Lecture 3: Direct Dark Matter Detection

00000000000000

DAVID G. CERDEÑO

We don't know yet what DM is... but we do know many of its properties

It is a NEW particle

- Neutral
- Stable on cosmological scales
- Reproduce the correct relic abundanc
- Not excluded by current searches
- No conflicts with BBN or stellar evolutior

Many candidates in Particle Physics

- Axions
- Weakly Interacting Massive Particles (W
- SuperWIMPs and Decaying DM
- WIMPzillas
- Asymmetric DM
- SIMPs, CHAMPs, SIDMs, ETCs...



MASS



DIRECT DARK MATTER SEARCHES: What can we measure?

NUCLEAR SCATTERING

- "Canonical" signature
- Elastic or Inelastic scattering
- Sensitive to m >1 GeV

ELECTRON SCATTERING

• Sensitive to light WIMPs

ELECTRON ABSORBPTION

• Very light (non-WIMP)

EXOTIC SEARCHES

- Axion-photon conversion in the atomic EM field
- Light Ionising Particles



Copper casing

.

0







-

Copper casing



-2V

0 000

0

Total mass: ~ 9 kg Physics run: 2009-2012

20 cm

00000000000

...............

..................



9.0 kg Ge (15 iZIPs x 600g)

The SuperCDMS Experiment

High purity Germanium crystals





Protected by a very clean shielding

LEAD POLYETHILENE

And an international team of ~100 scientists from 30 different instutions

SuperCDMS at SOUDAN



Flux of DM particles

We can easily estimate the flux of DM particles through the Earth. The DM typical velocity is of the order of $300 \text{ km s}^{-1} \sim 10^{-3} c$. Also, the local DM density is $\rho_0 = 0.3 \text{ GeV cm}^{-3}$, thus, the DM number density is $n = \rho/m$.

$$\phi = \frac{v\rho}{m} \approx \frac{10^7}{m} \,\mathrm{cm}^{-2} \,\mathrm{s}^{-1}$$
 (3.1)

Kinematics

$$E_R = \frac{1}{2} m_{\chi} v^2 \frac{4m_{\chi}m_N}{(m_{\chi} + m_N)^2} \frac{1 + \cos\theta}{2}$$

$$E_R^{max} = \frac{1}{2}m_{\chi}v^2 = \frac{1}{2}m_{\chi} \times 10^{-6} = \frac{1}{2}\left(\frac{m_{\chi}}{1\,\text{GeV}}\right)\,\text{keV}$$

Master formula for direct detection

We want to determine the number of nuclear recoils as a function of the recoil energy

$$\frac{dN}{dE_R} = t \, n \, v \, N_T \, \frac{d\sigma}{dE_R} \, .$$

n = DM number density t = time v = DM speed NT = number of targets

The DM speed is not unique, it is distributed according to f(v)

$$\frac{dN}{dE_R} = t n N_T \int_{v_{min}} v f(\vec{v}) \frac{d\sigma}{dE_R} d\vec{v} ,$$

$$v_{min} = \sqrt{m_{\chi} E_R / 2\mu_{\chi N}^2}$$

Using
$$N_T = M_T/m_N$$

 $n = \rho/m_\chi$
 $\epsilon = t M_T$



Conventional direct detection approach

$$R = \int_{E_T} dE_R \frac{\rho_0}{m_N m_\chi} \int_{v_{min}} v f(v) \frac{d\sigma_{WN}}{dE_R}(v, E_R) dv$$

Experimental setup

Target material (sensitiveness to different couplings)

Detection threshold

Astrophysical parameters

Local DM density Velocity distribution factor

Theoretical input

Differential cross section (of WIMPs with quarks)

Nuclear uncertainties

Conventional direct detection approach

$$R = \int_{E_T} dE_R \frac{\rho_0}{m_N m_\chi} \int_{v_{min}} v f(v) \frac{d\sigma_{WN}}{dE_R} (v, E_R) dv$$

Experimental setup

Target material (sensitiveness to different couplings)

Detection threshold

Astrophysical parameters

Local DM density Velocity distribution factor

Theoretical input

Differential cross section (of WIMPs with quarks)

Nuclear uncertainties

Experimental challenges:

- Discriminating Nuclear and Electron recoils
- Reduction of backgrounds
- Increment Target Size
- Low Energy threshold

WIMP expected fingerprint:

- Exponential spectrum
- Annual Modulation of the signal
- Directionality

Conventional direct detection approach

$$R = \int_{E_T} dE_R \frac{\rho_0}{m_N \, m_\chi} \int_{v_{min}}$$

$$vf(v) \frac{d\sigma_{WN}}{dE_R}(v, E_R) dv$$

Experimental setup

Target material (sensitiveness to different couplings)

Detection threshold

Astrophysical parameters

Local DM density Velocity distribution factor

Theoretical input

Differential cross section (of WIMPs with quarks)

Nuclear uncertainties

$$\left(\frac{d\sigma_{WN}}{dE_R}\right) = \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SI} + \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SD}$$

Spin-independent and **Spin-dependent** components, stemming from different microscopic interactions leading to different coherent factors

Detecting Dark Matter through elastic scattering with nuclei

We want to describe the (elastic) scattering cross section of DM particles with nuclei

 $\frac{d\sigma_{WN}}{dE_R}(v, E_R)$



But our microscopic theory generally provides the interaction with quarks and gluons

Quarks \rightarrow Nucleons (protons and neutrons)

Nucleons \rightarrow Nucleus Nuclear models (encoded in a Form Factor)

The WIMP-nucleus cross section has two components

$$\frac{d\sigma_{WN}}{dE_R} = \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SI} + \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SD}$$

Spin-independent contribution: scalar (or vector) coupling of WIMPs with quarks

$$\mathcal{L} \supset \alpha_q^S \bar{\chi} \chi \bar{q} q + \alpha_q^V \bar{\chi} \gamma_\mu \chi \bar{q} \gamma^\mu q$$

Total cross section with Nucleus scales as A² Present for all nuclei (favours heavy targets) and WIMPs

Spin-dependent contribution: WIMPs couple to the quark axial current

$$\mathcal{L} \supset lpha_q^A (\bar{\chi} \gamma^\mu \gamma_5 \chi) (\bar{q} \gamma_\mu \gamma_5 q)$$

Total cross section with Nucleus scales as J/(J+1)Only present for nuclei with $J \neq 0$ and WIMPs with spin WIMP-nucleus (elastic) scattering cross section

$$\frac{\mathrm{d}\sigma^{WN}}{\mathrm{d}E_R} = \frac{m_N}{2\mu_N^2 v^2} \left(\sigma_0^{SI,N} F_{SI}^2(E_R) + \sigma_0^{SD,N} F_{SD}^2(E_R)\right)$$

Where the spin-independent and spin-dependent contributions read

.

$$\sigma_0^{SI,N} = \frac{4\mu_N^2}{\pi} [Zf_p + (A - Z)f_n]^2,$$

$$\sigma_0^{SD,N} = \frac{32\mu_N^2 G_F^2}{\pi} [a_p S_p + a_n S_n]^2 \left(\frac{J+1}{J}\right)$$

The Form factor encodes the loss of coherence for large momentum exchange

$$F^{2}(q) = \left(\frac{3j_{1}(qR_{1})}{qR_{1}}\right)^{2} \exp(-q^{2}s^{2})$$

For
$$\sim$$
keV energies, F(q) \sim 1



Detecting Dark Matter through elastic scattering with nuclei

$$\frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma_{WN}}{dE_R}(v, E_R) \, dv$$

Minimal DM velocity for a recoil of energy E_R

$$v_{min}(E_R) = \sqrt{\frac{m_N E_R}{2\mu_{\chi N}^2}}$$

Isothermal spherical halo

$$f(\vec{v} + \vec{v}_{lag}) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma^3} exp\left(-\frac{(\vec{v} + \vec{v}_{lag})^2}{2\sigma^2}\right)$$

$$\sigma = 150 \text{ km s}^{-1}$$

 $v_{lag} = 230 \text{ km s}^{-1}$

Astrophysical parameters

Local DM density Velocity distribution factor Uncertainties in the Dark Halo affect significantly the prospects for direct detection

For example, there might be nonthermalised components: dark disk or streams



Kavanagh and Green 2013

Discriminating a DM signal: ENERGY SPECTRUM

DM scattering would leave an **exponential signal** in the differential rate

$$R = \int_{E_T}^{\infty} dE_R \frac{\rho_0}{m_N m_\chi} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma_{WN}}{dE_R}(v, E_R) dv$$

The slope is dependent on the DM mass and the target mass



17/09/2019

The challenge of low-mass WIMPs

• The signal is expected at very low recoil energies

Favours light targets

Low-threshold searches

- Usual DM targets are relatively heavy so the threshold has to be significantly reduced.
- Backgrounds are more difficult to discriminate (this is in general not a background-free search)
- Relies on the goodness of the background model and MC simulations



Discriminating a DM signal: ANNUAL MODULATION



The relative velocity of WIMPs in the Earth reference frame has an annual modulation.

This implies a modulation in the rate.

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} \approx \left(\frac{\mathrm{d}R}{\mathrm{d}E_R}\right) \left(1 + \Delta(E_R)\cos(\alpha(t))\right).$$

The modulation amplitude is small (~7%) and very sensitive to the details of the halo parameters DAMA (DAMA/LIBRA) signal on annual modulation

cumulative exposure 427,000 kg day (13 annual cycles) with Nal

$$\frac{\mathrm{d}R}{\mathrm{d}E_R} \approx \left(\frac{\mathrm{d}\bar{R}}{\mathrm{d}E_R}\right) \left[1 + \Delta(E_R)\cos\alpha(t)\right]$$





... however other experiments (CDMS, Xenon, CoGeNT, ZEPLIN, Edelweiss, ...) did not confirm (its interpretation in terms of WIMPs).

2-6 keV

CDMS did not see annual modulation

An analysis of CDMS II (Ge) data has shown no evidence of modulation.

This means a further constraint on CoGeNT claims



• CoGeNT: smaller amplitude of the DM modulation signal in second year of data Collar in IDM 2012

No modulation in ANAIS



Discriminating a DM signal: **DIRECTIONALITY**



Experimental challenges

Low-pressure TPC to measure direction

Large exposure needed (from current limits)

Characteristic dipole signal

- Poor resolution
- Low- number of WIMPs vs. Background J. Billard et al., 2010

Ring-like structure

- Requires low-recoil energies and heavy WIMPs
- Also aberration due to Earth's motion

Bozorgnia et al., 2012

Constraints on the DM-nucleus scattering cross section

Single or double phase noble gas detectors excel in searches at large DM masses XENON1T, LUX, Panda-X (Xe), DARKSIDE, DEAP (Ar) Easily scalable



Constraints on low-mass WIMPs

CDMSlite, SuperCDMS, Edelweiss, CDEX (Ge), CRESST (CaWO₄), NEWS-G (Ne) complete the search for WIMPs at low masses.

Low-threshold experiments (with smaller targets) are probing large areas of parameter space



Constraints on low-mass WIMPs

Using only the ionisation signal, liquid noble gas detectors (e.g., XENON, DARKSIDE) are also advancing on the search for lowmass WIMPs



DISCLAIMER:

THESE PLOTS ASSUME

- Isothermal Spherical Halo
- WIMP with only spin-independent interaction
- coupling to protons = coupling to neutrons
- elastic scattering

Astrophysical input and uncertainties

Uncertainties in the parameters describing the Dark Matter halo affect bounds and reconstruction



- Incorporating uncertainties is crucial in order to compare results among different experiments. Halo-independent analyses.
- Very relevant to combine direct and indirect detection constraints.
- Low mass region is especially sensitive

17/09/2019

Effect of the Gaia Sausage on direct detection searches

Existing bounds are affected (especially at low masses)

Predictions for directional searches slightly modified (dipole signal elongated)





Evans, O'Hare, McCabe 1810.11468

Theoretical prejudice

Example: "Isospin violation": the scattering amplitudes for proton and neutrons may interfere destructively

$$R = \sigma_p \sum_{i} \eta_i \frac{\mu_{A_i}^2}{\mu_p^2} I_{A_i} \left[Z + (A_i - Z) f_n / f_p \right]^2$$
$$f_n / f_p = -Z / (A - Z)$$

For Xe (Z=54, A~130) $\rightarrow f_n/f_p = -0.7$

The interference depends on the target nucleus

pends on the target
$$10^{-41}$$



 10^{-36}

XENON100 (Xe) and CDMS II (Si) results "reconciled" Frandsen et al. 2013

The effective interaction of DM particles with nuclei can be more diverse than previously considered

 $f_n/f_p = -0.7$

Are we being too simplistic in describing WIMP-nucleus interactions?

$$R = \int_{E_T}^{\infty} dE_R \frac{\rho_0}{m_N m_{\chi}} \int_{v_{min}}^{\infty} v f(v) \frac{d\sigma_{WN}}{dE_R} (v, E_R) dv$$
$$\frac{d\sigma_{WN}}{dE_R} = \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SI} + \left(\frac{d\sigma_{WN}}{dE_R}\right)_{SD}$$

Effective Field Theory approach

The most general effective Lagrangian contains up to 14 different operators that induce **6 types of response functions and two new interference terms**

Haxton, Fitzpatrick 2012-2014

$$\mathcal{L}_{\rm int}(\vec{x}) = c \ \Psi_{\chi}^*(\vec{x}) \mathcal{O}_{\chi} \Psi_{\chi}(\vec{x}) \ \Psi_N^*(\vec{x}) \mathcal{O}_N \Psi_N(\vec{x})$$

$$\begin{array}{ll}
\mathcal{O}_{1} = 1_{\chi} 1_{N} \\
\mathcal{O}_{3} = i \vec{S}_{N} \cdot \left[\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp} \right] \\
\mathcal{O}_{4} = \vec{S}_{\chi} \cdot \vec{S}_{N} \\
\mathcal{O}_{5} = i \vec{S}_{\chi} \cdot \left[\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp} \right] \\
\mathcal{O}_{6} = \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right] \\
\mathcal{O}_{7} = \vec{S}_{N} \cdot \vec{v}^{\perp} \\
\mathcal{O}_{8} = \vec{S}_{\chi} \cdot \vec{v}^{\perp} \\
\mathcal{O}_{9} = i \vec{S}_{\chi} \cdot \left[\vec{S}_{N} \times \frac{\vec{q}}{m_{N}} \right] \\
\vec{z}
\end{array}$$

$$\begin{array}{l}
\mathcal{O}_{10} = i \vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \\
\mathcal{O}_{11} = i \vec{S}_{\chi} \cdot \vec{q} \\
\mathcal{O}_{12} = \vec{S}_{\chi} \cdot \left[\vec{S}_{N} \times \vec{v}^{\perp} \right] \\
\mathcal{O}_{13} = i \left[\vec{S}_{\chi} \cdot \vec{v}^{\perp} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right] \\
\mathcal{O}_{14} = i \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \vec{v}^{\perp} \right] \\
\mathcal{O}_{15} = - \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\left(\vec{S}_{N} \times \vec{v}^{\perp} \right) \cdot \frac{\vec{q}}{m_{N}} \right] \\
\vec{z}
\end{array}$$

(x2) if we allow for different couplings to protons and neutrons (isoscalar and isovector)

17/09/2019

Effective Field Theory approach

 (\rightarrow)

The most general effective Lagrangian contains up to 14 different operators that induce **6 types of response functions and two new interference terms**

Haxton, Fitzpatrick 2012-2014

<hr/>

$$\mathcal{L}_{int}(\vec{x}) = c \ \Psi_{\chi}^{+}(\vec{x}) \mathcal{O}_{\chi} \Psi_{\chi}(\vec{x}) \ \Psi_{N}^{+}(\vec{x}) \mathcal{O}_{N} \Psi_{N}(\vec{x})$$
Spin-Indep.
$$\begin{array}{l} \mathcal{O}_{1} = 1_{\chi} 1_{N} \\ \mathcal{O}_{3} = i \vec{S}_{N} \cdot \left[\vec{q} \\ \overline{m}_{N} \times \vec{v}^{\perp} \right] \\ \mathcal{O}_{3} = i \vec{S}_{\chi} \cdot \vec{S}_{N} \\ \mathcal{O}_{4} = \vec{S}_{\chi} \cdot \vec{S}_{N} \\ \mathcal{O}_{5} = i \vec{S}_{\chi} \cdot \left[\vec{q} \\ \overline{m}_{N} \times \vec{v}^{\perp} \right] \\ \mathcal{O}_{6} = \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right] \\ \mathcal{O}_{6} = \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right] \\ \mathcal{O}_{7} = \vec{S}_{N} \cdot \vec{v}^{\perp} \\ \mathcal{O}_{8} = \vec{S}_{\chi} \cdot \vec{v}^{\perp} \\ \mathcal{O}_{9} = i \vec{S}_{\chi} \cdot \left[\vec{S}_{N} \times \frac{\vec{q}}{m_{N}} \right] \end{array}$$

$$\begin{array}{c} \mathcal{O}_{10} = i \vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \\ \mathcal{O}_{11} = i \vec{S}_{\chi} \cdot \vec{q}^{\perp} \\ \mathcal{O}_{12} = \vec{S}_{\chi} \cdot \left[\vec{S}_{N} \times \vec{v}^{\perp} \right] \\ \mathcal{O}_{13} = i \left[\vec{S}_{\chi} \cdot \vec{v}^{\perp} \right] \left[\vec{S}_{N} \cdot \vec{w}^{\perp} \right] \\ \mathcal{O}_{14} = i \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \vec{v}^{\perp} \right] \\ \mathcal{O}_{15} = - \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\left(\vec{S}_{N} \times \vec{v}^{\perp} \right) \cdot \frac{\vec{q}}{m_{N}} \right] \end{aligned}$$

(x2) if we allow for different couplings to protons and neutrons (isoscalar and isovector)

17/09/2019

Effective Field Theory approach

0

The most general effective Lagrangian contains up to 14 different operators that induce **6 types of response functions and two new interference terms**

 (\rightarrow) (\rightarrow)

Haxton, Fitzpatrick 2012-2014

 (\rightarrow)

$$\mathcal{L}_{int}(x) = c \Psi_{\chi}^{+}(x) \mathcal{O}_{\chi} \Psi_{\chi}(x) \Psi_{N}(x) \mathcal{O}_{N} \Psi_{N}(x)$$
Spin-Indep.

$$\mathcal{O}_{1} = \mathbf{1}_{\chi} \mathbf{1}_{N}$$

$$\mathcal{O}_{3} = i \vec{S}_{N} \cdot \left[\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp} \right]$$

$$\mathcal{O}_{3} = i \vec{S}_{N} \cdot \left[\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp} \right]$$

$$\mathcal{O}_{4} = \vec{S}_{\chi} \cdot \vec{S}_{N}$$

$$\mathcal{O}_{5} = i \vec{S}_{\chi} \cdot \left[\frac{\vec{q}}{m_{N}} \times \vec{v}^{\perp} \right]$$

$$\mathcal{O}_{6} = \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right]$$

$$\mathcal{O}_{12} = \vec{S}_{\chi} \cdot \vec{S}_{N} \times \vec{v}^{\perp}$$

$$\mathcal{O}_{13} = i \left[\vec{S}_{\chi} \cdot \vec{v}^{\perp} \right] \left[\vec{S}_{N} \cdot \frac{\vec{q}}{m_{N}} \right]$$

$$\mathcal{O}_{14} = i \left[\vec{S}_{\chi} \cdot \frac{\vec{q}}{m_{N}} \right] \left[\vec{S}_{N} \cdot \vec{v}^{\perp} \right]$$

$$\mathcal{O}_{9} = i \vec{S}_{\chi} \cdot \left[\vec{S}_{N} \times \frac{\vec{q}}{m_{N}} \right]$$

(x2) if we allow for different couplings to protons and neutrons (isoscalar and isovector)

These operators can be obtained as the non-relativistic limit of relativistic operators (e.g., starting from UV complete models)

Spin-0 DM particle + scalar mediator

$$\mathcal{L}_{S\phi q} = \partial_{\mu} S^{\dagger} \partial^{\mu} S - m_{S}^{2} S^{\dagger} S - \frac{\lambda_{S}}{2} (S^{\dagger} S)^{2} + \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m_{\phi}^{2} \phi^{2} - \frac{m_{\phi} \mu_{1}}{3} \phi^{3} - \frac{\mu_{2}}{4} \phi^{4} + i \bar{q} D q - m_{q} \bar{q} q - g_{1} m_{S} S^{\dagger} S \phi - \frac{g_{2}}{2} S^{\dagger} S \phi^{2} - h_{1} \bar{q} q \phi - i h_{2} \bar{q} \gamma^{5} q \phi,$$





 $\begin{array}{c} \begin{array}{c} \text{Usual "spin-independent"}\\ \text{independent"}\\ \text{contribution} \end{array} \\ \hline (S^{\dagger}S)(\bar{q}q) & \longrightarrow \left(\frac{h_{1}^{N}g_{1}}{m_{\phi}^{2}}\right) \mathcal{O}_{1} \\ \hline (S^{\dagger}S)(\bar{q}\gamma^{5}q) & \longrightarrow \left(\frac{h_{2}^{N}g_{1}}{m_{\phi}^{2}}\right) \mathcal{O}_{10} \end{array}$

Momentum-dependent "spin-dependent" contribution

We might MISS a DM signature

The spectrum from some interactions (momentum dependent) differs from the standard exponential signature

We might **misinterpret** a DM signature (if we reconstruct it with the usual templates)



A low threshold is extremely beneficial

We might MISS a DM signature

The spectrum from some interactions (momentum dependent) differs from the standard exponential signature

We might **misinterpret** a DM signature (if we reconstruct it with the usual templates)

We might **miss** a signature (if we misidentify it as a background)



A low threshold is extremely beneficial

Example: reconstruction in the usual SI-SD-mass plane

A single experiment cannot determine all the WIMP couplings, a combination of various targets is necessary.



Example: reconstruction in the usual SI-SD-mass plane

A single experiment cannot determine all the WIMP couplings, a combination of various targets is necessary.



$$\sigma_0^{SI} = 10^{-9} \text{ pb}$$
$$\sigma_0^{SD} = 10^{-5} \text{ pb}$$
$$m_W = 50 \text{ GeV}$$
$$\epsilon = 300 \text{ kg yr}$$

Germanium and Xenon might not be able to fully reconstruct the DM parameters

Example: reconstruction in the usual SI-SD-mass plane

A single experiment cannot determine all the WIMP couplings, a combination of various targets is necessary.



$$\sigma_0^{SI} = 10^{-9} \text{ pb}$$
$$\sigma_0^{SD} = 10^{-5} \text{ pb}$$
$$m_W = 50 \text{ GeV}$$
$$\epsilon = 300 \text{ kg yr}$$

Germanium and Xenon might not be able to fully reconstruct the DM parameters

Targets with different sensitivities to SI and SD cross section are needed (e.g., F, AI)