Heavy meson χPT
in finite volume

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Based on

Daniel Arndt and C-JDL, Phys Rev **D70**, 014503 (2004)

Outline

- Introduction – CKM Matrix and Lattice QCD.
- Heavy meson χPT (HMχPT).
- HMχPT in finite volume – physical picture.
- HMχPT in finite volume – technical aspect.
- Volume effects in B^0–$\bar{B}^0$ mixing and their impact on the global CKM fit.
- Conclusion.

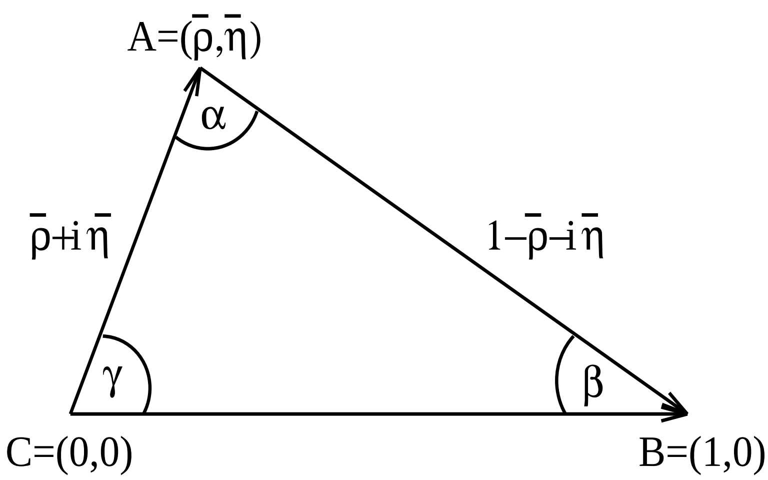
Introduction
The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}.$$

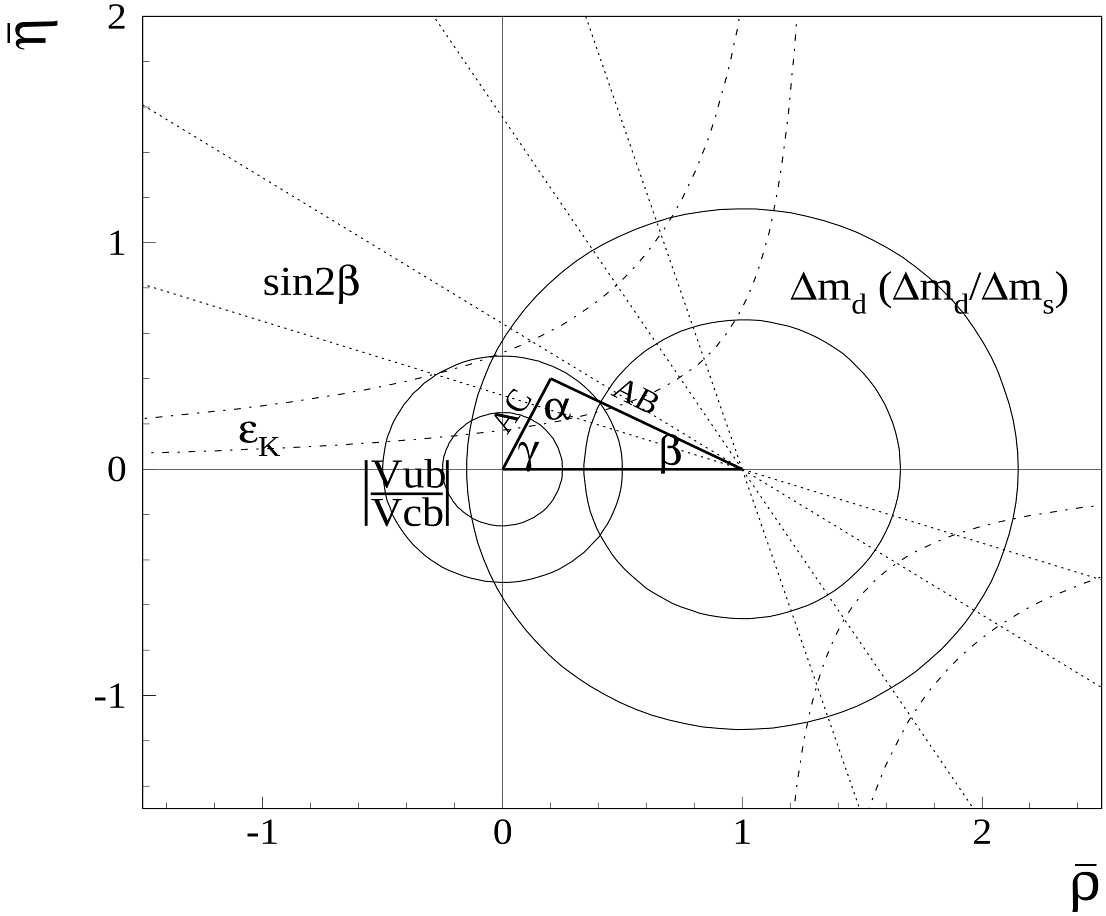
The Wolfenstein parameterisation

$$\hat{V}_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4).$$

Unitarity implies $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$.



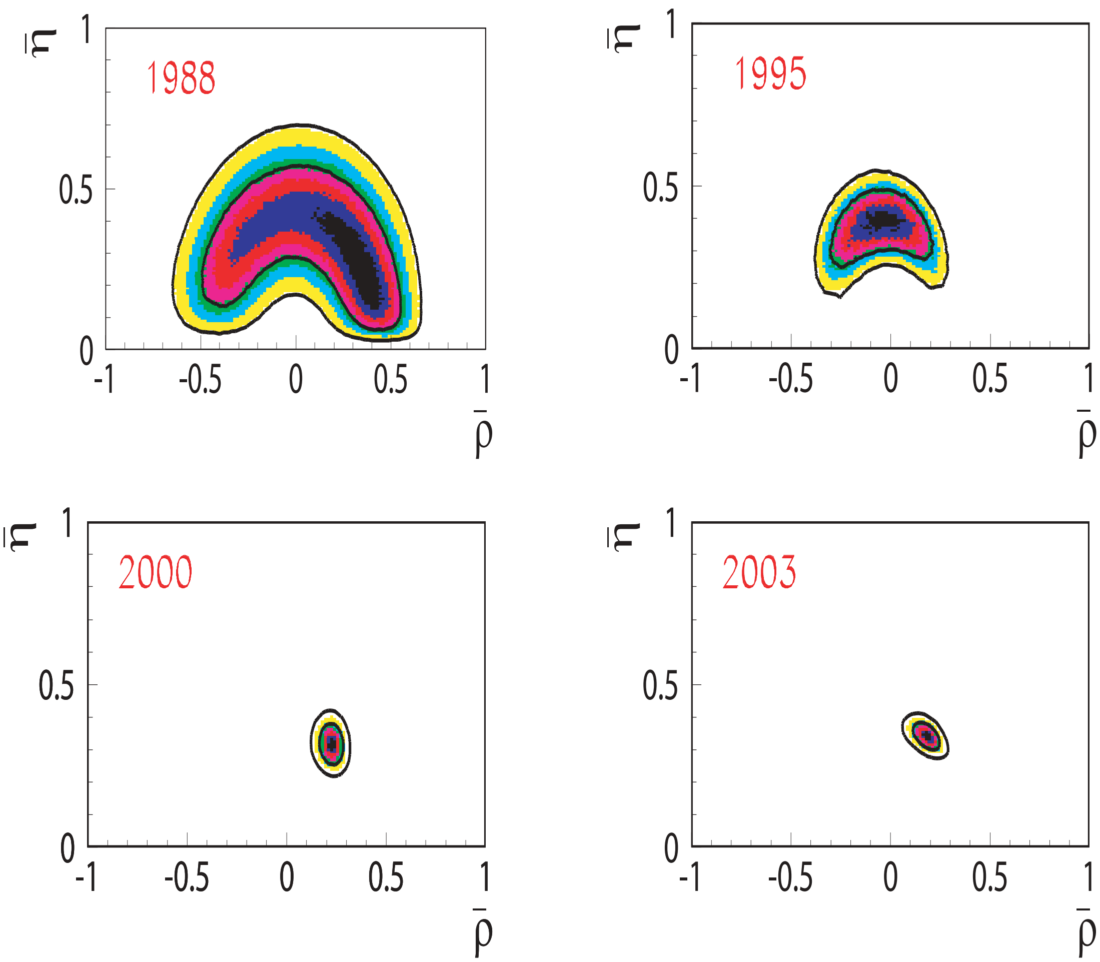
Introduction
The global CKM fit



Experimental result = V_{ij} (short-distance QCD effect)

$\times$(long-distance QCD matrix element)

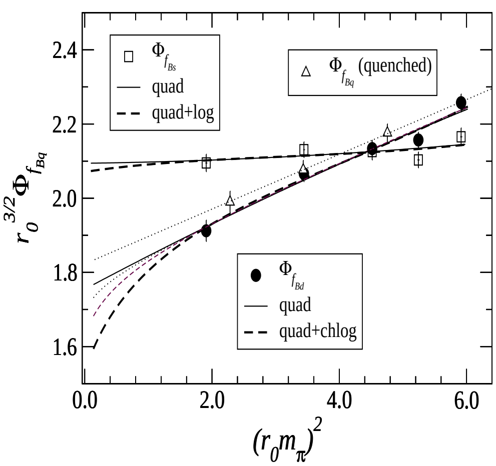
Lattice QCD
Working with experiments

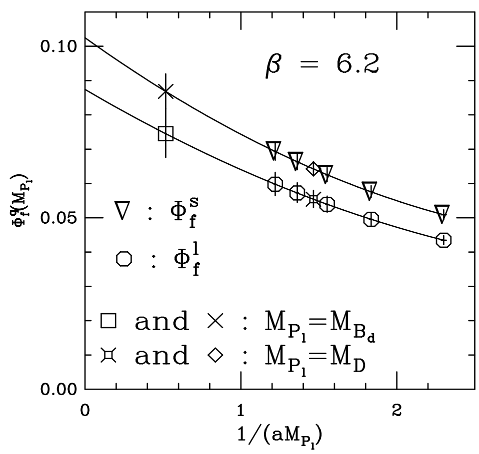


- Impressive progress.
- On-going efforts pursuing high precision.

Lattice QCD
Systematic errors in lattice QCD

- Unrealistic quark masses in the simulation





- Volume effects often depend on the meson masses $\rightarrow$ are they seriously amplified in the mass extrapolations?

- Other systematic effects include finite lattice spacing, (partial) quenching, *etc.*

HMχPT
The EFT in infinite volume

- Chiral and heavy-quark spin symmetries:
  - B, B^*: static sources emitting/absorbing π.
  - $p_\mu = M_B v_\mu + k_\mu \rightarrow$ Velocity superselection rule.

- Leading order:
  - Propagators: $\underbrace{\frac{i}{2(v\cdot k + i\epsilon)}}_{B_{(s)}}, \underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v\cdot k + i\epsilon)}}_{B^*_{(s)}}$

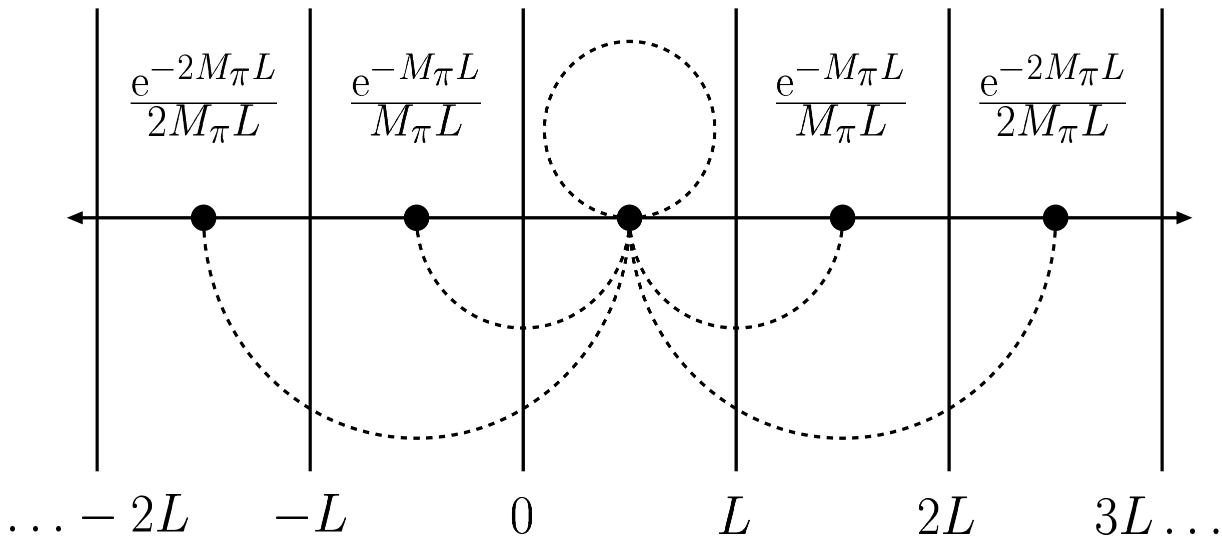
    $\xrightarrow{\text{position space}} \sim \theta(t)\delta^{(3)}(\vec{x})$
  - B–B^*–π and B^*–B^*–π couplings g.

- NLO in $1/M_B$ and M_π [discard $\mathcal{O}(M_\pi/M_B)$]:
  - Mass splittings from two counterterms
    1. $\Delta = M_{B^*} - M_B \sim \lambda_2/M_B$ (indep. of M_π).
    2. $\delta_s = M_{B_s} - M_B \sim \lambda_1(m_s - m_d)$ (indep. of M_B).
  - $\underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v\cdot k - \Delta + i\epsilon)}}_{B^*}, \underbrace{\frac{i}{2(v\cdot k - \delta_s + i\epsilon)}}_{B_s}, \underbrace{\frac{-i(g_{\mu\nu} - v_\mu v_\nu)}{2(v\cdot k - \Delta - \delta_s + i\epsilon)}}_{B_s^*}.$
  - Additional counterterms.

- In reality and on the lattice, Δ and δ_s are of $\mathcal{O}(M_\pi)$.

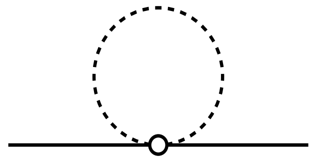
HMχPT in (large) finite volume
The physical picture

$T \to \infty$, finite L with periodic BC, B meson rest frame

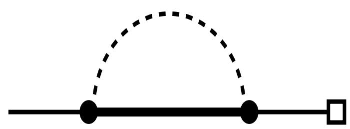


- In the limit $M_B \to \infty$, FV effects come entirely from pion wrapping around the world $\to$ $e^{-nM_\pi L}/(nM_\pi L)$.
- When $M_B \not\to \infty$, $\Delta = M_{B^*} - M_B \sim \mathcal{O}(M_\pi)$ alters FV effects by bringing the B^* off-shell, with $\delta t \sim 1/\Delta$:
  - FV effects decrease as Δ increases.
  - The alteration is controlled by M_π/Δ.
  - Notice $\Delta \sim 1/M_B$.
  - $\delta_s = M_{B_s} - M_B$ has similar effects.

HMχPT in finite volume
Momentum sums in the EFT

- 

$$\mathrm{FV}_{\mathrm{tadpole}} \sim \sum_{\vec{n}\neq\vec{0}} \frac{1}{nL}\int_0^\infty dk \frac{k\ \sin(nkL)}{\sqrt{k^2+M_\pi^2}} \overset{M_\pi L \gg 1}{\sim} \sum_{\vec{n}\neq\vec{0}} \frac{\mathrm{e}^{-nM_\pi L}}{nM_\pi L}$$

-  Need the integral/sum:

$$-i\frac{1}{L^3}\sum_{\vec{k}}\int \frac{dk_0}{2\pi}\frac{1}{(v_0 k_0-\Delta+i\epsilon)(k^2-M_\pi^2+i\epsilon)}$$

$$\mathrm{FV}_{\mathrm{sunset}} \sim \sum_{\vec{n}\neq\vec{0}} \frac{1}{nL}\int_0^\infty dk \frac{k\ \sin(nkL)}{\sqrt{k^2+M_\pi^2}(\sqrt{k^2+M_\pi^2}+\Delta)}$$

$$\overset{M_\pi L \gg 1}{\sim} \sum_{\vec{n}\neq\vec{0}} \frac{\mathrm{e}^{-nM_\pi L}}{nM_\pi L}\mathcal{A}(\Delta).$$

$\mathcal{A}$ is the alteration of volume effects due to the mass splittings amongst the heavy-light mesons.

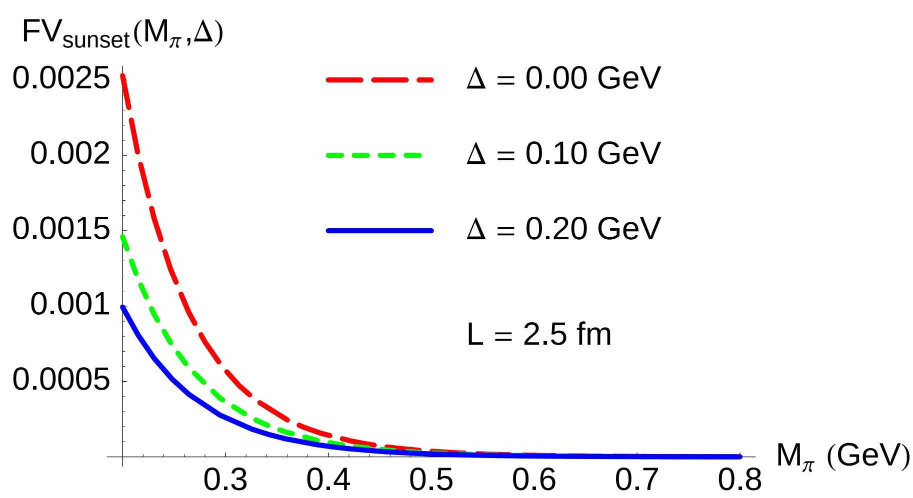
HMχPT in finite volume
Reproducing the physical picture

$$\mathcal{A} = \underbrace{\exp(z^2)}_{\mathcal{A}_0}[1 - \mathrm{Erf}(z)] + \sum_{j=1}^{\infty} \mathcal{A}_j(z) \left(\frac{1}{nM_\pi L}\right)^j,$$

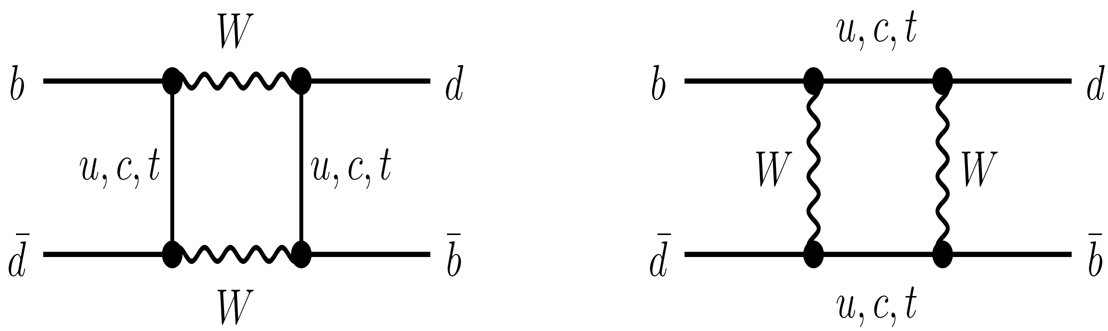
$$z = \frac{\Delta}{M_\pi}\sqrt{\frac{nM_\pi L}{2}}.$$

$\mathcal{A}$ has the properties:

1. $\mathcal{A}_j(z) \to$ polynomials in z and $Z^m \mathcal{A}_0$, such that,
2. $\mathcal{A} = 1$ when $\Delta = 0$ (No alteration).
3. $\mathcal{A} \approx \exp(z^2)\,[1 - \mathrm{Erf}(z)]$ when $z \ll 1$.
4. $\mathcal{A} \approx 1/z$ when $\Delta \sim M_\pi$.



B^0–$\bar{B}^0$ mixing
Phenomenology



- The top quark dominates the box $\rightarrow$ $|V_{td}|$.

- $\Delta m_d = \frac{G_F}{8\pi^2} M_W^2 |V_{td} V_{tb}^*|^2 \eta_B S_0 \left(\frac{m_t^2}{M_W^2}\right) C_B(\mu) |\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle| / M_B.$

- The operator $\mathcal{O}_d^{\Delta B=2} = [\bar{b}\gamma^\mu(1-\gamma_5)d][\bar{b}\gamma^\mu(1-\gamma_5)d].$

- Cleaner theory prediction in

$$\frac{\Delta m_s}{\Delta m_d} = \left|\frac{V_{ts}}{V_{td}}\right|^2 \left(\frac{M_{B_d}}{M_{B_s}}\right) \left|\frac{\langle \bar{B}_s | \mathcal{O}_s^{\Delta B=2}(\mu) | B_s \rangle}{\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle}\right|.$$

  $\rightarrow$ Accurate Δm_s from Tevatron and LHC.

- Conventionally $\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle = \frac{8}{3} M_{B_d}^2 f_{B_d}^2 B_{B_d}(\mu)$, and

$$\left|\frac{\langle \bar{B}_s | \mathcal{O}_s^{\Delta B=2}(\mu) | B_s \rangle}{\langle \bar{B}_d | \mathcal{O}_d^{\Delta B=2}(\mu) | B_d \rangle}\right| = \left(\frac{M_{B_s}}{M_{B_d}}\right)^2 \xi^2 = \left(\frac{M_{B_s}}{M_{B_d}}\right)^2 \xi_f^2 \; \xi_B.$$

B^0–$\bar{B}^0$ mixing
Analysis of finite volume effects

- Ensure $M_\pi L > 2.5$.

- Analyse the lattice setup (isospin limit):
  1. simulation performed at $(m_s)_{\mathrm{latt}} = (m_s)_{\mathrm{phys}}$.
  2. $(m_d)_{\mathrm{latt}}$ is varied.

  Therefore need to express $M_{K,\eta}$, in χPT, as

$$
\begin{aligned}
M_K^2 &= \frac{M_{s\bar{s}}^2 + M_\pi^2}{2}, \\
M_\eta^2 &= \frac{2M_{s\bar{s}}^2 + M_\pi^2}{3},
\end{aligned}
$$

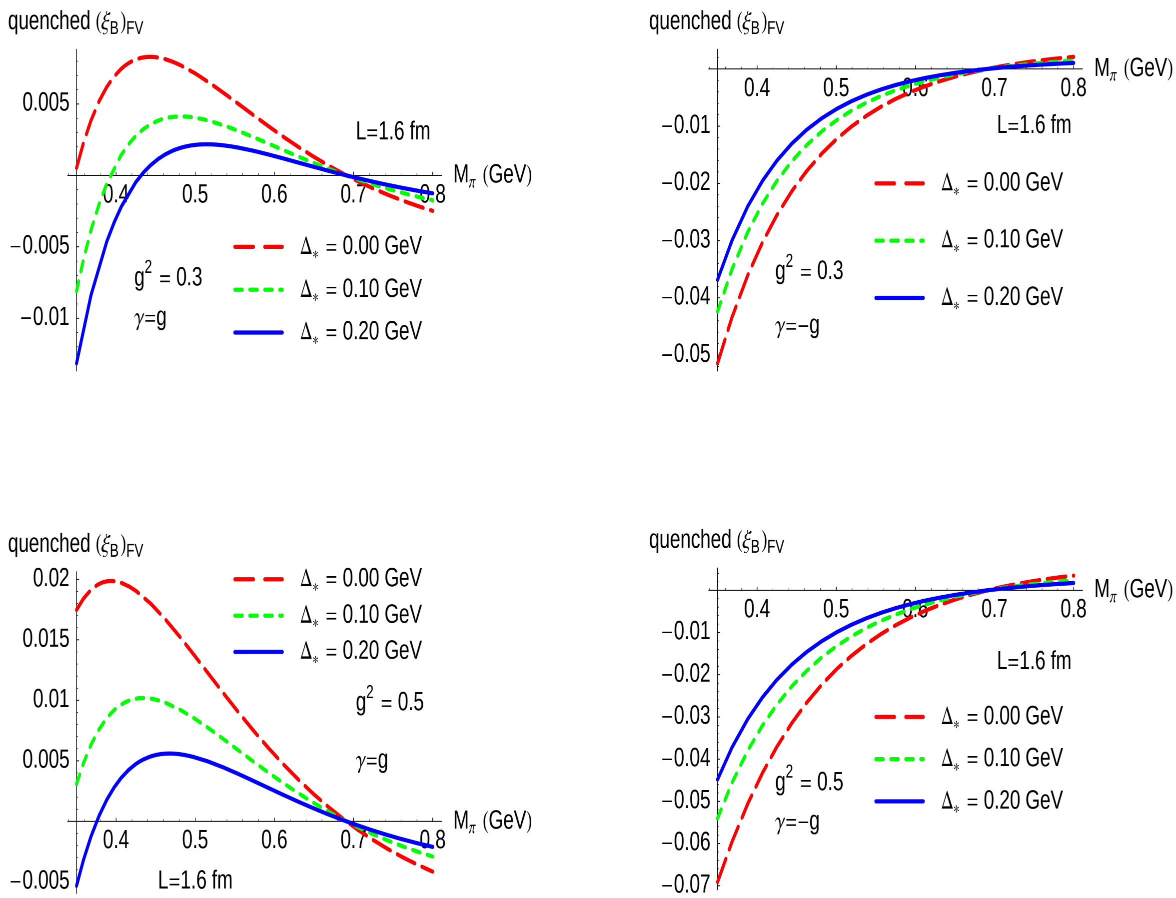
- In this scenario, $((m_s)_{\mathrm{phys}}, m_d) \to ((M_{s\bar{s}})_{\mathrm{phys}}, M_\pi)$:
  1. Fix $(M_{s\bar{s}})^2_{\mathrm{phys}} = (2M_K^2 - M_\pi^2)_{\mathrm{phys}}$
  2. In HMχPT, the SU(3) breaking effect

$$M_{B_s} - M_B \equiv \delta_s = \lambda_1 (M_{s\bar{s}}^2 - M_\pi^2),$$

  $\rightarrow$ Use physical M_{B_s}, M_B, $M_{s\bar{s}}$ and M_π to fix λ_1.

B^0–$\bar{B}^0$ mixing
Plots for FV effects in B_{B_s}/B_{B_d}

$(\xi_B)_{\text{FV}}$ in quenched QCD.



γ is the η'–B–B^* coupling.

It is clear that volume effects can be amplified in both light-quark and heavy-quark mass scalings.

B^0–$\bar{B}^0$ mixing
Impact on the global CKM fit

- Lattice 2003 review (A. Kronfeld, 2003) quotes

$$\xi = 1.23 \pm 0.10$$

- Our work on FV effects shows that in the parameter space of near past/future simulations:
  - QQCD data have ~ 5–7% FV effects.
  - PQQCD data have ~ 3–4% FV effects.

- Amplified in both light and heavy quark mass extrapolations/interpolations.

- They have not been taken into account in all the lattice calculations hitherto.

Conclusions and outloook

- The mass splittings amongst heavy-light mesons can alter FV effects.

- These effects can be amplified in both light and heavy quark mass extrapolations in lattice calculations.

- They can significantly enlarge the quoted error on ξ, hence have an impact on the global CKM fit.

- Same techniques in baryon systems

  * Nucleon mass and magnetic moment

    S.R.Beane, hep-lat/0403015.

  * Baryon axial charge

    S.R.Beane and M.J.Savage, hep-ph/0404131.

  * Twist-2 operators in proton SF

    W.Detmold and C-JDL, to appear.