Lectures on Flavour Physics

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Brief review of the flavour structure of the Standard Model

G. C. Branco and D. Emmanuel-Costa hep-ph/1402.4068

$$G_{\rm SM} \equiv {\rm SU}(3)_{\rm C} \times {\rm SU}(2)_{\rm L} \times {\rm U}(1)_{\rm Y}$$

Gauge interactions determined by covariant derivative

$$D_\mu = \partial_\mu - i g_s L^k G^k_\mu - i g T^j W^j_\mu - i g' y B_\mu$$

Weak hypercharge $Y \equiv Q - T_3$

$q_{iL} \equiv \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L$	$(3,2,1/6)$	$D_\mu q_L = \left(\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu - i\frac{g}{2}\tau_j W^j_\mu - i\frac{g'}{6}B_\mu\right) q_L\,,$
u_{iR}	$(3,1,2/3)$	$D_\mu u_R = \left(\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu - i\frac{2g'}{3}B_\mu\right) u_R\,,$
d_{iR}	$(3,1,-1/3)$	$D_\mu d_R = \left(\partial_\mu - i\frac{g_s}{2}\lambda_k G^k_\mu + i\frac{g'}{3}B_\mu\right) d_R\,,$
$\ell_{iL} \equiv \begin{pmatrix} \nu_i \\ e^-_i \end{pmatrix}_{\rm L}$	$(1,2,-1/2)$	$D_\mu \ell_L = \left(\partial_\mu - i\frac{g}{2}\tau_j W^j_\mu + i\frac{g'}{2}B_\mu\right) q_L\,,$
$e^-_{i\ R}$	$(1,1,-1)$	$D_\mu e^-_L = \left(\partial_\mu + i g B_\mu\right) e^-_R\,.$

There are no right-handed neutrinos: $\nu_R \sim (1,1,0)$

Brief review of the flavour structure of the Standard Model

Spontaneous breakdown of electroweak gauge symmetry into electromagnetism

$$\mathsf{SU}(3)_{\mathrm{C}} \times \mathsf{SU}(2)_{\mathrm{L}} \times \mathsf{U}(1)_{\mathrm{Y}} \longrightarrow \mathsf{SU}(3)_{\mathrm{C}} \times \mathsf{U}(1)_{\mathrm{e.m.}}$$

Higgs scalar field:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1/2)$$

Most general gauge invariant renormalisable scalar potential

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

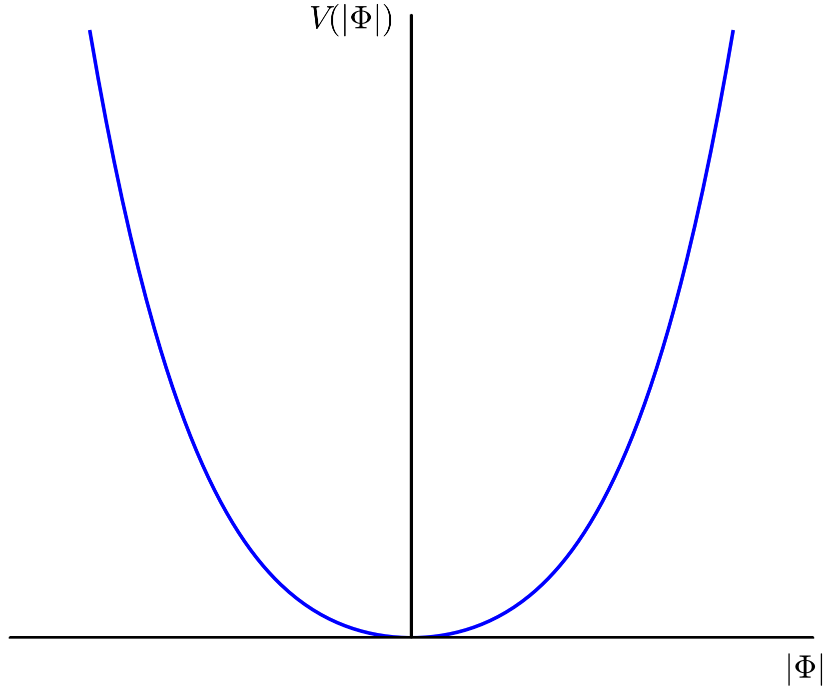
For $\lambda > 0$ the potential is bounded from below

For $\mu^2 > 0$ one has $\langle 0|\phi|0\rangle = 0$

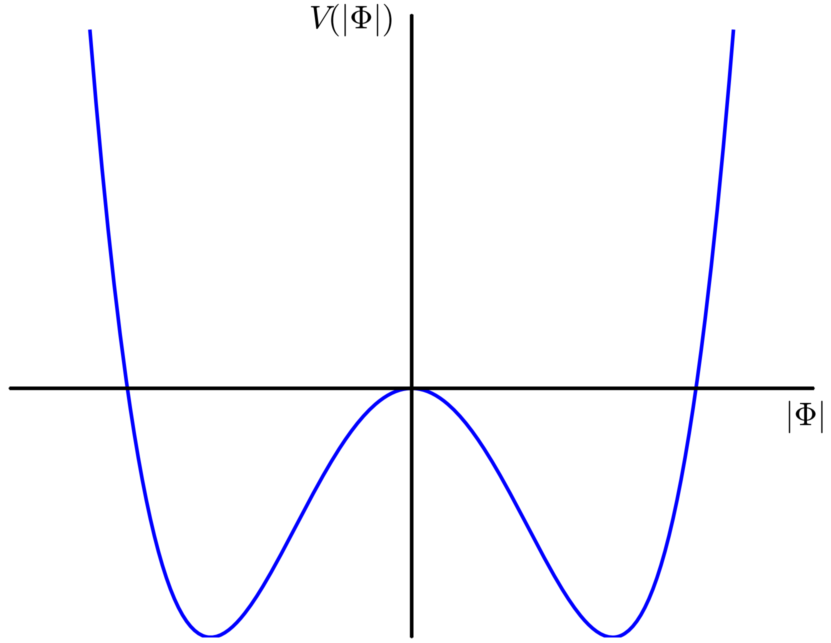
For $\mu^2 < 0$ one has $\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v \end{pmatrix}$

Spontaneous breaking of electroweak symmetry

Brief review of the flavour structure of the Standard Model



(a) $\lambda > 0,\ \mu^2 > 0$



(b) $\lambda > 0,\ \mu^2 < 0$

U(1) of electromagnetism remains unbroken:

$$Q = T_3 + Y \qquad Q = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} + \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\langle 0|\phi|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} \qquad Q \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} = 0$$

$$e^{i\alpha Q} \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} = \left[1 + i\alpha Q + \cdots\right] \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix}$$

Electric charge is conserved in the SM

Brief review of the flavour structure of the Standard Model

In the 2HDM electric charge is not automatically conserved. In general

$$\langle 0|\phi_1|0\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} v_1 \end{pmatrix}, \qquad \langle 0|\phi_2|0\rangle = \begin{pmatrix} \xi \\ \frac{1}{\sqrt{2}} v_2\, e^{i\theta} \end{pmatrix}$$

T. D. Lee

Need to choose a region of parameter space of the potential that does not violate electrical charge.

The same applies to Supersymmetric models

Comment: hypercharge is chosen in such a way as to lead to the correct electric charges

$$Y \equiv Q - T_3$$

$$Y_{u_L} = \frac{2}{3} - \frac{1}{2} = \frac{1}{6},$$
$$Y_{d_L} = -\frac{1}{3} + \frac{1}{2} = \frac{1}{6}$$

Puzzle

Brief review of the flavour structure of the Standard Model

Convenient parametrisation to describe SSB

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H + i\,G_0) \end{pmatrix}$$

The scalar fields $G^\pm$ and G_0 are massless states, the so-called Nambu-Goldstone bosons.

Brout-Englert-Higgs mechanism:

The charged bosons are absorbed in the longitudinal components of $W_\mu^\pm$

The neutral boson becomes the longitudinal component of Z_μ

$$M_W = \frac{g\,v}{2} \qquad Z_\mu = \cos\theta_W\, W_\mu^3 - \sin\theta_W\, B_\mu \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$$M_Z = \sqrt{g^2 + g'^2}\,\frac{v}{2} = \frac{M_W}{\cos\theta_W}$$

State orthogonal to Z_μ

$$A_\mu = \cos\theta_W\, B_\mu + \sin\theta_W\, W_\mu^3$$

photon remains massless

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings

Bare mass terms for fermions are not allowed due to gauge symmetry

$$m(\overline{\Psi_L}\Psi_R + \overline{\Psi_R}\Psi_L)$$

Suppose that one builds a Grand Unified Theory G containing the SM

G could be for instance SU(5) or SO(10)

Since fermions masses are not invariant under the gauge group of the SM fermions do not acquire mass in the breaking of G into the gauge group of the SM

This breaking usually occurs at a very high scale and the gauge bosons associated to the broken symmetry acquire mass at this high scale V>>v

Within the SM all fermion masses are protected by gauge symmetry

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont.)

In the SM fermions acquire mass through the Yukawa interactions

Yukawa interactions

$$-\mathscr{L}_Y = (Y_u)_{ij}\, \overline{q}_{iL}\, \tilde{\phi}\, u_{iR} + (Y_d)_{ij}\, \overline{q}_{iL}\, \phi\, d_{iR} + (Y_\ell)_{ij}\, \overline{\ell}_{iL}\, \phi\, e_{iR} + \text{H.c.}\,,$$

$$\tilde{\phi} \equiv i\tau_2\, \phi^\dagger$$

$Y_u,\ Y_d$ and Y_ℓ are arbitrary complex matrices in flavour space

$$\begin{aligned}
-\mathscr{L}_Y = {} & \frac{v}{\sqrt{2}}\,(Y_u)_{ij}\,\overline{u}_{iL}\,u_{iR} + \frac{v}{\sqrt{2}}\,(Y_d)_{ij}\,\overline{d}_{iL}\,d_{iR} + \frac{v}{\sqrt{2}}\,(Y_\ell)_{ij}\,\overline{e}_{iL}\,e_{iR} \\
& + \frac{(Y_u)_{ij}}{\sqrt{2}}\,\overline{u}_{iL}\,u_{iR}\,H + \frac{(Y_d)_{ij}}{\sqrt{2}}\,\overline{d}_{iL}\,d_{iR}\,H + \frac{(Y_\ell)_{ij}}{\sqrt{2}}\,\overline{e}_{iL}\,e_{iR}\,H \\
& - \frac{i\,(Y_u)_{ij}}{\sqrt{2}}\,\overline{u}_{iL}\,u_{iR}\,G^0 + \frac{i\,(Y_d)_{ij}}{\sqrt{2}}\,\overline{d}_{iL}\,d_{iR}\,G^0 + \frac{i\,(Y_\ell)_{ij}}{\sqrt{2}}\,\overline{e}_{iL}\,e_{iR}\,G^0 \\
& - (Y_u)_{ij}\,\overline{d}_{iL}\,u_{iR}\,G^- + (Y_d)_{ij}\,\overline{u}_{iL}\,d_{iR},\, G^+ + (Y_\ell)_{ij}\,\overline{\nu}_{iL}\,e_{iR}\,G^+ + \text{H.c.}\,.
\end{aligned}$$

$$\phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H + i\,G_0) \end{pmatrix}$$

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont)

Once a gauge transformation is performed in order to absorb the Nambu-Goldstone bosons

$$
\begin{aligned}
-\mathscr{L}_Y = & \, (m_u)_{ij}\, \overline{u_{iL}}\, u_{iR} + (m_d)_{ij}\, \overline{d_{iL}}\, d_{iR} + (m_\ell)_{ij}\, \overline{e_{iL}}\, e_{iR} \\
& + \frac{(Y_u)_{ij}}{\sqrt{2}}\, \overline{u_{iL}}\, u_{iR} H + \frac{(Y_d)_{ij}}{\sqrt{2}}\, \overline{d_{iL}}\, d_{iR} H + \frac{(Y_\ell)_{ij}}{\sqrt{2}}\, \overline{e_{iL}}\, e_{iR} H + \mathrm{H.c.}
\end{aligned}
$$

Exercise: In the Standard Model there are no Higgs mediated Flavour changing neutral currents. Explain why.

Quark and lepton mass matrices

$$
m_u \equiv \frac{v}{\sqrt{2}} Y_u\,, \qquad m_d \equiv \frac{v}{\sqrt{2}} Y_d\,, \qquad m_\ell \equiv \frac{v}{\sqrt{2}} Y_\ell
$$

arbitrary complex matrices

Gauge invariance does not constrain the flavour structure of the Yukawa couplings

In the SM there are no Higgs mediated Flavour Changing Neutral Currents at tree level

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont)

Yukawa couplings are the only couplings that can be complex

Mass terms:

$$-\mathscr{L}_m = (m_u)_{ij}\,\overline{u}^0_{iL}\,u^0_{i\,R} + (m_d)_{ij}\,\overline{d}^0_{iL}\,d^0_{i\,R} + (m_\ell)_{ij}\,\overline{e}^0_{iL}\,e^0_{i\,R}$$

the mass terms are diagonalised through biunitary transformations

The charged weak current is of the form:

$$-\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}}\left[\overline{u}^0_{iL}\,\gamma^\mu\,d^0_{iL} + \overline{\nu}^0_{iL}\,\gamma^\mu\,e^0_{iL}\right] W^+_\mu + \text{H.c.}\,.$$

We are still in what is called a weak basis

Weak basis transformations are transformations that leave the kinetic energy terms invariant as well as the gauge couplings of the fermions. The mass terms look different.

THE PHYSICS IS THE SAME

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont)

$$-\mathscr{L}_m = (m_u)_{ij}\, \overline{u}^0_{iL}\, u^0_{i\,R} + (m_d)_{ij}\, \overline{d}^0_{iL}\, d^0_{i\,R} + (m_\ell)_{ij}\, \overline{e}^0_{iL}\, e^0_{i\,R}$$

$$-\mathscr{L}_{\rm CC} = \frac{g}{\sqrt{2}} \left[\overline{u}^0_{iL}\, \gamma^\mu\, d^0_{iL} + \overline{\nu}^0_{iL}\, \gamma^\mu\, e^0_{iL}\right] W^+_\mu + {\rm H.c.}\,.$$

Most general weak basis (WB) transformation

$$\begin{aligned} u^0_L &= U_L\ u^{0\prime}_L; & u^0_R &= U^u_R\ u^{0\prime}_R \\ d^0_L &= U_L\ d^{0\prime}_L; & d^0_R &= U^d_R\ u^{0\prime}_R \\ e^0_L &= U^e_L\ e^{0\prime}_L; & e^0_R &= U^e_R\ e^{0\prime}_R \end{aligned}$$

The mass terms and the Yukawa couplings change but the physics does not change. Why? You will soon see why.

Important to note that textures zeros imposed in the mass matrices are no longer visible. Symmetries are no longer apparent. This renders the use of WB invariants very important

Exercise: Show that there is no loss of generality in choosing a WB where the mass matrices are Hermitian

Exercise: Show that there is no loss of generality in choosing a basis where up quark mass matrix is real diagonal (the same applies separately to the down quark mass matrix)

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont)

$$-\mathcal{L}_m = (m_u)_{ij}\, \overline{u}^0_{iL}\, u^0_{i\,R} + (m_d)_{ij}\, \overline{d}^0_{iL}\, d^0_{i\,R} + (m_\ell)_{ij}\, \overline{e}^0_{iL}\, e^0_{i\,R}$$

Diagonalisation of the mass terms:

$$u^0_{iL} = U^u_L\, u_{iL}\,; \qquad u^0_{iR} = U^u_R\, u_{iR}\,,$$
$$d^0_{iL} = U^d_L\, d_{iL}\,; \qquad d^0_{iR} = U^d_R\, d_{iR}\,,$$
$$e^0_{iL} = U^e_L\, e_{iL}\,; \qquad e^0_{iR} = U^e_R\, e_{iR}\,,$$

$$m_u \longrightarrow U^{u\,\dagger}_L\, m_u\, U^u_R = \mathrm{diag}\,(m_u, m_c, m_t)\,,$$
$$m_d \longrightarrow U^{d\,\dagger}_L\, m_d\, U^d_R = \mathrm{diag}\,(m_d, m_s, m_b)\,,$$
$$m_\ell \longrightarrow U^{e\,\dagger}_L\, m_\ell\, U^e_R = \mathrm{diag}\left(m_\ell, m_\mu, m_\tau\right)\,.$$

The fields $u_{L,R}, d_{L,R}, e_{L,R}$ are thus the mass eigenstates

In the mass eigenstate basis the charged currents become:

$$-\mathcal{L}_{\rm CC} = \frac{g}{\sqrt{2}}\left[\overline{u}_L\, \gamma^\mu\, U^{u\,\dagger}_L U^d_L\, d_L + \overline{\nu}^0_L\, \gamma^\mu\, U^e_L\, e_L\right] W^+_\mu + \mathrm{H.c.}$$

$$V \equiv U^{u\,\dagger}_L U^d_L$$

V is the Cabibbo-Kobayashi-Maskawa matrix

U^e_L is physically meaningless

why?

In the SM neutrinos are strictly massless ,
as a result there is no leptonic mixing

Fermion masses and mixings (cont)

Answer: Since neutrinos are massless in the SM one can always redefine them in such a way that this matrix is rotated away

$$\nu_L^0 \longrightarrow \nu_L = U_L^e \, \nu_L$$

The electromagnetic and the neutral currents are not affected by the transformations that diagonalise the mass terms

In a weak basis:

$$J^\mu_{\text{e.m.}} = \frac{2}{3}\left[\overline{u}^0_L \gamma^\mu u^0_L + \overline{u}^0_R \gamma^\mu u^0_R\right] - \frac{1}{3}\left[\overline{d}^0_L \gamma^\mu d^0_L + \overline{d}^0_R \gamma^\mu d^0_R\right] - \left[\overline{e}^0_L \gamma^\mu e^0_L + \overline{e}^0_R \gamma^\mu e^0_R\right]$$

In the mass eigenstate basis:

$$J'^\mu_{\text{e.m.}} = \frac{2}{3}\left[\overline{u}_L \gamma^\mu U^{u\dagger}_L U^u_L u_L + \overline{u}_R \gamma^\mu U^{u\dagger}_R U^u_R u_R\right] - \frac{1}{3}\left[\overline{d}_L \gamma^\mu U^{d\dagger}_L U^d_L d_L + \overline{d}_R \gamma^\mu U^{d\dagger}_R U^d_R d_R\right]$$
$$- \left[\overline{e}_L \gamma^\mu U^{e\dagger}_L U^e_L e_L + \overline{e}_R \gamma^\mu U^{e\dagger}_R U^e_R e_R\right] ,$$

Brief review of the flavour structure of the Standard Model

Fermion masses and mixings (cont)

Neutral currents Lagrangian in a weak basis

$$\mathscr{L}_{\rm NC} = \frac{g}{\cos\theta_W}\left[\overline{u}_L^0\,\gamma^\mu\,u_L^0 - \overline{d}_L^0\,\gamma^\mu\,d_L^0 + \overline{\nu}_L^0\,\gamma^\mu\,\nu_L^0 - \overline{e}_L\,\gamma^\mu\,e_L^0 - 2\sin^2\theta_W\,J^\mu_{\rm e.m.}\right] Z_\mu$$

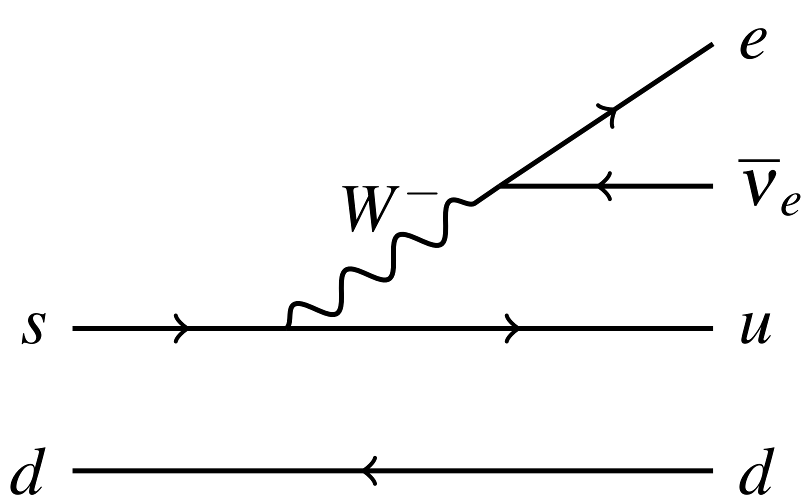
Neutral Currents Lagrangian in the mass eigenstate basis

$$\mathscr{L}'_{\rm NC} = \frac{g}{\cos\theta_W}\left[\overline{u}_L\,\gamma^\mu\,u_L - \overline{d}_L\,\gamma^\mu\,d_L + \overline{\nu}_L\,\gamma^\mu\,\nu_L - \overline{e}_L\,\gamma^\mu\,e_L - 2\sin^2\theta_W\,J^\mu_{\rm e.m.}\right] Z_\mu$$

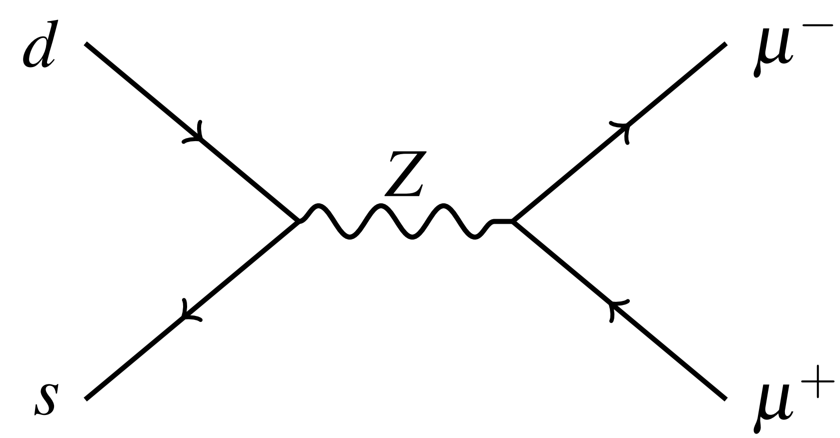
Flavour Changing Neutral currents are naturally absent at tree level in the SM

No tree level Z-mediated Flavour Changing Neutral Currents

Example 1. *The decay* $K_L^0 \to \mu^+\mu^-$ *has a branching ratio* extremely suppressed, *with respect to the decay* $K_L^0 \to \pi^+ e^- \overline{\nu_e}$. *If FCNC existed they would have branching ratios of the same order of magnitude which are shown in figure 2.*



(a) $K_L^0 \longrightarrow \pi^+ e^- \nu_e$



(b) Does not exist at tree-level in SM

In summary

In the SM there are no Z mediated Flavour Changing neutral currents at tree level

As we have seen there are also no Higgs mediated FCNC at tree level

FCNC appear at loop level and are, therefore, suppressed. They have played a crucial role in testing the Standard Model and in putting bounds on New Physics through meson mixing, rare K decays, rare B decays and CP violation.

All flavour changing transitions in the SM are mediated by charged weak currents with flavour mixing controlled by the CKM matrix

CKM has small off-diagonal elements

Fundamental properties of the CKM matrix

G. C Branco, L. Lavoura, J. P. Silva "CP Violation" Oxford University Press 1999

$$\mathscr{L}_{CC} = \left(\overline{u}\ \overline{c}\ \overline{t}\right)_L \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{H.c.}$$

The CKM matrix is complex but not all its phases have physical meaning

$$u_\alpha = e^{i\varphi_\alpha} u'_\alpha , \qquad d_k = e^{i\varphi_k} d'_k$$

There is freedom to rephase the mass eigenstate quark fields. As a result:

$$V'_{\alpha k} = e^{i(\varphi_k - \varphi_\alpha)} V_{\alpha k}$$

Only rephasing invariant quantities have physical meaning.
The simplest rephasing invariants of the CKM matrix are moduli and "quartets"

$$|V_{\alpha k}| \qquad Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \qquad \text{with } \alpha \neq \beta \text{ and } i \neq j.$$

Higher order Invariants can in general be written in terms of these .

Fundamental properties of the CKM matrix (cont)

Example of the construction of quartets

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

$$V_{us} V_{cb} V_{ub}^* V_{cs}^* = Q_{uscb}$$

$$V_{cd} V_{ts} V_{td}^* V_{cs}^* = Q_{cdts}$$

Exercise: Show that

$$V_{\alpha i} V_{\beta j} V_{\gamma k} V_{\alpha j}^* V_{\beta k}^* V_{\gamma i}^* = \frac{Q_{\alpha i \beta j} Q_{\beta i \alpha j}}{|V_{\beta i}|^2}.$$

CP Violation

Within the SM the only source of CP violation is the phase of the CKM matrix

There is a large redundancy in the number of parameters contained in mass matrices of the up and down quark still in a weak basis

Each 3X3 complex mass matrix will contain 18 parameters

A weak basis transformation:

$$u_L^0 = U_L\ u_L^{0'}; \qquad u_R^0 = U_R^u\ u_R^{0'}$$
$$d_L^0 = U_L\ d_L^{0'}; \qquad d_R^0 = U_R^d\ u_R^{0'}$$
$$e_L^0 = U_L^e\ e_L^{0'}; \qquad e_R^0 = U_R^e\ e_R^{0'}$$

allows to choose a weak basis where one of the matrices is real diagonal and the other is Hermitian

i.e. m_u = $\mathrm{diag}(m_u, m_c, m_t)$ $\qquad$ m_d is Hermitian

$$m_d = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{12}^* & m_{22} & m_{23} \\ m_{13}^* & m_{23}^* & m_{33} \end{pmatrix}$$

the only rephasing invariant phase is

$$\varphi \equiv \arg(m_{12}\, m_{23}\, m_{13}^*)$$

Number of independent parameters: three up quark masses, six moduli in the down quark mass and one phase. This equals the number of physical parameters.

CP Violation

$$\mathscr{L} = \mathscr{L}_{(\mathrm{CP})} + \mathscr{L}'$$

CP can be investigated in the fermion mass eigenstate or in a weak basis

In the fermion mass eigenstate

$$m_u = \mathrm{diag}\left(m_u, m_c, m_t\right) ,$$
$$m_d = \mathrm{diag}\left(m_d, m_s, m_b\right) ,$$

Most general CP transformation

$$(\mathrm{CP})\, W^{+\mu}(t,\vec{r})\, (\mathrm{CP})^{-1} = -e^{i\zeta_W} W^{-\mu}(t,-\vec{r}) ,$$
$$(\mathrm{CP})\, W^{-\mu}(t,\vec{r})\, (\mathrm{CP})^{-1} = -e^{-i\zeta_W} W^{+\mu}(t,-\vec{r}) ,$$
$$(\mathrm{CP})\, u_\alpha(t,\vec{r})\, (\mathrm{CP})^{-1} = e^{i\zeta_\alpha} \gamma^0 C \bar{u}_\alpha^{\mathsf{T}}(t,-\vec{r}) ,$$
$$(\mathrm{CP})\, d_k(t,\vec{r})\, (\mathrm{CP})^{-1} = e^{i\zeta_k} \gamma^0 C \bar{u}_k^{\mathsf{T}}(t,-\vec{r}) ,$$

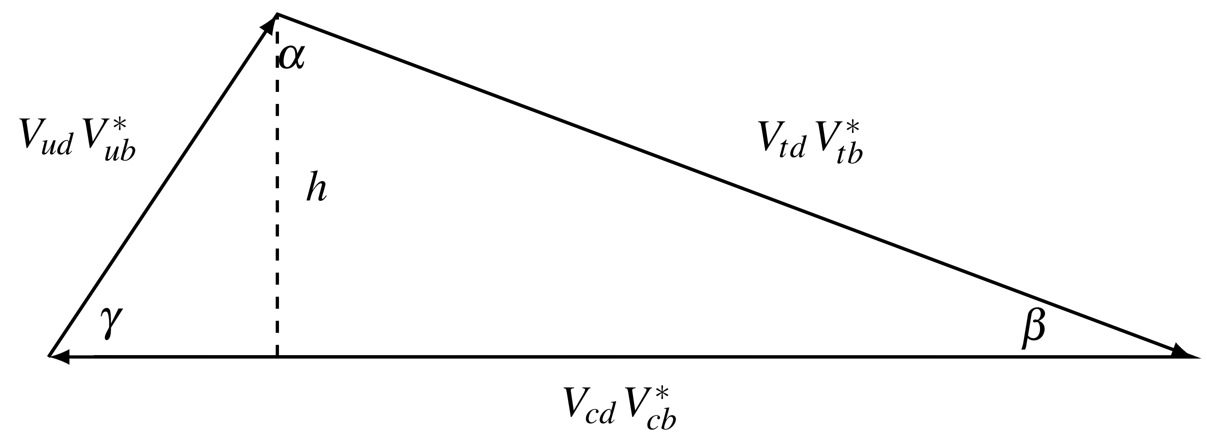
Constraint: $$V_{\alpha k}^{*} = e^{i(\zeta_W + \zeta_k - \zeta_\alpha)} V_{\alpha k}$$

There is CP violation in the SM if and only if any of the rephasing functions of the CKM matrix V is not real

CP Violation

Orthogonality between the first and third column of V

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



Unitarity triangle

The internal angles:

$$\alpha \equiv \arg\left[-V_{td}V_{ub}V_{ud}^*V_{tb}^*\right] = \arg(-Q_{ubtd})$$

$$\beta \equiv \arg\left[-V_{cd}V_{tb}V_{cb}^*V_{td}^*\right] = \arg(-Q_{tbcd}),$$

$$\gamma \equiv \arg\left[-V_{ud}V_{cb}V_{ub}^*V_{cd}^*\right] = \arg(-Q_{cbud})$$

are rephasing invariant

and: $\alpha + \beta + \gamma = \arg(-1) = \pi \pmod{\pi}$ by definition

The area of the triangle is given by $A = |V_{cd}V_{cb}^*|\,\dfrac{h}{2}$

$h = |V_{ud}V_{ub}^*|\sin\gamma$ using the above definition of gamma, we obtain:

$A = \dfrac{1}{2}|\mathrm{Im}\,Q_{udcb}|$ Since all $|\mathrm{Im}\,Q|$ are equal then all triangles have the same area.

CP Violation

Experimentally we know that:

$$|V_{\rm CKM}| \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \qquad \text{with } \lambda \approx 0.22$$

The six unitarity triangles are: (only two are not squashed)

$$\overset{\lambda}{V_{ud}V_{us}^*} + \overset{\lambda}{V_{cd}V_{cs}^*} + \overset{\lambda^5}{V_{td}V_{ts}^*} = 0\,, \qquad T_{ds}$$

$$\overset{\lambda^3}{V_{ud}V_{ub}^*} + \overset{\lambda^3}{V_{cd}V_{cb}^*} + \overset{\lambda^3}{V_{td}V_{tb}^*} = 0\,, \qquad T_{db}$$

$$\overset{\lambda^4}{V_{us}V_{ub}^*} + \overset{\lambda^2}{V_{cs}V_{cb}^*} + \overset{\lambda^2}{V_{ts}V_{tb}^*} = 0\,, \qquad T_{sb}$$

$$\overset{\lambda}{V_{ud}V_{cd}^*} + \overset{\lambda}{V_{cs}V_{cs}^*} + \overset{\lambda^5}{V_{ub}V_{cb}^*} = 0\,, \qquad T_{uc}$$

$$\overset{\lambda^3}{V_{ud}V_{td}^*} + \overset{\lambda^3}{V_{us}V_{ts}^*} + \overset{\lambda^3}{V_{ub}V_{td}^*} = 0\,, \qquad T_{ut}$$

$$\overset{\lambda^4}{V_{cd}V_{td}^*} + \overset{\lambda^2}{V_{cs}V_{ts}^*} + \overset{\lambda^2}{V_{tb}V_{tb}^*} = 0\,, \qquad T_{ct}$$

$$|{\rm Im}\,Q| = \left| \overset{\lambda^0}{V_{ud}} \overset{\lambda^3}{V_{ub}} \overset{\lambda}{V_{cd}} \overset{\lambda^2}{V_{cb}} \right| \sin\gamma\,.$$

(small)

Maximal possible ImQ:

$$V = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^* \\ 1 & \omega^* & \omega \end{pmatrix}$$

with $\omega \equiv \exp(i2\pi/3)$, yielding

$${\rm Im}\,Q = \frac{1}{6\sqrt{3}} \approx 0.096$$

CP Violation

Convenient parametrisation of CKM (standard parametrisation)

$$V(\theta_{12},\theta_{13},\theta_{23},\delta_{13}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}\,e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12}\,c_{13} & s_{12}\,c_{13} & s_{13}\,e^{-i\delta_{13}} \\ -s_{12}\,c_{23} - c_{12}\,s_{23}\,s_{13}e^{i\delta_{13}} & c_{12}\,c_{23} - s_{12}\,s_{23}\,s_{13}e^{i\delta_{13}} & s_{23}\,c_{13} \\ s_{12}\,s_{23} - c_{12}\,c_{23}\,s_{13}e^{i\delta_{13}} & -c_{12}\,s_{23} - s_{12}\,c_{23}\,s_{13}e^{i\delta_{13}} & c_{23}\,c_{13} \end{pmatrix}$$

$$s_{13} = |V_{ub}|\,, \qquad s_{12} = \frac{|V_{us}|}{\sqrt{1-|V_{ub}|^2}}\,, \qquad s_{23} = \frac{|V_{cb}|}{\sqrt{1-|V_{ub}|^2}}$$

the sines are related in a simple way to measurable quantities,
all data then has to fit a single parameter

CP Violation

Invariant approach

Bernabeu, Branco and Gronau, 1986

most general CP transformations which leave $\mathscr{L}_{(\mathrm{CP})}$ invariant

$$(\mathrm{CP})\, u_L^0\, (\mathrm{CP})^{-1} = e^{i\zeta_W} K_L \gamma^0 C \overline{u_L^0}^{\mathsf{T}},$$
$$(\mathrm{CP})\, d_L^0\, (\mathrm{CP})^{-1} = K_L \gamma^0 C \overline{d_L^0}^{\mathsf{T}},$$
$$(\mathrm{CP})\, u_R^0\, (\mathrm{CP})^{-1} = K_R^u \gamma^0 C \overline{u_R^0}^{\mathsf{T}},$$
$$(\mathrm{CP})\, d_R^0\, (\mathrm{CP})^{-1} = K_R^d \gamma^0 C \overline{d_R^0}^{\mathsf{T}},$$

conditions:

$$K_L^\dagger m_u K_R^u = m_u^*$$
$$K_L^\dagger m_d K_R^d = m_d^*$$

For $H_u \equiv m_u m_u^\dagger, \qquad H_d \equiv m_d m_d^\dagger$

$$K_L^\dagger H_u K_L = H_u^* = H_u^{\mathsf{T}}$$
$$K_L^\dagger H_d K_L = H_u^* = H_u^{\mathsf{T}}$$

implying:

$$K_L^\dagger [H_u, H_d] K_L = [H_u^{\mathsf{T}}, H_d^{\mathsf{T}}] = -[H_u, H_d]^{\mathsf{T}} \qquad \mathrm{Tr}[H_u, H_d]^r = 0. \qquad (r \text{ odd})$$

in the SM CP invariance implies $\mathrm{Tr}[H_u, H_d]^r = 0$ The minimal non trivial case is for r=3

Necessary and sufficient condition for CP conservation in the SM with three generations:

$$\mathrm{Tr}[H_u, H_d]^3 = 6i(m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2)(m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\, \mathrm{Im}\, Q$$

$$\det[H_u, H_d] = \frac{1}{3} tr[H_u, H_d]^3$$

$$J \equiv \mathrm{Im}\, Q_{uscb} = \mathrm{Im}\,(V_{us} V_{cb} V_{ub}^* V_{cs}^*) = c_{12} s_{12} c_{13}^2 s_{13} c_{23} s_{23} \sin\delta_{13}$$

Parameter counting of CKM for n_g

Number of independent parameters in the SM

V is an $n_g \times n_g$ unitarity matrix

n_g^2 parameters in an $n_g \times n_g$ unitary matrix

$(2n_g - 1)$ phases can be absorbed by rephasing

$$N_{param} = n_g^2 - (2n_g - 1) = (n_g - 1)^2$$

an $n_g \times n_g$ orthogonal matrix is parametrised by

$n_g(n_g - 1)/2$ rotation angles (Euler angles)

$$N_{angle} = \frac{1}{2} n_g (n_g - 1)$$

$$N_{phase} = N_{param} - N_{angle} = \frac{1}{2}(n_g - 1)(n_g - 2)$$

this is the number of physical phases

Parameter counting of CKM for n_g (cont.)

$N_{phase} = \frac{1}{2}(n_g - 1)(n_g - 2)$ physical phases

$n_g = 2$ $\quad N_{phase} = 0$ no CP violating phase

$n_g = 3$ $\quad N_{phase} = 1$ one CP violating phase

In terms of quartets

$$\begin{pmatrix} V_{ud} & V_{us} \\ V_{cd} & V_{cs} \end{pmatrix}$$

Two generations $\quad Q_{udcs} = V_{ud} V_{cs} V_{us}^* V_{cd}^*$

orthogonality relation $\quad V_{ud} V_{cd}^* + V_{us} V_{cs}^* = 0$

$Q_{uds} = - V_{us} V_{cs}^* V_{cs} V_{us}^* = -|V_{us}|^2 |V_{cs}|^2$ real !

Three generations, orthogonality of first two rows of V

$$V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* = 0$$

Exercise: Show that the imaginary part of all quartets has the same modulus

Wolfenstein Parametrisation of CKM

See PDG

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Definitions:

$$s_{12} = \lambda = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}}, \qquad s_{23} = A\lambda^2 = \lambda\left|\frac{V_{cb}}{V_{us}}\right|,$$

$$s_{13}e^{i\delta} = V_{ub}^* = A\lambda^3(\rho + i\eta) = \frac{A\lambda^3(\bar\rho + i\bar\eta)\sqrt{1-A^2\lambda^4}}{\sqrt{1-\lambda^2}[1-A^2\lambda^4(\bar\rho + i\bar\eta)]}.$$

These relations ensure that $\bar\rho + i\bar\eta = -(V_{ud}V_{ub}^*)/(V_{cd}V_{cb}^*)$ is phase convention independent, and the CKM matrix written in terms of λ, A, $\bar\rho$, and $\bar\eta$ is unitary to all orders in λ.

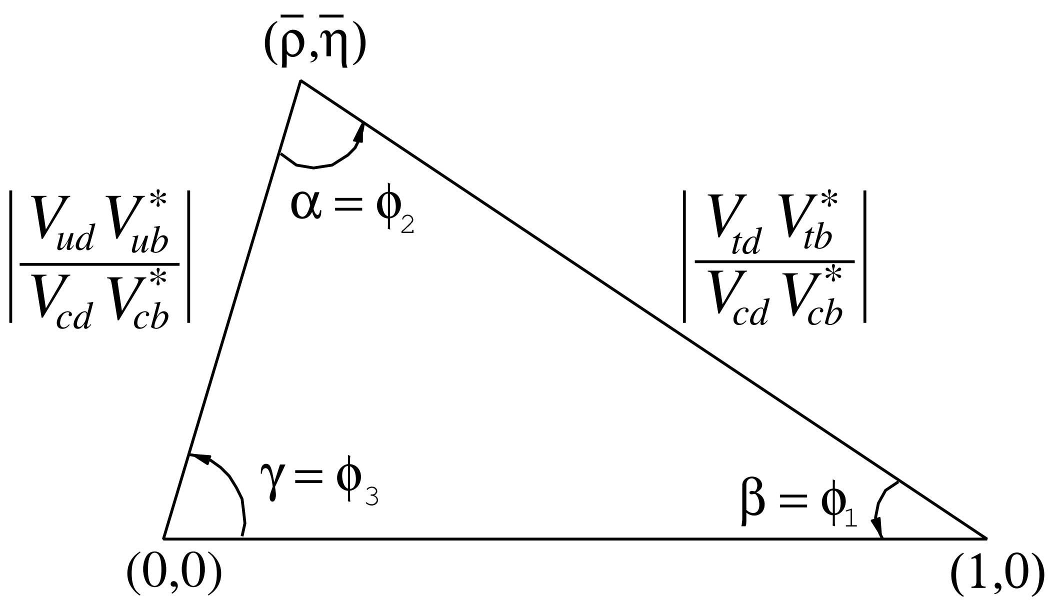
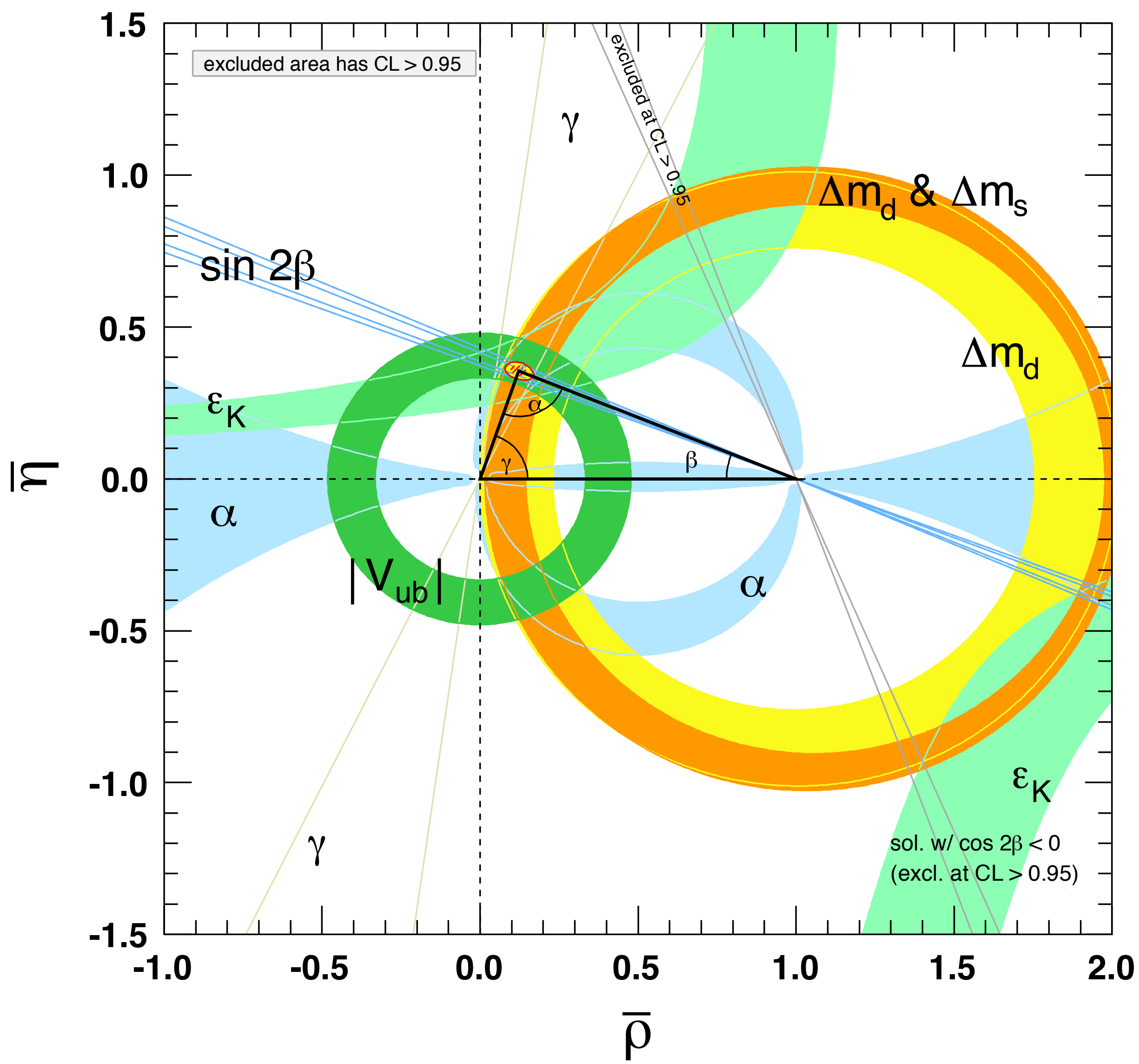


Figure 12.1: Sketch of the unitarity triangle.

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

Each side is divided by the best know side of the UT: $V_{cd}V_{cb}^*$

The standard way of testing the compatibility of the SM with the existing data consists of adopting the Wolfenstein parametrization and plotting in the $\bar{\rho}, \bar{\eta}$ plane the constraints derived from various experimental inputs



From PDG

Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

The CKM elements can be most precisely determined using a global fit to all available measurements and imposing the SM constraints (e.g. three generation unitarity). The fit must also use theory predictions for hadronic matrix elements which sometimes have significant uncertainties. There are several approaches to combining the experimental data CKM fitter, UT fit (see references in PDG)

$$\lambda = 0.22453 \pm 0.00044\,, \qquad A = 0.836 \pm 0.015$$
$$\bar{\rho} = 0.122^{+0.018}_{-0.017}\,, \qquad \bar{\eta} = 0.355^{+0.012}_{-0.011}\,.$$

Fit results for the magnitudes of all nine elements of CKM

$$V_{\rm CKM} = \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix}$$

Violating 3X3 unitarity

F. J. Botella, G, C. Branco, M. Nebot, MNR, 2003

What is the best strategy to perform precision test of the SM mechanism of flavour mixing and CP violation, while at the same time searching for New Physics?

Throughout the paper, we will assume that the tree level weak decays are dominated by the SM W-exchange diagrams, thus implying that the extraction of $|V_{us}|$, $|V_{ub}|$ and $|V_{cb}|$ from experiment continues to be valid even in the presence of New Physics (NP). We will allow for contributions from NP in processes like B_d^0–$\overline{B}_d^0$ mixing and B_s^0–$\overline{B}_s^0$ mixing, as well as in penguin diagrams. Since the SM contributes to these processes only at loop level, the effects of NP are more likely to be detectable. Examples of processes which are sensitive to NP, are the CP asymmetries corresponding to the decays $B_d^0 \to J/\Psi K_s$ and $B_d^0 \to \pi^+\pi^-$ which are affected by NP contributions to B_d^0–$\overline{B}_d^0$ mixing. Significant contributions to B_d^0–$\overline{B}_d^0$ and B_s^0–$\overline{B}_s^0$ mixing can arise in many of the extensions of the SM, such as models with vector-like quarks [11,12] and supersymmetric extensions of the SM

Violating 3X3 unitarity (cont)

Alternative approach to the one presented before: choose a complete set of rephasing invariant phases and use 3x3 unitarity of CKM to derive a set of exact relations written in terms of measurable quantities, namely moduli of CKM and and arguments of rephasing invariant quartets.

Since all relations are exact and written in terms of measurable quantities, they are particularly suited to perform precise tests of the SM

Suppose one drops the requirement of 3x3 unitarity, how many parameters are there in the 3x3 CKM matrix?

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} & \cdots \\ V_{cd} & V_{cs} & V_{cb} & \cdots \\ V_{td} & V_{ts} & V_{tb} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

9 moduli and 4 rephasing invariant phases (9-5)

A convenient choice for the four independent rephasing invariant phases is

$$\begin{aligned} \beta &\equiv \arg\left(-V_{cd}V_{tb}V_{cb}^*V_{td}^*\right), \\ \gamma &\equiv \arg\left(-V_{ud}V_{cb}V_{ub}^*V_{cd}^*\right), \\ \chi = \beta_s &\equiv \arg\left(-V_{cb}V_{ts}V_{cs}^*V_{tb}^*\right), \\ \chi' &\equiv \arg\left(-V_{us}V_{cd}V_{ud}^*V_{cs}^*\right), \end{aligned}$$

It is useful to adopt the following phase convention (no loss of generality):

$$\arg(V) = \begin{pmatrix} 0 & \chi' & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\chi & 0 \end{pmatrix}$$

We are left with four phases

Exact relations

From six unitarity relations corresponding to orthogonality of different rows and of different columns

$$(uc)\quad \sin\chi' = \frac{|V_{ub}||V_{cb}|}{|V_{us}||V_{cs}|}\sin\gamma,$$

$$(ut)\quad |V_{ud}||V_{td}|\sin\beta - |V_{us}||V_{ts}|\sin(\chi'-\chi) - |V_{ub}||V_{tb}|\sin\gamma = 0,$$

$$(ct)\quad \sin\chi = \frac{|V_{td}||V_{cd}|}{|V_{ts}||V_{cs}|}\sin\beta,$$

$$(db)\quad \frac{|V_{ub}|}{|V_{td}|} = \frac{\sin\beta}{\sin\gamma}\frac{|V_{tb}|}{|V_{ud}|},$$

$$(ds)\quad \sin\chi' = \frac{|V_{td}||V_{ts}|}{|V_{ud}||V_{us}|}\sin(\beta+\chi),$$

$$(sb)\quad \frac{\sin\chi}{\sin(\gamma+\chi')} = \frac{|V_{us}||V_{ub}|}{|V_{ts}||V_{tb}|},$$

A few additional relations:

$$(db)\quad |V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|}\frac{\sin\beta}{\sin(\gamma+\beta)},$$

$$(db)\quad |V_{td}| = \frac{|V_{cd}||V_{cb}|}{|V_{tb}|}\frac{\sin\gamma}{\sin(\gamma+\beta)},$$

$$(sb)\quad \sin\chi = \frac{|V_{us}||V_{ub}|}{|V_{cs}||V_{cb}|}\sin(-\chi+\chi'+\gamma).$$

Given the experimental knowledge on the size of the various moduli of the CKM matrix elements, some of the above relations can be, to an excellent approximation, substituted by simpler ones

An interesting relation is obtained by combining:

$$\frac{\sin\chi}{\sin(\gamma+\chi')} = \frac{|V_{us}||V_{ub}|}{|V_{ts}||V_{tb}|}, \qquad \textbf{with} \qquad |V_{ub}| = \frac{|V_{cd}||V_{cb}|}{|V_{ud}|}\frac{\sin\beta}{\sin(\gamma+\beta)},$$

$$\sin\chi = \frac{|V_{us}||V_{cd}||V_{cb}|}{|V_{ts}||V_{tb}||V_{ud}|}\frac{\sin\beta\sin(\gamma+\chi')}{\sin(\gamma+\beta)}.$$

This equation is the exact version of Aleksan-London and Kayser relation (1994):

$$\sin\chi \simeq \frac{|V_{us}|^2}{|V_{ud}|^2}\frac{\sin\beta\sin\gamma}{\sin(\gamma+\beta)}. \qquad \simeq \quad \lambda^2\eta$$

Discussed by Silva and Wolfenstein (1996)

Violations of these relations can occur with or without violations of unitarity

Blackboard

Can one have extensions of the SM where deviations from unitarity of the CKM occur? Can they be sufficiently suppressed?

YES!

Introducing vector-like quarks. Quark field content of the model:

Branco, Morozumi, Parada, MNR (1993)

$$(u^0 d^0)_{Li}, \quad i=1,\ldots,n \ ,$$

$$D^0_{Lp}, \quad p=1,\ldots,N_d \ ,$$

$$U^0_{Lq}, \quad q=1,\ldots,N_u \ ,$$

$$D^0_{R\alpha}, \quad \alpha=1,\ldots,n+N_d \ ,$$

$$U^0_{R\beta}, \quad \beta=1,\ldots,n+N_u \ .$$

The quark mass terms are

$$\mathcal{L}_M = \overline{u}^0_{Li}(m_u)_{i\beta}U^0_{R\beta} + \overline{U}^0_{Lq}(M_U)_{q\beta}U^0_{R\beta} + \overline{d}^0_{Li}(m_d)_{i\alpha}D^0_{R\alpha} + D^0_{Lp}(M_D)_{p\alpha}D^0_{R\alpha}$$

Let us take for definiteness one down and one up vector-like quark:

$$\mathcal{L}_g = \mathcal{L}_W + \mathcal{L}_Z ,$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \bar{u}_{L\alpha} V^{\text{CKM}}_{\alpha\beta} \gamma_\mu d_{L\beta} W^\mu ,$$

$$\mathcal{L}_Z = \frac{g}{2\cos\theta_W} [z^u_{\alpha\beta} \bar{u}_{L\alpha} \gamma^\mu u_{L\beta} - z^d_{\alpha\beta} \bar{d}_{L\alpha} \gamma^\mu d_{L\beta} - \sin^2\theta_W J^\mu_{\text{em}}] Z_\mu \quad (\alpha,\beta = 1, \ldots, 4)$$

where u_α, d_β are mass eigenstates and

$$V^{\text{CKM}}_{\alpha\beta} = \sum_{i=1}^{3} U^*_{i\alpha} W_{i\beta} ,$$

$$z^u_{\alpha\beta} = \delta_{\alpha\beta} - U^*_{4\alpha} U_{4\beta} ,$$

$$z^d_{\alpha\beta} = \delta_{\alpha\beta} - W^*_{4\alpha} W_{4\beta} ,$$

where U and W denote the matrices which relate the weak and mass eigenstates:

$$\begin{bmatrix} u^0_i \\ U^0 \end{bmatrix}_L = U \begin{bmatrix} u_i \\ T \end{bmatrix}_L , \quad \begin{bmatrix} d^0_i \\ D^0 \end{bmatrix}_L = W \begin{bmatrix} d_i \\ B \end{bmatrix}_L$$

Because of the presence of the vectorlike quarks there are flavor-changing neutral currents which are closely connected to the deviations from unitarity in V^{CKM}. Indeed, using the unitarity of U and W, one readily obtains

$$(VV^\dagger)_{\alpha\beta} = z^u_{\alpha\beta} ,$$

$$(V^\dagger V)_{\alpha\beta} = z^d_{\alpha\beta} \quad (V \equiv V^{\text{CKM}})$$

Deviations from unitarity of V and ZFCNC are related and are suppressed in the standard quark sector by the ratio of the standard quark masses by vector like quark masses

A Common Origin for all CP Violations

Branco, Parada, MNR, 2003

All CP violating phenomena may have a common origin

- CP violation in the quark sector

- CP Violation in the leptonic sector

- Generation of Baryon Asymmetry of the Universe

- Solution to Strong CP Problem

Spontaneous CP violation

Blackboard

$$D^0 \to -D^0, \quad S \to -S$$
$$\psi_l^0 \to i\psi_l^0, \quad e_R^0 \to ie_R^0, \quad \nu_R^0 \to i\nu_R^0$$

Yukawa Terms

$$\mathcal{L}_Y = \mathcal{L}_q + \mathcal{L}_l$$
$$\mathcal{L}_q = \overline{\psi_q^0} G_u \phi \; u_R^0 + \overline{\psi_q^0} G_d \tilde{\phi} \; d_R^0 +$$
$$+(f_q S + f_q{}' S^*) \overline{D_L^0} d_R^0 + \tilde{M} \overline{D_L^0} D_R^0 + h.c.$$
$$\mathcal{L}_l = \overline{\psi_l^0} G_l \phi \; e_R^0 + \overline{\psi_l^0} G_\nu \tilde{\phi} \; \nu_R^0 +$$
$$\frac{1}{2} \nu_R^{0T} C (f_\nu S + + f_\nu{}' S^*) \nu_R^0 + h.c.$$

$$V_{CKM}{}^{-1} \; h \; V_{CKM} = d^2 \tag{6}$$

$$h \equiv m_d^0 m_d^{0\dagger} - (m_d^0 M_D{}^\dagger M_D \; m_d^{0\dagger}) / \overline{M}^2 \tag{7}$$

where $d^2 = \text{diag}(m_d^2, m_s^2, m_b^2)$, $m_d^0 = \frac{v}{\sqrt{2}} \; G_d$, $\overline{M}^2 = M_D M_D{}^\dagger + \tilde{M}^2$ and $M_D = \frac{V}{\sqrt{2}}(f_+^q \cos(\alpha) + i f_-^q \sin(\alpha))$, with $f_\pm \equiv f_q \pm f_q{}'$.