“Neutrino tutorials” TAE 2019

1. *Neutrino oscillations:* The general expression for neutrino oscillations in vacuum is given by:

$$
\begin{aligned}
\mathrm{P}(\nu_\beta \to \nu_\alpha) &= \sum_{i,j} U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j} e^{i\Delta m^2_{ij} L/2E} \\
&= \delta_{\alpha\beta} - 4 \sum_{i>j} \mathrm{Re}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E}) \\
&\qquad + 2 \sum_{i>j} \mathrm{Im}(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E}) \quad , \qquad (1)
\end{aligned}
$$

where $\Delta m^2_{ij} = m^2_i - m^2_j$ is the mass squared difference between the neutrino mass eigenstates, L is the distance between neutrino production and detection, E is the neutrino energy and

$$
U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad , \qquad (2)
$$

is the neutrino mixing matrix and where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

a) “Atmospheric regime” neutrino oscillations. For atmospheric neutrinos L/E is small, so that $\Delta m^2_{21} L/E \sim 0$ can be neglected. Then Δm^2_{31} $= \Delta m^2_{21} + \Delta m^2_{32} \sim \Delta m^2_{32}$. The small mixing angle, θ_{13} also plays only a subleading role and can be set to 0 as a good approximation. With these two approximations compute the oscillation probabilities of $\mathrm{P}(\nu_e \to \nu_e)$, $\mathrm{P}(\nu_e \to \nu_\mu)$, $\mathrm{P}(\nu_\mu \to \nu_\mu)$ and $\mathrm{P}(\nu_\mu \to \nu_e)$, which are the relevant probabilities for atmospheric neutrino oscillations since ν_τ cannot be detected at those energies. Show that through $\mathrm{P}(\nu_\mu \to \nu_\mu)$ θ_{23} and $|\Delta m^2_{31}|$ can be measured. These measurements have been confirmed (and improved) at several accelerator experiments (K2K, MINOS and T2K) which produce beams of ν_μ from accelerated pions and search for $\mathrm{P}(\nu_\mu \to \nu_\mu)$.

b) “Solar regime” neutrino oscillations. In the “solar regime” L/E is much larger so that $\Delta m^2_{21} L/E \sim 1$ and cannot be neglected. But very low energies are needed for this regime, so only ν_e can be detected. Compute $\mathrm{P}(\nu_e \to \nu_e)$ in this regime using only the approximation that $\theta_{13} \sim 0$. Show that with this oscillation probability Δm^2_{21} and θ_{12} can be measured. This oscillation has been seen by the KamLAND experiment that detected the $\bar{\nu}_e$ flux from several nuclear reactors in Japan. These measurements agree well with those of solar neutrinos in the regime of very strong and adiabatic matter effects.

c) θ_{13}. In 2012, θ_{13} was discovered by several reactor experiments searching for $\mathrm{P}(\bar{\nu}_e \to \bar{\nu}_e)$ with a small value of L/E so $\Delta m^2_{21} L/E \sim 0$. Compute this oscillation probability and show that it depends on θ_{13}.

d) CP violation. In order to observe CP violation the interference between the "solar" and the "atmospheric" regime must be observed in an appearance channel. The most promising channel for this measurement, the "golden channel", is the P($\nu_e \to \nu_\mu$) channel. This oscillation probability expanded up to second order in the small parameters θ_{13} and $\Delta m^2_{21} L/E$ can be found for instance in hep-ph/0002108 (eq. (7) for vacuum oscillations and (16) for matter). Which value of L/E would you choose to optimize the sensitivity of an oscillation experiment to CP violation?

2. *Seesaw mechanisms:* There are three ways to realize at tree level the Weinberg $d = 5$ effective operator for neutrino masses through different extensions of the SM and after integrating out the new heavy degrees of freedom:

- **The type-I Seesaw** introduces heavy fermion singlets N_R^i to the SM particle content which are allowed to have a Yukawa coupling with the Lepton doublets ℓ_L^α and a Majorana mass:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{N}_R \not{D} N_R - \frac{1}{2}\overline{N_R^i}\,(M_N)_{ij}\,N_R^{cj} - (Y_N)_{i\alpha}\,\overline{N_R^i}\tilde{\phi}^\dagger \ell_L^\alpha + \text{h.c.}\,,$$

- **The type-II Seesaw** introduces a scalar $SU(2)_L$ triplet Δ to the SM particle content which is allowed to have a Yukawa coupling with the Lepton doublets, a coupling to the SM Higgs and a mass term:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + (D_\mu \Delta)^\dagger\, D^\mu \Delta + (Y_\Delta)_{\alpha\beta}\,\overline{\ell_L^{c\,\alpha}}\vec{\tau}\ell_L^\beta \Delta + \mu_\Delta \tilde{\phi}^\dagger\,(\vec{\tau}\Delta)^\dagger\,\phi - \Delta^\dagger M_\Delta^2 \Delta + \text{h.c.}$$

- **The type-III Seesaw** introduces fermion $SU(2)_L$ triplets Σ_R^i to the SM particle content which are allowed to have a Yukawa coupling with the Lepton doublets and a Majorana mass:

$$\mathcal{L} = \mathcal{L}_{\rm SM} + i\overline{\Sigma}_R \not{D} \Sigma_R - \frac{1}{2}\overline{\Sigma_R^i}\,(M_\Sigma)_{ij}\,\Sigma_R^{cj} - (Y_\Sigma)_{i\alpha}\,\overline{\Sigma_R^i}\tilde{\phi}^\dagger \vec{\tau}\ell_L^\alpha + \text{h.c.}\,,$$

  where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the Pauli matrices and $\tilde{\phi} = i\tau_2\phi^*$ with ϕ the SM Higgs field.

All types of Seesaw mechanisms (I, II and III) lead to the Weinberg $d = 5$ effective operator for neutrino masses upon integrating out their heavy degrees of freedom. However, all of them lead to different $d = 6$ operators. Discuss the type of $d = 6$ operators that would be induced after integrating out the heavy particles for each of the Seesaw mechanisms. What would be the phenomenological implications of these operators and the differences between them?