partial A_\nu}{\partial n^\mu} - \frac{\partial A_\mu}{\partial n^\nu} = \epsilon^{\mu\nu\lambda} n^\lambda.$

Then

$$(\epsilon^{\mu\rho\sigma} n^\rho)\epsilon^{\mu\nu\lambda} n^\lambda \dot{n}^\nu = B_\mu \cdot (\epsilon^{\mu\rho\sigma} n^\rho)$$

$$\underset{\underset{d_t\vec{n}^2=0}{\parallel}}{n^\rho \dot{n}^\rho} n^\sigma - \underset{\underset{1}{\parallel}}{\underbrace{n_\rho n_\rho}} \dot{n}^\sigma = (\vec{B}\times\vec{n})^\sigma. \qquad (\epsilon^{\mu\rho\sigma}\epsilon^{\mu\nu\lambda} = \delta_{\rho\nu}\delta_{\sigma\lambda} - \delta_{\rho\lambda}\delta_{\sigma\nu}$$

$$\Rightarrow \quad \dot{\vec{S}} = -\vec{B}\times\vec{S} \qquad (\vec{S} = s\vec{n} \text{ as before}).$$

Can view $A[\vec{m}]$ as vector potential of a magnetic monopole in spin space ~ various aspects follow from this (cf arXiv:cond-mat/9907002).

For many spins: the field $\vec{m}[\vec{r}]$ sees such a term.

So, find an effective Lagrangian

$$\mathcal{L}_{FM} = s\frac{N}{A}\int d^2r\left\{\dot{n}^\mu A_\mu[\vec{n}] - \underbrace{\Delta_v(\vec{r},t) n_z(\vec{r},t)}_{\substack{\text{“magnetic field” on valleys} \\ \text{(from uniform / random strain)}}}\right\} - \frac{1}{2}\int d^2\vec{r}\left\{\rho_s(\nabla\vec{n})^2 - \beta n_z^2\right\}$$

* Could have just written this down for <u>any</u> (easy-axis) FM.

Where did QH physics go?

<u>II.B Topological Terms</u>

Consider dragging an electron through the system.

Since system is FM, 'pseudospin' tracks the local $\vec{n}$

$$\mathcal{L}_e = -e\dot{x}^\mu A^\mu + s\dot{n}^\nu A^\nu[\vec{n}]$$

$$= -e\dot{x}^\mu(A^\mu + a^\mu) \quad \text{where} \quad a^\mu = -\frac{\Phi_0}{2\pi} s \frac{\partial n^\nu}{\partial x^\mu} A^\nu[\vec{n}]$$

$$\Phi_0 = \frac{h}{e} \sim \frac{2\pi}{e} \quad \text{if } \hbar = 1.$$

"Additional" fictitious magnetic field seen by test electron:

$$b = \epsilon^{\alpha\beta}\partial_\alpha a_\beta = -\frac{\Phi_0}{2\pi}\, s\, \epsilon^{\alpha\beta}\frac{\partial}{\partial x^\alpha}\left\{ \frac{\partial n^\nu}{\partial x^\beta} A^\nu[\vec{n}] \right\}$$

$$= -\frac{\Phi_0}{2\pi}\, s\, \epsilon^{\alpha\beta}\left\{ \cancelto{0 \text{ b/c } \epsilon^{\alpha\beta}}{\frac{\partial^2 n^\nu}{\partial x^\alpha \partial x^\beta}} A^\nu + \frac{\partial n^\nu}{\partial x^\beta}\underbrace{\frac{\partial A^\nu[\vec{n}]}{\partial x^\alpha}}_{= \frac{\partial A^\nu}{\partial n^\mu}\cdot\frac{\partial n^\mu}{\partial x^\alpha}} \right\}.$$

$$= -\frac{\Phi_0}{2\pi}\, s\, \epsilon^{\alpha\beta}\underbrace{\frac{\partial n^\mu}{\partial x^\alpha}\frac{\partial n^\nu}{\partial x^\beta}}_{\text{antisymmetric in } \mu\leftrightarrow\nu}\frac{\partial A^\nu}{\partial n^\mu}.$$

$$= -\frac{\Phi_0}{2\pi}\, s\, \epsilon^{\alpha\beta}\frac{\partial n^\mu}{\partial x^\alpha}\frac{\partial n^\nu}{\partial x^\beta}\frac{1}{2}\underbrace{F^{\mu\nu}}_{= \epsilon^{\mu\nu\sigma} n^\sigma} \equiv -\Phi_0\, \tilde{\rho}_{top}$$

Where, for $s = 1/2$ we define

$$\tilde{\rho}_{top} = \frac{1}{8\pi}\epsilon^{\alpha\beta}\epsilon^{abc} n^a \partial_\alpha n^b \partial_\beta n^c \sim \text{Pontryagin density.}$$

As electron moves past texture, sees adiabatically varying a_μ

$\Rightarrow$ sees an effective field b.

$\Rightarrow$ extra charge density from this is $\delta\rho = \sigma_{xy}\cdot b = \nu\frac{e^2}{h}\cdot\frac{h}{e}\tilde{\rho}_{top} = \nu e\tilde{\rho}_{top}$

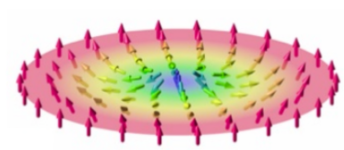
↳ Laughlin argument.

Define "topological charge" $Q_{top} = \int d^2r\, \tilde{\rho}_{top}$

$\Rightarrow$ Extra charge $\delta Q = \nu e Q_{top}$

Spin textures w/ nonzero topological charge carry electrical charge.

A $Q_{top} = 1$ texture is a Skyrmion.



A more complex term ('Hopf term') gives the skyrmion Fermi statistics.

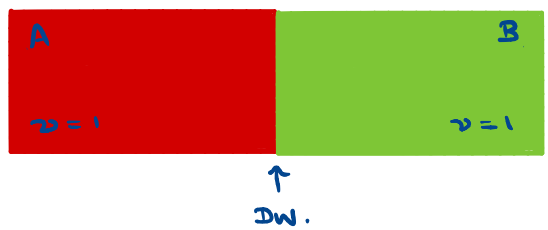
Effective field theory must incorporate these (+ terms $\int V(\vec{r}-\vec{r}')\,\delta\tilde{\rho}_{top}(\vec{r})\,\delta\tilde{\rho}_{top}(\vec{r}')$)

[Basic reason: charge + valley don't commute within the Landau level].

III Domain Wall Physics

Recall we have an Ising anisotropy of our order parameter

Natural topological defects: not skyrmions, but domain walls.



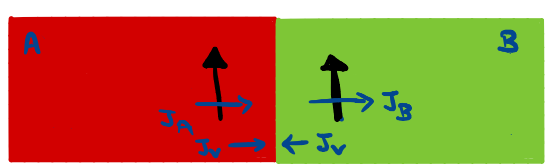
Domain walls carry topologically-protected edge modes (unsurprising!)

One slick way to see this + get quantum numbers: use $\sigma_{xy} \neq 0$.

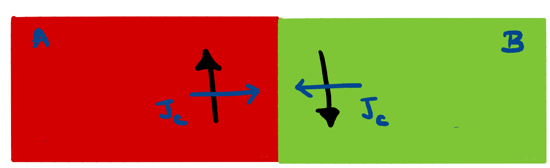
Observe 2 conserved U(1) charges: total charge $\propto \rho_A + \rho_B$
" valley $\propto \rho_A - \rho_B$

Apply $\vec{E}$ field

Apply valley $\vec{E}$-field. ($+\vec{E}$ on A, $-\vec{E}$ on B).



valley inflow to DW



charge inflow to DW.

$\Rightarrow$ at $\nu = 1$, DW must have gapless charge/valley modes.

Goal: derive an effective theory + understand symmetry protection.

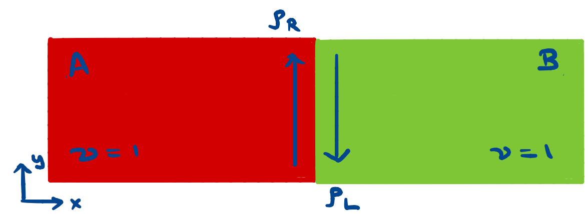
Intuitively: 2 counterpropagating modes ~ Luttinger liquid (Luttinger parameter not set by bulk filling, unlike chiral Luttinger liquids at QH edges).

~ but b/c counterpropagating, why is there protection of gaplessness?

Strategy: use bosonization: represent R/L moving densities in terms of phase fields ϕ, θ [cf Giamarchi's book]
"right movers" from valley A, "left movers" from valley B.

Trick: in Landau level, $X \sim p_y$ (coordinate $\perp^r$ wall $\sim$ momentum parallel to it).

$$\left.\begin{aligned} \rho_R(q_y) &\sim \sum_X c^+_{X+q_y,A}\, c_{X,A} \\ \rho_L(q_y) &\sim \sum_X c^+_{X+q_y,B}\, c_{X,B} \,. \end{aligned}\right\}$$
only nontrivial near the DW. (standard bosoniz^n.)



$\rightarrow \rho_{R,L}(y)$ by Fourier transform.

Keeping only H_0:

$$H_{DW} \sim v_F^0 \int dy \left[(\rho_R + \rho_L)^2 + (\rho_R - \rho_L)^2 \right]$$

most general density-density terms from $\mathcal{H}_0$, $v_F^0 \sim e^2/\epsilon$.

Position of DW is not specified $\Rightarrow \begin{bmatrix} \rho_A \rightarrow \rho_A + \epsilon \\ \rho_B \rightarrow \rho_B - \epsilon \end{bmatrix}$ is a symmetry (transl. $\perp^r$ to wall).

$\Rightarrow$ Can't write any term of the form $(\rho_R - \rho_L)^2$

$H_{DW} \sim \frac{\pi}{2} v_F^0 \int dy (\rho_R + \rho_L)^2 \longrightarrow$ singular Luttinger liquid (!).

$\rightarrow$ actually, to create DW, have "valley field" gradient Γ, of form.

$\delta H_0 \sim v_F^0 \sum_X \Gamma \cdot X \cdot (c^+_{X,A} c_{X,A} - c^+_{X,B} c_{X,B})$. (breaks shift symmetry).

Since $X \sim q_y$, this is like the usual fermion kinetic energy.

A standard result is that for linear dispersion, have

$$\delta H_{DW} \sim \pi \Gamma \int dy \, \left(\rho_R^2(y) + \rho_L^2(y) \right) = \pi\Gamma \int dy \left\{ \tfrac{1}{2} (\rho_R + \rho_L)^2 + \tfrac{1}{2} (\rho_R - \rho_L)^2 \right\}$$

$$H_{DW} + \delta H_{DW} \sim \frac{1}{2}\pi \int dy \left\{ (v_F^0 + \Gamma)(\rho_R + \rho_L)^2 + \Gamma (\rho_R - \rho_L)^2 \right\}$$

Bosonization: $\rho_R + \rho_L \equiv -\frac{1}{\pi} \nabla \phi \qquad \rho_R - \rho_L \equiv \frac{1}{\pi} \nabla \theta$.

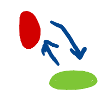
$$[\phi(y), \nabla\theta(y')] = i\,\delta(y - y')$$

$$H_{DW} + \delta H_{DW} = \frac{1}{2\pi} \int dy \left\{ (v_F^0 + \Gamma)(\nabla\phi)^2 + \Gamma (\nabla\theta)^2 \right\}$$

$$= \frac{v_F}{2\pi} \int dy \left\{ \frac{1}{K} (\nabla\phi)^2 + K (\nabla\theta)^2 \right\}$$

where $\frac{v_F}{K} = v_F^0 + \Gamma$; $v_F K = \Gamma$ $\Rightarrow$ $v_F = \sqrt{(v_F^0 + \Gamma)\Gamma}$, $K = \sqrt{\frac{\Gamma}{v_F^0 + \Gamma}}$

Note $v_F, K \to 0$ as $\Gamma \to 0$ (singular limit) mentioned earlier.
for $\Gamma \ll 1$, $K < 1$ ~ repulsive interactions.

So far: only considered H_0 terms. What about H_1?  + h.c.

~ renormalizes couplings, but could induce backscattering ($\cos\theta$, $\cos\phi$).

[in bosonization: "forward" scattering changes v_F, K;
"backscattering" generates cosine terms (try to gap system)]

Conserved quantities: total particle #: $\mathcal{N} = N_A + N_B$; pseudospin $I^z = \frac{1}{2}(N_A - N_B)$.

Recall
$$\nabla\phi \sim -\pi[\rho_R + \rho_L] \sim -\pi[\rho_A + \rho_B]$$
$$\nabla\theta \sim -\pi[\rho_R - \rho_L] \sim -\pi[\rho_A - \rho_B]$$

U(1) generators of our symmetries linked to fields via:

$$\mathcal{N} = \int dy \, (\rho_A + \rho_B) \sim -\frac{1}{\pi} \int dy \, \nabla\phi$$

$$I^z = \int dy \, \frac{(\rho_A - \rho_B)}{2} \sim -\frac{1}{2\pi} \int dy \, \nabla\theta .$$

Use the commutation relation and $[A, e^{iB}] = i[A,B]\, e^{iB}$
(valid if $[[A,B],B] = 0$) then

$$[\mathcal{N}, e^{\pm i\theta}] = \mp e^{\pm i\theta} \; ; \; [I^z, e^{\pm i\phi}] = \mp e^{\pm i\phi}$$

$$\left.\begin{aligned} &\sim e^{\mp i\theta} \text{ are raising / lowering operators for } \mathcal{N} \\ &\quad e^{\mp i\phi} \text{ are raising / lowering operators for } I^z \end{aligned}\right\} \text{ forbidden by U(1) symmetry.}$$

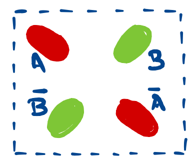
$\Rightarrow$ "defects" of one field (e.g. $e^{i\theta}$ creates defect of ϕ and vice-versa) carry quantum numbers of the other field.

$\Rightarrow$ as long as charge conservation + valley $U(1)$ are preserved, domain wall is gapless (since cosines are forbidden).

Topological symmetry protection ~ similar to $U(1)\times U(1)$ limit of TI (see by "folding" at DW).

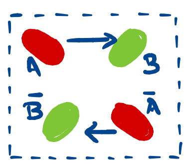
IV. A richer example

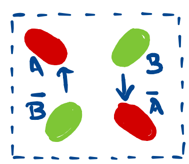
Consider a 4-valley system



Can build $\nu = 2$ DW between $A\bar{A}$ and $B\bar{B}$ regions (this is natural b/c strain acts the same way on $A\bar{A}$ and on $B\bar{B}$)

Hierarchy of scales: leading terms same as before, but new terms appear at $O((a/\ell_B)^2)$:





Nominally: 4 conserved $U(1)$s $(N_A, N_{\bar{A}}, N_B, N_{\bar{B}})$.

But the new terms violate conservation of $N_A + N_{\bar{A}} - N_B - N_{\bar{B}}$.

So rearrange as

$$\mathcal{N} = N_A + N_B + N_B + N_{\bar{B}} \quad ✓$$

$$Q^z = \tfrac{1}{2}(N_A + N_B - N_{\bar{A}} - N_{\bar{B}}) \quad ✓$$

$$P^z = \tfrac{1}{2}(N_A - N_B - N_{\bar{A}} + N_{\bar{B}}) \quad ✓$$

all conserved!

$$I^z = \tfrac{1}{2}(N_A - N_B + N_{\bar{A}} - N_{\bar{B}}) \quad \times \text{ not conserved}$$

Can derive a 2-channel Luttinger liquid [$A\bar{A}B\bar{B} \sim R\uparrow R\downarrow L\uparrow L\downarrow$].

$$\mathcal{H}_{eff} = \sum_{\zeta=\rho,\sigma} \frac{u_\zeta}{2\pi}\int dy \left[\frac{1}{K_\zeta}(\nabla\phi_\zeta)^2 + K_\zeta(\nabla\theta_\zeta)^2 \right].$$

with

$$\nabla\phi_\rho \sim \rho_A + \rho_B + \rho_{\bar{A}} + \rho_{\bar{B}} \qquad \nabla\theta_\rho \sim \rho_A - \rho_B + \rho_{\bar{A}} - \rho_{\bar{B}}$$
$$\nabla\phi_\sigma \sim \rho_A + \rho_B - \rho_{\bar{A}} - \rho_{\bar{B}} \qquad \nabla\theta_\sigma \sim \rho_A - \rho_B - \rho_{\bar{A}} - \rho_{\bar{B}}.$$

so

$$\mathcal{N} \sim \int dy \,\nabla\phi_\rho \;;\; Q^z \sim \int dy\,\nabla\phi_\sigma \;;\; P^z \sim \int dy\,\nabla\theta_\sigma \;;\; I^z \sim \int dy\,\nabla\theta_\rho$$

$$[\mathcal{N}, e^{\pm i\theta_\rho/\sqrt{2}}] = \mp e^{\pm i\theta_\rho/\sqrt{2}} \qquad [P^z, e^{\pm i\sqrt{2}\phi_\sigma}] = \mp e^{\pm i\sqrt{2}\phi_\sigma}$$
$$[Q^z, e^{\pm i\sqrt{2}\theta_\sigma}] = \mp e^{\pm i\sqrt{2}\theta_\sigma} \qquad [I^z, e^{\pm i\sqrt{2}\phi_\rho}] = \mp e^{\pm i\sqrt{2}\phi_\rho}.$$

But I^z only conserved modulo 2!

$\Rightarrow$ $\cos(2\sqrt{2}\phi_\rho)$ is an allowed perturbation.

$$\mathcal{H}_{eff} \to \mathcal{H}_{eff} + g\int dy \cos(2\sqrt{2}\phi_\rho).$$

For $K \ll 1$, $g \to \infty$, "pins" ϕ_ρ. Since ϕ_ρ is conjugate to θ_ρ, when ϕ_ρ orders, θ_ρ is disordered.

$$\Rightarrow \left\langle e^{\frac{i}{\sqrt{2}}\theta_\rho(y)} e^{\frac{-i}{\sqrt{2}}\theta_\rho(y')} \right\rangle \sim e^{-|y-y'|/\xi} \qquad \text{charge gap!}$$

<u>But</u> gapless valley mode persists ~ valley charge separation.

[EXPERIMENTAL DATA].

<u>If time</u>: field theoretic derivation for $\nu = 1$.

x —— x .